ECE1658 Assignment 1

Derivation Explanations:

Position Vectors r_i:

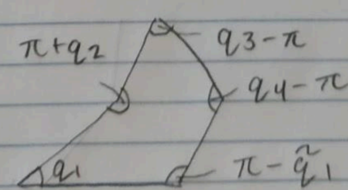
The position vector r_1 is located at the stance foot so its position is $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. We define rotation matrices C_{12}, C_{23}, C_{34}, C_{45}, and C_{36} where $C \in \mathbb{R}^{2\times 2}$ and C is a valid rotation matrix and C_{ij} represents the rotation between mass i and mass j. Each rotation matrix represents an angle, q_1, q_2, q_3, q_4, and q_5 respectively. Note that $C_{i\,i+1}$ corresponds to q_i except in the case of C_{36} where there is an angle q_5 between m_3 and m_6. For positions r_2 to r_5, they are connected in a continuous chain of links, so $r_i = r_{i-1} + C_{1i}\begin{bmatrix} \ell_{r_i} \\ 0 \end{bmatrix}$, where C_{1i} is the rotation from m_1 to m_i which can be formed by multiplying rotation matrices $C_{12} \dots C_{i-1\,i}$. ℓ_{r_i} represents the link length between m_i and m_{i-1} and it is multiplied by C_{1i} to get the correct orientation and added to the previous position to get the correct position. r_6 is a special case where the previous position is r_3 instead of r_5 and we use the rotation matrix C_{36} instead.

Potential Function $P(q)$

The potential function $P(q) = \sum_{i=1}^{6} m_i g r_{i,y}$ since the mass of the robot is distributed as 6 point masses at each of the robot joints so the potential energy of each point mass (which is the mass multiplied by the height of the mass multiplied by gravity) can be calculated and summed together.

Relabelling Function $T(q)$

First the expressions $\tilde{q}_i(q)$ were determined. The expressions $\tilde{q}_2 = -q_4 + 2\pi$, $\tilde{q}_3 = -q_3 + 2\pi$ and $\tilde{q}_4 = -q_3 + 2\pi$ can be determined pretty easily using inspection of Figure 2. The expression $\tilde{q}_5 = q_5 - q_3 + \pi$ can be determined as well by noticing that the angle between q_3 and q_5 is π away from $\tilde{q}_5$. The expression $\tilde{q}_1 = q_1 + q_2 + q_3 + q_4 - 3\pi$ was hardest to determine. Notice that the legs of the robot form a 5-point shape. All interior angles of a 5-point shape must sum to 3π. A figure of the shape is shown below:



So an expression for the 5-point shape is $q_1 + (\pi + q_2) + (q_3 - \pi) + (q_4 - \pi) + (\pi - \tilde{q}_1) = 3\pi$. Isolating for $\tilde{q}_1$ we get $\tilde{q}_1 = q_1 + q_2 + q_3 + q_4 - 3\pi$. Converting all the expressions to matrix form we get:

$$T(q) = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix}}_{q} + \underbrace{\begin{bmatrix} -3\pi \\ 2\pi \\ 2\pi \\ 2\pi \\ \pi \end{bmatrix}}_{d} = \begin{bmatrix} q_1 + q_2 + q_3 + q_4 - 3\pi \\ -q_4 + 2\pi \\ -q_3 + 2\pi \\ -q_2 + 2\pi \\ -q_3 + q_5 + \pi \end{bmatrix}$$