Weekly Homework 2

Kevin Christensen
Introduction to Abstract Math

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Definition 2.1. An integer is even if n=2k for some integer k

Definition 2.2. An integer is odd if n=2k+1 for some integer k

Definition 2.11. An integer n divides the integer m, written n|m, if and only if there exists k $\in \mathbb{Z}$ such that m=nk. We may also say that m is divisible by n.

Fact 2.3.1. Sums and products of integers are integers.

Problem 2.10. Either prove, or provide a counterexample to, the statement "If at least one of a pair of integers is even, then their product is even."

Proof. Consider m,n $\in \mathbb{Z}$, such that n is even. By Definition 2.1, we know n=2k.

$$\begin{aligned} n * m &= (2k) * m \\ &= 2(km) \end{aligned}$$

By Fact 2.3.2, we know that km is an integer. Therefore, $2km$ is even by Definition 2.1. □

Theorem 2.15. Assume n,m,a $\in \mathbb{Z}$. If a|n, then a|nm.

Proof. Assume n,m,a $\in \mathbb{Z}$, and that a|n. By Definition 2.11, in order for the statement a|mn to be true, we know that $\frac{mn}{a}$ must be an integer.

$$\begin{aligned} \frac{mn}{a} &= \frac{m(ak)}{a} && \text{(By Definition 2.11)} \\ &= mk \end{aligned}$$

By Fact 2.3.1, we know that mk is an integer. Therefore, we know a|mn is true. □