

Weekly Homework 2

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Introduction to Abstract Math

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Definition 2.11. An integer n divides the integer m , written $n|m$, if and only if there exists $k \in \mathbb{Z}$ such that $m=nk$. We may also say that m is divisible by n .

Fact 2.3.1. Sums and products of integers are integers.

Theorem 2.18. Assume $n, m, a \in \mathbb{Z}$. If a divides n , then a divides $-n$.

Proof. Theorem 2.15 states that if $a|n$, then $a|mn$. If $m=-1$, then $mn = (-1)n = -n$. Therefore, if $a|n$, then $a|m$. \square

Question 2.21. Assume $a, b, m \in \mathbb{Z}$. Determine whether the following statement holds sometimes, always, or never.

If ab divides m , then a divides m and b divides m .

Justify with a proof or counterexample.

Proof. By Definition 2.11, we know $m = abk = (ak)b = (bk)a$. By Fact 2.3.2, we know that both ak and bk are integers, which means that if $ab|m$, then $a|m$ and $b|m$. Therefore the statement is always true. \square