Weekly Homework 8

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Introduction to Abstract Math

March 17, 2021

Theorem 4.17. Let A and B be sets. If $A \subseteq B$ then $B^c \subseteq A^c$.

Proof. We will proceed with a proof by contradiction. Let $A \subseteq B$ and $x \in B^c$. Assume $x \in A$. By Definition 4.6, if $x \in A$ then $x \in B$. However, this contradicts our assumption that $x \in B^c$. By this, we know that if $x \in B^c$, then $x \notin A$, which means by Definition 4.13 that $x \in A^c$. Because x is arbitrary, we can say by Definition 4.6 that $B^c \subseteq A^c$.
Therefore, if $A \subseteq B$ then $B^c \subseteq A^c$. $\square$

Theorem 4.26. Let S and T be sets. Then $\mathcal{P}(S) \cup \mathcal{P}(T) \subseteq \mathcal{P}(S \cup T)$.

Proof. Let $A \in \mathcal{P}(S) \cup \mathcal{P}(T)$ be an arbitrary set and $x \in A$ be an arbitrary element. By Definition 4.13, $A \in \mathcal{P}(S)$ or $A \in \mathcal{P}(T)$, so by Definition 4.22 $x \in S$ or $x \in T$. This means that $x \in S \cup T$, and therefore that $A \in \mathcal{P}(S \cup T)$.
Because A is arbitrary, we use Definition 4.6 to say that $\mathcal{P}(S) \cup \mathcal{P}(T) \subseteq \mathcal{P}(S \cup T)$. $\square$