Daily Homework 1-25-21

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Introduction to Abstract Math

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Theorem 2.18. Assume $n, a \in \mathbb{Z}$. If a divides n then a divides -n

Proof. Theorem 2.15 states that if$a \mid n$ then $a \mid nm$ some integer m. If m = -1, then $n \times m = -n$, therfore $a \mid -n$ □

Theorem 2.19. Assume $n, m, a \in \mathbb{Z}$. If a divides m and a divides n, then a divides m + n

Proof. Note that if $a \mid n$ then $n = a \times k \in \mathbb{Z}$ and if $a \mid n$ then $m = a \times j \in \mathbb{Z}$. We know that $m + n = a \times k + a \times j = a \times (k + j)$, by fact 2.3 we know that k+j is an integer, so by Definition 2.11 we know that $a \mid n + m$ □

Theorem 2.20. Assume $n, m, a \in \mathbb{Z}$. If a divides m and a divides n, then a divides m - n

Proof. Note that if $a \mid n$ then $n = a \times k \in \mathbb{Z}$ and if $a \mid n$ then $m = a \times j \in \mathbb{Z}$. We know that $m + n = a \times k - a \times j = a \times (k - j)$, by fact 2.3 we know that k-j is an integer, so by Definition 2.11 we know that $a \mid n - m$. □

Theorem 2.21. Assume $a, b, m \in \mathbb{Z}$. Determine whether the following statement holds sometimes, always, or never. If ab divides m, then a divides m and b divides m. Justify with a proof or counterexample.

Proof. Note that if $ab \mid m$ then $m = a \times b \times k \in \mathbb{Z}$. Note $a \mid m$ equals $m = a \times (b \times k)$, and $b \mid m$ equals $m = b \times (a \times k)$, by Fact 2.3 $a \times k$ and $b \times k$ are integers so by Definition 2.11 $a \mid m$ and $b \mid m$. In conclusion we can see that the statement should always be true □

Theorem 2.22. Assume $a, b, c \in \mathbb{Z}$. are such that a divides b and b divides c, then a divides c.

Proof. Note $a \mid b$ then $b = a \times k \in \mathbb{Z}$ and $b \mid c$ then $c = b \times j \in \mathbb{Z}$. Note $c = b \times j = a \times k \times j$, and by Fact 2.3 $k \times j$ is an integer, so by Definition 2.11 $a \mid c$. □

Theorem 2.23. If $n \in \mathbb{Z}$ then 3 divides n + (n + 1) + (n + 2).

Proof. Note $n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$. By Fact 2.3 (n+1) is an integer, so by Definition 2.11, $3 \mid (n + (n + 1) + (n + 2))$. □