

# Daily Homework 1-25-21

Calvin Nowlen, Kevin Christensen  
Introduction to Abstract Math

January 27, 2021

**Problem 2.21.** Assume  $a, b, m \in \mathbb{Z}$ . Determine whether the following statement holds sometimes, always, or never. If  $ab$  divides  $m$ , then  $a$  divides  $m$  and  $b$  divides  $m$ . Justify with a proof or counterexample.

*Proof.* Note that if  $ab \mid m$  then  $m = a \times b \times k \in \mathbb{Z}$ . Note  $a \mid m$  equals  $m = a \times (b \times k)$ , and  $b \mid m$  equals  $m = b \times (a \times k)$ , by Fact 2.3  $a \times k$  and  $b \times k$  are integers so by Definition 2.11  $a \mid m$  and  $b \mid m$ . In conclusion we can see that the statement should always be true  $\square$

**Exercise 3.2.** Determine whether the following are propositions or not. Explain

- (a) (All cars are red) Yes, because it is a complete sentence that we can say is true or false
- (b) (Every person whose name begins with J has the name Joe.) Yes, because it is a statement that we can say is true or false
- (c) ( $x^2 = 4$ ) No, we do not know what  $x$  is, it is not specified.
- (d) (There exists an  $x$  such that  $x^2 = 4$ ) Yes, this is a specific mathematical statement we can say is true or false
- (e) (for all real numbers  $x$ ,  $x^2 = 4$ ) Yes, this is a specific mathematical statement we can say is true or false
- (f) ( $\sqrt{2}$  is an irrational number) Yes, because it is a statement that we can say is true or false
- (g) ( $p$  is prime) No, because we do not know what  $p$  is.
- (h) (Led Zeppelin is the best band of all time.) No, because we can not categorically say the statement is true or false.

**Question 3.5.** What's the difference between  $A \implies B$  and  $A \iff B$ ? Can you give an example of two statements  $A, B$  where  $A \implies B$  is true but  $A \iff B$  is false? Can you give an example of two statements  $A, B$  where  $A \iff B$  is true but  $A \implies B$  is false?

*Proof.* The first statement claims that A implies B. The Second claims that if A is true then B is true. The Statement, x is an even number  $> 2$ , implies the statement, x is not a prime number, but the second does not imply the first. You cannot have two statements where A is true if and only if B is true, and have A not imply B is true.  $\square$

**Exercise 3.6.** Describe the meaning of  $\neg(A \vee B)$  and  $\neg(A \wedge B)$

*Proof.* The first states that the proposition is true if and only if both A and B are false, while the second states that the proposition is true if and only if at least one of A and B are false  $\square$

**Exercise 3.7.** Let A represent “6 is an even number” and B represent “6 is a multiple of 4.” First, explain why A and B are statements. Then, express each of the following statements in ordinary English sentences. Which of these statements are true? Why?

*Proof.* Both A and B are sentences that are either true or false, and by Definition 3.1 they are statements.  $\square$

- (a)  $(A \wedge B)$  The statement, A and B, is true if and only if both A and B are true, this statement is false, because B is false
- (b)  $(A \vee B)$  The statement, A or B, is true if and only if one of the statements, A or B, is true. This statement is true because A is true.
- (c)  $(\neg A)$  The statement, not A, is true if and only if A is false. The statement is false because A is true.
- (d)  $(\neg B)$  The statement, not B, is true if and only if B is false. The statement is true because B is false.
- (e)  $(\neg(A \wedge B))$  The statement, not A and B, is true if and only if at least one of the statements A and B are false. The statement is true because B is false.
- (f)  $(\neg(A \vee B))$  The statement, not A or B, is true if and only if both statements A and B are false. The statement is false because A is true.
- (g)  $(A \implies B)$  The statement, If A then B, is true if and only if B is true whenever A is true. The statement is false because A is true, but B is not true

**Exercise 3.12.** Create a truth table for each of  $A \vee B$ ,  $\neg A$ ,  $\neg(A \wedge B)$ , and  $\neg A \wedge \neg B$ .

A	B	$A \vee B$	A	$\neg A$	A	B	$A \wedge B$	$\neg(A \wedge B)$	A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
T	T	T	T	F	T	T	T	F	T	T	F	F	F
T	F	T	T	F	T	F	F	T	T	F	F	T	F
F	T	T	F	T	F	T	F	T	F	T	T	F	F
F	F	F	F	T	F	F	F	T	F	F	T	T	T