Weekly Homework 6

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Introduction to Abstract Math

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Question 3.61. For all integers, $3n^2 + n + 14$ is even.

Proof. For this proof, we will need to prove two claims. First is that the sum of an odd integer and an odd integer is even, which we will prove now. Assume $a, b \in \mathbb{Z}$ are odd. By Definition 2.2, we can write $a = 2k+1$ and $b = 2j+1$. We calculate $a+b = 2k+2j+2 = 2(k+j+1)$, which is an even integer by Definition 2.1. Therefore, the sum of two odd integers is even.
The second is that the sum of two even integers is even, which we will prove now. Assume $a, b \in Z$ are even. By Definition 2.1, we can write $a = 2k$ and $b = 2j$. We calculate $a + b = 2k + 2j = 2(k + j)$, which is an even integer by Definition 2.1. Therefore, the sum of two even integers is even.
Now we will prove that $3n^2 + n + 14$ is even. We know by Fact 2.3.1, integers can be either even or odd, but not both. Let us consider both cases in turn.
Assume n is even. We know by Theorem 2.5 that n^2 is even. We proved in Problem 2.8 that the product of an odd integer and an even integer is even, so we know that $3n^2$ is even. By our earlier proof of the sums of even integers, we know that the sum $3n^2 + n$ is an even integer, and by extension we know that $(3n^2 + n) + 14$ is also an even integer. Therefore, if n is even, the full expression is even.
Now we consider the case the n is odd. Assume $n \in Z$ is odd. We know by our work in Problem 2.9 that $3n^2$ is odd. We know by our earlier proof of the sums of odd integers that $3n^2 + n$ is an even integer. Since $3n^2 + 2$ is even and 14 is even we know by our proof of the sums of even integers that $3n^2 + n + 14$ is even. Therefore, if n is odd, the full expression is even.
Thus we have proved that $3n^2 + n + 14$ is even for all integers n.

$\square$

Theorem 4.8. Let S be a set. Then $S \subseteq S$ and $\emptyset \subseteq S$.

Proof. Assume $x \in S$. Since $x \in S$ for any arbitrary x, so we know that $S \subseteq S$. Now assume $y \in \emptyset$. Because the empty set contains no values, there are no possible values for y, so every value of y is contained in S. Therefore, $\emptyset \subseteq S$. $\square$