Daily Homework 2.2

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Introduction to Abstract Math

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Theorem 2.15. Assume n,m,a $\in \mathbb{Z}$. If a|n, then a|mn.

Proof. By Definition 2.11, we can write mn = m(ak), where k $\in \mathbb{Z}$.

$$\begin{aligned}\frac{mn}{a} &= \frac{m(ak)}{a} \\ &= km.\end{aligned}$$

We know by Fact 2.3 that km is an integer. Therefore, by Definition 2.11, a|mn. □

Corollary 2.16. Assume n,a $\in \mathbb{Z}$. If a divides n, then a divides n^2.

Proof. By Theorem 2.15, we know that if a|n then a|mn. If m = n, then mn = n^2. Therefore, since a|n, we know that a|n^2. □

Problem 2.17. Assume n,a $\in \mathbb{Z}$. Consider the statement:

$$\boxed{\text{If a divides } n^2 \text{ then a divides n.}}$$

Is this statement always true? Is it always false? Prove that your answers are correct by giving particular examples and/or general arguments.

Proof. This statement is only sometimes true.
Consider a case described by Corollary 2.16. Here, a|n^2 and a|n. This is a case where the statement is true.
Now consider the case where a = 8 and n = 4. n^2/a = 16/8 = 2 $\in \mathbb{Z}$, so a|n^2. However, n/a = 4/8 = 1/2 $\notin \mathbb{Z}$. In this case, the statement is false because a divides n^2 but not n. □

Theorem 2.18. Assume n,a $\in \mathbb{Z}$. If a divides n, then a divides -n.

Proof. Let m = -1. By Theorem 2.15, if a|n, a|mn. Therefore, if a divides n, a also divides -n.

□