Daily Homework 6.1

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Introduction to Abstract Math

February 24, 2021

Theorem 3.61. For all integers $3n^2 + n + 14$ is even.

Proof. There are two possibilities for n, either n is even or n is odd. If n is even, we express it as $n = 2k$ and substitute that into the equation, giving us $3n^2 + n + 14 = 12k^2 + 2k + 14 = 2(6k^2 + k + 7)$. By Definition 2.1, this is an even integer.
If n is odd, we can express it as $n = 2k + 1$. This gives us the equation $3n^2 + n + 14 = 12k^2 + 12k + 3 + 2k + 1 + 14 = 2(6k^2 + 7k + 9)$. By Definition 2.1, this is also an even integer. Because both possibilities for n make $3n^2 + n + 14$ even, we know that the statement is true for all integers n. $\square$

Exercise 4.4. Unpack each of the following sets and see if you can find a simple description of the elements that each set contains.

1. A = $\{\ x \in \mathbb{N} \mid x = 3k \text{ for some } k \in N\ \}$
2. B = $\{\ t \in \mathbb{R} \mid t^2 \leq 2\ \}$
3. C = $\{\ t \in \mathbb{Z} \mid t^2 \leq 2\ \}$
4. D = $\{\ m \in \mathbb{R} \mid m = 1 - \frac{1}{n}, \text{ where } n \in N\ \}$

Solution.

1. A is the set of natural numbers that are divisible by 3.
2. B is the set of rational numbers that are between -$\sqrt{2}$ and $\sqrt{2}$.
3. C is the set of integers that are between -$\sqrt{2}$ and $\sqrt{2}$.
4. D is the set of real numbers between 0 and 2.

$\square$

Exercise 4.5. Write each of the following sentences using set builder notation.

1. The set of all real numbers less than -$\sqrt{2}$.

2. The set of all real numbers greater than -12 and less than or equal to 42.
3. The set of all even natural numbers.

Solution.

1. A $= \{ \ x \in \mathbb{R} \mid x < -\sqrt{2} \ \}$
2. B $= \{ \ y \in \mathbb{R} \mid -12 < x \leq 42 \ \}$
3. C $= \{ \ z \in \mathbb{N} \mid z = 2k, k \in \mathbb{Z} \ \}$

$\square$

Problem 4.7. Suppose A and B are sets. Describe a skeleton proof for proving that $A \subseteq B$.

Solution. Let $x \in A$.

... [*Use definitions and known results to prove* $x \in B$.] ...

By Definition 4.6, $A \subseteq B$.

$\square$

Theorem 4.8. Let S be a set. Then $S \subseteq S$ and $\emptyset \subseteq S$.

Proof.
Assume $x \in S$. For any chosen x, we know that $x \in S$. Therefore, by Definition 4.6, $S \subseteq S$. Since $\emptyset$ contains no elements, we can always take the entirety of the empty set from any set containing 0 or more elements. Therefore, $\emptyset \in S$. $\square$