Weekly Homework 1

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Introduction to Abstract Math

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Definition 2.1. An integer is even if n=2k for some integer k

Definition 2.2. An integer is odd if n=2k+1 for some integer k

Fact 2.3.1. Sums and products of integers are integers.

Theorem 2.4. If n $\in \mathbb{Z}$ then the sum of n and n+1 is odd.

Proof. Let n be an integer. Notice that n+(n+1)=2n+1. By Definition 2.2, we see that 2n+1 is odd. □

Problem 2.6. Either prove, or provide a counterexample to, the statement "The sum of an even integer and an odd integer is odd."

Proof. Let n be an even integer and m be an odd integer. By Definitions 2.1 and 2.2 respectively, we can write n as 2k and m as 2j+1.

$$\begin{aligned} n + m &= 2k + 2j + 1 \\ &= 2(k + j) + 1 \end{aligned}$$

We know that (k+j) is an integer by Fact 2.3.1. Therefore, we show by Definition 2.2 that 2(k+j)+1 is odd for all values of k and j. □