Daily Homework 2-1-21

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Introduction to Abstract Math

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Definition 3.18. The converse of $A \implies B$ is $B \implies A$.

Exercise 3.19. Provide an example of a true conditional proposition whose converse is false.

Proof. The statement "if $x = 5$, then x is odd" is a true conditional statement. However, the converse of the statement is "if x is odd, then $x = 5$," and this statement is not true. □

Definition 3.18. The inverse of $A \implies B$ is $\neg A \implies \neg B$.

Exercise 3.21. Provide an example of a true conditional proposition whose inverse is false.

Proof. Let A be "$x = 5$" and B be "x is odd". The inverse of the statement $A \implies B$ is "if $x \neq 5$, then x is not odd". This is a false statement because x could be an odd number that is not 5. □

Exercise 3.23. Let A represent "6 is an even number" and B represent "6 is a multiple of 4". Express the following in ordinary English sentences:

1. The converse of $A \implies B$.
2. The contrapositive of $A \implies B$.

Proof.

1. If "6 is a multiple of 4" then "6 is an even number".
2. If "6 is not a multiple of 4" then "6 is not an even number".

□

Exercise 3.24. Find the converse and the contrapositive of the following statement: "If a person lives in Flagstaff then that person lives in Arizona."

Proof.

1. Converse: If a person lives in Arizona, then that person lives in Flagstaff.
2. Contrapositive: If a person doesn't live in Arizona, then that person does not live in Flagstaff.

□

Theorem 3.25. The implication $A \implies B$ is equivalent to its contrapositive.

Proof.

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg B$	$\neg A$	$\neg B \implies \neg A$
T	T	T
T	F	F
F	T	T
F	F	T

We see that the truth tables for $A \implies B$ and $\neg B \implies \neg A$ are the same. Therefore, by Definition 3.14, the two statements are logically equivalent. $\square$