Daily Homework 2-5-2021

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Introduction to Abstract Math

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Definition 3.34. We write $\mathbb{N} = 1, 2, 3, 4, ...$ for the set of natural numbers or positive integers.

Theorem 3.32. Suppose $n \in \mathbb{Z}$. If n^2 is even then n is even. Prove by contradiction.

Proof. Assume n^2 is even and n is odd. We proved in problem 2.9 that the product of an odd integer and an odd integer is odd, which means that if n is odd, then n^2 is odd. However, this contradicts that n^2 is even. $\square$

Theorem 3.33. If x is a real number in $[0, \pi/2]$, then $\sin\ x + \cos\ x \geq 1$. Prove by contradiction

Proof. Assume x is a real number in $[0, \pi/2]$ and that $\sin\ x + \cos\ x < 1$. We know that $|sinx|, |cosx| \in [0, 1]$ for any $x \in R$. The $\sin\ x$ and the $\cos\ x$ represent the length of the legs of a right triangle whose hypotenuse is 1, and which has an inner angle of x. The sum of the length of any two sides of a triangle is greater than or equal to the length of the third side. Therefore, $|sinx| + |cosx| \geq 1$. If $\sin\ x + \cos\ x < 1$, then $sinx \vee cosx < 1$. This occurs when $x \in R$ in $(\pi/2, 2\pi)$. This contradicts our statement that x is a real number in $[0, \pi/2]$. $\square$

Theorem 3.35. Assume $x, y \in N$. if x divides y then $x \leq y$.

Proof. Assume $x \mid y$ and that $x > y$. By Definition 2.11 there exists some integer k such that $y = xk$. We know that x and y are positive integers, and for $y = xk$ to hold k must also be positive. However, if $x > y$, product of x and any natural number would be greater than y. We cannot find any $k \in \mathbb{Z}$ such that $y = xk$, therefore contradicting that $x \mid y$. $\square$

Problem 3.36. Is Theorem 3.35 true if we only assume $x, y \in \mathbb{Z}$? Give a proof or a counterexample.

Proof. We will proceed with a counterexample. If $x = 5$ and $y = -15$, then $x \mid y$ because $y = (-3)x$ by Definition 2.11. However, in this case $x > y$, contradicting the assertion in Theorem 3.35. $\square$