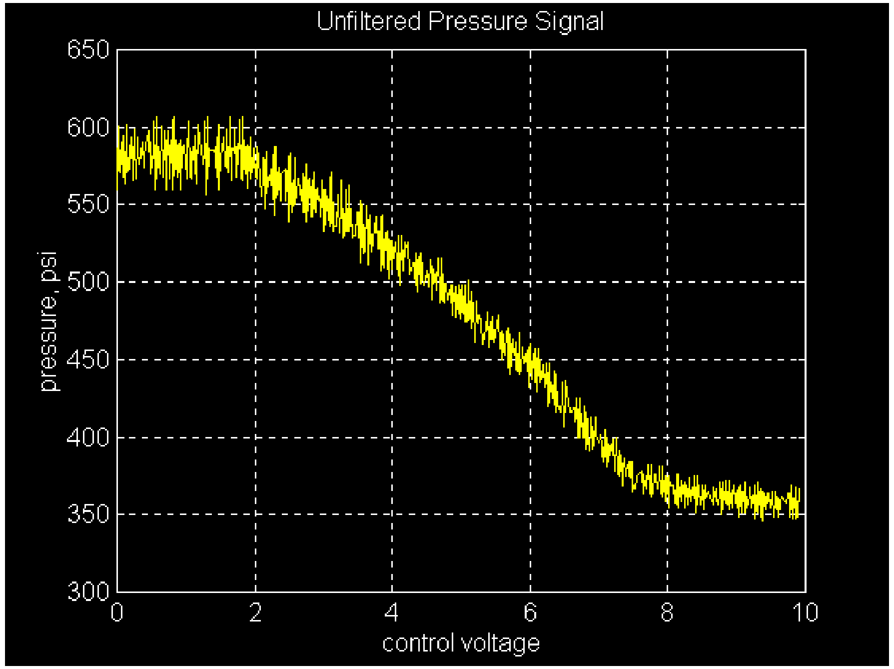
Signal Conditioning

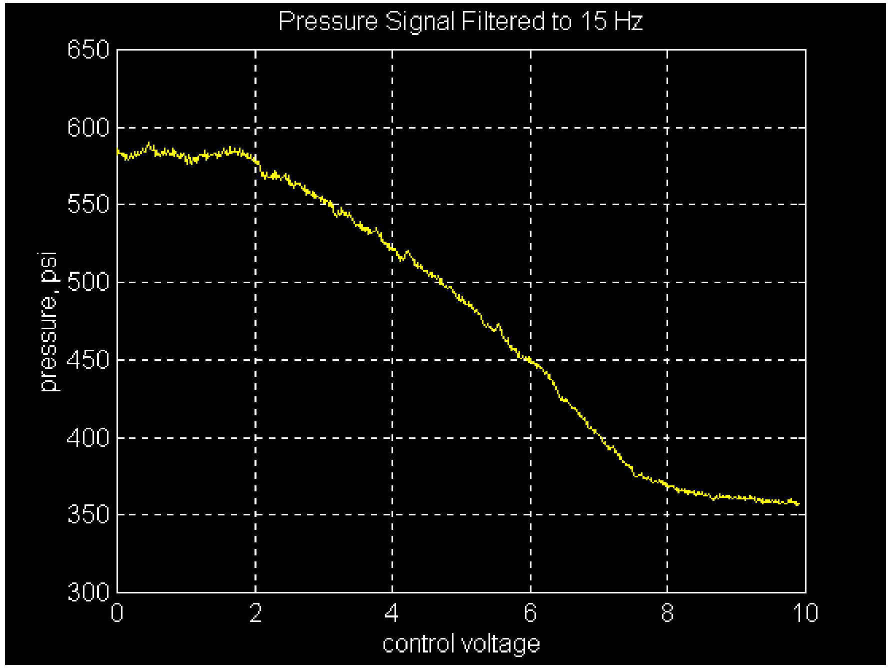
ENGG 6150: Bio-Instrumentation

Signal Conditioning

- Passive Filters
  - Simple Low pass filter
  - Simple High pass filter
  - N-section LC ladder circuit (low-pass filter prototypes)
    - Butterworth filter
    - Equal-ripple filter "Chebyshev"
  - Transformation from Low Pass to High Pass filter
  - Transformation from Low Pass to Band Pass filter
- Active Filters
  - Low pass filter
  - Cascaded low pass filter
  - Sallen Key low pass filter
  - High pass filter
  - Cascaded high pass filter
  - Sallen Key low pass filter

What is a Filter?

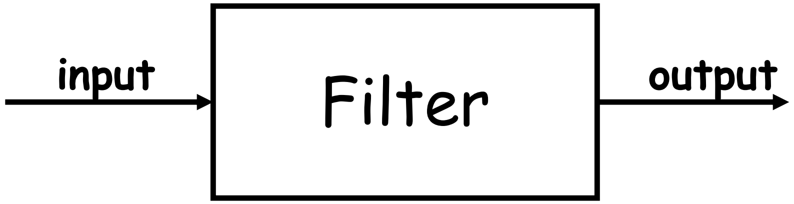




Filtering:

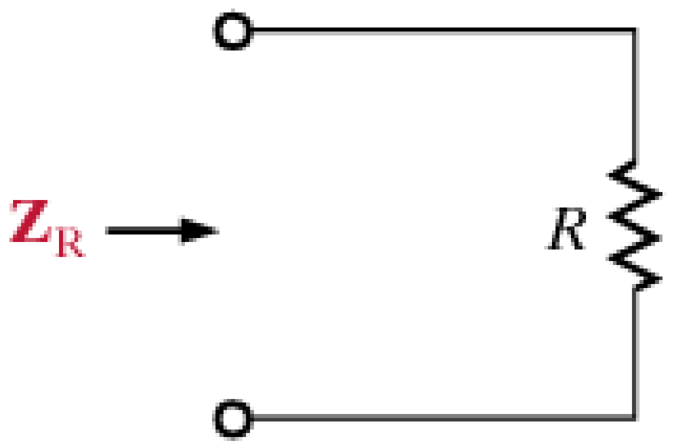
Certain desirable features are **retained**

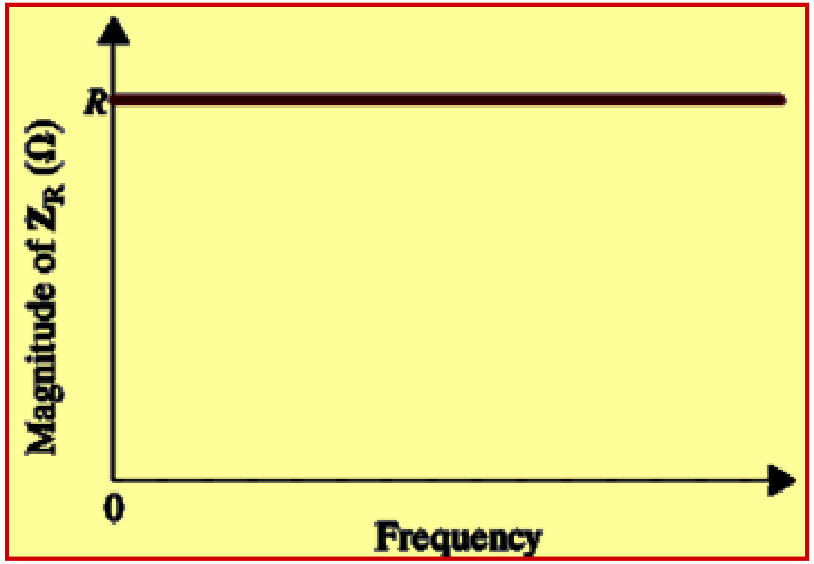
Other undesirable features are **suppressed**

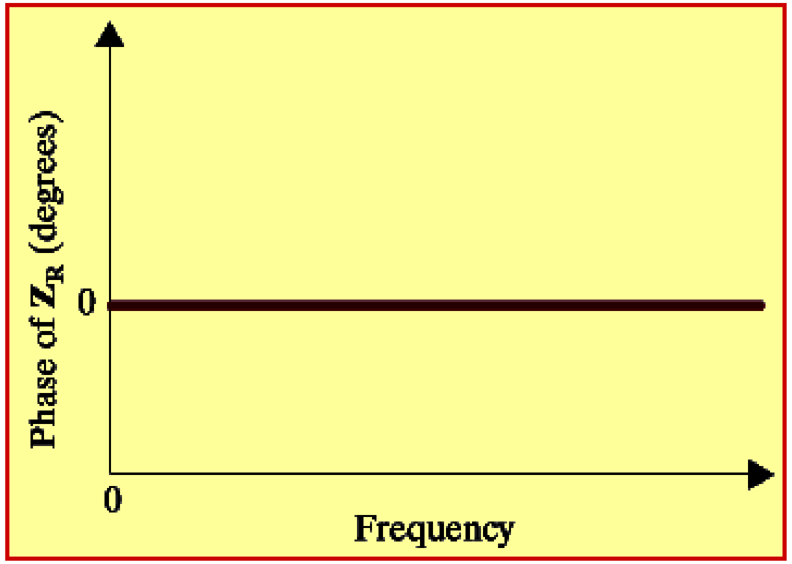


Resistors

Filters are circuits that are capable of *passing signals within a band* of frequencies while *rejecting or blocking* signals of frequencies *outside this band*. This property of filters is also called “frequency selectivity”.





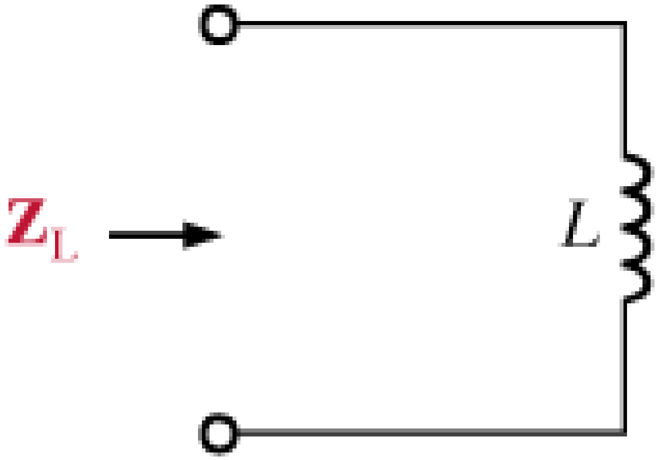


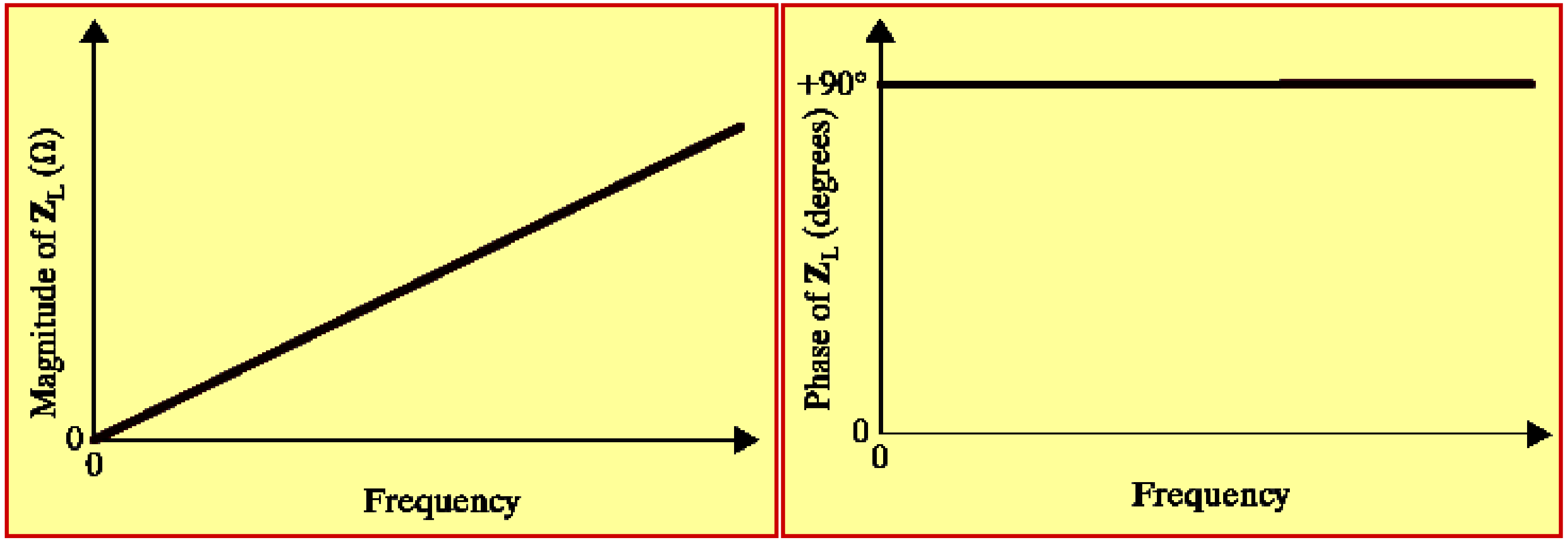
$$Z_R = R = R\angle 0^\circ$$

$$Magnitude\ of\ Z_R = |Z_R| = R$$

$$Phase\ of\ Z_R = 0^o$$

Inductors



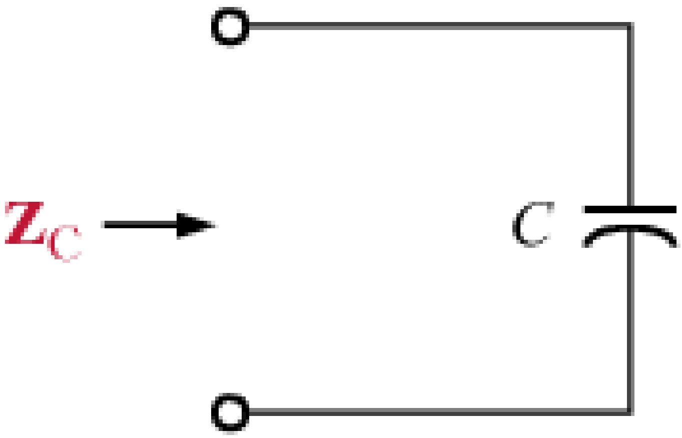


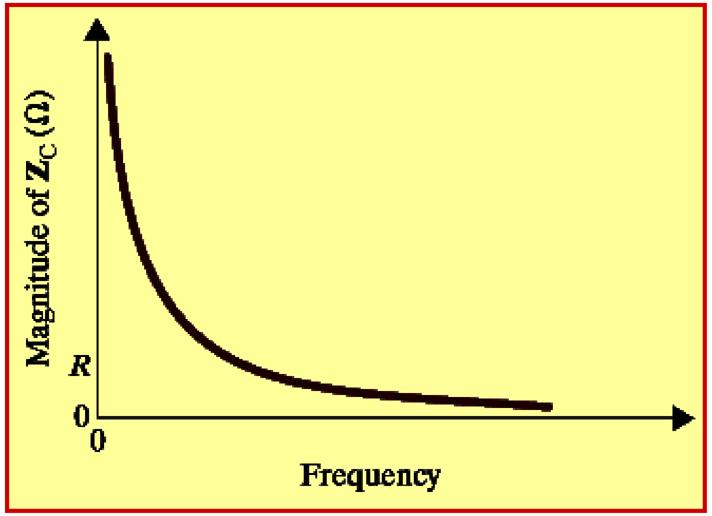
$$Z_L = j\omega L = \omega L \angle 90^\circ$$

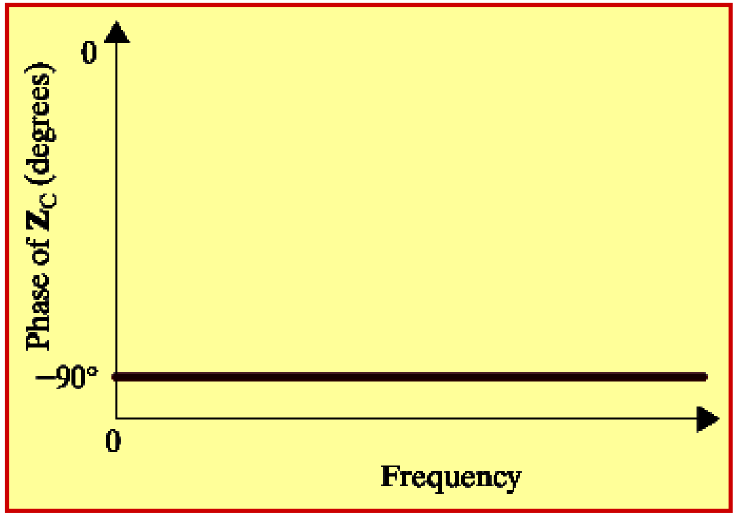
$$Magnitude\ of\ Z_L = |Z_L| = \omega L$$

$$Phase\ of\ Z_L = +90^o$$

Capacitors





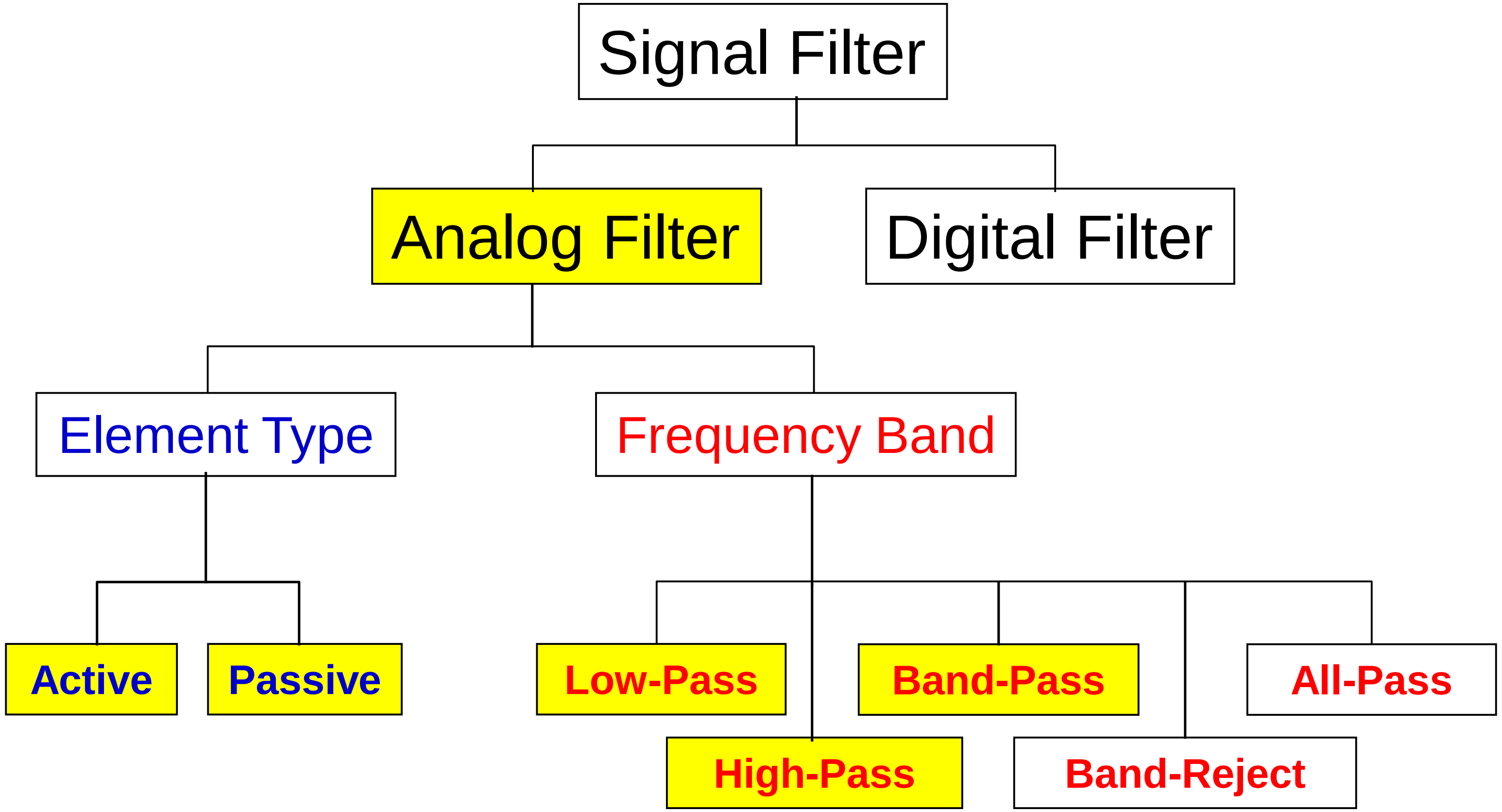


$$Z_C = 1/j\omega C = 1/\omega C^{\angle -90^\circ}$$

$$Magnitude\,of\ Z_C = |Z_C| = 1/\omega C$$

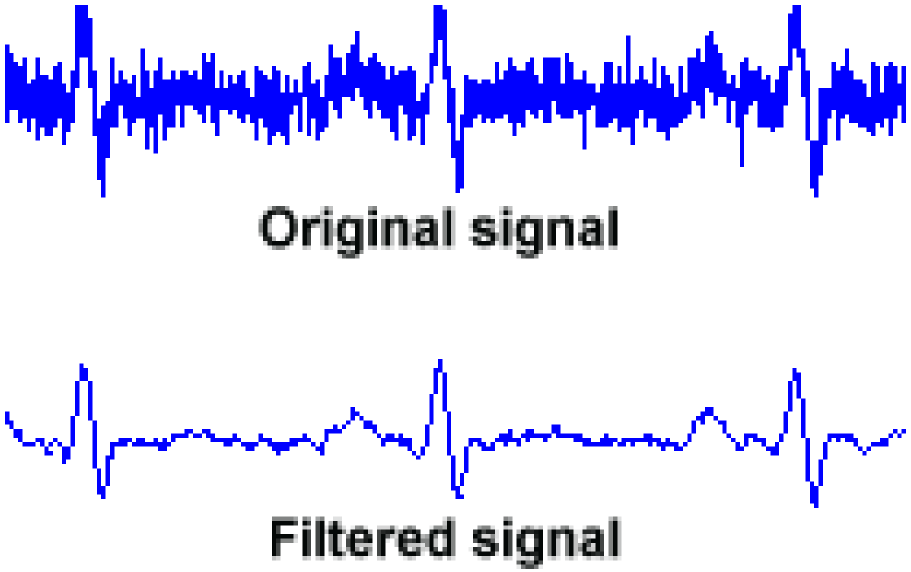
$$Phase\ of\ \ Z_C = -90^o$$

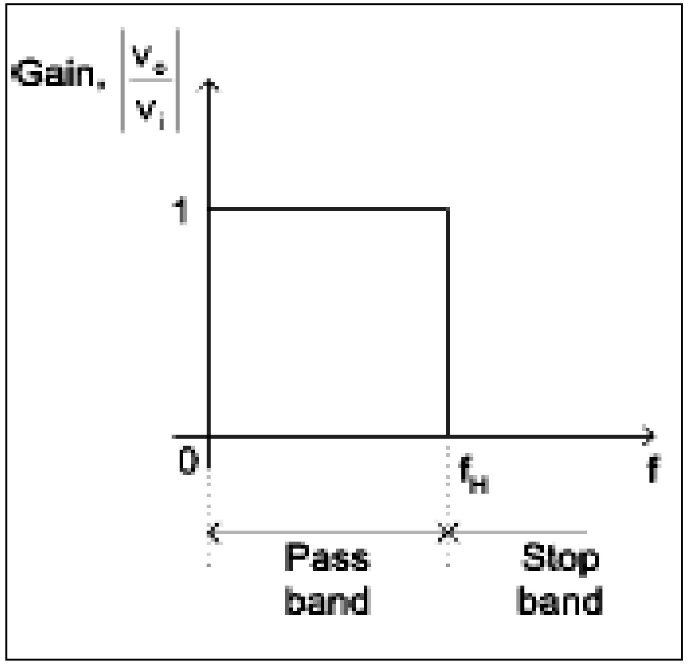
Classification of Filters



Low-Pass Filter

➢ An ideal low-pass filter is a filter that passes frequencies from 0Hz to certain frequency called the critical frequency f_H and completely attenuates all other frequencies.





Ideal response

Example 1

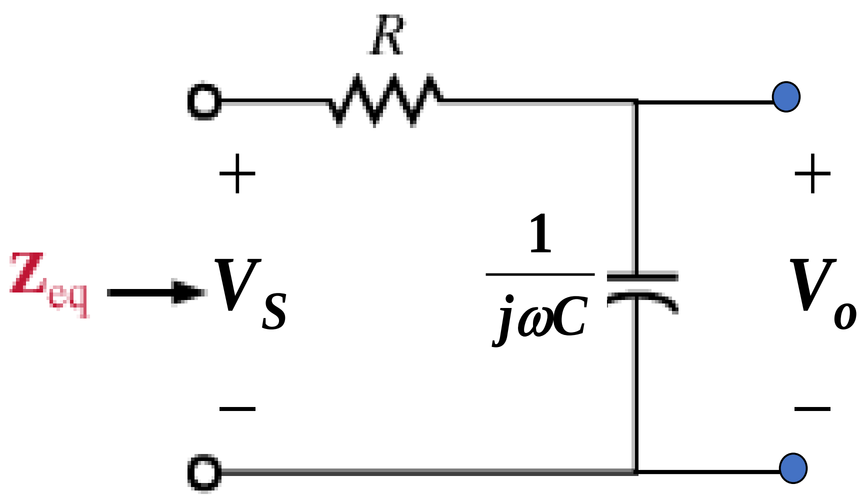
Find the voltage gain and frequency response of the *RC* circuit.

SOLUTION:

<u>The voltage gain:</u>

$$H(\omega) = \frac{V_o}{V_s} = \frac{(1/j\omega C)}{R + (1/j\omega C)} = \frac{1}{1 + j\omega RC}$$

Set: $$\omega_c = \frac{1}{RC}$$



<u>Magnitude is:</u> $$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

<u>Phase is:</u>

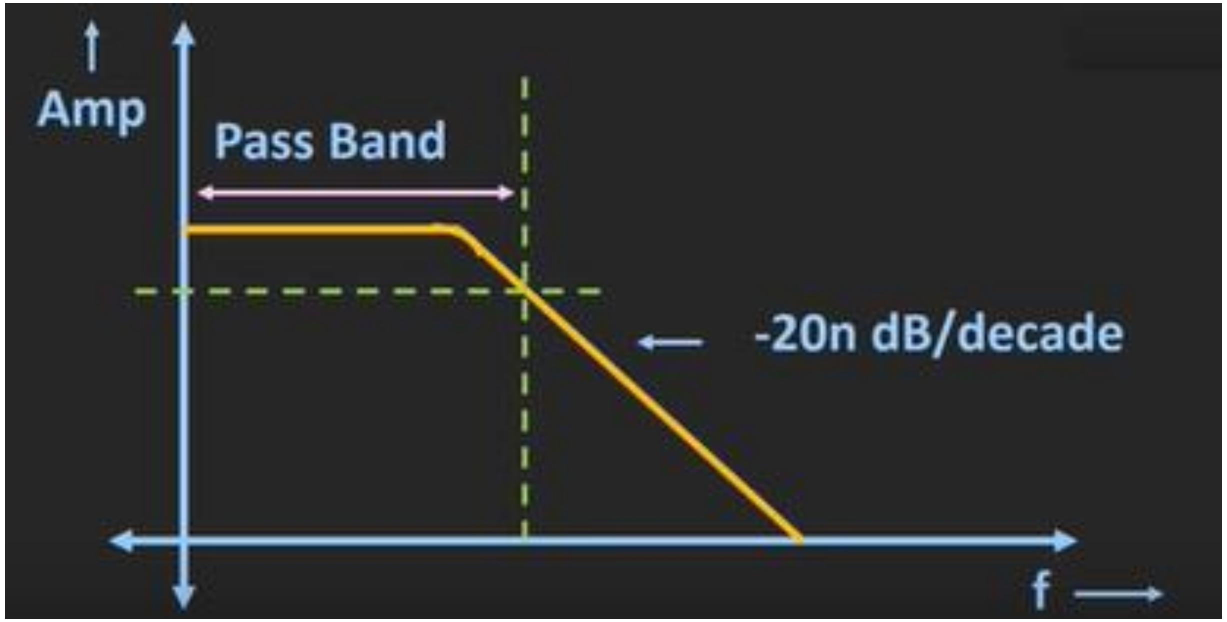
$$\phi = -tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= -tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

ω/ω_c	$\lvert H(\omega)\rvert$	ϕ
0	1.0	0
1.0	0.707	-45^o
10	0.1	-84^o
100	0.01	-89^o

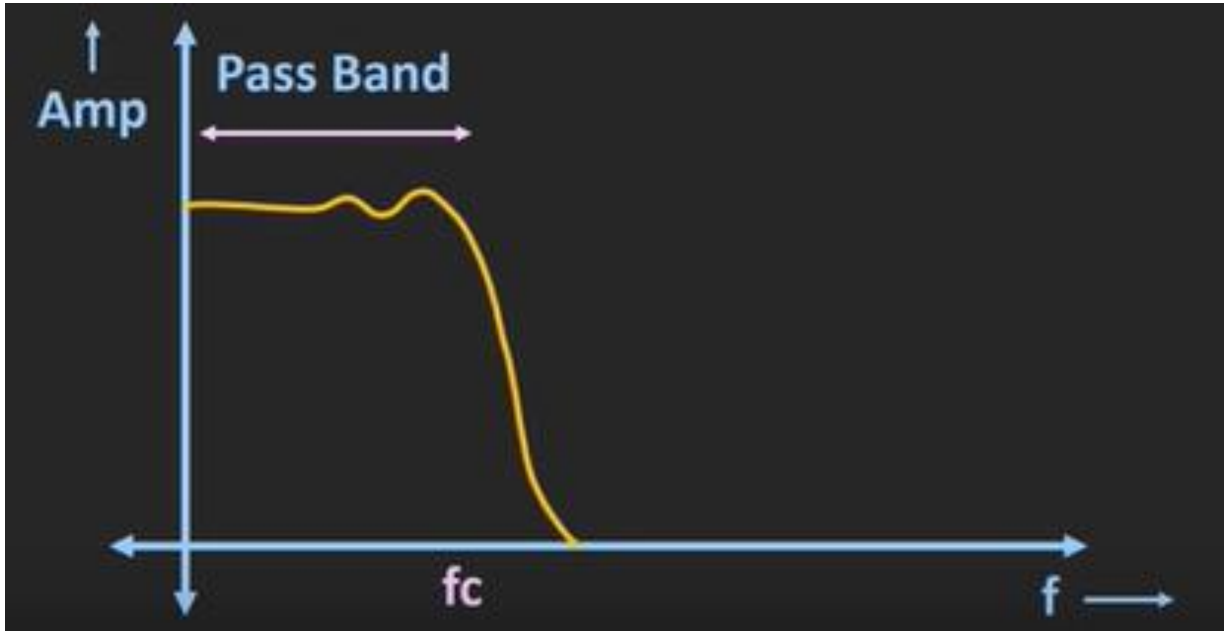
Butterworth Filter Approximation

- It provides the maximally flat approximation.
- The roll off is at -20dB/decade for the first order, -40dB/decade for the second order and so on.



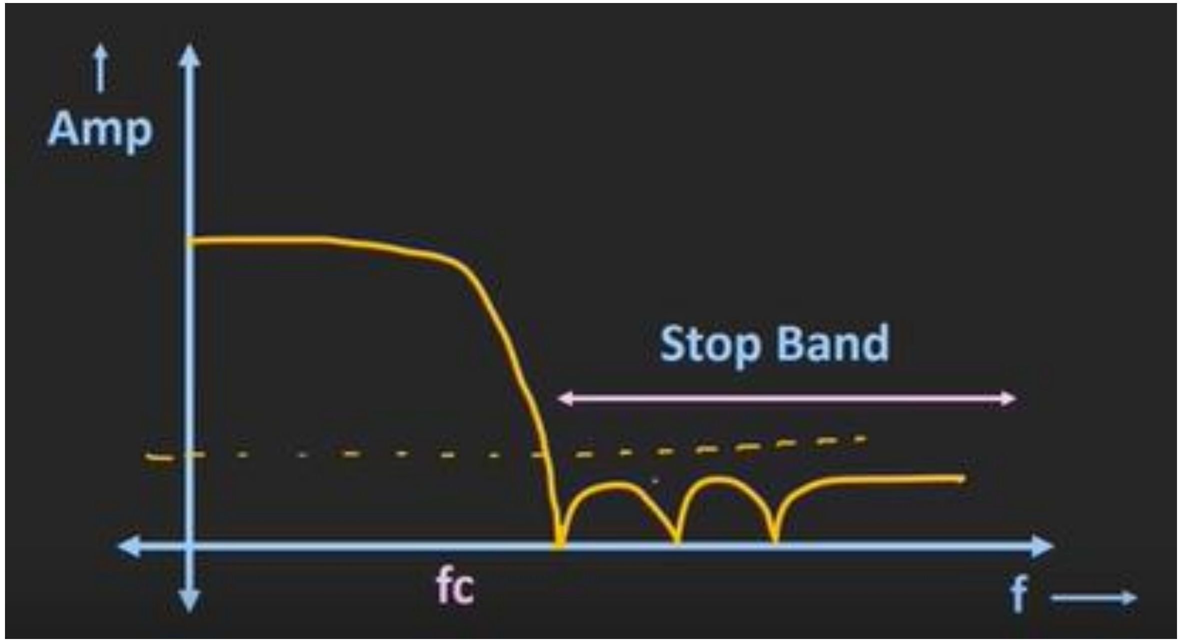
Chebyshev Filter Approximation

- It provides some ripples in the pass band. The number of ripples = n/2 where n is the order of the filter.
- However, in stop band there will be no ripples.
- The roll off is much faster than the Butter worth filter.



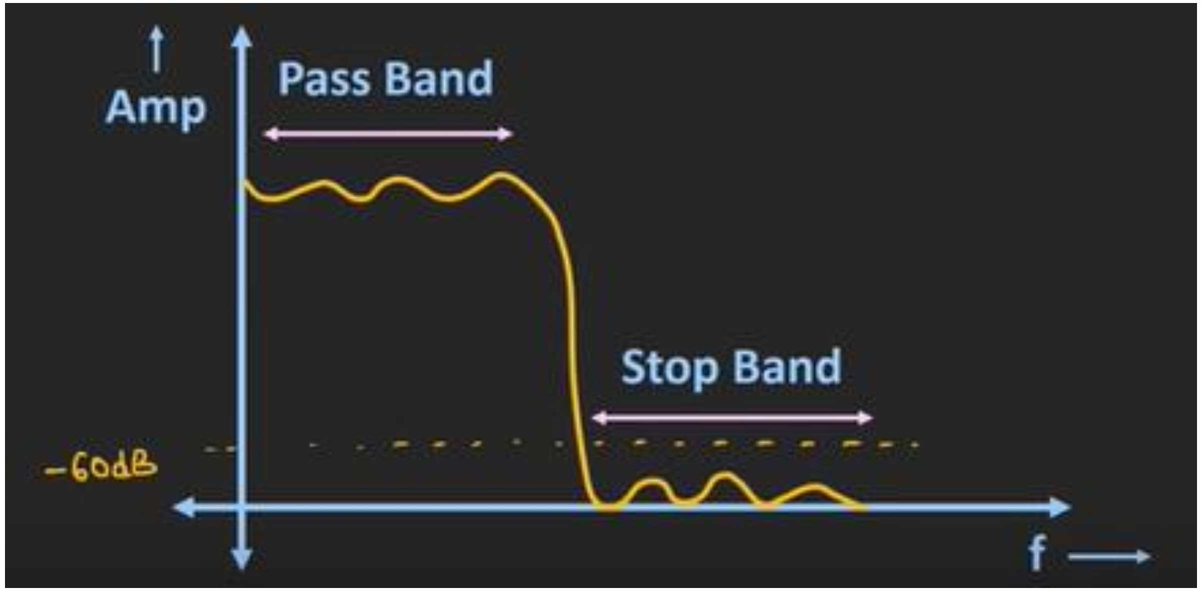
Inverse Chebyshev Filter Approximation

- It provides some ripples in the stop band.
- However, in pass band there will be no ripples.
- The roll off is much faster than the Butter worth filter.



Elliptic Filter Approximation

- It provides some ripples in both the pass and stop bands.
- The roll off is the fastest among all filters.



Low-Pass Filter

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_o}\right)^2}} \quad \Longrightarrow \quad |H(j\omega)|_{\text{dB}} = 20\log\left[\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_o}\right)^2}}\right]$$

When $\omega = 0 \quad \Longrightarrow \quad |H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{0}{\omega_o}\right)^2}} = 1 \quad \Longrightarrow \quad |H(j\omega)|_{\text{dB}} = 20\log 1 = 0\ \text{dB}$

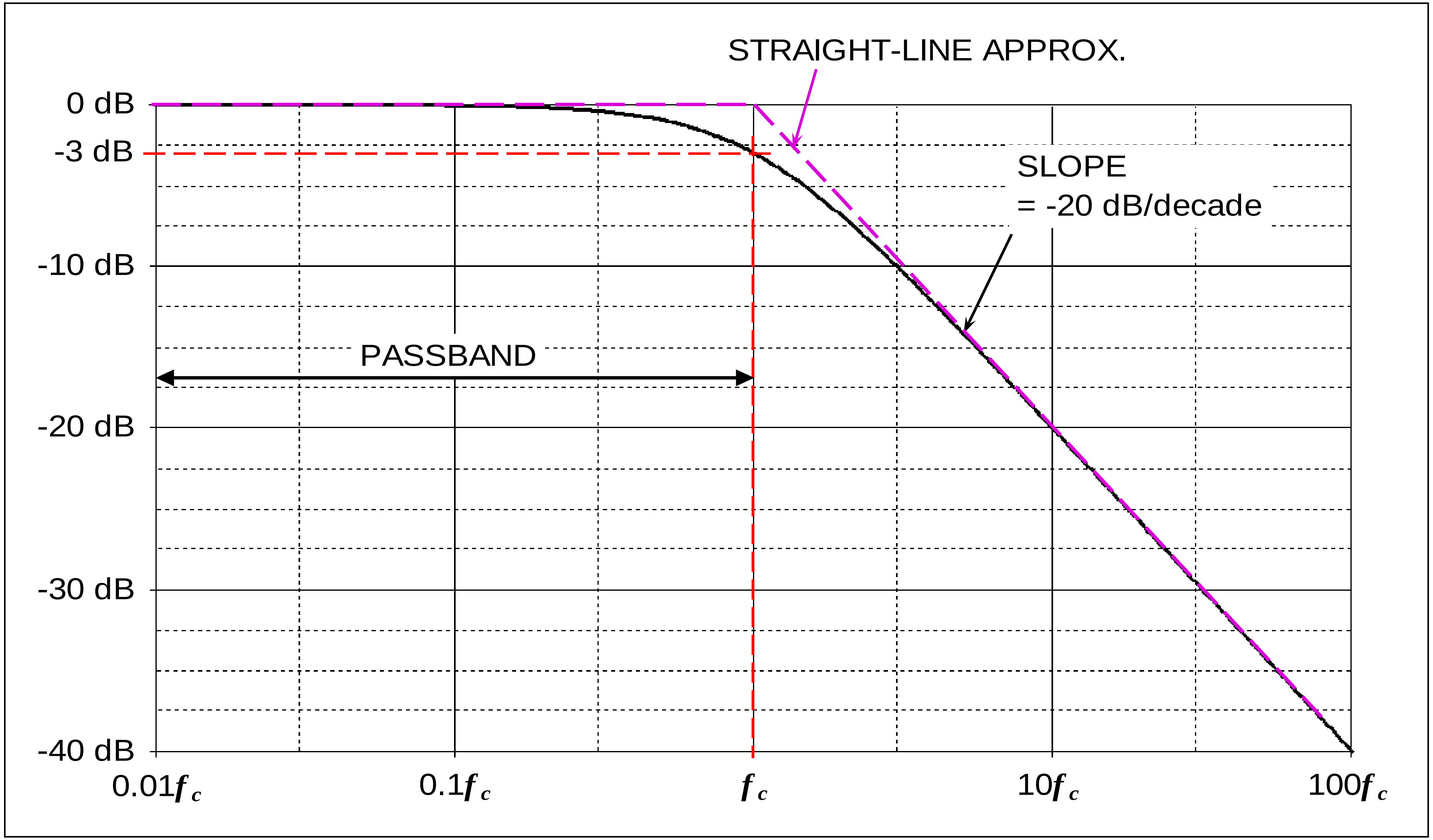
When $\omega = \omega_c = \omega_o = \frac{1}{CR} \quad \Longrightarrow \quad |H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega_o}\right)^2}} = \frac{1}{\sqrt{1+1}} = 0.707$

$$\Longrightarrow \quad |H(j\omega)|_{\text{dB}} = 20\log 0.707 = -3\ \text{dB}$$

$\omega_c = \frac{1}{CR}$ is known as 3-dB or cutoff frequency (rad/s)

Low-Pass Filter

Response curve

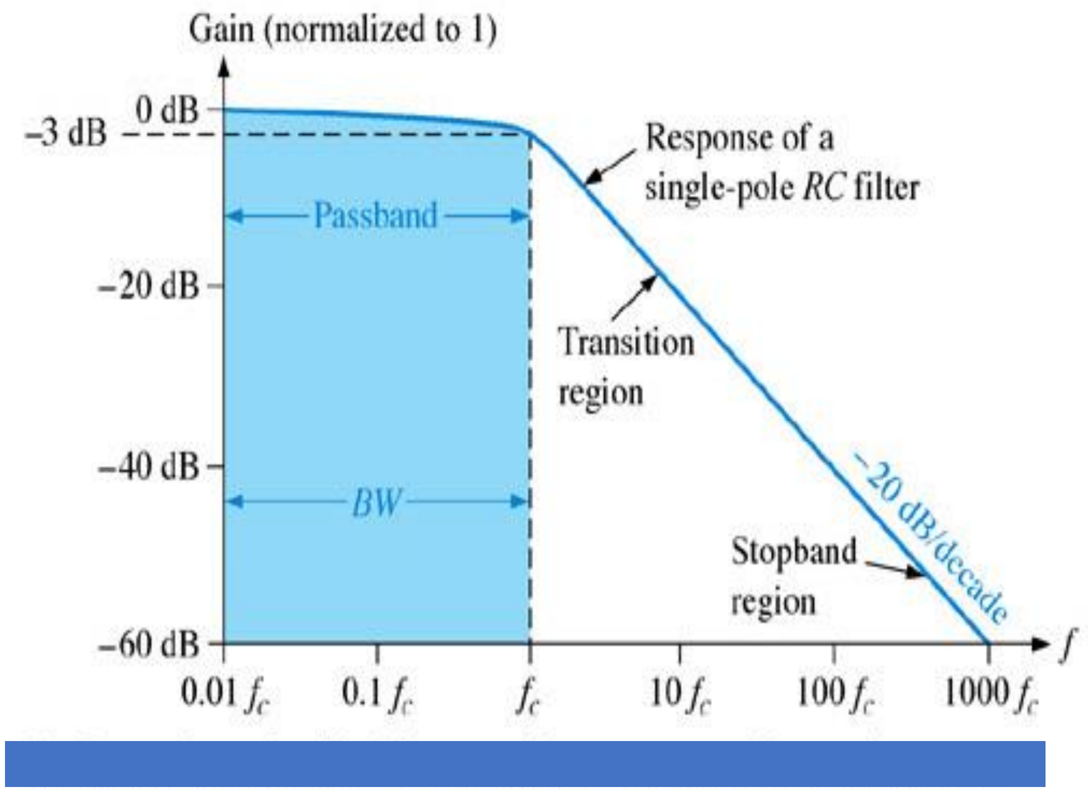


Low-Pass Filter

➢ An actual low-pass filter is a filter that passes most of frequencies from 0Hz to certain frequency and significantly attenuates other frequencies.

➢ **Passband** of a filter is the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than -3 dB of attenuation).

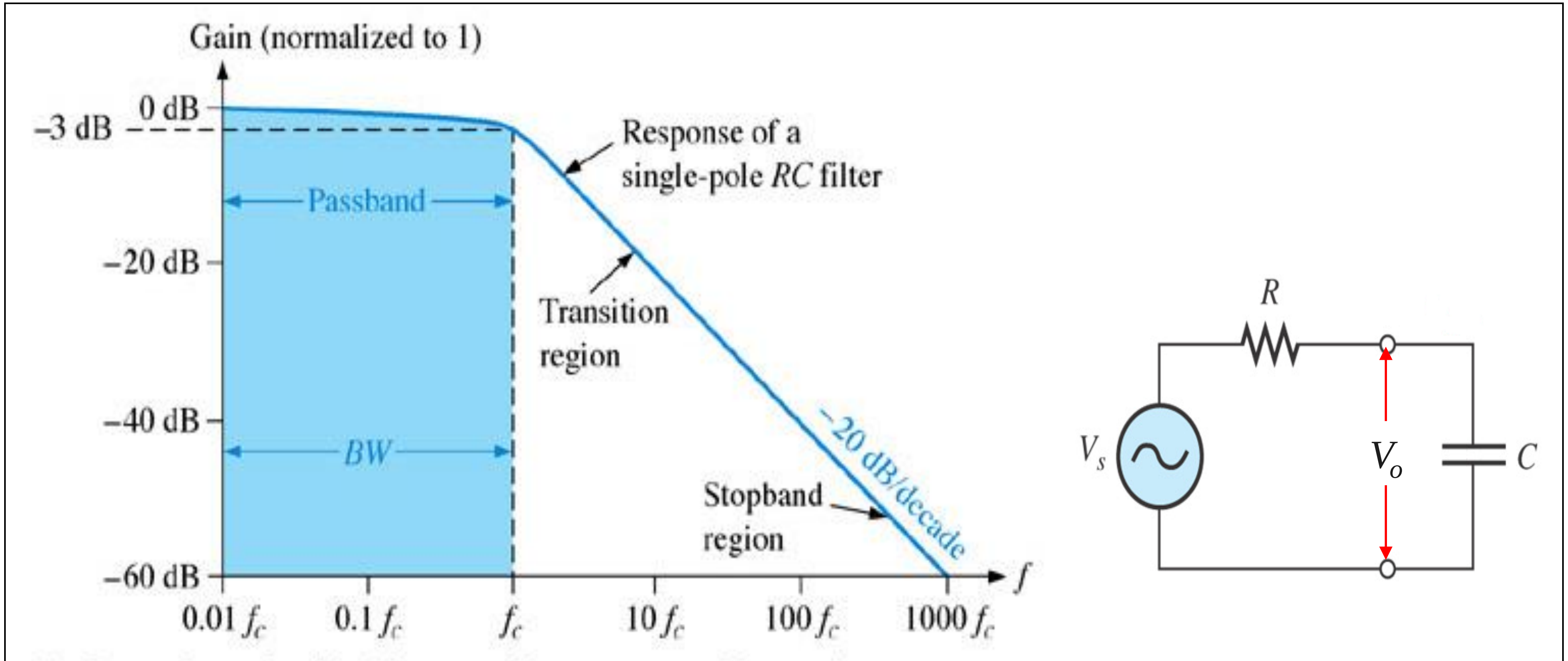
➢ **Transition region** shows the area where the fall-off occurs.



Actual response

➢ **Stopband** is the range of frequencies that have the most attenuation.

➢ **Critical frequency, f_c**, (also called the cutoff frequency) defines the end of the passband and normally specified at the point where the response drops – 3 dB (70.7%) from the passband response.



(a) Comparison of an ideal low-pass filter response with actual response

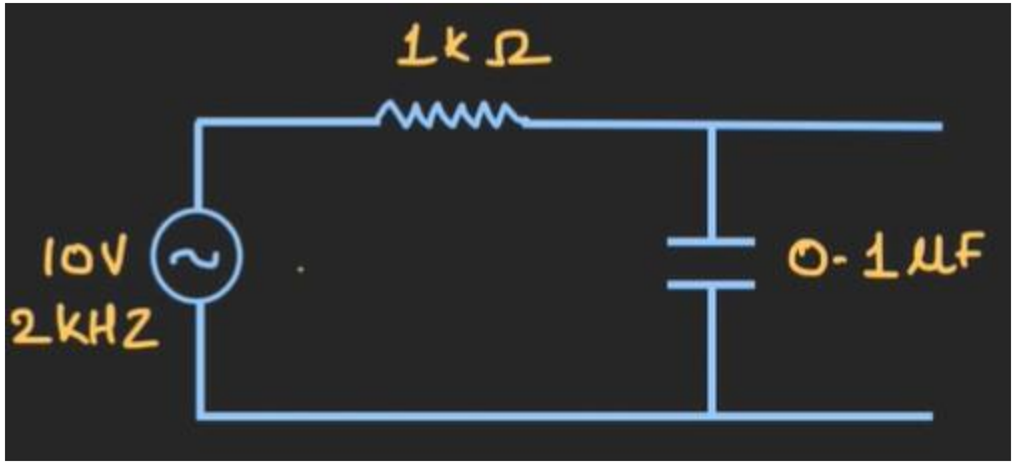
(b) Basic low-pass circuit

- At low frequencies, X_C is very high and the capacitor circuit can be considered as open circuit. Under this condition, $V_o = V_{in}$ or $A_V = 1$ (unity).

- At very high frequencies, X_C is very low and the V_o is small as compared with V_{in}. Hence the gain falls and drops off gradually as the frequency is increased.

Example 2

Find the cut off frequency and output voltage at 2kHz.

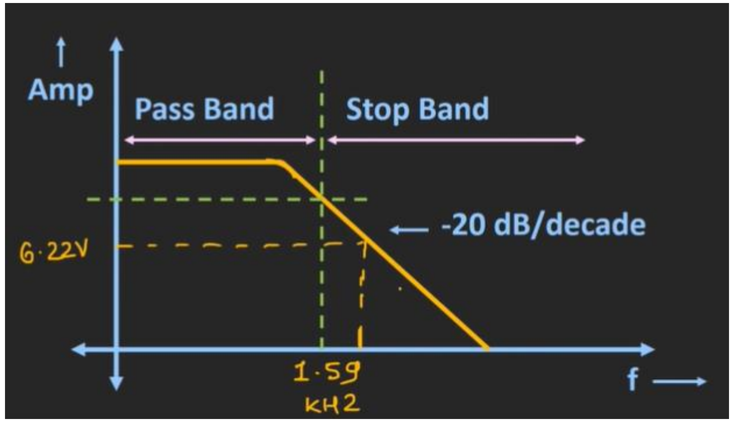


SOLUTION:

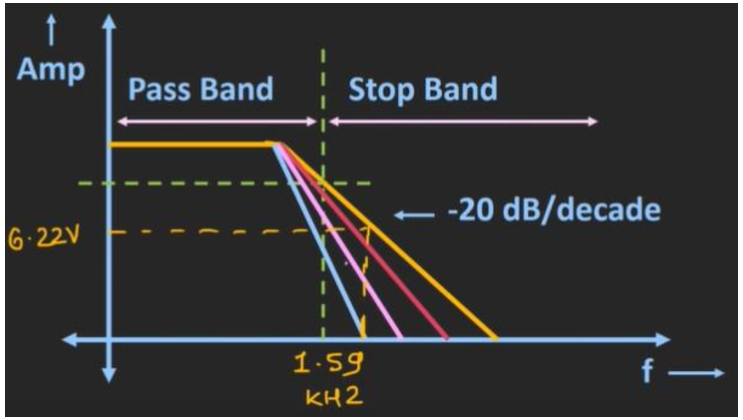
<u>The cut off frequency:</u>

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi 1k \times 0.1\mu} = 1.59kHz$$

$$V_{out} = \frac{1}{\sqrt{1 + (\omega RC)^2}} V_{in} = \frac{1}{\sqrt{1 + (2\pi \times 2k \times 1k \times 0.1\mu)^2}} 10V = 6.22V$$

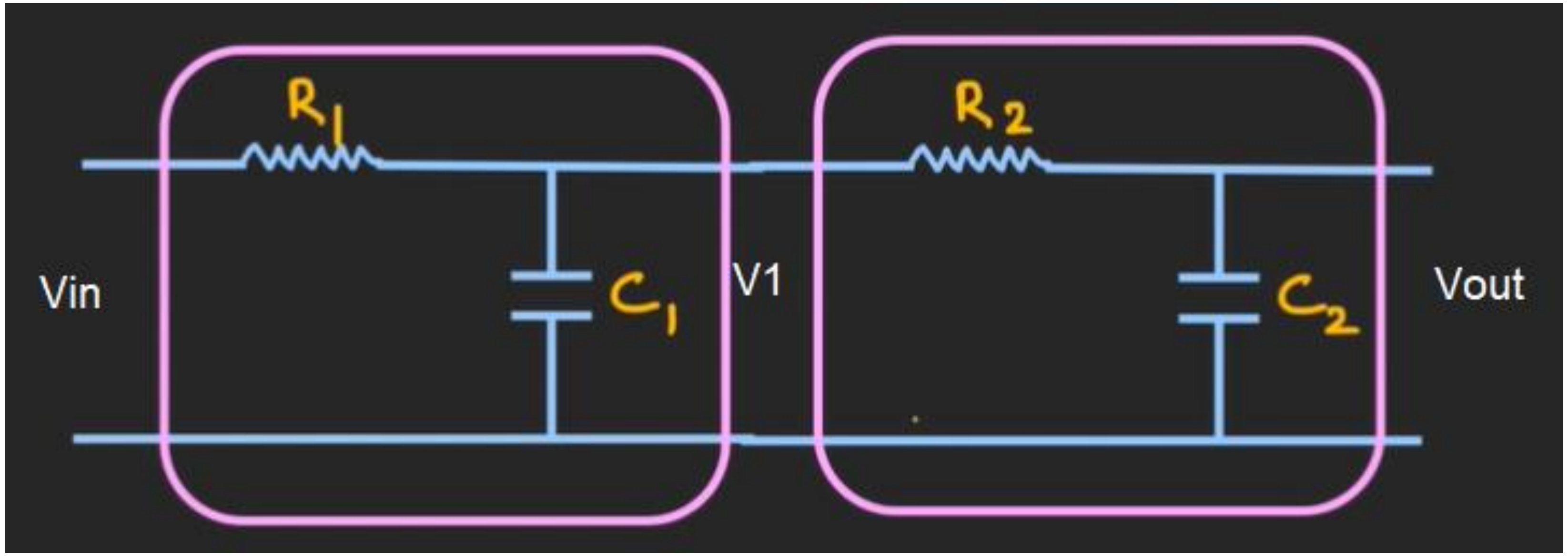


For more attenuation



Higher Order Low-Pass Filter

➢ Higher order low pass filters can be achieved by cascading multiple RC filters as shown below.



Second order low pass filter

Higher Order Low-Pass Filter

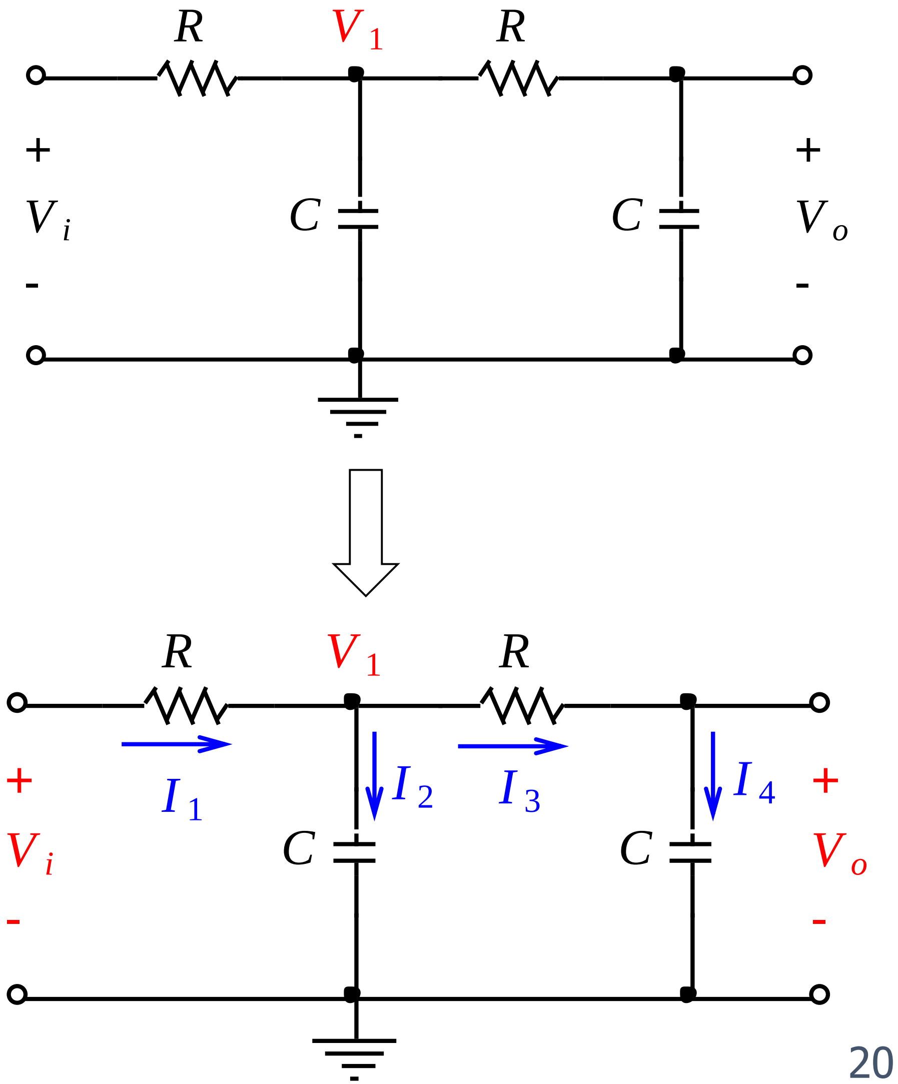
- Derive the transfer function for this second order passive filter.

Note that s = jω

$$\frac{V_i - V_1}{R} = sCV_1 + \frac{V_1 - V_o}{R}$$

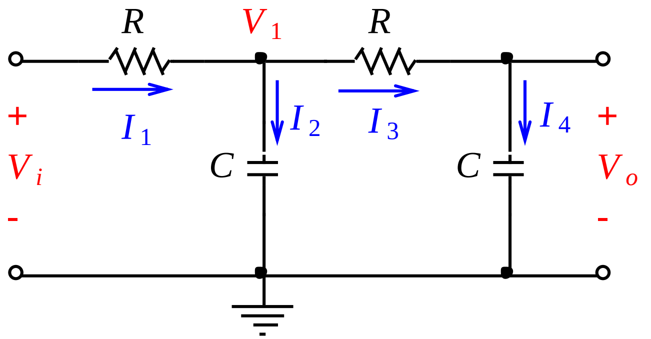
$$\frac{V_i}{R} = \left(\frac{sCR + 2}{R}\right)V_1 - \frac{V_o}{R}$$

$$\frac{V_1 - V_o}{R} = sCV_o \quad\Rightarrow\quad V_1 = (sCR + 1)V_o$$



Higher Order Low-Pass Filter

Since:



$$\frac{V_i}{R} = \left(\frac{sCR+2}{R}\right)V_1 - \frac{V_o}{R}$$

And:

$$V_1 = (sCR+1)V_o \quad \Rightarrow \quad \frac{V_i}{R} = \left(\frac{sCR+2}{R}\right)(sCR+1)V_o - \frac{V_o}{R}$$

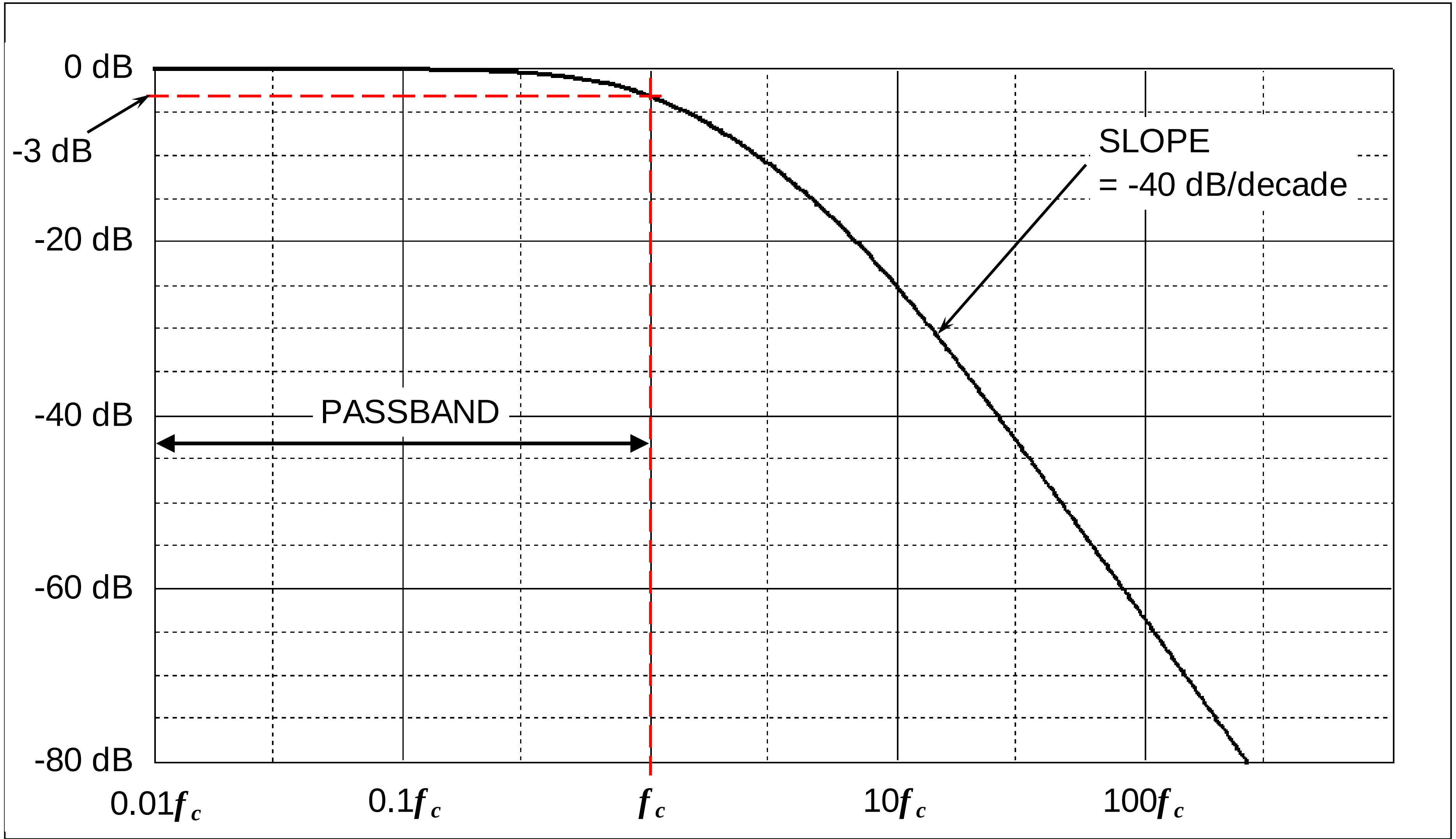
$$\Rightarrow \quad H(s) \equiv \frac{V_o}{V_i} = \frac{1}{(sCR)^2 + 3sCR + 1}$$

$$H(s) = \frac{1}{(sCR)^2 + 3sCR + 1} \quad \Rightarrow \quad H(s) = \frac{\frac{1}{(CR)^2}}{s^2 + s\frac{3}{CR} + \frac{1}{(CR)^2}}$$

Or $$H(s) = \frac{\omega_o^2}{s^2 + s3\omega_o + \omega_o^2} \qquad \left[\omega_o = \frac{1}{CR}\right]$$

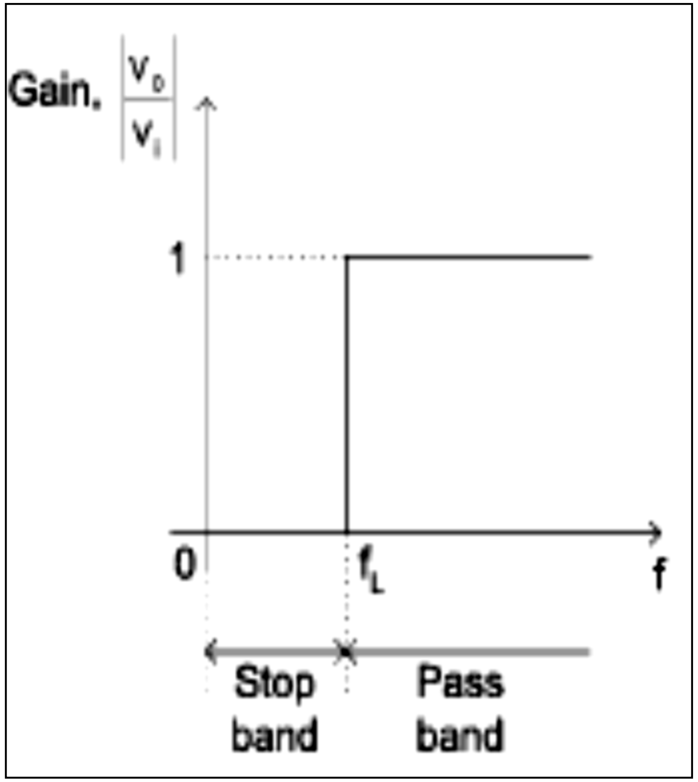
Higher Order Low-Pass Filter

Response curve



High-Pass Filter Response

➢ An ideal high-pass filter is a filter that completely attenuates all frequencies **below** certain frequency and passes all frequencies **above** that frequency.



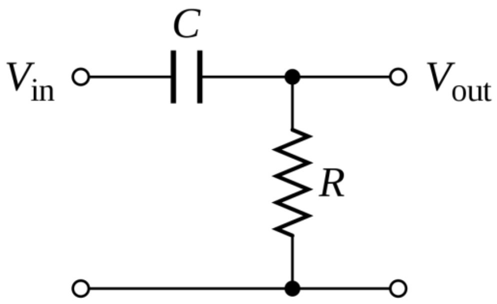
Ideal response

Example 3

Find the voltage gain and frequency response of the *RC* circuit.

SOLUTION:

$$V_{out} = \frac{R}{\left(R + \frac{1}{j\omega C}\right)} V_{in} \longrightarrow V_{out} = \frac{j\omega RC}{(j\omega RC + 1)} V_{in}$$



<u>The voltage gain:</u> $$\frac{V_{out}}{V_{in}} = \frac{j\omega RC}{(j\omega RC + 1)}$$

Set: $$\omega_c = \frac{1}{RC}$$

<u>Magnitude is:</u> $$|H(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} = \frac{(\omega/\omega_c)}{\sqrt{1 + (\omega/\omega_c)^2}}$$

<u>Phase is:</u>

$$\emptyset = 90 - tan^{-1}\left(\frac{\omega RC}{1}\right)$$

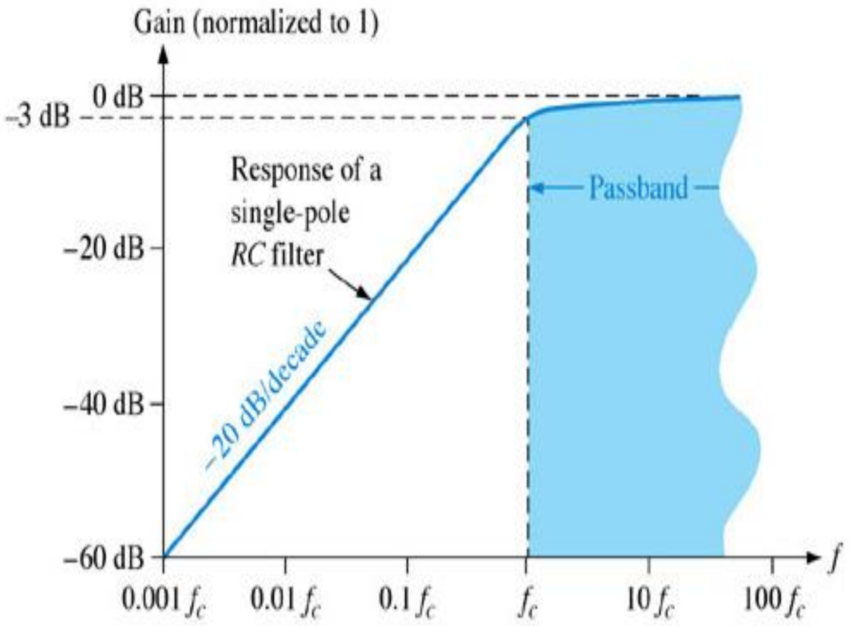
$$= 90 - tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

$\omega/\ \omega_c$	$\|H(\omega)\|$	ϕ
0	0.0	90
1.0	0.707	45°
10	0.995	6°
100	0.9999	1°

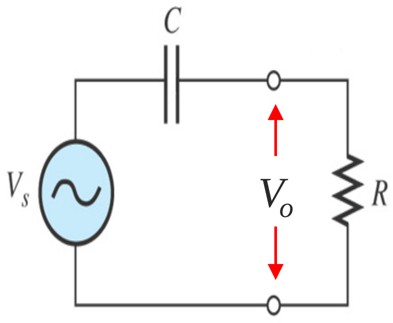
- At $\omega = 1/(RC)$, the magnitude of the voltage gain is $1/\sqrt{2}$.
- This is called the critical frequency of a high-pass RC filter

High-Pass Filter Response

- An actual high-pass filter is a filter that significantly attenuates or rejects all frequencies **below** f_c and passes all frequencies **above** f_c.
- The passband of a high-pass filter is all frequencies above the critical frequency.



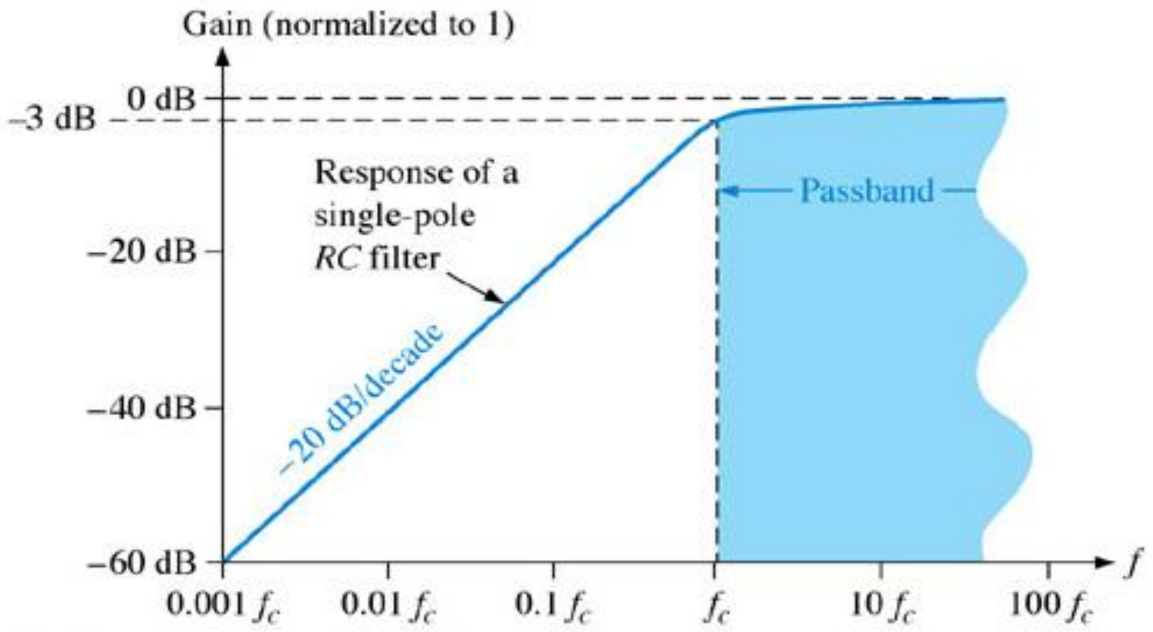
(a) Comparison of an ideal high-pass filter response with actual response



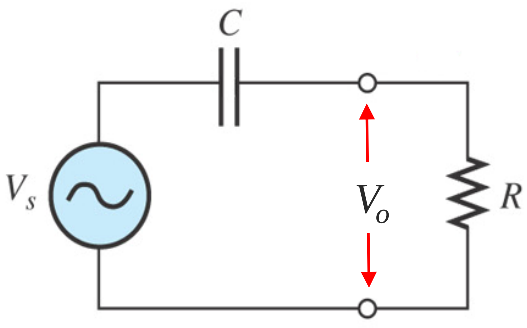
(b) Basic high-pass circuit

Actual response

High-Pass Filter Response



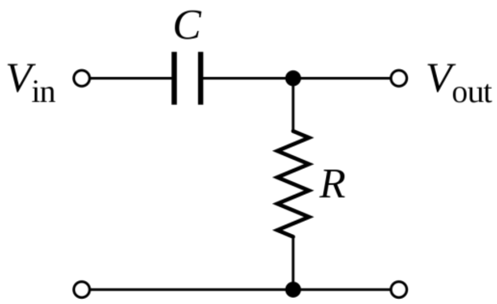
(a) Comparison of an ideal high-pass filter response with actual response



(b) Basic high-pass circuit

- At low frequencies, X_C is very high and the capacitor circuit can be considered as open circuit. Under this condition, $V_o = 0$.
- At very high frequencies, X_C is very low (acts like short circuit) and the V_o is equal to V_{in}. Hence the gain increases gradually as the frequency is increased till $A_V = 1$ (unity).

Example 4



- Design a high pass filter with a cut-off frequency of 10 kHz.

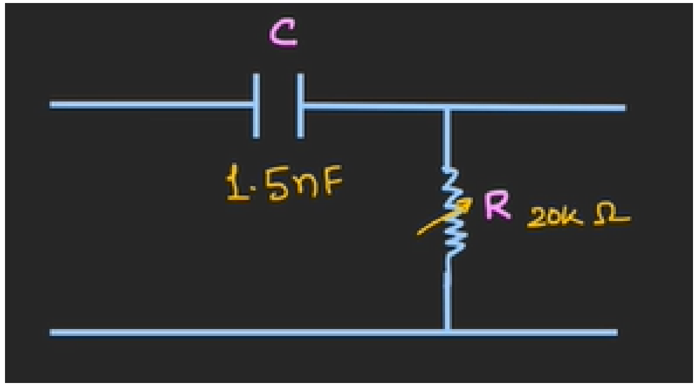
SOLUTION:

Select the value of R to be 10 kΩ. The question why this value? This is for the following reasons:

a) Small value of R in Ohms, will lead to a loading effect to the connected circuit.

b) High values of R in MΩ will lead to small value of C comparable to the parasitic capacitance.

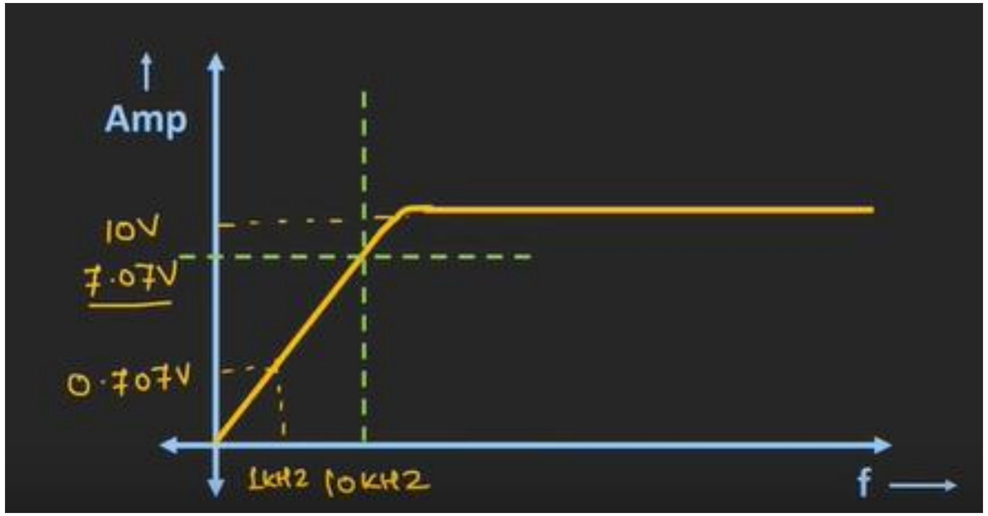
$$C = \frac{1}{2\pi Rf} = \frac{1}{2\pi 10k \times 10k} = 1.59nF$$

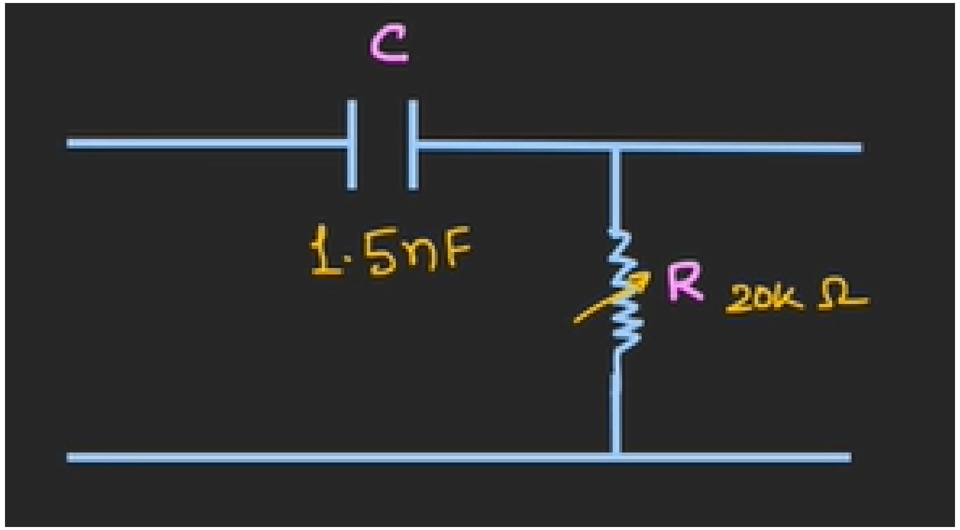


- In practice, we will have 1.5 nF and not 1.59nF, so we can adjust the 10kHz cut-off frequency using 20kΩ variable resistance.

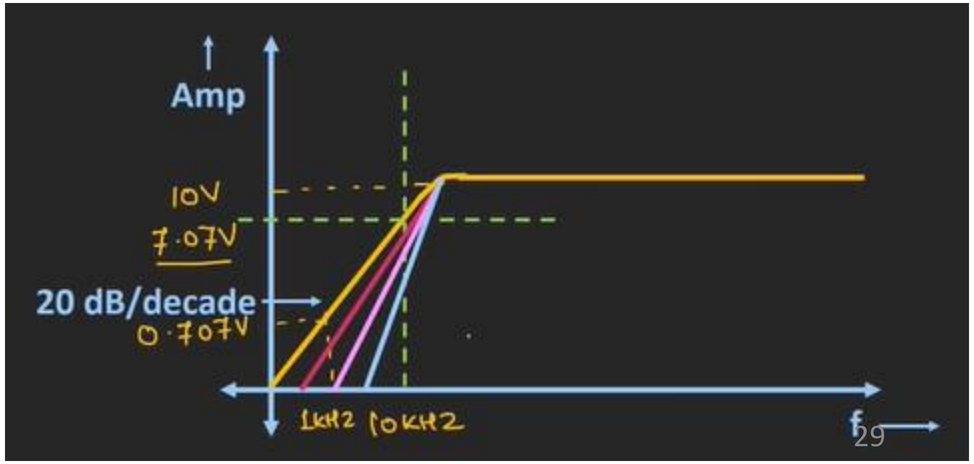
Example 4, Continue

So if we have an input of 10V, the output will be affected as follows:



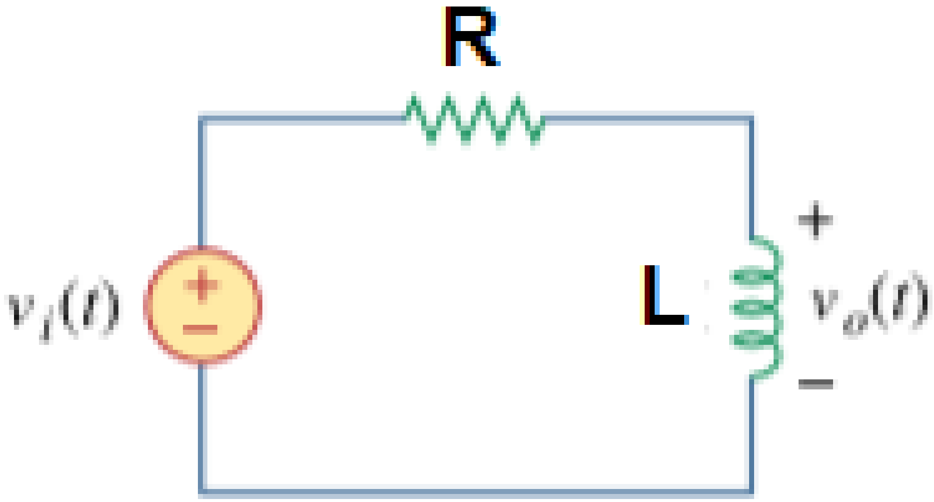


- The attenuation will be 20dB/decade or the output voltage will drop by one tenth when the frequency increases by one tenth.
- By having higher order filters, the attenuation will be much higher



Example 5

What is the type of the filter shown below?



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L}$$

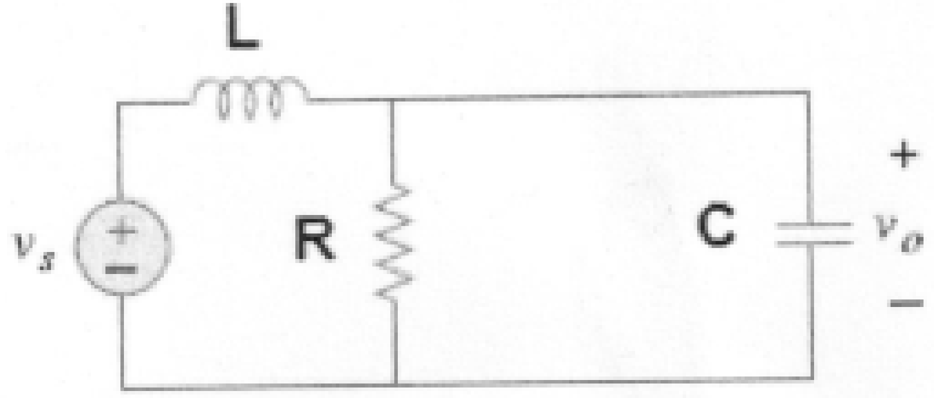
$$H(0) = 0 \ \text{ and } \ H(\infty) = 1$$

This is a high pass filter

Example 6

What is the type of the filter shown below?

$$\frac{V_o}{V_s} = \frac{R \parallel 1/j\omega C}{R \parallel 1/j\omega C + j\omega L} = \frac{\dfrac{R/j\omega C}{R + 1/j\omega C}}{\dfrac{R/j\omega C}{R + 1/j\omega C} + j\omega L}$$

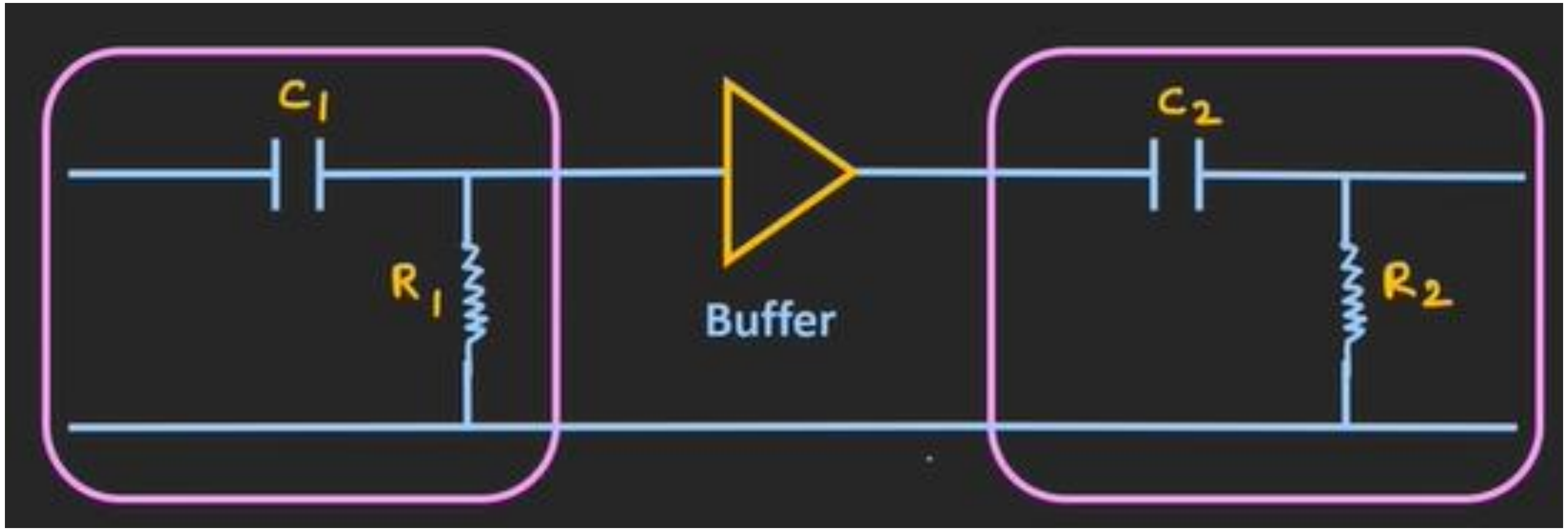


$$\xrightarrow{\times (R + 1/j\omega C)} \; = \frac{R/j\omega C}{R/j\omega C + j\omega L (R + 1/j\omega C)} \times \frac{j\omega C}{j\omega C}$$

$$= \frac{R}{R + j\omega L(j\omega RC + 1)} = \frac{R}{R - \omega^2 RLC + j\omega L}$$

$$\left.\begin{array}{l} @\,\omega = 0 \Rightarrow \left|\frac{V_o}{V_s}\right| = 1 \\ @\,\omega = \infty \Rightarrow \left|\frac{V_o}{V_s}\right| = 0 \end{array}\right\} \text{L.P.F.}$$

Higher Order High Pass Filter

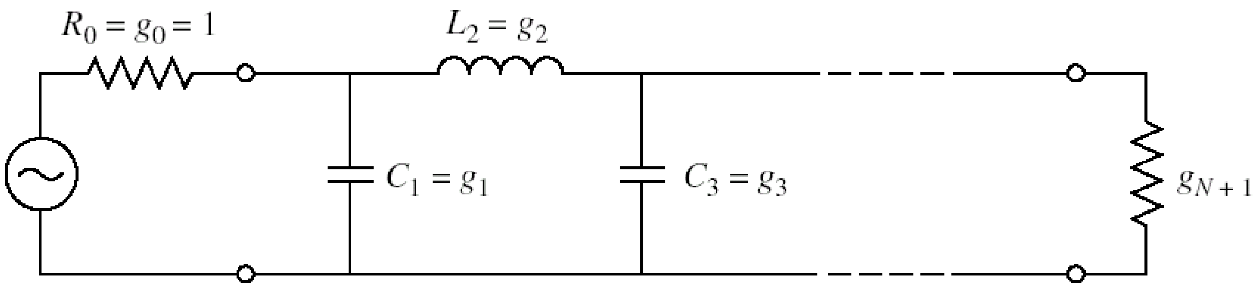


➢ Higher order high pass filters can be achieved by cascading multiple RC filters as shown below.

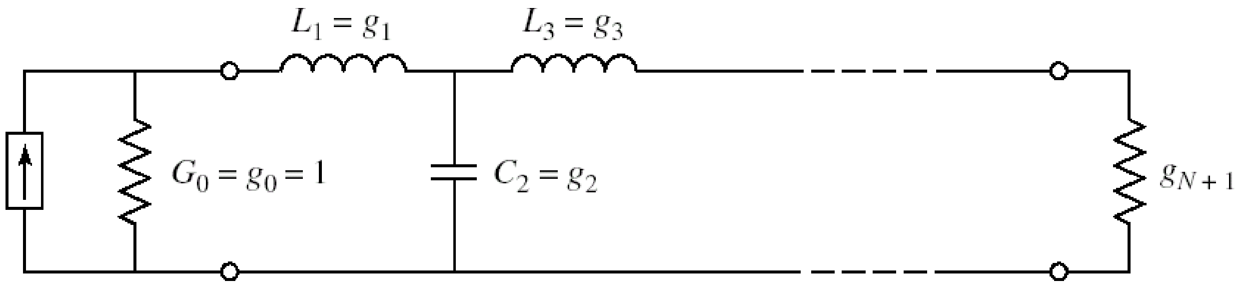
➢ The buffer is used to minimize the loading effect between the different stages of the filter.

N-section LC ladder circuit (low-pass filter prototypes)

Prototype beginning with serial element

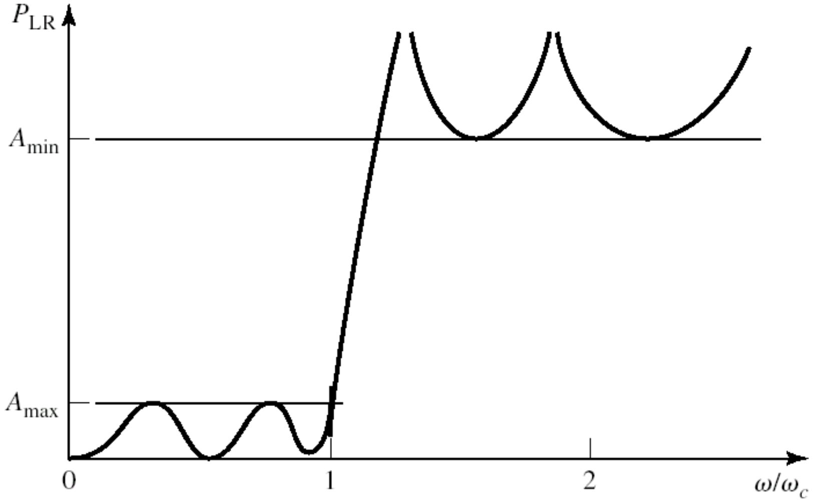


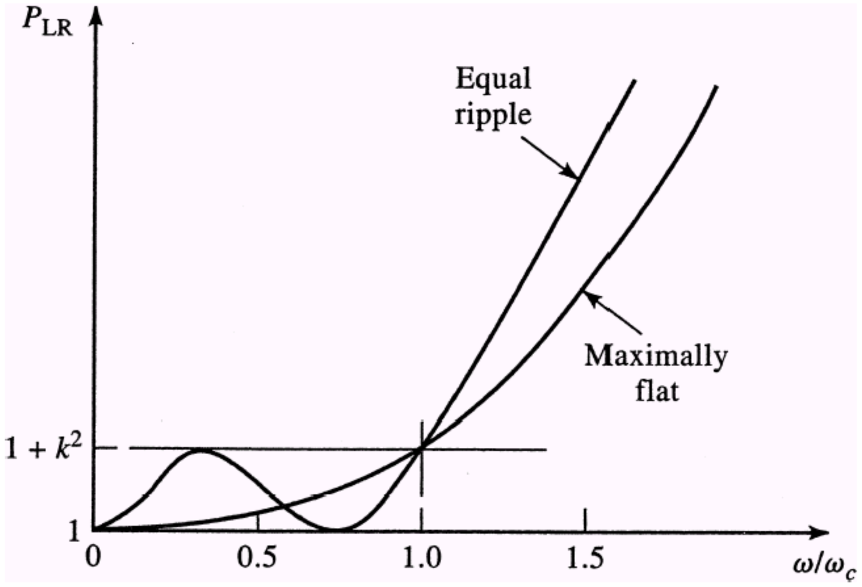
Prototype beginning with shunt element



Type of Responses for n-Section Prototype filter

- Maximally flat or Butterworth
- Equal ripple or Chebyshev
- Elliptic function
- Linear phase

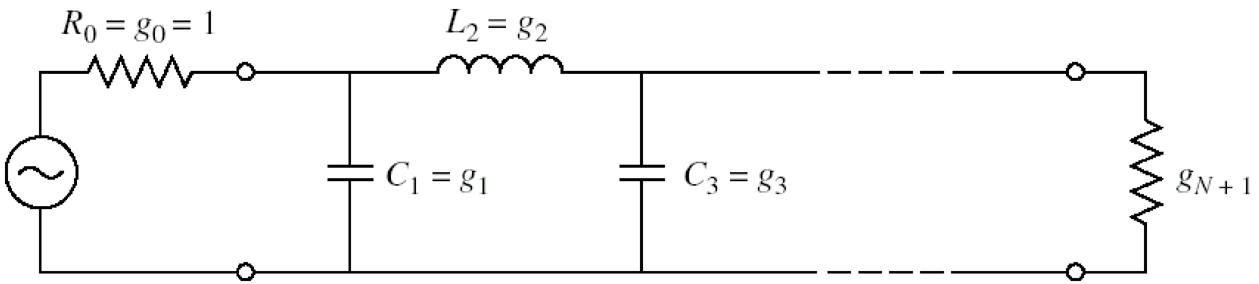




Scaling for low-pass

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

$$C_k = \frac{1}{Z_o g_k \omega_c} \qquad L_k = \frac{Z_o}{g_k \omega_c}$$



Maximally flat or Butterworth filter

For low -pass power ratio response

$$H(\omega)=\left(1+C^2\left(\frac{\omega}{\omega_c}\right)^{2n}\right)^{-1}$$

where

C=1 for -3dB cutoff point

n= order of filter

ω_c= cutoff frequency

No of order (or no of elements)

$$n=\frac{\log_{10}\left(10^{A/10}-1\right)}{2\log_{10}\left(\omega_1/\omega_c\right)}$$

Where A is the attenuation at ω_1 point and $\omega_1>\omega_c$

Prototype elements

$$g_0 = g_{n+1} = 1 \quad \begin{cases} \text{Series } R=Z_o \\ \text{Shunt } G=1/Z_o \end{cases}$$

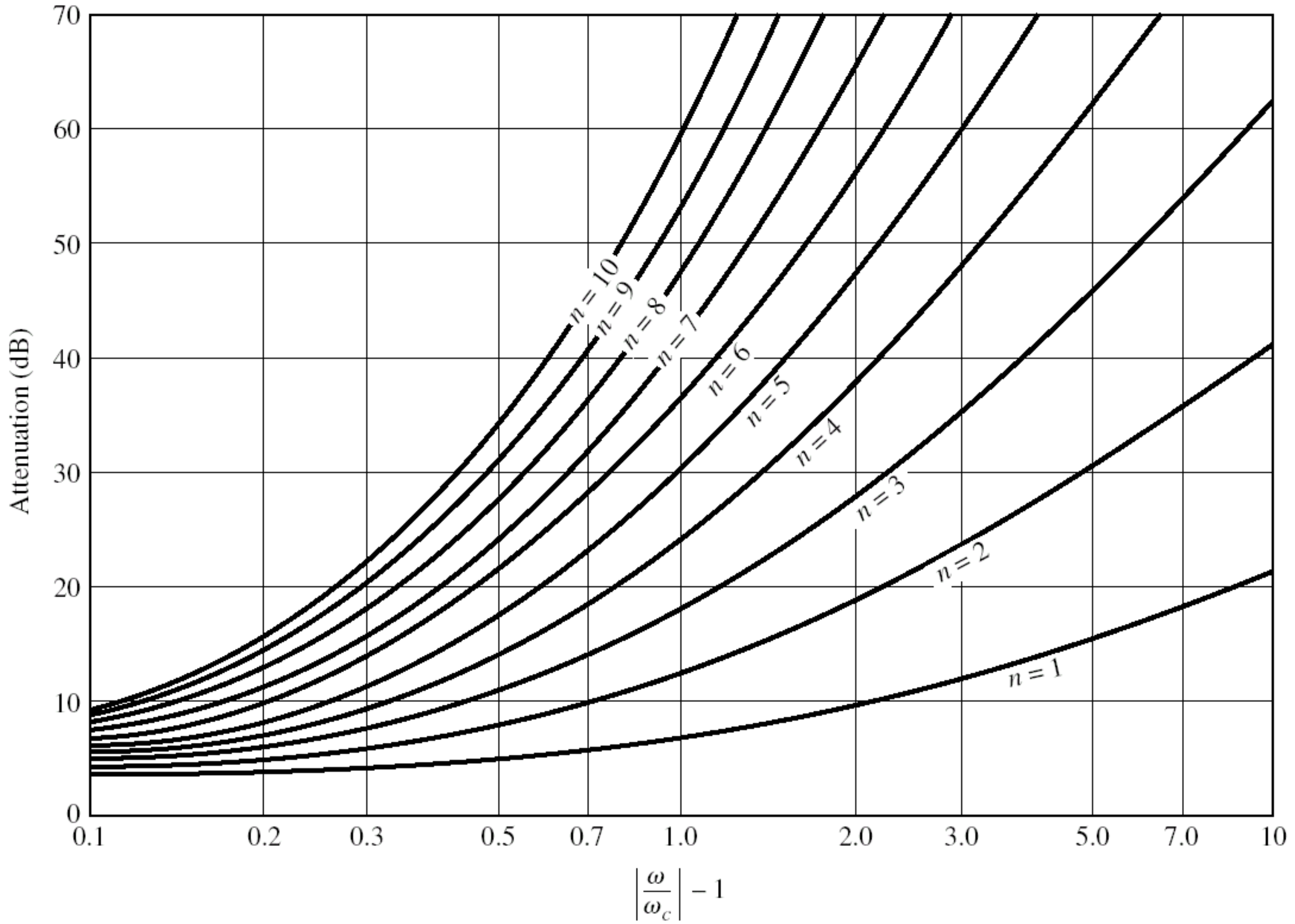
$$g_k = 2\sin\left[\frac{(2k-1)\pi}{2n}\right]$$

$$L_k = \frac{Z_o g_k}{\omega_c}$$ Series element

$$C_k = \frac{g_k}{Z_o\omega_c}$$ Shunt element

k= 1,2,3.......n

Defining the Order of the Maximally flat or Butterworth filter



Example 8

Calculate the inductance and capacitance values for a maximally-flat low-pass filter that has a 3dB bandwidth of 400MHz. The filter is to be connected to 50 ohm source and load impedance. The filter must has a high attenuation of 20 dB at 1 GHz.

Solution

First , determine the number of elements

$$n = \frac{\log_{10}\left(10^{A/10} - 1\right)}{2\log_{10}\left(\omega_1 / \omega_c\right)}$$

$$= \frac{\log_{10}\left(10^{20/10} - 1\right)}{2\log_{10}\left(1000 / 400_c\right)} > 2.51$$

Thus choose an integer value , I.e n=3

Prototype values

$g_0 = g_{3+1} = 1$

$$g_1 = 2\sin\left[\frac{(2-1)\pi}{2\times 3}\right] = 1$$

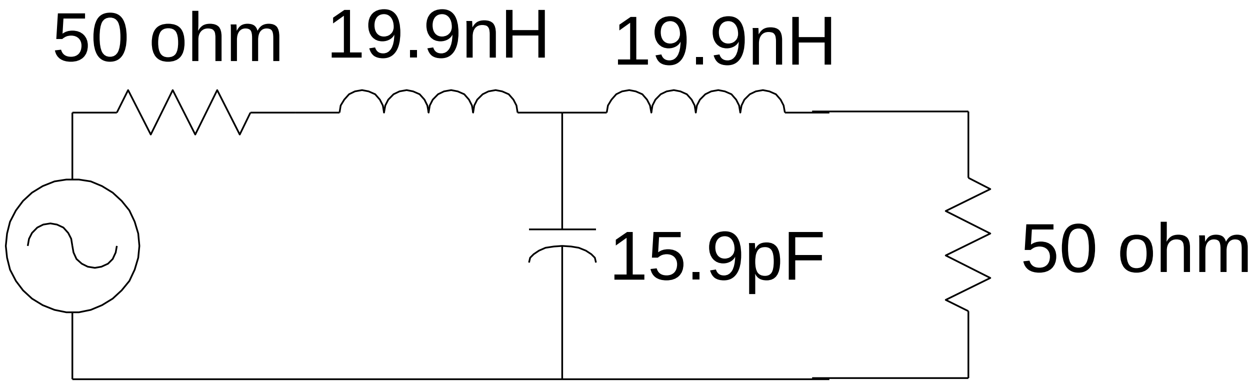
$$g_2 = 2\sin\left[\frac{(2\times 2-1)\pi}{2\times 3}\right] = 2$$

$$g_3 = 2\sin\left[\frac{(2\times 3-1)\pi}{2\times 3}\right] = 1$$

Continue

$$L_3 = L_1 = \frac{Z_o g_1}{\omega_c} = \frac{50\times 1}{2\times\pi\times 400\times 10^6} = 19.9nH$$

$$C_2 = \frac{g_2}{Z_o \omega_c} = \frac{2}{50\times 2\times\pi\times 400\times 10^6} = 15.9\,pF$$

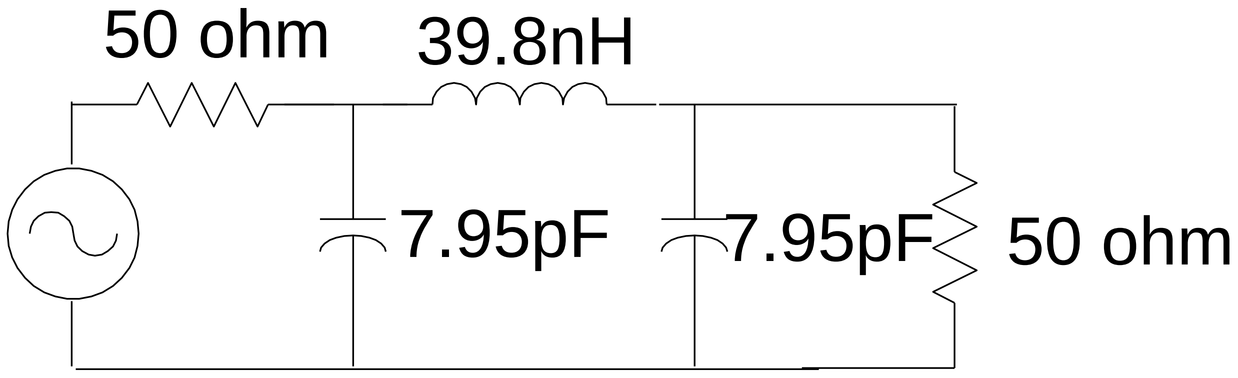


Continue

Or

$$C_3 = C_1 = \frac{g_1}{Z_o\omega_c} = \frac{1}{50\times 2\times \pi \times 400\times 10^6} = 7.95\,pF$$

$$L_2 = \frac{Z_o g_2}{\omega_c} = \frac{50\times 2}{2\times \pi \times 400\times 10^6} = 39.8\,nH$$



Equal-ripple Filter

For low -pass power ratio response

$$H(\omega)=\left(1+F_o C_n{}^2\left(\frac{\omega}{\omega_c}\right)\right)^{-1}$$

where

$C_n(x)$=Chebyshev polynomial for n order

and argument of x where $x=\left(\frac{\omega}{\omega_c}\right)$

n= order of filter

ω_c= cutoff frequency

F_o=constant related to passband ripple

$$F_o=10^{Lr/10}-1$$

Where Lr is the ripple attenuation in pass-band

Chebyshev polynomial

$$C_o(x)=1$$

$$C_1(x)=x$$

$$C_n(x)=2xC_{n-1}(x)\text{-}C_{n\text{-}2}(x)$$

$$C_n(1)=1 \qquad i.e\ \omega=\omega_c$$

Continue

Prototype elements

$$g_0 = 1 \qquad g_1 = \frac{2a_1}{F_2}$$

$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}$$ For k=2,..,n

$$g_{n+1} = \begin{cases} 1 & for\ n\ odd \\ \coth^2(F_1) & for\ n\ even \end{cases}$$

$$L_k = \frac{Z_o g_k}{\omega_c}$$ Series element

$$C_k = \frac{g_k}{Z_o \omega_c}$$ Shunt element

where

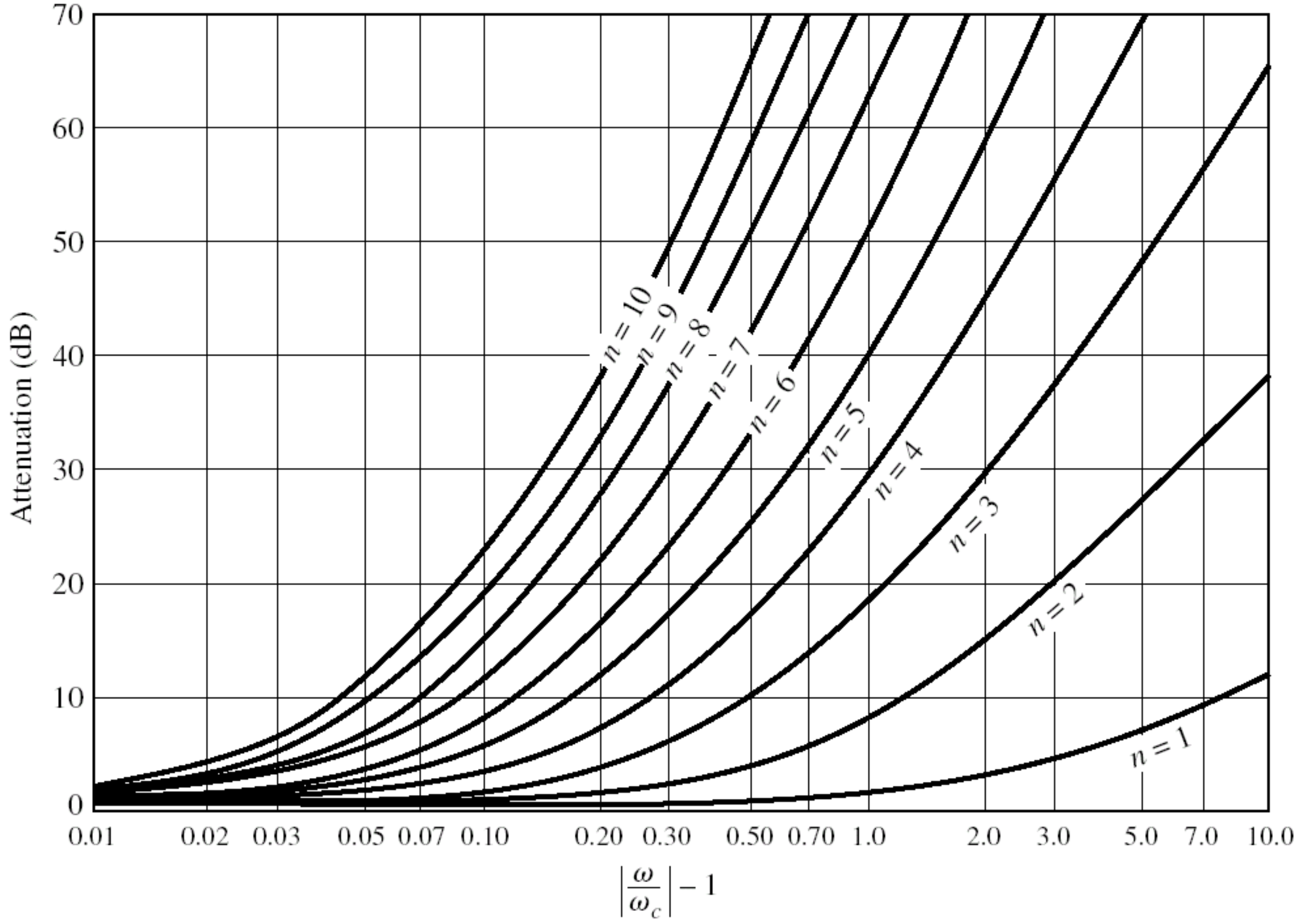
$$F_1 = \frac{1}{4}\ln\left\{\coth\left(\frac{Lr}{17.372}\right)\right\}$$

$$F_2 = \sinh\left(\frac{2F_1}{n}\right)$$

$$a_k = \sin\left\{\frac{(2k-1)\pi}{2n}\right\} \qquad k = 1,2,\dots.n$$

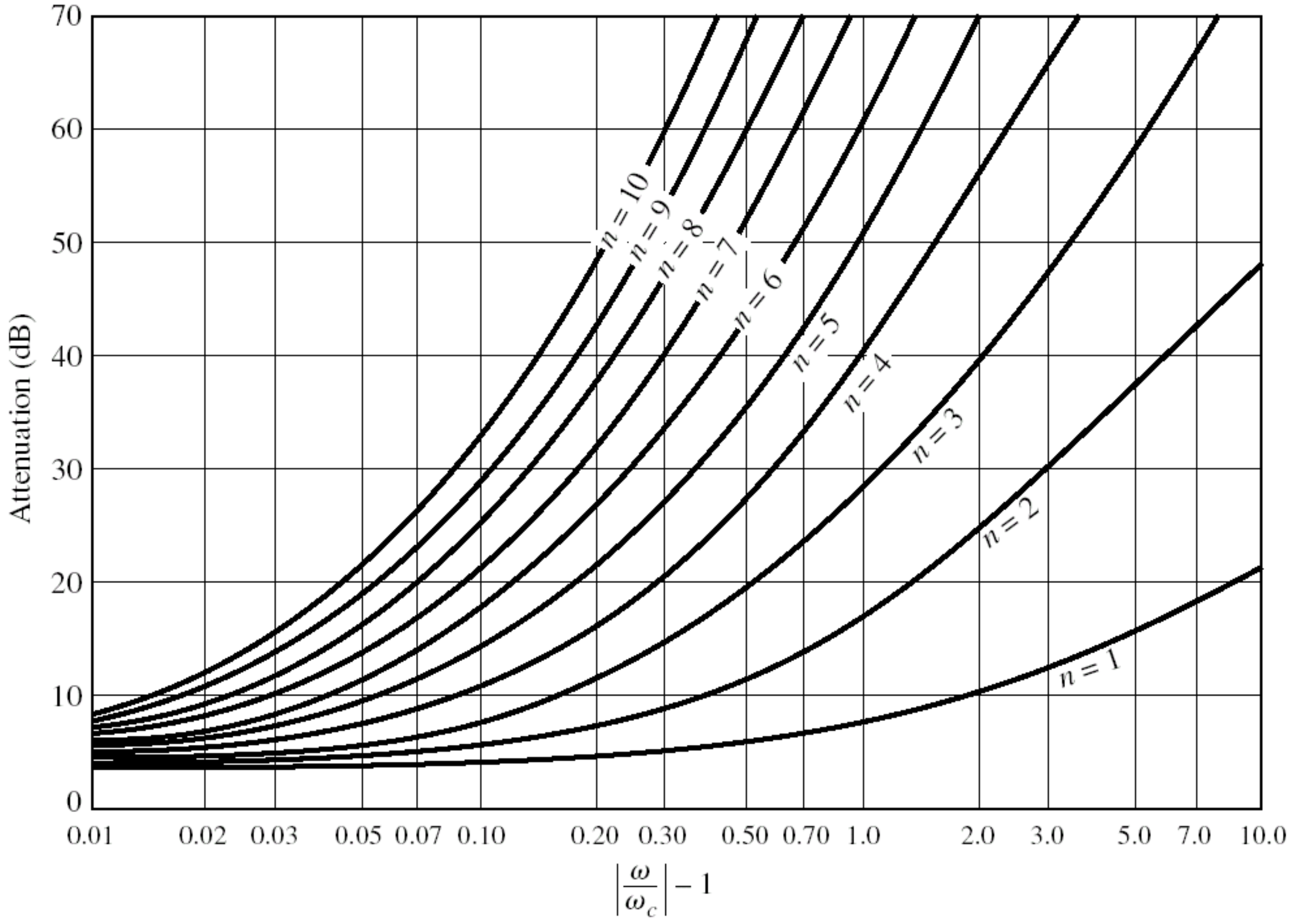
$$b_k = F_2^2 + \sin^2\left(\frac{k\pi}{n}\right) \qquad k = 1,2,\dots.n$$

Equal-ripple Filter
“prototype 0.5 dB ripple level”



(a)

Equal-Ripple Filter
“prototype 3 dB ripple level”



(b)

Example 9

Design a 3 section Chebyshev low-pass filter that has a ripple of 0.05dB and cutoff frequency of 1 GHz.

From the formula given we have

F_1=1.4626 F_2= 1.1371

a_1=0.5 a_2=1.0 a_3=0.5

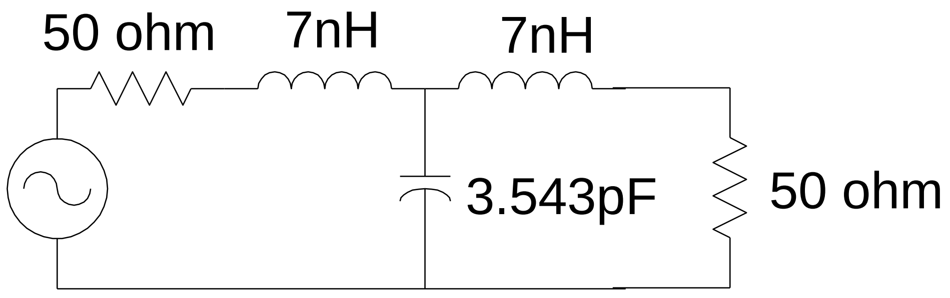
b_1=2.043 b_2=2.043

$g_1 = g_3 = 0.8794$

g_2= 1.1132

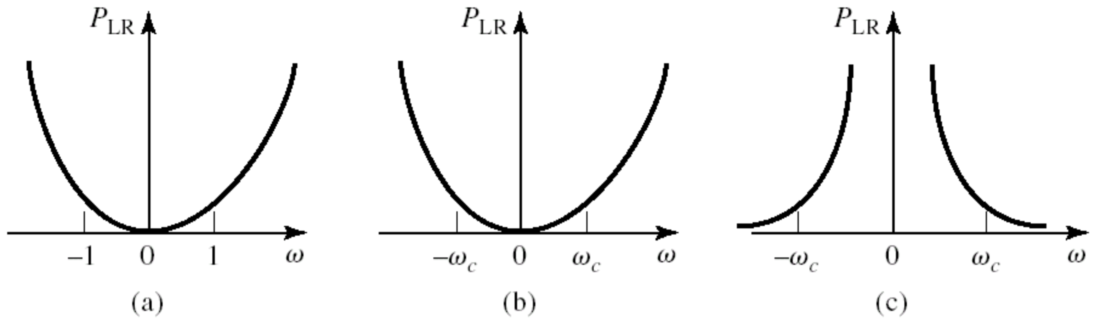
$$L_1 = L_3 = \frac{50 \times 0.8794}{2\pi \times 10^9} = 7\,nH$$

$$C_2 = \frac{1.1132}{50 \times 2\pi \times 10^9} = 3.543\,pF$$



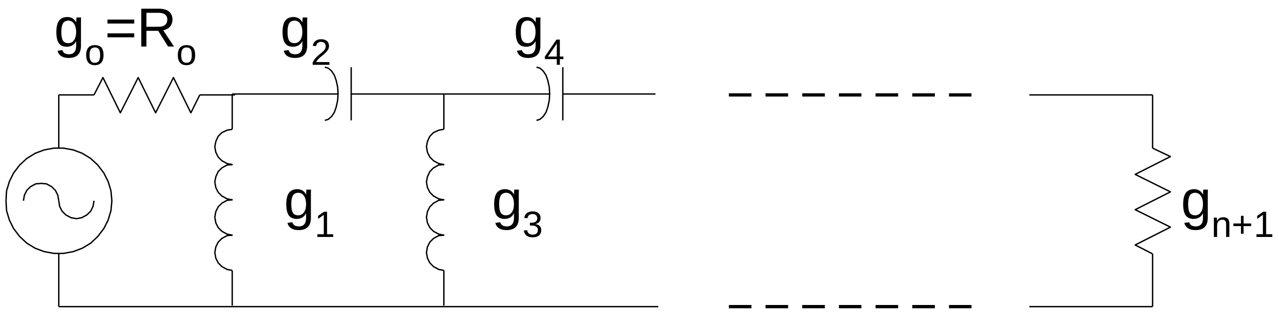
Transformation from Low-Pass to High-Pass

$$\omega \longleftarrow -\frac{\omega_c}{\omega}$$



- Series inductor L_k must be replaced by capacitor C'_k
- Shunts capacitor C_k must be replaced by inductor L'_k

$$C_k = \frac{1}{Z_o g_k \omega_c} \qquad L_k = \frac{Z_o}{g_k \omega_c} \qquad L_k^{'} = \frac{R_o}{\omega_c C_k} \quad C_k^{'} = \frac{1}{R_o \omega_c L_k}$$



Transformation from Low-Pass to Band-Pass

$$\omega \leftarrow \frac{1}{\Omega}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)$$ where $$\Omega = \frac{\omega_2 - \omega_1}{\omega_o}$$ and $$\omega_o = \sqrt{\omega_1 \omega_2}$$

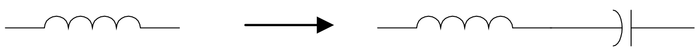
Now we consider the series inductor

$$jX = j\frac{1}{\Omega}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)L_k = j\frac{1}{\Omega}\frac{\omega}{\omega_o}L_k - j\frac{1}{\Omega}\frac{\omega_o}{\omega}L_k = j\omega L'_{sk} - \frac{j}{\omega C'_{sk}}$$

$$L_{sk} = \frac{L_k}{\Omega\omega_o} \qquad C_{sk} = \frac{\Omega}{\omega_o L_k} \qquad L_k = Z_o g_k$$

normalized

- Thus , series inductor L_k must be replaced by serial L_{sk} and C_{sk}



Impedance= series

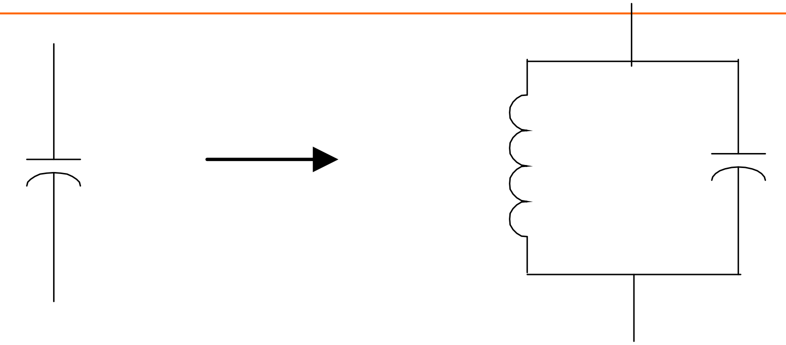
Continue

Now we consider the shunt capacitor

$$jB_k = j\frac{1}{\Omega}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)C_k = j\frac{1}{\Omega}\frac{\omega}{\omega_o}C_k - j\frac{1}{\Omega}\frac{\omega_o}{\omega}C_k = j\omega C'_{pk} - \frac{j}{\omega L'_{pk}}$$

$$L_{pk} = \frac{\Omega}{\omega_o C_k} \qquad C_{pk} = \frac{C_k}{\Omega\omega_o} \qquad C_k = \frac{g_k}{Z_o}$$

- Shunts capacitor C_k must be replaced by parallel L_{pk} and C_{pk}



Admittance= parallel

Transformation from low-pass to band-stop

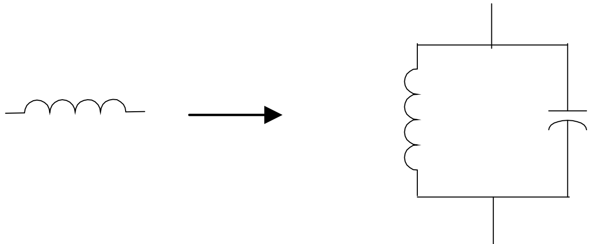
$$\omega \leftarrow -\Omega\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^{-1} \quad \text{where } \Omega = \frac{\omega_2 - \omega_1}{\omega_o} \quad \text{and } \omega_o = \sqrt{\omega_1 \omega_2}$$

Now we consider the series inductor --convert to admittance

$$j\frac{1}{X_k} = j\frac{1}{\Omega L_k}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) = j\frac{1}{\Omega}\frac{\omega}{\omega_o L_k} - j\frac{1}{\Omega}\frac{\omega_o}{\omega L_k} = j\omega C'_{pk} - \frac{j}{\omega L'_{pk}}$$

$$L_{pk} = \frac{\Omega L_k}{\omega_o} \qquad C_{pk} = \frac{1}{\omega_o \Omega L_k} \qquad L_k = Z_o g_k$$

- Thus , series inductor L_k must be replaced by parallel L_{pk} and C_{kp}



admittance = parallel

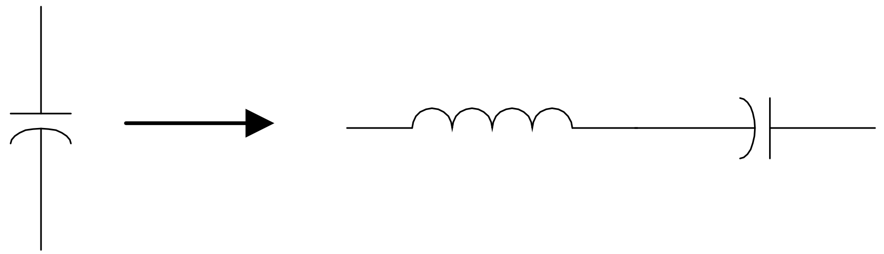
Continue

Now we consider the shunt capacitor --> convert to impedance

$$j\frac{1}{B_k} = j\frac{1}{\Omega C_k}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) = j\frac{1}{\Omega}\frac{\omega}{\omega_o C_k} - j\frac{1}{\Omega}\frac{\omega_o}{\omega C_k} = j\omega L'_{sk} - \frac{j}{\omega C'_{sk}}$$

$$L_{sk} = \frac{1}{\Omega \omega_o C_k} \qquad C_{sk} = \frac{\Omega C_k}{\omega_o} \qquad C_k = \frac{g_k}{Z_o}$$

- Shunts capacitor C_k must be replaced by series L_{sk} and C_{sk}



Example 10

Design a band-pass filter having a 0.5 dB ripple response, with N=3. The center frequency is 1GHz, the bandwidth is 10%, and the impedance is 50Ω.

Solution

From table 8.4 Pozar pg 406.

$g_o=1$, $g_1=1.5963$, $g_2=1.0967$, $g_3= 1.5963$, $g_4= 1.000$

Let's first and third elements are equivalent to series inductance and $g_1=g_3$, thus

$$L_{s1} = L_{s3} = \frac{Z_o g_1}{\Omega \omega_o} = \frac{50\times 1.5963}{0.1\times 2\pi \times 10^9} = 127nH$$

$$\boxed{L_k = Z_o g_k}$$

$$C_{s1} = C_{s3} = \frac{\Omega}{\omega_o Z_o g_1} = \frac{0.1}{2\pi \times 10^9 \times 50\times 1.5963} = 0.199pF$$

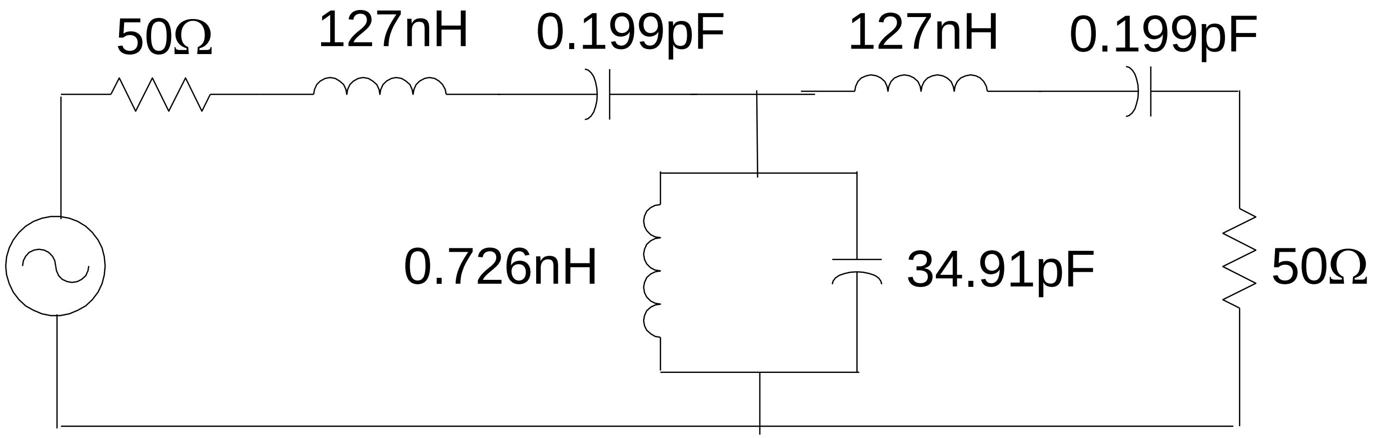
Continue

Second element is equivalent to parallel capacitance, thus

$$\boxed{C_k = \frac{g_k}{Z_o}}$$

$$L_{p2} = \frac{\Omega Z_o}{\omega_o g_2} = \frac{0.1 \times 50}{2\pi \times 10^9 \times 1.0967} = 0.726nH$$

$$C_{p2} = \frac{g_2}{Z_o \Omega \omega_o} = \frac{1.0967}{50 \times 0.1 \times 2\pi \times 10^9} = 34.91pF$$



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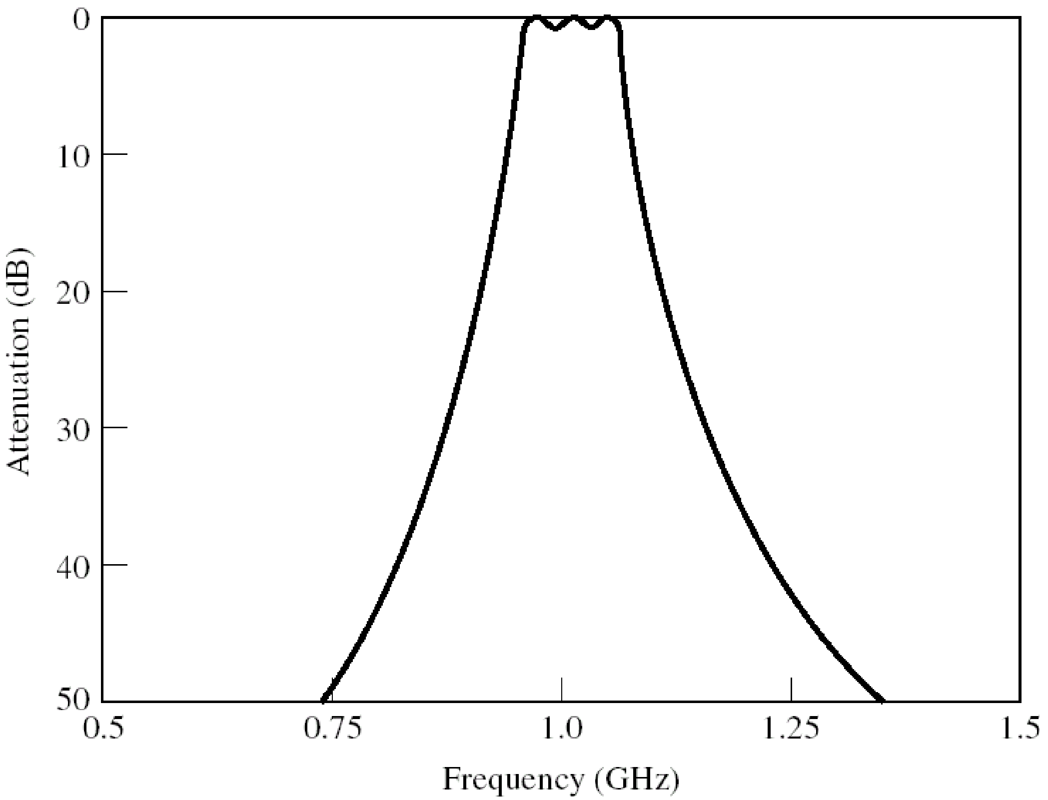
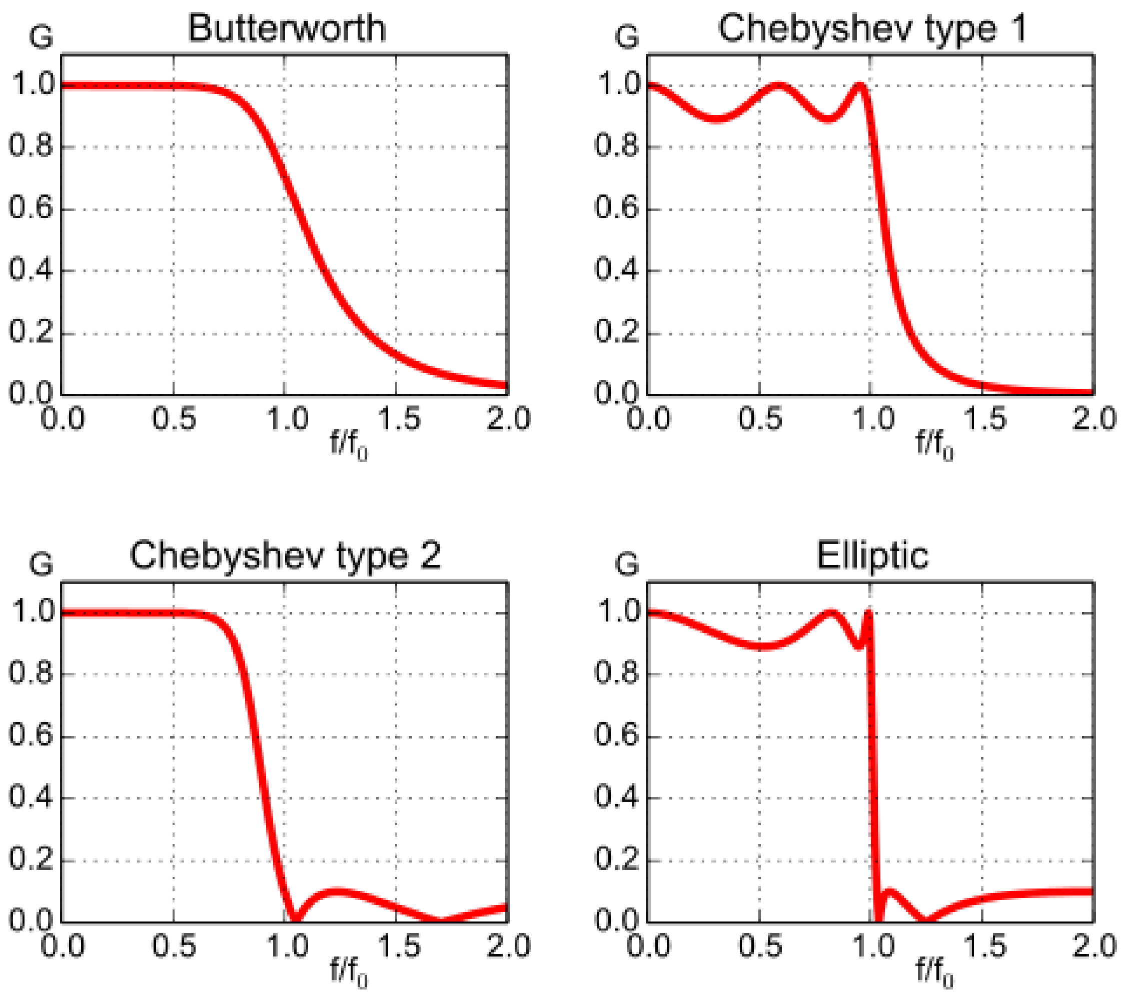
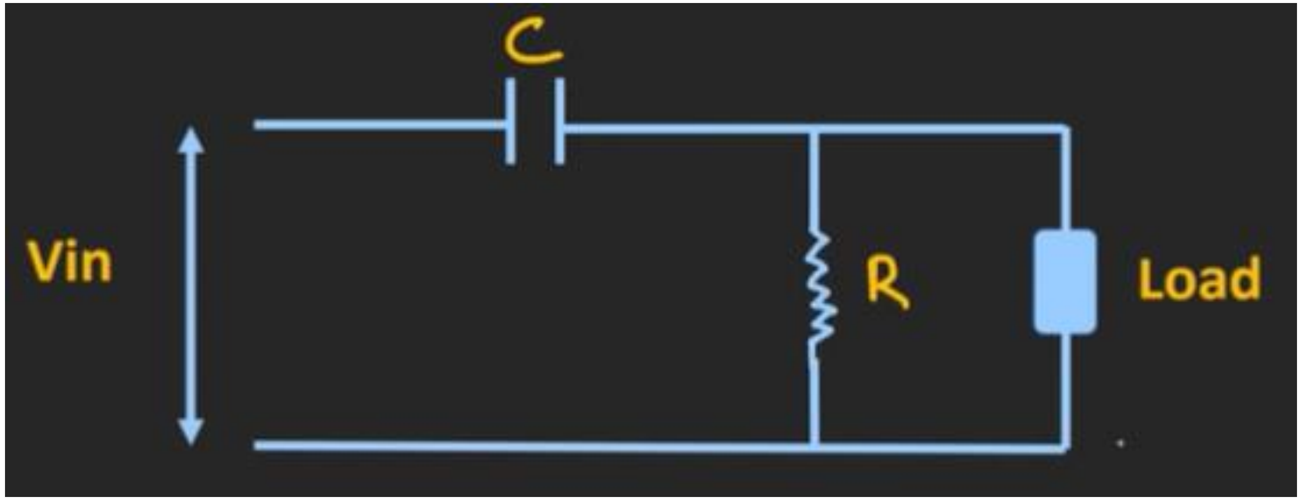


Comparison of Filter Reponses



Type	Passband	Stopband	Roll-off	Step Response
Butterworth	Flat	Monotonic	Good	Good
Chebyshev	Rippled	Monotonic	Very Good	Poor
Inverse Chebyshev	Flat	Rippled	Very Good	Good
Elliptic	Rippled	Rippled	Best	Poor

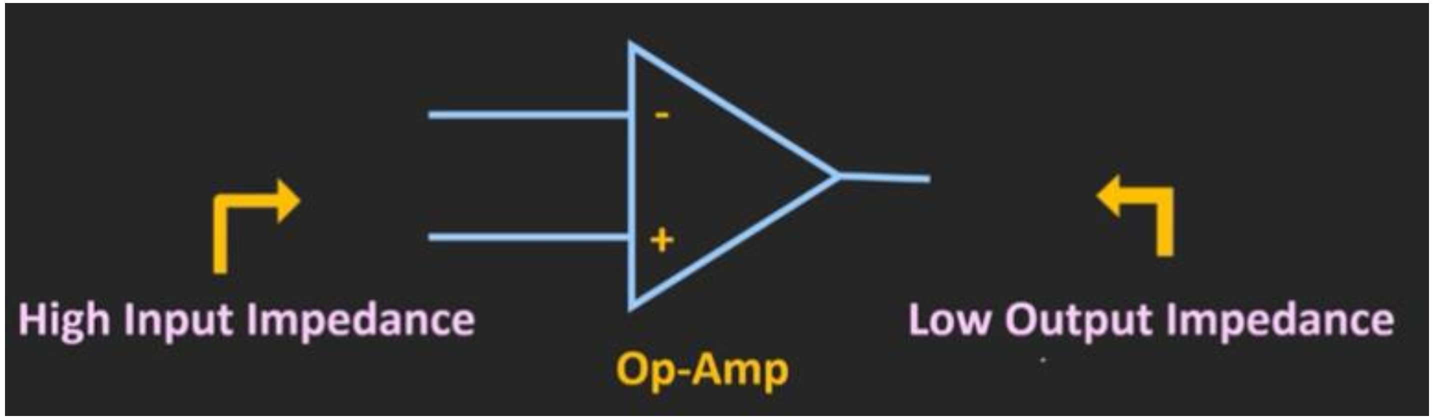
Active vs. Passive Filters



- The maximum magnitude of the voltage transfer function of a passive filter is at best equal to 1.
- Active filters incorporate amplifiers with the filter elements so that the filter has gain.
- The load in passive filters will impact the value of R. For example in this high pass filter, the load will alter the value of R and hence the cut-off frequency

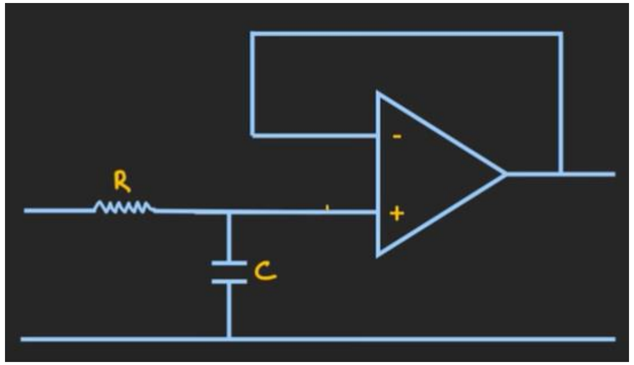
Active Low Pass Filters

- The Op-Amp has a high input impedance and low output impedance.



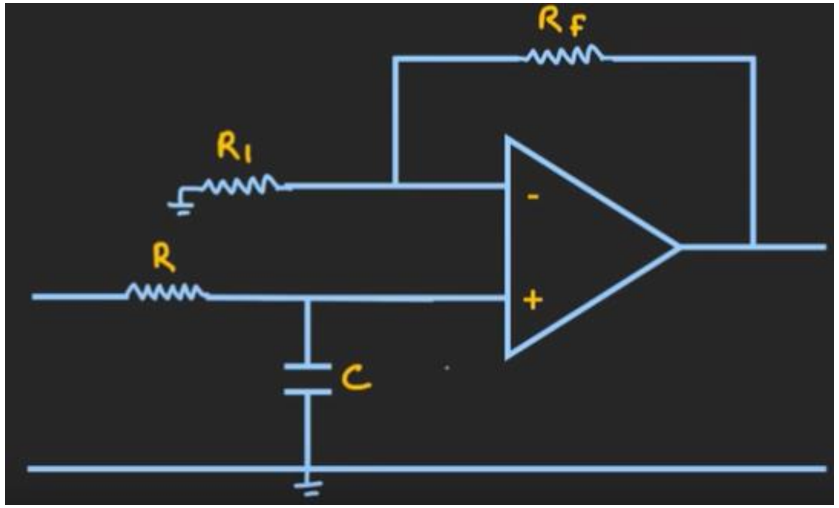
So it can be used as a buffer to isolate the different stages of the filter.

But this configuration has no gain



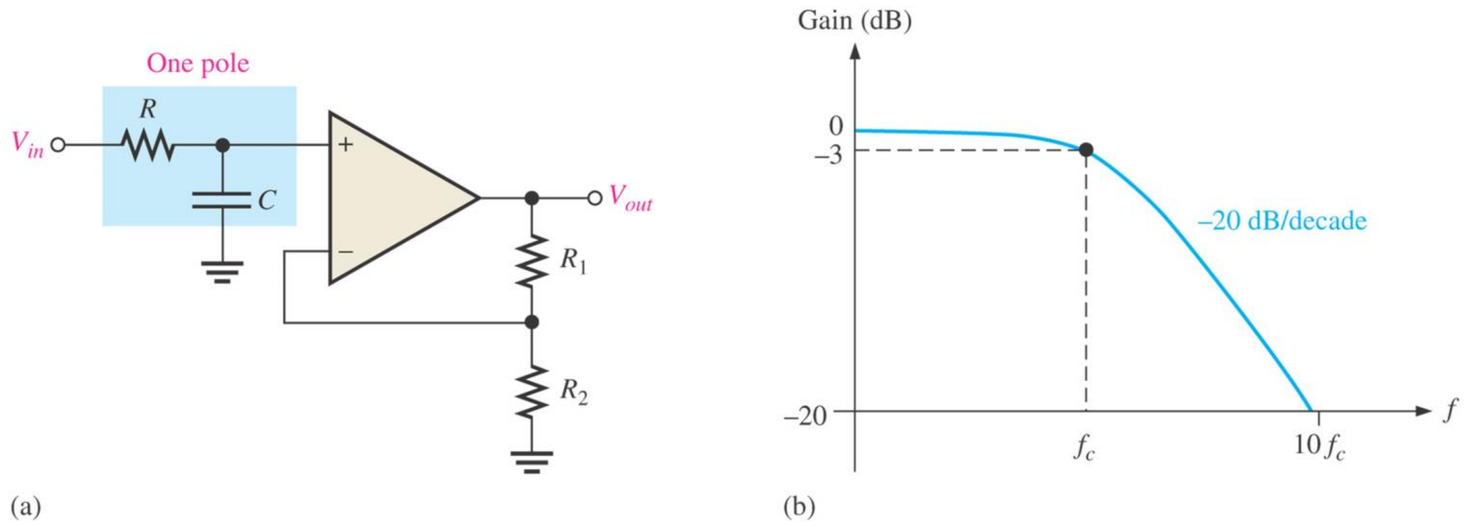
Active Low Pass Filters

- To have a gain for the filter, the Op-Amp can be connected as a negative feedback with positive gain (non-inverting amplifier) as shown below:



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+\left(\frac{f}{fc}\right)^2}} \times \left(1+\frac{R_f}{R_1}\right)$$

Active Low Pass Filters



One-pole (first-order) low-pass filter.

➢ The critical frequency, f_c is determined by the values of **R** and **C** in the frequency-selective RC circuit.

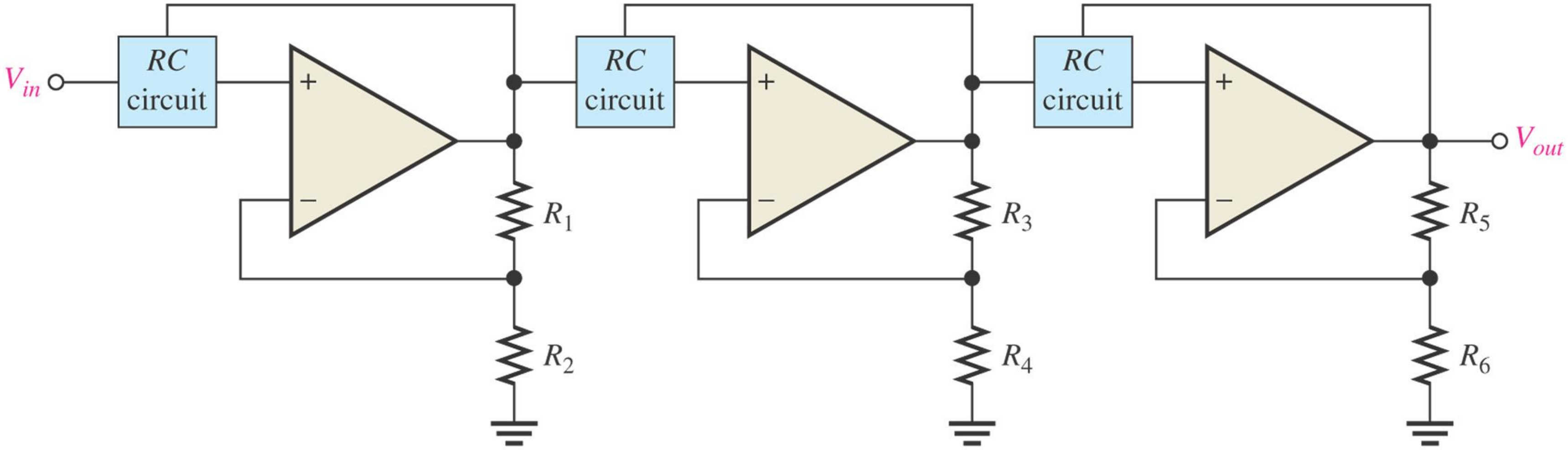
$$\boxed{f_c = \frac{1}{2\pi RC}}$$

➢Each RC set of filter components represents a pole.

➢ This filter provides a roll-off rate of -20 dB/decade above the critical frequency. Greater roll-off rates can be achieved with more poles.

Active Low Pass Filters

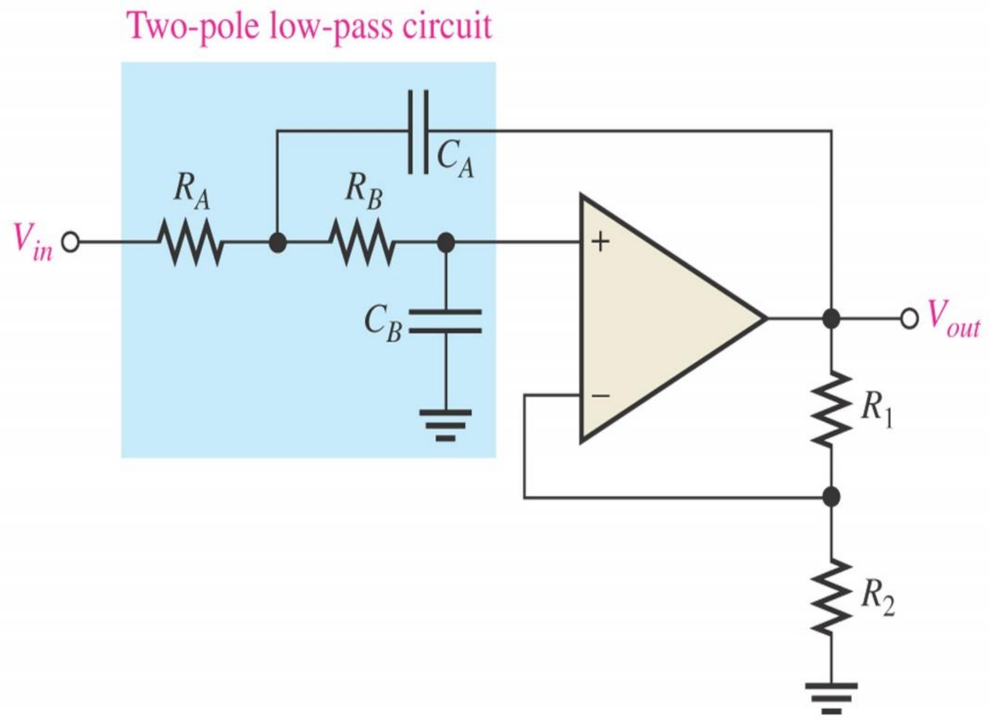
➢ The number of filter poles can be increased by ***cascading***. To obtain a filter with three poles, cascade a two-pole with one-pole filters.



Three-pole (third-order) low-pass filter.

Sallen-Key Low-Pass Filter

- Sallen-Key is one of the most common configurations for a second order (two-pole) filter.



Basic Sallen-Key low-pass filter.

- There are two low-pass RC circuits that provide a roll-off of -40 dB/decade above f_c.
- One RC circuit consists of R_A and C_A, and the second circuit consists of R_B and C_B.

Sallen-Key Low-Pass Filter

- The critical frequency for the Sallen-Key filter is :

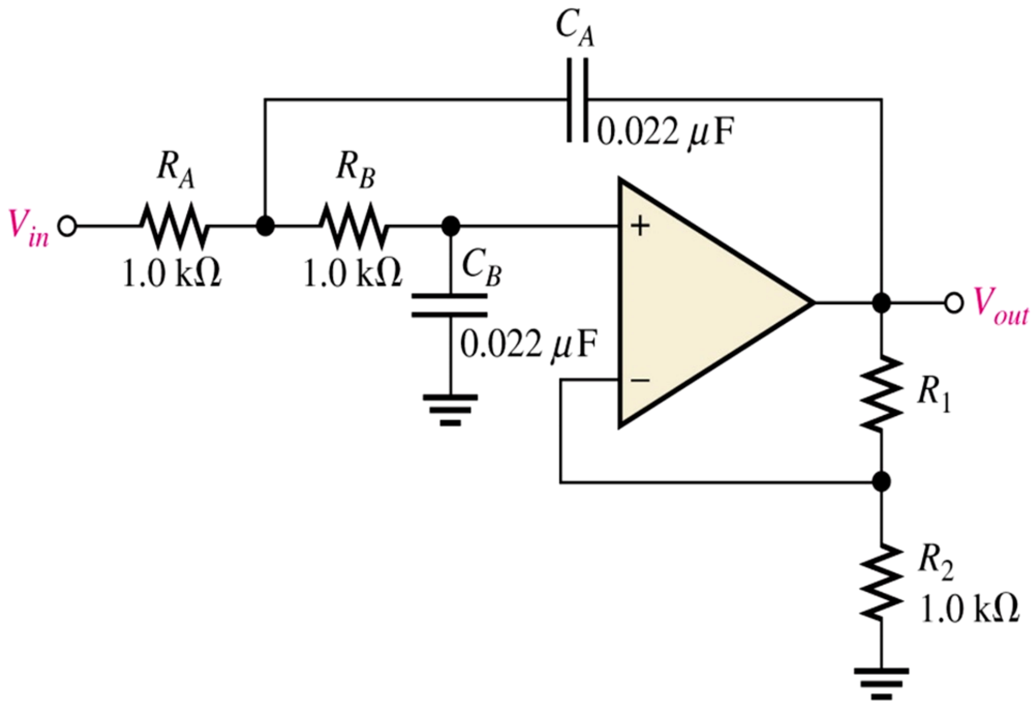
$$\boxed{f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}}$$

- For $R_A = R_B = R$ and $C_A = C_B = C$, thus the critical frequency :

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

Example 11

- Determine critical frequency
- Set the value of R_1 for Butterworth response by giving that Butterworth response for second order is 0.586



- Critical frequency

$$\boxed{f_c = \frac{1}{2\pi RC} = 7.23kHz}$$

- Butterworth response given

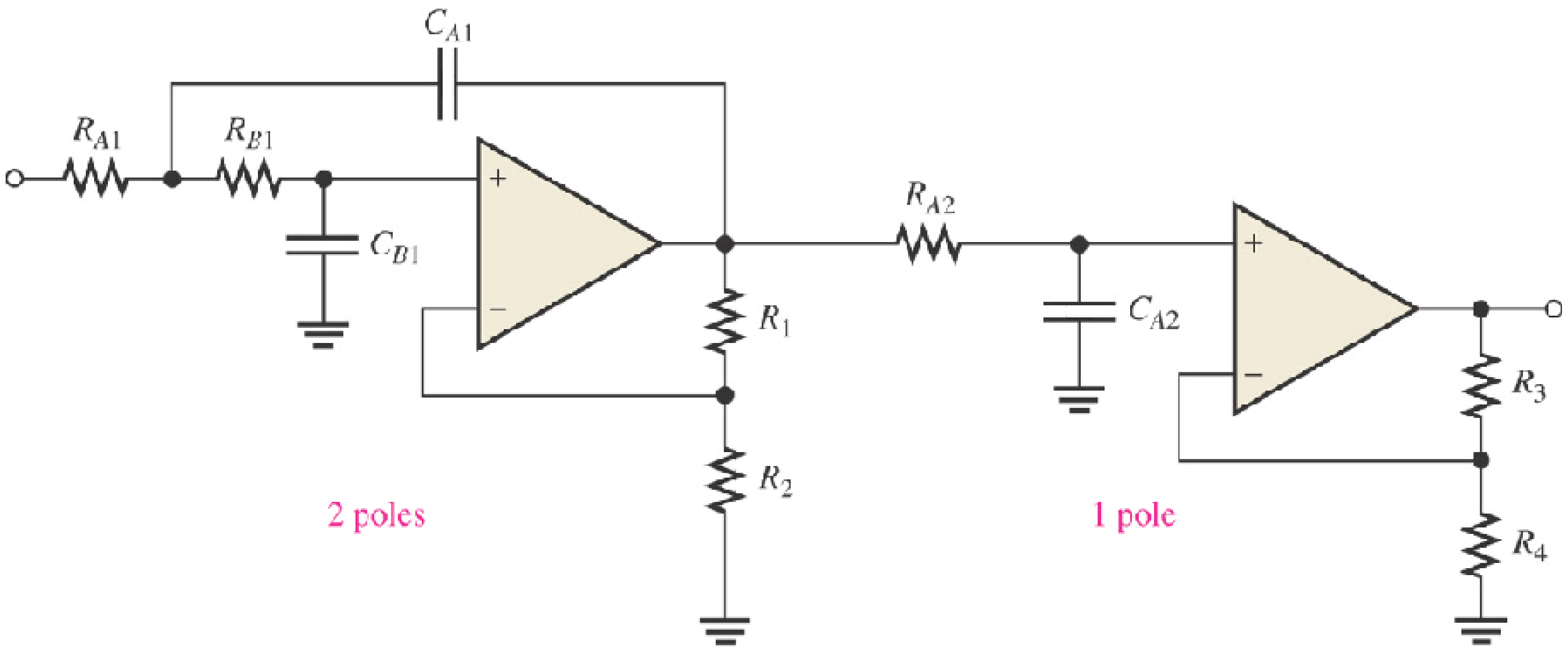
  $R_1/R_2 = 0.586$

$$\boxed{R_1 = 0.586R_2}$$

$$\boxed{R_1 = 586k\Omega}$$

Cascading Low-Pass Filter

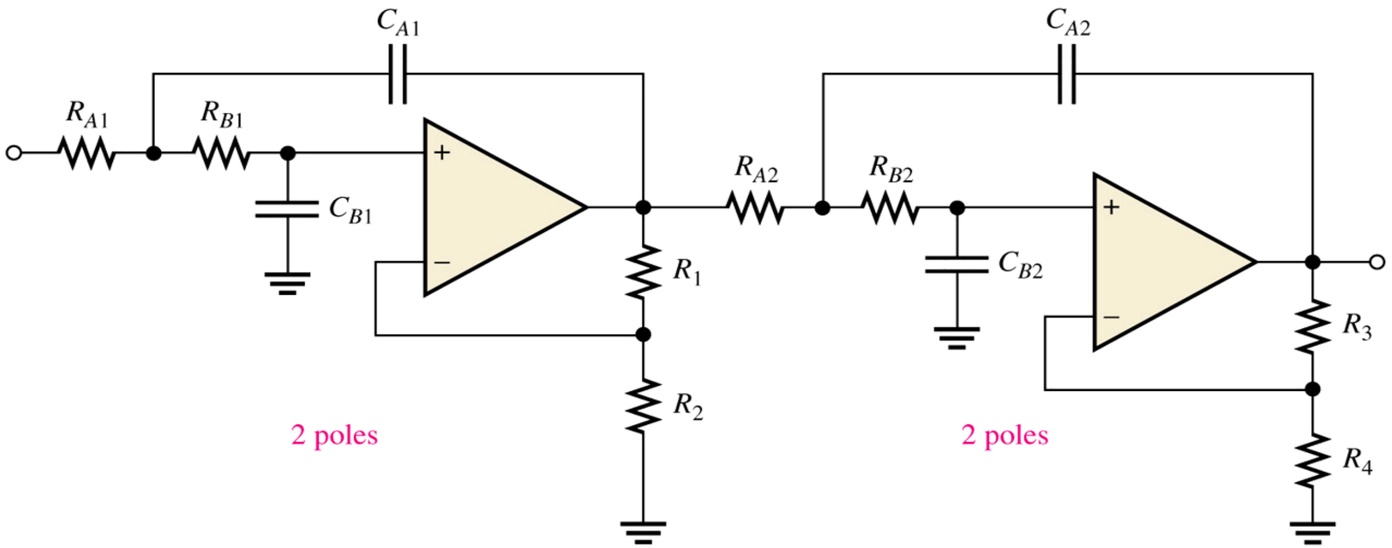
➢ A three-pole filter is required to provide a roll-off rate of -60 dB/decade. This is done by cascading a two-pole Sallen-Key low-pass filter and a single-pole low-pass filter.



Cascaded low-pass filter: third-order configuration.

Cascading Low-Pass Filter

➢ A four-pole filter is required to provide a roll-off rate of -80 dB/decade. This is done by cascading a two-pole Sallen-Key low-pass filter and a two-pole Sallen-Key low-pass filter.

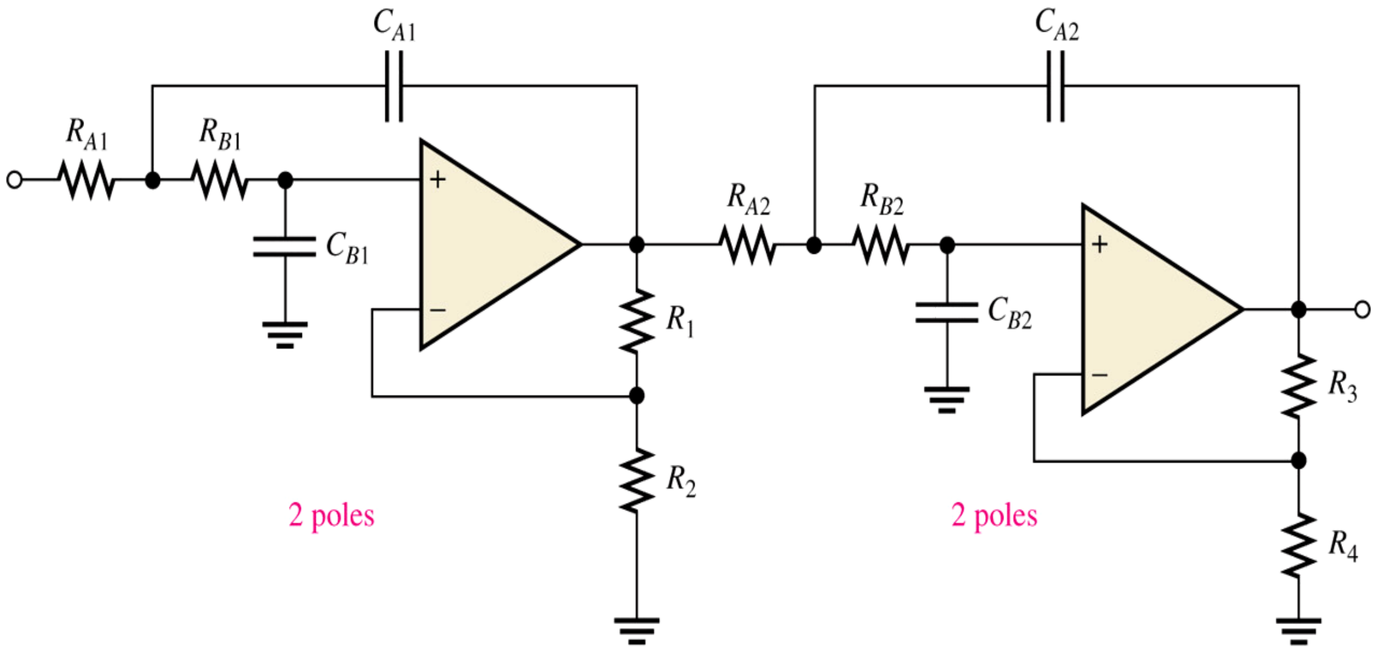


(b) Fourth-order configuration

Cascaded low-pass filter: fourth-order configuration.

Example 12

- Determine the capacitance values required to produce a critical frequency of 2680 Hz if all resistors in RC low pass circuit is 1.8kΩ



(b) Fourth-order configuration

$$f_c = \frac{1}{2\pi RC}$$

$$C = \frac{1}{2\pi f_c R} = 0.033\mu F$$

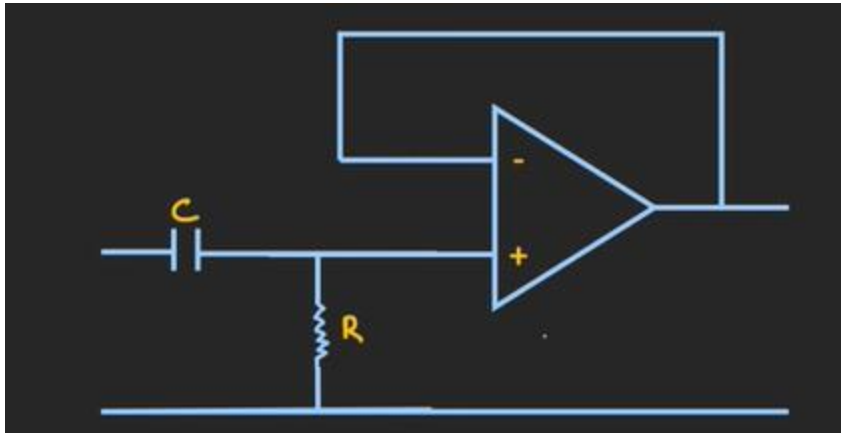
$$C_{A1}=C_{B1}=C_{A2}=C_{B2}=0.033\mu f$$

- Both stages must have the same f_c. Assume equal-value of capacitor

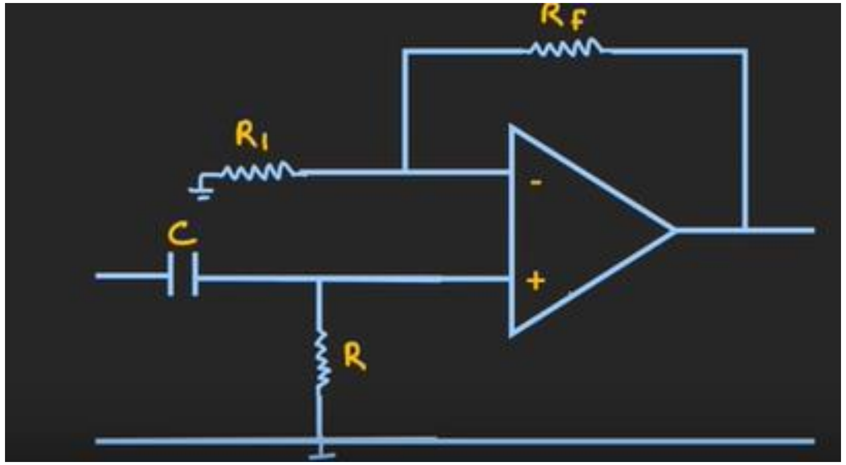
Active High Pass Filters

- Similarly to the design of low pass active filter, we can design high pass active filter.

High pass filter with no gain.



High pass filter with gain.



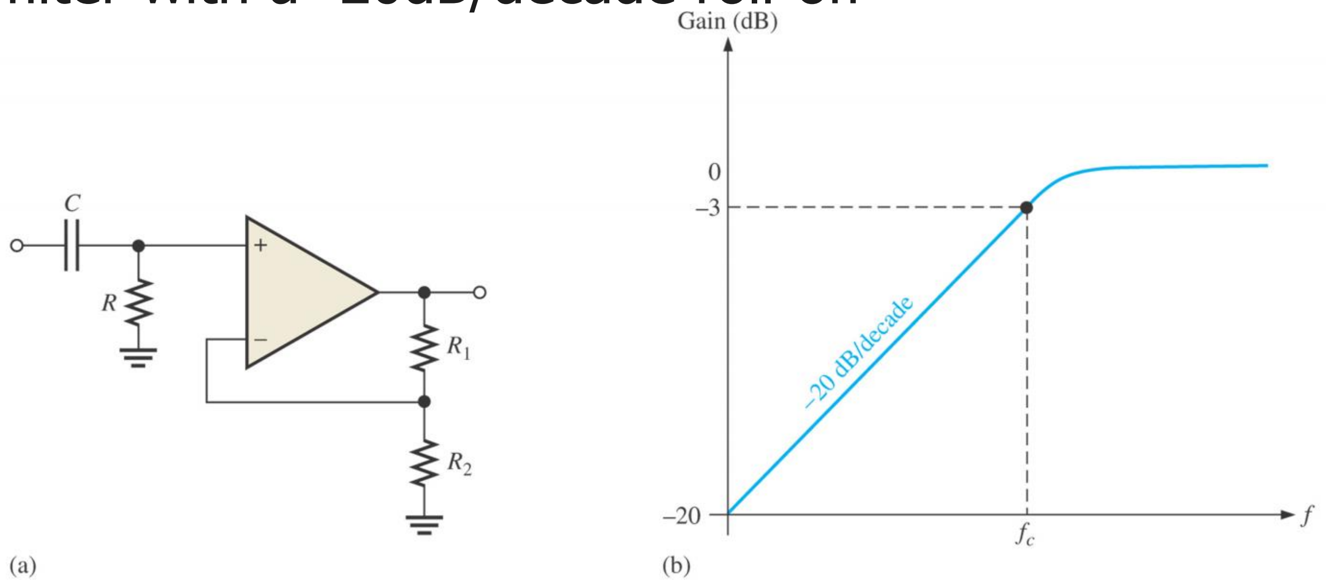
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\left(\frac{f}{fc}\right)}{\sqrt{1+\left(\frac{f}{fc}\right)^2}} \times \left(1+\frac{R_f}{R_1}\right)$$

Single-Pole High Pass Filter

- In high-pass filters, the roles of the capacitor and resistor are reversed in the RC circuits as shown from Figure (a). The negative feedback circuit is the same as for the low-pass filters.
- Figure (b) shows a high-pass active filter with a -20dB/decade roll-off

- The critical frequency of the single-pole filter is :

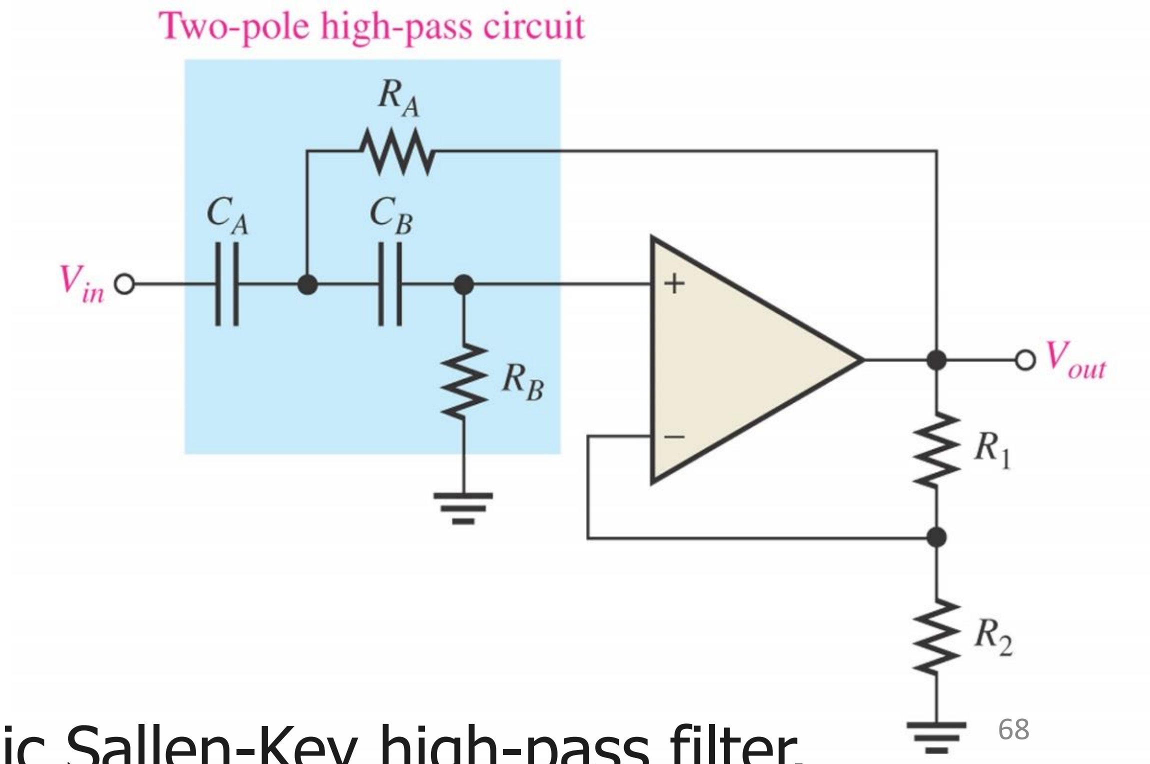
$$\boxed{f_c = \frac{1}{2\pi RC}}$$



Single-pole active high-pass filter and response curve.

Sallen-Key High-Pass Filter

- Components R_A, C_A, R_B, and C_B form the second order (two-pole) frequency-selective circuit.
- The position of the resistors and capacitors in the frequency-selective circuit are opposite in low pass configuration.
- There are two high-pass RC circuits that provide a roll-off of -40 dB/decade above fc
- The response characteristics can be optimized by proper selection of the feedback resistors, R_1 and R_2.



Basic Sallen-Key high-pass filter.

Sallen-Key High-Pass Filter

- The critical frequency for the Sallen-Key filter is :

$$\boxed{f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}}$$

- For R_A = R_B = R and C_A = C_B = C, thus the critical frequency :

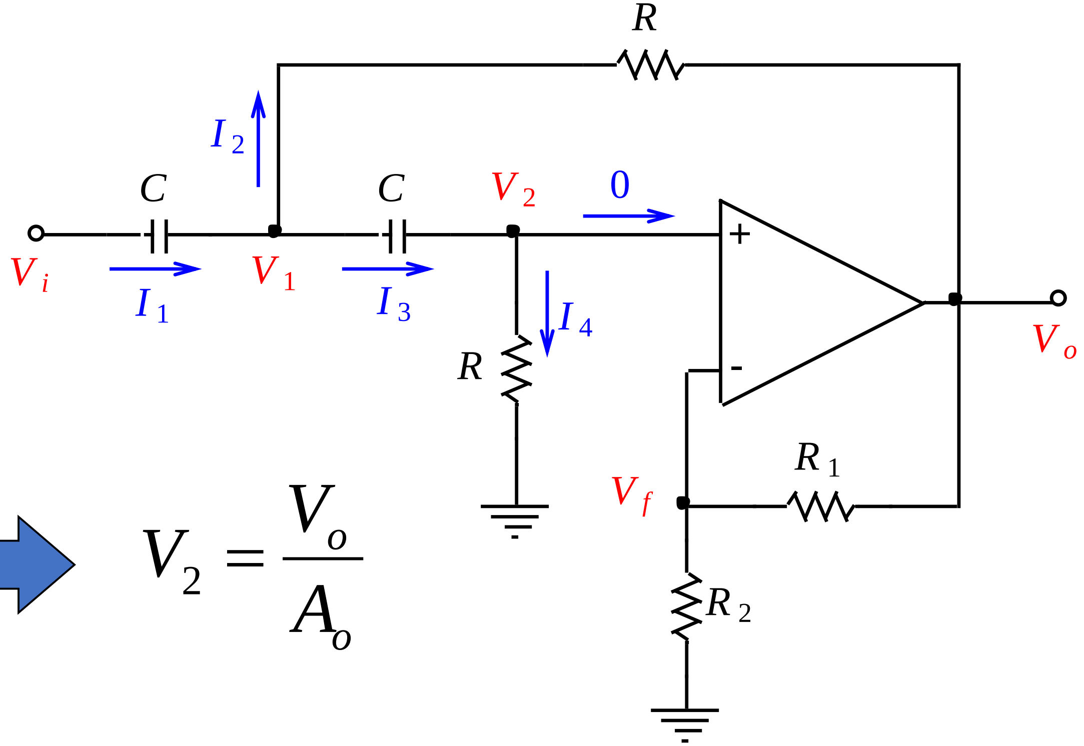
$$\boxed{f_c = \frac{1}{2\pi RC}}$$

Sallen-Key High-Pass Filter

$$V_2 = V_f$$

$$V_f = \left(\frac{R_2}{R_1 + R_2}\right) V_o$$

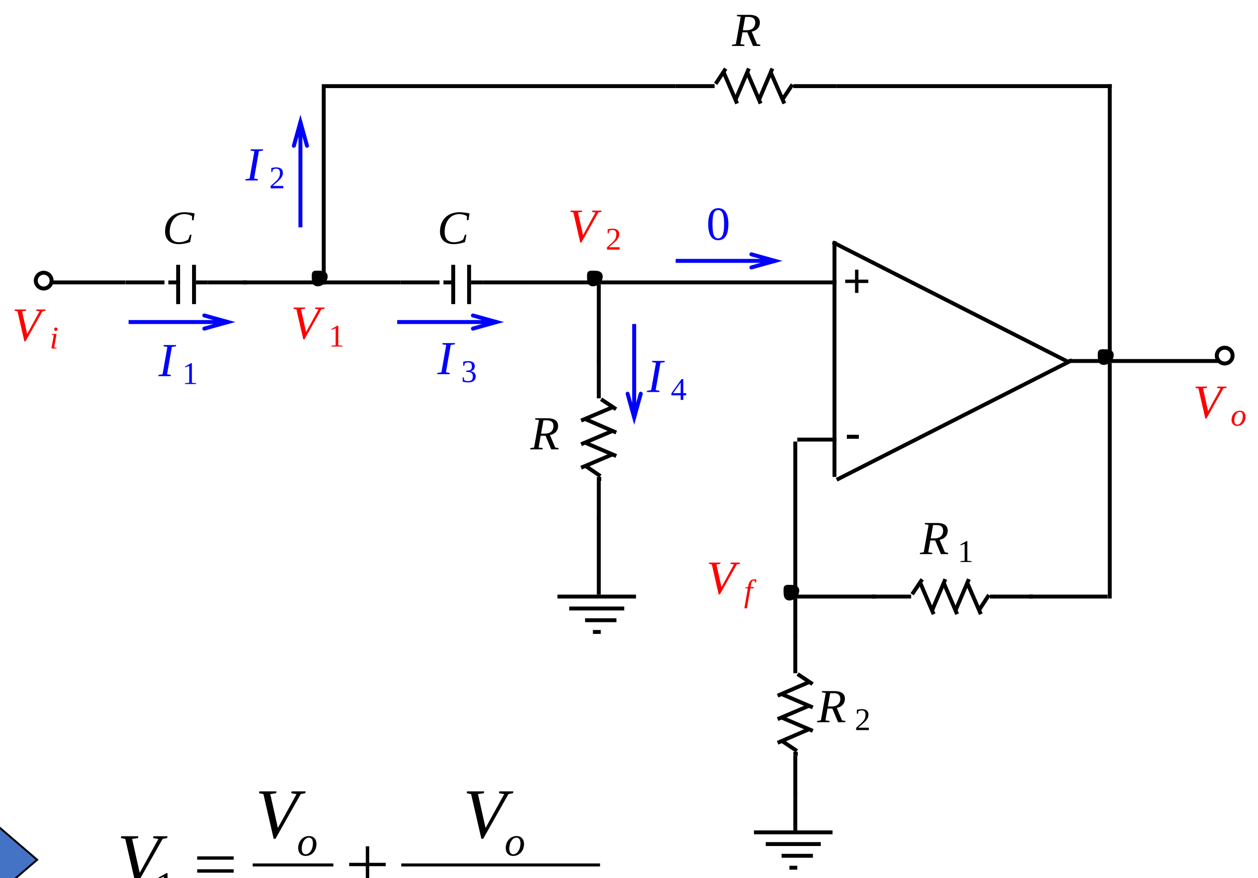
$$A_o \equiv \frac{V_o}{V_2} = \frac{V_o}{V_f} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2} \quad \Rightarrow \quad V_2 = \frac{V_o}{A_o}$$



At node V_1 $\quad I_1 = I_2 + I_3$

$$sC(V_i - V_1) = \frac{V_1 - V_o}{R} + sC(V_1 - V_2) \quad \Rightarrow \quad sC(V_i - V_1) = \frac{V_1 - V_o}{R} + sC\left(V_1 - \frac{V_o}{A_o}\right)$$

Sallen-Key High-Pass Filter



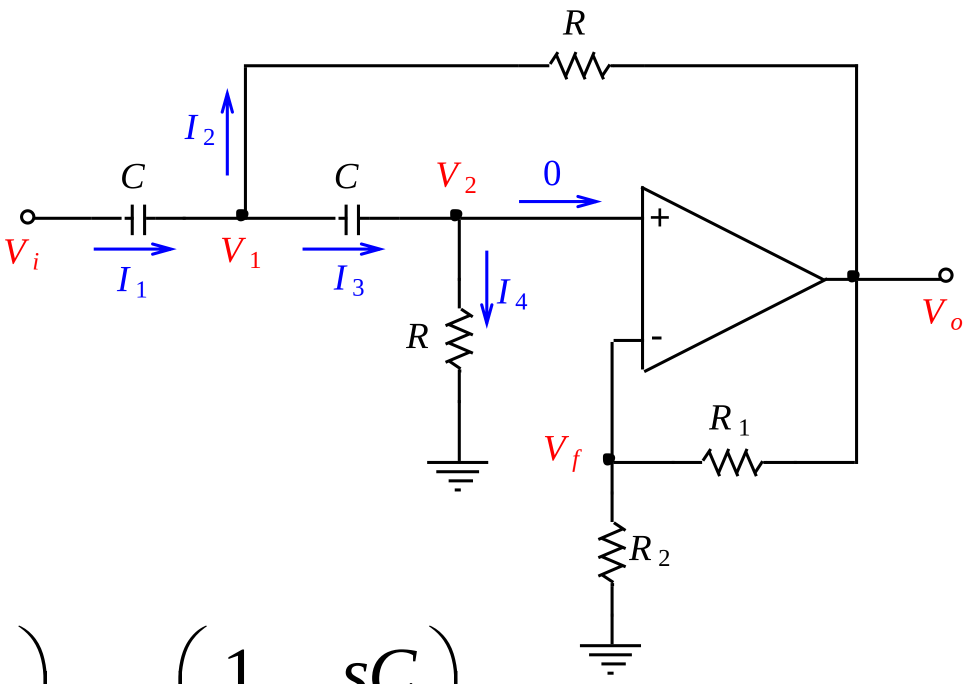
At node V_2

$$I_3 = I_4$$

$$sC(V_1 - V_2) = \frac{V_2}{R}$$

$$sC\left(V_1 - \frac{V_o}{A_o}\right) = \frac{V_o}{A_o R} \quad \Rightarrow \quad V_1 = \frac{V_o}{A_o} + \frac{V_o}{A_o sCR}$$

Sallen-Key High-Pass Filter



$$sC(V_i - V_1) = \frac{V_1 - V_o}{R} + sC\left(V_1 - \frac{V_o}{A_o}\right)$$

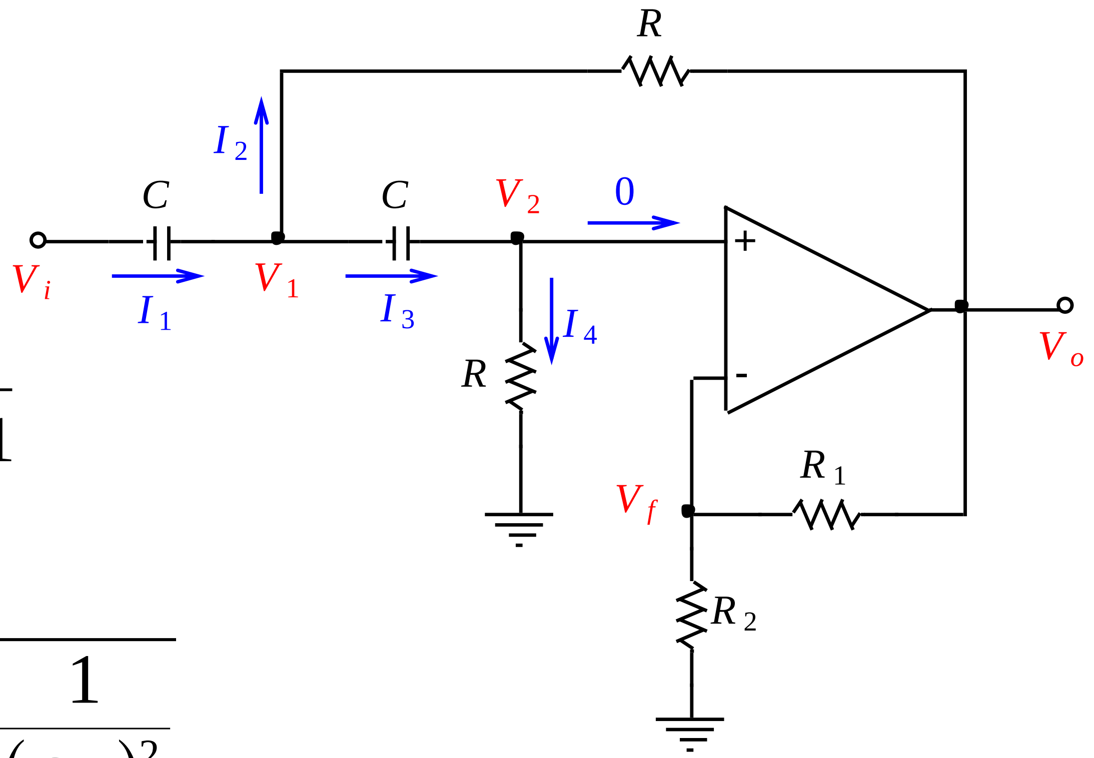
$$V_1 = \frac{V_o}{A_o} + \frac{V_o}{A_o sCR}$$

$$sCV_i = \left(\frac{1}{R} + s2C\right)\left(\frac{1}{A_o} + \frac{1}{A_o sCR}\right)V_o - \left(\frac{1}{R} + \frac{sC}{A_o}\right)V_o$$

$$sCV_i = \left(\frac{(sCR)^2 + (3 - A_o)sCR + 1}{A_o sCR^2}\right)V_o$$

Sallen-Key High-Pass Filter

$$sCV_i = \left(\frac{(sCR)^2 + (3 - A_o)sCR + 1}{A_o sCR^2}\right)V_o$$



$$H(s) \equiv \frac{V_o}{V_i} = \frac{A_o(sCR)^2}{(sCR)^2 + (3 - A_o)sCR + 1}$$

$$= \frac{A_o s^2}{s^2 + \frac{(3 - A_o)}{CR}s + \frac{1}{(CR)^2}}$$

$$\omega_o = \frac{1}{CR}$$

$$= \frac{A_o s^2}{s^2 + (3 - A_o)\omega_o s + \omega_o^2}$$

Sallen-Key High-Pass Filter

$$= \frac{A_o s^2}{s^2 + (3 - A_o)\omega_o s + \omega_o^2}$$

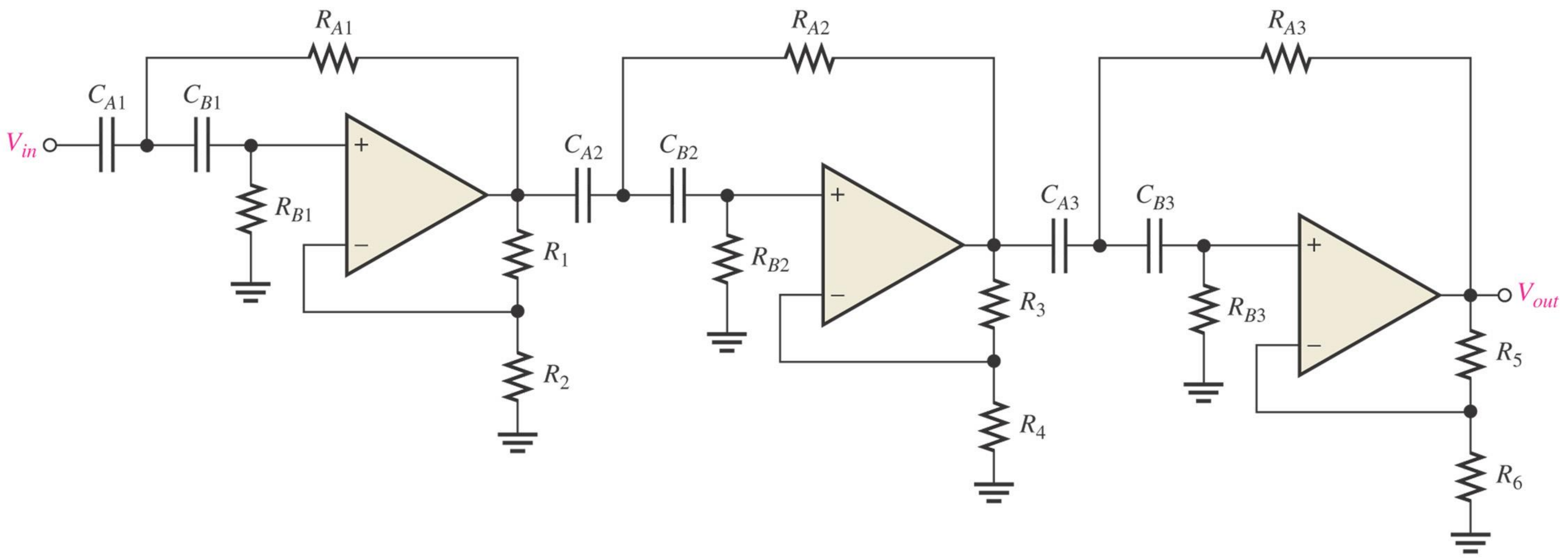
$\alpha = 3 - A_o$ = Damping Factor

$$H(s) = \frac{A_o s^2}{s^2 + \alpha \omega_o s + \omega_o^2}$$

$$\alpha = 3 - A_o \quad \Rightarrow \quad = 3 - \left(1 + \frac{R_1}{R_2}\right) \quad \Rightarrow \quad \alpha = 2 - \frac{R_1}{R_2}$$

Cascading High-Pass Filter

- As with the low-pass filter, first- and second-order high-pass filters can be cascaded to provide three or more poles and thereby create faster roll-off rates.
- A six-pole high-pass filter consisting of three Sallen-Key two-pole stages with the roll-off rate of -120 dB/decade.



Sixth-order high-pass filter