

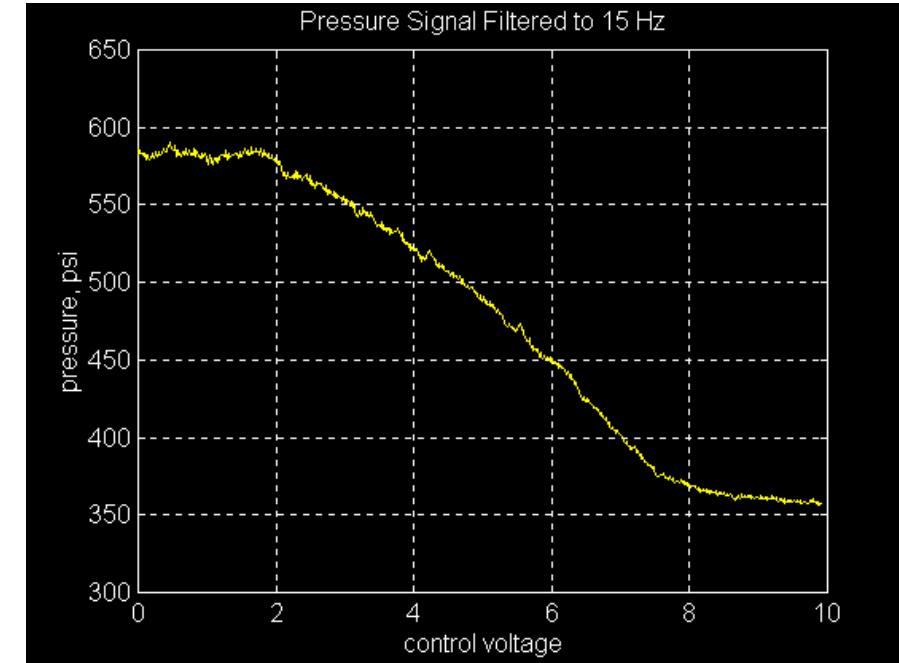
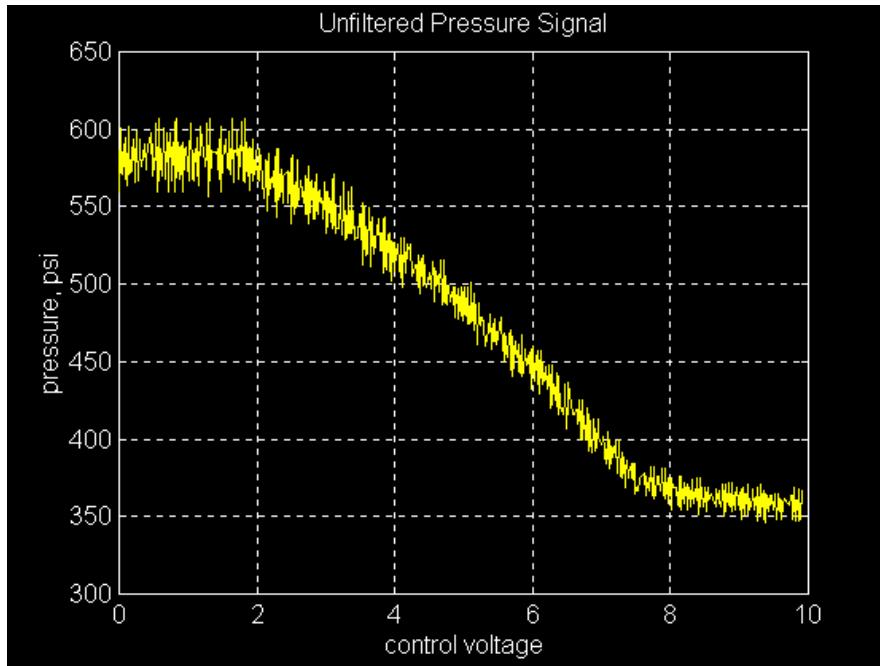
Signal Conditioning

ENGG 6150: Bio-Instrumentation

Signal Conditioning

- Passive Filters
 - Simple Low pass filter
 - Simple High pass filter
 - N-section LC ladder circuit (low-pass filter prototypes)
 - Butterworth filter
 - Equal-ripple filter "Chebyshev"
 - Transformation from Low Pass to High Pass filter
 - Transformation from Low Pass to Band Pass filter
- Active Filters
 - Low pass filter
 - Cascaded low pass filter
 - Sallen Key low pass filter
 - High pass filter
 - Cascaded high pass filter
 - Sallen Key low pass filter

What is a Filter?



Filtering:

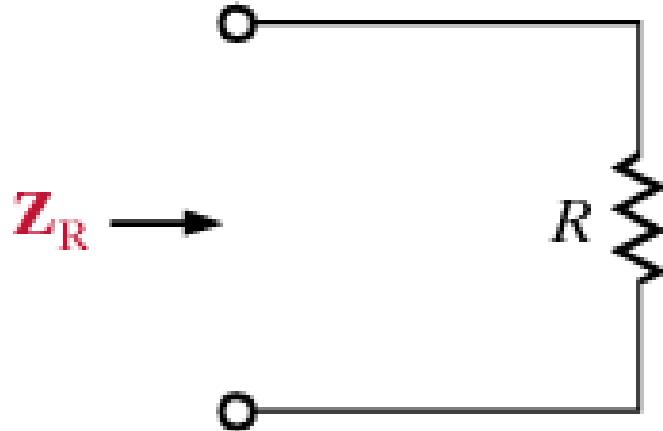
Certain desirable features are **retained**

Other undesirable features are **suppressed**



Resistors

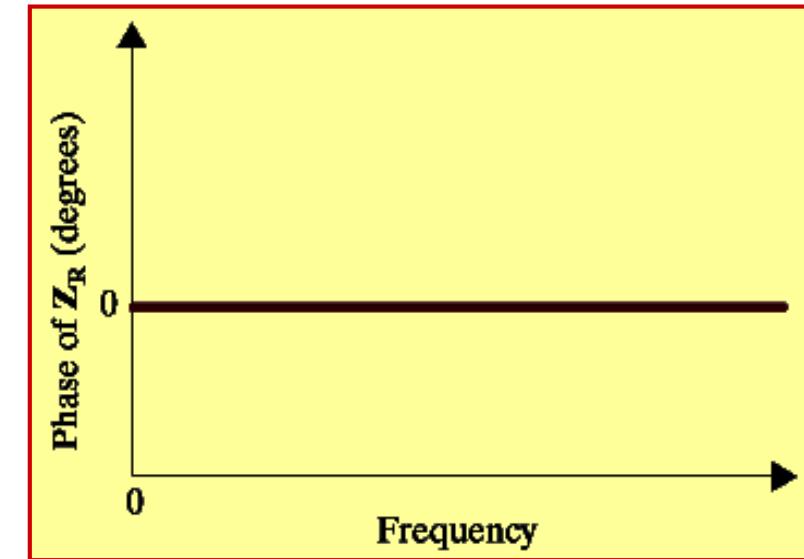
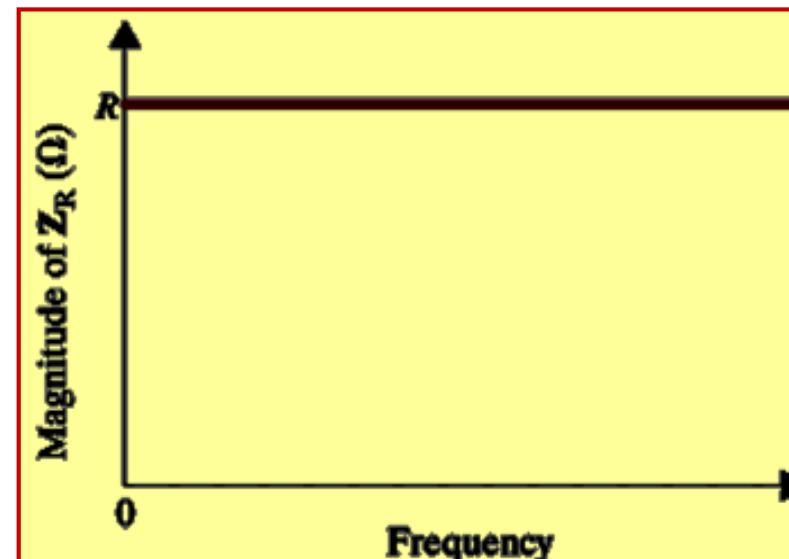
Filters are circuits that are capable of *passing signals within a band* of frequencies while *rejecting or blocking signals of frequencies outside this band*. This property of filters is also called “**frequency selectivity**”.



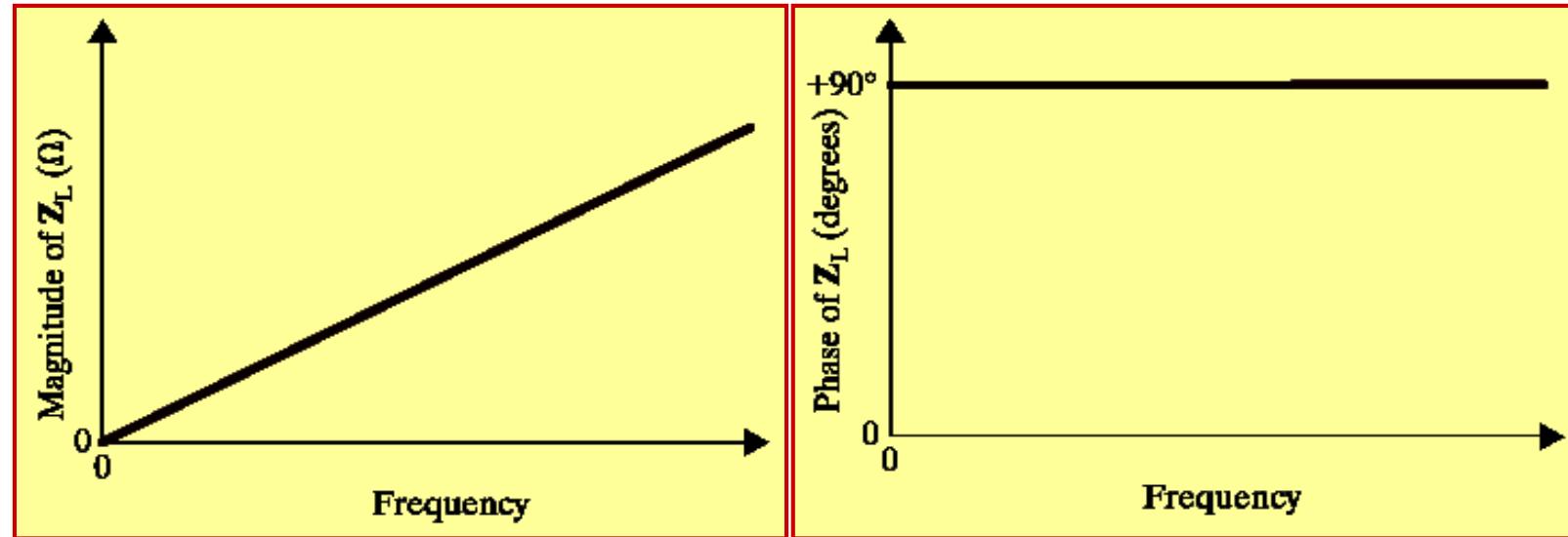
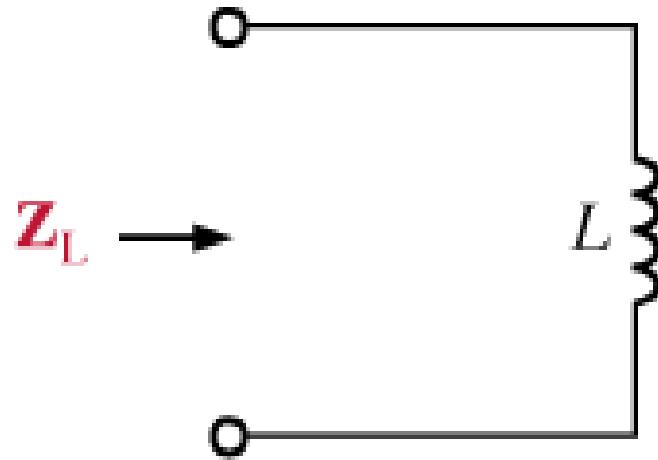
$$Z_R = R = R\angle 0^\circ$$

$$\text{Magnitude of } Z_R = |Z_R| = R$$

$$\text{Phase of } Z_R = 0^\circ$$



Inductors

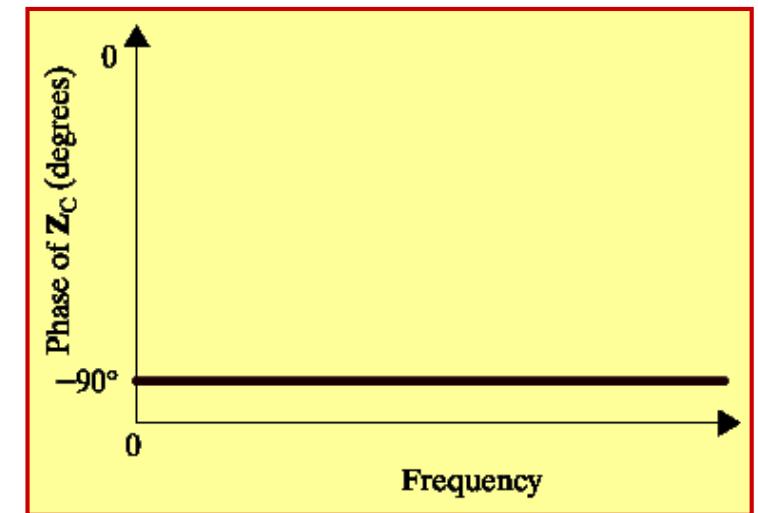
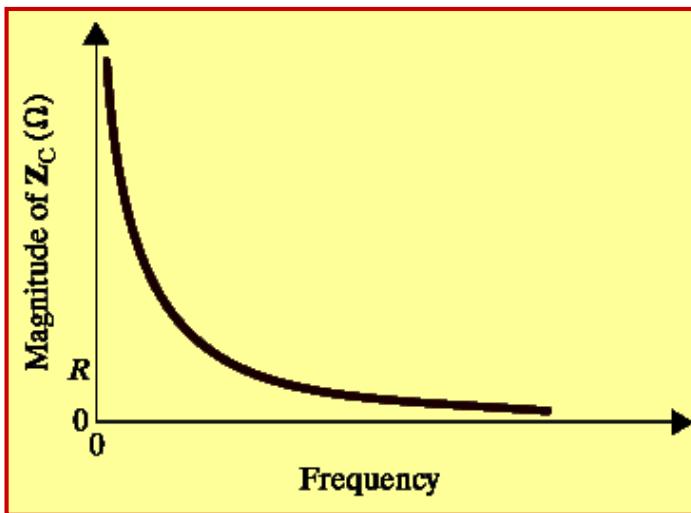
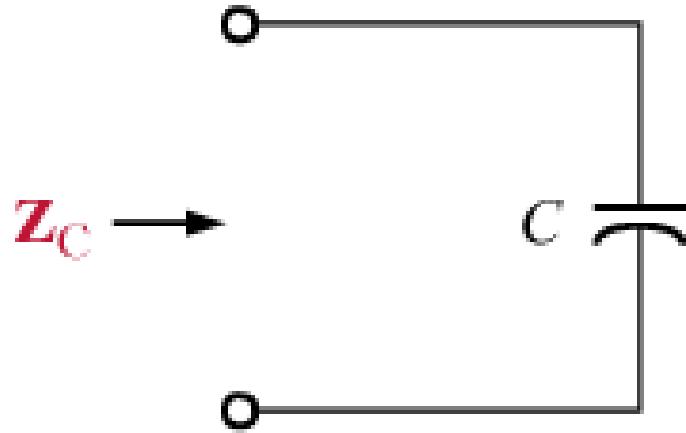


$$Z_L = j\omega L = \omega L \angle 90^\circ$$

Magnitude of Z_L = $|Z_L| = \omega L$

Phase of Z_L = $+90^\circ$

Capacitors

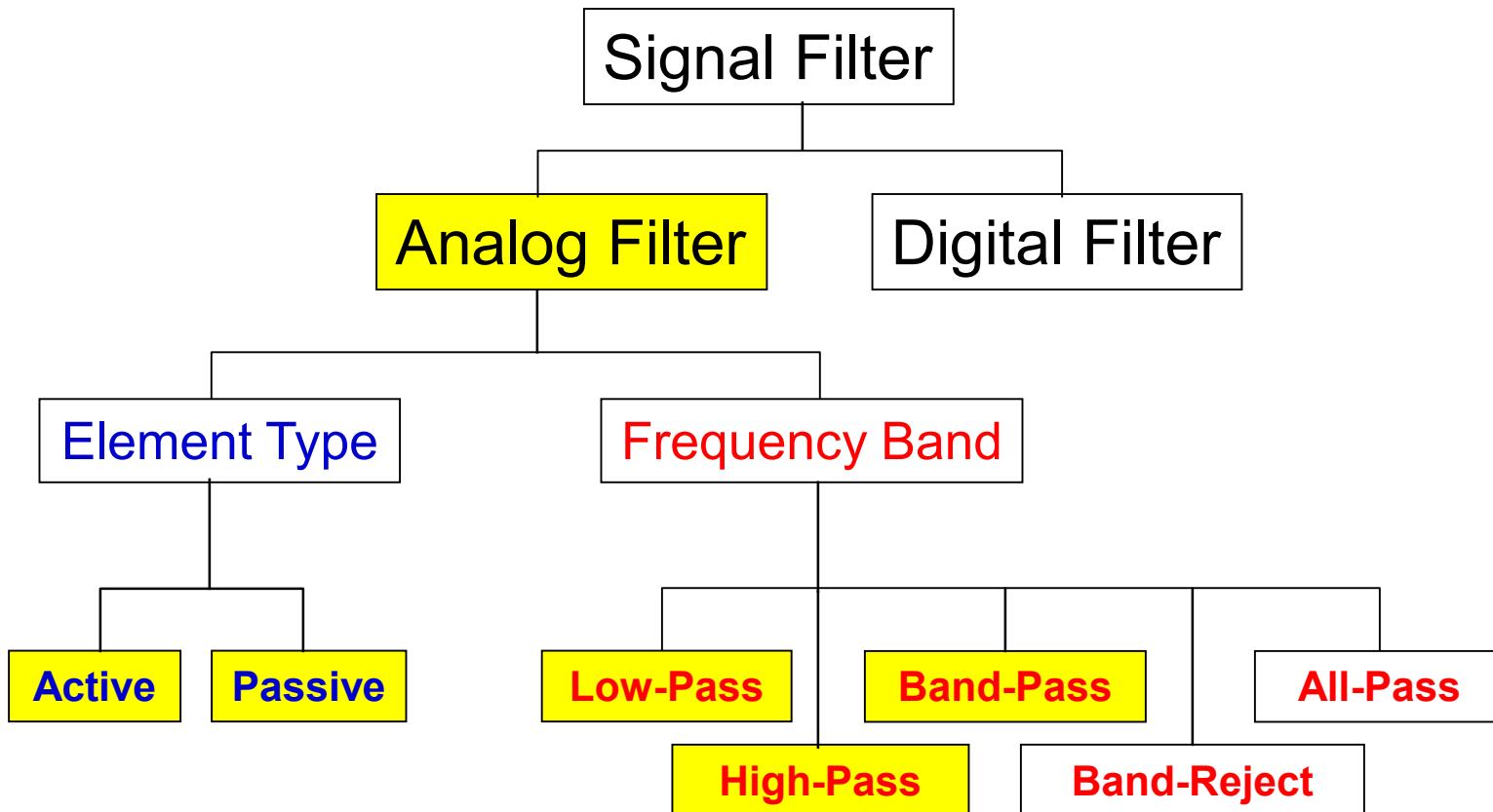


$$Z_C = 1 / j\omega C = 1 / \omega C e^{-j90^\circ}$$

Magnitude of Z_C = $|Z_C| = 1 / \omega C$

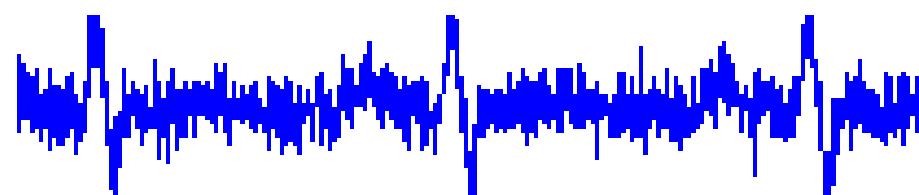
Phase of Z_C = -90°

Classification of Filters

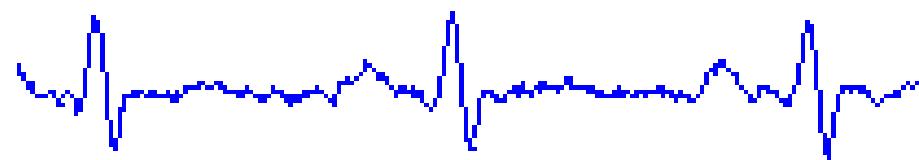


Low-Pass Filter

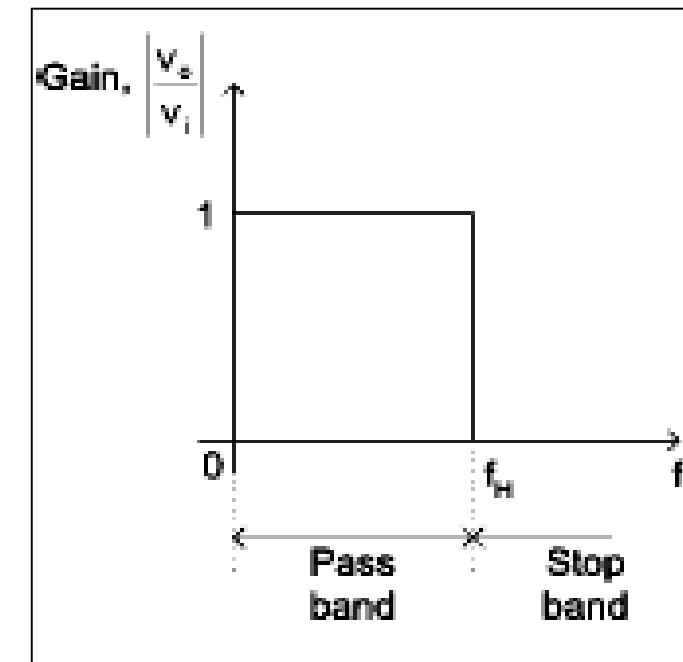
- An ideal **low-pass filter** is a filter that passes frequencies from 0Hz to certain frequency called the critical frequency f_H and completely attenuates all other frequencies.



Original signal



Filtered signal



Ideal response

Example 1

Find the voltage gain and frequency response of the RC circuit.

SOLUTION:

The voltage gain:

$$H(\omega) = \frac{V_o}{V_s} = \frac{(1/j\omega C)}{R + (1/j\omega C)} = \frac{1}{1 + j\omega RC}$$

$$\text{Set: } \omega_c = \frac{1}{RC}$$

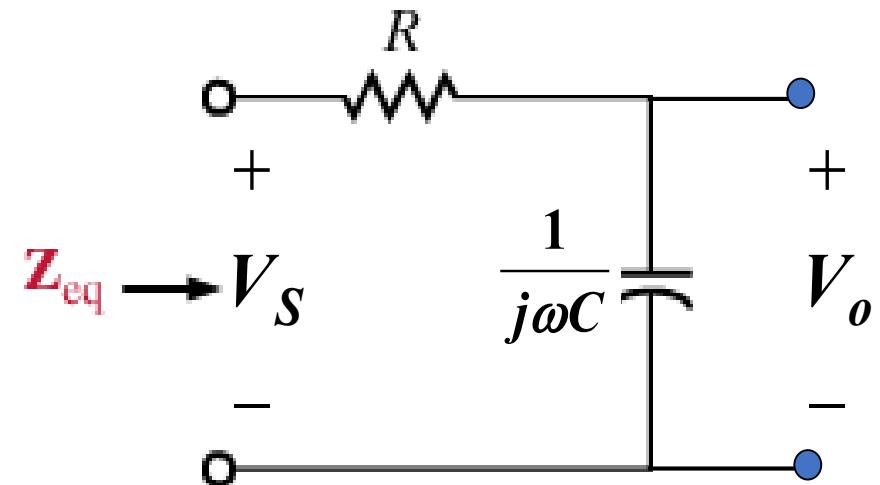
Magnitude is:

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

Phase is:

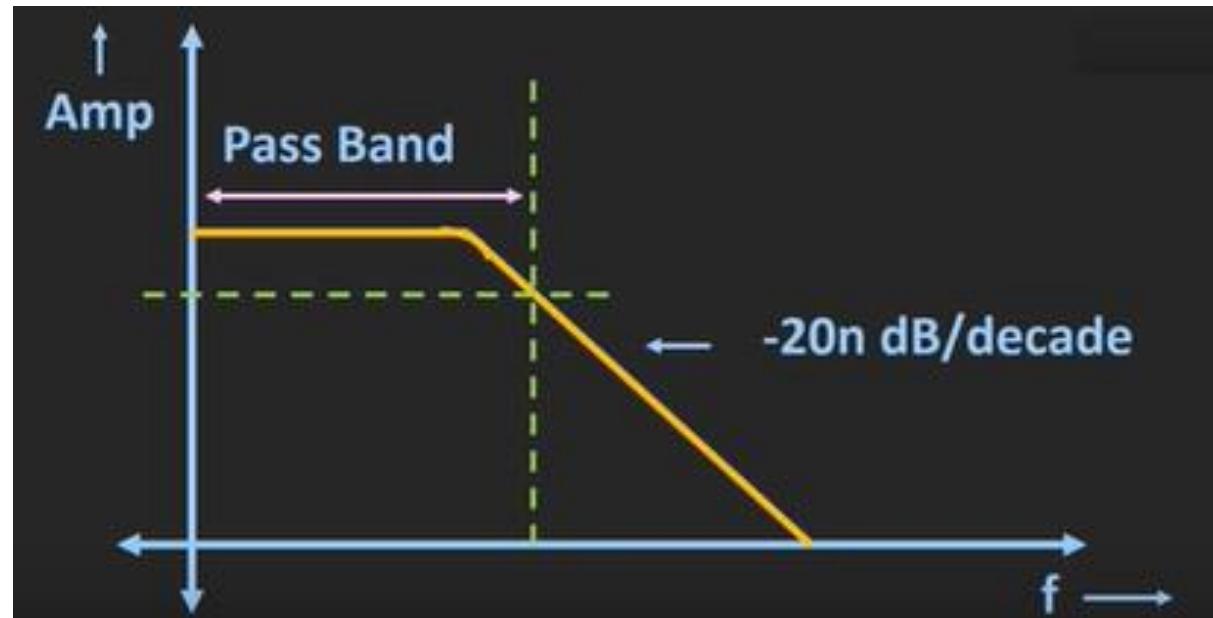
$$\begin{aligned}\phi &= -\tan^{-1}\left(\frac{\omega RC}{1}\right) \\ &= -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)\end{aligned}$$

ω/ω_c	$ H(\omega) $	ϕ
0	1.0	0
1.0	0.707	-45°
10	0.1	-84°
100	0.01	-89°



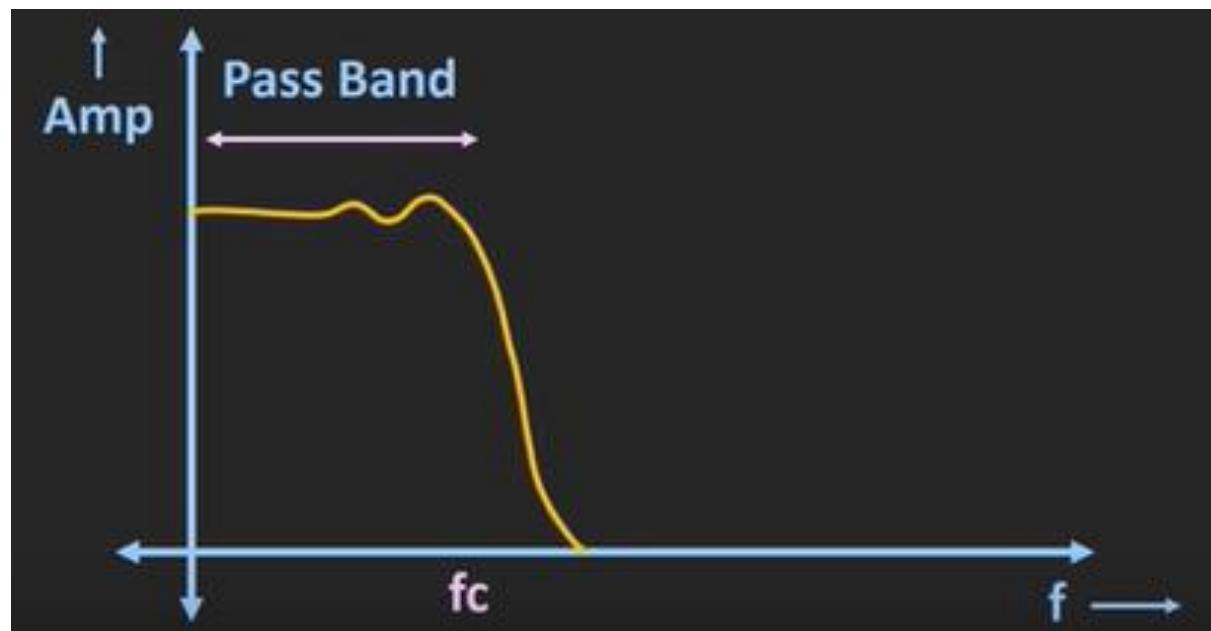
Butterworth Filter Approximation

- It provides the maximally flat approximation.
- The roll off is at -20dB/decade for the first order, -40dB/decade for the second order and so on.



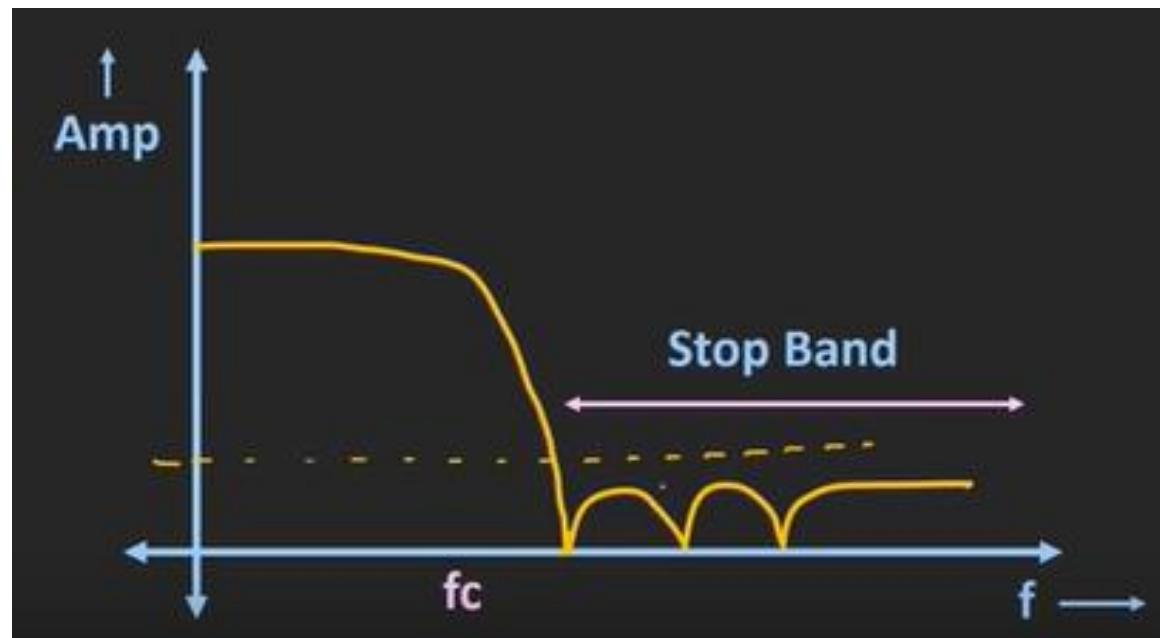
Chebyshev Filter Approximation

- It provides some ripples in the pass band. The number of ripples = $n/2$ where n is the order of the filter.
- However, in stop band there will be no ripples.
- The roll off is much faster than the Butter worth filter.



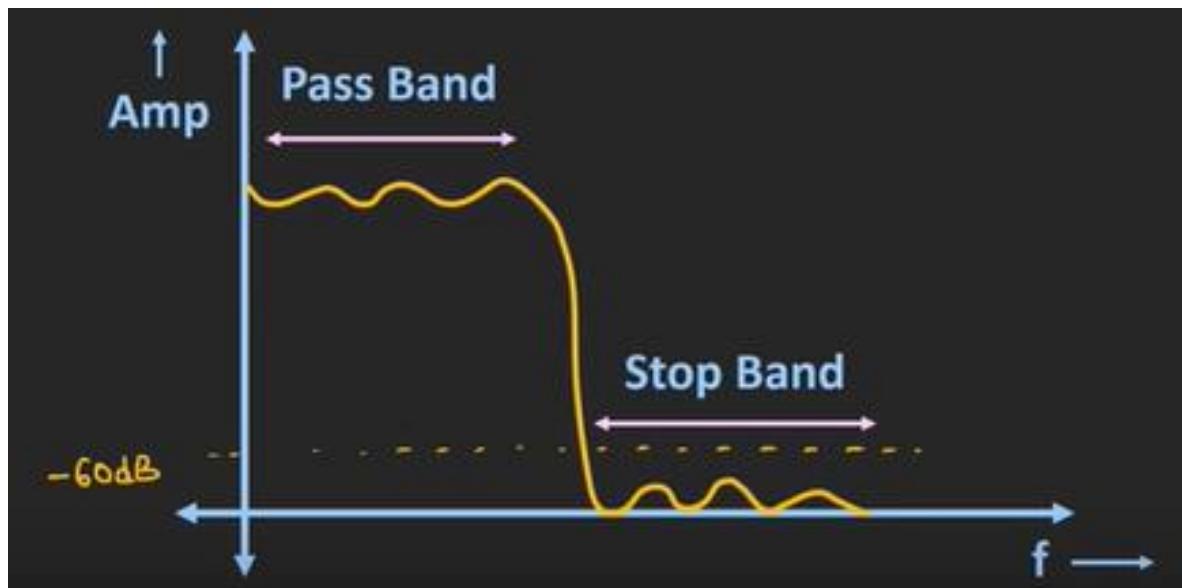
Inverse Chebyshev Filter Approximation

- It provides some ripples in the stop band.
- However, in pass band there will be no ripples.
- The roll off is much faster than the Butter worth filter.



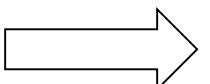
Elliptic Filter Approximation

- It provides some ripples in both the pass and stop bands.
- The roll off is the fastest among all filters.



Low-Pass Filter

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}}$$



$$|H(j\omega)|_{\text{dB}} = 20 \log \left[\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}} \right]$$

When $\omega = 0 \rightarrow |H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{0}{\omega_o}\right)^2}} = 1 \rightarrow |H(j\omega)|_{\text{dB}} = 20 \log 1 = 0 \text{ dB}$

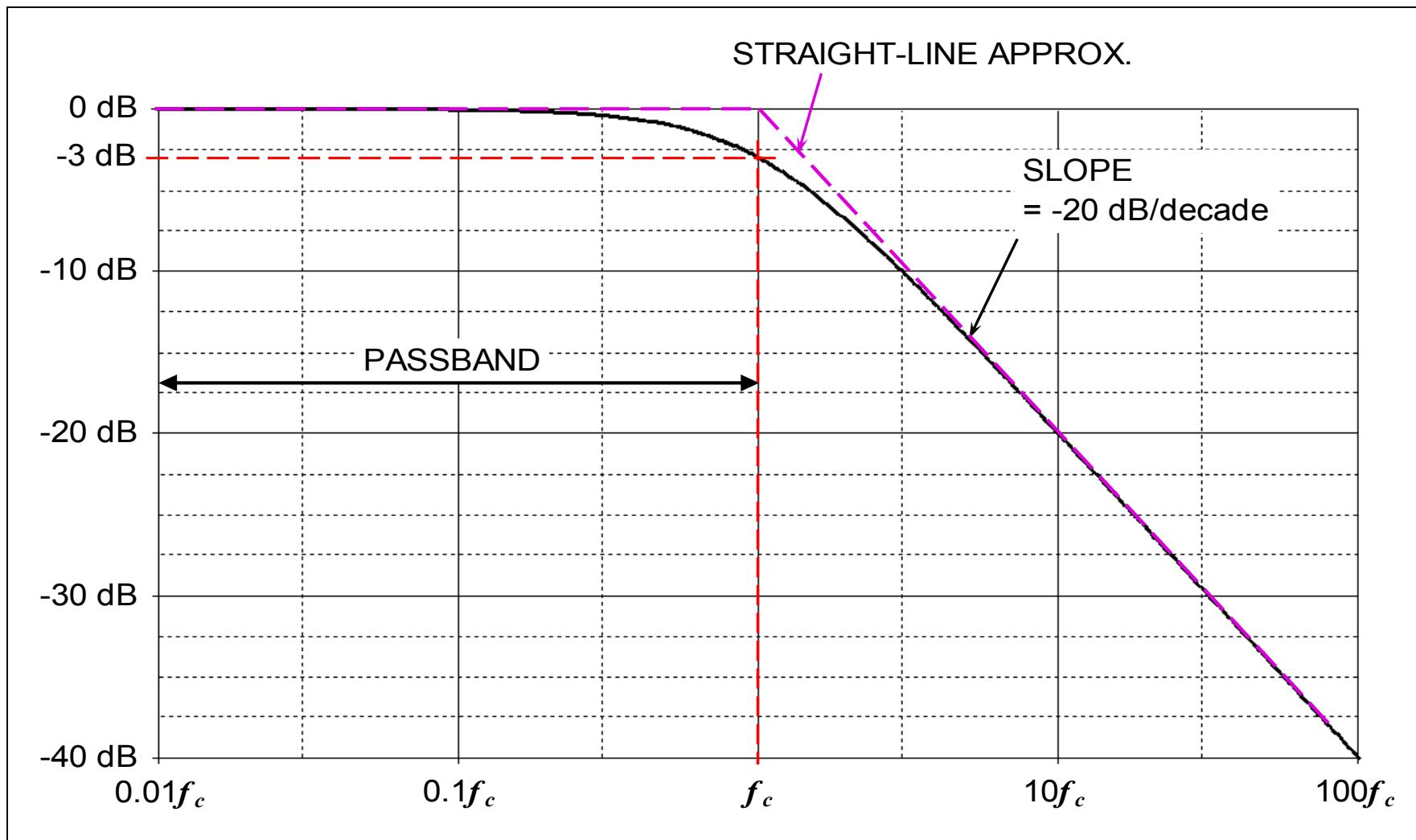
When $\omega = \omega_c = \omega_o = \frac{1}{CR} \rightarrow |H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega_o}\right)^2}} = \frac{1}{\sqrt{1+1}} = 0.707$

$\rightarrow |H(j\omega)|_{\text{dB}} = 20 \log 0.707 = -3 \text{ dB}$

$\omega_c = \frac{1}{CR}$ is known as 3-dB or cutoff frequency (rad/s)

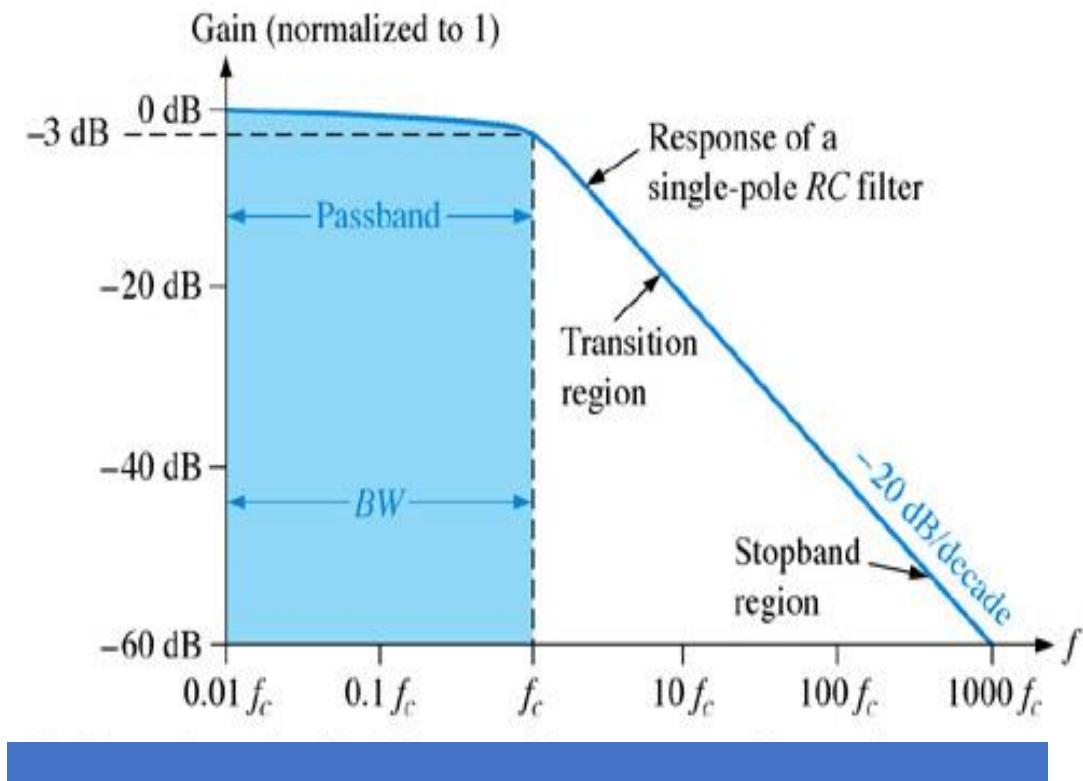
Low-Pass Filter

Response curve

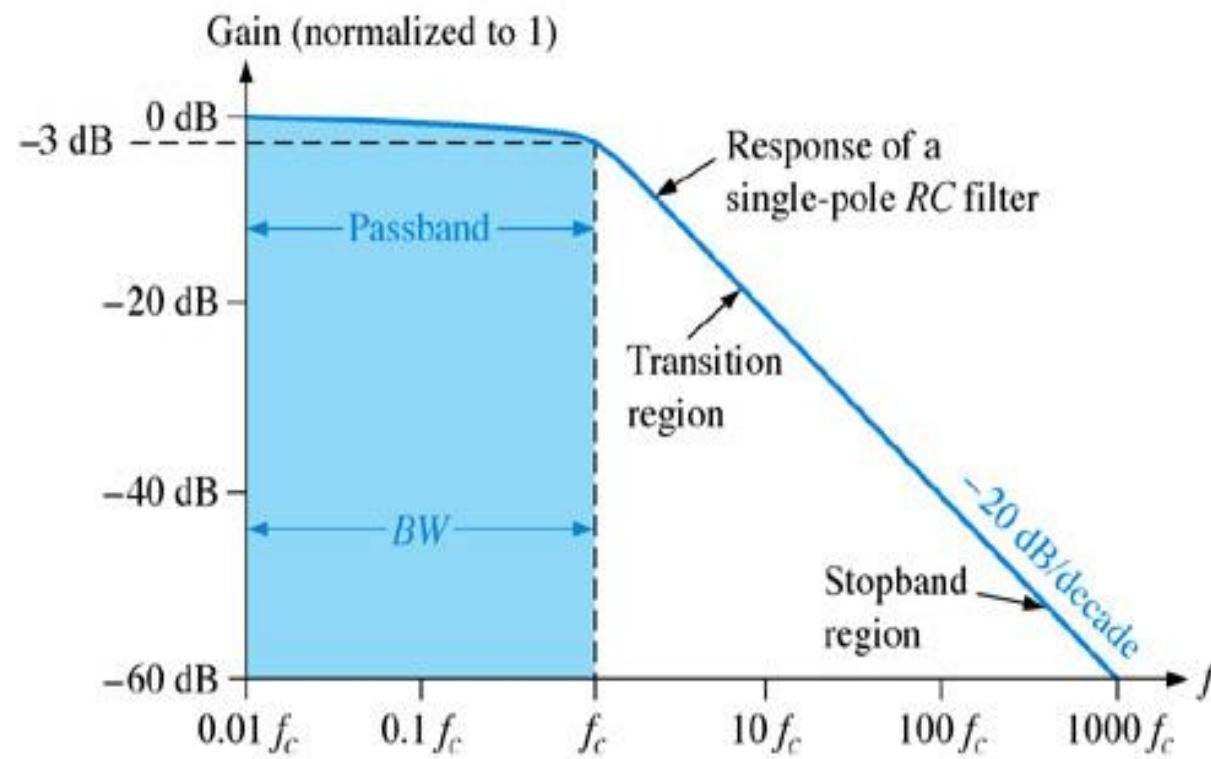


Low-Pass Filter

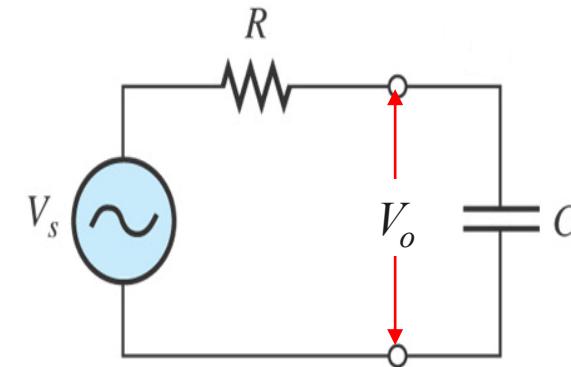
- An actual **low-pass filter** is a filter that passes most of frequencies from 0Hz to certain frequency and significantly attenuates other frequencies.
- **Passband** of a filter is the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than -3 dB of attenuation).
- **Transition region** shows the area where the fall-off occurs.
- **Stopband** is the range of frequencies that have the most attenuation.
- **Critical frequency, f_c** , (also called the cutoff frequency) defines the end of the passband and normally specified at the point where the response drops – 3 dB (70.7%) from the passband response.



Actual response



(a) Comparison of an ideal low-pass filter response with actual response



(b) Basic low-pass circuit

- At low frequencies, X_C is very high and the capacitor circuit can be considered as open circuit. Under this condition, $V_o = V_{in}$ or $A_V = 1$ (unity).
- At very high frequencies, X_C is very low and the V_o is small as compared with V_{in} . Hence the gain falls and drops off gradually as the frequency is increased.

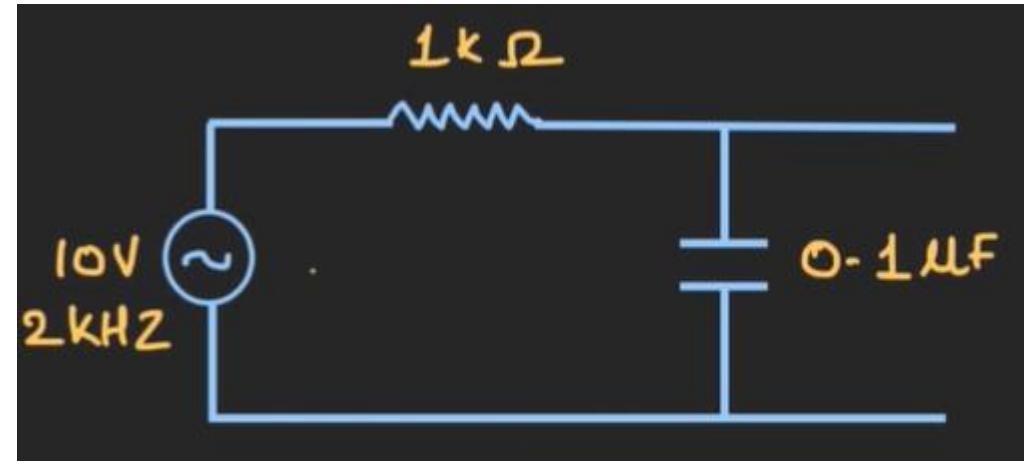
Example 2

Find the cut off frequency and output voltage at 2kHz.

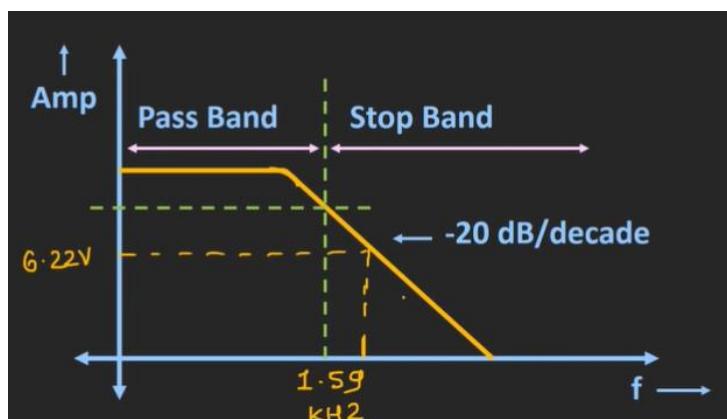
SOLUTION:

The cut off frequency:

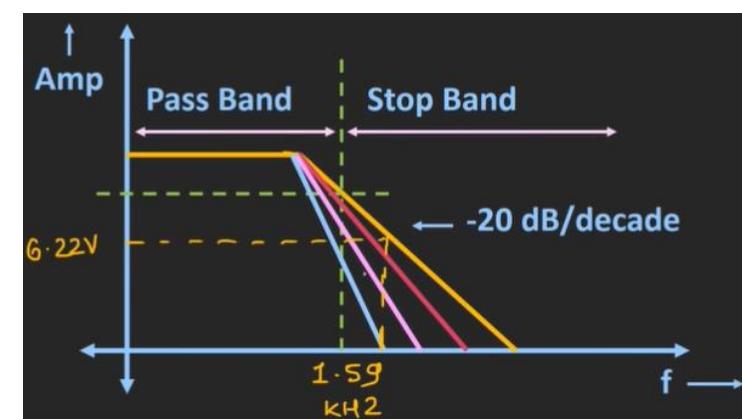
$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi 1k \times 0.1\mu} = 1.59\text{kHz}$$



$$V_{out} = \frac{1}{\sqrt{1 + (\omega RC)^2}} V_{in} = \frac{1}{\sqrt{1 + (2\pi \times 2k \times 1k \times 0.1\mu)^2}} 10V = 6.22V$$

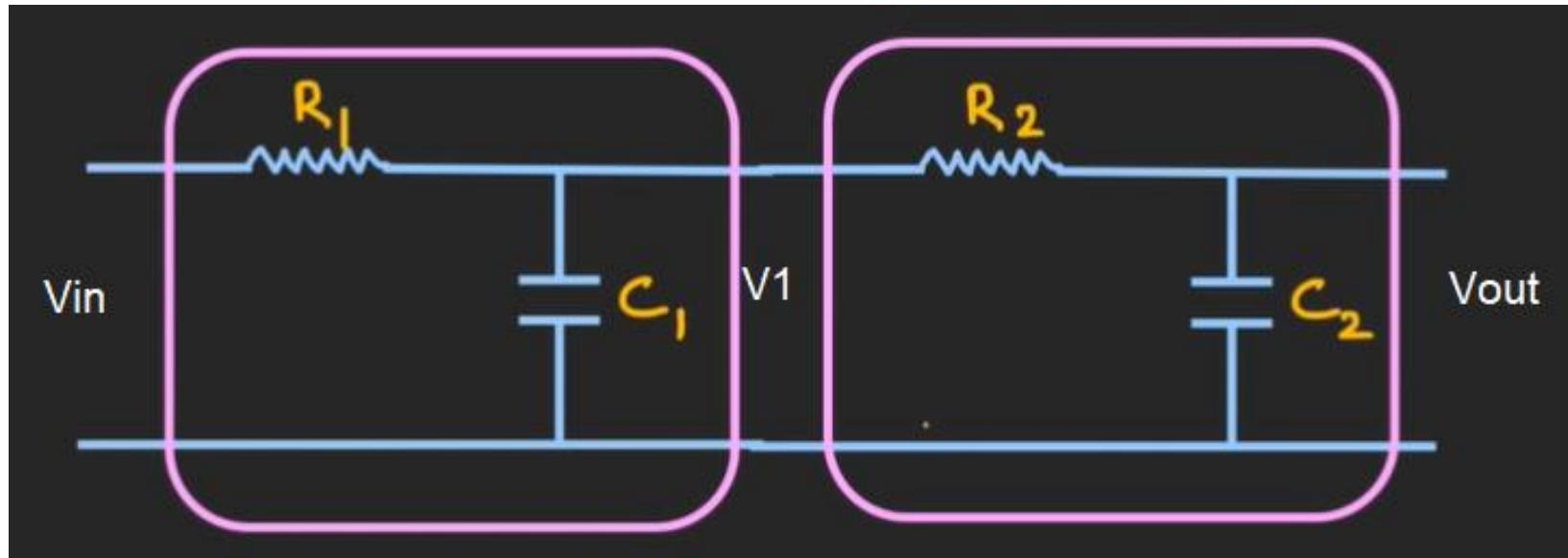


For more attenuation



Higher Order Low-Pass Filter

- Higher order low pass filters can be achieved by cascading multiple RC filters as shown below.



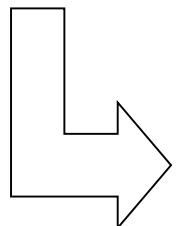
Second order low pass filter

Higher Order Low-Pass Filter

- Derive the transfer function for this second order passive filter.

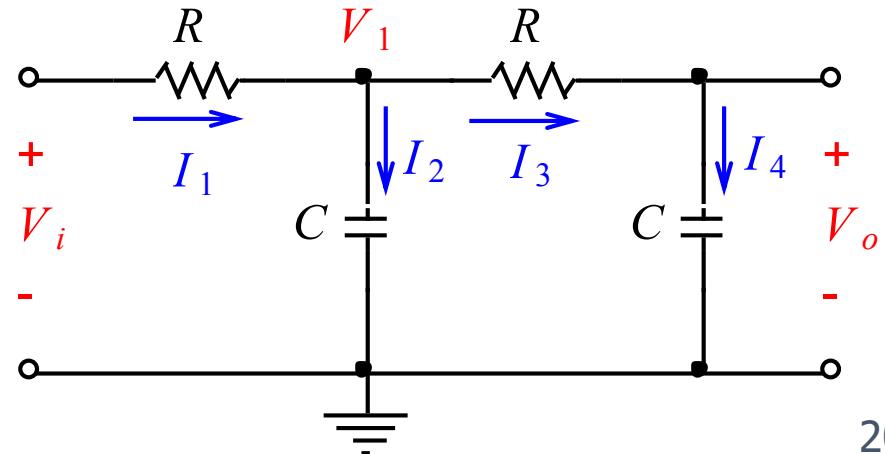
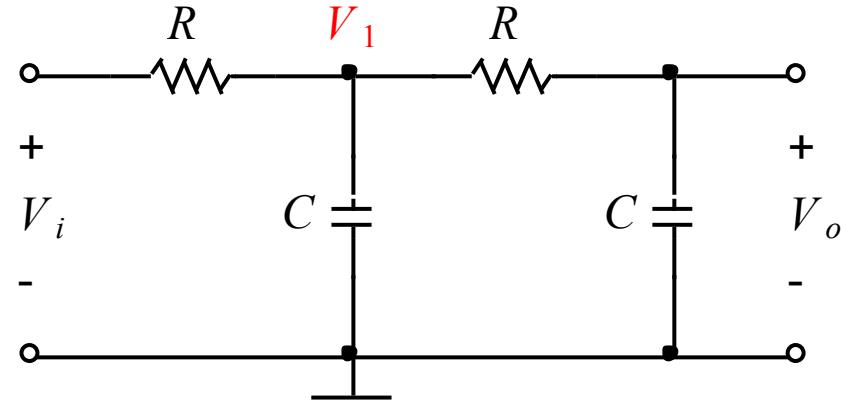
Note that $s = j\omega$

$$\frac{V_i - V_1}{R} = sCV_1 + \frac{V_1 - V_o}{R}$$



$$\frac{V_i}{R} = \left(\frac{sCR + 2}{R} \right) V_1 - \frac{V_o}{R}$$

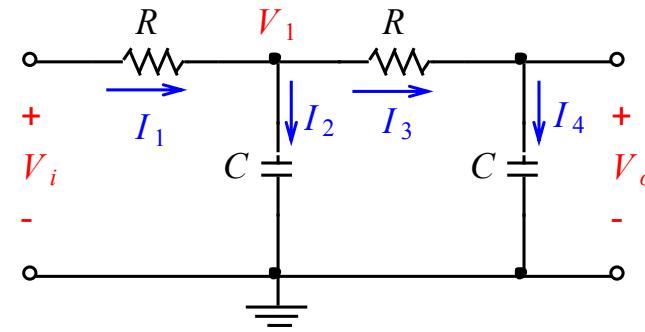
$$\frac{V_1 - V_o}{R} = sCV_o \quad \longrightarrow \quad V_1 = (sCR + 1)V_o$$



Higher Order Low-Pass Filter

Since:

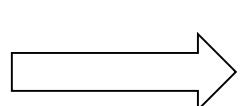
$$\frac{V_i}{R} = \left(\frac{sCR + 2}{R} \right) V_1 - \frac{V_o}{R}$$



And:

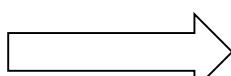
$$V_1 = (sCR + 1)V_o \quad \longrightarrow$$

$$\frac{V_i}{R} = \left(\frac{sCR + 2}{R} \right) (sCR + 1)V_o - \frac{V_o}{R}$$



$$H(s) \equiv \frac{V_o}{V_i} = \frac{1}{(sCR)^2 + 3sCR + 1}$$

$$H(s) = \frac{1}{(sCR)^2 + 3sCR + 1}$$



$$H(s) = \frac{\frac{1}{(CR)^2}}{s^2 + s\frac{3}{CR} + \frac{1}{(CR)^2}}$$

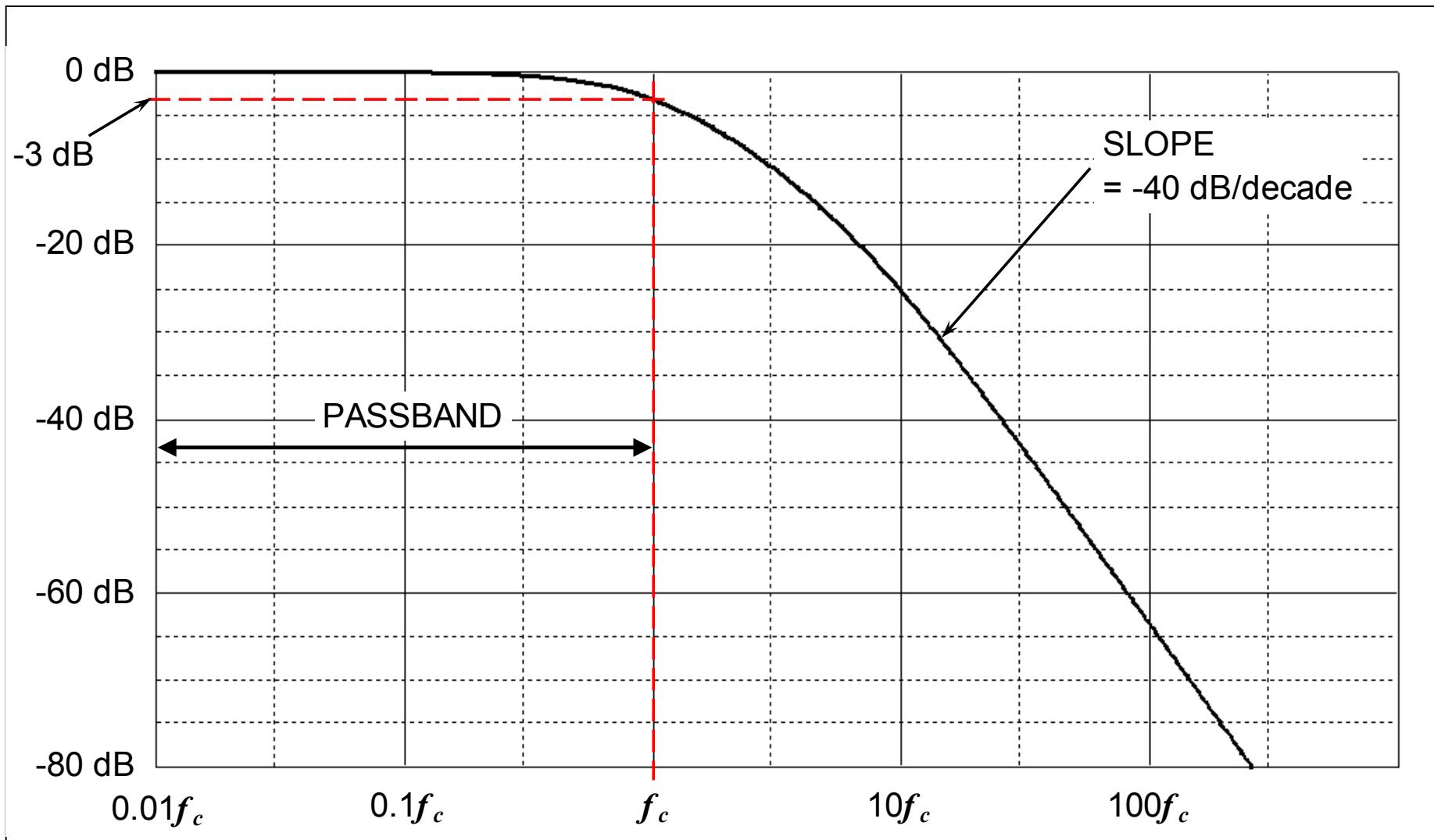
Or

$$H(s) = \frac{\omega_o^2}{s^2 + s3\omega_o + \omega_o^2}$$

$$\left[\omega_o = \frac{1}{CR} \right]$$

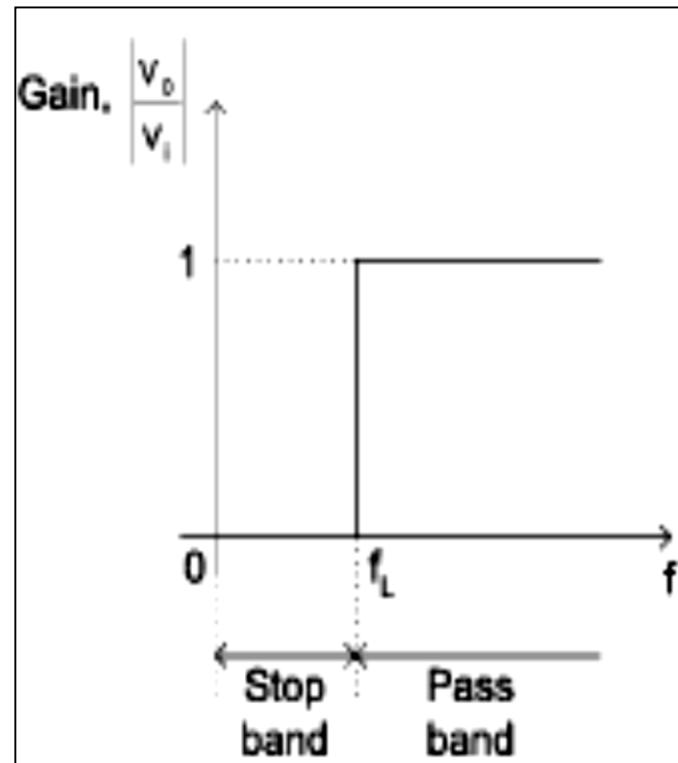
Higher Order Low-Pass Filter

Response curve



High-Pass Filter Response

- An ideal **high-pass filter** is a filter that completely attenuates all frequencies **below** certain frequency and passes all frequencies **above** that frequency.



Ideal response

Example 3

Find the voltage gain and frequency response of the RC circuit.

SOLUTION:

$$V_{out} = \frac{R}{\left(R + \frac{1}{j\omega C}\right)} V_{in} \longrightarrow V_{out} = \frac{j\omega RC}{(j\omega RC + 1)} V_{in}$$

The voltage gain: $\frac{V_{out}}{V_{in}} = \frac{j\omega RC}{(j\omega RC + 1)}$

Set: $\omega_c = \frac{1}{RC}$

Magnitude is:

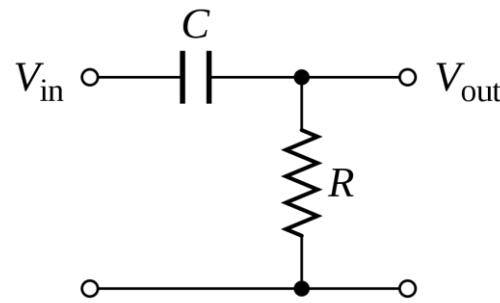
$$|H(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} = \frac{(\omega/\omega_c)}{\sqrt{1 + (\omega/\omega_c)^2}}$$

Phase is:

$$\phi = 90 - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= 90 - \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

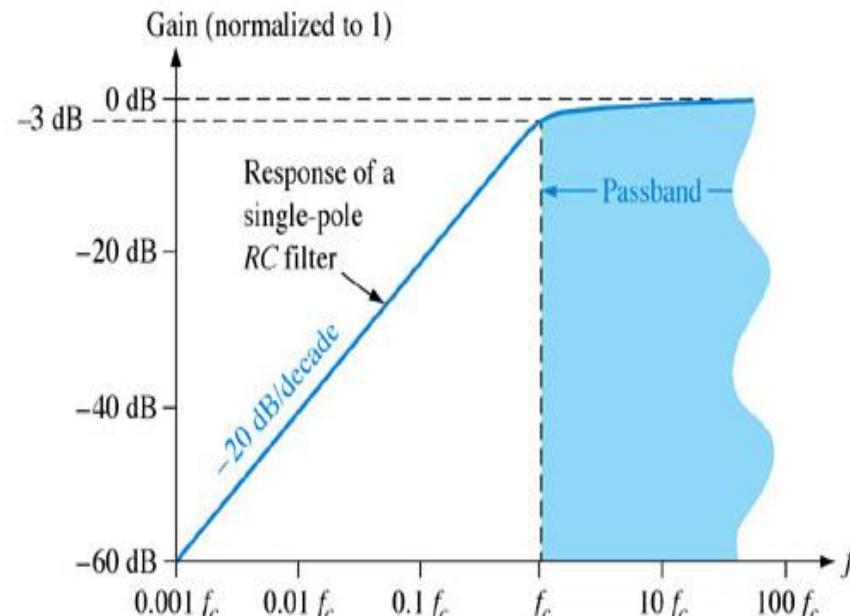
ω/ω_c	$ H(\omega) $	ϕ
0	0.0	90
1.0	0.707	45°
10	0.995	6°
100	0.9999	1°



- At $\omega = 1/(RC)$, the magnitude of the voltage gain is $1/\sqrt{2}$.
- This is called the critical frequency of a high-pass RC filter

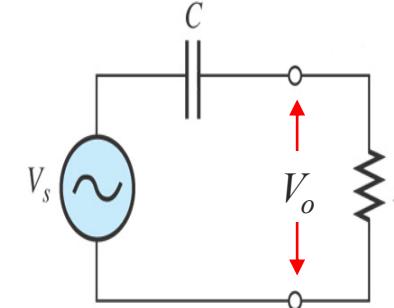
High-Pass Filter Response

- An actual **high-pass filter** is a filter that significantly attenuates or rejects all frequencies **below** f_c and passes all frequencies **above** f_c .
- The passband of a high-pass filter is all frequencies above the critical frequency.



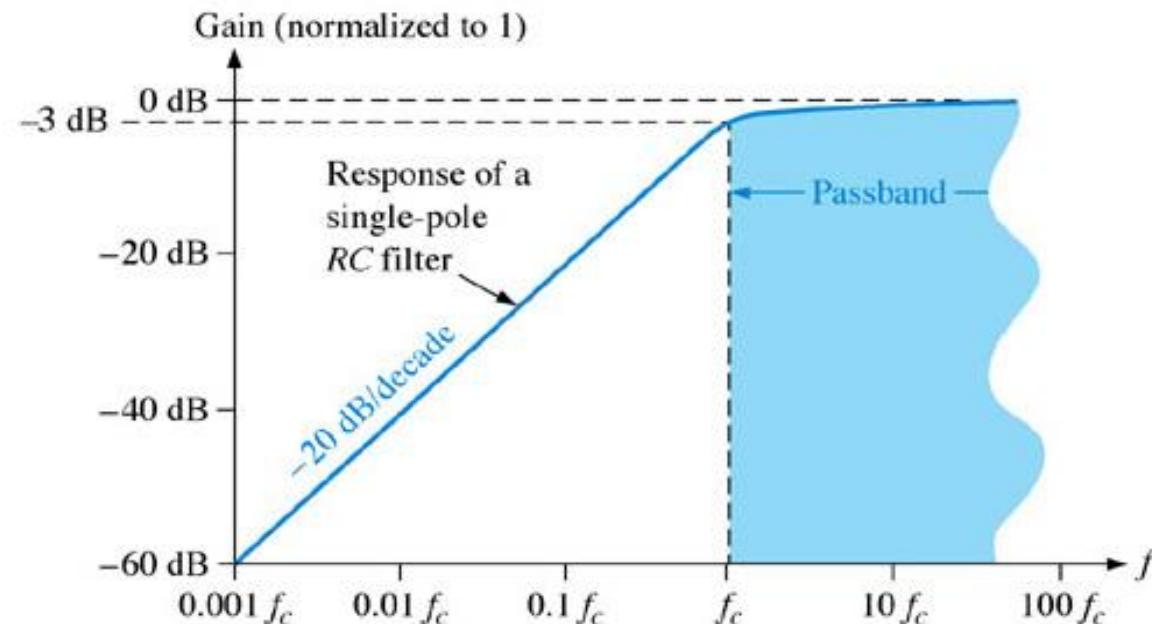
(a) Comparison of an ideal high-pass filter response with actual response

Actual response

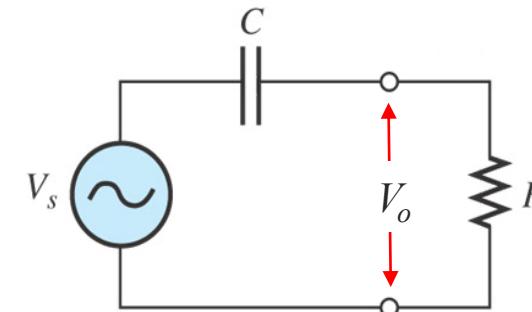


(b) Basic high-pass circuit

High-Pass Filter Response



(a) Comparison of an ideal high-pass filter response with actual response



(b) Basic high-pass circuit

- At low frequencies, X_C is very high and the capacitor circuit can be considered as open circuit. Under this condition, $V_o = 0$.
- At very high frequencies, X_C is very low (acts like short circuit) and the V_o is equal to V_{in} . Hence the gain increases gradually as the frequency is increased till $A_v = 1$ (unity).

Example 4

- Design a high pass filter with a cut-off frequency of 10 kHz.

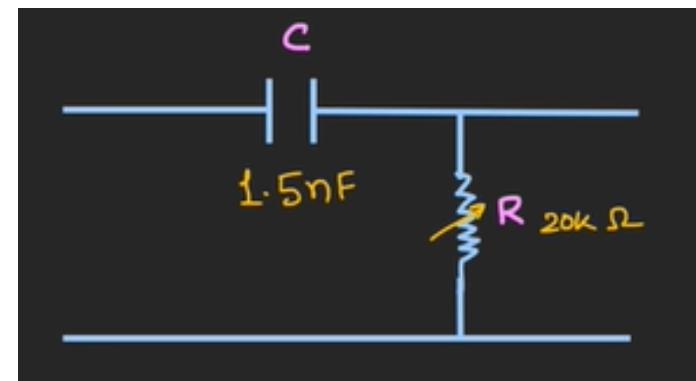
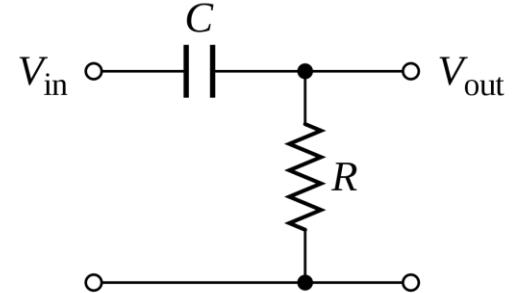
SOLUTION:

Select the value of R to be 10 k Ω . The question why this value? This is for the following reasons:

- a) Small value of R in Ohms, will lead to a loading effect to the connected circuit.
- b) High values of R in M Ω will lead to small value of C comparable to the parasitic capacitance.

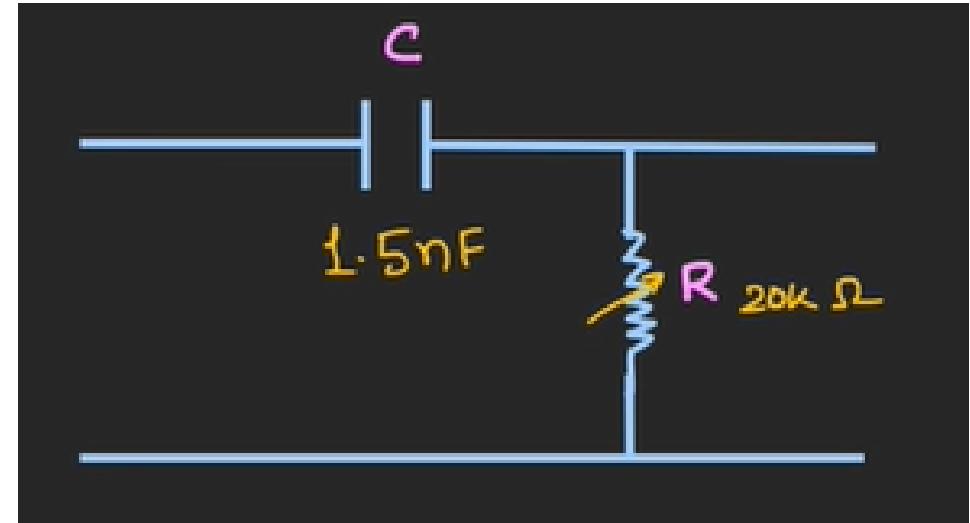
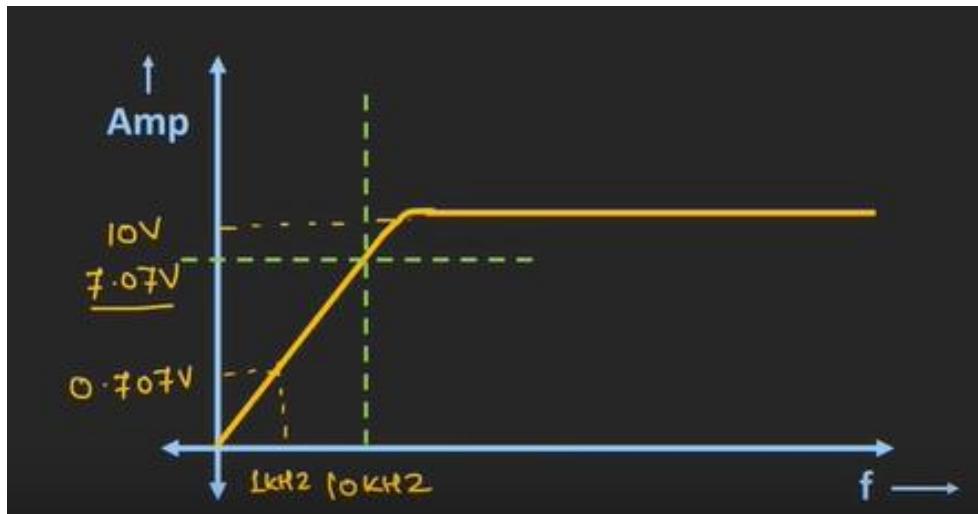
$$C = \frac{1}{2\pi R f} = \frac{1}{2\pi 10k \times 10k} = 1.59nF$$

- In practice, we will have 1.5 nF and not 1.59nF, so we can adjust the 10kHz cut-off frequency using 20k Ω variable resistance.

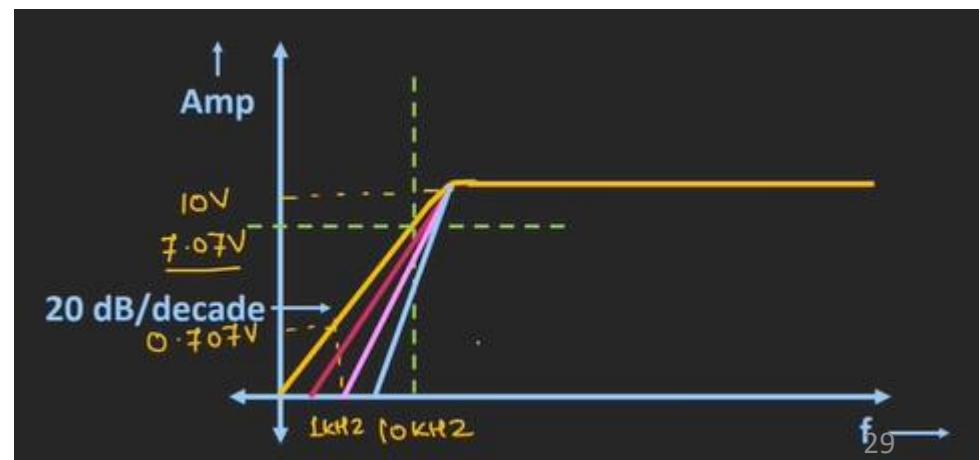


Example 4, Continue

So if we have an input of 10V, the output will be affected as follows:



- The attenuation will be 20dB/decade or the output voltage will drop by one tenth when the frequency increases by one tenth.
- By having higher order filters, the attenuation will be much higher



Example 5

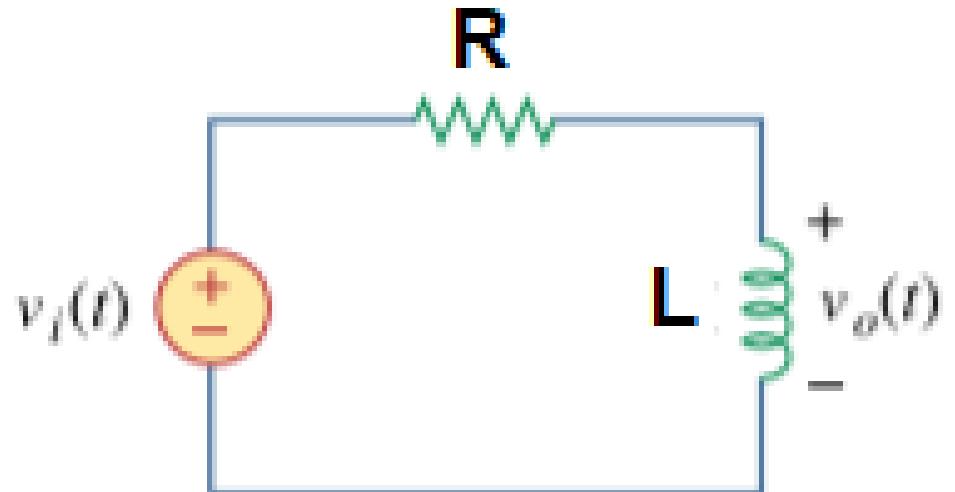
What is the type of the filter shown below?

$$H(\omega) = \frac{V_o}{V_i} = \frac{j\omega L}{R + j\omega L}$$

$$H(0) = 0 \text{ and } H(\infty) = 1$$



This is a high pass filter



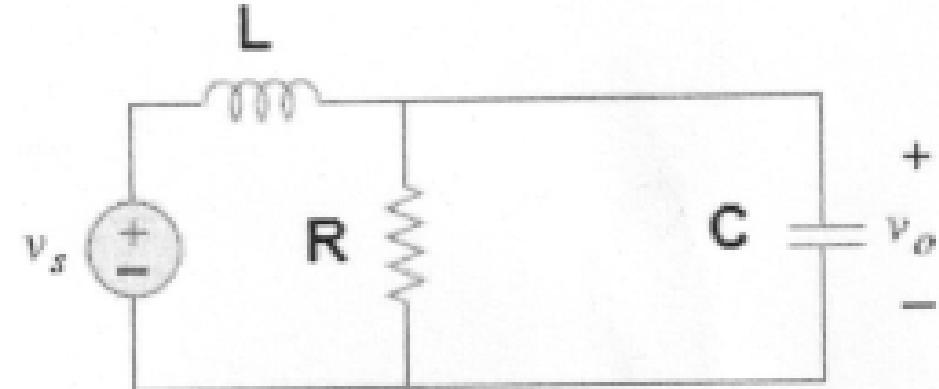
Example 6

What is the type of the filter shown below?

$$\frac{V_o}{V_s} = \frac{\frac{R // 1/j\omega C}{R // 1/j\omega C + j\omega L}}{\frac{R // 1/j\omega C}{R + 1/j\omega C} + j\omega L}$$

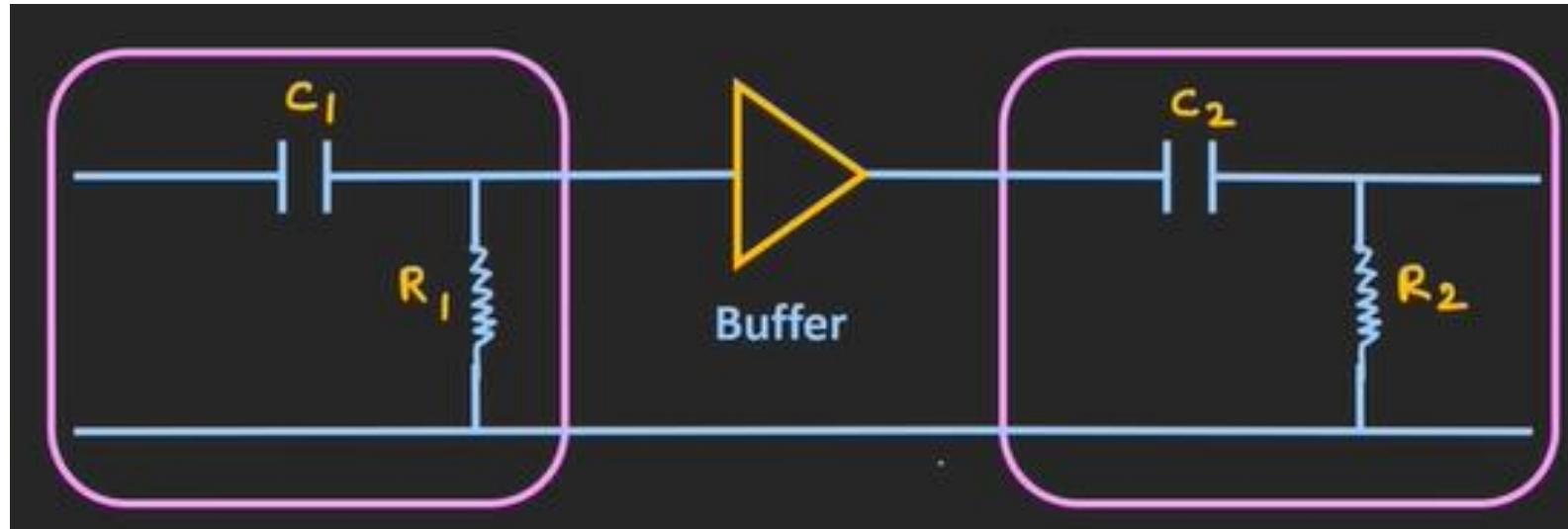
$$X(R + 1/j\omega C) = \frac{R/j\omega C}{R/j\omega C + j\omega L(R + 1/j\omega C)} * \frac{j\omega C}{j\omega C}$$

$$= \frac{R}{R + j\omega L(j\omega RC + 1)} = \frac{R}{R - \omega^2 RLC + j\omega L}$$



$$\left. \begin{aligned} @ \omega = 0 & \Rightarrow \left| \frac{V_o}{V_s} \right| = \frac{1}{R} \\ @ \omega = \infty & \Rightarrow \left| \frac{V_o}{V_s} \right| = 0 \end{aligned} \right\} \text{L.P.F.}$$

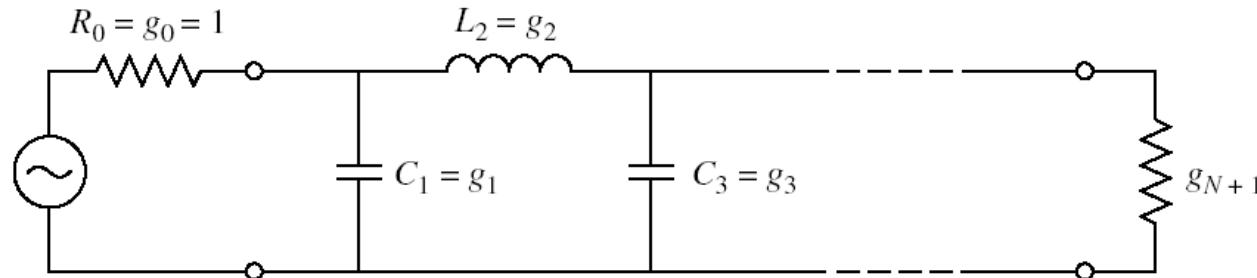
Higher Order High Pass Filter



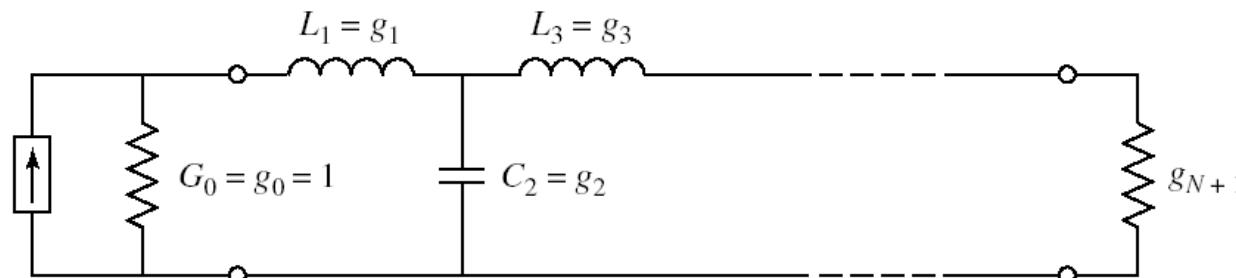
- Higher order high pass filters can be achieved by cascading multiple RC filters as shown below.
- The buffer is used to minimize the loading effect between the different stages of the filter.

N-section LC ladder circuit (low-pass filter prototypes)

Prototype beginning with serial element

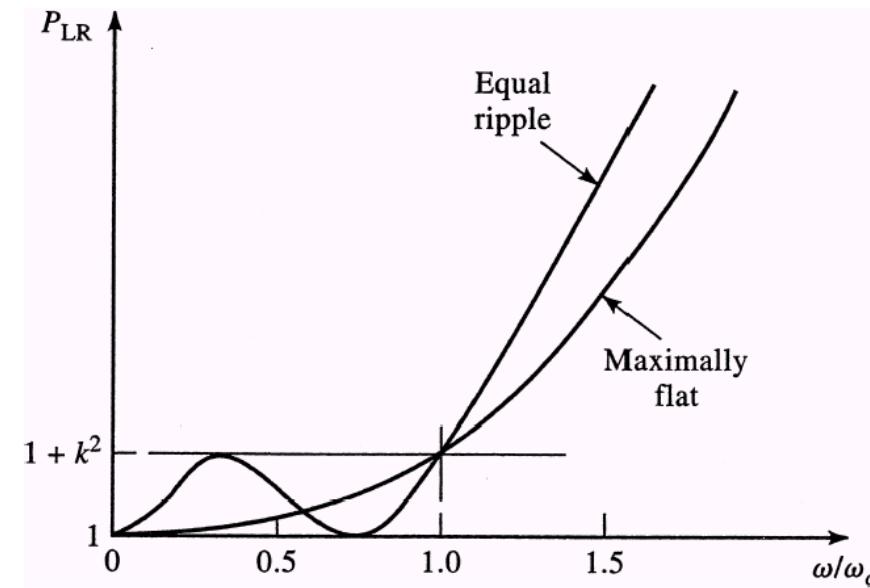
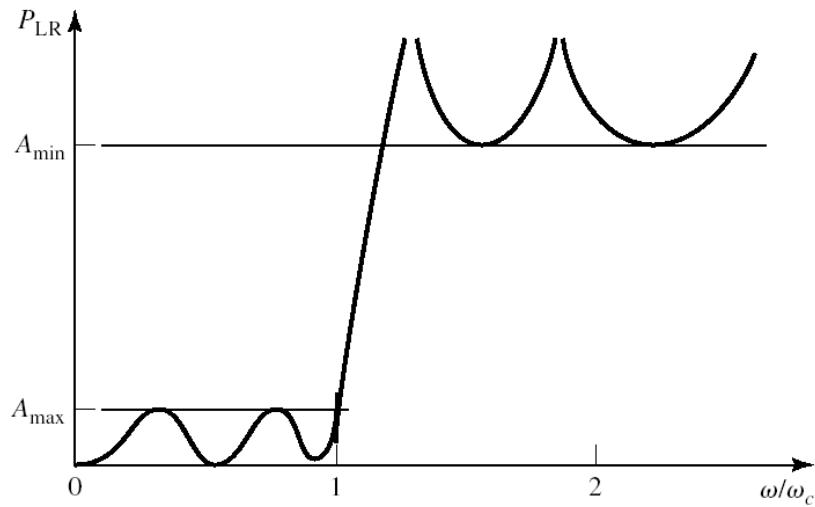


Prototype beginning with shunt element



Type of Responses for n-Section Prototype filter

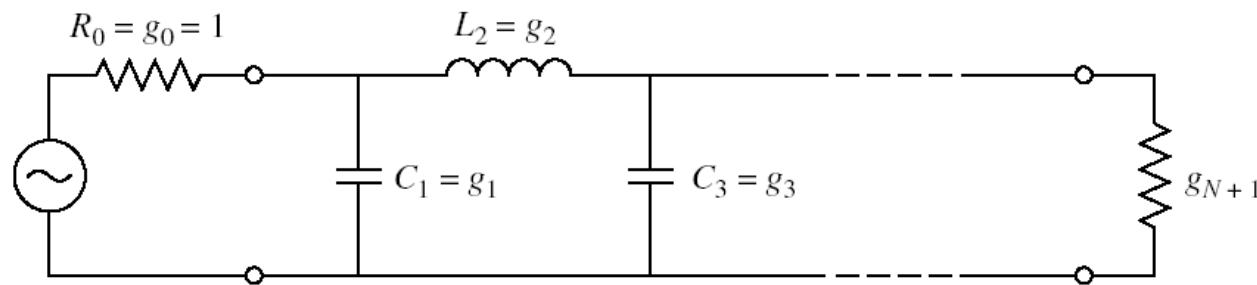
- Maximally flat or Butterworth
- Equal ripple or Chebyshev
- Elliptic function
- Linear phase



Scaling for low-pass

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

$$C_k = \frac{1}{Z_o g_k \omega_c} \quad L_k = \frac{Z_o}{g_k \omega_c}$$



Maximally flat or Butterworth filter

For low -pass power ratio response

$$H(\omega) = \left(1 + C^2 \left(\frac{\omega}{\omega_c} \right)^{2n} \right)^{-1}$$

where

C=1 for -3dB cutoff point

n= order of filter

ω_c = cutoff frequency

No of order (or no of elements)

$$n = \frac{\log_{10}(10^{A/10} - 1)}{2 \log_{10}(\omega_1 / \omega_c)}$$

Where A is the attenuation at ω_1 point and $\omega_1 > \omega_c$

Prototype elements

$$g_0 = g_{n+1} = 1 \quad \begin{cases} \text{Series } R = Z_o \\ \text{Shunt } G = 1/Z_o \end{cases}$$

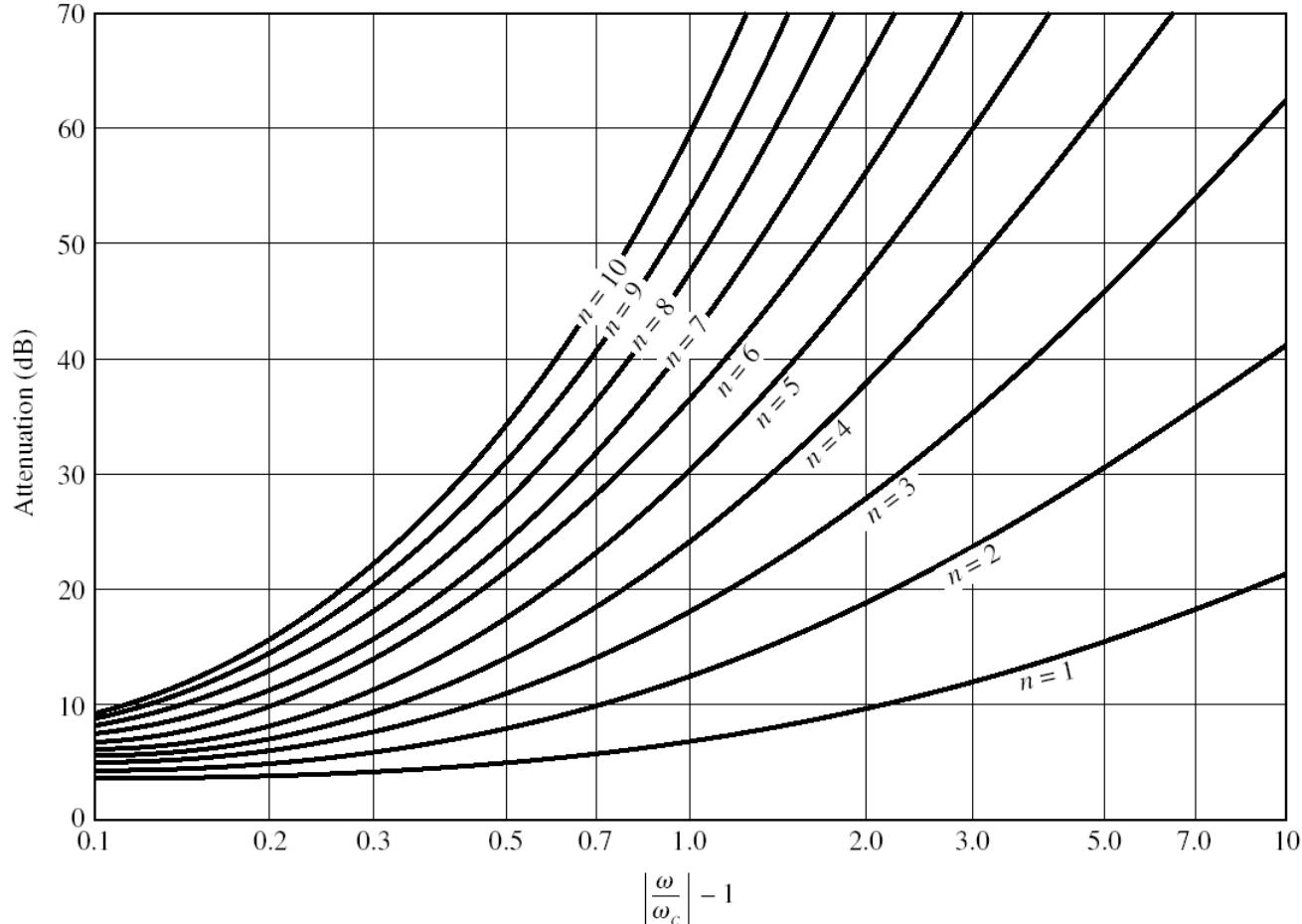
$$g_k = 2 \sin \left[\frac{(2k-1)\pi}{2n} \right]$$

$$L_k = \frac{Z_o g_k}{\omega_c} \quad \text{Series element}$$

$$C_k = \frac{g_k}{Z_o \omega_c} \quad \text{Shunt element}$$

k= 1,2,3.....n

Defining the Order of the Maximally flat or Butterworth filter



Example 8

Calculate the inductance and capacitance values for a maximally-flat low-pass filter that has a 3dB bandwidth of 400MHz. The filter is to be connected to 50 ohm source and load impedance. The filter must have a high attenuation of 20 dB at 1 GHz.

Solution

First , determine the number of elements

$$n = \frac{\log_{10}(10^{A/10} - 1)}{2 \log_{10}(\omega_1 / \omega_c)}$$
$$= \frac{\log_{10}(10^{20/10} - 1)}{2 \log_{10}(1000/400_c)} > 2.51$$

Thus choose an integer value , i.e n=3

Prototype values

$$g_0 = g_{3+1} = 1$$

$$g_1 = 2 \sin \left[\frac{(2-1)\pi}{2 \times 3} \right] = 1$$

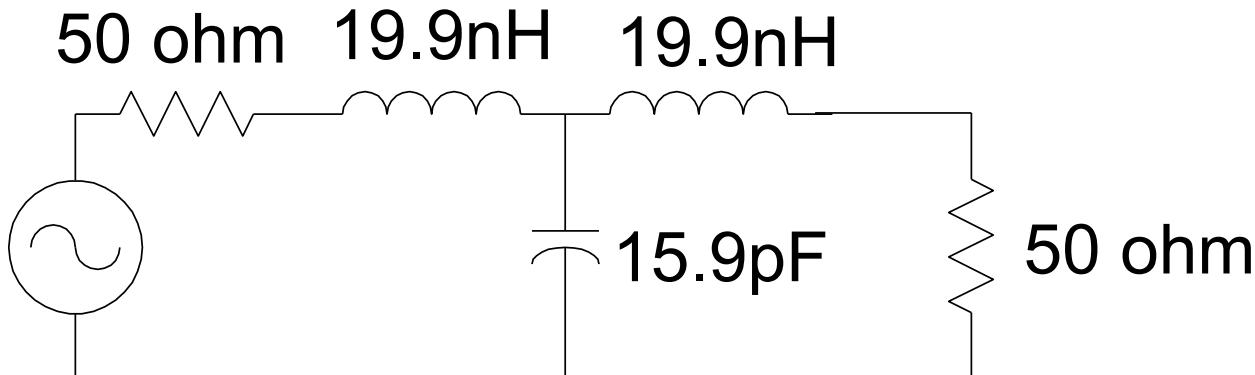
$$g_2 = 2 \sin \left[\frac{(2 \times 2 - 1)\pi}{2 \times 3} \right] = 2$$

$$g_3 = 2 \sin \left[\frac{(2 \times 3 - 1)\pi}{2 \times 3} \right] = 1$$

Continue

$$L_3 = L_1 = \frac{Z_o g_1}{\omega_c} = \frac{50 \times 1}{2 \times \pi \times 400 \times 10^6} = 19.9 \text{nH}$$

$$C_2 = \frac{g_2}{Z_o \omega_c} = \frac{2}{50 \times 2 \times \pi \times 400 \times 10^6} = 15.9 \text{pF}$$

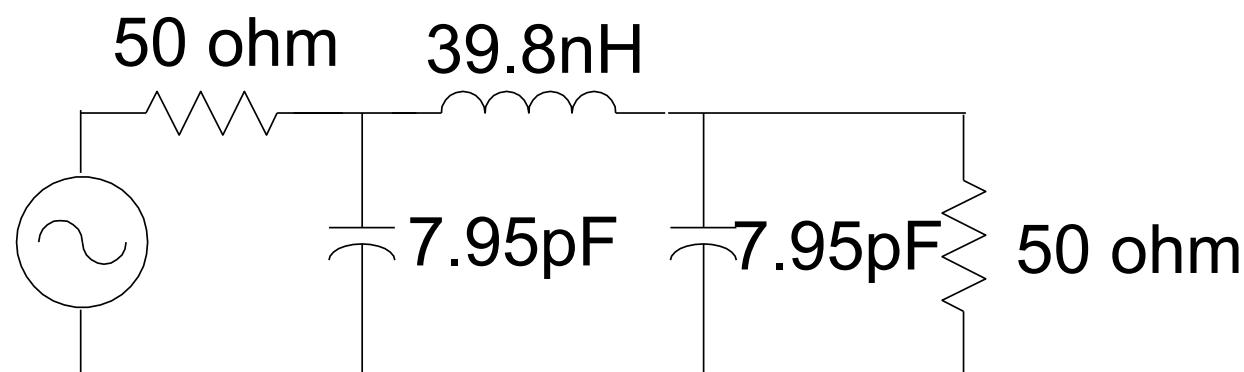


Continue

Or

$$C_3 = C_1 = \frac{g_1}{Z_o \omega_c} = \frac{1}{50 \times 2 \times \pi \times 400 \times 10^6} = 7.95 \text{ pF}$$

$$L_2 = \frac{Z_o g_2}{\omega_c} = \frac{50 \times 2}{2 \times \pi \times 400 \times 10^6} = 39.8 \text{ nH}$$



Equal-ripple Filter

For low -pass power ratio response

$$H(\omega) = \left(1 + F_o C_n^2 \left(\frac{\omega}{\omega_c} \right) \right)^{-1}$$

where

$C_n(x)$ =Chebyshev polynomial for n order

and argument of x where $x = \left(\frac{\omega}{\omega_c} \right)$

n = order of filter

ω_c = cutoff frequency

F_o =constant related to passband ripple

$$F_o = 10^{Lr/10} - 1$$

Where Lr is the ripple attenuation in pass-band

Chebyshev polynomial

$$C_0(x) = 1$$

$$C_1(x) = x$$

$$C_n(x) = 2x C_{n-1}(x) - C_{n-2}(x)$$

$$C_n(1) = 1 \quad i.e \omega = \omega_c$$

Continue

Prototype elements

$$g_0 = 1 \quad g_1 = \frac{2a_1}{F_2}$$
$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}} \quad \text{For } k=2,..,n$$

$$g_{n+1} = \begin{cases} 1 & \text{for } n \text{ odd} \\ \coth^2(F_1) & \text{for } n \text{ even} \end{cases}$$

$$L_k = \frac{Z_o g_k}{\omega_c} \quad \text{Series element}$$

$$C_k = \frac{g_k}{Z_o \omega_c} \quad \text{Shunt element}$$

where

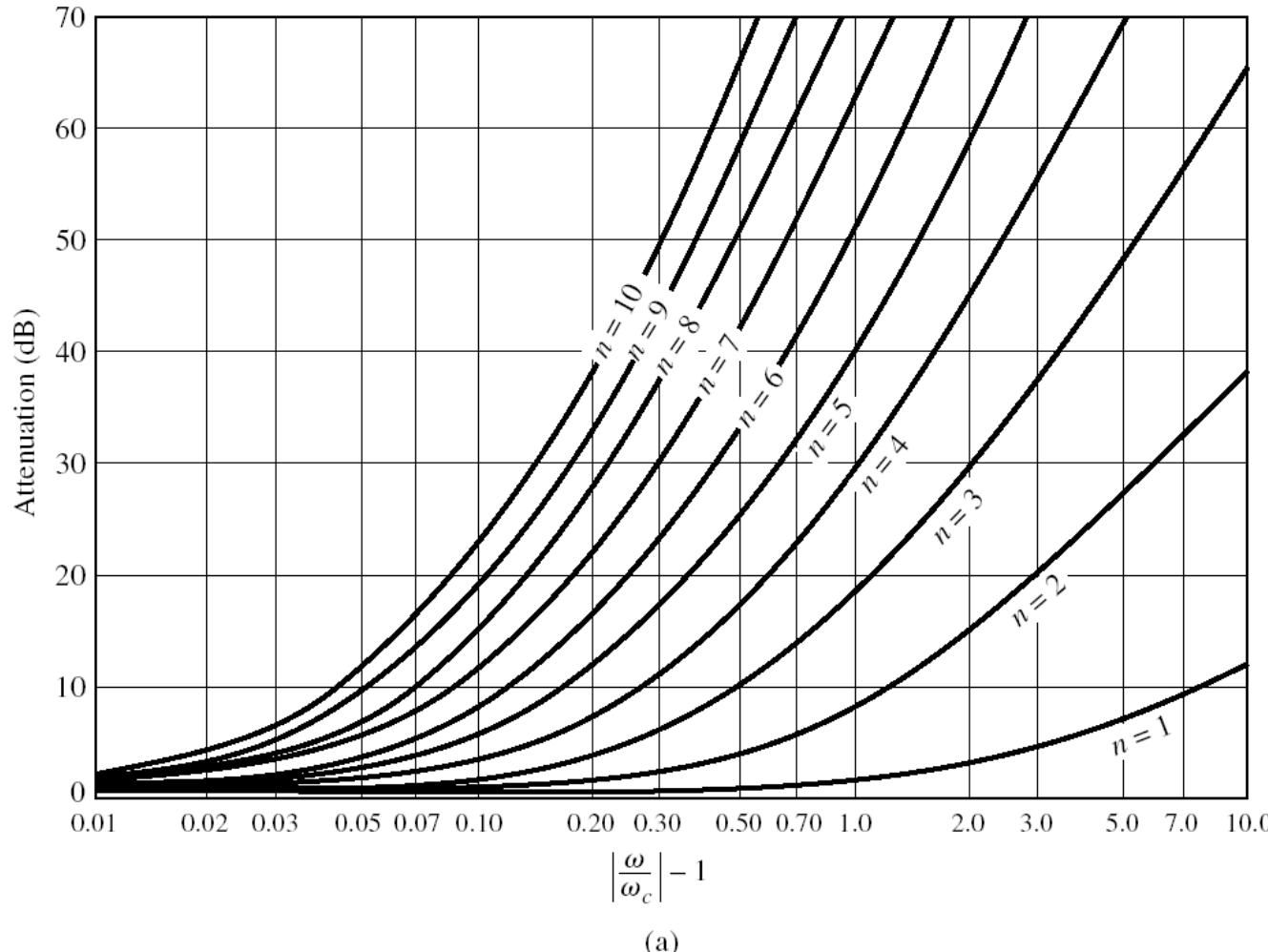
$$F_1 = \frac{1}{4} \ln \left\{ \coth \left(\frac{Lr}{17.372} \right) \right\}$$

$$F_2 = \sinh \left(\frac{2F_1}{n} \right)$$

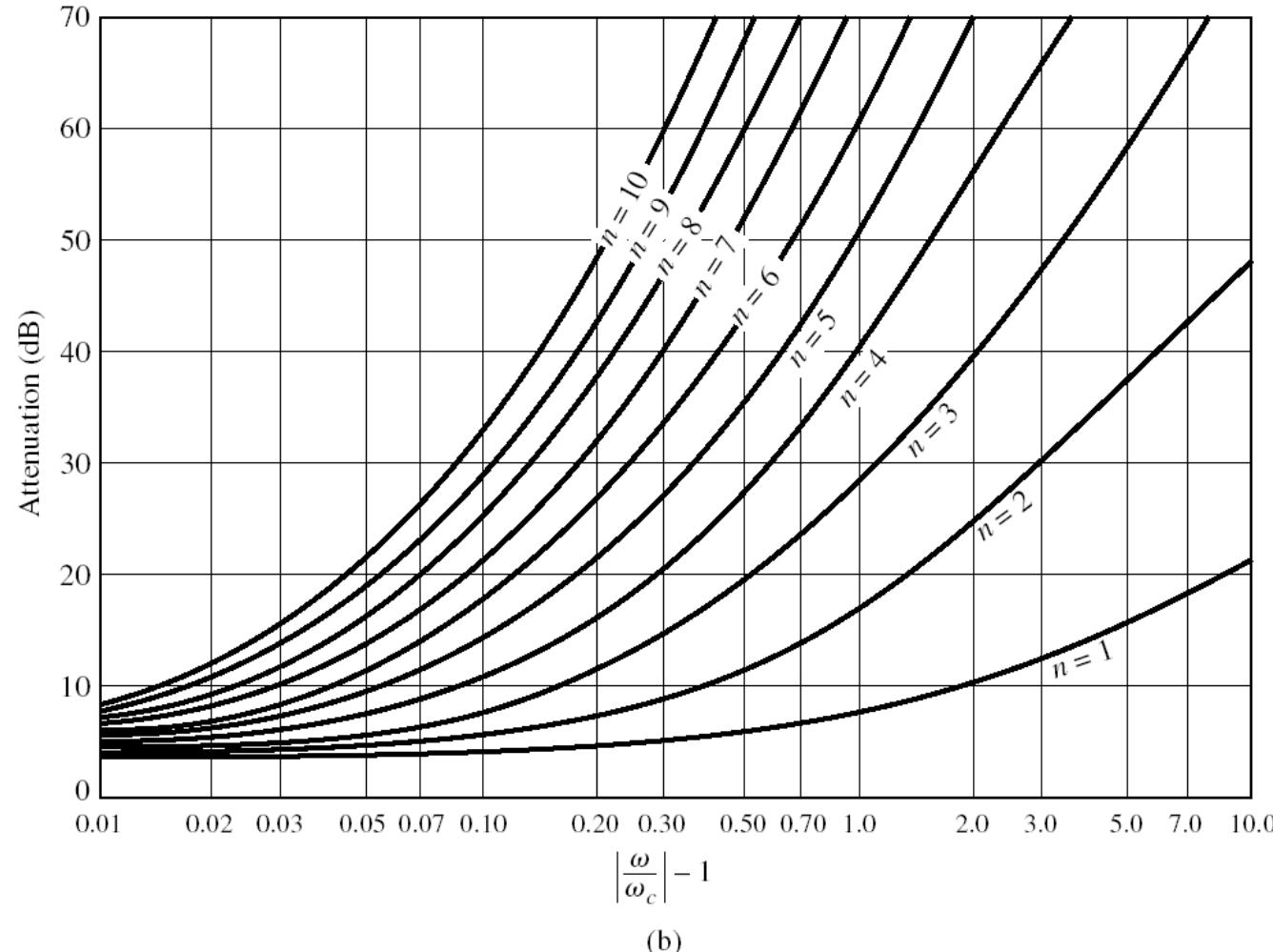
$$a_k = \sin \left\{ \frac{(2k-1)\pi}{2n} \right\} \quad k = 1, 2, \dots, n$$

$$b_k = F_2^2 + \sin^2 \left(\frac{k\pi}{n} \right) \quad k = 1, 2, \dots, n$$

Equal-ripple Filter “prototype 0.5 dB ripple level”



Equal-Ripple Filter “prototype 3 dB ripple level”



(b)

Example 9

Design a 3 section Chebyshev low-pass filter that has a ripple of 0.05dB and cutoff frequency of 1 GHz.

From the formula given we have

$$F_1 = 1.4626$$

$$F_2 = 1.1371$$

$$a_1 = 0.5 \quad a_2 = 1.0 \quad a_3 = 0.5$$

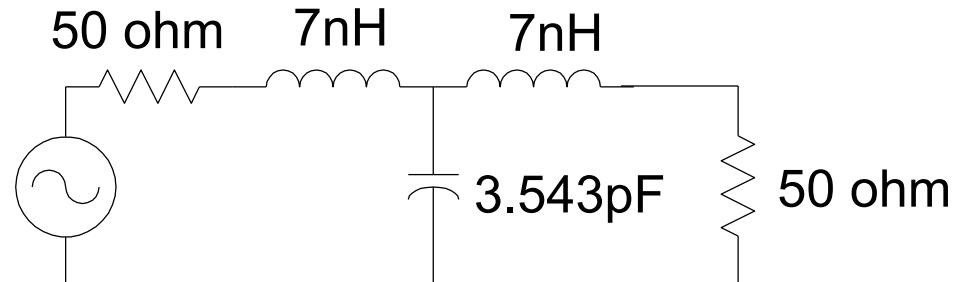
$$b_1 = 2.043 \quad b_2 = 2.043$$

$$g_1 = g_3 = 0.8794$$

$$g_2 = 1.1132$$

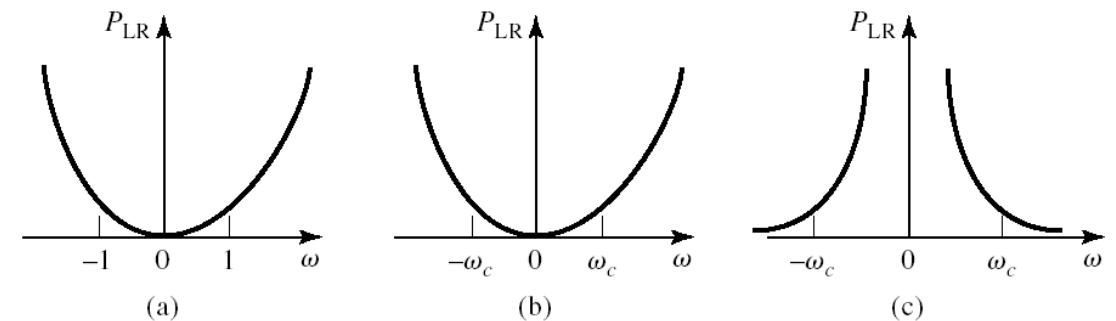
$$L_1 = L_3 = \frac{50 \times 0.8794}{2\pi \times 10^9} = 7 \text{ nH}$$

$$C_2 = \frac{1.1132}{50 \times 2\pi \times 10^9} = 3.543 \text{ pF}$$



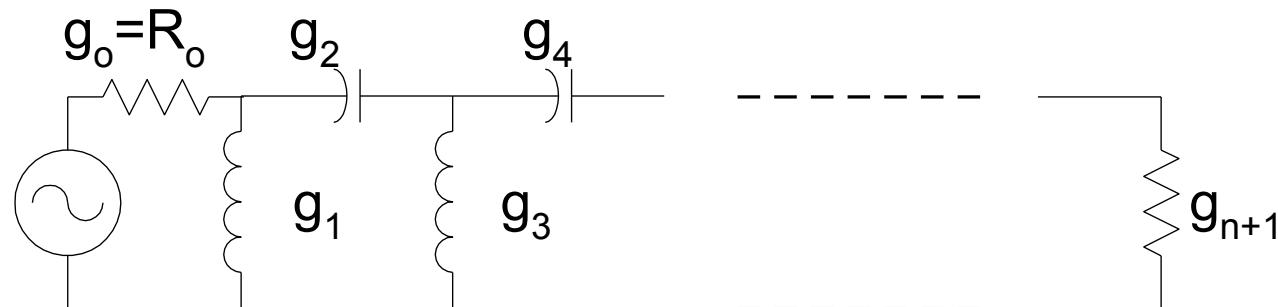
Transformation from Low-Pass to High-Pass

$$\omega \leftarrow \frac{\omega_c}{\omega}$$



- Series inductor L_k must be replaced by capacitor C'_k
- Shunts capacitor C_k must be replaced by inductor L'_k

$$C_k = \frac{1}{Z_o g_k \omega_c} \quad L_k = \frac{Z_o}{g_k \omega_c} \quad L'_k = \frac{R_o}{\omega_c C_k} \quad C'_k = \frac{1}{R_o \omega_c L_k}$$



Transformation from Low-Pass to Band-Pass

$$\omega \leftarrow \frac{1}{\Omega} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) \quad \text{where} \quad \Omega = \frac{\omega_2 - \omega_1}{\omega_o} \quad \text{and} \quad \omega_o = \sqrt{\omega_1 \omega_2}$$

Now we consider the series inductor

$$jX = j \frac{1}{\Omega} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) L_k = j \frac{1}{\Omega} \frac{\omega}{\omega_o} L_k - j \frac{1}{\Omega} \frac{\omega_o}{\omega} L_k = j\omega L'_{sk} - \frac{j}{\omega C'_{sk}}$$

$$L_{sk} = \frac{L_k}{\Omega \omega_o} \quad C_{sk} = \frac{\Omega}{\omega_o L_k} \quad L_k = Z_o g_k \boxed{\text{normalized}}$$

- Thus , series inductor L_k must be replaced by serial L_{sk} and C_{sk}



Impedance= series

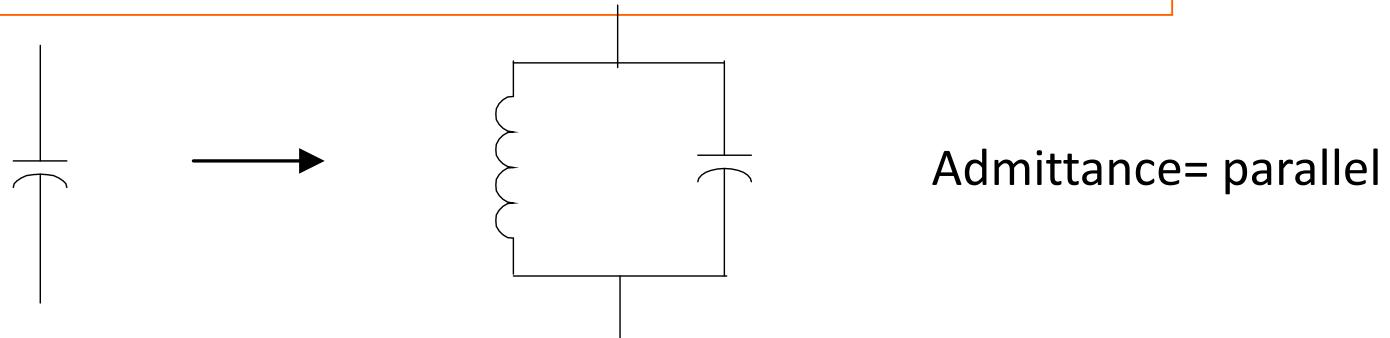
Continue

Now we consider the shunt capacitor

$$jB_k = j \frac{1}{\Omega} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) C_k = j \frac{1}{\Omega} \frac{\omega}{\omega_o} C_k - j \frac{1}{\Omega} \frac{\omega_o}{\omega} C_k = j\omega C'_{pk} - \frac{j}{\omega L'_{pk}}$$

$$L_{pk} = \frac{\Omega}{\omega_o C_k} \quad C_{pk} = \frac{C_k}{\Omega \omega_o} \quad C_k = \frac{g_k}{Z_o}$$

- Shunts capacitor C_k must be replaced by parallel L_{pk} and C_{pk}



Transformation from low-pass to band-stop

$$\omega \leftarrow -\Omega \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)^{-1}$$

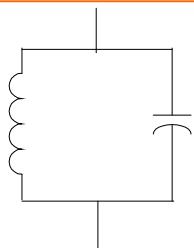
where $\Omega = \frac{\omega_2 - \omega_1}{\omega_o}$ and $\omega_o = \sqrt{\omega_1 \omega_2}$

Now we consider the series inductor --convert to admittance

$$j \frac{1}{X_k} = j \frac{1}{\Omega L_k} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) = j \frac{1}{\Omega} \frac{\omega}{\omega_o L_k} - j \frac{1}{\Omega} \frac{\omega_o}{\omega L_k} = j \omega C'_{pk} - \frac{j}{\omega L'_{pk}}$$

$$L_{pk} = \frac{\Omega L_k}{\omega_o} \quad C_{pk} = \frac{1}{\omega_o \Omega L_k} \quad L_k = Z_o g_k$$

- Thus , series inductor L_k must be replaced by parallel L_{pk} and C_{kp}



admittance = parallel

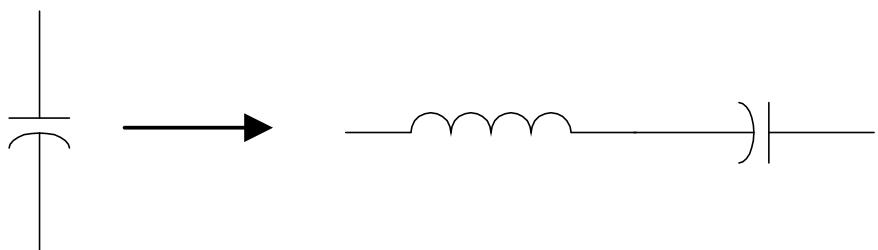
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Now we consider the shunt capacitor --> convert to impedance

$$j \frac{1}{B_k} = j \frac{1}{\Omega C_k} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) = j \frac{1}{\Omega} \frac{\omega}{\omega_o C_k} - j \frac{1}{\Omega} \frac{\omega_o}{\omega C_k} = j \omega L'_{sk} - \frac{j}{\omega C'_{sk}}$$

$$L_{sk} = \frac{1}{\Omega \omega_o C_k} \quad C_{sk} = \frac{\Omega C_k}{\omega_o} \quad C_k = \frac{g_k}{Z_o}$$

- Shunts capacitor C_k must be replaced by series L_{sk} and C_{sk}



Example 10

Design a band-pass filter having a 0.5 dB ripple response, with N=3. The center frequency is 1GHz, the bandwidth is 10%, and the impedance is 50Ω.

Solution

From table 8.4 Pozar pg 406.

$$g_0 = 1, g_1 = 1.5963, g_2 = 1.0967, g_3 = 1.5963, g_4 = 1.000$$

Let's first and third elements are equivalent to series inductance and $g_1 = g_3$, thus

$$L_{s1} = L_{s3} = \frac{Z_o g_1}{\Omega \omega_o} = \frac{50 \times 1.5963}{0.1 \times 2\pi \times 10^9} = 127nH$$

$$L_k = Z_o g_k$$

$$C_{s1} = C_{s3} = \frac{\Omega}{\omega_o Z_o g_1} = \frac{0.1}{2\pi \times 10^9 \times 50 \times 1.5963} = 0.199 pF$$

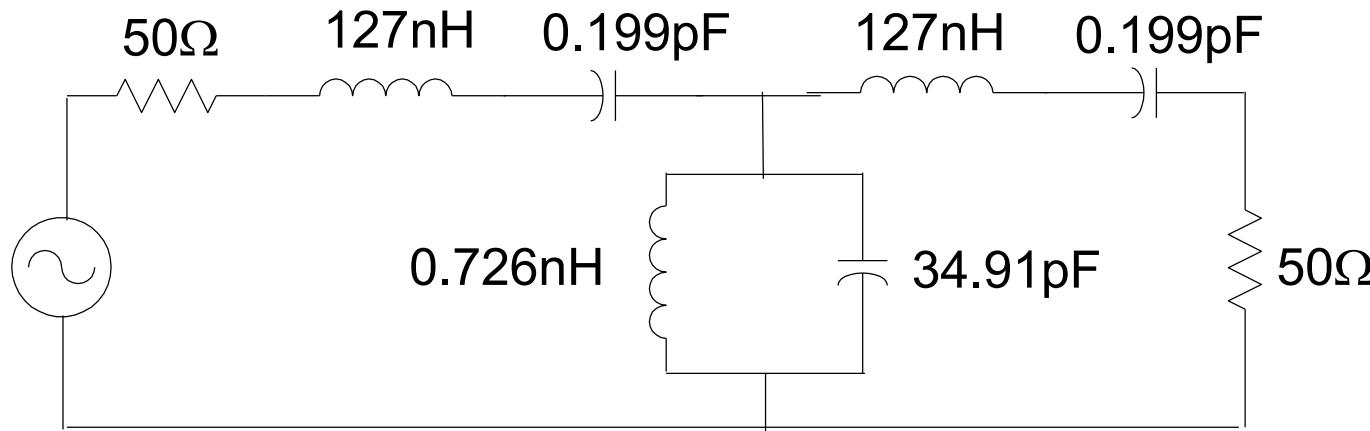
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Second element is equivalent to parallel capacitance, thus

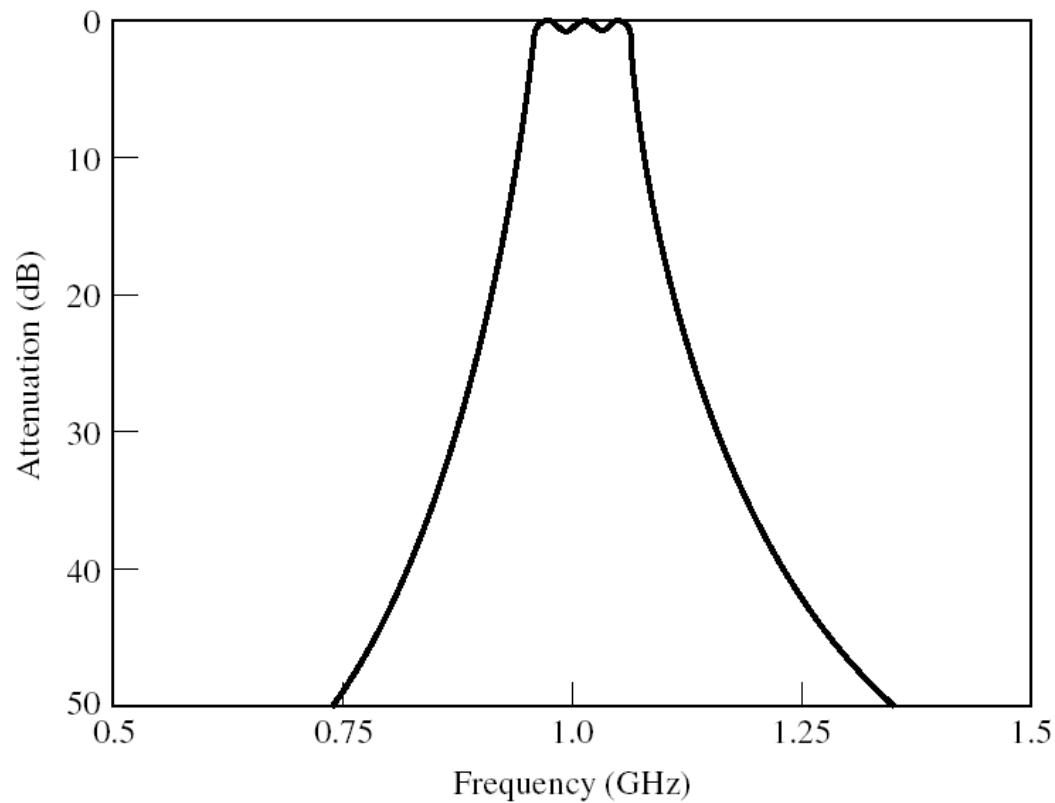
$$L_{p2} = \frac{\Omega Z_o}{\omega_o g_2} = \frac{0.1 \times 50}{2\pi \times 10^9 \times 1.0967} = 0.726nH$$

$$C_k = \frac{g_k}{Z_o}$$

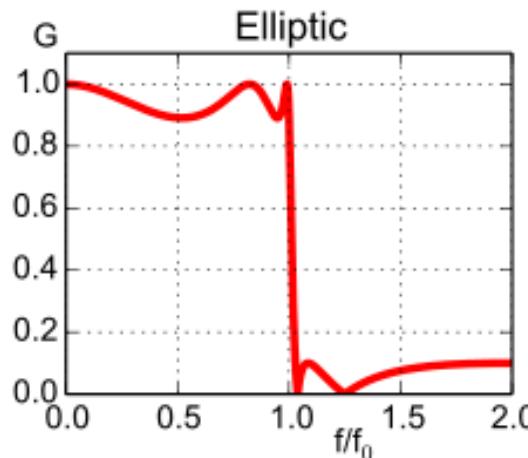
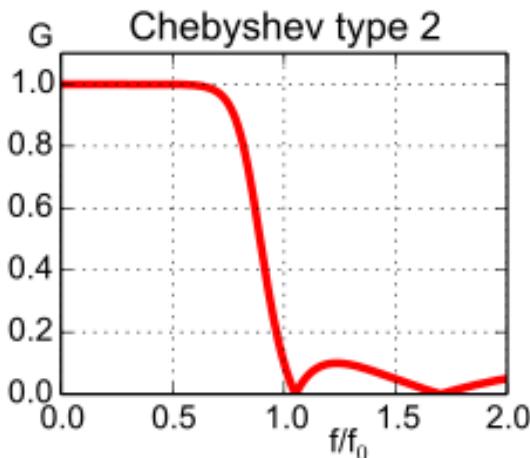
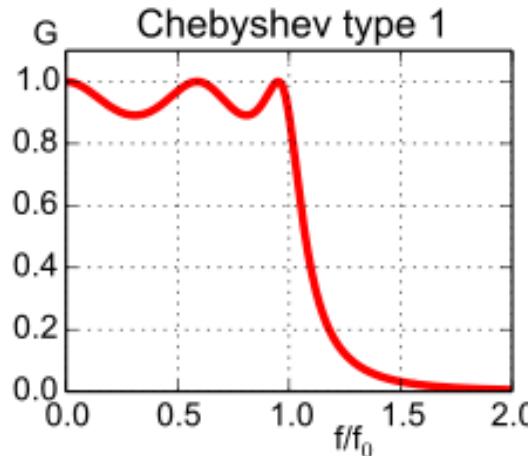
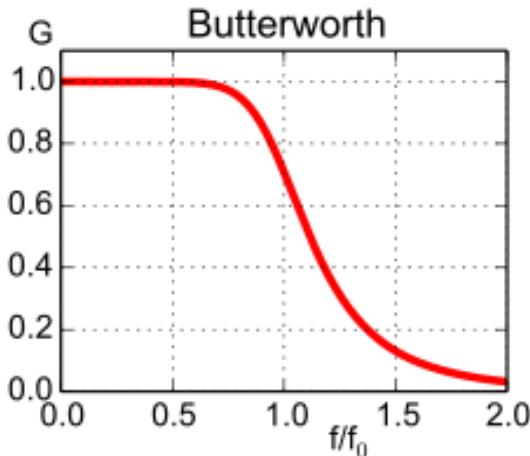
$$C_{p2} = \frac{g_2}{Z_o \Omega \omega_o} = \frac{1.0967}{50 \times 0.1 \times 2\pi \times 10^9} = 34.91pF$$



Continue

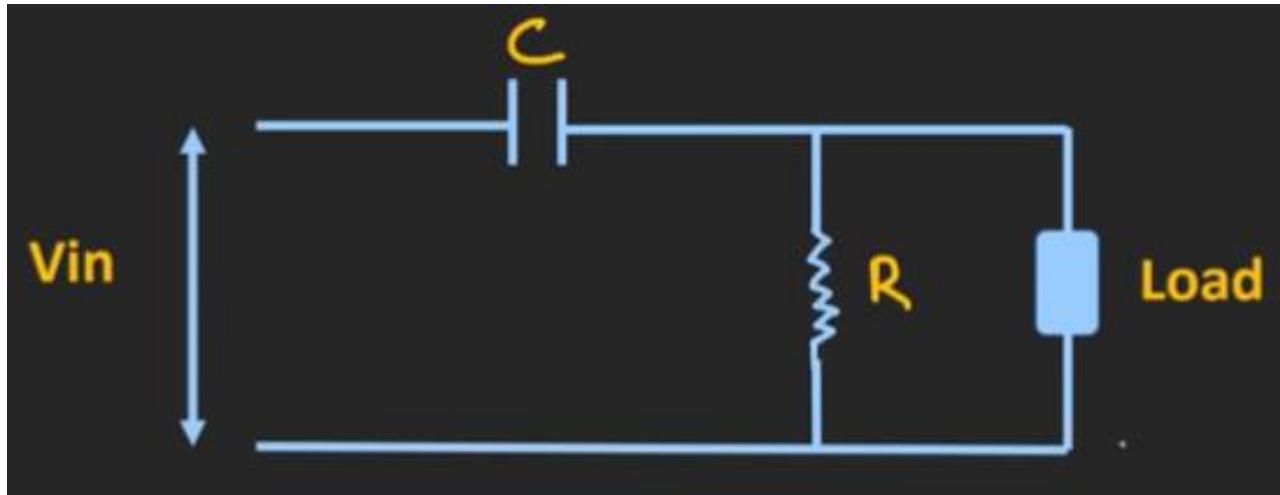


Comparison of Filter Responses



Type	Passband	Stopband	Roll-off	Step Response
Butterworth	Flat	Monotonic	Good	Good
Chebyshev	Rippled	Monotonic	Very Good	Poor
Inverse Chebyshev	Flat	Rippled	Very Good	Good
Elliptic	Rippled	Rippled	Best	Poor

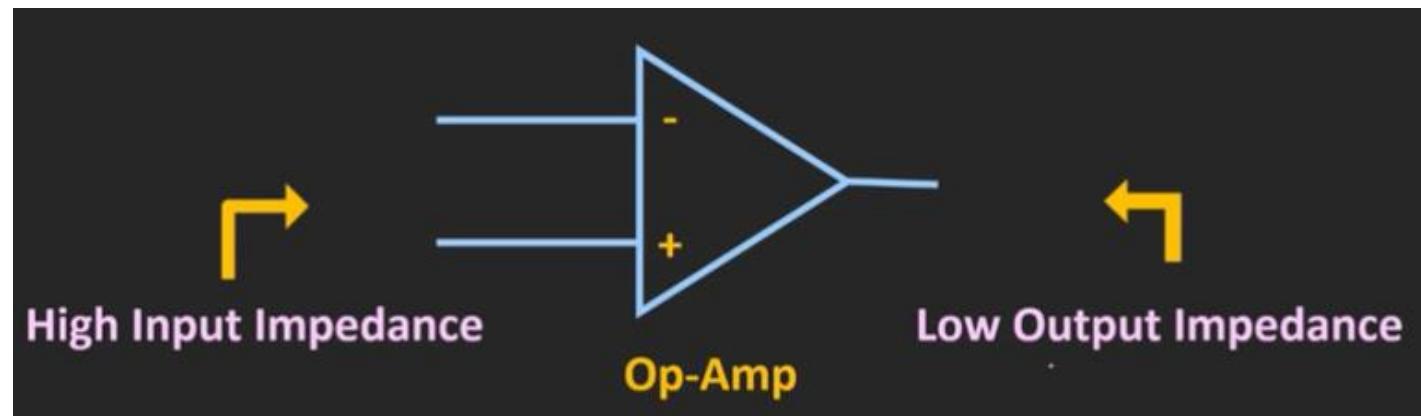
Active vs. Passive Filters



- The maximum magnitude of the voltage transfer function of a passive filter is at best equal to 1.
- Active filters incorporate amplifiers with the filter elements so that the filter has gain.
- The load in passive filters will impact the value of R . For example in this high pass filter, the load will alter the value of R and hence the cut-off frequency

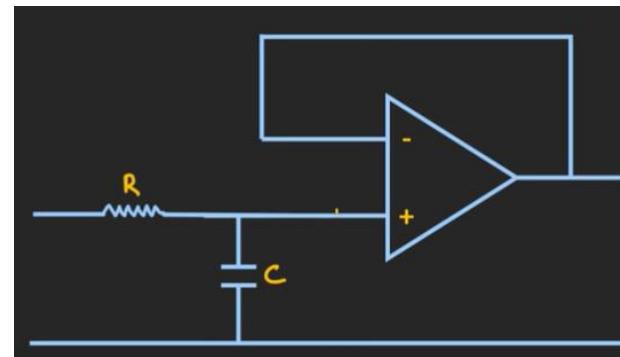
Active Low Pass Filters

- The Op-Amp has a high input impedance and low output impedance.



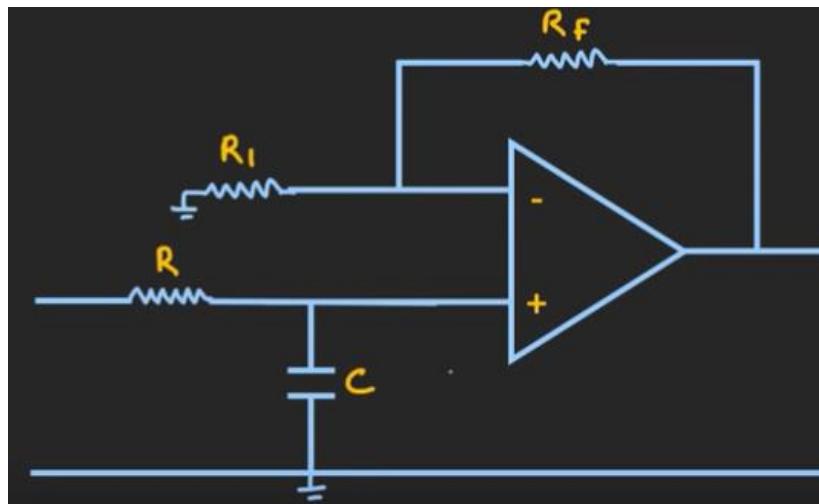
So it can be used as a buffer to isolate the different stages of the filter.

But this configuration has no gain



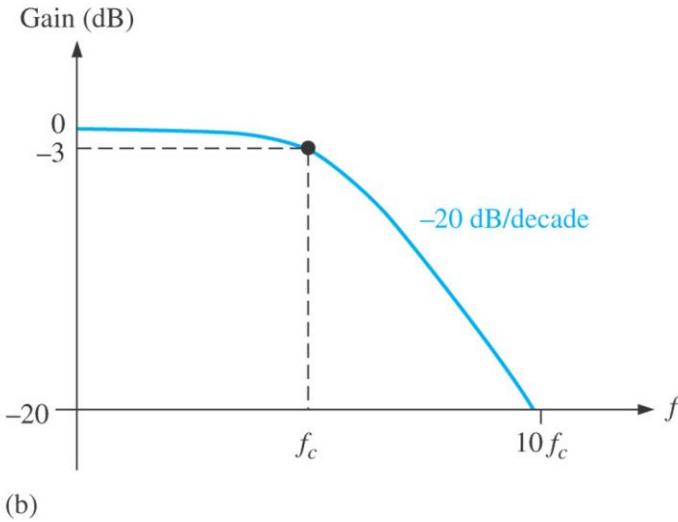
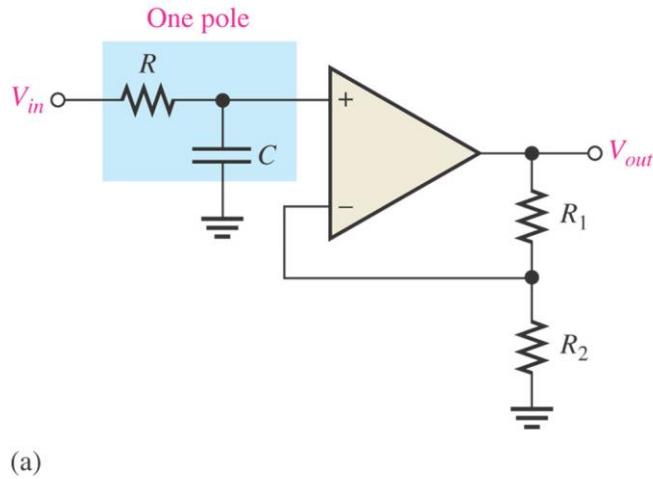
Active Low Pass Filters

- To have a gain for the filter, the Op-Amp can be connected as a negative feedback with positive gain (non-inverting amplifier) as shown below:



$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + \left(\frac{f}{fc} \right)^2}} \times \left(1 + \frac{R_f}{R_1} \right)$$

Active Low Pass Filters



One-pole (first-order) low-pass filter.

- The **critical frequency, f_c** is determined by the values of **R** and **C** in the frequency-selective RC circuit.

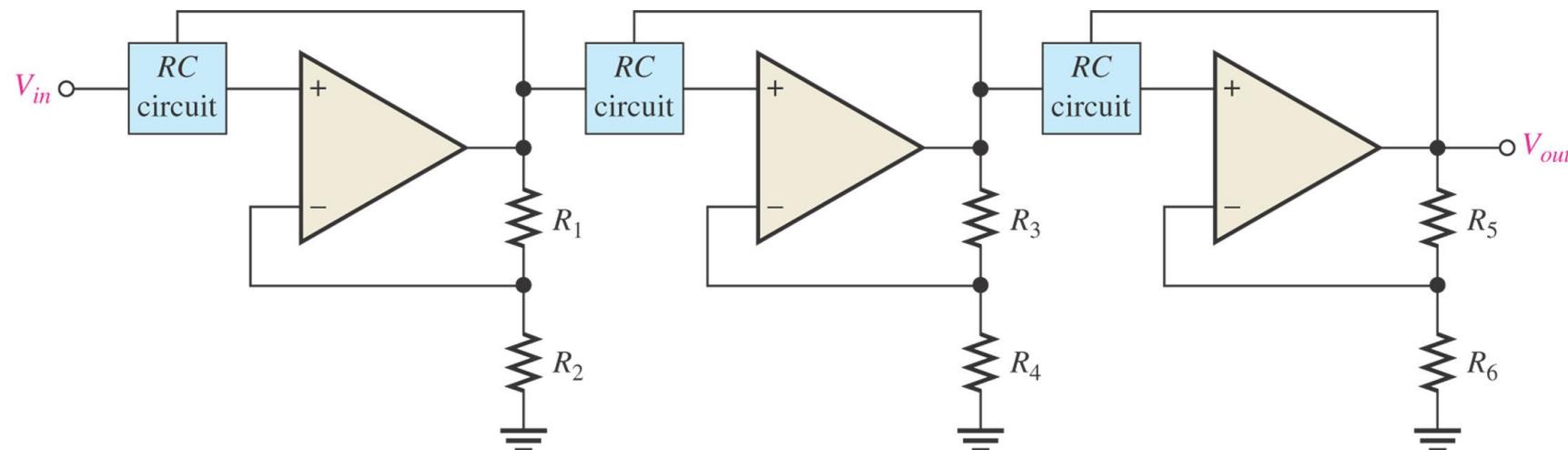
$$f_c = \frac{1}{2\pi RC}$$

- Each **RC** set of filter components represents a **pole**.

- This filter provides a roll-off rate of -20 dB/decade above the critical frequency. **Greater roll-off rates** can be achieved with **more poles**.

Active Low Pass Filters

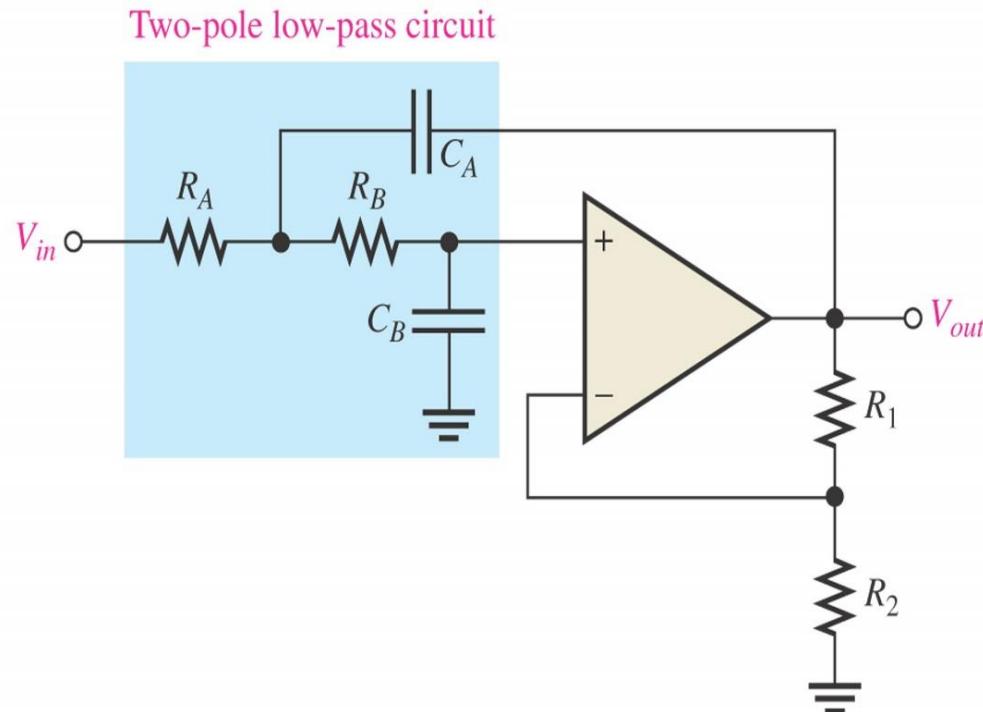
- The number of filter poles can be increased by **cascading**. To obtain a filter with three poles, cascade a two-pole with one-pole filters.



Three-pole (third-order) low-pass filter.

Sallen-Key Low-Pass Filter

- Sallen-Key is one of the most common configurations for a second order (two-pole) filter.



Basic Sallen-Key low-pass filter.

- There are two low-pass RC circuits that provide a roll-off of -40 dB/decade above f_c .
- One RC circuit consists of R_A and C_A , and the second circuit consists of R_B and C_B .

Sallen-Key Low-Pass Filter

- The critical frequency for the Sallen-Key filter is :

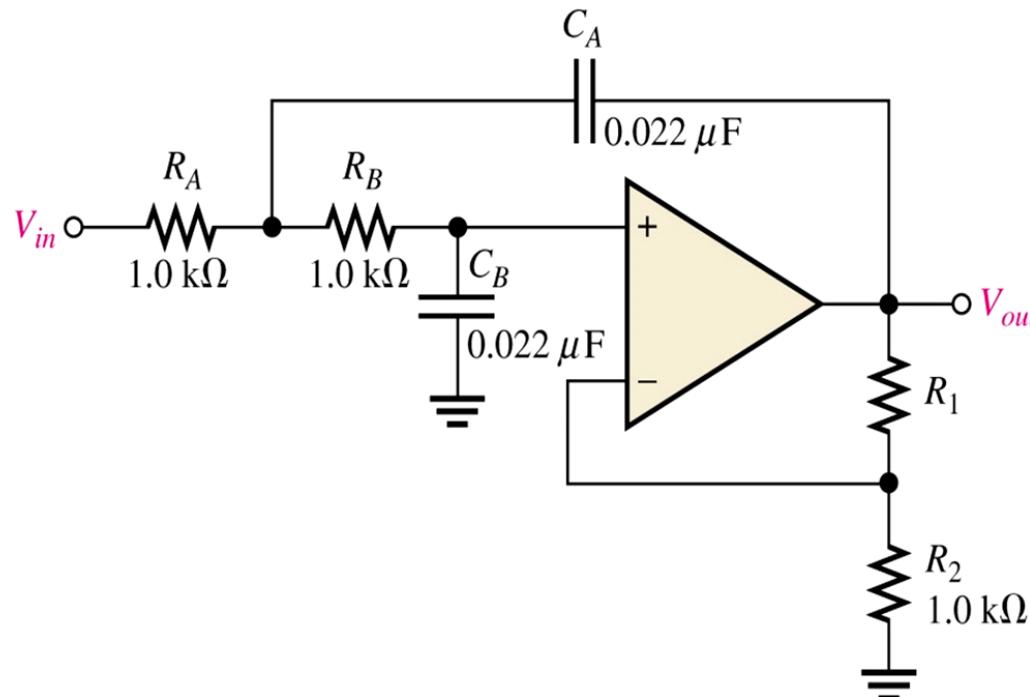
$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

- For $R_A = R_B = R$ and $C_A = C_B = C$, thus the critical frequency :

$$f_c = \frac{1}{2\pi R C}$$

Example 11

- Determine critical frequency
- Set the value of R_1 for Butterworth response by giving that Butterworth response for second order is 0.586



- Critical frequency

$$f_c = \frac{1}{2\pi RC} = 7.23\text{ kHz}$$

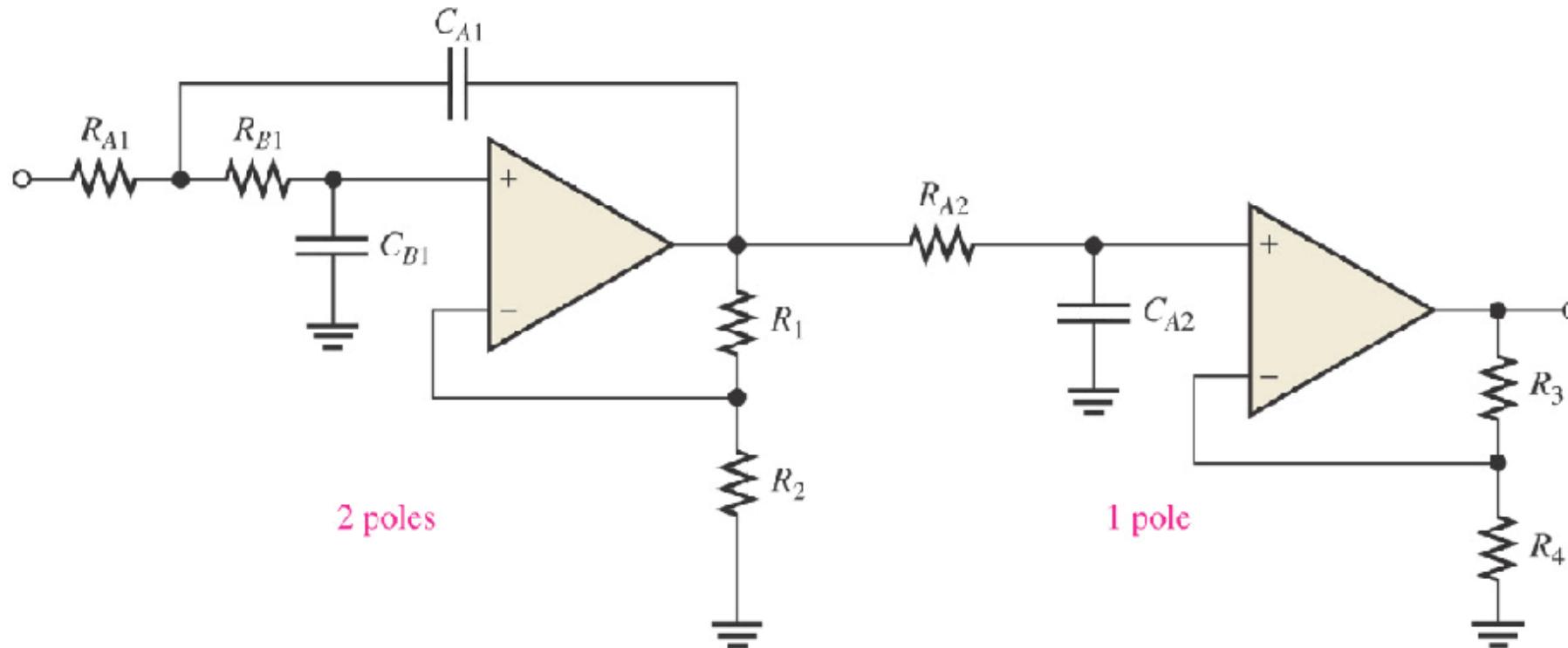
- Butterworth response given $R_1/R_2 = 0.586$

$$R_1 = 0.586R_2$$

$$R_1 = 586\text{ k}\Omega$$

Cascading Low-Pass Filter

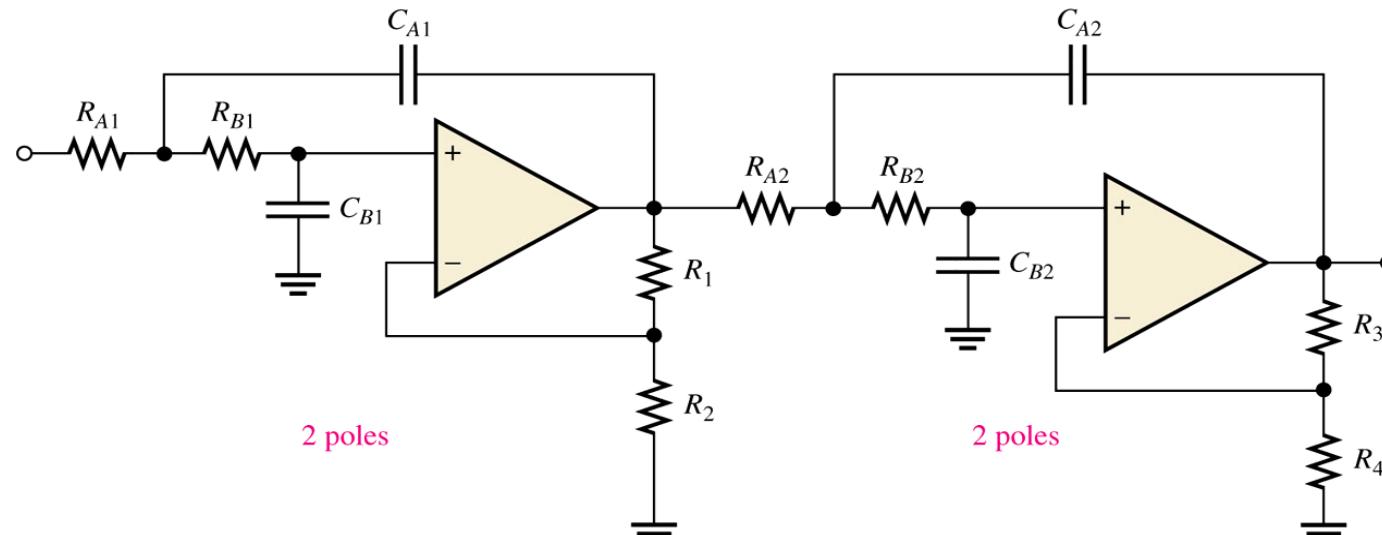
- A three-pole filter is required to provide a roll-off rate of -60 dB/decade. This is done by cascading a two-pole Sallen-Key low-pass filter and a single-pole low-pass filter.



Cascaded low-pass filter: third-order configuration.

Cascading Low-Pass Filter

- A four-pole filter is required to provide a roll-off rate of -80 dB/decade. This is done by cascading a two-pole Sallen-Key low-pass filter and a two-pole Sallen-Key low-pass filter.

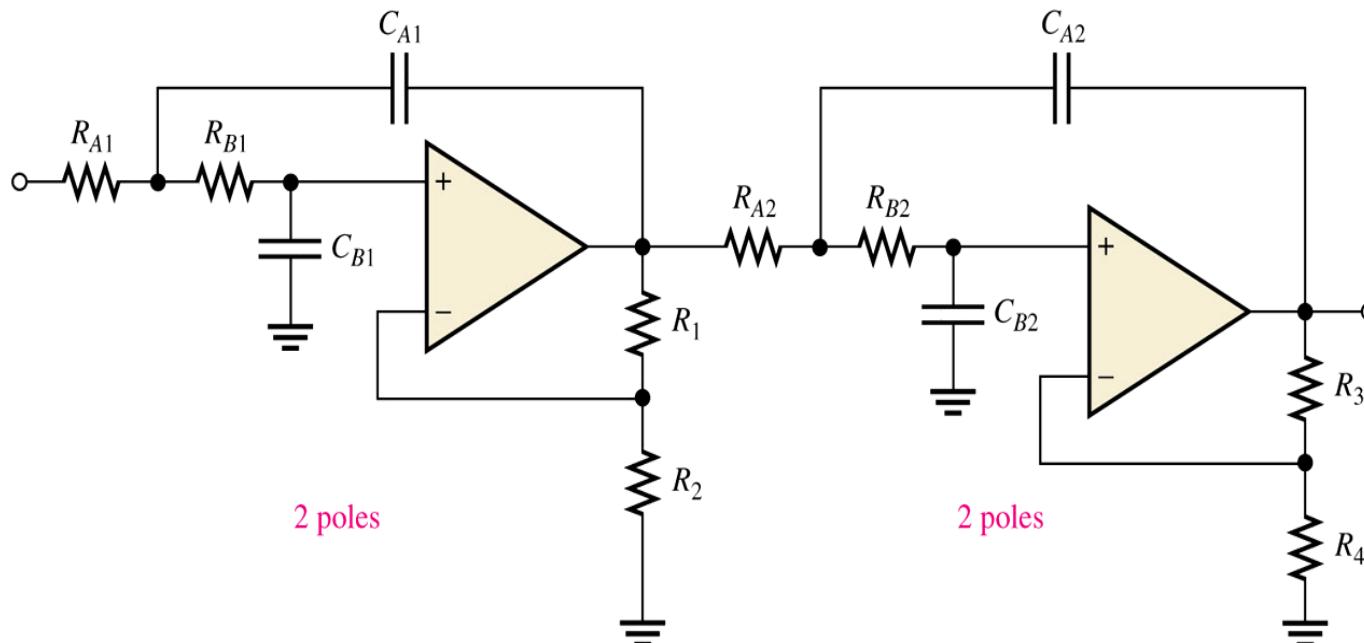


(b) Fourth-order configuration

Cascaded low-pass filter: fourth-order configuration.

Example 12

- Determine the capacitance values required to produce a critical frequency of 2680 Hz if all resistors in RC low pass circuit is $1.8\text{k}\Omega$



(b) Fourth-order configuration

- Both stages must have the same f_c . Assume equal-value of capacitor

$$f_c = \frac{1}{2\pi RC}$$

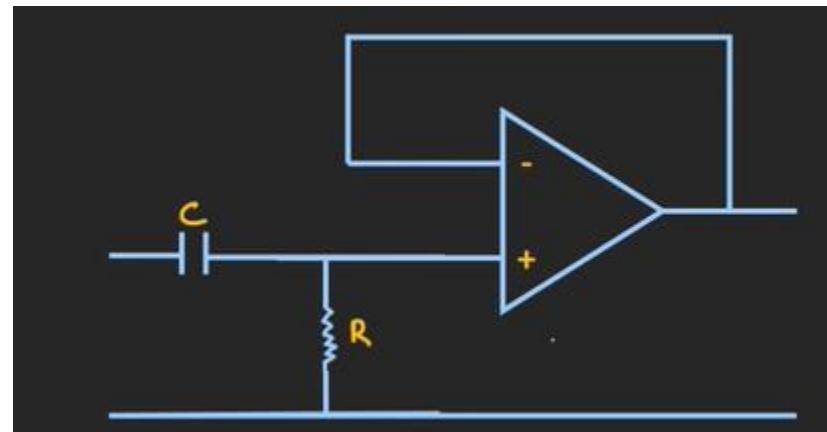
$$C = \frac{1}{2\pi f_c R} = 0.033\mu\text{F}$$

$$C_{A1} = C_{B1} = C_{A2} = C_{B2} = 0.033\mu\text{F}$$

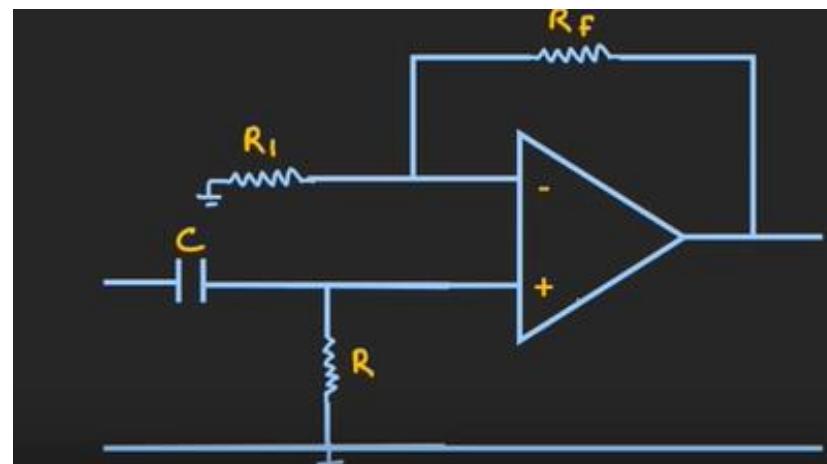
Active High Pass Filters

- Similarly to the design of low pass active filter, we can design high pass active filter.

High pass filter with no gain.



High pass filter with gain.



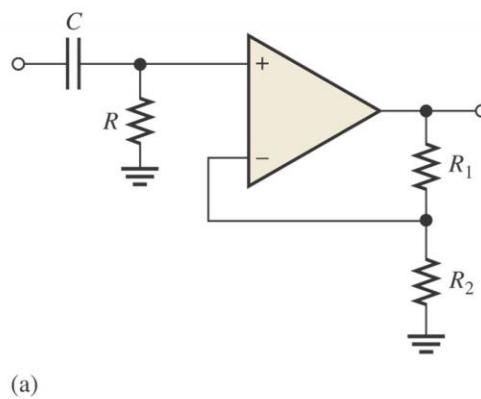
$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{\left(\frac{f}{f_c} \right)}{\sqrt{1 + \left(\frac{f}{f_c} \right)^2}} \times \left(1 + \frac{R_f}{R_1} \right)$$

Single-Pole High Pass Filter

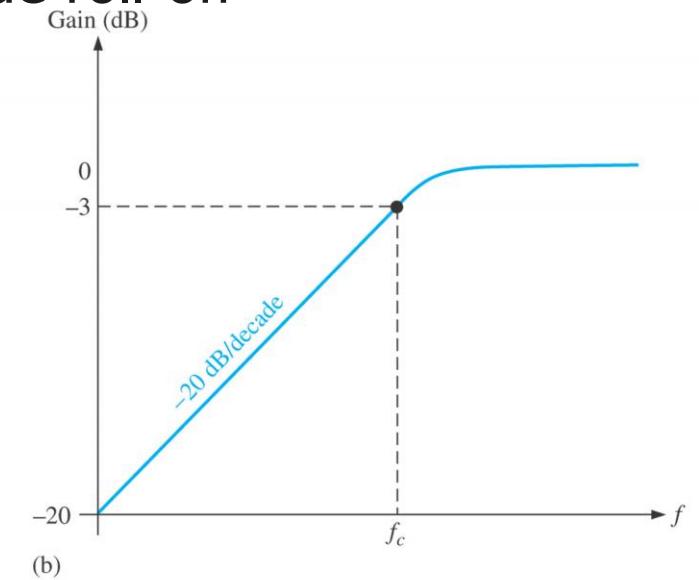
- In high-pass filters, the roles of the **capacitor** and **resistor** are **reversed** in the RC circuits as shown from Figure (a). The negative feedback circuit is the same as for the low-pass filters.
- Figure (b) shows a high-pass active filter with a -20dB/decade roll-off

- The critical frequency of the single-pole filter is :

$$f_c = \frac{1}{2\pi RC}$$



(a)

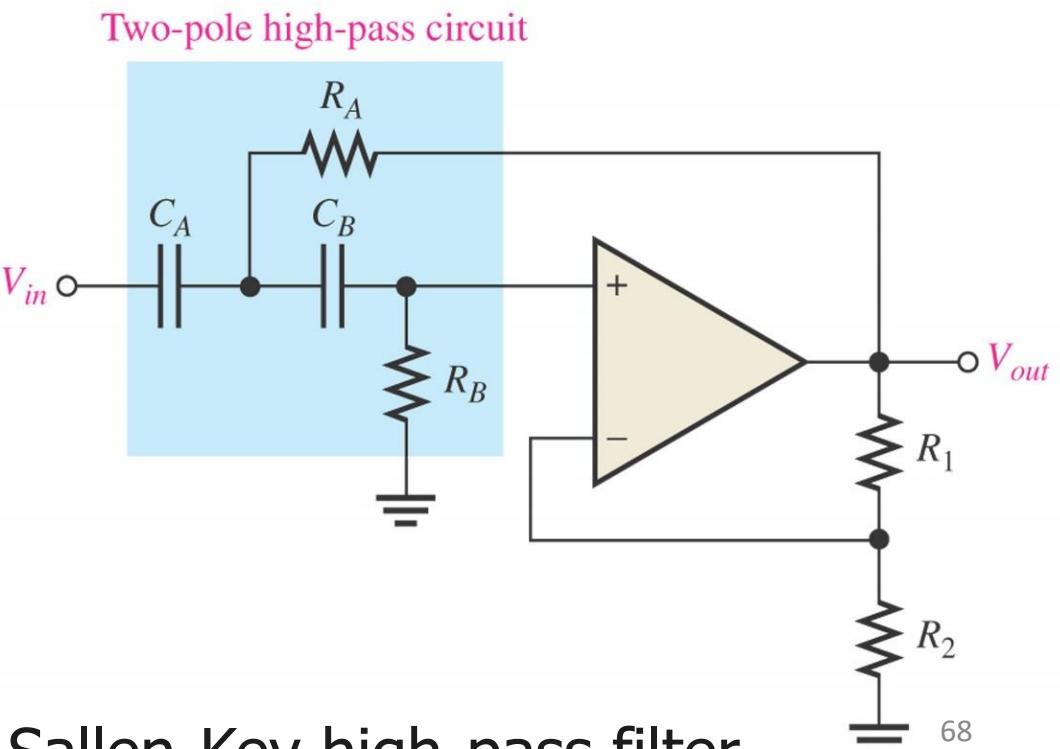


(b)

Single-pole active high-pass filter and response curve.

Sallen-Key High-Pass Filter

- Components R_A , C_A , R_B , and C_B form the **second order** (two-pole) frequency-selective circuit.
- The position of the resistors and capacitors in the frequency-selective circuit are **opposite** in low pass configuration.
- There are two high-pass RC circuits that provide a **roll-off of -40 dB/decade above f_c**
- The **response characteristics** can be optimized by proper selection of the **feedback resistors**, R_1 and R_2 .



Sallen-Key High-Pass Filter

- The critical frequency for the Sallen-Key filter is :

$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

- For $R_A = R_B = R$ and $C_A = C_B = C$, thus the critical frequency :

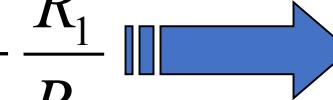
$$f_c = \frac{1}{2\pi R C}$$

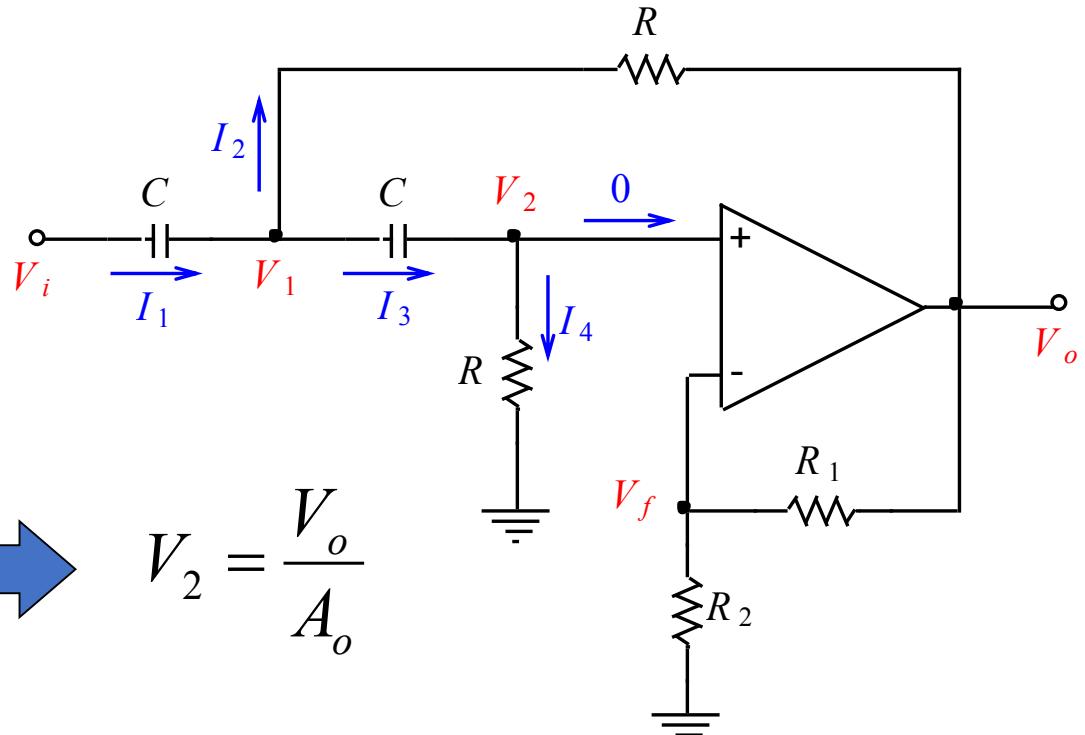
Sallen-Key High-Pass Filter

$$V_2 = V_f$$

$$V_f = \left(\frac{R_2}{R_1 + R_2} \right) V_o$$

$$A_o \equiv \frac{V_o}{V_2} = \frac{V_o}{V_f} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

≡ 



At node V_1 $I_1 = I_2 + I_3$

$$sC(V_i - V_1) = \frac{V_1 - V_o}{R} + sC(V_1 - V_2)$$

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$$sC(V_i - V_1) = \frac{V_1 - V_o}{R} + sC\left(V_1 - \frac{V_o}{A_o}\right)$$

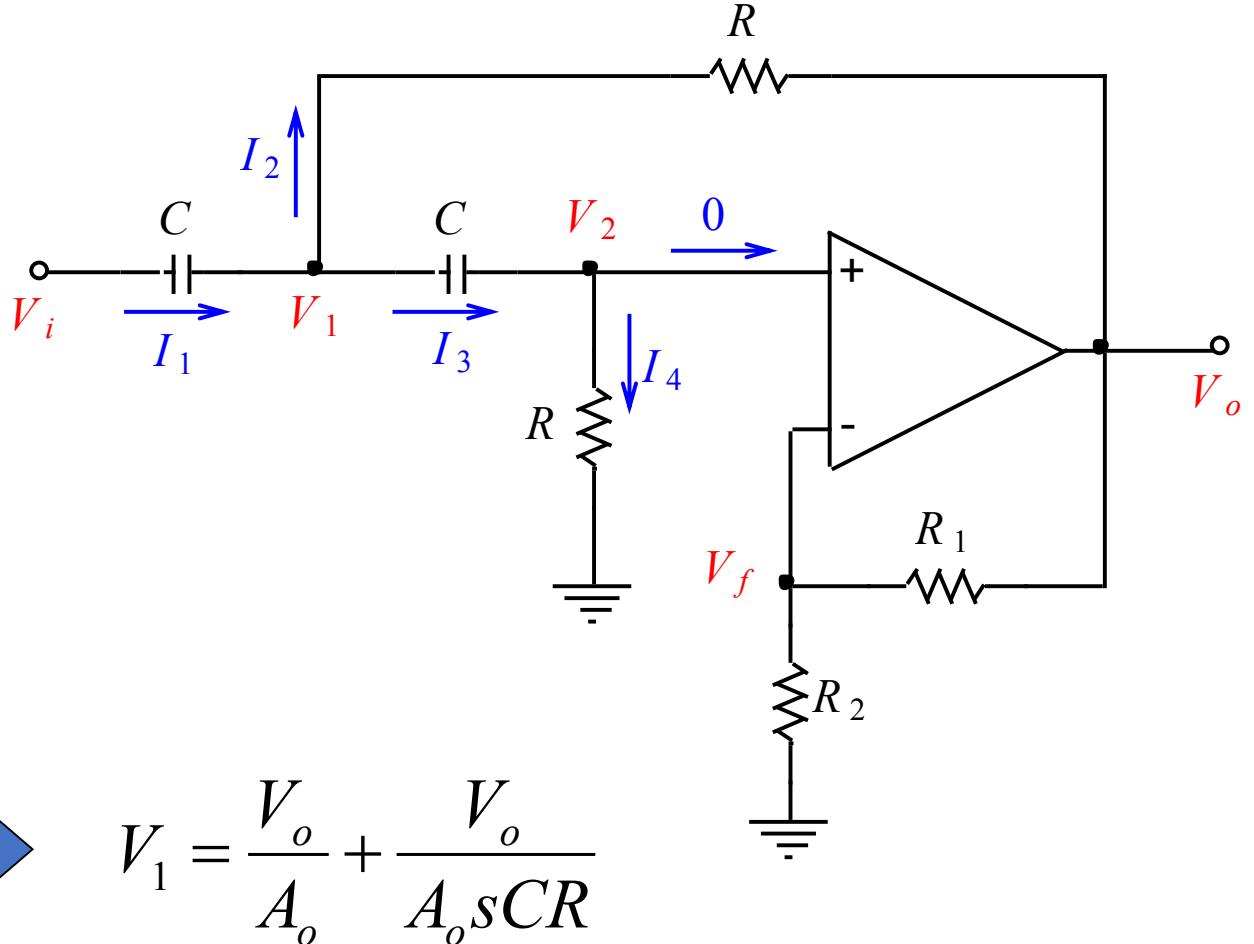
Sallen-Key High-Pass Filter

At node V_2

$$I_3 = I_4$$

$$sC(V_1 - V_2) = \frac{V_2}{R}$$

$$sC\left(V_1 - \frac{V_o}{A_o}\right) = \frac{V_o}{A_o R}$$



$$V_1 = \frac{V_o}{A_o} + \frac{V_o}{A_o sCR}$$

Sallen-Key High-Pass Filter

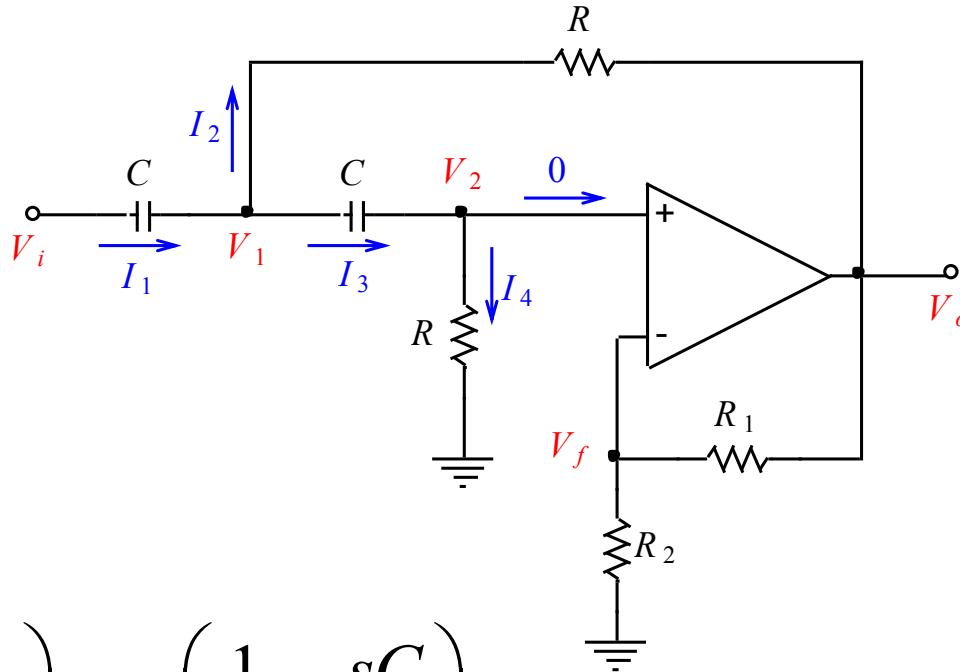
$$sC(V_i - V_1) = \frac{V_1 - V_o}{R} + sC\left(V_1 - \frac{V_o}{A_o}\right)$$

$$V_1 = \frac{V_o}{A_o} + \frac{V_o}{A_o s C R}$$

$$sCV_i = \left(\frac{1}{R} + s2C \right) \left(\frac{1}{A_o} + \frac{1}{A_o s C R} \right) V_o - \left(\frac{1}{R} + \frac{sC}{A_o} \right) V_o$$



$$sCV_i = \left(\frac{(sCR)^2 + (3 - A_o)sCR + 1}{A_o s C R^2} \right) V_o$$

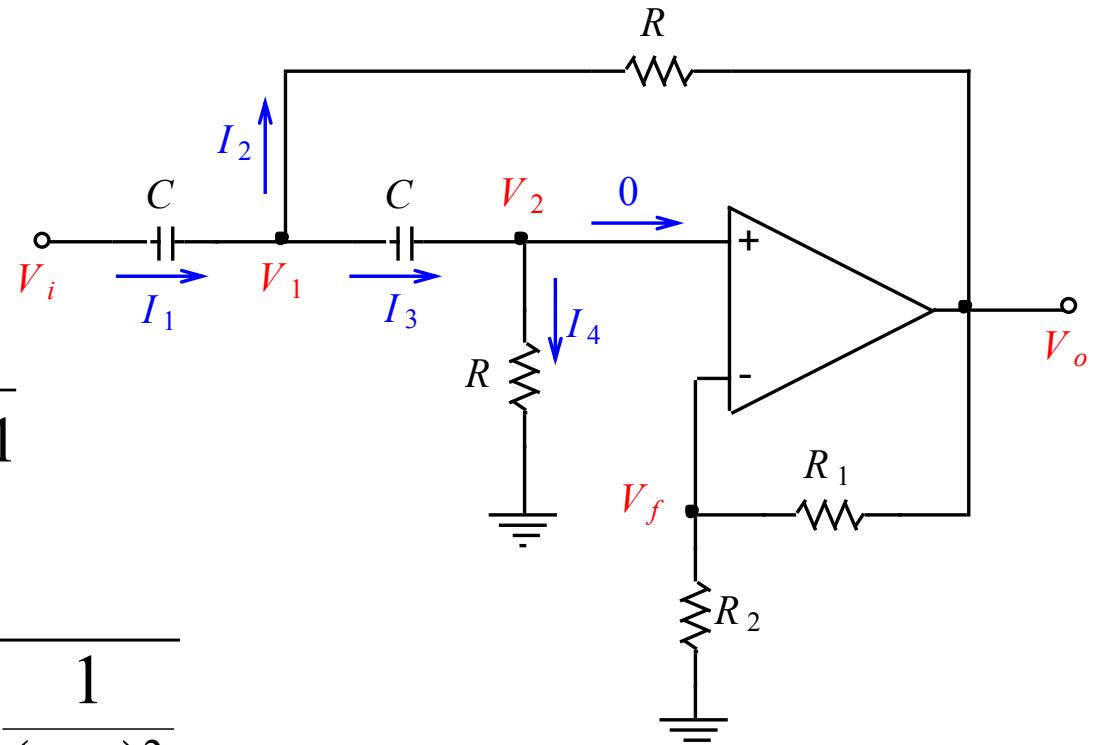


Sallen-Key High-Pass Filter

$$sCV_i = \left(\frac{(sCR)^2 + (3 - A_o)sCR + 1}{A_o sCR^2} \right) V_o$$

$$H(s) \equiv \frac{V_o}{V_i} = \frac{A_o (sCR)^2}{(sCR)^2 + (3 - A_o)sCR + 1}$$

$$\begin{aligned} \omega_o &= \frac{1}{CR} & \xrightarrow{\quad\quad\quad} &= \frac{A_o s^2}{s^2 + \frac{(3 - A_o)}{CR}s + \frac{1}{(CR)^2}} \\ & & \xrightarrow{\quad\quad\quad} &= \frac{A_o s^2}{s^2 + (3 - A_o)\omega_o s + \omega_o^2} \end{aligned}$$



Sallen-Key High-Pass Filter

$$= \frac{A_o s^2}{s^2 + (3 - A_o)\omega_o s + \omega_o^2}$$

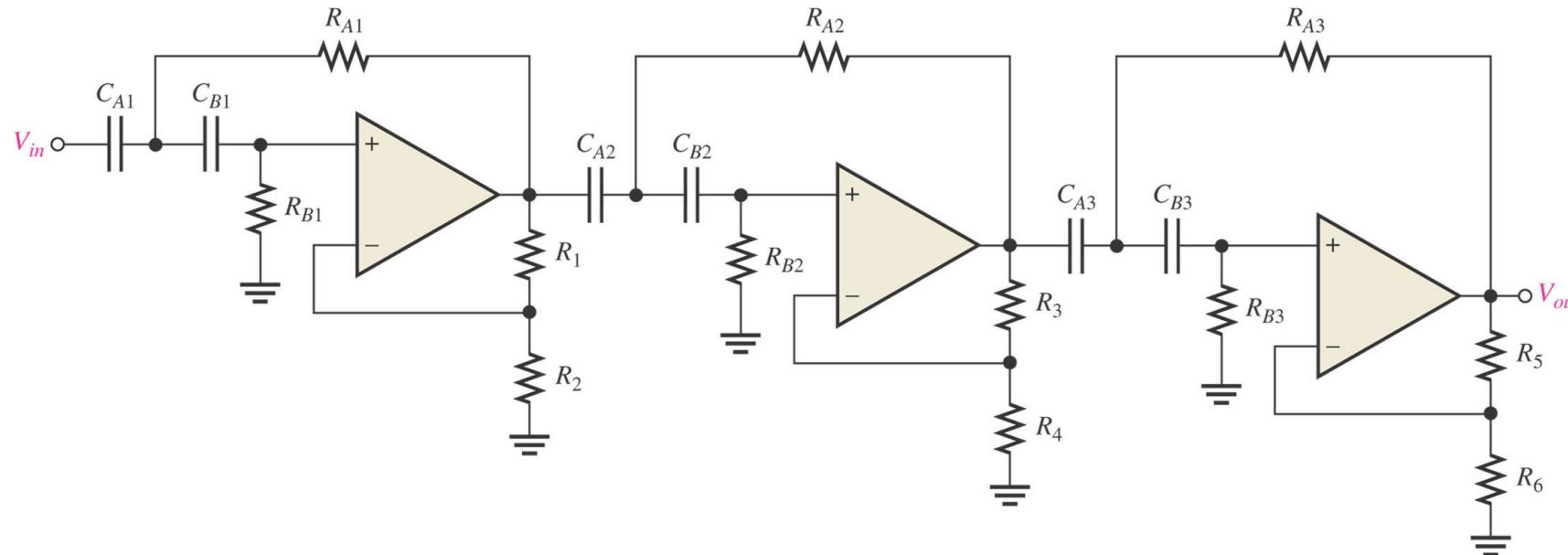
$\alpha = 3 - A_o$ = Damping Factor

$$H(s) = \frac{A_o s^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

$$\alpha = 3 - A_o \rightarrow = 3 - \left(1 + \frac{R_1}{R_2}\right) \rightarrow \alpha = 2 - \frac{R_1}{R_2}$$

Cascading High-Pass Filter

- As with the low-pass filter, first- and second-order high-pass filters can be cascaded to provide three or more poles and thereby create faster roll-off rates.
- A **six-pole high-pass filter** consisting of **three Sallen-Key two-pole stages** with the roll-off rate of **-120 dB/decade**.



Sixth-order high-pass filter