Filtering Variational Quantum Eigensolver

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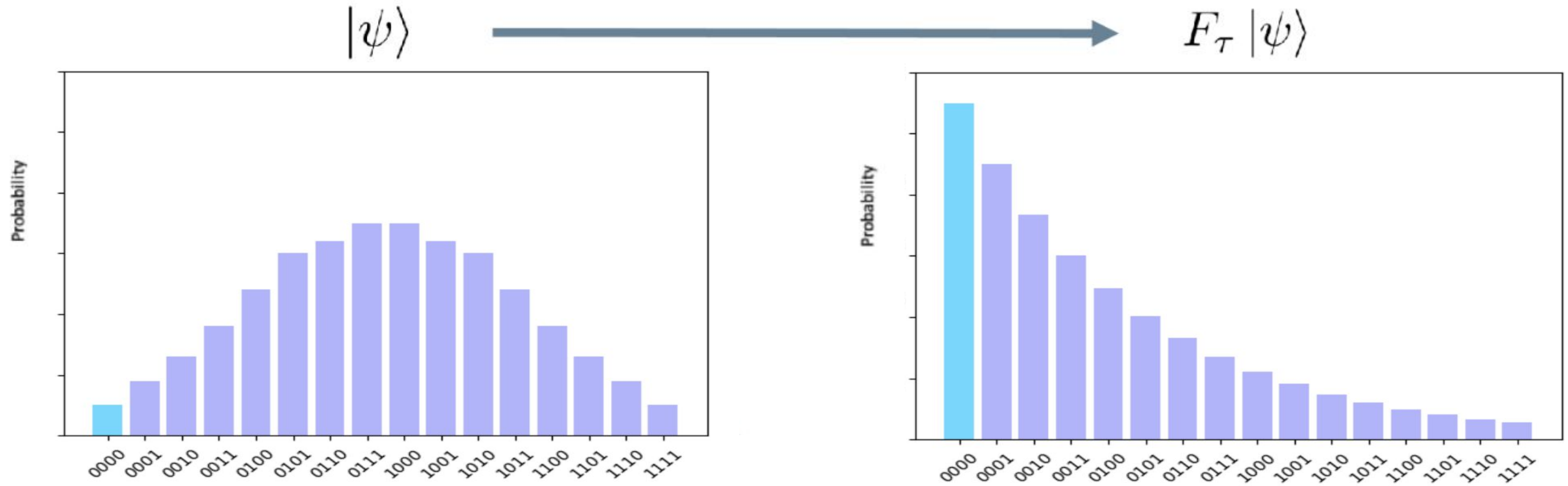
ICTP - Quantinuum Hackathon

April 23, 2023

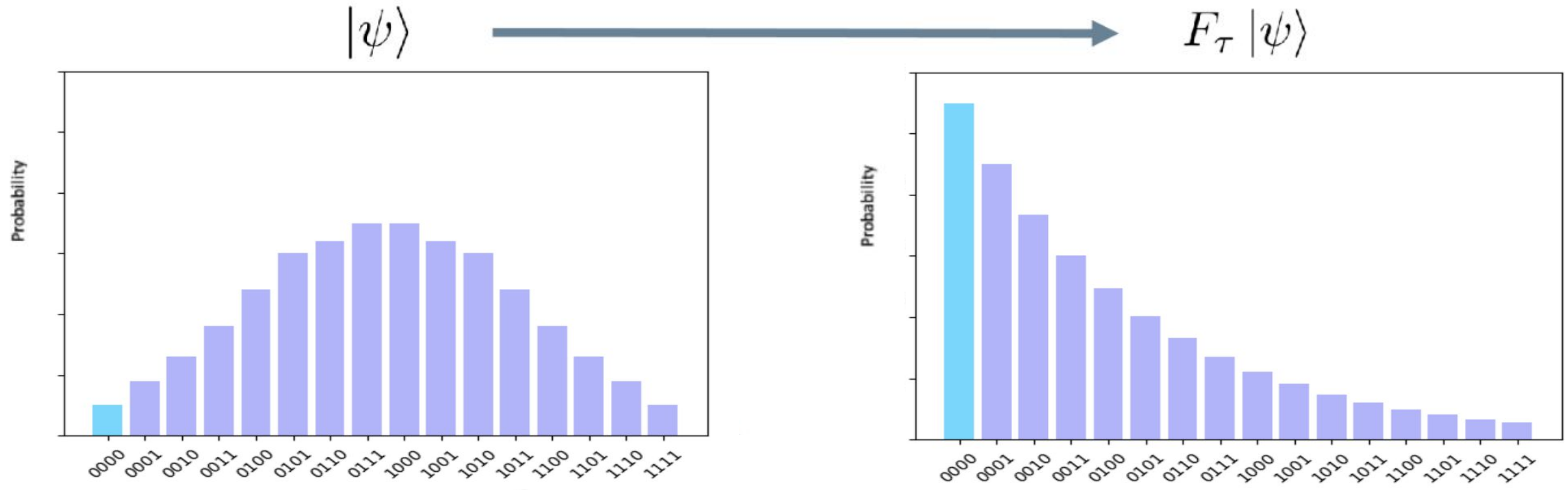
QUANTINUUM

Visualizing F-VQE



$$|\psi(\theta_0)\rangle \to |\psi(\theta_1)\rangle \approx F_\tau|\psi(\theta_0)\rangle \to \cdots \to |\psi(\theta_T)\rangle \approx F_\tau|\psi(\theta_{T-1})\rangle \approx F_\tau^T|\psi(\theta_0)\rangle$$

Visualizing F-VQE



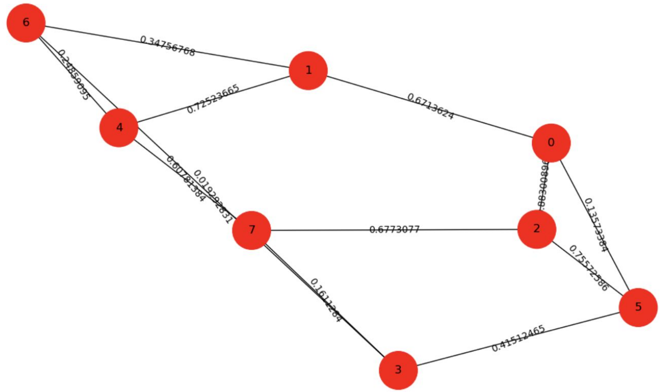
$$\mathcal{C}_t(\boldsymbol{\theta}) = \frac{1}{2}\left\| \, |\psi(\boldsymbol{\theta})\rangle - |F_t\psi_{t-1}\rangle \, \right\|^2$$

Approach to project

Approach to project

- Generation of Random Instances of Max Cut
- Cost function and bounds
- Implementation of F-VQE using QuJax (exact gradients)
- Ansatz development (expressibility)
- Implementation of F-VQE via parameter shift rule and circuit sampling

$$\left.\frac{\partial \mathcal{C}_t(\boldsymbol{\theta})}{\partial \theta_j}\right|_{\boldsymbol{\theta}_{t-1}} = -\frac{\langle F_t \rangle_{\psi_{t-1}^{j+}} - \langle F_t \rangle_{\psi_{t-1}^{j-}}}{4\sqrt{\langle F_t^2 \rangle_{\psi_{t-1}}}}$$

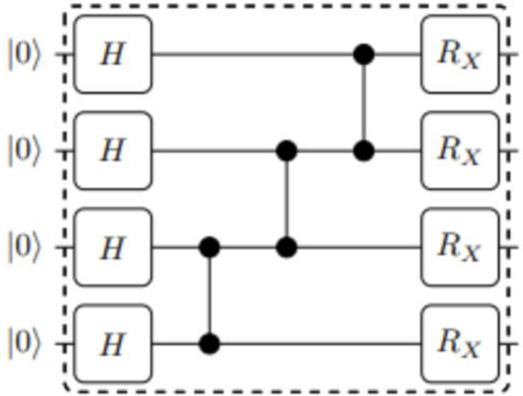


$$C = \Sigma_{n=1}^{N} \omega_{n_1,n_2} * (g(n_1) \oplus g(n_2))$$

Exploration and Analysis

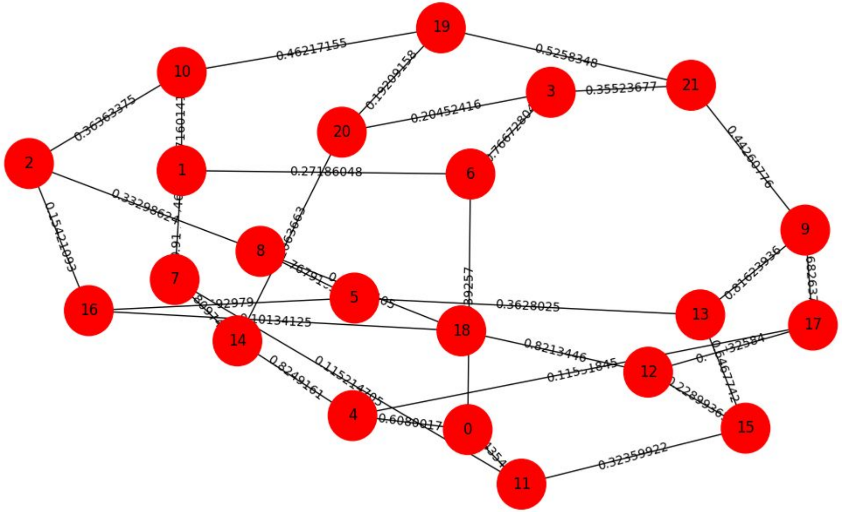
- Study of different ansatz & problem sizes for generality and scalability
- Attempts of various filtering functions for best performance
- Exploration of changes in constant and variable Tau values and learning rates
- Comparison of F-VQE performance between noiseless and noisy backends

$$F_\tau = f(H;t) = \begin{cases} H^{-\tau} \\ e^{-H\tau} \\ \cos^\tau(H) \end{cases}$$

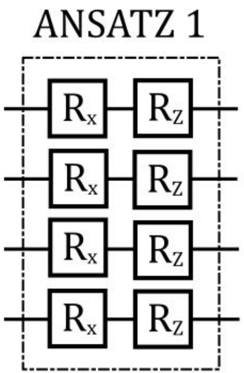


MaxCut Graph Generation

- Simple Combinatorial Optimization problem
- Straightforward cost function to generate
- Can be mapped easily to QUBO
- Black Box function rather than Hamiltonian

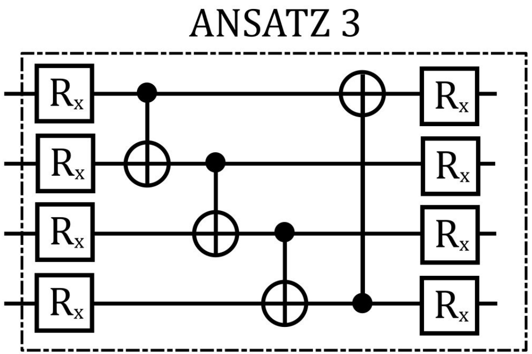


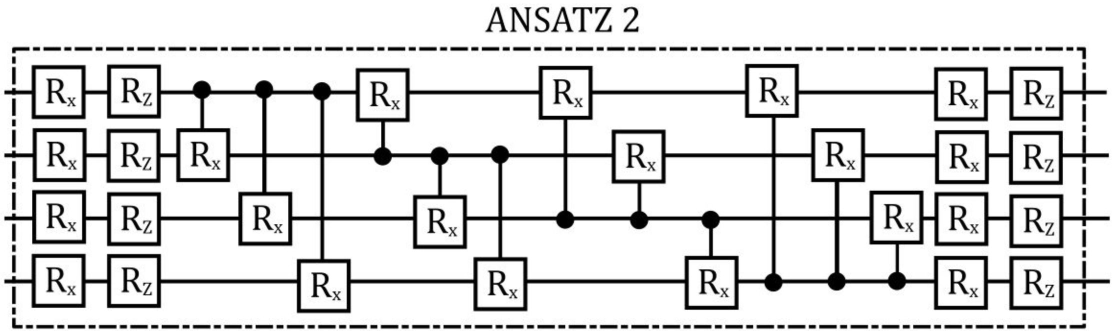
Ansatz



Expressibility and entangling capability of parameterized quantum circuits for hybrid quantum-classical algorithms

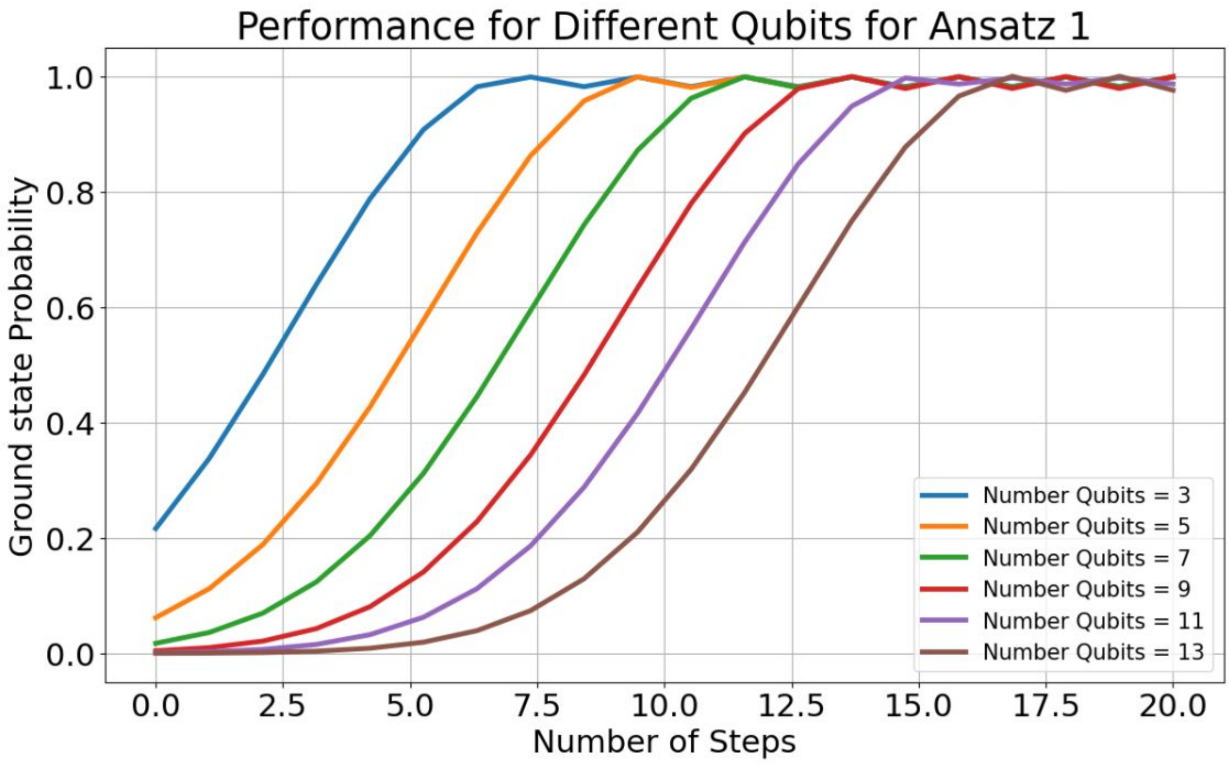
Sukin Sim,[1,2,*] Peter D. Johnson,[2] and Alán Aspuru-Guzik[2,3,4,5,†]



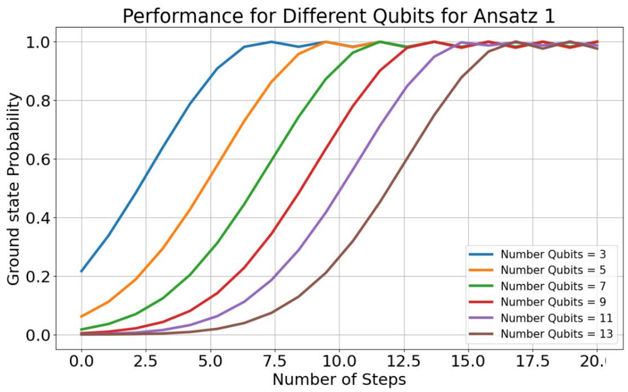


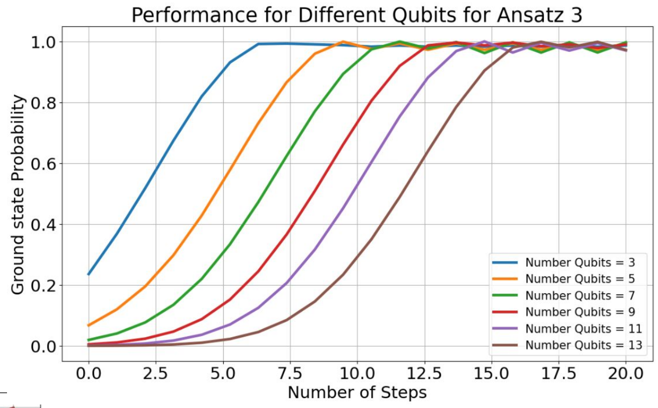
Expressibility of the circuits

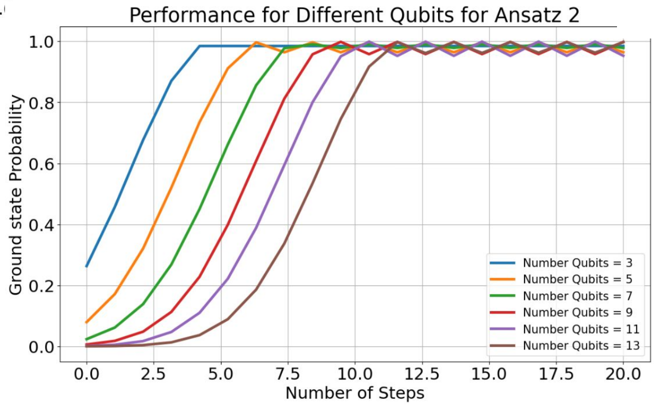
- Successful convergence with few qubits.
- Decreasing convergence time with scalability.
- Expressibility to faster convergence trade with simulation time.



Expressibility of the circuits

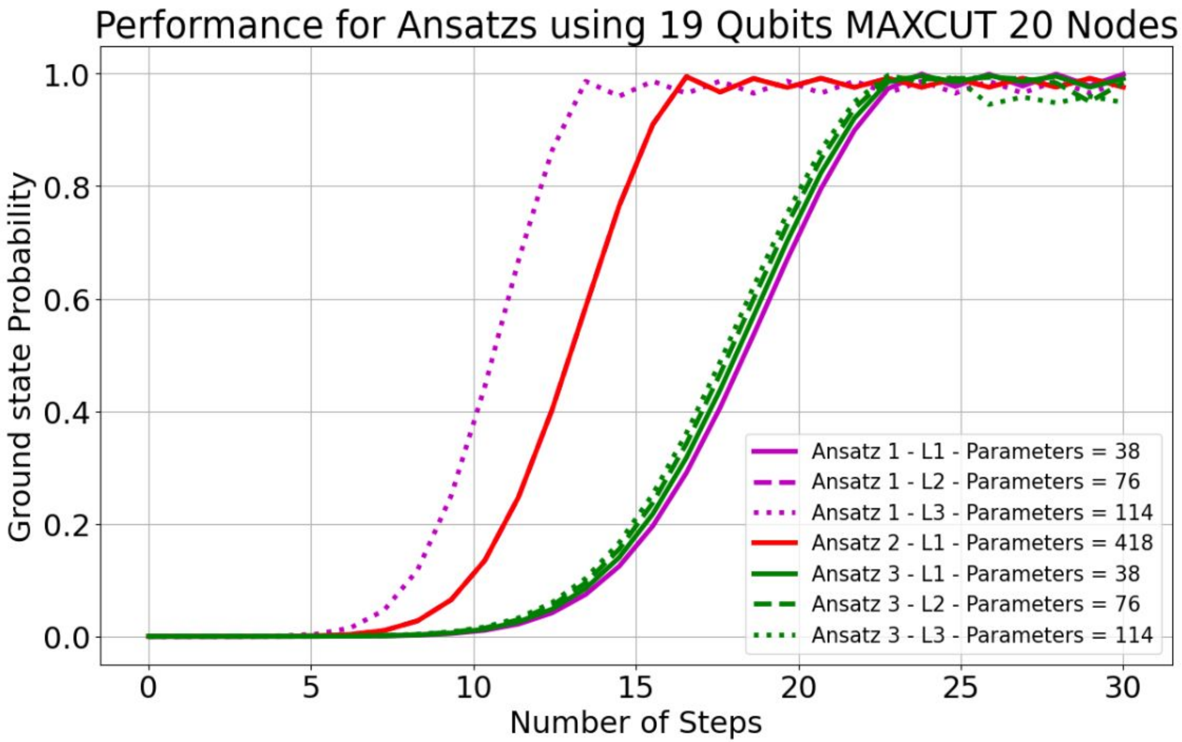






Max number of Qubits?

- Different Ansatz:
- Tests based on # parameters:
- Different number of layers:
- Fast convergence rate:

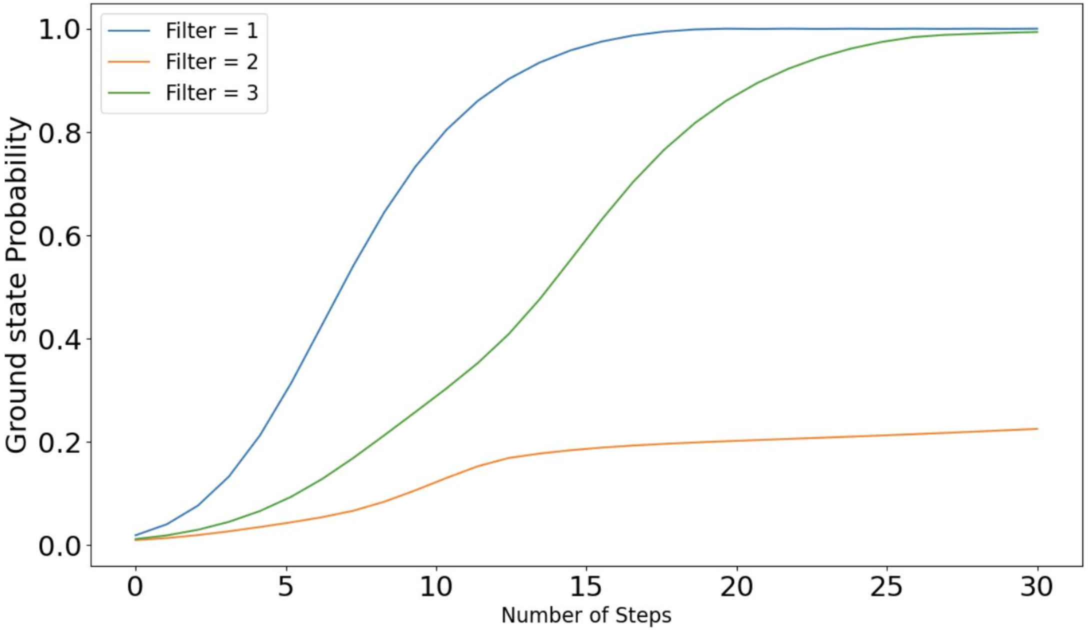


Building F-VQE

What about the filters?

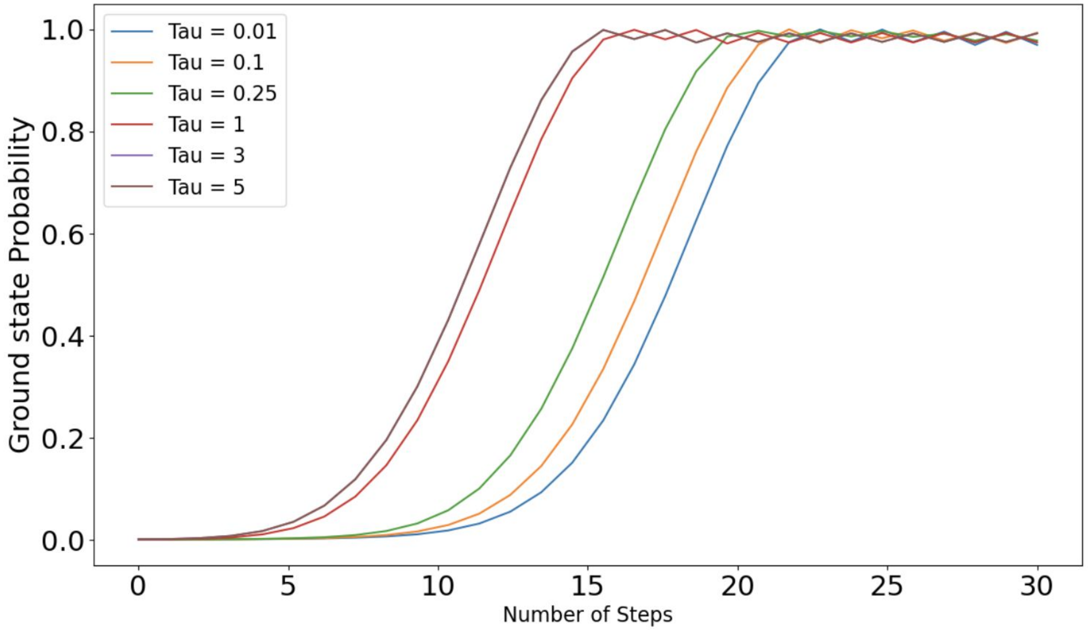
Scalability of Composite Filter Functions

- Tested combinations of inverse, exponential, and cosine filter functions
- No composite filter function converged for 9+ qubits
- **Inverse** was most efficient of pure filter functions
- Inverse○Exponential was best composite filter, converging for 7 qubits



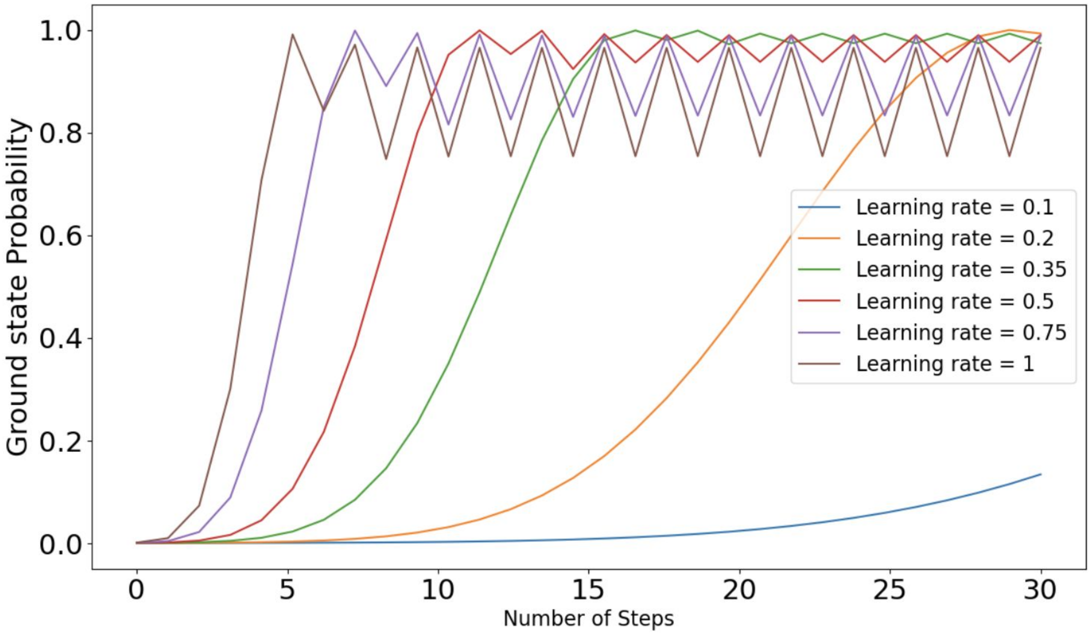
Comparison of different Tau values

- Tested constant Tau values of 0.01, 0.1, 0.25, 1, 3, 5
- Tested variable Taus both decreasing and increasing as GS probability increases
- Higher Taus converge faster
- Difference larger as qubit count increases
- Variable Tau provided <u>no difference</u> up to 19 qubits



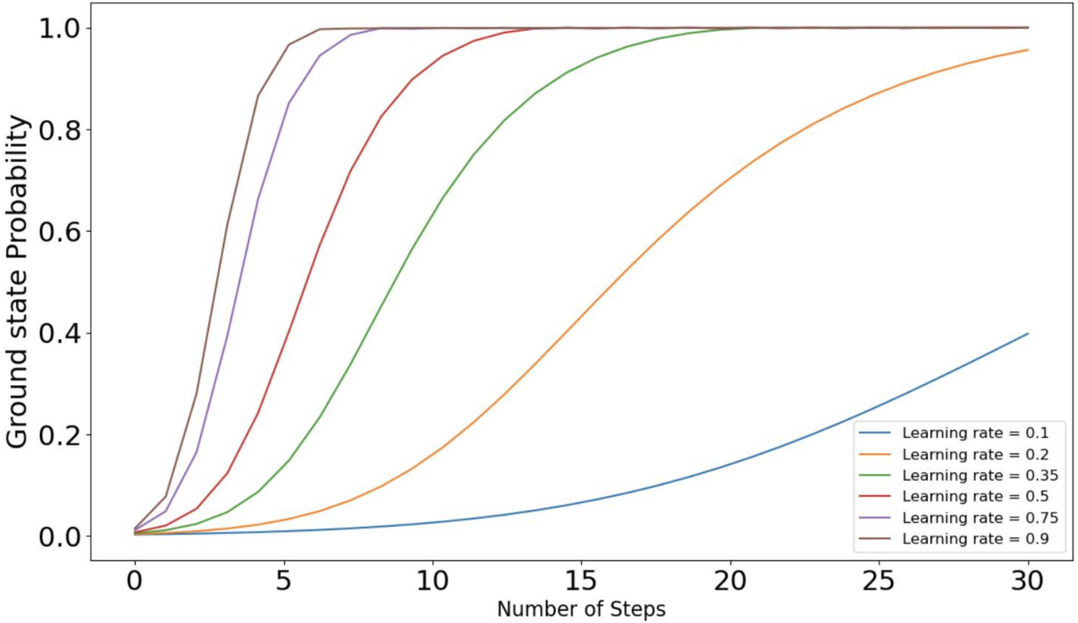
Comparison of constant learning rates

- Tested constant LR values between 0 and 1
- The best performing learning rate is about 0.35
- High learning rates over-adjust, don't converge
- Rates higher than 1.57 ($\pi/2$) converge to probability 0

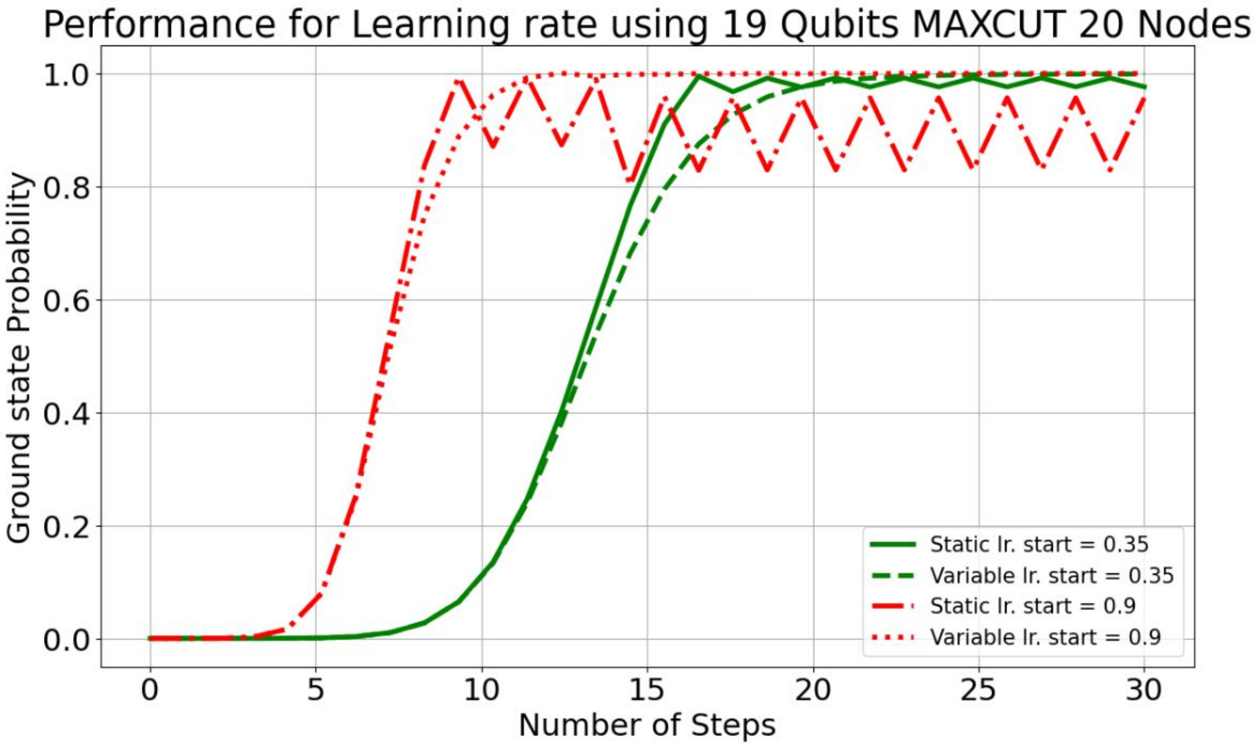


Comparison of variable (adaptive) learning rates

- Tested decrease of LR proportional to probability and **prob^2** (best)
- Higher learning rate values are able to converge smoothly
- Maximum converging LR a function of # qubits
- LR up to 0.9 converges at 19 qubits

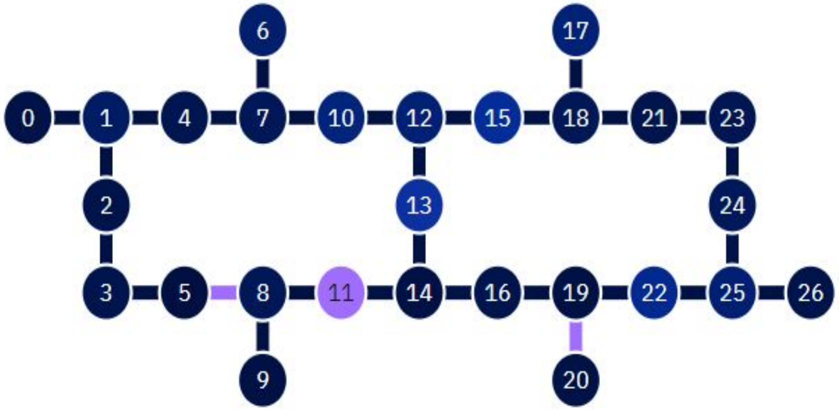


Max number of qubits with different learning rate



Noisy backend simulation

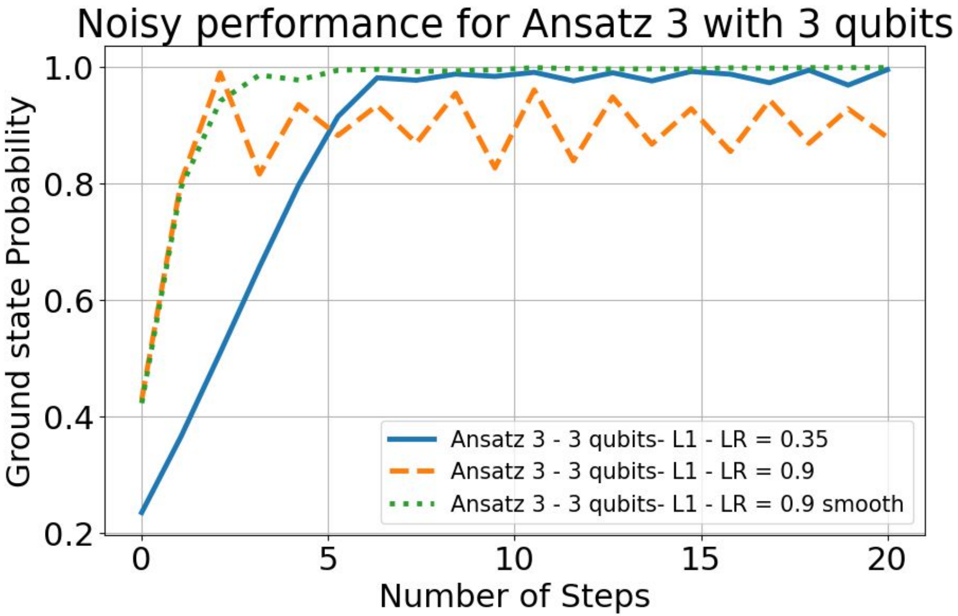
For the noisy simulation, our team used the noise model of IBMQ-Hanoi

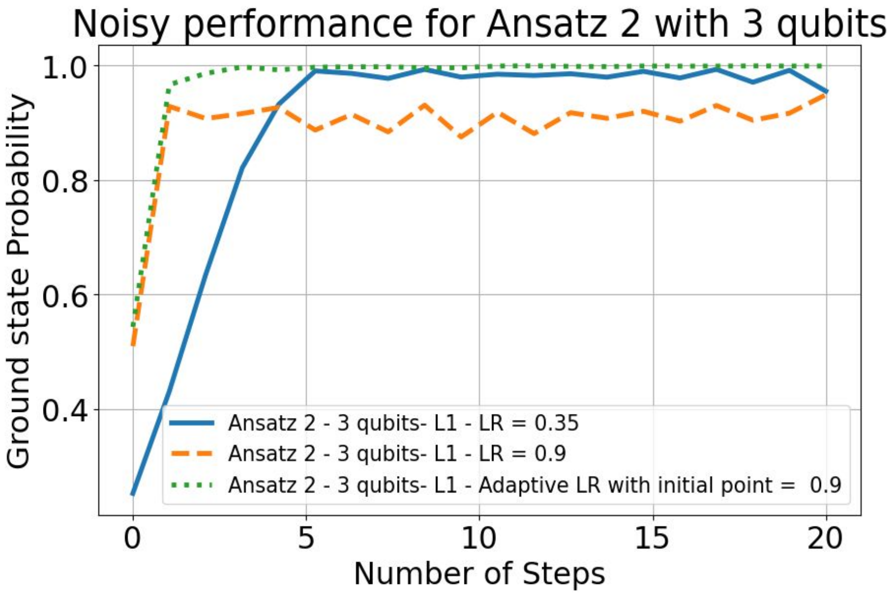


- Simulation with 3, 5, 7, 9 and 11 qubits for proposed Ansatzs
- Study noise effect on adaptive learning rate method

Noisy backend simulation

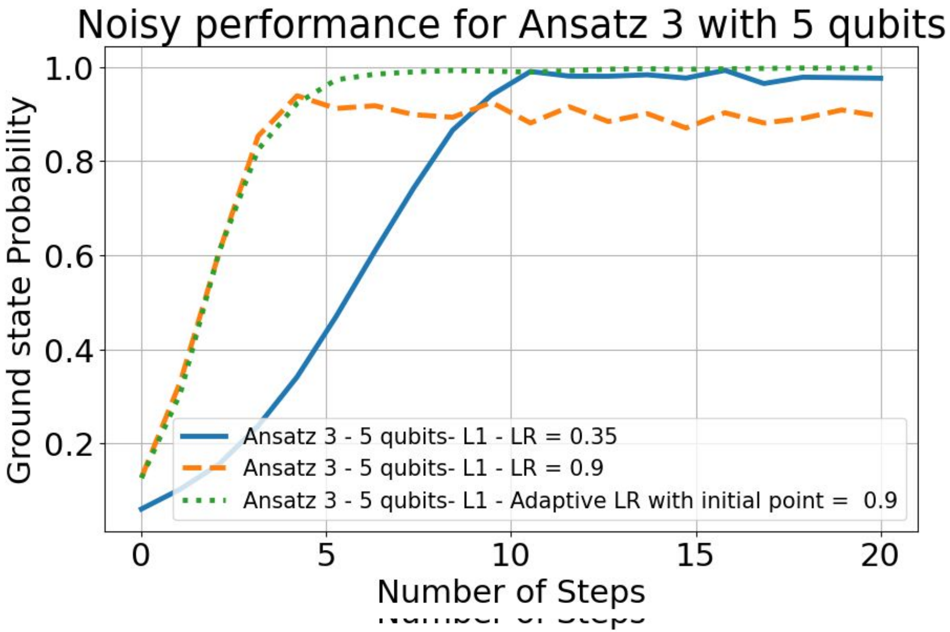
Noisy simulation with 3 qubits

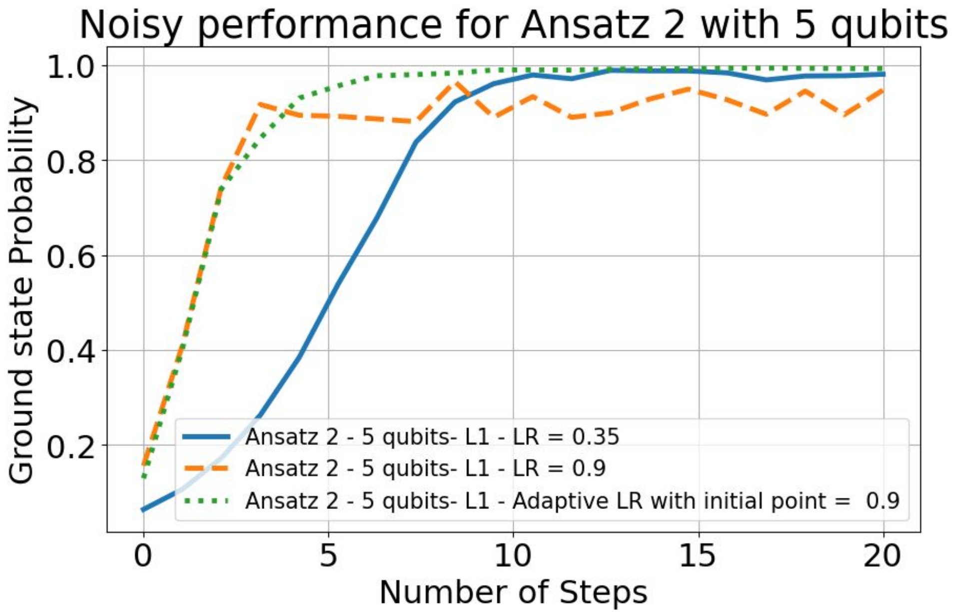




Noisy backend simulation

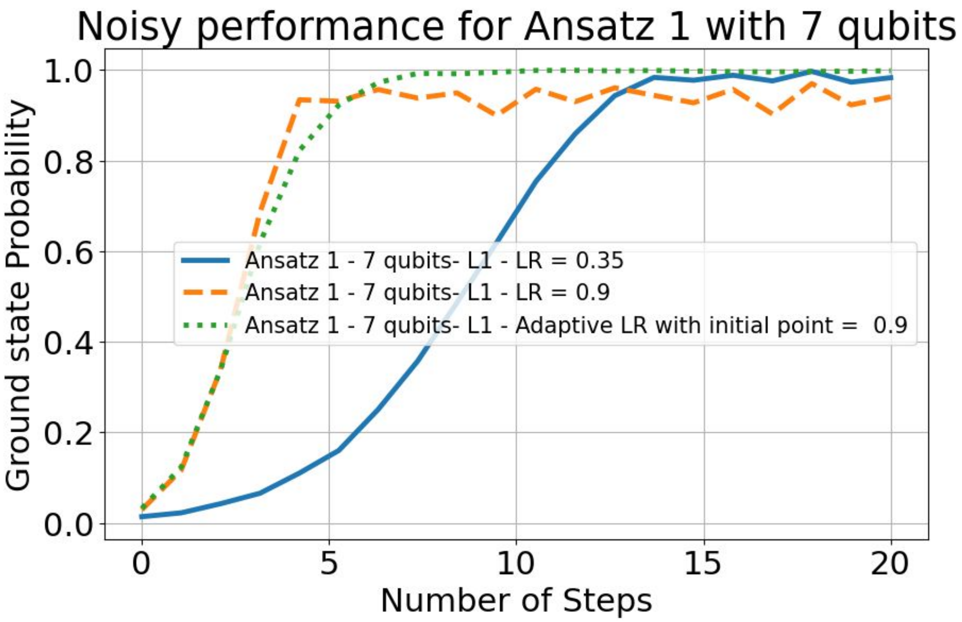
Noisy simulation with 5 qubits

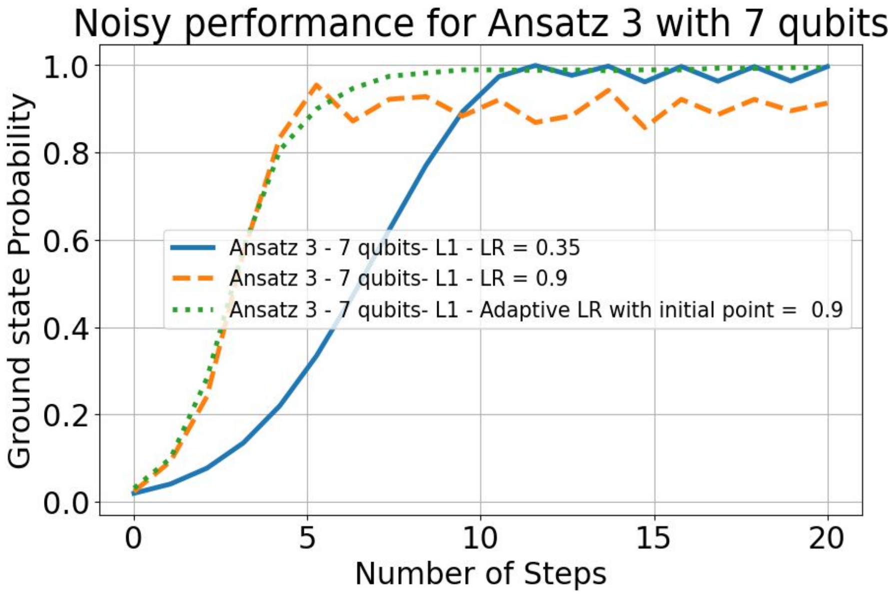




Noisy backend simulation

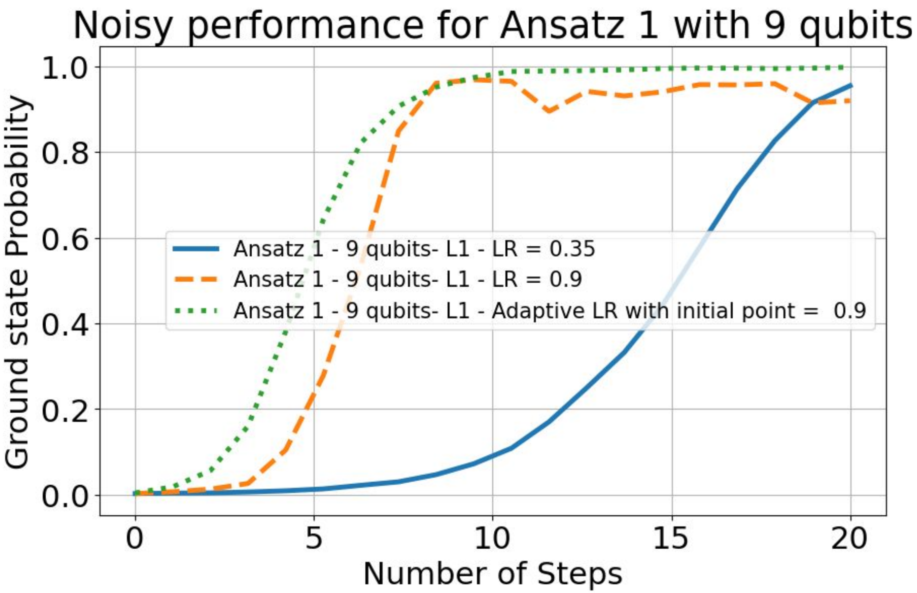
Noisy simulation with 7 qubits

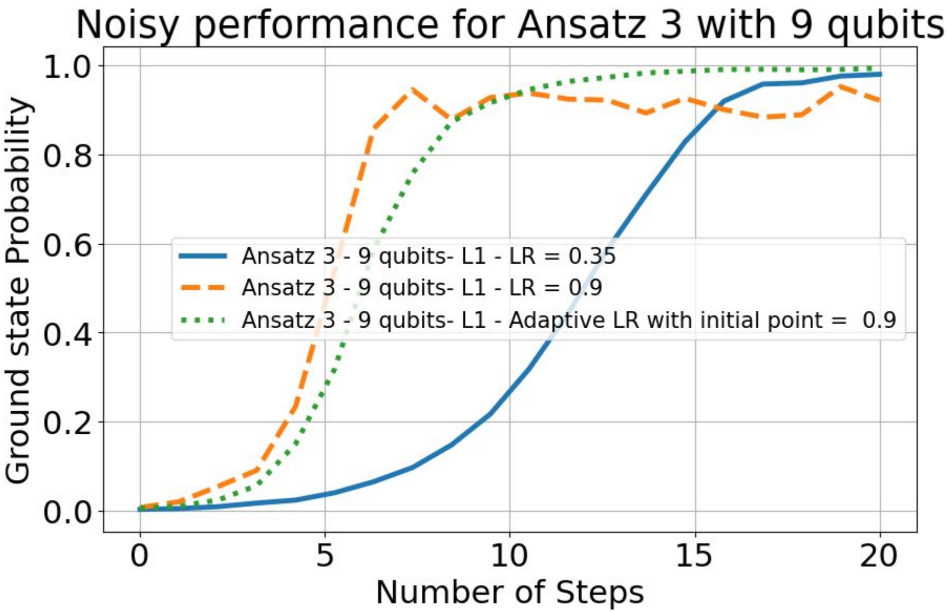




Noisy backend simulation

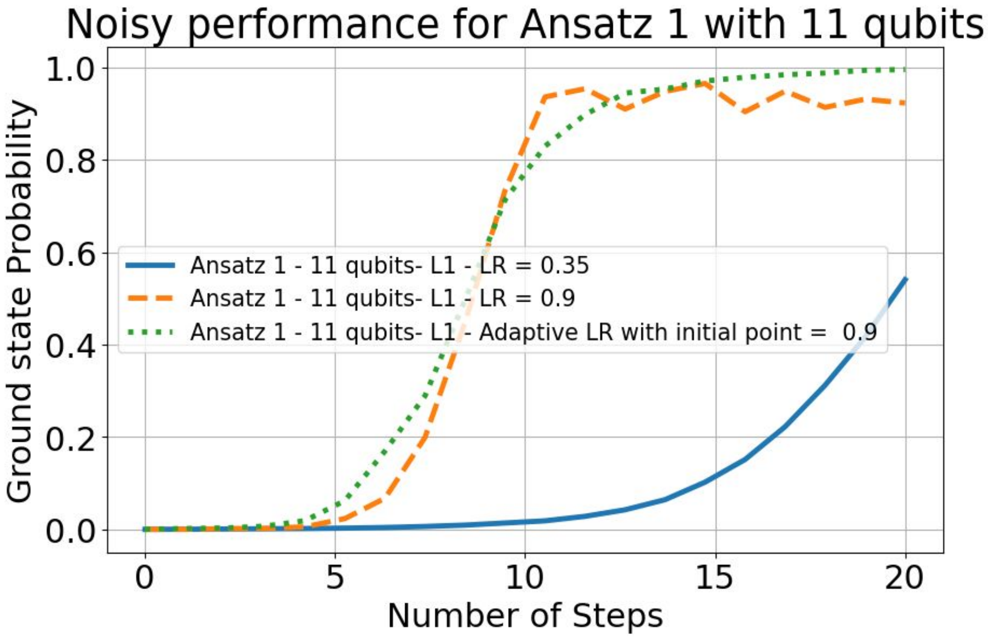
Noisy simulation with 9 qubits

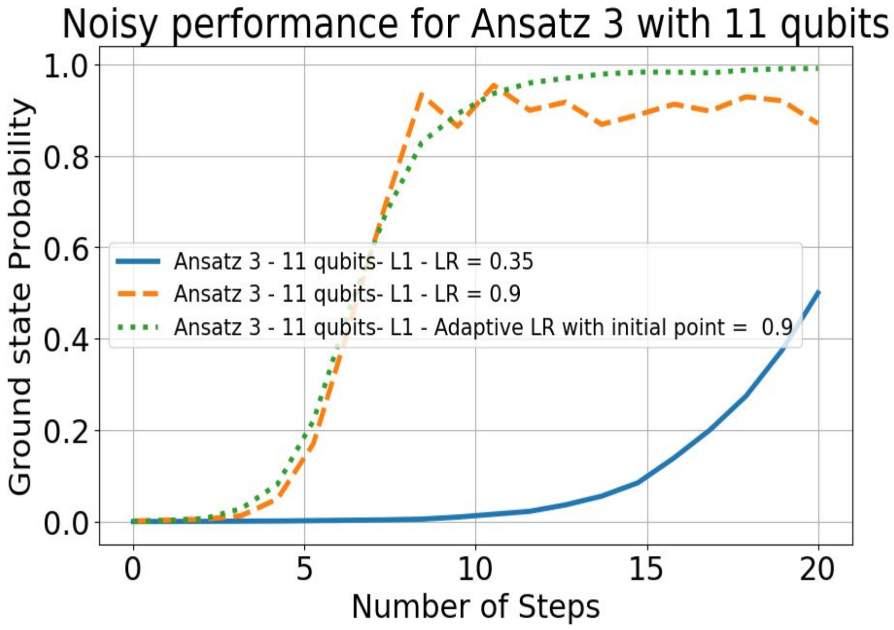




Noisy backend simulation

Noisy simulation with 11 qubits





Noisy backend simulation

From the noisy simulation, we can see that:

- Our system is not affected much by noise
- Shallow ansatz in a noisy environment can still converge
- Our system can potentially scale to higher number of qubits
- Further analysis of adaptive LR can work to cancel out the noise

Conclusion

- Displayed the versatility of FVQE
- Proposed a **new approach** to smooth convergence.
- Demonstrated that robustness for noise backend allowed for higher qubit simulations.

Questions?

Questions?

References

[1] Sim, S., Johnson, P. D., & Aspuru-Guzik, A. (2019). Expressibility and entangling capability of parameterized quantum circuits for hybrid quantum-classical algorithms. Advanced Quantum Technologies, 2(12), 1900070.

[2] Farhi, E., Goldstone, J., & Gutmann, S. (2014). A quantum approximate optimization algorithm. arXiv preprint arXiv:1411.4028.

[3] L.Zhou, et al., Quantum Approximate Optimization Algorithms: Performance, Mechanism, and Implementation on Near-Term Devices. Arxiv e-prints, art. Arxiv:1812.01041 (2018).

[4] D. Amaro, et al., Filtering Variational Quantum Algorithms for Combinatorial Optimization, Quantum Sci. Technol. 7, 015021 (2022)

[5] Samuel Duffield, et al., qujax: Simulating quantum circuits with JAX, 2022, https://github.com/CQCL/qujax.

[6] Sivarajah, S., Dilkes, S., Cowtan, A., Simmons, W., Edgington, A., & Duncan, R. (2020). t| ket⟩: a retargetable compiler for NISQ devices. Quantum Science and Technology, 6(1), 014003.