

HW13

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Question 1

Use integration by substitution to solve the integral below.

$$\int 4e^{-7x} dx$$

Answer 1

$$-\frac{4}{7}e^{-7x} + C$$

Steps using substitution:

$$U = -7x, dU = -7dx, dx = \frac{dU}{-7} \text{ so,}$$

$$4 \int e^U \frac{dU}{-7} = \frac{4}{-7} \int e^U dU = -\frac{4}{7}e^U + C \text{ thus, } -\frac{4}{7}e^{-7x} + C$$

Question 2

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

Answer 2

To find a function, take the integral of the rate:

$$\begin{aligned} & \int \left(\frac{-3150}{t^4} - 220 \right) dt \\ & \int (-3150t^{-4} - 220) dt \\ & \int -3150t^{-4} dt - \int 220 dt \\ & -3150 \int t^{-4} dt - 220 \int dt \\ & -3150 \frac{-1}{3} t^{-3} - 220t \end{aligned}$$

$$N(t) = \frac{1050}{t^3} - 220t + c$$

Solve for c using the initial condition at day 1 $N(1) = 6530$,

$$6530 = \frac{1050}{1^3} - 220(1) + c$$

$$6530 = 1050 - 220 + c$$

$$6530 - 1050 + 220 = c$$

$$5700 = c$$

Result:

$$N(t) = \frac{1050}{t^3} - 220t + 5700$$

Question 3

Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$

Answer 3

Question 4

Find the area of the region bounded by the graphs of the given equations $y_1 = x_1^2 - 2x_1 - 2$ and $y_2 = x_2 + 2$.

Enter your answer below.

Answer 4

Question 5

A beauty supply store expects to sell 110 flat irons during the next year. It costs \$3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

Answer 5

Question 6

Use integration by parts to solve the integral below.

$$\int \ln(9x) \times x^6 dx$$

Answer 6

Question 7

Determine whether $f(x)$ is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral.

$$f(x) = \frac{1}{6x}$$

Answer 7