The Perception of Present: A Study of Inconsistency in Discount Rate with the General Quasi-Hyperbolic Discounting Model

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Abstract

In this paper, we examine heterogeneity in individual perceptions of the present in an attempt to understand better the dynamic inconsistency (the non-exponentially discounted utility) in time preferences. Quasi-Hyperbolic Discounting models are often used to study dynamic inconsistency. Recent studies have found that the time horizon of the defined present (Is the present an hour, a day, or longer?) impacts the observed magnitude of dynamic inconsistency. We propose a more general framework of the conventional Quasi-Hyperbolic Discounting model, which helps to identify the prospective present and its corresponding inconsistency. We applied our General Quasi-Hyperbolic Discounting (GQHD) model to two existing Convex Time Budget, a commonly employed experimental design to study time preference, experimental data sets. The results suggest empirical evidence supporting the conjecture that the perceptional present, which we consider the sense of the present from a subjective point of view, lasts longer than the immediate present (hours or days). The GQHD estimates of present bias parameters varies substantially across individuals, yielding different specifications of the "present" in our structural model. We find that experimentally observed choices are more likely to exhibit secondary present (when we consider the "present" can be longer than the immediate present) inconsistency than the primary present (when we only consider the immediate present as the "present") inconsistency.

1 Introduction

Time is a fundamental concept in decision-making and understanding the trade-off between present and future consumption is critical to studying economic behavior. To understand how we make decisions on a time horizon, we study the time preference and apply the discoveries to many policies (health, saving, and retirement decisions, etc. ; Uzawa 1969; Fuchs, 1980; Finke and Huston 2013). It is crucial for the research in economics to understand and model time preference accurately. We often find present bias behavior (a tendency that people give stronger weight to the payoffs that are closer to the present than the future) in time preference among individuals. To model the inconsistency behaviors (O'Donoghue and Rabin 1999). The Quasi-Hyperoboslic model became one of the most commonly adopted approaches in modeling present bias behaviors (Thaler 1981; Frederick, Loewenstein, and O'donoghue 2002).

"How soon is now?" is one of the most actively controversial topics in the Quasi-Hyperbolic Discounting (QHD) model-based time preference literature (Glimcher, Kable, Louie 2007; Balakrishnan, Haushofer, and Jakiela 2017; DellaVigna 2018; Ericson, and Laibson 2019). To explore the question "nowness," we linked time inconsistency with the perceptional sense of time, then found the threshold that distinguishes the present and the future in the QHD framework. The threshold we find challenges the conventional belief that present bias driven time inconsistency only applies to a relatively short period of time. Our results show that disregarding the importance of time perception in a revealed preference study leads to misleading conclusions about time inconsistency. With our newly proposed General Quasi-Hyperbolic Discounting (GQHD) approach and its empirical application, we find a link between time perception and dynamic inconsistency, which helps us to give a more precise answer about when time inconsistencies occur.

Conventionally, empirical models set a boundary that distinguishes present and future, an approach which limits our ability to observe present bias to immediate consumption choices. In other words, the subjects are being observational present biased if and only if the present bias happened precisely according to the constraint artificial boundary that specifies their present and future. In lab experiments, the design of immediate pay-off varies for different studies from hours, days, weeks, to years. These studies find disagreement about the existence of the present bias to the dynamic inconsistency related research questions (Angeletos, Laibson, Repetto, Tobacman, Weinberg 2001; McClure, Ericson, Laibson, Loewenstein, and Cohen 2007; Augenblick, Niederle, and Sprenger 2015; Augenblick 2017). Other research finds that the time preferences also depend on the subjective sense of time (Angeletos, Laibson, Repetto, Tobacman, and Weinberg 2001; dos Santos and Martinez 2018). Balakrishnan et al. (2017) explore the timing of lab payments and find that present bias only exists when participants are paid at the immediate conclusion of the experimental session. However, a structural model that restricts the notion of the present to the immediate now may over restrict the notion and lead to errors, such as unobserved dynamic inconsistency in time preference studies (Imai, Rutter, and Camerer 2019).

To find a more general definition of time horizon that better explains the data, we divide our study into the following sections. First, we develop a new framework that generalizes the QHD model with a more flexible definition of the present sense along with its well-defined characterizations (secondary present bias and secondary future bias behavior in time preference). This general format comes with a modified β_τ parameter of the QHD that captures the perceptional present alone with its inconsistency. This theoretical work closely follows the Convex Time Budget (CTB) design to find empirical evidence of perceptional dynamic inconsistency from existing experimental data. We also demonstrate that potential ignorance of the perception present in time preference research leads to imprecise judgment of the types of inconsistency with both derived intuition and simulated data. Next, we applied the GQHD method with a nonlinear least square estimation approach to lab experiment data from Carvalho, Meier, and Wang 2016 (CMW16), and Andreoni, and Sprenger 2012a (AS12), with different strengths of restrictions of the utility assumptions along with robustness check of various functional forms. We find strong evidence to support the existence of a more diversified perspective of present in the observational data set than the previously assumption that the present means "now". In fact, the average perceptional present lasts for days or even months. Most importantly, when the present is correctly specified in the model, conclusions about the direction of time inconsistency may differ from previous studies.

The main contribution of our research is to establish the importance of considering the perceptional sense of the present. Failing to do so leads to incorrect analysis of present and future bias behaviors which undermines policy efforts to improve behaviors related to time discounting. We propose a General Quasi-Hyperbolic Discounting method that addresses both perception of the present and its corresponding dynamic consistency. This method allows us to assess time inconsistency more precisely at both aggregated level and individual level in the perceptional time scale.

2 Background

Time preference measures how individuals evaluate receiving a good at an earlier time compared to receiving it at a later time. It is essential for economists to understand it, then to explain how individuals, even the whole population smoothing out the consumption. Historically, we tried to model time preference with many different models. Samuelson started this path with his proposed Exponential Discounting (ED) utility model (Samuelson 1937; Koopmans 1960). The ED model assumes that individuals' valuation of consumption depreciates at a constant rate δ over time (3). Researchers have shown that behavior is often inconsistent with the ED model (Ainslie 1975; Thaler 1981; Frederick, Loewenstein, and O'donoghue 2002). Ainslie (1975) first showed the inconsistency in time preferences, and illustrated greater discounting for short-run consumption than long-run consumption, a phenomenon called present bias. As an example, suppose we have the following two choices:

Example.

A. \$ 10 today vs B. \$ 11 in a week, and A'. \$ 10 in 50 weeks vs B'. \$ 11 in 51 weeks

There are many decision makers choose A over B, then choose B' over A'. There are many well-known theoretical models in both economics and psychology studies that address this documented observational reversal behavior in time preference (Thaler 1981; Frederick, Loewenstein, and O'donoghue 2002). For instance the proportional discounting model (Ainslie, and Herrnstein 1981), the power discounting model (Harvey 1986), the hyperbolic discounting ($\frac{1}{1+(\kappa t)}$) model (Mazur 1987; Loewenstein, and Prelec 1992), and the constant sensitivity model (Ebert and Prelec 2007). Among the discounting models, Hyperbolic Discounting becomes the most commonly chosen one to model this time-inconsistency due to the accuracy of explaining this reversal behavior.

The Laibson (1997) successfully approximated the hyperbolic discounting model with the Quasi-Hyperbolic Discounting function ((1) ;Phelps and Pollak 1968) in a discrete time preference study.

The important advantage of this approximation is that the quasi-hyperbolic model (1) has a β parameter that represents a departure from Samuelson's exponential discounting model (3) and separates time discounting into distinct processes for present and future rewards. Conventionally, we are constraining the β parameter to be strictly less than one to indicate present bias in behavior. However, some recent research has found evidence that time preferences exhibits future bias characteristics (Aycinena, Blazsek, Rentschler, and Sandoval 2015; Corbett 2016; Aycinena, and Rentschler 2018). Additionally, a meta-analysis among these Quasi-Hyperbolic Discounting (QHD) models with the Convex Time Budget (CTB) design found evidence of over-reporting present bias in existing studies (Imai, Rutter, and Camerer 2019). Future biased time preference behaviors are not only found in CTB studies are also discovered in other non-CTB experimental designs (Takeuchi 2011; Jackson, and Yariv 2014). Thus, previous results suggest that the true direction of dynamic inconstancy remains to be conclusively established.

In addition to research on time inconsistency parameters, there is another stream of research which focuses on establishing how one's perspective (subjective) time scale affects the measurement of dynamic inconsistency. The subjective time can be measured by the ability to distinguish the proportional difference between three months, one year, and three years on a physical object or measured by the ability to accurately tell how long a minute is (Zauberman, Kim, and Malkoc 2009; Bradford, Dolan, Galizzi 2013; Brocas, Carrillo, and Tarrasó 2018) Research shows that the degree of hyperbolic discounting (present bias) is reduced significantly when the dynamic inconsistency is measured by subjective time perception. In other words, people are less inconsistent when we measure time in their perspective time scale (Zauberman, Kim, and Malkoc 2009). Additional to the inconsistency of time preference, the results can also be extended to the estimation of the discounting parameter δ. The mental representation of the delays is correlated to our ability to delay consumption to the future. If the delay in time is exaggerated in once mind (for example, if the mental presentation of one day is longer than the physics definition of one day), then we will observe more deprecation of the delayed award in value. The more we shrink the time in our perspective sense, the more we will discount the valuation of future consumption (Brocas, Carrillo, and Tarrasó 2018).

Given the fact that time is a conceptually subjective matter, the notion of present bias should also be attached to such conjecture. The present (future) biased behavior could also occur on a perceptional time scale. Most of the time, the definition of the present is predetermined by the experimental design, rather than discovered by the empirical work. The definition of the present can vary in length from different studies (Angeletos, Laibson, Repetto, Tobacman, Weinberg 2001; McClure, Ericson, Laibson, Loewenstein, and Cohen 2007; Augenblick, Niederle, and Sprenger 2015; Augenblick 2017). With the different experimental design of "immediate present" and future, studies find the inconstancy decay of discounting can be diverse base on the particular setting. These discussions lead to another open-ended research question, which is when and where we can observe present bias on the time horizon (DellaVigna 2018; Ericson, and Laibson 2019). It is debatable about when the present is. Augenblick et al. (2015) found present bias behavior when considering an interval between a few minutes and weeks as the present for exerting effort experiment. On the other hand, another research observed the present bias behavior if considering a few hours as the present for a monetary award in the experiment. (Augenblick 2017; Balakrishnan, Haushofer, and Jakiela 2017).

The CTB experiment is commonly employed design of dynamic inconsistency time preference in both laboratory and field economic studies (Andreoni, and Sprenger 2012a; Gine, Goldberg, Silverman and Yang 2012; Carvalho, Meier, and Wang 2016). The CTB experiments ask the subjects to make decisions about dividing their award into two accounts, the sooner and the delayed account. If a certain amount of award is located in the delayed account, the subjects will be paid with that certain amount plus interest. However, there will be no interest for the amount they put in the sooner account when they are getting paid. Two separate awards will be issued to the subjects according to the definition of the "sooner" and the "delay" in the experiment. One of the essential advantages of the CTB design is the subject can split the reward into two accounts on a continuous scale. Therefore it allows the structural model to estimate the curvature parameter (elasticity of substitution) of the utility more precisely. Thereby we can estimate the discounting parameter more accurately (4). Given the fixiblbility of the CTB design, a considerable amount of economics (Atalay, Bakhtiar, Cheung, Slonim 2014; Liu, Meng, Wang 2014; Cheung 2015; Alan, Ertac 2015; Carvalho, Meier, Wang 2016; Balakrishnan, Haushofer, Jakiela 2017; Kuhn, Kuhn, Villeval 2017; Luhrmann, Serra-Garcia, Winter 2018) and psychology (Yang and Carlsson 2016; Hoel, Schwab, Hoddinott 2016; Lindner and Rose 2017) studies combine the Convex Time Budget and Quasi-Hyperbolic Discounting to study revealed time preference. These studies come with different treatment dimensions in the experiments to either suggest improvement for the design or provide policy applications. If the QHD model does not precisely characterize the time preference, the validities of these policies would be undermined. We will follow the CTB design closely to further develop the QHD model with the perception of present extension in the **The Theoretical Framework** section to address our concern of the existing time preference studies.

3 The Theoretical Framework

3.1 the General Quasi-Hyperbolic Discounting Model

To study dynamic inconsistency in the revealed time preference, we apply the Quasi-Hyperbolic Discounting (QHD) function to an additive time separable utility function. The QHD model has a β parameter after the first consumption period (1). The β allows the Exponential Discounting (ED) model (3) to mimic the feature of Hyperbolic Discounting. It captures the departure of the inconsistent discounting from the ED function (Strotz 1956; Phelps and Pollak 1968; Laibson 1997; and O'Donoghue and Rabin 1999a). The interpretation of this departure is limited by where we are placing the β parameter. In empirical and experimental studies, we interpret the β as the inconsistency of discounting rate of all future consumption $(x_t, where t \in \{t_1, t_2 ... t_n\})$ compared to the consumption (x_{t_0}) of the soonest observational period (1).

$$u(x_{t_0}) + \beta \sum_{t=t_1}^{t_n} \delta^t u(x_t) \tag{1}$$

where the time $t \in \{t_1, t_2 ... t_n\}$ depends on when we observe the consumption, and x_{t_0} is the initial consumption level at t_0, and x_ts' is all future consumption. We call the β a dynamic inconsistency parameter, which identifies the deviation from the exponential discounting parameter δ (1).

To characterize dynamic inconsistency as a perceptional notion, we modified QHD into a more general functional form ((2); Echenique, Imai, and Saito 2019), and introduce a new parameter $\beta_{\tau_{t_i}}$ (β_τ for simplicity) where t_i can take different values in $\{t_0, t_1 ... t_n\}$ to adjust for an individual's specific perception of the present. We define τ_{t_i} (τ for simplicity) as the ***Preset Threshold*** that distinguishes a subject's perception of present and future. We name β_τ as the ***Perceptional Dynamic Inconsistency Parameter***. The β_τ not only characterizes the direction of bias (either present bias ($\beta_\tau < 1$) or future bias ($\beta_\tau > 1$)) with its magnitude, but also captures the individual's perceptional distinction of the present and the future with the τ. Specifically, the dynamic inconsistency parameter β_τ starts to appear in the formula (2) if and only if the assigned time of sooner consumption and later consumption crosses the individual perceptional present threshold τ. The functional form in (2) is more general on choosing the value of t_i compare to the QHD model (1), and we call it the General Quasi-Hyperbolic Discounting (GQHD).

$$\sum_{t=t_0}^{t_i} \delta^t u(x_t) + \beta_{\tau_{t_i}} \sum_{t=t_i+1}^{t_n} \delta^t u(x_t) \tag{2}$$

where $t \in \{t_0, t_1, ...t_n\}$, which depends on the experimental design, and $\tau_{t_i} \in \{\tau_{t_0}, \tau_{t_1}, ...\tau_{t_n}\}$, $t_0 \leq \tau_{t_0} < t_1$, $t_1 \leq \tau_{t_1} < t_2$, ... $t_n \leq \tau_{t_n}$. Each τ_{t_i} (τ for simplicity) can take any arbitrary value within the given interval, and it would not affect the specification of the structural model for the empirical work. If $\tau < t_0$ or $\tau \geq t_n$, then the QHD utility reduces to the exponential discounting utility (3). Therefore, the estimation of β equals 1 has two possible interpretations. Either the subject has a dynamic consistency utility, or the design failed to trigger the subject's sense that distinguishes his or her perception of present and future (see ***Property 1***).

$$\sum_{t=t_0}^{t_n} \delta^t u(x_t) \tag{3}$$

We present some unique definitions of the GQHD model that corresponds to the Convex Time Budget (CTB) experiment.

Definition 1: ***Time*** $t \in [0, t_n] = T$ is a continuous variable that describes the progress domain of a CTB experiment from the very moment when we observe subject's time preference ($t = 0$) until the moment when the very last award is scheduled to be delivered ($t = t_n$) to the subjects.

Definition 2: ***Now*** $t = 0$ is a discrete definition that defines the very moment when we observe the time preference behavior[1]. It doesn't take a long time (minutes) to collect the choices from a CTB experiment, and given a long enough time horizon (months) for the last award to be delivered (Andreoni and Sprenger 2012a); so we define Now as discrete arbitrarily.

Considering the revealed time preference data is collect at ***Now***, we will label the observed utility function given the data set as $U_0(\cdot_t)$ to keep the consistency of the notation in the theoretical framework. Notice that we do not have to assume the utility is time-invariant $(U_t(\cdot_t) \equiv U(\cdot_t), \forall t \in T)$ to estimate the parameters for the empirical work. However, to further extend the GQHD model to O'Donoghue and Rabin's procrastination studies (1999a; 1999b; 2001) or to develop any policy implication of the empirical results, imposing the time-invariant assumption is necessary.

Definition 3: ***Delivery Date*** $t = t_i \in \{t_0, t_1, ...t_n\}$, (where $i = 0, 1, 2, ...n$) is a discrete variable that identifies the time when the reward is scheduled to be delivered to the subjects.

One important note we need to emphasize is that there exists a small time gap (ϵ) between the preference is revealed, and the first reward is delivered $(0 + \epsilon = t_0, \epsilon > 0)$. For example, the subjects may need to fill out a questionnaire after the CTB experiment, or it takes some time for the electronic transaction to process. How soon is now type of research is addressing the question about how small ϵ should be, then $U_0(\cdot_t) \equiv U_{0+\epsilon}(\cdot_t)$ (Balakrishnan, Haushofer, and Jakiela 2017). Kirby (1997) concerns not to distinguish the difference between the instant award and the nearly instant award. In our framework, we also assume $0 < t_0$.

Definition 4: ***Present Threshold*** $\tau \in \{\tau_{t_0}, \tau_{t_1}, \tau_{t_2}, ...\tau_{t_n}\}$ $(t_i < \tau_{t_i} \leq t_{i+1}, i \in \{0, 1, 2, ...n-1\}$ or $t_i < \tau_{t_i}, i = n)$ is a discrete variable that captures a value of time, which distinguishes the ***Present*** and the ***Future***.

The ***Present Threshold*** (τ_{t_i}) can take any value between the interval $(t_i, t_{i+1}]$ because consumption level is not observed for any available ***Delivery Date*** in T. To take any value within the interval will not change the the estimation strategy of any structural model. Additionally, the ***Present Threshold*** can be further decomposed into three subcategories the ***Primary Present Threshold*** (τ_{t_0} for QHD model), the ***Secondary Present Threshold*** (τ_{t_i} $i \in \{1, 2, 3, ...n-1\}$ for GQHD model), and the ***Observational Dynamic Consistency Threshold***. (τ_{t_n} for ED model). The τ_{t_1} is equivalent to the conventional Quasi-Hyperbolic Discounting model (Phelps and Pollak 1968); τ_{t_n} is equivalent to the exponential discounting model (Koopmans 1960); the

[1]If we extend time $t \in \mathbb{R}$, then the ***Past*** can be defined as $t \in (-\infty, 0) = T_h$ (h for history).

Secondary Present Threshold is unique to our General Quasi-Hyperbolic Discounting model. A subject's time preference is GQHD rationalizable if he or she has a unique and stable τ corresponding to revealing decisions at ***Now***. The subscript τ indicates the actual ***time*** location of the ***Perceptional Dynamic Inconsistency Parameter*** β_τ (2).

Definition 5: The ***Present*** $t \in [0, \tau] = T_p$ is an interval of time (t) between ***Now*** and the ***Present Threshold*** (τ). We denote the individual ***Perceptional Present*** as $T_{\tilde{p}}$.

Definition 6: The ***Future*** $t \in (\tau, t_n] = T_f$ is an interval of time (t) between the ***Present Threshold*** (τ) and the date that the latest award is scheduled to be delivered (t_n). We denote the individual ***Perceptional Future*** as $T_{\tilde{f}}$[2].

In the framework of GQHD model, the ***Now*** and the ***Present*** are different. ***Now*** is the lower bound to the ***Present***. On the contrary, the upper bound of the ***Present*** is determined by the individual's time perception. Previously, we consider the t_0 as the upper bound of the present in the structural models' estimation strategy. The GQHD model is more general compare to the QHD in the sense that we propose a more flexible upper bound of the ***Present*** to detect how long it would last. Simultaneously, the ***Future*** is no longer defined as all ***Delivery Date*** that is not t_0, but as the ***Time*** (t) after the ***Present Threshold***. We define $t \in [0, \tau_0] = T_{p_0}$ as the ***Primary Present***, $t \in [0, \tau_i] = T_{p_i}$ as the ***Secondary Present***, $t \in (\tau_0, t_n] = T_{f_0}$ as the ***Primary Future***, and $t \in (\tau_i, t_n] = T_{f_i}$ as the ***Secondary Future*** (where $i \in \{1, 2, 3, ...n-1\}$).

3.2 The Behavioral Characterizations of the GQHD Model

With a formal extension of the QHD model, the ***Perceptional Dynamic Inconsistency Parameter*** β_τ differs the ***Perceptional Present*** from the ***Perceptional Future*** in the GQHD model. As a result of the flexible location of the β_τ, the GQHD model has some unique properties compared to the ED and QHD models. We demonstrate five properties of the GQHD model to characterize the behavior in the revealed time preference.

Property 1: The revealed time preference exhibits **Observational Dynamic Consistent** if $\beta_\tau = 1$.

Generally, we emphasize the dynamic consistency is observational because the structural model fits the data in the give experimental time horizon. The inconsistency in time preference might

[2] If we extend time $t \in \mathbb{R}$, then the ***Future*** can be defined as $t \in (\tau, \infty) = T_f$.

happens beyond the experimental ***time*** domain. The locations of β_τ are constraint by the way that we collect data in the experiment. The β_τ exists in the structural model if and only if $t_0 \leq \tau < t_n$. The structural model cannot detect β_τ, if the τ exists at a location further than t_n. Similarly, the structural model is not sensitive to the time inconsistency behavior if τ exists at a location that is sooner than t_0. Balakrishnan et al. (2017) address this $\tau < t_0$ issue with a modified design of the CTB. However, the $t_n \leq \tau$ question remains to be discovered. $ED \equiv QHD \equiv GQHD$, if $\beta_\tau = \beta = 1$ or $\tau < t_0$ or $t_n \leq \tau$.

Property 2: The revealed time preference exhibits **Primary Present Bias** if $t_0 \leq \tau < t_1$, and $\beta_\tau < 1$.

Property 3: The revealed time preference exhibits **Primary Future Bias** if $t_0 \leq \tau < t_1$, and $\beta_\tau > 1$.

The primary bias model, either present bias or future bias, is equivalent to the conventional setup (Laibson 1997; Andreoni and Sprenger 2012a). The additional explanation we provide to this traditional notion is that we abolish the restriction of the sense of threshold that differs present and future at the individual level. We distinguish the present and future, not by a single value in the timeline, say today verse any day that is not today. Instead, we consider any arbitrary $\tau < t_1$ can be interpenetrated as an individual's perception of the present. $QHD \equiv GQHD$, if $t_0 \leq \tau < t_1$.

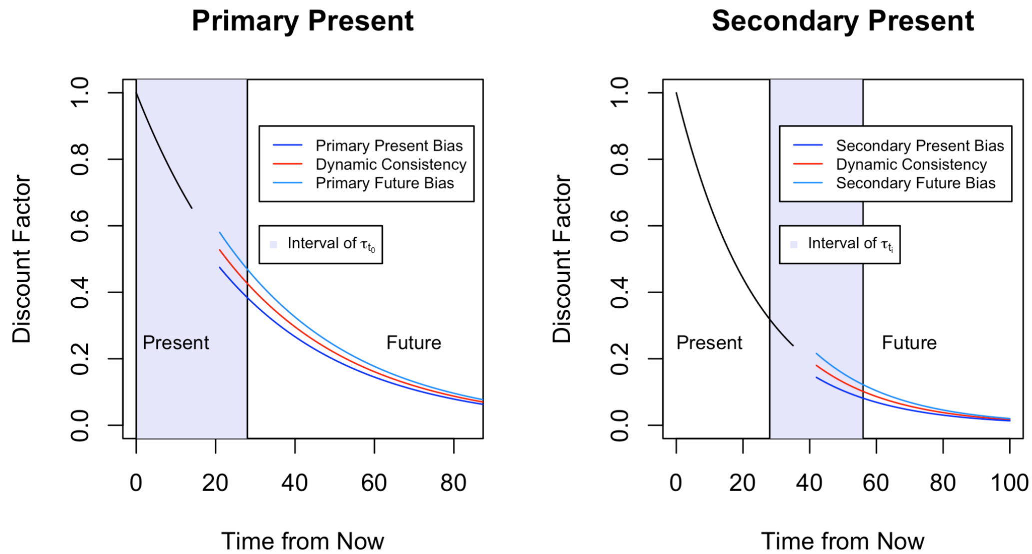
Property 4: The revealed time preference exhibits **Secondary Present Bias** if $t_1 \leq \tau < t_n$, and $\beta_\tau < 1$.

Property 5: The revealed time preference exhibits **Secondary Future Bias** if $t_1 \leq \tau < t_n$, and $\beta_\tau > 1$.

The secondary bias model is a general extension of the primary bias model. We allow the τ to take the value greater than t_1, which is the nearest observed consumption. The general format could detect the inconsistency time preference between present and future with a relaxed assumption that individual subject has different time perception (Zauberman, Kim, and Malkoc 2009). The subjective time perception would affect the way we describe time inconsistency severely.

There is a gap between the theoretical framework and the structural empirical model proposed by the CTB protocol. The gap commonly exists for all of the intertemporal choices studies; the simple version (Fisher 1930), or any of its variations (Benhabib, Bisin, and Schotter 2010; Takeuchi 2011; Jackson, and Yariv 2014; Andreoni, and Sprenger 2012a; Montiel Olea and Strzalecki 2014).

Figure 1: Primary Present and Secondary Present



The subjects make decisions in the discrete-time slots. The consumption in the potential interval of τ area is unobserved. We use the structural model to fit the observations and then describe the subjects' dynamic time inconsistency. Theoretically, the primary present model and secondary present model are equally compelling to explain time inconsistency with the assumption that within the shaded area, the subjects have a inconsistency time preference. The advantage of the GQHD model is to initiate β_τ at different time spot, then find the most unexpected kinky consumption among all others (Figure 1).

3.3 Empirical Strategy

Given the general theoretical framework of the convex time budgets (CTB) in time preference studies (Andreoni and Sprenger 2012), we assume the subjects in the CTB experiment maximizing an additively time separable constant relative risk aversion (CRRA) utility function of consumption ($U(x)$) between two ***Delivery Date*** t_i, and t_i+k. Such utility function is discounted with a GQHD function. Consider a subject maximizing the GQHD-CRRA utility at ***Now*** (4), subject to a given time budget constraint in the experiments (5).

$$U_0(x_{t_i}, x_{t_i+k}) = \delta^{t_i} \frac{1}{1-\eta}(x_{t_i}+\omega)^{1-\eta} + \beta_\tau^{\mathbb{1}\{t_i \le \tau < t_i+k\}} \delta^{t_i+k} \frac{1}{1-\eta}(x_{t_i+k}+\omega)^{1-\eta} \tag{4}$$

where we denote the one-period discount factor as δ, the curvature parameter as $1-\eta$, the background consumption as ω, which we assume it as constant for every period. Lastly, x_{t_i} and x_{t_i+k} denote the consumption that allocate into the sooner account and the later account correspondingly.

$$x_{t_i} + (1+r)^{-1} x_{t_i+k} = m \tag{5}$$

where we denote the r as the interest rate through k days and m as the total endowment for the current trail. Then the optimal demand function of sooner consumption is: (6)

$$x^*_{t_i} = \begin{cases} \frac{(\delta^k(1+r))^{\eta^{-1}}-1}{1+(1+r)(\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\delta^k(1+r))^{\eta^{-1}}}{1+(1+r)(\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } \tau < t_i \text{ or } t_i + k \leq \tau \\ \frac{(\beta_\tau\delta^k(1+r))^{\eta^{-1}}-1}{1+(1+r)(\beta_\tau\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\beta_\tau\delta^k(1+r))^{\eta^{-1}}}{1+(1+r)(\beta_\tau\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } t_i \leq \tau < t_i + k \end{cases} \tag{6}$$

To illustrate the strategy of finding the optimal τ, we provide an example under the assumptions of the empirical framework. Consider a subject's three-period discounting function exhibit the following pattern $D_\tau(t_0, t_1, t_2) = (\delta^{t_0}, \delta^{t_1}, \beta\delta^{t_2})$, which is a secondarily bias time preference behavior. Suppose we correctly specified the structure model with a sequence of time-dependent discount factors $\hat{D}_\tau(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$. We can recover the true value of δ by solving the equation between the marginal rate of substitution and the price ratio of x_{t_0} and x_{t_1}. Here we denote the growth interest rate as r_{t_0,t_1} (7).

$$\frac{\partial U_0/\partial x_{t_1}}{\partial U_0/\partial x_{t_0}} = \frac{1}{1+r_{t_0,t_1}} \implies \delta^* = (\frac{1}{1+r_{t_0,t_1}})^{\frac{1}{t_1-t_0}}(\frac{x_{t_1}}{x_{t_0}})^{\frac{1-\eta}{t_1-t_0}} \tag{7}$$

Similarly, we can recover the true value of β by solving the equation between the marginal rate of substitution and the price ratio of x_{t_1} and x_{t_2}. We denote the growth interest rate as r_{t_1,t_2}. Here notice β^* (or $\beta^{*t_2-t_1}$) is strictly greater than 0 (8).

$$\frac{\partial U_0/\partial x_{t_2}}{\partial U_0/\partial x_{t_1}} = \frac{1}{1+r_{t_1,t_2}} \implies \beta^{*t_2-t_1} = (\frac{1}{1+r_{t_1,t_2}})^{\frac{1}{t_2-t_1}} (\frac{x_{t_2}}{x_{t_1}})^{\frac{1-\eta}{t_2-t_1}} (\frac{1}{1+r_{t_0,t_1}})^{\frac{-1}{t_1-t_0}} (\frac{x_{t_1}}{x_{t_0}})^{\frac{\eta-1}{t_1-t_0}} \quad (8)$$

Contrarily, suppose we follow the QHD set up and misspecify the structure model with $\hat{D}_{\tau'}(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\beta}\hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$. Then misleadingly equating the misspecified estimation with the true parameter $D_\tau(.) = \hat{D}_{\tau'}(.)$, we can derive $\hat{\delta}$ in (9). For simplicity, we consider $\delta^{t_0} = 1$.

$$\frac{\hat{\beta}\hat{\delta}^{t_2}}{\hat{\beta}\hat{\delta}^{t_1}} = \frac{\beta\delta^{t_2}}{\delta^{t_1}} \implies \hat{\delta} = \beta^{\frac{1}{t_2-t_1}}\delta \implies \begin{cases} \hat{\delta} > \delta, & \text{if} \quad \beta > 1 \\ \hat{\delta} < \delta, & \text{if} \quad \beta < 1 \end{cases} \quad (9)$$

For $\hat{\delta}$, we can use the equation (8) to quantify the magnitude of overestimation or underestimation. With a similar approach, we can derive $\hat{\beta}$ in (10).

$$\frac{\hat{\beta}\hat{\delta}^{t_1}}{\hat{\delta}^{t_0}} = \frac{\delta^{t_1}}{\delta^{t_0}} \implies \hat{\beta} = \frac{\delta^{t_1-t_0}}{\hat{\delta}^{t_1-t_0}} \implies \begin{cases} \hat{\beta} < 1 < \beta, & \text{if} \quad \beta > 1 \\ \hat{\beta} > 1 > \beta, & \text{if} \quad \beta < 1 \end{cases} \quad (10)$$

The result above has an intuition that can guide us to initiate the β_τ at an optimal location that provides a more precise inference for the time preference (9 and 10). Suppose we misplace a secondary future bias subject's initial β_τ earlier than the true location, then we are likely to interpret this subject's behavior as primary present bias. As pushing τ to the further locations, we would observe a decreasing pattern of $\hat{\delta}$, along with an increasing trend of $\hat{\beta}_\tau$. Ideally, we can continuously push the τ to a further location until we observe the $\hat{\beta}_\tau > 1$. Similarly, if we place a secondary present biased subject's τ at a closer location, then we should expect an increasing trend of the $\hat{\delta}$ along with a decreasing pattern of the $\hat{\beta}_\tau$ as we push the τ to the later position. The optimal initial location in the summation of the β_τ in the summation is obtained when the $\hat{\beta}_\tau < 1$ in this scenario. As many dynamic discrete choice studies demonstrated, the β and δ vary accordingly with a systematic mechanism in QHD (Laibson, Repetto, and Tobacman 2007; Arcidiacono and Miller 2008; Mahajan and Tarozzi 2012; Fang and Wang 2015; Abbring, Daljord, and Iskhakov 2018). This grand truth also applies to the Convex Time Budget, since the choices are still in a dynamic setting. Our intuitive derivation reveals a potential confound classification of the ***Primary Present Bias*** and the ***Secondary Future Bias*** (the ***Primary Future Bias*** and the ***Secondary Present Bias***). However, the confusion of mixing different types of behaviors have two directions ((9), (10); and (14), (15) from Appendix A). Besides, the trend of changing in β and δ becomes more complicated when dynamic choices are made in many periods. To find the models that give a more profound explanation of the behaviors, we would provide more detailed findings with the empirical methods. To estimate the parameters, we will employ the non-linear least square (NLS) strategy (Andreoni and Sprenger 2012a; See the **Aggregate Level Results** section).

4 Simulated and Experimental Data

We acquired two data sets with the Convex Time Budget design to detect the perception of present and its corresponding time inconsistency to see the robustness of the primary present model (Andreoni and Sprenger 2012a (AP12); Carvalho, Meier, and Wang 2016 (CMW16)). In the following sections, we will mainly focus on the CMW16 data set because we find more server changes in point estimation of β_τ and δ in CMW16 than AP12 (See Table 9 in Appendix B for AP12 results). Since we focus on the design and the data set from the CMW16 data set, we also simulated 21 different data sets according to the CTB design to either demonstrate the intuition or confirm the findings in our studies. Since we focus on the design and the data set from the CMW16 data set, we also simulated 21 different data sets according to the CTB design to either demonstrate the intuition or confirm the findings in our studies. We will go through the details about the simulations when needed. Before any analysis, it is necessary for us to learn about the features of the Convex Time Budget design in CMW16.

In Carvalho et al. (2016), the participants are asked to allocate their experimental budget of \$500 into two payment accounts. The sooner account (t_i), and its paired later account (t_i+k). There are three paired accounts with $(t_i, t_i+k) \in \{(28\ days, 84\ days), (28\ days, 84\ days), (today, 28\ days)\}$[3]. For each paired accounts, subjects make 4 choices of how they would like to split 500 dollars into sooner and later accounts. The delayed (later) account has its associated interest rate of 0%, 0.5%,

Table 1: The CTB Design in CMW16

choice	t_i	$t_i + k$	**r**	**m**
1	28 days	84 days	0%	$500
2	28 days	84 days	0.5%	$500
3	28 days	84 days	1%	$500
4	28 days	84 days	3%	$500
5	28 days	56 days	0%	$500
6	28 days	56 days	0.5%	$500
7	28 days	56 days	1%	$500
8	28 days	56 days	3%	$500
9	today	28 days	0%	$500
10	today	28 days	0.5%	$500
11	today	28 days	1%	$500
12	today	28 days	3%	$500

1%, and 3% respectively at the payday. In total, each subject make 12 choices (we have 12 observations for each subject). The researchers randomly select one out of twelve choices to issue the reward to the subjects at the paydays. If the choice has a nonzero amount in the later account, then they will be paid with the associated interest rate **r** (Table 1). They restricted the sample for 1061 subjects that finish all 12 trials in the data set. Our analysis will impose the same restriction on these participants who complete all 12 trials in the experiments. They observed a slight primary present bias for the subjects at the aggregate level from a tow-limit Tobit model with a CARA utility setup (Carvalho, Meier, and Wang 2016).

The CTB design in CMW16 allows us to set the ***Present Thresholds*** (τ) at four different time slots in the GQHD specification. Firstly at τ_0, the subject can consider any day between today and 28 days later as his or her perceptional present. Consider the ***Primary Present*** (T_{p_0}) directly come from the QHD model. Secondly, τ_{28} is when a subject's τ can take any day between 28 days and 56 days arbitrarily. Then, τ_{56} has a range between 56 days and 84 days. These specifications consider the ***Secondary Present*** (T_{p_i}), which is the innovation of the GQHD. Lately, the τ_{84} indicates the ***Present Thresholds*** surpasses 84 days, and the specification claps back to the ED model.

We illustrate the systematic changes in the β and the δ parameters in the **Empirical Strategy**

[3]In the CTB design of CMW16, the payment dates are labeled as today, 4 weeks, and 8 weeks. We denote 4 weeks as 28 days, 8 weeks and 56 days, and 12 weeks as 84 days to be consistent with our structural model. We also consider t_i for today as $t_0 = 0$ to fit the model.

Table 2: Estimations of 18 Simulated Data Sets (CMW16)

500 Subjects		τ_0	τ_{28}	τ_{56}	τ_0	τ_{28}	τ_{56}
$\mathbf{1-\eta}$		$\boldsymbol{\beta_{\tau_{56}} = 1.1\ \delta = 0.9995}$			$\boldsymbol{\beta_{\tau_{56}} = 1.05\ \delta = 0.9995}$		
0.9	$\hat{\beta}$	0.9622	0.9999	1.1000	0.9811	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0013	0.9995	1.0003	1.0003	0.9995
0.85	$\hat{\beta}$	0.9626	1.0007	1.1000	0.9814	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0011	0.9995	1.0003	1.0003	0.9995
0.8	$\hat{\beta}$	0.9624	0.9992	1.1000	0.9815	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0011	0.9995	1.0003	1.0003	0.9995
$\mathbf{1-\eta}$		$\boldsymbol{\beta_{\tau_{56}} = 1.1\ \delta = 0.9985}$			$\boldsymbol{\beta_{\tau_{56}} = 1.05\ \delta = 0.9985}$		
0.9	$\hat{\beta}$	0.9608	1.0000	1.1000	0.9807	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9999	0.9985	0.9992	0.9991	0.9985
0.85	$\hat{\beta}$	0.9614	1.0000	1.1000	0.9808	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9999	0.9985	0.9992	0.9991	0.9985
0.8	$\hat{\beta}$	0.9617	1.0000	1.1000	0.9809	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9998	0.9985	0.9992	0.9991	0.9985
$\mathbf{1-\eta}$		$\boldsymbol{\beta_{\tau_{56}} = 1.1\ \delta = 0.9975}$			$\boldsymbol{\beta_{\tau_{56}} = 1.05\ \delta = 0.9975}$		
0.9	$\hat{\beta}$	0.9612	1.0000	1.1000	0.9810	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9986	0.9975	0.9982	0.9980	0.9975
0.85	$\hat{\beta}$	0.9617	1.0000	1.1000	0.9810	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9987	0.9975	0.9982	0.9981	0.9975
0.8	$\hat{\beta}$	0.9618	1.0000	1.1000	0.9811	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9987	0.9975	0.9982	0.9981	0.9975

section. However, that derived intuition was limited to three-period consumption observations. The design of CMW 16 has four periods (today, 28 days, 56 days, and 84 days) instead of three. We will confirm if the systematic changes in the β and the δ parameters remain unchanged from 18 simulated data sets corresponding to the CTB in CMW16. To begin with, we simulated the following 18 data sets to detect the validity of the intuition that the structural model tends to underestimate the β parameter if we assume the perception of the present ends right after the soonest consumption rather than terminates at later possible locations. Suppose we have two sets of secondary future biased time preference populations with $\beta_{\tau_{56}}$ equals to either 1.1 or 1.05 ($\beta_{\tau_{56}} \in B = \{1.1, 1.05\}$). We simulated 9 data sets for each of these two secondary future biased populations with selected curvature ($1 - \eta \in H = \{0.9, 0.85, 0.8\}$) parameters and discount parameters ($\delta \in$

$D = \{0.9995, 0.9985, 0.9975\}$). The annual discount rates $(1/\delta)^{365} - 1$ are 20.03%, 72.96%, and 149.34% for the selected discount parameter 0.9995, 0.9985, and 0.9975 respectively. We simulated 500 subjects' optimal consumption level $x^*_{t_i}$ at date t_i (6) according to the CMW16 design with a random normally distributed error ($\boldsymbol{e} \overset{i.i.d}{\sim} \boldsymbol{N}(0, 0.01)$) for all 18 data sets ($H \times B \times D$). The simulated results are presented in Table 2. As a consequence, we observed a strong tendency of the increasing trend in the estimated β and the decreasing trend of the estimated δ as we move the τ to the true location τ_{56} from the QHD assumed τ_0. Such tendency suggests the degree of present bias can be overestimated if we treat the subjective present unanimously as the ***Primary Present*** (T_{p_i}), especially for those who exhibit ***Secondary Future Bias*** ($\beta_{\tau_{56}} > 1$) in their time preference (see the **Analysis and Results** section for the detailed statistical approach). The findings coincide with the grand truth reached in (9) and (10).

5 Analysis and Results

5.1 Aggregate Level Results

To estimate the parameters, we employed a similar non-linear least square (NLS) strategy proposed by Andreoni and Sprenger (2012a). We specified the indicating function of the β to identify the perceptional present accordingly. Furthermore, we estimate the model with the assumption of the time-independent $\omega \approx 0$ in (6) consider the decision-maker is not necessarily narrowly brackets in the experiment (Rabin and Weizsacker 2009). The differences between restricting $\omega = 0$ or not are statistically negligible, and we present the results of the estimations without such restriction in Table 3.

From Table 3, We find the estimation of the β in the primary model is equal to 0.9516, which suggests the subjects exhibits ***Primary Present Bias***, which is consistent with the results in CMW16. However, as we relocate the tau into further location, we find a clear increasing trend in the estimation of β and a decreasing trend in δ estimation. This pattern suggests that there are more subjects' behaviors that follow the ***Secondary Future Bias*** instead of the ***Primary Present Bias***. The future bias is observed at τ_{56}. We chose three different criteria to compare the

Table 3: Aggregate Level Estimation for 1061 Subjects (CMW16)

Model	τ_0	τ_{28}	τ_{56}	τ_{84}
$\hat{\beta}$	0.9516	1.0010	1.1280	
	(0.0064)	(0.0086)	(0.0121)	
$\hat{\delta}$	0.9979	0.9976	0.9962	0.9976
	(0.0001)	(0.0002)	(0.0002)	(0.0001)
$1-\hat{\eta}$	0.7364	0.7224	0.7561	0.7224
	(0.2111)	(0.2319)	(0.2205)	(0.2319)
AIC	165404.5	165463.7	165308.8	165461.7
BIC	165441.7	165500.9	165346	165491.5
RSS	326667281	328189670	324220845	328189845
Observations	1061	1061	1061	1061

overall performance of different models (Akaike information criterion (AIC), Bayesian Information Criterion (BIC), and Residual Sum of Squares (RSS)). These three criteria also provide evidence that supports $\beta_{\tau_{56}}$ is the most desirable model to describe the data at the aggregate level. The model of $\beta_{\tau_{56}}$has the lowest AIC, BIC, and RSS, given all of the possible locations of the τ we detected.

Since the credibility of our structural model relies heavily on the existence of well-behaved (monotonic, quasi-concave, continuous, and non-satiated) utility functions, we next evaluate individual participants' data for consistency with the requirements of rationality, exclude observations that fail to meet the criteria, repeat the analysis and compare results. A commonly used approach for detecting the existence of well-behaved utility is satisfying the Generalized Axiom of Revealed Preference (GARP) and its non-parametric test (Afriat 1967, Afriat 1972, Varian 1982, and Varian 1991). This approach will not work, however, due to the original design of the CTB experiments since the commodities from different trials are not nested, and the time budget lines barely intersect with one other. We, therefore, employed the Law of demand test which requires that if a trial that has a higher interest rate for the later account than the previous trial, we should observe the consumption level of the later account is at least as much as the previous trial. This basic consistency approach has been utilized by the earlier studies to detect for rationality (Gine, Goldberg, Silverman and Yang 2012; Balakrishnan, Haushofer, and Jakiela 2017; Echenique, Imai, and Saito 2019).

Table 4: Aggregate Level Estimation for 452 Subjects (CMW16)

Model	$\boldsymbol{\tau_0}$	$\boldsymbol{\tau_{28}}$	$\boldsymbol{\tau_{56}}$	$\boldsymbol{\tau_{84}}$
$\hat{\boldsymbol{\beta}}$	0.9625	1.0000	1.1010	
	(0.0064)	(0.0085)	(0.0123)	
$\hat{\boldsymbol{\delta}}$	0.9977	0.9974	0.9964	0.9974
	(0.0001)	(0.0002)	(0.0002)	(0.0001)
$\mathbf{1}-\hat{\boldsymbol{\eta}}$	0.8450	0.8392	0.8534	0.8392
	(0.0826)	(0.0848)	(0.1129)	(0.0848)
AIC	71257	71291.83	71199.9	71289.83
BIC	71290	71324.82	71232.89	71316.22
RSS	160888352	161924583	159203278	161924667
Observations	452	452	452	452

We find that 452 out of 1061 subjects' choice set strictly follows the law of demand and repeat our analysis with only these subsets Table 3. We found these people, whose time preference follows the law of demand, also seems to exhibit present bias when we set β_τ at t_0 on an aggregated level. Additionally, there is an increasing trend of the β estimator and a decreasing trend of the δ estimator as we move τ to later possible locations (τ_{28}, τ_{56}, and τ_{84}). We observe the best performance when τ_{84} according to AIC, BIC, and RSS criterion, which suggests the subjects are more likely to have a secondary future biased time preference than a primary present biased at the aggregate level (Table 4).

To consider the concentration of these dynamically inconsistent behaviors at the individual level, and how it would affect the aggregate level estimation, it is necessary to sort out these subjects with dynamically consistent time preferences. The Strong Axiom of Revealed Exponentially Discounted Utility (SAR-EDU). The SAR-EDU suggests that if a finite time revealed time preference data set is EDU rational, then the time preference can be rationalized with an exponential discounted well-behaved utility function, where delta can take the value of 1. The additional finding suggests the test's result is aligned with parametric estimations. If the revealed time preference data set passes the SAR-EDU test, then the empirical estimation of the β_τ is likely to be close to 1 (Echenique, Imai, and Saito 2019).

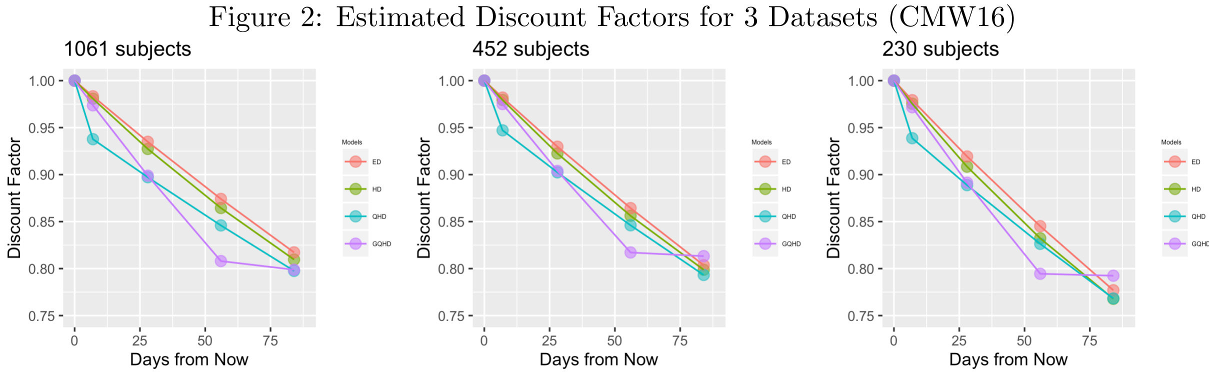
We rerun the analysis with the 230 subjects whose choice set, which did not pass the SAR-EDU test within the 452 subjects. These 230 people's revealed time preference choice set is theoretically

Table 5: Aggregate Level Estimation for 230 Subjects (CMW16)

Model	τ_0	τ_{28}	τ_{56}	τ_{84}
$\hat{\beta}$	0.9560 (0.0067)	1.0020 (0.0086)	1.1190 (0.0151)	
$\hat{\delta}$	0.9974 (0.0001)	0.9970 (0.0002)	0.9959 (0.0003)	0.9970 (0.0001)
$1-\hat{\eta}$	0.8819 (0.0503)	0.8779 (0.0485)	0.8884 (0.1179)	0.8779 (0.0485)
AIC **BIC** **RSS**	35711.12 35740.73 67006296	35757.27 35786.88 68136084	35637.66 35667.27 65246300	35755.34 35779.04 68137966
Observations	230	230	230	230

unrationalizable with dynamic consistency well-behaved utilities (EU). The trend of β estimation and δ estimations remain unchanged. The model τ_{56} produced the best overall performance among all models (Table 5). We will investigate how different restrictions change the concentrations of the different types of dynamically inconsistent behavior and how such concentration could affect the aggregate level conclusion in the **Individual Level Results** section.

Figure 2: Estimated Discount Factors for 3 Datasets (CMW16)



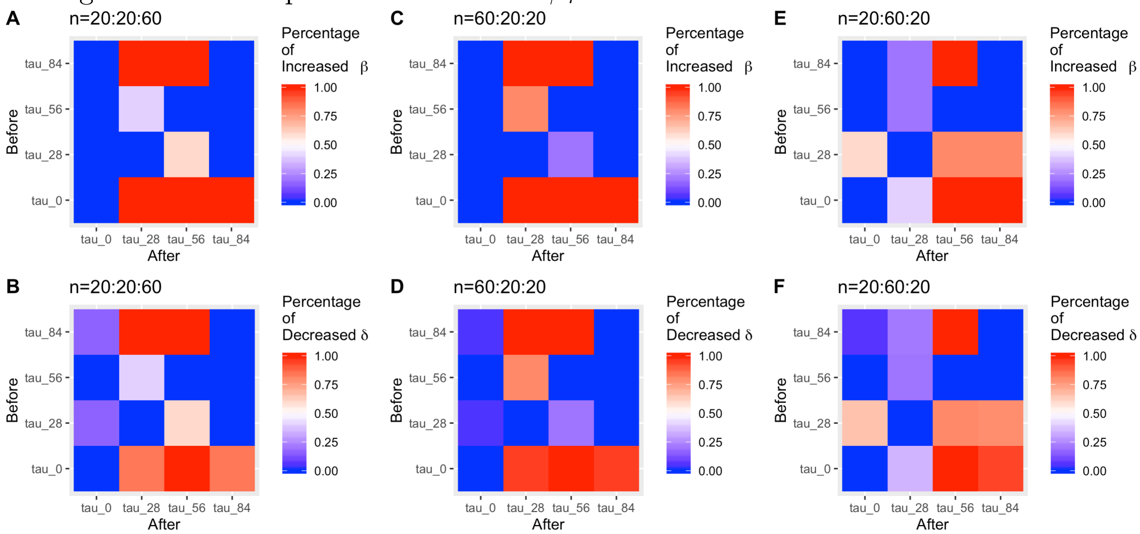
To make the comparisons between the different discounting models, we present the estimated discount factors among the ED, the HD (the Hyperbolic Discounting Model: see the results from from Appendix C), the QHD, and the GQHD models in Figure 2. We see a departure at the initial days of the HD model and the QHD models for all three different data sets. This departure indicates that the initial sharp drop of the QHD model was likely to catch a later seemly inconsistency discount rate caused by the underestimation of the δ, rather than to present the inconsistent discounting rates between the present and the future. The GQHD model indeed captures some unusual behavior in the discount function at the aggregate level for all different data sets. The GQHD model indeed captures some unusual behavior of the curvature in the discount function (Takeuchi 2012) at the aggregate level for all different data sets. It has the overall best performance among all four different models. An additional investigation at the individual level is necessary to explain what we find at the aggregated level.

5.2 Individual Level Results

At the aggregate level, the people with secondary future biased time preference is dominating the time preference behavior in CMW16 data set ($\beta_{\tau_{56}} > 1$ models have the best overall performance). We can also reach this intuition by unanimously observing the same pattern of trends of estimations of β and δ parameters when relocating τ to later positions. To verify how the concentration of the different behavior changes the pattern of the estimations, we newly simulated three data sets (100 simulated subjects each) with different proportion combinations of primary present biased, time consistent, secondary future biased subjects. The proportions of primary present biased, time consistent, and secondary future biased subjects for these three simulated data sets are 2:2:6 (secondary future biased dominating), 2:6:2 (time consistent), and 6:6:2 (primary present biased dominating) respectively. For the simulated data, the η takes a uniform distribution between 0.87 and 0.89 ($1 - \eta \in [0.87, 0.89]$), and the δ takes a uniform distribution between 0.9997 and 0.9999 ($(1/\delta)^{365} - 1 \in [20.03\%, 149.34\%]$). We also assumed the background consumption ω is 0 for all of the simulated subjects. The biased parameter β are uniformly generated between 0.95 and 0.99 for the primary present biased ($\beta_{\tau_0} \in [0.95, 0.99]$) simulated subjects, and between 1.05 and 1.1 for the secondary future biased ($\beta_{\tau_{56}} \in [1.05, 1.1]$) simulated subjects. The β are equal to 1 for the time consistent time preference ($\beta_{\tau_{28}} = 1$) simulated subjects. Then we estimated the β and δ parameters with the NLS approach for these simulated subjects.

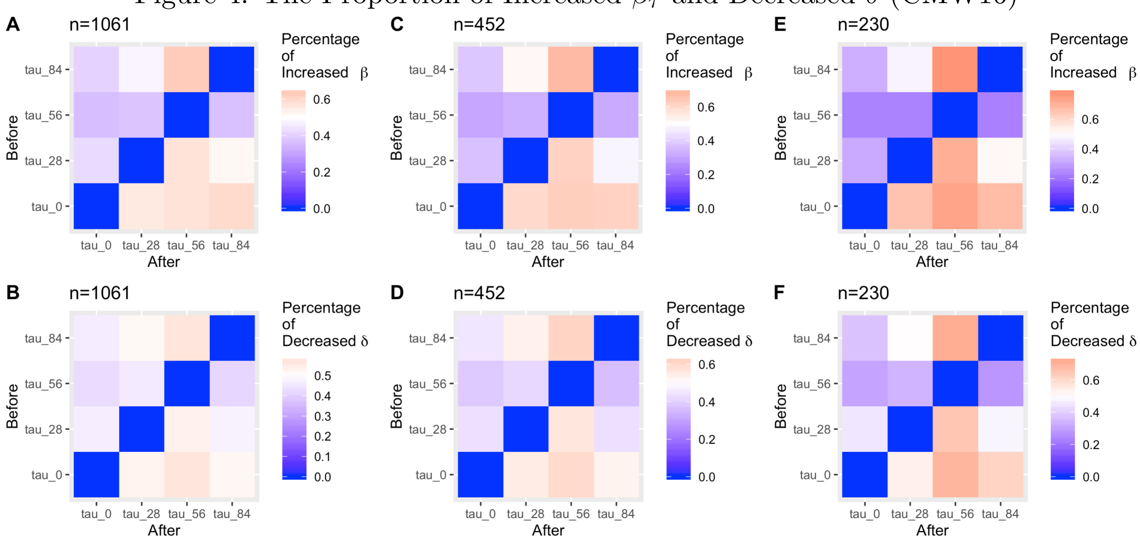
To visualize the relationship between the change of the estimations and the proportion of the different types of time preference, we plotted a heat map of the proportions of increased β for all pairwise comparisons of two different β_τ identified structural models. For example, suppose we estimate the β_{τ_0} as the first model (labeled as before), and we estimate the $\beta_{\tau_{28}}$ as the second model (labeled as after). The pixel for β_{τ_0} (before) and $\beta_{\tau_{28}}$ (after) represents the percentage of

Figure 3: The Proportion of Increased β_τ and Decreased δ for the Simulated Data



the increased β for the second model compare to the first model. We apply the same rule for the percentage of decreased δ estimations. We can recognize the different patterns in the heat maps when compare plots A and B (secondary future biased dominating), vs. C and D (time consistent) vs. E and F (primary present biased dominating) in Figure 4. We will use the simulated heat maps as the reference to trace the hypothetical concentration of different behaviors in CMW16 data.

Figure 4: The Proportion of Increased β_τ and Decreased δ (CMW16)



As a comparison, we plot the heat maps of the NLS estimations of the β and δ parameters for all three CMW 16 subsets (1061, 452, and 230) in the same style as the 300 simulated subjects. For the full sample (n=1061) and the other two sub-samples (n=452, or 230), the observed proportion of the increased β is always surpassing 50 % when comparing the closer τ (before) models (τ_0, τ_0, and τ_{28}) to the further τ (after) models (τ_{28}, τ_{56}, and τ_{56}) respectively. The decreasing trend of the estimated δ is also dominating (surpass 50%) for all samples as if we focus on the counterpart of the β pixels. We can see this clear pattern in the lower triangular region (bottom right) of the heat maps in Figure 4. We can also find this coherent similarity of the color pattern in plots A and B from Figure 3. The similarity in color patters indicates the subjects in CMW16 are likely to be dominated by the secondary future biased time preference rather than the reported primary present biased behaviors (Carvalho, Meier, and Wang 2016). Our finding explains the reason that there was an additional 68 subjects' time preference became Future-Biased QHD rational (28.94 percent increase). Still, only 9 more subjects became Present-Biased QHD rational (3.83 percent increase) when the τ was relocated at later positions in CMW16 data (Echenique, Imai, and Saito 2019)[4]. More importantly, we observed denser and denser color pixels as more restricting assumptions were imposed on the CMW16 data. This tendency pinpoints that the failure of separating different behavior with the theoretical framework would lead to the misconceptions at the aggregate level.

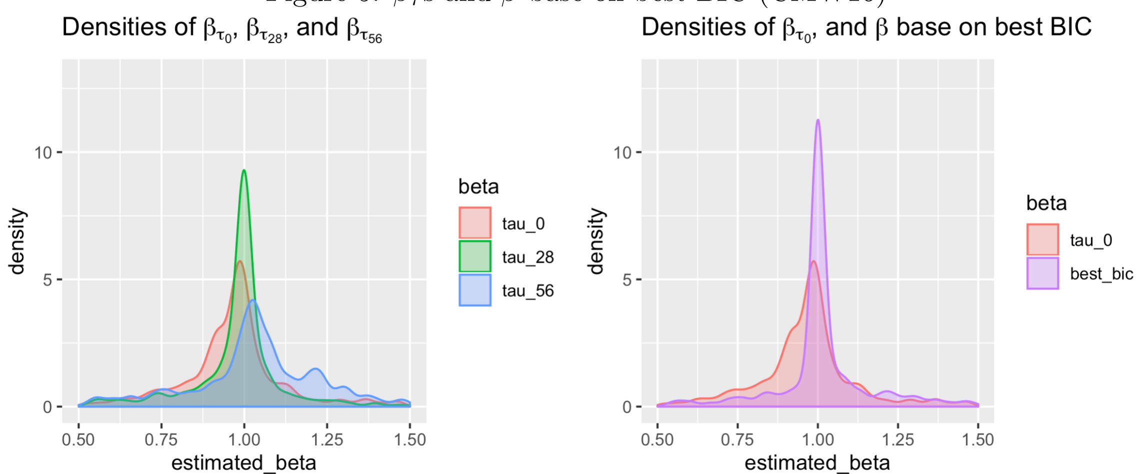
Table 6: Best Model based AIC, BIC, and RSS for Individual Subject (CMW16)

Criterion	**Observations**	τ_0	τ_{28}	τ_{56}	τ_{84}
best AIC	1061	150 14.14%	105 9.90%	439 41.38%	367 34.59%
	452	58 12.83%	47 10.40%	204 45.13%	143 31.64%
	230	30 13.04%	25 10.87%	124 53.91%	51 22.17%
best BIC	1061	139 13.10%	91 8.58%	417 39.30%	414 39.02%
	452	54 11.95%	43 9.51%	197 43.58%	158 34.96%
	230	29 12.61%	24 10.43%	121 52.61%	56 24.35%
best RSS	1061	218 20.55%	180 16.97%	588 55.42%	75 7.07%
	452	85 18.81%	67 14.82%	260 57.52%	40 8.85%
	230	43 18.70%	38 16.52%	143 62.17%	6 2.61%

[4] Echenique et al. (2019) observed 235 strictly QHD rational time preferences among 1061 subjects from the CMW16 when considering τ_{t_0}.

The result of the comparison between the simulations and the experimental data indicates some of the model selection criteria (AIC, BIC, and RSS) are sensitive in terms of detecting the true location of the ***Present Threshold*** (τ). Therefore, the AIC, BIC, and RSS are also calculated for NLS models at the individual subject level. The number and the percentage of the best-performed model based on three different criteria by the given subsets are presented in the Table 6. The result suggests that τ_{56} has the most robust performance, especially when we imposing stronger assumptions on the samples. Overall, the secondary biased (τ_{28} or τ_{56}) models (either present biased or future biased) are the majority of the best-performed models among all subsets, which is different from the traditional assumption of τ_0.

Figure 5: β_τs and β base on best BIC (CMW16)



The best estimation of β_τ can be decomposed based on the best location of the perspective of the present according to the revealed choice of time preference at the subject level. Figure 5(left) depicts the distribution of estimated beta at the different underlying assumptions of the perspective present. The distribution is skewed to the right as the assumptions of the perspective present is pushed to the future. The further we assume the perspective present is, the more likely we consider the subjects as future biased. Then we merged all 1061 estimated β_τ (13.67% of the primary present behaved subject, 46.59% of secondary present behaved subjects, and 38.74% time consistent subjects) based on the BIC (Figure 5(right)). The over distribution is shifted to the right for the best BIC β_τ compare the conventional setup β_{τ_0} with one-sided Mann–Whitney U test's p-value of 0.0001 (alternative less than).

5.3 Robustness Check with the CARA Utility Specification

There are two major concerns of the estimation strategy for the CTB design along with the non-linear least square approach. First of all, the NLS could create the bias for parameter estimations due to not adjusting for corner solutions. We already addressed this concern with the simulations. We find that the tendency of change for both δ and β remains stable with or without the censoring. Another concern is that the current result can be driven by the CRRA utility functional form. Therefore, we specify the structural model with the constant absolute risk aversion (CARA) utility along with the additive discounting function in (11):

$$U_0(x_{t_i}, x_{t_i+k}) = \delta^t(-exp(-\alpha(x_{t_i} + \omega))) + \beta_\tau^{\mathbb{1}_{\{t_i \le \tau < t_i+k\}}} \delta^{t_i+k}(-exp(-\alpha(x_{t_i+k} + \omega))). \tag{11}$$

With the CARA specification, the solution of the optimization takes (12):

$$x_{t_i}^* = \begin{cases} [\frac{k\ln(\delta)}{-\alpha} + \frac{\ln(1+r)}{-\alpha} + m]\frac{1}{1+(1+r)}, & \text{if } \tau < t_i \text{ or } t_i + k \le \tau \\ [\frac{\ln(\beta_\tau)}{-\alpha} + \frac{k\ln(\delta)}{-\alpha} + \frac{\ln(1+r)}{-\alpha} + m]\frac{1}{1+(1+r)}, & \text{if } t_i \le \tau < t_i + k \end{cases} \tag{12}$$

As suggested by Andreoni and Sprenger (2012a), the parameter can be recovered by a two-step tow-limit Tobit model. However, the Tobit model's specification cannot be estimated due to the multicollinearity problem when τ_{56} is assumed with the CMW16 design. Then, we employed the NLS approach again to estimate the CARA specification. We find the robust result of the increasing trend of β and decreasing trend of δ estimations as we push the position of τ from τ_0 to τ_{56} given the CARA utility. The $\beta_{\tau_{56}}$ is also the most desired model among all others for the BIC criterion for all three subsets of CMW16. Our results hold across the different functional forms of the utility (Table 7).

Table 7: CARA Estimations of All Three Subsets (CMW16)

CARA Models (Obs)		τ_0	τ_{28}	τ_{56}	τ_{84}
	$\hat{\beta}$	0.9496	1.0000	1.1340	
	$\hat{\delta}$	0.9978	0.9975	0.9960	0.9975
	$\hat{\alpha}$	0.0012	0.0012	0.0011	0.0012
1061	**BIC**	165459.9	165517.2	165365.5	165507.8
	$\hat{\beta}$	0.9596	0.9999	1.1078	
	$\hat{\delta}$	0.9976	0.9973	0.9961	0.9973
	$\hat{\alpha}$	0.0007	0.0008	0.0007	0.0008
452	**BIC**	71305.98	71342.05	71245.46	71333.45
	$\hat{\beta}$	0.9507	1.0022	1.1280	
	$\hat{\delta}$	0.9973	0.9969	0.9956	0.9969
	$\hat{\alpha}$	0.0006	0.0006	0.0005	0.0006
230	**BIC**	35754.3	35808.37	35669.66	35800.51

5.4 The Average of the Perceptional Present

To find an revealing description of the central tendency, we calculate the average of the perceptional present ($\bar{T}_{\tilde{p}}$) among all of the subjects. (13).

$$\bar{T}_{\tilde{p}} = \sum_{i=1}^{n-1} W_{\tau_{t_i}} (\frac{t_i + t_{i+1}}{2}) \tag{13}$$

where $W_{\tau_{t_i}}$ is a weight function that proportion of the dynamic inconsistency subject whose perception of the present is likely to be located at τ_{t_i}. We consider the middle point of the interval $[t_i, t_{i+1})$ as the optimal location of τ_{t_i} for simplicity. Notice that the **Observational Dynamic Consistent** subject has no observed $\bar{T}_{\tilde{p}}$.

We find that $\bar{T}_{\tilde{p}}(CMW16) = 47.59$ days and $\bar{T}_{\tilde{p}}(AP12) = 65.44$ days, respectively. The results suggest that the observed kinky point of chosen consumption in these lab experiments happened far away from the conventional assumption that the time preference discounted inconsistently immediately after the nearest to now award. Our empirical method shows that it is more precise to discover how far could the perception of the present lasts rather than to question how soon does it end (Balakrishnan, Haushofer, and Jakiela 2017).

5.5 Dynamic Inconsistent in Perceptional Time Scale

Ignorance of the perceptional sense of the present would lead to a misleading conclusion of the directions of the time inconsistency when we evaluate the data in depth. We present the final breakdown of different time preference behaviors base on the BIC criteria in Table 8. For the CMW16 experiment, despite the small proportion of dynamic consistency behavior, the percentage of future bias (both primary and secondary) adds up to 32.80%, which is higher the 28.18% of the present bias (both primary and secondary). We conclude the future bias time preference is dominating the population in the CMW16, which is different from the reported present bias behavior in their study (Carvalho, Meier, and Wang 2016). Similar, the proportion of the observed perceptional present biased behavior in AS12 is 52.57%, which is higher than the 36.05% of future bias behavior. When time inconsistency is measure in a procreational scale, we also reached a different conclusion from the original discovered slightly future biased behavior in Andreoni and Sprenger (2012a). When time is considered on a perceptional scale and explored in more details, our results are not consistent with the discovery of the previous studies.

Table 8: the Breakdown of Different Time Preference Behaviors (CMW16, AP12)

Research	**CMW16**		**AS12**	
BIC	**count**	**percentage**	**count**	**percentage**
Dynamic Consistency	414	39.02%	11	11.34%
Primary Present Bias	91	8.58%	3	3.09%
Primary Future Bias	48	4.52%	7	7.22%
Secondary Present Bias	208	19.60%	48	49.48%
Secondary Future Bias	300	28.28%	28	28.87%
Total	1061	100%	97	100%

6 Conclusion and Discussion

We developed an extension of the QHD model and added the concept of perceptional time scale to the Qausi-Hyperbolic Discounting function. By adjusting for the perceptional of present feature in time preference, we developed the QHD into the General Qausi-Hyperbolic Discounting model. With the application of our GQHD model, the results point to two conclusions. First of all, the perception of the present is a critical component of measuring inconsistency in time preference. Disregarding the importance of the perceptional dimension of time would lead to a misleading conclusion of the time inconsistency. The secondary future bias behavior could be misleadingly treated as the primary present bias behavior. The secondary future bias behavior could be misleadingly treated as the primary present bias behavior. Simultaneously, the secondary present bias behavior could be misleadingly treated as the primary future bias behavior. Simply assuming time preference discounted inconsistency only after the immediate payoff without any hypothetical test, we either fail to observe the inconsistency or fail to distinguish the present bias from the future bias. The misconception of the direction of the bias could sufficiently undermine the conclusions and challenge the validity of the policy application in time preference studies. If a policy is intended to address the primary present bias issue; however, it is applied to the secondary future bias population; this policy may further intensify future bias behavior

Secondly, with the introduction of a flexible parameter β_τ in the empirical work, we were able to detect the length of the perception of the present at both aggregate and individual levels. The results indicate the perceptional present reaches far beyond immediate soon (t_0). Constraining the β parameter at its initial place limits the ability of the empirical model to fit the data—additionally, it constrains the QHD model to give a more precise explanation of the time preference behavior. The GQHD model provides a more insightful interpretation of the time inconsistency along with additional information about when it happens.

By adding a perception dimension of time in the Quasi-Hyperbolic Discounting, we provide a more general method to differ the observed the primary present bias from its quasi-twin secondary future bias in time preference. We answered the research question of "what is the present," then discussed how the observed bias of the time preference changed toward this discovered perceptional present. Our findings advocates a necessity of consideration of the perceptional present for future dynamic inconsistency in time preference studies. However, our conclusion has its constraint to a certain extent.

First, the possible locations of the structural model in our identifications heavily rely on the design of CMW16 and AP12. Both of these studies focus on time inconsistency for the monetary incentive. To have a more versatile measurement of how late the present could last and even further generalize our discussion beyond monetary incentive, we need an adaptive budget set design to resolve the problems of preventing corner solutions, then detecting the present beyond money(Imai and Camerer 2018). More importantly, the current literature of time preference in economics is present-biased-focused. The research which finds the evidence of the existence of future bias behavior either challenges the design fails to prevail the myopic behavior (Shu 2008) or provides little explanation of the abnormal finding (Aycinena, Blazsek, Rentschler, and Sandoval 2015; Corbett 2016; Aycinena, and Rentschler 2018). On the other hand, some studies in both economics and phonology emphasize the importance of future bias (Dougherty, 2015; Greene and Sullivan 2015; Dorsey 2017). Suppose a new design can successfully control the uncertainty of future income and the higher interest rates are positively correlated with the probability of triggering perceptional future bias, we can link our GQHD model to early studies of over-saving and retirement research to find some additional practical applications (Diamond and Köszegi 2003; Salanié and Treich 2006; Zhang 2013).

Second, we find the optimal duration for the present in the experiment. Still, our finding was not intended to explain if the observed future bias is caused by the risk-averse driven behavior concerning the uncertainty of getting the reward after the experiment (Andreoni and Sprenger 2012b). Additionally, this optimal duration we find is based on our modified version of QHD specification. We chose the model mainly to keep the beta parameter in a discrete setting, then compare our results with the findings in related literature. Other models can also identify the turning point in the discounting function (for instance, Loewenstein and Prelec 1992). It is also compelling to see the relationship between the perceptional present and the reversal point of the discounting factor in other models.

Third, the criteria and visualization methods we used successfully identify the optimal location of the preset threshold and the concentration of different behaviors in the population. However, we are uncertain how accurate and sensitive are these methods, especially for mild inconsistency in time preference.

Lastly, This study is a complement of ”how soon is now” and ”now or as soon as possible” (Glimcher, Kable, and Louie 2007; Balakrishnan, Haushofer, and Jakiela 2017). It’s necessary to conduct a conurbation of our studies to comprehend if the unobserved present bias is caused by present is too soon or we fail to consider present could last longer in perceptional scale. We will leave these concerns to the future work.

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Appendix A Misspecification with the Secondary Models

We also provide some institutions of the consequence of misspecified secondary present models for the primary present behaviors. Consider a subject's three-period discounting function exhibit the following secondarily bias pattern $D_\tau(t_0, t_1, t_2) = (\delta^{t_0}, \beta\delta^{t_1}, \beta\delta^{t_2})$. Nevertheless, we specified a secondary present model with a sequence of time-dependent discount factors $\hat{D}_{\tau'}(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$.

With a similar approach in (9) and (8), we can derive $\hat{\delta}$ in (14), and $\hat{\beta}$ in (15). For simplicity, we consider $\delta^{t_0} = 1$.

$$\frac{\hat{\delta}^{t_1}}{\hat{\delta}^{t_0}} = \frac{\beta\delta^{t_1}}{\delta^{t_0}} \implies \hat{\delta} = \beta^{\frac{1}{t_1 - t_0}}\delta \implies \begin{cases} \hat{\delta} > \delta, & \text{if} \quad \beta > 1 \\ \hat{\delta} < \delta, & \text{if} \quad \beta < 1 \end{cases} \tag{14}$$

$$\frac{\hat{\beta}\hat{\delta}^{t_2}}{\hat{\delta}^{t_1}} = \frac{\beta\delta^{t_2}}{\beta\delta^{t_1}} \implies \hat{\beta} = \frac{\delta^{t_2 - t_1}}{\hat{\delta}^{t_2 - t_1}} \implies \begin{cases} \hat{\beta} < 1 < \beta, & \text{if} \quad \beta > 1 \\ \hat{\beta} > 1 > \beta, & \text{if} \quad \beta < 1 \end{cases} \tag{15}$$

The equations in (8), (9), (14) and (15) shows that the ***Primary Present Bias*** and the ***Secondary Future Bias*** create very similar trends (sign) when pushing τ to later positions in the empirical models. The ***Primary Future Bias*** and the ***Secondary Present Bias*** tends to have the same character.

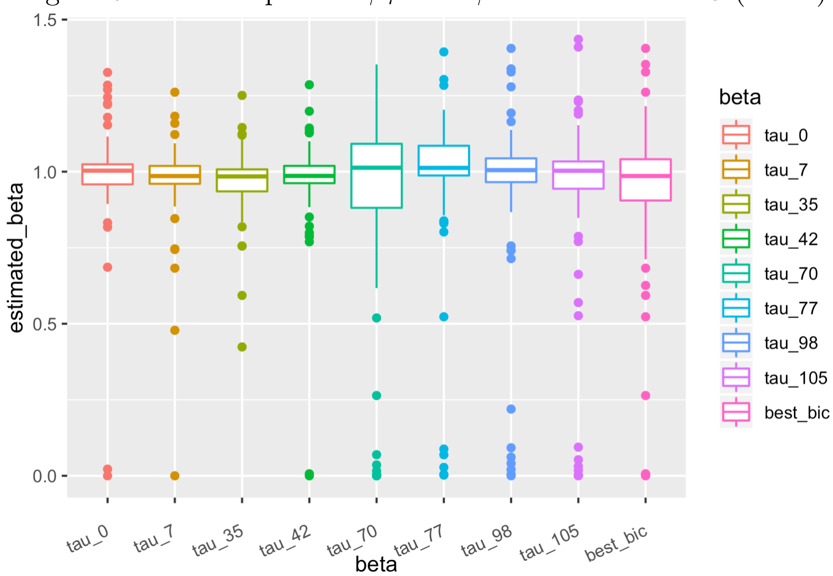
Appendix B AP12 Results

Table 9: Aggregate Level Estimation for 97 Subjects (AP12)

Model	τ_0	τ_7	τ_{35}	τ_{42}	τ_{70}	τ_{77}	τ_{98}	τ_{105}	τ_{133}
$\hat{\beta}$	1.0090 (0.0084)	0.9965 (0.0074)	0.9864 (0.0115)	0.9801 (0.0124)	0.9864 (0.0118)	1.0220 (0.0104)	1.0200 (0.0098)	0.9991 (0.0143)	
$\hat{\delta}$	0.9990 (0.0001)	0.9990 (0.0001)	0.9992 (0.0002)	0.9992 (0.0002)	0.9991 (0.0001)	0.9988 (0.0001)	0.9989 (0.0001)	0.9990 (0.0001)	0.9990 (0.0001)
$1-\hat{\eta}$	0.8737 (0.0101)	0.8750 (0.0098)	0.8760 (0.0097)	0.8743 (0.0099)	0.8725 (0.0105)	0.8762 (0.0096)	0.8754 (0.0097)	0.8746 (0.0099)	0.8746 (0.0099)
AIC	43351.08	43352.02	43350.86	43349.66	43350.88	43347.48	43348.17	43352.23	43350.24
BIC	43382.98	43383.93	43382.76	43381.57	43382.79	43379.38	43380.08	43384.14	43375.76
RSS	5244675	5245806	5244408	5242974	5244441	5240351	5241182	5246063	5246068
Observations	97	97	97	97	97	97	97	97	97

The change in point estimations of β in AP12 is mild compare to CMW 16 (between 0.9801 and 1.0220 in Table 9). This finding consistent with Echenique et al. (2019), which finds around 51% (49 out of 97) of that subjects passes the Strong Axiom of Revealed Exponentially Discounted Utility (SAR-EDU) test. Their results indicate the overall revealed preference either likely to exhibit time consistency behavior or mild time inconsistency in the imperial estimations.

Figure 6: The Box-plots of β_τs and β base on best BIC (AP12)



The over distribution is shifted to the left for the best BIC βs compare the conventional setup β_{τ_0} with on-sided Mann–Whitney U test's p-value of 0.0181 (alternative greater than).

Appendix C The Hyperbolic Discounting Model

Consider a Hyperbolic Discounting function $D(t_i) = \frac{1}{1+(\kappa t_i)}$, then the CRRA utility of the CTB takes the functional from in (16).

$$U_0(x_{t_i}, x_{t_i+k}) = \frac{1}{1+\kappa(t_i)}\frac{1}{1-\eta}(x_{t_i}+\omega)^{1-\eta} + \frac{1}{1+\kappa(t_i+k)}\frac{1}{1-\eta}(x_{t_i+k}+\omega)^{1-\eta} \tag{16}$$

Then the optimal demand function of sooner consumption would be (17) accordingly.

$$x^*_{t_i} = \frac{(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}} - \frac{1}{1+\kappa(t_i)}}{\frac{1}{1+\kappa(t_i)} + (1+r)(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}(\omega) + \frac{(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}{\frac{1}{1+\kappa(t_i)} + (1+r)(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}(m) \tag{17}$$

Table 10: Aggregate Level Estimations for Hyperbolic Discounting Model (CMW16)

$\hat{\boldsymbol{\kappa}}$	0.0028	0.0030	0.0035
	(0.0002)	(0.0002)	(0.0002)
$\mathbf{1-\hat{\boldsymbol{\eta}}}$	0.7274	0.8416	0.8796
	(0.2317)	(0.0872)	(0.0511)
AIC	165440.7	71272.12	35729.4
BIC	165470.5	71298.52	35753.09
RSS	327649101	161396915	67500389
Observations	1061	452	230

As a result, the performance (measured with AIC, BIC, and RSS) of the HD model is better than the ED model for all three data sets, which suggests the overall behavior is in favor of the inconsistent dynamic models. The departure between the HD model and the QHD model provides some evidence against the immediate present bias behavior.