

# The Perception of Present: A Study of Inconsistency in Discount Rate with the General Quasi-Hyperbolic Discounting Model

Kevin Zou

## Abstract

In this paper, we examine heterogeneity in individual perceptions of the present in an attempt to understand better the dynamic inconsistency (the non-exponentially discounted utility) in time preferences. Quasi-Hyperbolic Discounting models are often used to study dynamic inconsistency. Recent studies have found that the time horizon of the defined present (Is the present an hour, a day, or longer?) impacts the observed magnitude of dynamic inconsistency. We propose a more general framework of the conventional Quasi-Hyperbolic Discounting model, which helps to identify the prospective present and its corresponding inconsistency. We applied our General Quasi-Hyperbolic Discounting (GQHD) model to two existing Convex Time Budget, a commonly employed experimental design to study time preference, experimental data sets. The results suggest empirical evidence supporting the conjecture that the perceptual present, which we consider the sense of the present from a subjective point of view, lasts longer than the immediate present (hours or days). The GQHD estimates of present bias parameters varies substantially across individuals, yielding different specifications of the “present” in our structural model. We find that experimentally observed choices are more likely to exhibit secondary present (when we consider the “present” can be longer than the immediate present) inconsistency than the primary present (when we only consider the immediate present as the “present”) inconsistency.

# 1 Introduction

Time is a fundamental concept in decision-making and understanding the trade-off between present and future consumption is critical to studying economic behavior. To understand how we make decisions on a time horizon, we study the time preference and apply the discoveries to many policies (health, saving, and retirement decisions, etc. ; Uzawa 1969; Fuchs, 1980; Finke and Huston 2013). It is crucial for the research in economics to understand and model time preference accurately. We often find present bias behavior (a tendency that people give stronger weight to the payoffs that are closer to the present than the future) in time preference among individuals. To model the inconsistency behaviors (O'Donoghue and Rabin 1999). The Quasi-Hyperbolic model became one of the most commonly adopted approaches in modeling present bias behaviors (Thaler 1981; Frederick, Loewenstein, and O'donoghue 2002).

“How soon is now?” is one of the most actively controversial topics in the Quasi-Hyperbolic Discounting (QHD) model-based time preference literature (Glimcher, Kable, Louie 2007; Balakrishnan, Haushofer, and Jakiela 2017; DellaVigna 2018; Ericson, and Laibson 2019). To explore the question “nowness,” we linked time inconsistency with the perceptual sense of time, then found the threshold that distinguishes the present and the future in the QHD framework. The threshold we find challenges the conventional belief that present bias driven time inconsistency only applies to a relatively short period of time. Our results show that disregarding the importance of time perception in a revealed preference study leads to misleading conclusions about time inconsistency. With our newly proposed General Quasi-Hyperbolic Discounting (GQHD) approach and its empirical application, we find a link between time perception and dynamic inconsistency, which helps us to give a more precise answer about when time inconsistencies occur.

Conventionally, empirical models set a boundary that distinguishes present and future, an approach which limits our ability to observe present bias to immediate consumption choices. In other words, the subjects are being observationally present biased if and only if the present bias happened precisely according to the constraint artificial boundary that specifies their present and future. In lab experiments, the design of immediate pay-off varies for different studies from hours, days, weeks, to years. These studies find disagreement about the existence of the present bias to the dynamic inconsistency related research questions (Angeletos, Laibson, Repetto, Tobacman,

Weinberg 2001; McClure, Ericson, Laibson, Loewenstein, and Cohen 2007; Augenblick, Niederle, and Sprenger 2015; Augenblick 2017). Other research finds that the time preferences also depend on the subjective sense of time (Angeletos, Laibson, Repetto, Tobacman, and Weinberg 2001; dos Santos and Martinez 2018). Balakrishnan et al. (2017) explore the timing of lab payments and find that present bias only exists when participants are paid at the immediate conclusion of the experimental session. However, a structural model that restricts the notion of the present to the immediate now may over restrict the notion and lead to errors, such as unobserved dynamic inconsistency in time preference studies (Imai, Rutter, and Camerer 2019).

To find a more general definition of time horizon that better explains the data, we divide our study into the following sections. First, we develop a new framework that generalizes the QHD model with a more flexible definition of the present sense along with its well-defined characterizations (secondary present bias and secondary future bias behavior in time preference). This general format comes with a modified  $\beta_\tau$  parameter of the QHD that captures the perceptual present alone with its inconsistency. This theoretical work closely follows the Convex Time Budget (CTB) design to find empirical evidence of perceptual dynamic inconsistency from existing experimental data. We also demonstrate that potential ignorance of the perception present in time preference research leads to imprecise judgment of the types of inconsistency with both derived intuition and simulated data. Next, we applied the GQHD method with a nonlinear least square estimation approach to lab experiment data from Carvalho, Meier, and Wang 2016 (CMW16), and Andreoni, and Sprenger 2012a (AS12), with different strengths of restrictions of the utility assumptions along with robustness check of various functional forms. We find strong evidence to support the existence of a more diversified perspective of present in the observational data set than the previously assumption that the present means “now”. In fact, the average perceptual present lasts for days or even months. Most importantly, when the present is correctly specified in the model, conclusions about the direction of time inconsistency may differ from previous studies.

The main contribution of our research is to establish the importance of considering the perceptual sense of the present. Failing to do so leads to incorrect analysis of present and future bias behaviors which undermines policy efforts to improve behaviors related to time discounting. We propose a General Quasi-Hyperbolic Discounting method that addresses both perception of the present and its corresponding dynamic consistency. This method allows us to assess time in-

consistency more precisely at both aggregated level and individual level in the perceptual time scale.

## 2 Background

Time preference measures how individuals evaluate receiving a good at an earlier time compared to receiving it at a later time. It is essential for economists to understand it, then to explain how individuals, even the whole population smoothing out the consumption. Historically, we tried to model time preference with many different models. Samuelson started this path with his proposed Exponential Discounting (ED) utility model (Samuelson 1937; Koopmans 1960). The ED model assumes that individuals' valuation of consumption depreciates at a constant rate  $\delta$  over time (3). Researchers have shown that behavior is often inconsistent with the ED model (Ainslie 1975; Thaler 1981; Frederick, Loewenstein, and O'donoghue 2002). Ainslie (1975) first showed the inconsistency in time preferences, and illustrated greater discounting for short-run consumption than long-run consumption, a phenomenon called present bias. As an example, suppose we have the following two choices:

### Example.

A. \$ 10 today vs B. \$ 11 in a week, and A'. \$ 10 in 50 weeks vs B'. \$ 11 in 51 weeks

There are many decision makers choose A over B, then choose B' over A'. There are many well-known theoretical models in both economics and psychology studies that address this documented observational reversal behavior in time preference (Thaler 1981; Frederick, Loewenstein, and O'donoghue 2002). For instance the proportional discounting model (Ainslie, and Herrnstein 1981), the power discounting model (Harvey 1986), the hyperbolic discounting ( $\frac{1}{1+(\kappa t)}$ ) model (Mazur 1987; Loewenstein, and Prelec 1992), and the constant sensitivity model (Ebert and Prelec 2007). Among the discounting models, Hyperbolic Discounting becomes the most commonly chosen one to model this time-inconsistency due to the accuracy of explaining this reversal behavior.

The Laibson (1997) successfully approximated the hyperbolic discounting model with the Quasi-Hyperbolic Discounting function ((1) ;Phelps and Pollak 1968) in a discrete time preference study.

The important advantage of this approximation is that the quasi-hyperbolic model (1) has a  $\beta$  parameter that represents a departure from Samuelson’s exponential discounting model (3) and separates time discounting into distinct processes for present and future rewards. Conventionally, we are constraining the  $\beta$  parameter to be strictly less than one to indicate present bias in behavior. However, some recent research has found evidence that time preferences exhibits future bias characteristics (Aycinena, Blazsek, Rentschler, and Sandoval 2015; Corbett 2016; Aycinena, and Rentschler 2018). Additionally, a meta-analysis among these Quasi-Hyperbolic Discounting (QHD) models with the Convex Time Budget (CTB) design found evidence of over-reporting present bias in existing studies (Imai, Rutter, and Camerer 2019). Future biased time preference behaviors are not only found in CTB studies are also discovered in other non-CTB experimental designs (Takeuchi 2011; Jackson, and Yariv 2014). Thus, previous results suggest that the true direction of dynamic inconstancy remains to be conclusively established.

In addition to research on time inconsistency parameters, there is another stream of research which focuses on establishing how one’s perspective (subjective) time scale affects the measurement of dynamic inconsistency. The subjective time can be measured by the ability to distinguish the proportional difference between three months, one year, and three years on a physical object or measured by the ability to accurately tell how long a minute is (Zauberman, Kim, and Malkoc 2009; Bradford, Dolan, Galizzi 2013; Brocas, Carrillo, and Tarrasó 2018) Research shows that the degree of hyperbolic discounting (present bias) is reduced significantly when the dynamic inconsistency is measured by subjective time perception. In other words, people are less inconsistent when we measure time in their perspective time scale (Zauberman, Kim, and Malkoc 2009). Additional to the inconsistency of time preference, the results can also be extended to the estimation of the discounting parameter  $\delta$ . The mental representation of the delays is correlated to our ability to delay consumption to the future. If the delay in time is exaggerated in one’s mind (for example, if the mental presentation of one day is longer than the physics definition of one day), then we will observe more depreciation of the delayed award in value. The more we shrink the time in our perspective sense, the more we will discount the valuation of future consumption (Brocas, Carrillo, and Tarrasó 2018).

Given the fact that time is a conceptually subjective matter, the notion of present bias should also be attached to such conjecture. The present (future) biased behavior could also occur on a

perceptual time scale. Most of the time, the definition of the present is predetermined by the experimental design, rather than discovered by the empirical work. The definition of the present can vary in length from different studies (Angeletos, Laibson, Repetto, Tobacman, Weinberg 2001; McClure, Ericson, Laibson, Loewenstein, and Cohen 2007; Augenblick, Niederle, and Sprenger 2015; Augenblick 2017). With the different experimental design of “immediate present” and future, studies find the inconstancy decay of discounting can be diverse base on the particular setting. These discussions lead to another open-ended research question, which is when and where we can observe present bias on the time horizon (DellaVigna 2018; Ericson, and Laibson 2019). It is debatable about when the present is. Augenblick et al. (2015) found present bias behavior when considering an interval between a few minutes and weeks as the present for exerting effort experiment. On the other hand, another research observed the present bias behavior if considering a few hours as the present for a monetary award in the experiment. (Augenblick 2017; Balakrishnan, Haushofer, and Jakiela 2017).

The CTB experiment is commonly employed design of dynamic inconsistency time preference in both laboratory and field economic studies (Andreoni, and Sprenger 2012a; Gine, Goldberg, Silverman and Yang 2012; Carvalho, Meier, and Wang 2016). The CTB experiments ask the subjects to make decisions about dividing their award into two accounts, the sooner and the delayed account. If a certain amount of award is located in the delayed account, the subjects will be paid with that certain amount plus interest. However, there will be no interest for the amount they put in the sooner account when they are getting paid. Two separate awards will be issued to the subjects according to the definition of the “sooner” and the “delay” in the experiment. One of the essential advantages of the CTB design is the subject can split the reward into two accounts on a continuous scale. Therefore it allows the structural model to estimate the curvature parameter (elasticity of substitution) of the utility more precisely. Thereby we can estimate the discounting parameter more accurately (4). Given the flexibility of the CTB design, a considerable amount of economics (Atalay, Bakhtiar, Cheung, Slonim 2014; Liu, Meng, Wang 2014; Cheung 2015; Alan, Ertac 2015; Carvalho, Meier, Wang 2016; Balakrishnan, Haushofer, Jakiela 2017; Kuhn, Kuhn, Villeval 2017; Luhrmann, Serra-Garcia, Winter 2018) and psychology (Yang and Carlsson 2016; Hoel, Schwab, Hoddinott 2016; Lindner and Rose 2017) studies combine the Convex Time Budget and Quasi-Hyperbolic Discounting to study revealed time preference. These studies come with

different treatment dimensions in the experiments to either suggest improvement for the design or provide policy applications. If the QHD model does not precisely characterize the time preference, the validities of these policies would be undermined. We will follow the CTB design closely to further develop the QHD model with the perception of present extension in the **The Theoretical Framework** section to address our concern of the existing time preference studies.

### 3 The Theoretical Framework

#### 3.1 the General Quasi-Hyperbolic Discounting Model

To study dynamic inconsistency in the revealed time preference, we apply the Quasi-Hyperbolic Discounting (QHD) function to an additive time separable utility function. The QHD model has a  $\beta$  parameter after the first consumption period (1). The  $\beta$  allows the Exponential Discounting (ED) model (3) to mimic the feature of Hyperbolic Discounting. It captures the departure of the inconsistent discounting from the ED function (Strotz 1956; Phelps and Pollak 1968; Laibson 1997; and O'Donoghue and Rabin 1999a). The interpretation of this departure is limited by where we are placing the  $\beta$  parameter. In empirical and experimental studies, we interpret the  $\beta$  as the inconsistency of discounting rate of all future consumption ( $x_t, \text{where } t \in \{t_1, t_2 \dots t_n\}$ ) compared to the consumption ( $x_{t_0}$ ) of the soonest observational period (1).

$$u(x_{t_0}) + \beta \sum_{t=t_1}^{t_n} \delta^t u(x_t) \quad (1)$$

where the time  $t \in \{t_1, t_2 \dots t_n\}$  depends on when we observe the consumption, and  $x_{t_0}$  is the initial consumption level at  $t_0$ , and  $x_t$ 's is all future consumption. We call the  $\beta$  a dynamic inconsistency parameter, which identifies the deviation from the exponential discounting parameter  $\delta$  (1).

To characterize dynamic inconsistency as a perceptual notion, we modified QHD into a more general functional form ((2); Echenique, Imai, and Saito 2019), and introduce a new parameter  $\beta_{\tau_{t_i}}$  ( $\beta_\tau$  for simplicity) where  $t_i$  can take different values in  $\{t_0, t_1 \dots t_n\}$  to adjust for an individual's

specific perception of the present. We define  $\tau_{t_i}$  ( $\tau$  for simplicity) as the **Preset Threshold** that distinguishes a subject's perception of present and future. We name  $\beta_\tau$  as the **Perceptual Dynamic Inconsistency Parameter**. The  $\beta_\tau$  not only characterizes the direction of bias (either present bias ( $\beta_\tau < 1$ ) or future bias ( $\beta_\tau > 1$ )) with its magnitude, but also captures the individual's perceptual distinction of the present and the future with the  $\tau$ . Specifically, the dynamic inconsistency parameter  $\beta_\tau$  starts to appear in the formula (2) if and only if the assigned time of sooner consumption and later consumption crosses the individual perceptual present threshold  $\tau$ . The functional form in (2) is more general on choosing the value of  $t_i$  compare to the QHD model (1), and we call it the General Quasi-Hyperbolic Discounting (GQHD).

$$\sum_{t=t_0}^{t_i} \delta^t u(x_t) + \beta_{\tau_{t_i}} \sum_{t=t_i+1}^{t_n} \delta^t u(x_t) \quad (2)$$

where  $t \in \{t_0, t_1, \dots, t_n\}$ , which depends on the experimental design, and  $\tau_{t_i} \in \{\tau_{t_0}, \tau_{t_1}, \dots, \tau_{t_n}\}$ ,  $t_0 \leq \tau_{t_0} < t_1$ ,  $t_1 \leq \tau_{t_1} < t_2$ , ...  $t_n \leq \tau_{t_n}$ . Each  $\tau_{t_i}$  ( $\tau$  for simplicity) can take any arbitrary value within the given interval, and it would not affect the specification of the structural model for the empirical work. If  $\tau < t_0$  or  $\tau \geq t_n$ , then the QHD utility reduces to the exponential discounting utility (3). Therefore, the estimation of  $\beta$  equals 1 has two possible interpretations. Either the subject has a dynamic consistency utility, or the design failed to trigger the subject's sense that distinguishes his or her perception of present and future (see **Property 1**).

$$\sum_{t=t_0}^{t_n} \delta^t u(x_t) \quad (3)$$

We present some unique definitions of the GQHD model that corresponds to the Convex Time Budget (CTB) experiment.

**Definition 1: Time**  $t \in [0, t_n] = T$  is a continuous variable that describes the progress domain of a CTB experiment from the very moment when we observe subject's time preference ( $t = 0$ ) until the moment when the very last award is scheduled to be delivered ( $t = t_n$ ) to the subjects.

**Definition 2: Now**  $t = 0$  is a discrete definition that defines the very moment when we observe



the time preference behavior<sup>1</sup>. It doesn't take a long time (minutes) to collect the choices from a CTB experiment, and given a long enough time horizon (months) for the last award to be delivered (Andreoni and Sprenger 2012a); so we define *Now* as discrete arbitrarily.

Considering the revealed time preference data is collect at ***Now***, we will label the observed utility function given the data set as  $U_0(\cdot_t)$  to keep the consistency of the notation in the theoretical framework. Notice that we do not have to assume the utility is time-invariant ( $U_t(\cdot_t) \equiv U(\cdot_t), \forall t \in T$ ) to estimate the parameters for the empirical work. However, to further extend the GQHD model to O'Donoghue and Rabin's procrastination studies (1999a; 1999b; 2001) or to develop any policy implication of the empirical results, imposing the time-invariant assumption is necessary.

**Definition 3: *Delivery Date***  $t = t_i \in \{t_0, t_1, \dots, t_n\}$ , (where  $i = 0, 1, 2, \dots, n$ ) is a discrete variable that identifies the time when the reward is scheduled to be delivered to the subjects.

One important note we need to emphasize is that there exists a small time gap ( $\epsilon$ ) between the preference is revealed, and the first reward is delivered ( $0 + \epsilon = t_0, \epsilon > 0$ ). For example, the subjects may need to fill out a questionnaire after the CTB experiment, or it takes some time for the electronic transaction to process. How soon is now type of research is addressing the question about how small  $\epsilon$  should be, then  $U_0(\cdot_t) \equiv U_{0+\epsilon}(\cdot_t)$  (Balakrishnan, Haushofer, and Jakiela 2017). Kirby (1997) concerns not to distinguish the difference between the instant award and the nearly instant award. In our framework, we also assume  $0 < t_0$ .

**Definition 4: *Present Threshold***  $\tau \in \{\tau_{t_0}, \tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}\}$  ( $t_i < \tau_{t_i} \leq t_{i+1}, i \in \{0, 1, 2, \dots, n-1\}$  or  $t_i < \tau_{t_i}, i = n$ ) is a discrete variable that captures a value of time, which distinguishes the ***Present*** and the ***Future***.

The ***Present Threshold*** ( $\tau_{t_i}$ ) can take any value between the interval  $(t_i, t_{i+1}]$  because consumption level is not observed for any available ***Delivery Date*** in  $T$ . To take any value within the interval will not change the the estimation strategy of any structural model. Additionally, the ***Present Threshold*** can be further decomposed into three subcategories the ***Primary Present Threshold*** ( $\tau_{t_0}$  for QHD model), the ***Secondary Present Threshold*** ( $\tau_{t_i} i \in \{1, 2, 3, \dots, n-1\}$  for GQHD model), and the ***Observational Dynamic Consistency Threshold***. ( $\tau_{t_n}$  for ED model). The  $\tau_{t_1}$  is equivalent to the conventional Quasi-Hyperbolic Discounting model (Phelps and Pollak 1968);  $\tau_{t_n}$  is equivalent to the exponential discounting model (Koopmans 1960); the

---

<sup>1</sup>If we extend time  $t \in \mathbb{R}$ , then the ***Past*** can be defined as  $t \in (-\infty, 0) = T_h$  ( $h$  for history).

*Secondary Present Threshold* is unique to our General Quasi-Hyperbolic Discounting model. A subject's time preference is GQHD rationalizable if he or she has a unique and stable  $\tau$  corresponding to revealing decisions at *Now*. The subscript  $\tau$  indicates the actual *time* location of the *Perceptual Dynamic Inconsistency Parameter*  $\beta_\tau$  (2).

**Definition 5:** The *Present*  $t \in [0, \tau] = T_p$  is an interval of time ( $t$ ) between *Now* and the *Present Threshold* ( $\tau$ ). We denote the individual *Perceptual Present* as  $T_{\bar{p}}$ .

**Definition 6:** The *Future*  $t \in (\tau, t_n] = T_f$  is an interval of time ( $t$ ) between the *Present Threshold* ( $\tau$ ) and the date that the latest award is scheduled to be delivered ( $t_n$ ). We denote the individual *Perceptual Future* as  $T_{\bar{f}}$ <sup>2</sup>.

In the framework of GQHD model, the *Now* and the *Present* are different. *Now* is the lower bound to the *Present*. On the contrary, the upper bound of the *Present* is determined by the individual's time perception. Previously, we consider the  $t_0$  as the upper bound of the present in the structural models' estimation strategy. The GQHD model is more general compare to the QHD in the sense that we propose a more flexible upper bound of the *Present* to detect how long it would last. Simultaneously, the *Future* is no longer defined as all *Delivery Date* that is not  $t_0$ , but as the *Time* ( $t$ ) after the *Present Threshold*. We define  $t \in [0, \tau_0] = T_{p_0}$  as the *Primary Present*,  $t \in [0, \tau_i] = T_{p_i}$  as the *Secondary Present*,  $t \in (\tau_0, t_n] = T_{f_0}$  as the *Primary Future*, and  $t \in (\tau_i, t_n] = T_{f_i}$  as the *Secondary Future* (where  $i \in \{1, 2, 3, \dots, n-1\}$ ).

### 3.2 The Behavioral Characterizations of the GQHD Model

With a formal extension of the QHD model, the *Perceptual Dynamic Inconsistency Parameter*  $\beta_\tau$  differs the *Perceptual Present* from the *Perceptual Future* in the GQHD model. As a result of the flexible location of the  $\beta_\tau$ , the GQHD model has some unique properties compared to the ED and QHD models. We demonstrate five properties of the GQHD model to characterize the behavior in the revealed time preference.

**Property 1:** The revealed time preference exhibits **Observational Dynamic Consistent** if  $\beta_\tau = 1$ .

Generally, we emphasize the dynamic consistency is observational because the structural model fits the data in the give experimental time horizon. The inconsistency in time preference might

---

<sup>2</sup>If we extend time  $t \in \mathbb{R}$ , then the *Future* can be defined as  $t \in (\tau, \infty) = T_f$ .

happens beyond the experimental *time* domain. The locations of  $\beta_\tau$  are constraint by the way that we collect data in the experiment. The  $\beta_\tau$  exists in the structural model if and only if  $t_0 \leq \tau < t_n$ . The structural model cannot detect  $\beta_\tau$ , if the  $\tau$  exists at a location further than  $t_n$ . Similarly, the structural model is not sensitive to the time inconsistency behavior if  $\tau$  exists at a location that is sooner than  $t_0$ . Balakrishnan et al. (2017) address this  $\tau < t_0$  issue with a modified design of the CTB. However, the  $t_n \leq \tau$  question remains to be discovered.  $ED \equiv QHD \equiv GQHD$ , if  $\beta_\tau = \beta = 1$  or  $\tau < t_0$  or  $t_n \leq \tau$ .

**Property 2:** The revealed time preference exhibits **Primary Present Bias** if  $t_0 \leq \tau < t_1$ , and  $\beta_\tau < 1$ .

**Property 3:** The revealed time preference exhibits **Primary Future Bias** if  $t_0 \leq \tau < t_1$ , and  $\beta_\tau > 1$ .

The primary bias model, either present bias or future bias, is equivalent to the conventional setup (Laibson 1997; Andreoni and Sprenger 2012a). The additional explanation we provide to this traditional notion is that we abolish the restriction of the sense of threshold that differs present and future at the individual level. We distinguish the present and future, not by a single value in the timeline, say today verse any day that is not today. Instead, we consider any arbitrary  $\tau < t_1$  can be interpenetrated as an individual's perception of the present.  $QHD \equiv GQHD$ , if  $t_0 \leq \tau < t_1$ .

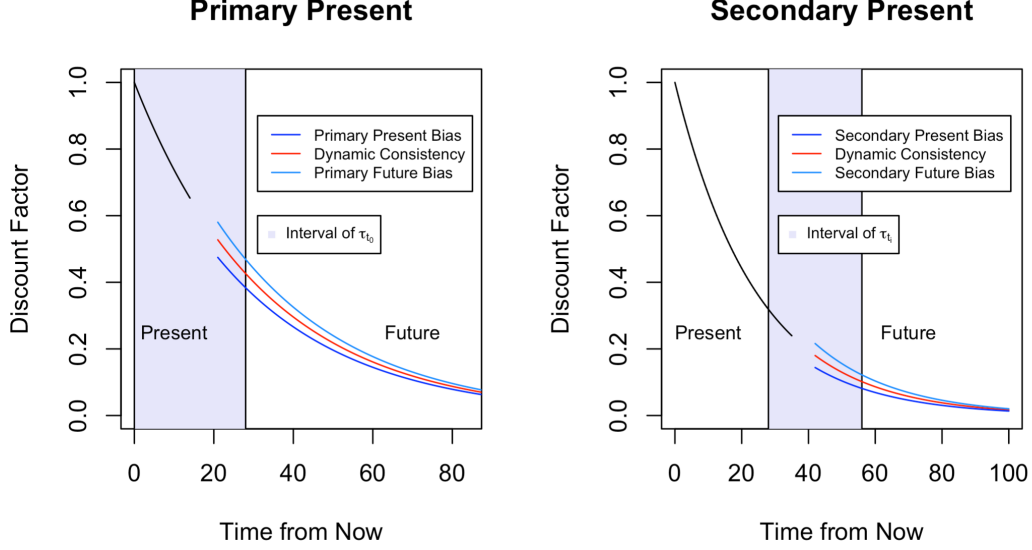
**Property 4:** The revealed time preference exhibits **Secondary Present Bias** if  $t_1 \leq \tau < t_n$ , and  $\beta_\tau < 1$ .

**Property 5:** The revealed time preference exhibits **Secondary Future Bias** if  $t_1 \leq \tau < t_n$ , and  $\beta_\tau > 1$ .

The secondary bias model is a general extension of the primary bias model. We allow the  $\tau$  to take the value greater than  $t_1$ , which is the nearest observed consumption. The general format could detect the inconsistency time preference between present and future with a relaxed assumption that individual subject has different time perception (Zauberman, Kim, and Malkoc 2009). The subjective time perception would affect the way we describe time inconsistency severely.

There is a gap between the theoretical framework and the structural empirical model proposed by the CTB protocol. The gap commonly exists for all of the intertemporal choices studies; the simple version (Fisher 1930), or any of its variations (Benhabib, Bisin, and Schotter 2010; Takeuchi 2011; Jackson, and Yariv 2014; Andreoni, and Sprenger 2012a; Montiel Olea and Strzalecki 2014).

Figure 1: Primary Present and Secondary Present



The subjects make decisions in the discrete-time slots. The consumption in the potential interval of  $\tau$  area is unobserved. We use the structural model to fit the observations and then describe the subjects' dynamic time inconsistency. Theoretically, the primary present model and secondary present model are equally compelling to explain time inconsistency with the assumption that within the shaded area, the subjects have a inconsistency time preference. The advantage of the GQHD model is to initiate  $\beta_\tau$  at different time spot, then find the most unexpected kinky consumption among all others (Figure 1).

### 3.3 Empirical Strategy

Given the general theoretical framework of the convex time budgets (CTB) in time preference studies (Andreoni and Sprenger 2012), we assume the subjects in the CTB experiment maximizing an additively time separable constant relative risk aversion (CRRA) utility function of consumption ( $U(x)$ ) between two **Delivery Date**  $t_i$ , and  $t_i + k$ . Such utility function is discounted with a GQHD function. Consider a subject maximizing the GQHD-CRRA utility at **Now** (4), subject to a given time budget constraint in the experiments (5).

$$U_0(x_{t_i}, x_{t_i+k}) = \delta^{t_i} \frac{1}{1-\eta} (x_{t_i} + \omega)^{1-\eta} + \beta_\tau^{\mathbb{1}_{\{t_i \leq \tau < t_i+k\}}} \delta^{t_i+k} \frac{1}{1-\eta} (x_{t_i+k} + \omega)^{1-\eta} \quad (4)$$

where we denote the one-period discount factor as  $\delta$ , the curvature parameter as  $1 - \eta$ , the background consumption as  $\omega$ , which we assume it as constant for every period. Lastly,  $x_{t_i}$  and  $x_{t_i+k}$  denote the consumption that allocate into the sooner account and the later account correspondingly.

$$x_{t_i} + (1 + r)^{-1} x_{t_i+k} = m \quad (5)$$

where we denote the  $r$  as the interest rate through  $k$  days and  $m$  as the total endowment for the current trail. Then the optimal demand function of sooner consumption is: (6)

$$x_{t_i}^* = \begin{cases} \frac{(\delta^k(1+r))^{\eta^{-1}} - 1}{1 + (1+r)(\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\delta^k(1+r))^{\eta^{-1}}}{1 + (1+r)(\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } \tau < t_i \text{ or } t_i + k \leq \tau \\ \frac{(\beta_\tau \delta^k(1+r))^{\eta^{-1}} - 1}{1 + (1+r)(\beta_\tau \delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\beta_\tau \delta^k(1+r))^{\eta^{-1}}}{1 + (1+r)(\beta_\tau \delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } t_i \leq \tau < t_i + k \end{cases} \quad (6)$$

To illustrate the strategy of finding the optimal  $\tau$ , we provide an example under the assumptions of the empirical framework. Consider a subject's three-period discounting function exhibit the following pattern  $D_\tau(t_0, t_1, t_2) = (\delta^{t_0}, \delta^{t_1}, \beta \delta^{t_2})$ , which is a secondarily bias time preference behavior. Suppose we correctly specified the structure model with a sequence of time-dependent discount factors  $\hat{D}_\tau(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\delta}^{t_1}, \hat{\beta} \hat{\delta}^{t_2})$ . We can recover the true value of  $\delta$  by solving the equation between the marginal rate of substitution and the price ratio of  $x_{t_0}$  and  $x_{t_1}$ . Here we denote the growth interest rate as  $r_{t_0, t_1}$  (7).

$$\frac{\partial U_0 / \partial x_{t_1}}{\partial U_0 / \partial x_{t_0}} = \frac{1}{1 + r_{t_0, t_1}} \implies \delta^* = \left( \frac{1}{1 + r_{t_0, t_1}} \right)^{\frac{1}{t_1 - t_0}} \left( \frac{x_{t_1}}{x_{t_0}} \right)^{\frac{1 - \eta}{t_1 - t_0}} \quad (7)$$

Similarly, we can recover the true value of  $\beta$  by solving the equation between the marginal rate of substitution and the price ratio of  $x_{t_1}$  and  $x_{t_2}$ . We denote the growth interest rate as  $r_{t_1, t_2}$ . Here notice  $\beta^*$  (or  $\beta^{*t_2 - t_1}$ ) is strictly greater than 0 (8).

$$\frac{\partial U_0 / \partial x_{t_2}}{\partial U_0 / \partial x_{t_1}} = \frac{1}{1 + r_{t_1, t_2}} \implies \beta^{*t_2 - t_1} = \left( \frac{1}{1 + r_{t_1, t_2}} \right)^{\frac{1}{t_2 - t_1}} \left( \frac{x_{t_2}}{x_{t_1}} \right)^{\frac{1 - \eta}{t_2 - t_1}} \left( \frac{1}{1 + r_{t_0, t_1}} \right)^{\frac{-1}{t_1 - t_0}} \left( \frac{x_{t_1}}{x_{t_0}} \right)^{\frac{\eta - 1}{t_1 - t_0}} \quad (8)$$

Contrarily, suppose we follow the QHD set up and misspecify the structure model with  $\hat{D}_{\tau'}(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\beta}\hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$ . Then misleadingly equating the misspecified estimation with the true parameter  $D_{\tau}(\cdot) = \hat{D}_{\tau'}(\cdot)$ , we can derive  $\hat{\delta}$  in (9). For simplicity, we consider  $\delta^{t_0} = 1$ .

$$\frac{\hat{\beta}\hat{\delta}^{t_2}}{\hat{\beta}\hat{\delta}^{t_1}} = \frac{\beta\delta^{t_2}}{\delta^{t_1}} \implies \hat{\delta} = \beta^{\frac{1}{t_2 - t_1}} \delta \implies \begin{cases} \hat{\delta} > \delta, & \text{if } \beta > 1 \\ \hat{\delta} < \delta, & \text{if } \beta < 1 \end{cases} \quad (9)$$

For  $\hat{\delta}$ , we can use the equation (8) to quantify the magnitude of overestimation or underestimation. With a similar approach, we can derive  $\hat{\beta}$  in (10).

$$\frac{\hat{\beta}\hat{\delta}^{t_1}}{\hat{\delta}^{t_0}} = \frac{\delta^{t_1}}{\delta^{t_0}} \implies \hat{\beta} = \frac{\delta^{t_1 - t_0}}{\hat{\delta}^{t_1 - t_0}} \implies \begin{cases} \hat{\beta} < 1 < \beta, & \text{if } \beta > 1 \\ \hat{\beta} > 1 > \beta, & \text{if } \beta < 1 \end{cases} \quad (10)$$

The result above has an intuition that can guide us to initiate the  $\beta_{\tau}$  at an optimal location that provides a more precise inference for the time preference (9 and 10). Suppose we misplace a secondary future bias subject's initial  $\beta_{\tau}$  earlier than the true location, then we are likely to interpret this subject's behavior as primary present bias. As pushing  $\tau$  to the further locations, we would observe a decreasing pattern of  $\hat{\delta}$ , along with an increasing trend of  $\hat{\beta}_{\tau}$ . Ideally, we can continuously push the  $\tau$  to a further location until we observe the  $\hat{\beta}_{\tau} > 1$ . Similarly, if we place a secondary present biased subject's  $\tau$  at a closer location, then we should expect an increasing trend of the  $\hat{\delta}$  along with a decreasing pattern of the  $\hat{\beta}_{\tau}$  as we push the  $\tau$  to the later position. The optimal initial location in the summation of the  $\beta_{\tau}$  in the summation is obtained when the  $\hat{\beta}_{\tau} < 1$  in this scenario. As many dynamic discrete choice studies demonstrated, the  $\beta$  and  $\delta$  vary accordingly with a systematic mechanism in QHD (Laibson, Repetto, and Tobacman 2007;

Arcidiacono and Miller 2008; Mahajan and Tarozi 2012; Fang and Wang 2015; Abbring, Daljord, and Iskhakov 2018). This grand truth also applies to the Convex Time Budget, since the choices are still in a dynamic setting. Our intuitive derivation reveals a potential confound classification of the *Primary Present Bias* and the *Secondary Future Bias* (the *Primary Future Bias* and the *Secondary Present Bias*). However, the confusion of mixing different types of behaviors have two directions ((9), (10); and (14), (15) from Appendix A). Besides, the trend of changing in  $\beta$  and  $\delta$  becomes more complicated when dynamic choices are made in many periods. To find the models that give a more profound explanation of the behaviors, we would provide more detailed findings with the empirical methods. To estimate the parameters, we will employ the non-linear least square (NLS) strategy (Andreoni and Sprenger 2012a; See the **Aggregate Level Results** section).

## 4 Simulated and Experimental Data

We acquired two data sets with the Convex Time Budget design to detect the perception of present and its corresponding time inconsistency to see the robustness of the primary present model (Andreoni and Sprenger 2012a (AP12); Carvalho, Meier, and Wang 2016 (CMW16)). In the following sections, we will mainly focus on the CMW16 data set because we find more server changes in point estimation of  $\beta_\tau$  and  $\delta$  in CMW16 than AP12 (See Table 9 in Appendix B for AP12 results). Since we focus on the design and the data set from the CMW16 data set, we also simulated 21 different data sets according to the CTB design to either demonstrate the intuition or confirm the findings in our studies. Since we focus on the design and the data set from the CMW16 data set, we also simulated 21 different data sets according to the CTB design to either demonstrate the intuition or confirm the findings in our studies. We will go through the details about the simulations when needed. Before any analysis, it is necessary for us to learn about the features of the Convex Time Budget design in CMW16.

In Carvalho et al. (2016), the participants are asked to allocate their experimental budget of \$500 into two payment accounts. The sooner account ( $t_i$ ), and its paired later account ( $t_i+k$ ). There are three paired accounts with  $(t_i, t_i+k) \in \{(28 \text{ days}, 84 \text{ days}), (28 \text{ days}, 84 \text{ days}), (today, 28 \text{ days})\}$ <sup>3</sup>. For each paired accounts, subjects make 4 choices of how they would like to split 500 dollars into

Table 1: The CTB Design in CMW16

choice	$t_i$	$t_i + k$	$\mathbf{r}$	$\mathbf{m}$
1	28 days	84 days	0%	\$500
2	28 days	84 days	0.5%	\$500
3	28 days	84 days	1%	\$500
4	28 days	84 days	3%	\$500
5	28 days	56 days	0%	\$500
6	28 days	56 days	0.5%	\$500
7	28 days	56 days	1%	\$500
8	28 days	56 days	3%	\$500
9	today	28 days	0%	\$500
10	today	28 days	0.5%	\$500
11	today	28 days	1%	\$500
12	today	28 days	3%	\$500

sooner and later accounts. The delayed (later) account has its associated interest rate of 0%, 0.5%, 1%, and 3% respectively at the payday. In total, each subject make 12 choices (we have 12 observations for each subject). The researchers randomly select one out of twelve choices to issue the reward to the subjects at the paydays. If the choice has a nonzero amount in the later account, then they will be paid with the associated interest rate  $\mathbf{r}$  (Table 1). They restricted the sample for 1061 subjects that finish all 12 trials in the data set. Our analysis will impose the same restriction on these participants who complete all 12 trials in the experiments. They observed a slight primary present bias for the subjects at the aggregate level from a tow-limit Tobit model with a CARA utility setup (Carvalho, Meier, and Wang 2016).

The CTB design in CMW16 allows us to set the *Present Thresholds* ( $\tau$ ) at four different time slots in the GQHD specification. Firstly at  $\tau_0$ , the subject can consider any day between today and 28 days later as his or her perceptual present. Consider the *Primary Present* ( $T_{p_0}$ ) directly come from the QHD model. Secondly,  $\tau_{28}$  is when a subject's  $\tau$  can take any day between 28 days and 56 days arbitrarily. Then,  $\tau_{56}$  has a range between 56 days and 84 days. These specifications consider the *Secondary Present* ( $T_{p_i}$ ), which is the innovation of the GQHD. Lately, the  $\tau_{84}$  indicates the *Present Thresholds* surpasses 84 days, and the specification claps back to the ED model.

<sup>3</sup>In the CTB design of CMW16, the payment dates are labeled as today, 4 weeks, and 8 weeks. We denote 4 weeks as 28 days, 8 weeks and 56 days, and 12 weeks as 84 days to be consistent with our structural model. We also consider  $t_i$  for today as  $t_0 = 0$  to fit the model.



Table 2: Estimations of 18 Simulated Data Sets (CMW16)

500 Subjects		$\tau_0$	$\tau_{28}$	$\tau_{56}$	$\tau_0$	$\tau_{28}$	$\tau_{56}$
$1 - \eta$		$\beta_{\tau_{56}} = 1.1 \delta = \mathbf{0.9995}$			$\beta_{\tau_{56}} = 1.05 \delta = \mathbf{0.9995}$		
<b>0.9</b>	$\hat{\beta}$	0.9622	0.9999	1.1000	0.9811	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0013	0.9995	1.0003	1.0003	0.9995
<b>0.85</b>	$\hat{\beta}$	0.9626	1.0007	1.1000	0.9814	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0011	0.9995	1.0003	1.0003	0.9995
<b>0.8</b>	$\hat{\beta}$	0.9624	0.9992	1.1000	0.9815	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0011	0.9995	1.0003	1.0003	0.9995
$1 - \eta$		$\beta_{\tau_{56}} = 1.1 \delta = \mathbf{0.9985}$			$\beta_{\tau_{56}} = 1.05 \delta = \mathbf{0.9985}$		
<b>0.9</b>	$\hat{\beta}$	0.9608	1.0000	1.1000	0.9807	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9999	0.9985	0.9992	0.9991	0.9985
<b>0.85</b>	$\hat{\beta}$	0.9614	1.0000	1.1000	0.9808	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9999	0.9985	0.9992	0.9991	0.9985
<b>0.8</b>	$\hat{\beta}$	0.9617	1.0000	1.1000	0.9809	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9998	0.9985	0.9992	0.9991	0.9985
$1 - \eta$		$\beta_{\tau_{56}} = 1.1 \delta = \mathbf{0.9975}$			$\beta_{\tau_{56}} = 1.05 \delta = \mathbf{0.9975}$		
<b>0.9</b>	$\hat{\beta}$	0.9612	1.0000	1.1000	0.9810	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9986	0.9975	0.9982	0.9980	0.9975
<b>0.85</b>	$\hat{\beta}$	0.9617	1.0000	1.1000	0.9810	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9987	0.9975	0.9982	0.9981	0.9975
<b>0.8</b>	$\hat{\beta}$	0.9618	1.0000	1.1000	0.9811	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9987	0.9975	0.9982	0.9981	0.9975

We illustrate the systematic changes in the  $\beta$  and the  $\delta$  parameters in the **Empirical Strategy** section. However, that derived intuition was limited to three-period consumption observations. The design of CMW 16 has four periods (today, 28 days, 56 days, and 84 days) instead of three. We will confirm if the systematic changes in the  $\beta$  and the  $\delta$  parameters remain unchanged from 18 simulated data sets corresponding to the CTB in CMW16. To begin with, we simulated the following 18 data sets to detect the validity of the intuition that the structural model tends to underestimate the  $\beta$  parameter if we assume the perception of the present ends right after the soonest consumption rather than terminates at later possible locations. Suppose we have two sets of secondary future biased time preference populations with  $\beta_{\tau_{56}}$  equals to either 1.1 or 1.05 ( $\beta_{\tau_{56}} \in B = \{1.1, 1.05\}$ ). We simulated 9 data sets for each of these two secondary future biased populations

with selected curvature ( $1 - \eta \in H = \{0.9, 0.85, 0.8\}$ ) parameters and discount parameters ( $\delta \in D = \{0.9995, 0.9985, 0.9975\}$ ). The annual discount rates  $(1/\delta)^{365} - 1$  are 20.03%, 72.96%, and 149.34% for the selected discount parameter 0.9995, 0.9985, and 0.9975 respectively. We simulated 500 subjects' optimal consumption level  $x_{t_i}^*$  at date  $t_i$  (6) according to the CMW16 design with a random normally distributed error ( $e \stackrel{i.i.d}{\sim} N(0, 0.01)$ ) for all 18 data sets ( $H \times B \times D$ ). The simulated results are presented in Table 2. As a consequence, we observed a strong tendency of the increasing trend in the estimated  $\beta$  and the decreasing trend of the estimated  $\delta$  as we move the  $\tau$  to the true location  $\tau_{56}$  from the QHD assumed  $\tau_0$ . Such tendency suggests the degree of present bias can be overestimated if we treat the subjective present unanimously as the **Primary Present** ( $T_{p_i}$ ), especially for those who exhibit **Secondary Future Bias** ( $\beta_{\tau_{56}} > 1$ ) in their time preference (see the **Analysis and Results** section for the detailed statistical approach). The findings coincide with the grand truth reached in (9) and (10).

## 5 Analysis and Results

### 5.1 Aggregate Level Results

To estimate the parameters, we employed a similar non-linear least square (NLS) strategy proposed by Andreoni and Sprenger (2012a). We specified the indicating function of the  $\beta$  to identify the perceptual present accordingly. Furthermore, we estimate the model with the assumption of the time-independent  $\omega \approx 0$  in (6) consider the decision-maker is not necessarily narrowly brackets in the experiment (Rabin and Weizsacker 2009). The differences between restricting  $\omega = 0$  or not are statistically negligible, and we present the results of the estimations without such restriction in Table 3.

From Table 3, We find the estimation of the  $\beta$  in the primary model is equal to 0.9516, which suggests the subjects exhibits **Primary Present Bias**, which is consistent with the results in CMW16. However, as we relocate the tau into further location, we find a clear increasing trend in the estimation of  $\beta$  and a decreasing trend in  $\delta$  estimation. This pattern suggests that there are more subjects' behaviors that follow the **Secondary Future Bias** instead of the **Primary**

Table 3: Aggregate Level Estimation for 1061 Subjects (CMW16)

Model	$\tau_0$	$\tau_{28}$	$\tau_{56}$	$\tau_{84}$
$\hat{\beta}$	0.9516 (0.0064)	1.0010 (0.0086)	1.1280 (0.0121)	
$\hat{\delta}$	0.9979 (0.0001)	0.9976 (0.0002)	0.9962 (0.0002)	0.9976 (0.0001)
$1 - \hat{\eta}$	0.7364 (0.2111)	0.7224 (0.2319)	0.7561 (0.2205)	0.7224 (0.2319)
<b>AIC</b>	165404.5	165463.7	165308.8	165461.7
<b>BIC</b>	165441.7	165500.9	165346	165491.5
<b>RSS</b>	326667281	328189670	324220845	328189845
<b>Observations</b>	1061	1061	1061	1061

**Present Bias.** The future bias is observed at  $\tau_{56}$ . We chose three different criteria to compare the overall performance of different models (Akaike information criterion (AIC), Bayesian Information Criterion (BIC), and Residual Sum of Squares (RSS)). These three criteria also provide evidence that supports  $\beta_{\tau_{56}}$  is the most desirable model to describe the data at the aggregate level. The model of  $\beta_{\tau_{56}}$  has the lowest AIC, BIC, and RSS, given all of the possible locations of the  $\tau$  we detected.

Since the credibility of our structural model relies heavily on the existence of well-behaved (monotonic, quasi-concave, continuous, and non-satiated) utility functions, we next evaluate individual participants' data for consistency with the requirements of rationality, exclude observations that fail to meet the criteria, repeat the analysis and compare results. A commonly used approach for detecting the existence of well-behaved utility is satisfying the Generalized Axiom of Revealed Preference (GARP) and its non-parametric test (Afriat 1967, Afriat 1972, Varian 1982, and Varian 1991). This approach will not work, however, due to the original design of the CTB experiments since the commodities from different trials are not nested, and the time budget lines barely intersect with one other. We, therefore, employed the Law of demand test which requires that if a trial that has a higher interest rate for the later account than the previous trial, we should observe the consumption level of the later account is at least as much as the previous trial. This basic consistency approach has been utilized by the earlier studies to detect for rationality (Gine, Goldberg, Silverman and Yang 2012; Balakrishnan, Haushofer, and Jakiela 2017; Echenique, Imai, and Saito

2019).

Table 4: Aggregate Level Estimation for 452 Subjects (CMW16)

Model	$\tau_0$	$\tau_{28}$	$\tau_{56}$	$\tau_{84}$
$\hat{\beta}$	0.9625 (0.0064)	1.0000 (0.0085)	1.1010 (0.0123)	
$\hat{\delta}$	0.9977 (0.0001)	0.9974 (0.0002)	0.9964 (0.0002)	0.9974 (0.0001)
$1 - \hat{\eta}$	0.8450 (0.0826)	0.8392 (0.0848)	0.8534 (0.1129)	0.8392 (0.0848)
<b>AIC</b>	71257	71291.83	71199.9	71289.83
<b>BIC</b>	71290	71324.82	71232.89	71316.22
<b>RSS</b>	160888352	161924583	159203278	161924667
<b>Observations</b>	452	452	452	452

We find that 452 out of 1061 subjects' choice set strictly follows the law of demand and repeat our analysis with only these subsets Table 3. We found these people, whose time preference follows the law of demand, also seems to exhibit present bias when we set  $\beta_\tau$  at  $t_0$  on an aggregated level. Additionally, there is an increasing trend of the  $\beta$  estimator and a decreasing trend of the  $\delta$  estimator as we move  $\tau$  to later possible locations ( $\tau_{28}$ ,  $\tau_{56}$ , and  $\tau_{84}$ ). We observe the best performance when  $\tau_{84}$  according to AIC, BIC, and RSS criterion, which suggests the subjects are more likely to have a secondary future biased time preference than a primary present biased at the aggregate level (Table 4).

To consider the concentration of these dynamically inconsistent behaviors at the individual level, and how it would affect the aggregate level estimation, it is necessary to sort out these subjects with dynamically consistent time preferences. The Strong Axiom of Revealed Exponentially Discounted Utility (SAR-EDU). The SAR-EDU suggests that if a finite time revealed time preference data set is EDU rational, then the time preference can be rationalized with an exponential discounted well-behaved utility function, where delta can take the value of 1. The additional finding suggests the test's result is aligned with parametric estimations. If the revealed time preference data set passes the SAR-EDU test, then the empirical estimation of the  $\beta_\tau$  is likely to be close to 1 (Echenique, Imai, and Saito 2019).

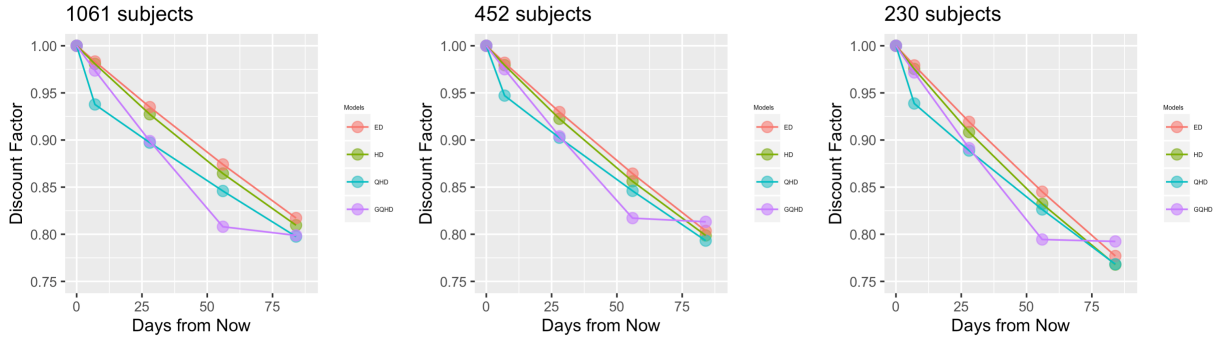
We rerun the analysis with the 230 subjects whose choice set, which did not pass the SAR-EDU

Table 5: Aggregate Level Estimation for 230 Subjects (CMW16)

Model	$\tau_0$	$\tau_{28}$	$\tau_{56}$	$\tau_{84}$
$\hat{\beta}$	0.9560 (0.0067)	1.0020 (0.0086)	1.1190 (0.0151)	
$\hat{\delta}$	0.9974 (0.0001)	0.9970 (0.0002)	0.9959 (0.0003)	0.9970 (0.0001)
$1 - \hat{\eta}$	0.8819 (0.0503)	0.8779 (0.0485)	0.8884 (0.1179)	0.8779 (0.0485)
<b>AIC</b>	35711.12	35757.27	35637.66	35755.34
<b>BIC</b>	35740.73	35786.88	35667.27	35779.04
<b>RSS</b>	67006296	68136084	65246300	68137966
<b>Observations</b>	230	230	230	230

test within the 452 subjects. These 230 people’s revealed time preference choice set is theoretically unrationalizable with dynamic consistency well-behaved utilities (EU). The trend of  $\beta$  estimation and  $\delta$  estimations remain unchanged. The model  $\tau_{56}$  produced the best overall performance among all models (Table 5). We will investigate how different restrictions change the concentrations of the different types of dynamically inconsistent behavior and how such concentration could affect the aggregate level conclusion in the **Individual Level Results** section.

Figure 2: Estimated Discount Factors for 3 Datasets (CMW16)



To make the comparisons between the different discounting models, we present the estimated discount factors among the ED, the HD (the Hyperbolic Discounting Model: see the results from from Appendix C), the QHD, and the GQHD models in Figure 2. We see a departure at the initial days of the HD model and the QHD models for all three different data sets. This departure indicates that the initial sharp drop of the QHD model was likely to catch a later seemingly inconsistency discount rate caused by the underestimation of the  $\delta$ , rather than to present the inconsistent

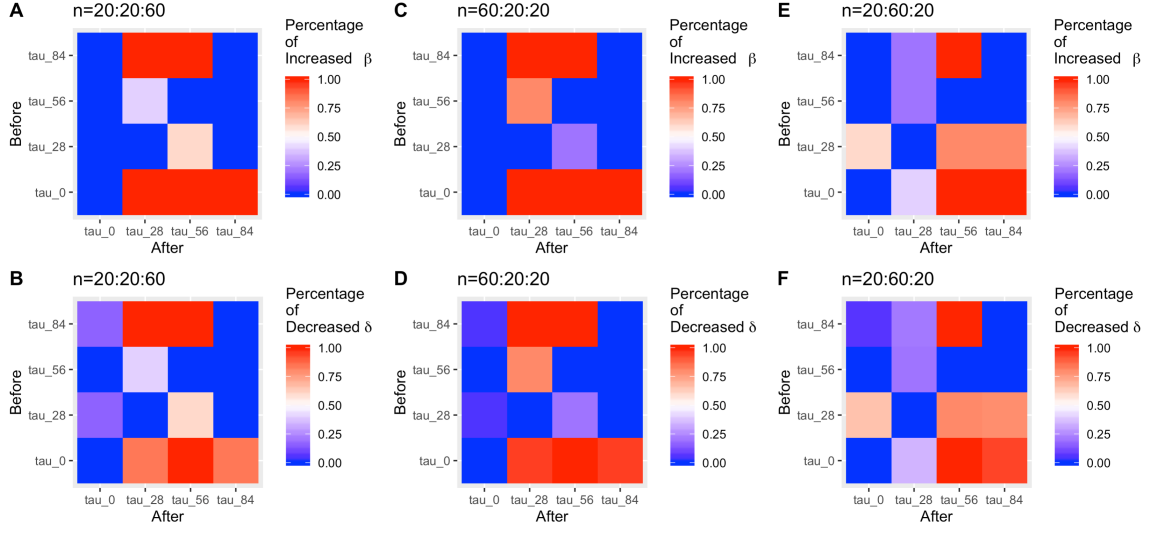
discounting rates between the present and the future. The GQHD model indeed captures some unusual behavior in the discount function at the aggregate level for all different data sets. The GQHD model indeed captures some unusual behavior of the curvature in the discount function (Takeuchi 2012) at the aggregate level for all different data sets. It has the overall best performance among all four different models. An additional investigation at the individual level is necessary to explain what we find at the aggregated level.

## 5.2 Individual Level Results

At the aggregate level, the people with secondary future biased time preference is dominating the time preference behavior in CMW16 data set ( $\beta_{\tau_{56}} > 1$  models have the best overall performance). We can also reach this intuition by unanimously observing the same pattern of trends of estimations of  $\beta$  and  $\delta$  parameters when relocating  $\tau$  to later positions. To verify how the concentration of the different behavior changes the pattern of the estimations, we newly simulated three data sets (100 simulated subjects each) with different proportion combinations of primary present biased, time consistent, secondary future biased subjects. The proportions of primary present biased, time consistent, and secondary future biased subjects for these three simulated data sets are 2:2:6 (secondary future biased dominating), 2:6:2 (time consistent), and 6:6:2 (primary present biased dominating) respectively. For the simulated data, the  $\eta$  takes a uniform distribution between 0.87 and 0.89 ( $1 - \eta \in [0.87, 0.89]$ ), and the  $\delta$  takes a uniform distribution between 0.9997 and 0.9999 ( $((1/\delta)^{365} - 1) \in [20.03\%, 149.34\%]$ ). We also assumed the background consumption  $\omega$  is 0 for all of the simulated subjects. The biased parameter  $\beta$  are uniformly generated between 0.95 and 0.99 for the primary present biased ( $\beta_{\tau_0} \in [0.95, 0.99]$ ) simulated subjects, and between 1.05 and 1.1 for the secondary future biased ( $\beta_{\tau_{56}} \in [1.05, 1.1]$ ) simulated subjects. The  $\beta$  are equal to 1 for the time consistent time preference ( $\beta_{\tau_{28}} = 1$ ) simulated subjects. Then we estimated the  $\beta$  and  $\delta$  parameters with the NLS approach for these simulated subjects.

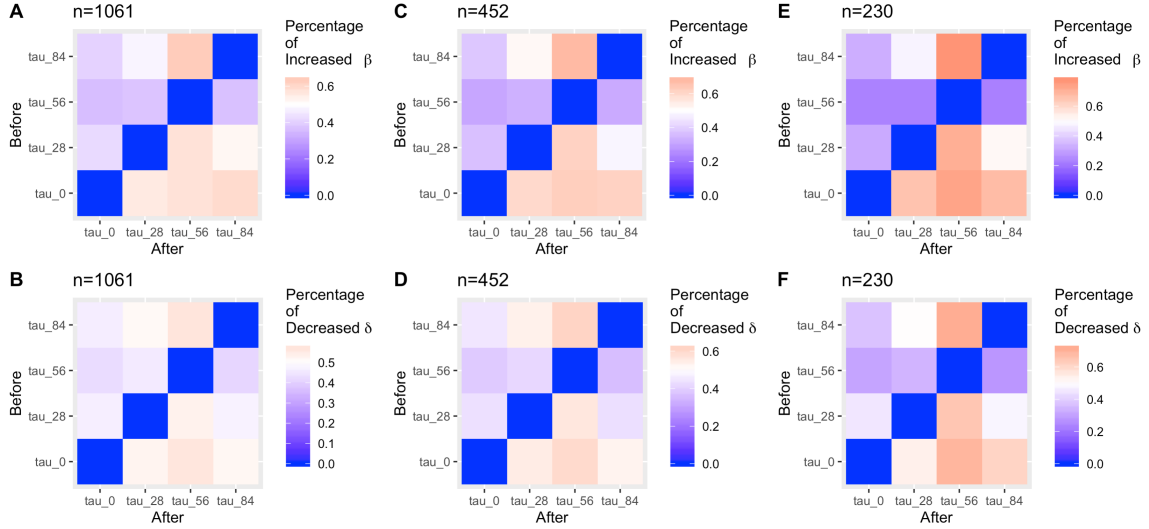
To visualize the relationship between the change of the estimations and the proportion of the different types of time preference, we plotted a heat map of the proportions of increased  $\beta$  for all pairwise comparisons of two different  $\beta_{\tau}$  identified structural models. For example, suppose we estimate the  $\beta_{\tau_0}$  as the first model (labeled as before), and we estimate the  $\beta_{\tau_{28}}$  as the second

Figure 3: The Proportion of Increased  $\beta_\tau$  and Decreased  $\delta$  for the Simulated Data



model (labeled as after). The pixel for  $\beta_{\tau_0}$  (before) and  $\beta_{\tau_{28}}$  (after) represents the percentage of the increased  $\beta$  for the second model compare to the first model. We apply the same rule for the percentage of decreased  $\delta$  estimations. We can recognize the different patterns in the heat maps when compare plots A and B (secondary future biased dominating), vs. C and D (time consistent) vs. E and F (primary present biased dominating) in Figure 4. We will use the simulated heat maps as the reference to trace the hypothetical concentration of different behaviors in CMW16 data.

Figure 4: The Proportion of Increased  $\beta_\tau$  and Decreased  $\delta$  (CMW16)



As a comparison, we plot the heat maps of the NLS estimations of the  $\beta$  and  $\delta$  parameters for all three CMW 16 subsets (1061, 452, and 230) in the same style as the 300 simulated subjects. For

the full sample ( $n=1061$ ) and the other two sub-samples ( $n=452$ , or  $230$ ), the observed proportion of the increased  $\beta$  is always surpassing 50 % when comparing the closer  $\tau$  (before) models ( $\tau_0$ ,  $\tau_0$ , and  $\tau_{28}$ ) to the further  $\tau$  (after) models ( $\tau_{28}$ ,  $\tau_{56}$ , and  $\tau_{56}$ ) respectively. The decreasing trend of the estimated  $\delta$  is also dominating (surpass 50%) for all samples as if we focus on the counterpart of the  $\beta$  pixels. We can see this clear pattern in the lower triangular region (bottom right) of the heat maps in Figure 4. We can also find this coherent similarity of the color pattern in plots A and B from Figure 3. The similarity in color patterns indicates the subjects in CMW16 are likely to be dominated by the secondary future biased time preference rather than the reported primary present biased behaviors (Carvalho, Meier, and Wang 2016). Our finding explains the reason that there was an additional 68 subjects' time preference became Future-Biased QHD rational (28.94 percent increase). Still, only 9 more subjects became Present-Biased QHD rational (3.83 percent increase) when the  $\tau$  was relocated at later positions in CMW16 data (Echenique, Imai, and Saito 2019)<sup>4</sup>. More importantly, we observed denser and denser color pixels as more restricting assumptions were imposed on the CMW16 data. This tendency pinpoints that the failure of separating different behavior with the theoretical framework would lead to the misconceptions at the aggregate level.

The result of the comparison between the simulations and the experimental data indicates some of the model selection criteria (AIC, BIC, and RSS) are sensitive in terms of detecting the true location of the *Present Threshold* ( $\tau$ ). Therefore, the AIC, BIC, and RSS are also calculated for NLS models at the individual subject level. The number and the percentage of the best-performed model based on three different criteria by the given subsets are presented in the Table 6. The result suggests that  $\tau_{56}$  has the most robust performance, especially when we imposing stronger assumptions on the samples. Overall, the secondary biased ( $\tau_{28}$  or  $\tau_{56}$ ) models (either present biased or future biased) are the majority of the best-performed models among all subsets, which is different from the traditional assumption of  $\tau_0$ .

The best estimation of  $\beta_\tau$  can be decomposed based on the best location of the perspective of the present according to the revealed choice of time preference at the subject level. Figure 5(left) depicts the distribution of estimated beta at the different underlying assumptions of the perspective present. The distribution is skewed to the right as the assumptions of the perspective present is

---

<sup>4</sup>Echenique et al. (2019) observed 235 strictly QHD rational time preferences among 1061 subjects from the CMW16 when considering  $\tau_0$ .



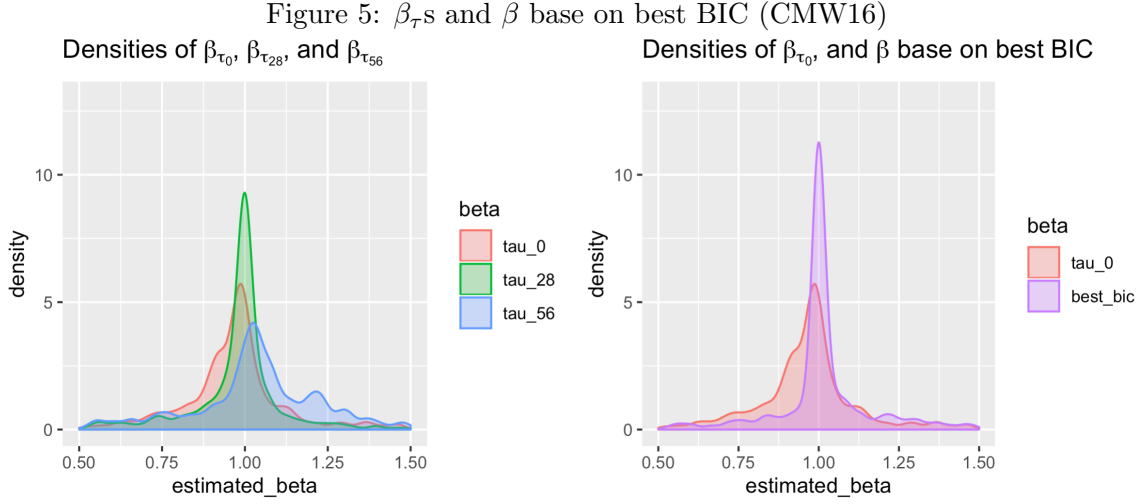
Table 6: Best Model based AIC, BIC, and RSS for Individual Subject (CMW16)

Criterion	Observations	$\tau_0$	$\tau_{28}$	$\tau_{56}$	$\tau_{84}$
best AIC	1061	150 14.14%	105 9.90%	439 41.38%	367 34.59%
	452	58 12.83%	47 10.40%	204 45.13%	143 31.64%
	230	30 13.04%	25 10.87%	124 53.91%	51 22.17%
best BIC	1061	139 13.10%	91 8.58%	417 39.30%	414 39.02%
	452	54 11.95%	43 9.51%	197 43.58%	158 34.96%
	230	29 12.61%	24 10.43%	121 52.61%	56 24.35%
best RSS	1061	218 20.55%	180 16.97%	588 55.42%	75 7.07%
	452	85 18.81%	67 14.82%	260 57.52%	40 8.85%
	230	43 18.70%	38 16.52%	143 62.17%	6 2.61%

pushed to the future. The further we assume the perspective present is, the more likely we consider the subjects as future biased. Then we merged all 1061 estimated  $\beta_\tau$  (13.67% of the primary present behaved subject, 46.59% of secondary present behaved subjects, and 38.74% time consistent subjects) based on the BIC (Figure 5(right)). The over distribution is shifted to the right for the best BIC  $\beta_\tau$  compare the conventional setup  $\beta_{\tau_0}$  with one-sided Mann–Whitney U test’s p-value of 0.0001 (alternative less than).

### 5.3 Robustness Check with the CARA Utility Specification

There are two major concerns of the estimation strategy for the CTB design along with the non-linear least square approach. First of all, the NLS could create the bias for parameter estimations due to not adjusting for corner solutions. We already addressed this concern with the simulations. We find that the tendency of change for both  $\delta$  and  $\beta$  remains stable with or without the censoring. Another concern is that the current result can be driven by the CRRA utility functional form. Therefore, we specify the structural model with the constant absolute risk aversion (CARA) utility



along with the additive discounting function in (11):

$$U_0(x_{t_i}, x_{t_i+k}) = \delta^t(-\exp(-\alpha(x_{t_i} + \omega))) + \beta_\tau^{\mathbb{1}_{\{t_i \leq \tau < t_i+k\}}} \delta^{t_i+k}(-\exp(-\alpha(x_{t_i+k} + \omega))). \quad (11)$$

With the CARA specification, the solution of the optimization takes (12):

$$x_{t_i}^* = \begin{cases} \left[ \frac{k \ln(\delta)}{-\alpha} + \frac{\ln(1+r)}{-\alpha} + m \right] \frac{1}{1+(1+r)}, & \text{if } \tau < t_i \text{ or } t_i + k \leq \tau \\ \left[ \frac{\ln(\beta_\tau)}{-\alpha} + \frac{k \ln(\delta)}{-\alpha} + \frac{\ln(1+r)}{-\alpha} + m \right] \frac{1}{1+(1+r)}, & \text{if } t_i \leq \tau < t_i + k \end{cases} \quad (12)$$

As suggested by Andreoni and Sprenger (2012a), the parameter can be recovered by a two-step tow-limit Tobit model. However, the Tobit model's specification cannot be estimated due to the multicollinearity problem when  $\tau_{56}$  is assumed with the CMW16 design. Then, we employed the NLS approach again to estimate the CARA specification. We find the robust result of the increasing trend of  $\beta$  and decreasing trend of  $\delta$  estimations as we push the position of  $\tau$  from  $\tau_0$  to  $\tau_{56}$  given the CARA utility. The  $\beta_{\tau_{56}}$  is also the most desired model among all others for the BIC criterion for all three subsets of CMW16. Our results hold across the different functional forms of the utility

(Table 7).

Table 7: CARA Estimations of All Three Subsets (CMW16)					
CARA Models (Obs)		$\tau_0$	$\tau_{28}$	$\tau_{56}$	$\tau_{84}$
$\hat{\beta}$		0.9496	1.0000	1.1340	
$\hat{\delta}$		0.9978	0.9975	0.9960	0.9975
$\hat{\alpha}$		0.0012	0.0012	0.0011	0.0012
1061	<b>BIC</b>	165459.9	165517.2	165365.5	165507.8
$\hat{\beta}$		0.9596	0.9999	1.1078	
$\hat{\delta}$		0.9976	0.9973	0.9961	0.9973
$\hat{\alpha}$		0.0007	0.0008	0.0007	0.0008
452	<b>BIC</b>	71305.98	71342.05	71245.46	71333.45
$\hat{\beta}$		0.9507	1.0022	1.1280	
$\hat{\delta}$		0.9973	0.9969	0.9956	0.9969
$\hat{\alpha}$		0.0006	0.0006	0.0005	0.0006
230	<b>BIC</b>	35754.3	35808.37	35669.66	35800.51

#### 5.4 The Average of the Perceptual Present

To find an revealing description of the central tendency, we calculate the average of the perceptual present ( $\bar{T}_{\bar{p}}$ ) among all of the subjects. (13).

$$\bar{T}_{\bar{p}} = \sum_{i=1}^{n-1} W_{\tau_{t_i}} \left( \frac{t_i + t_{i+1}}{2} \right) \quad (13)$$

where  $W_{\tau_{t_i}}$  is a weight function that proportion of the dynamic inconsistency subject whose perception of the present is likely to be located at  $\tau_{t_i}$ . We consider the middle point of the interval  $[t_i, t_{i+1})$  as the optimal location of  $\tau_{t_i}$  for simplicity. Notice that the **Observational Dynamic Consistent** subject has no observed  $\bar{T}_{\bar{p}}$ .

We find that  $\bar{T}_{\bar{p}}(CMW16) = 47.59$  days and  $\bar{T}_{\bar{p}}(AP12) = 65.44$  days, respectively. The results suggest that the observed kinky point of chosen consumption in these lab experiments happened far away from the conventional assumption that the time preference discounted inconsistently immediately after the nearest to now award. Our empirical method shows that it is more precise to discover how far could the perception of the present lasts rather than to question how soon does it

end (Balakrishnan, Haushofer, and Jakiela 2017).

## 5.5 Dynamic Inconsistent in Perceptual Time Scale

Ignorance of the perceptual sense of the present would lead to a misleading conclusion of the directions of the time inconsistency when we evaluate the data in depth. We present the final breakdown of different time preference behaviors base on the BIC criteria in Table 8. For the CMW16 experiment, despite the small proportion of dynamic consistency behavior, the percentage of future bias (both primary and secondary) adds up to 32.80%, which is higher the 28.18% of the present bias (both primary and secondary). We conclude the future bias time preference is dominating the population in the CMW16, which is different from the reported present bias behavior in their study (Carvalho, Meier, and Wang 2016). Similar, the proportion of the observed perceptual present biased behavior in AS12 is 52.57%, which is higher than the 36.05% of future bias behavior. When time inconsistency is measure in a procreational scale, we also reached a different conclusion from the original discovered slightly future biased behavior in Andreoni and Sprenger (2012a). When time is considered on a perceptual scale and explored in more details, our results are not consistent with the discovery of the previous studies.

Table 8: the Breakdown of Different Time Preference Behaviors (CMW16, AP12)

<b>Research</b>	<b>CMW16</b>		<b>AS12</b>	
<b>BIC</b>	<b>count</b>	<b>percentage</b>	<b>count</b>	<b>percentage</b>
<b>Dynamic Consistency</b>	414	39.02%	11	11.34%
<b>Primary Present Bias</b>	91	8.58%	3	3.09%
<b>Primary Future Bias</b>	48	4.52%	7	7.22%
<b>Secondary Present Bias</b>	208	19.60%	48	49.48%
<b>Secondary Future Bias</b>	300	28.28%	28	28.87%
<b>Total</b>	1061	100%	97	100%

## 6 Conclusion and Discussion

We developed an extension of the QHD model and added the concept of perceptual time scale to the Qausi-Hyperbolic Discounting function. By adjusting for the perceptual of present feature in time preference, we developed the QHD into the General Qausi-Hyperbolic Discounting

model. With the application of our GQHD model, the results point to two conclusions. First of all, the perception of the present is a critical component of measuring inconsistency in time preference. Disregarding the importance of the perceptual dimension of time would lead to a misleading conclusion of the time inconsistency. The secondary future bias behavior could be misleadingly treated as the primary present bias behavior. The secondary future bias behavior could be misleadingly treated as the primary present bias behavior. Simultaneously, the secondary present bias behavior could be misleadingly treated as the primary future bias behavior. Simply assuming time preference discounted inconsistency only after the immediate payoff without any hypothetical test, we either fail to observe the inconsistency or fail to distinguish the present bias from the future bias. The misconception of the direction of the bias could sufficiently undermine the conclusions and challenge the validity of the policy application in time preference studies. If a policy is intended to address the primary present bias issue; however, it is applied to the secondary future bias population; this policy may further intensify future bias behavior

Secondly, with the introduction of a flexible parameter  $\beta_\tau$  in the empirical work, we were able to detect the length of the perception of the present at both aggregate and individual levels. The results indicate the perceptual present reaches far beyond immediate soon ( $t_0$ ). Constraining the  $\beta$  parameter at its initial place limits the ability of the empirical model to fit the data—additionally, it constrains the QHD model to give a more precise explanation of the time preference behavior. The GQHD model provides a more insightful interpretation of the time inconsistency along with additional information about when it happens.

By adding a perception dimension of time in the Quasi-Hyperbolic Discounting, we provide a more general method to differ the observed the primary present bias from its quasi-twin secondary future bias in time preference. We answered the research question of “what is the present,” then discussed how the observed bias of the time preference changed toward this discovered perceptual present. Our findings advocates a necessity of consideration of the perceptual present for future dynamic inconsistency in time preference studies. However, our conclusion has its constraint to a certain extent.

First, the possible locations of the structural model in our identifications heavily rely on the design of CMW16 and AP12. Both of these studies focus on time inconsistency for the monetary incentive. To have a more versatile measurement of how late the present could last and even further

generalize our discussion beyond monetary incentive, we need an adaptive budget set design to resolve the problems of preventing corner solutions, then detecting the present beyond money (Imai and Camerer 2018). More importantly, the current literature of time preference in economics is present-biased-focused. The research which finds the evidence of the existence of future bias behavior either challenges the design fails to prevail the myopic behavior (Shu 2008) or provides little explanation of the abnormal finding (Aycinena, Blazsek, Rentschler, and Sandoval 2015; Corbett 2016; Aycinena, and Rentschler 2018). On the other hand, some studies in both economics and phonology emphasize the importance of future bias (Dougherty, 2015; Greene and Sullivan 2015; Dorsey 2017). Suppose a new design can successfully control the uncertainty of future income and the higher interest rates are positively correlated with the probability of triggering perceptual future bias, we can link our GQHD model to early studies of over-saving and retirement research to find some additional practical applications (Diamond and Köszegi 2003; Salanié and Treich 2006; Zhang 2013).

Second, we find the optimal duration for the present in the experiment. Still, our finding was not intended to explain if the observed future bias is caused by the risk-averse driven behavior concerning the uncertainty of getting the reward after the experiment (Andreoni and Sprenger 2012b). Additionally, this optimal duration we find is based on our modified version of QHD specification. We chose the model mainly to keep the beta parameter in a discrete setting, then compare our results with the findings in related literature. Other models can also identify the turning point in the discounting function (for instance, Loewenstein and Prelec 1992). It is also compelling to see the relationship between the perceptual present and the reversal point of the discounting factor in other models.

Third, the criteria and visualization methods we used successfully identify the optimal location of the preset threshold and the concentration of different behaviors in the population. However, we are uncertain how accurate and sensitive are these methods, especially for mild inconsistency in time preference.

Lastly, This study is a complement of "how soon is now" and "now or as soon as possible" (Glimcher, Kable, and Louie 2007; Balakrishnan, Haushofer, and Jakiela 2017). It's necessary to conduct a conurbation of our studies to comprehend if the unobserved present bias is caused by present is too soon or we fail to consider present could last longer in perceptual scale. We will

leave these concerns to the future work.

## References

- [1] Abbring, J. H., Daljord, Ø., & Iskhakov, F. (2018). Identifying present-biased discount functions in dynamic discrete choice models. Mimeo, CentER, Tilburg. 3.
- [2] Ainslie, G. (1975). Specious reward: a behavioral theory of impulsiveness and impulse control. *Psychological bulletin*, 82(4), 463.
- [3] Ainslie, G., & Herrnstein, R. J. (1981). Preference reversal and delayed reinforcement. *Animal Learning & Behavior*, 9(4), 476-482.
- [4] Afriat, S. N. (1967). The construction of utility functions from expenditure data. *International economic review*, 8(1), 67-77.
- [5] Afriat, S. N. (1972). Efficiency estimation of production functions. *International economic review*, 568-598.
- [6] Alan, S., & Ertac, S. (2015). Patience, self-control and the demand for commitment: Evidence from a large-scale field experiment. *Journal of Economic Behavior & Organization*, 115, 111-122.
- [7] Andreoni, J., & Sprenger, C. (2012a). Estimating time preferences from convex budgets. *American Economic Review*, 102(7), 3333-56.
- [8] Andreoni, J., & Sprenger, C. (2012b). Risk preferences are not time preferences. *American Economic Review*, 102(7), 3357-76.
- [9] Angeletos, G. M., Laibson, D., Repetto, A., Tobacman, J., & Weinberg, S. (2001). The hyperbolic consumption model: Calibration, simulation, and empirical evaluation. *Journal of Economic perspectives*, 15(3), 47-68.
- [10] Arcidiacono, P., & Miller, R. A. (2008). CCP estimation of dynamic discrete choice models with unobserved heterogeneity. *Tepper School of Business*, 319.
- [11] Atalay, K., Bakhtiar, F., Cheung, S., & Slonim, R. (2014). Savings and prize-linked savings accounts. *Journal of Economic Behavior & Organization*, 107, 86-106.
- [12] Augenblick, N. (2017). Short-Term Time Discounting of Unpleasant Tasks. Working Paper, UC Berkeley.
- [13] Augenblick, N., Niederle, M., & Sprenger, C. (2015). Working over time: Dynamic inconsistency in real effort tasks. *The Quarterly Journal of Economics*, 130(3), 1067-1115.



- [14] Aycinena, D., de Economia Experimental, C. V. S., Blazsek, S., Rentschler, L., & Sandoval, B. (2015). Risk, discounting and demand for intra-household control for recipients of conditional cash transfers.
- [15] Aycinena, D., & Rentschler, L. (2018). Discounting and digit ratio: Low 2D: 4D predicts patience for a sample of females. *Frontiers in behavioral neuroscience*, 11, 257.
- [16] Balakrishnan, U., Haushofer, J., & Jakiela, P. (2017). How soon is now? Evidence of present bias from convex time budget experiments (No. w23558). National Bureau of Economic Research.
- [17] Benhabib, J., Bisin, A., & Schotter, A. (2010). Present-bias, quasi-hyperbolic discounting, and fixed costs. *Games and Economic Behavior*, 69(2), 205-223.
- [18] Bradford, W. D., Dolan, P., & Galizzi, M. M. (2013). Looking ahead: subjective time perception and individual time discounting. Available at SSRN 2369916.
- [19] Brocas, I., Carrillo, J. D., & Tarrasó, J. (2018). How long is a minute?. *Games and Economic Behavior*, 111, 305-322.
- [20] Carvalho, L. S., Meier, S., & Wang, S. W. (2016). Poverty and economic decision-making: Evidence from changes in financial resources at payday. *American economic review*, 106(2), 260-84.
- [21] Cheung, S. L. (2015). Risk preferences are not time preferences: on the elicitation of time preference under conditions of risk: comment. *American Economic Review*, 105(7), 2242-60.
- [22] Corbett, C. H. (2016). "Preferences for Effort and Their Applications," Ph.D. thesis, University of Oregon.
- [23] DellaVigna, S. (2018). Structural behavioral economics. In *Handbook of Behavioral Economics: Applications and Foundations 1* (Vol. 1, pp. 613-723). North-Holland.
- [24] Diamond, P., & Köszegi, B. (2003). Quasi-hyperbolic discounting and retirement. *Journal of Public Economics*, 87(9-10), 1839-1872.
- [25] dos Santos, L. S., & Martinez, A. S. (2018). Inconsistency and subjective time dilation perception in intertemporal decision making. *Frontiers in Applied Mathematics and Statistics*, 4, 54.
- [26] Dorsey, D. (2017). Future-Bias: A (Qualified) Defense. *Pacific Philosophical Quarterly*, 98, 351-373.
- [27] Dougherty, T. (2015). Future-bias and practical reason. Ann Arbor, MI: Michigan Publishing, University of Michigan Library.

- [28] Ebert, J. E., & Prelec, D. (2007). The fragility of time: Time-insensitivity and valuation of the near and far future. *Management science*, 53(9), 1423-1438.
- [29] Echenique, F., Imai, T., & Saito, K. (2019). Testable Implications of Models of Intertemporal Choice: Exponential Discounting and Its Generalizations.
- [30] Kuhn, M. A., Kuhn, P., & Villeval, M. C. (2017). Decision-environment effects on intertemporal financial choices: How relevant are resource-depletion models?. *Journal of Economic Behavior & Organization*, 137, 72-89.
- [31] Ericson, K. M., & Laibson, D. (2019). Intertemporal choice. In *Handbook of Behavioral Economics: Applications and Foundations 1* (Vol. 2, pp. 1-67). North-Holland.
- [32] Fang, H., & Wang, Y. (2015). Estimating dynamic discrete choice models with hyperbolic discounting, with an application to mammography decisions. *International Economic Review*, 56(2), 565-596.
- [33] Finke, M. S., & Huston, S. J. (2013). Time preference and the importance of saving for retirement. *Journal of Economic Behavior & Organization*, 89, 23-34.
- [34] Fisher, I. (1930). *Theory of interest: as determined by impatience to spend income and opportunity to invest it*. Augustusm Kelly Publishers, Clifton.
- [35] Frederick, S., Loewenstein, G., & O'donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of economic literature*, 40(2), 351-401.
- [36] Fuchs, V. R. (1980). Time preference and health: an exploratory study (No. w0539). National Bureau of Economic Research.
- [37] Giné, X., Goldberg, J., Silverman, D., & Yang, D. (2012). Revising commitments: Field evidence on the adjustment of prior choices. The World Bank.
- [38] Glimcher, P. W., Kable, J., & Louie, K. (2007). Neuroeconomic studies of impulsivity: Now or just as soon as possible?. *American Economic Review*, 97(2), 142-147.
- [39] Greene, P., & Sullivan, M. (2015). Against time bias. *Ethics*, 125(4), 947-970.
- [40] Harvey, C. M. (1986). Value functions for infinite-period planning. *Management science*, 32(9), 1123-1139.
- [41] Hoel, J. B., Schwab, B., & Hoddinott, J. (2016). Self-control exertion and the expression of time preference: Experimental results from Ethiopia. *Journal of Economic Psychology*, 52, 136-146.

- [42] Imai, T., & Camerer, C. F. (2018). Estimating Time Preferences from Budget Set Choices Using Optimal Adaptive Design.
- [43] Imai, T., Rutter, T. A., & Camerer, C. (2019). Meta-Analysis of Present-Bias Estimation using Convex Time Budgets.
- [44] Jackson, M. O., & Yariv, L. (2014). Present bias and collective dynamic choice in the lab. *American Economic Review*, 104(12), 4184-4204.
- [45] Kirby, K. N. (1997). Bidding on the future: Evidence against normative discounting of delayed rewards. *Journal of Experimental Psychology: General*, 126(1), 54.
- [46] Kuhn, M. A., Kuhn, P., & Villeval, M. C. (2017). Decision-environment effects on intertemporal financial choices: How relevant are resource-depletion models?. *Journal of Economic Behavior & Organization*, 137, 72-89.
- [47] Koopmans, T. C. (1960). Stationary ordinal utility and impatience. *Econometrica: Journal of the Econometric Society*, 287-309.
- [48] Laibson, D. (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, 112(2), 443-478.
- [49] Laibson, D., Repetto, A., & Tobacman, J. (2007). Estimating discount functions with consumption choices over the lifecycle (No. w13314). National Bureau of Economic Research.
- [50] Liu, E. M., Meng, J., & Wang, J. T. Y. (2014). Confucianism and preferences: Evidence from lab experiments in Taiwan and China. *Journal of Economic Behavior & Organization*, 104, 106-122.
- [51] Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *The Quarterly Journal of Economics*, 107(2), 573-597.
- [52] Luhrmann, M., Serra-Garcia, M., & Winter, J. (2018). The impact of financial education on adolescents' intertemporal choices. *American Economic Journal: Economic Policy*, 10(3), 309-32.
- [53] Mahajan, A., & Tarozi, A. (2012). Time inconsistency, expectations and technology adoption: The case of insecticide treated nets.
- [54] Mazur, J. E. (1987). An adjusting procedure for studying delayed reinforcement. *Commons, ML.; Mazur, JE.; Nevin, JA*, 55-73.

- [55] McClure, S. M., Ericson, K. M., Laibson, D. I., Loewenstein, G., & Cohen, J. D. (2007). Time discounting for primary rewards. *Journal of neuroscience*, 27(21), 5796-5804.
- [56] Montiel Olea, J. L., & Strzalecki, T. (2014). Axiomatization and measurement of quasi-hyperbolic discounting. *The Quarterly Journal of Economics*, 129(3), 1449-1499.
- [57] North, D. C. (1994). Economic performance through time. *The American economic review*, 84(3), 359-368.
- [58] O'Donoghue, T., & Rabin, M. (1999a). Doing it now or later. *American economic review*, 89(1), 103-124.
- [59] O'Donoghue, T., & Rabin, M. (1999b). Incentives for procrastinators. *The Quarterly Journal of Economics*, 114(3), 769-816.
- [60] O'Donoghue, T., & Rabin, M. (2001). Choice and procrastination. *The Quarterly Journal of Economics*, 116(1), 121-160.
- [61] Phelps, E. S., & Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth. *The Review of Economic Studies*, 35(2), 185-199.
- [62] Rabin, M., & Weizsäcker, G. (2009). Narrow bracketing and dominated choices. *American Economic Review*, 99(4), 1508-43.
- [63] Salanié, F., & Treich, N. (2006). Over-savings and hyperbolic discounting. *European Economic Review*, 50(6), 1557-1570.
- [64] Samuelson, P. A. (1937). A note on measurement of utility. *The review of economic studies*, 4(2), 155-161.
- [65] Shu, S. B. (2008). Future-biased search: the quest for the ideal. *Journal of Behavioral Decision Making*, 21(4), 352-377.
- [66] Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *The review of economic studies*, 23(3), 165-180.
- [67] Takeuchi, K. (2011). Non-parametric test of time consistency: Present bias and future bias. *Games and Economic Behavior*, 71(2), 456-478.
- [68] Takeuchi, K. (2012). Time discounting: the concavity of time discount function: an experimental study. *Journal of Behavioral Economics and Finance*, 5, 2-9.

- [69] Uzawa, H. (1969). Time preference and the Penrose effect in a two-class model of economic growth. *Journal of Political Economy*, 77(4, Part 2), 628-652.
- [70] Varian, H. R. (1982). The nonparametric approach to demand analysis. *Econometrica: Journal of the Econometric Society*, 945-973.
- [71] Varian, H. R. (1991). Goodness-of-fit for revealed preference tests. Ann Arbor: Department of Economics, University of Michigan.
- [72] Zauberma, G., Kim, B. K., Malkoc, S. A., & Bettman, J. R. (2009). Discounting time and time discounting: Subjective time perception and intertemporal preferences. *Journal of Marketing Research*, 46(4), 543-556.
- [73] Zhang, L. (2013). Saving and retirement behavior under quasi-hyperbolic discounting. *Journal of Economics*, 109(1), 57-71.

## Appendix A Misspecification with the Secondary Models

We also provide some institutions of the consequence of misspecified secondary present models for the primary present behaviors. Consider a subject's three-period discounting function exhibit the following secondarily bias pattern  $D_\tau(t_0, t_1, t_2) = (\delta^{t_0}, \beta\delta^{t_1}, \beta\delta^{t_2})$ . Nevertheless, we specified a secondary present model with a sequence of time-dependent discount factors  $\hat{D}_{\tau'}(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$ .

With a similar approach in (9) and (8), we can derive  $\hat{\delta}$  in (14), and  $\hat{\beta}$  in (15). For simplicity, we consider  $\delta^{t_0} = 1$ .

$$\frac{\hat{\delta}^{t_1}}{\hat{\delta}^{t_0}} = \frac{\beta\delta^{t_1}}{\delta^{t_0}} \implies \hat{\delta} = \beta^{\frac{1}{t_1-t_0}} \delta \implies \begin{cases} \hat{\delta} > \delta, & \text{if } \beta > 1 \\ \hat{\delta} < \delta, & \text{if } \beta < 1 \end{cases} \quad (14)$$

$$\frac{\hat{\beta}\hat{\delta}^{t_2}}{\hat{\delta}^{t_1}} = \frac{\beta\delta^{t_2}}{\beta\delta^{t_1}} \implies \hat{\beta} = \frac{\delta^{t_2-t_1}}{\hat{\delta}^{t_2-t_1}} \implies \begin{cases} \hat{\beta} < 1 < \beta, & \text{if } \beta > 1 \\ \hat{\beta} > 1 > \beta, & \text{if } \beta < 1 \end{cases} \quad (15)$$

The equations in (8), (9), (14) and (15) shows that the **Primary Present Bias** and the **Secondary Future Bias** create very similar trends (sign) when pushing  $\tau$  to later positions in the empirical models. The **Primary Future Bias** and the **Secondary Present Bias** tends to have the same character.

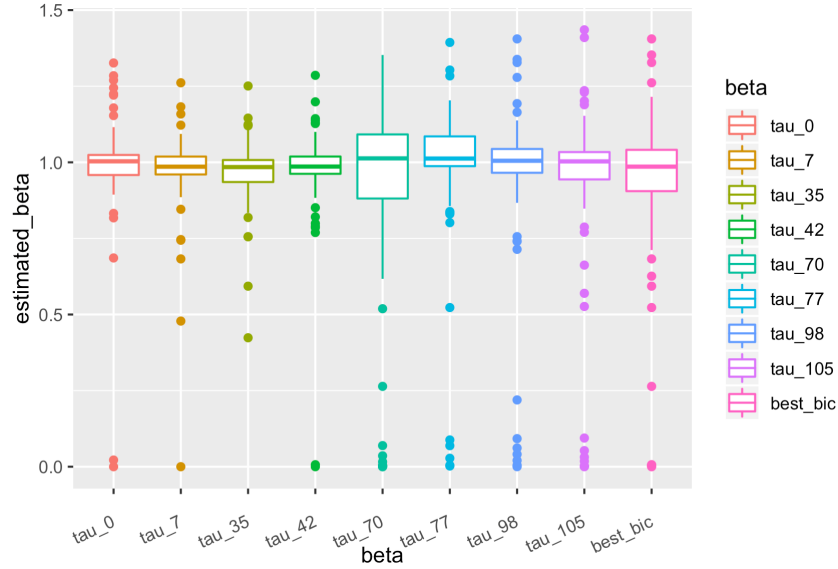
## Appendix B AP12 Results

Table 9: Aggregate Level Estimation for 97 Subjects (AP12)

Model	$\tau_0$	$\tau_7$	$\tau_{35}$	$\tau_{42}$	$\tau_{70}$	$\tau_{77}$	$\tau_{98}$	$\tau_{105}$	$\tau_{133}$
$\hat{\beta}$	1.0090 (0.0084)	0.9965 (0.0074)	0.9864 (0.0115)	0.9801 (0.0124)	0.9864 (0.0118)	1.0220 (0.0104)	1.0200 (0.0098)	0.9991 (0.0143)	
$\hat{\delta}$	0.9990 (0.0001)	0.9990 (0.0001)	0.9992 (0.0002)	0.9992 (0.0002)	0.9991 (0.0001)	0.9988 (0.0001)	0.9989 (0.0001)	0.9990 (0.0001)	0.9990 (0.0001)
$1 - \hat{\eta}$	0.8737 (0.0101)	0.8750 (0.0098)	0.8760 (0.0097)	0.8743 (0.0099)	0.8725 (0.0105)	0.8762 (0.0096)	0.8754 (0.0097)	0.8746 (0.0099)	0.8746 (0.0099)
AIC	43351.08	43352.02	43350.86	43349.66	43350.88	43347.48	43348.17	43352.23	43350.24
BIC	43382.98	43383.93	43382.76	43381.57	43382.79	43379.38	43380.08	43384.14	43375.76
RSS	5244675	5245806	5244408	5242974	5244441	5240351	5241182	5246063	5246068
Observations	97	97	97	97	97	97	97	97	97

The change in point estimations of  $\beta$  in AP12 is mild compare to CMW 16 (between 0.9801 and 1.0220 in Table 9). This finding consistent with Echenique et al. (2019), which finds around 51% (49 out of 97) of that subjects passes the Strong Axiom of Revealed Exponentially Discounted Utility (SAR-EDU) test. Their results indicate the overall revealed preference either likely to exhibit time consistency behavior or mild time inconsistency in the imperial estimations.

Figure 6: The Box-plots of  $\beta_{\tau}$ s and  $\beta$  base on best BIC (AP12)



The over distribution is shifted to the left for the best BIC  $\beta$ s compare the conventional setup  $\beta_{\tau_0}$  with on-sided Mann–Whitney U test’s p-value of 0.0181 (alternative greater than).

## Appendix C The Hyperbolic Discounting Model

Consider a Hyperbolic Discounting function  $D(t_i) = \frac{1}{1+(\kappa t_i)}$ , then the CRRA utility of the CTB takes the functional from in (16).

$$U_0(x_{t_i}, x_{t_i+k}) = \frac{1}{1+\kappa(t_i)} \frac{1}{1-\eta} (x_{t_i} + \omega)^{1-\eta} + \frac{1}{1+\kappa(t_i+k)} \frac{1}{1-\eta} (x_{t_i+k} + \omega)^{1-\eta} \quad (16)$$

Then the optimal demand function of sooner consumption would be (17) accordingly.

$$x_{t_i}^* = \frac{(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}} - \frac{1}{1+\kappa(t_i)}}{\frac{1}{1+\kappa(t_i)} + (1+r)(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}(\omega) + \frac{(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}{\frac{1}{1+\kappa(t_i)} + (1+r)(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}(m) \quad (17)$$

Table 10: Aggregate Level Estimations for Hyperbolic Discounting Model (CMW16)

$\hat{\kappa}$	0.0028 (0.0002)	0.0030 (0.0002)	0.0035 (0.0002)
$1 - \hat{\eta}$	0.7274 (0.2317)	0.8416 (0.0872)	0.8796 (0.0511)
<b>AIC</b>	165440.7	71272.12	35729.4
<b>BIC</b>	165470.5	71298.52	35753.09
<b>RSS</b>	327649101	161396915	67500389
<b>Observations</b>	1061	452	230

As a result, the performance (measured with AIC, BIC, and RSS) of the HD model is better than the ED model for all three data sets, which suggests the overall behavior is in favor of the inconsistent dynamic models. The departure between the HD model and the QHD model provides some evidence against the immediate present bias behavior.