Group Size Effect in Sequential Multi-battle Contests: An Experimental Study

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Abstract

This paper presents the result of a laboratory experiment that investigates the group size effect in the sequential multi-battle contest. We first generalized Konrad and Kovenock's (2009) result to multi-player ($n \geq 2$). Then we conducted an n-player ($n = 3$ or 6) multi-battle experiment to compare the all-pay auction and the Tullock lottery side by side. When the number of contestants is increased in a sequential multi-battle contest, we get the following result: First, in battle 1, the boundary distribution is distorted in the all-pay auction, but only the no entry rate is increased in the Tullock lottery. Second, when the players are not in the lead, the dropout rate is higher with more players in the game. Third, even though the total number of battles has not increased significantly due to the increased number of players, more players adopt the dormant-reenter strategy. Finally, when the number of contestants increases, so does the winner effort, the maximum battle effort, and the total contest effort. In general, the theory in the Tullock lottery provides a more accurate prediction than in the all-pay auction.

JEL Classification: C72; C73; C91; C92; D44; D72.

Keywords: Group Size; Sequential Contest; All-pay Auction; Tullock Lottery; Experiments.

1 Introduction

The number of contestants in many sequential multi-battle competitions is not limited to two. For instance, the primary election in the United States, auto racing (Formula One and NASCAR), multisport races (triathlon and biathlon), and combined track and field events (decathlon and heptathlon) are all considered sequential multi-battle contests with more than two players. Since many competitions have more than two contestants, the group size of a game becomes an essential component in research.

It is common to consider a multi-battle contest as dynamic or sequential. In early works, the structure of sequential multi-battle contest is applied to study R&D competition. Fudenberg et al. (1983) study a non-constant sum multi-battle contest of two firms. In Tullock (1980, 2001), two identical players simultaneously expand efforts in a one-shot contest. The winner of the contest is determined probabilistically. In Harris and Vickers (1985), two firms sequentially expend effort to decrease their continuous distance to the finish line. The continuous distance to the finish line becomes a series of battles in Harris and Vickers (1987). Klumpp and Polborn (2006) generalized two-player sequential multi-battle Tullock lottery contest results with different sensitivity parameters ($r \leq 1$). The two-players Tullock multi-battle contest's result is extended into a multi-player race ($n > 2$) in Doğan et al., 2018. Regarding another most commonly used contest success function (CSF) of the Colonel Blotto game, Baye et al. (1996) employed an all-pay auction in the one-shot contest with multi-player ($n \geq 2$) and asymmetrical valuation. Konrad and Kovenock (2009) developed the all-pay auction (Baye et al. 1996) into a multi-battle contest with two players. Deck and Sheremeta (2012) use a special case in Konrad and Kovenock (2009) to examine a two-payer game of the weakest-link system. Gelder (2014) introduced a prenatal structural into the two-player all-pay auction sequential multi-battle contest. In the theory portion of this paper, we generalized the result of the multi-battle auction contest into $n (n \geq 2)$ players.

The number of players is critical in contest research. In theory, the effect of the group size depends on the structure of a contest. The expected effort decreases as group size increases in the one-shot all-pay or Tullock lottery contest (Konrad, 2009). On the other hand, Gerchak and He

(2003) point out that the noise distribution affects the expected effort in a rank-order tournament.

Table 1: Group Size Experiments

Year	Author(s)	Contest type	Players n	Contests
2003	Harbring and Irlenbusch	All-pay auction	2, 3, 6	20
2004	Orrison, Schotter, and Weigelt	Tournaments	2, 4, 6	20
2006	Gneezy and Smorodinsky	All-pay auction	4, 8, 12	10
2011	Sheremeta	Tullock lottery	2, 4	60
2012	Morgan, Orzen, and Sefton	Tullock lottery	2, 3, 4, 5	50
2013	Ernst and Thöni	All-pay auction	2, 3	10
2014	Lim, Matros, and Turocy	Tullock lottery	2, 4, 9	10
2020	List, Van Soest, Stoop, and Zhou	Tournaments	2, 4, 8	4
2021	Hudja	Innovation	2, 4	30
2021	Fallucchi, Niederreiter, and Riccaboni	Tullock lottery	3, 5	60
2021	Peeters, Rao, and Wolk	Proportional-prize	2, 3, 4	20

Many studies have been conducted to see how group size affects behavior in experiments. We list some of the experimental contest researches that focus on the group size effect perspective in Table 1. In Tullock contests, Sheremeta (2011) and Morgan et al. (2012) find the expected effort decrease as the number of players increase. However, Lim et al. (2014) find the expected effort is not sensitive to the group size. Fallucchi et al. (2021) discovered the fraction of zero effort increases as the group size of a contest increases. In all-pay auctions, Gneezy and Smorodinsky (2006) find when there is more player in the game, the average bids decrease. Nevertheless, Harbring and Irlenbusch (2003) point out that a weak increase in the mean bid increases the group size in the contest. In the tournament contest, Orrison et al. (2004) find that the mean effort is n-invariant if the noise is uniformly distributed. Unlike the finding in Orrison et al. (2004), List et al. (2010) find mean effort decreases when contestant's number increases. Ernst and Thöni's analysis (2013) indicates that the participants' behavior becomes closer to theory prediction for larger group size. When considering the proportional-prize rule in the contest, Petters et al. (2021) observe that the mechanism of common knowledge of realization probabilities operates better with a larger group size. Lastly, Hudija (2021) detects that increasing player numbers result in more innovations in an innovation contest; meanwhile, individual innovation attempts decrease when enlarging the group size. Generally, all the existing experimental contest studies the group size effect in a static framework because of the winning threshold[1]is always $T = 1$ (including Hudija, 2021).

[1]The winning threshold T of a contest indicates the required number of battles for a player to claim victory in a game. For example, the winning threshold of a two-player best-of-three game is $T = 2$.

Table 2: Multi-battle Experiments

Year	Author(s)	Contest type	Players n	Threshold T	Contests
2002	Zizzo	Tullock lottery	2	10	2
2013	Mago, Sheremeta, and Yates	Tullock lottery	2	2	20
2017	Mago and Sheremeta	Tullock lottery	2	2	12
2017	Gelder and Kovenock	All-pay auction	2	4	20
2019	Mago and Sheremeta	All-pay auction	2	2	12
2019	Mago and Razzolini	Tullock lottery	2	3	15

Table 2 presents the laboratory research on sequential multi-battle contests. The first experiment to study the sequential Tullock multi-battle contest is Zizzo (2002). When the winning threshold is large ($T = 10$), they find the leaders do not bid significantly more than the follower; additionally, the followers are not necessarily dropped out from the race when the gap increases. Mago et al. (2013) find evidence supporting the strategic momentum, in which the winner of battle 1 bids more than the loser when $T = 2$. They also discovered that the inclusion of the intermediate prize and reduction of the role of luck (increase r) led to greater Tullock contest effort. Mago and Razzolini (2019) show that women exert more effort when their opponents are women for a $T = 3$ Tullock experiment. Sequential multi-battle contests ($T = 2$) create higher spending when compared to simultaneous multi-battle contests (Mago and Sheremeta 2019). A limited number of experiments study sequential multi-battle contests using the all-pay auction as the CSF. Gelder and Kovenock (2017) explored the escalation of conflict effort with losing penalty in sequential all-pay auction contest ($T = 4$). Mago and Sheremeta (2017) discussed the rurally studied underbidding behavior in the first battle of a sequential all-pay multi-battle contest ($T = 2$). All current sequential multi-battle experiments study the game with two players' interactions ($n = 2$).

Lastly, we look at laboratory experiments that directly compare the contest success function. When comparing the all-pay auction to the Tullock lottery in the one-shot contest, Davis and Reilly (1998) and Potters et al. (1998) demonstrate that the subjects bid more in the all-pay auction. Mago et al. (2013) illustrate that increasing the sensitivity parameter r results in higher bids in the Tullock contest. Suppose we consider the proportional-prize rule as a variant of contest success function; Cason et al. (2010) provide evidence suggesting proportional-prize generates more effort than the all-pay auction. The Tullock lottery contests generate more effort than the proportional-prize rule Cason et al. (2020).

This paper contributes to the existing literature in the following three dimensions. First, we generalize the theory result of sequential all-lay auction multi-battle contest from Konrad and Kovenock 2009 to n ($n \geq 2$) players. Second, we extend the laboratory contest experiment in group size to a multi-battle setup ($T > 1$). Third, we provide a side-by-side compression between the all-pay auction ($r = \infty$) and the Tullock lottery ($r = 1$) in the sequential multi-battle contest.

2 Theory and Hypotheses

2.1 Theoretical Model

We study sequential multi-battle contests with n players. Consider a set of risk-nurture players $N = \{1, 2...n\}$ (where $n \geq 2$) compete for a prize in a series of sequential battles.

Let V denote the prize of winning the contest, and V_i denote the valuation of V for player i, for every $i \in N$. We assume that $V_1 \geq V_2 ... \geq V_n$, and V_i is publicly known to all players[2].

To win a contest, the player i has to be the first one in n players who wins T battles ($T \geq 2$), and we call T a winning threshold. We take $T = 2$ to meet the minimum requirements of a multi-battle contest in our experimental design.

We use a battle node t to specify each player i's current state $(s_{1t}, s_{2t}...s_{nt})$, where $t = \{1, 2...m...n+1\}$ and $s_{it} = 0$ *or* 1. More explicitly, $s_{it} = 1$ denotes player i has won a battle at current battle node t, and $s_{it} = 0$ denotes player i has not won any battle at current battle node t. Therefore $\sum_{i=1}^{n} s_{it} = t - 1$. For simplicity, we call battle node t battle t, and the contest is terminated at battle node m.

Here we denote m as the terminal battle node, and we have $T \leq m \leq n(T-1)+1$. Suppose $T = 2$, and we have $2 \leq m \leq n+1$.

At each battle t, player i choose a costly effort $e_{it} \in [0, V]$ to expend [3]in order to win the battle node t.

[2]For the design purpose of the experiment, we consider $V = V_1 = V_2 ... = V_n$. In theory, we generalized the multi-battle all-pay auction contest result with asymmetrical valuations.

Consider the cost of the effort takes a linear functional form (Klumpp and Polborn, 2006; Konrad and Kovenock, 2009; Gelder, 2014; Doğan et al., 2018):

$$c(e_{it}) = e_{it}$$

To determine player i's probability of winning battle t, we applied the following contest success function (CSF):

$$p_{it}(e^r_{it}, e^r_{-it}) = \frac{e^r_{it}}{\sum_{j=1}^{n} e^r_{jt}}$$

where r is the sensitivity parameter that measures how individual i's winning probability is affected by their effort[4]. We consider the two extreme cases: the all-pay auction when $r = \infty$ (Hillman and Samet, 1987; Hirshleifer and Riley, 1978; Nalebuff and Stiglitz, 1982; Hillman and Riley, 1989; Baye et al. 1996) and the Tullock lottery when $r = 1$ (Tullock 1980, 2001).

The payoff for player i is:

$$\pi_i = \begin{cases} V - \sum_{t=1}^{m} e_{it} & \text{if player } i \text{ wins the contest} \\ -\sum_{t=1}^{m} e_{it} & \text{otherwise} \end{cases}$$

2.2 Theoretical Result

Theorem. Given a well-defined terminal battle node m, any battle node t that arises in a sequential all-pay auction multi-battle contest can be a standard all-pay auction contest.

In a sequential all-pay auction contest, we denote a player i's expected payoff at a battle node t as $v_{it} = v_i(s_{1t}, s_{2t}...s_{nt})$. Let $(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$, for $k \in N$, denote a path from the battle node $(s_{1t}...s_{kt}...s_{nt})$ to the battle node $(s_{1t}...s_{kt}+1...s_{nt})$. Let $p_t(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$ denote the probability that the path $(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$ is realized. Then $p_t(s_{1t}...s^{\downarrow}_{kt}...s_{nt}) = 0$ implies that the probability of player k winning battle node t is zero. Denote $t_{k\downarrow}$ as the consequence node of player k wins battle node t. For simplicity, we denote $p_t(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$ as $p_{t_{k\downarrow}}$.

[3] In the experiments, participants bid a whole number between 0 and 1000, and the minimum positive bid is 1 token.

[4] When $\sum_{j=1}^{n} e^r_{jt} = 0$, we consider $p_{it} = \frac{1}{n}$.

Lemma. Given a well-defined terminal battle node m, with only one player having a positive value v_{it}, at most, one player has a positive value v_{it} in each node t.

Proof of Lemma

We assume that there are two players in a battle node $t = (s_{1t}, s_{2t} ... s_{nt})$ with positive expected payoffs. We denote these two payers as player i and player j, for $i, j \in N$ and $i \neq j$. Suppose $t_{i\downarrow} = (s_{1t} ... s_{it}+1 ... s_{nt})$ and $t_{j\downarrow} = (s_{1t} ... s_{jt}+1 ... s_{nt})$, we assume that $v_{it_{i\downarrow}} > v_{jt_{j\downarrow}}$. In any equilibrium, we have $p_{t_{j\downarrow}} \cdot v_{jt_{j\downarrow}} - e_{jt} = 0$. Therefore at least one path with a positive possibility that leads player j to obtain a positive expected payoff must exist. We assume that path $(s_{1t} ... s^{\downarrow}_{kt} ... s_{nt})$ satisfies this condition. Then we have $v_{kt_{k\downarrow}} > 0$ and $v_{jt_{k\downarrow}} > 0$. Continuing the process, we have at least two players with positive values in terminal battle node m, which is a contradiction to the assumption of the terminal battle node. Therefore we have proved the Lemma. Using the **Lemma**, we can prove that the players who lose in a single battle will have a zero expected payoff. Therefore the **Theorem** is proved.

From the **Theorem** of multi-battle auction contest[5], we can see in battle 1; each player bids an expected effort of $\frac{V}{n}$. For battle 2, all players except the winner of battle 1 bid 0. The winner of battle 1 bids the minimum effort allowed to secure the contest-winning. For any post-battle 2 ($t > 2$), the players who have the current state s=1 will bid an expected effort of $\frac{V}{t-1}$. All other players drop out from the contest when $t > 2$.

As a comparison, in a multi-battle Tullock lottery contest (Doğan et al., 2018)[6], each player bids $\frac{(n-1)(21n^2-24n+6)V}{4n^2(3n-2)^2}$ in battle 1. In battle 2, the winner of battle 1 bids $(2n-1)\frac{(3n-3)V}{4(3n-2)^2}$, and the players, who lost battle 1, bid $\frac{(3n-3)V}{4(3n-2)^2}$. For all t ($t > 2$, post-battle 2), the players, who have a current $s = 1$, bid $\frac{(t-2)V}{(t-1)^2}$; otherwise, players drop out from the contest.

The expected total effort in a multi-battle all-pay auction is V, which is n-invariant. On the contrary, the expected total effort in a multi-battle Tullock lottery contest is $\frac{(12n^3-35n^2+20n-3)V}{2n(3n-2)^2}$, which is increasing as n increases and less than V.

[5] In the unique symmetric Nash equilibrium, players' bids follow the CDF of $\frac{e}{V}^{\frac{1}{n-1}}$ in battle 1 and the leading players' bids follow the CDF of $\frac{e}{V}^{\frac{1}{t-2}}$ in post-battle 2.

[6] The results come from Doğan et al. (2018). **Proposition 1** and **Proposition 2** when $T = 2$ and $n \geq 2$.

2.3 Predictions and Hypotheses

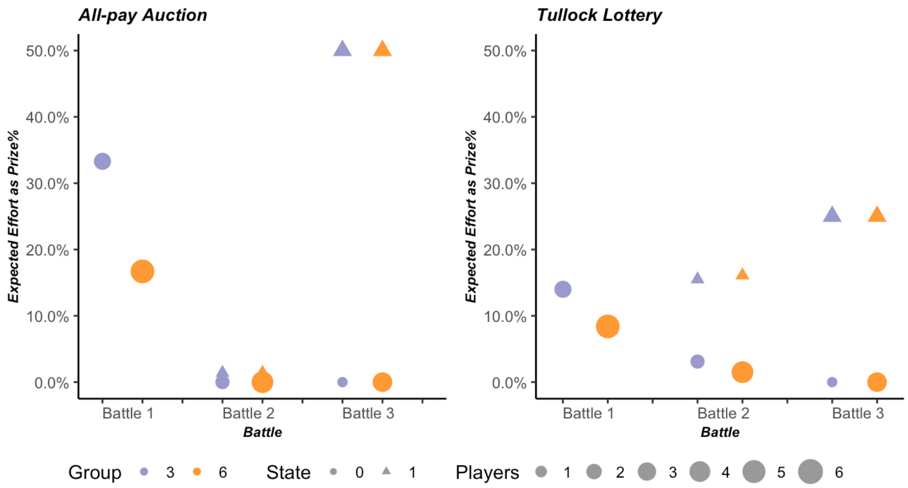

Figure 1: Predictions of Effort as Proportion of the Prize, when $n = 3$ and 6.

We design an experiment to examine the group size effect in a multi-battle contest. Meanwhile, we will detect how CSFs affect the group size effect in a multi-battle setup. Figure 1 displays the theoretical prediction when n takes 3 and 6 in the all-pay auction and the Tullock lottery multi-battle contests. The statistical analysis will test the following six hypotheses based on the theoretical predictions:

Hypothesis 1. In battle 1, the expected effort decreases as player number n increases of both all-pay auction and Tullock lottery.

Hypothesis 2. In battle 2, there is no group size effect in the all-pay auction contest. In battle 2 of the Tullock lottery, the effort increases as n increases when $s = 1$ but decreases as n increase when $s = 0$.

Hypothesis 3. In any post-battle 2, there is no group size effect for all-pay auction and Tullock lottery regardless of the current state.

Hypothesis 4. For both CSFs in post-battle 1, players with leading positions bid more.

Hypothesis 5. Only when in battle 1 or post-battle 2 and s=1 players bid more in the all-pay auction than in the Tullock lottery.

Hypothesis 6. To win a contest, the winner of a game exerts more effort when there are fewer contestants. The direction holds for both CSFs.

Hypothesis 7. The expected total effort in the all-pay auction is n-invariant. On the contrary, the expected total effort in the Tullock lottery increases when n increases.

3 Experiment Design and Procedures

We designed the experiments to study the group size effect in the multi-battle contests. The goal of the experiment is to compare strategic behaviors when the group size is different, further, to compare the difference in the size effect when the CSFs are different. We employed a 2 (n=3 and n=6) by 2 (all-pay auction and Tullock lottery) design for the experiment. We recruited 120 subjects to participate in 12 sessions (between 6 to 12 subjects) in the Virginia Tech Econ Lab. All subjects are students, staff, and faculty at Virginia Tech.

For each session, we randomly selected a fixed group size treatment in the contest, where n = 3 or 6. Participants played 12 all-pay auction multi-battle contests and 12 Tullock lottery multi-battle contests as phase one of the experimental session. The groups were anonymously reassigned after each contest, and subjects did not know their opponents in a game. We randomized the order in which the CSF of the games is played first in an experimental session. After reading the instructions for the games, subjects started to play the 24 contests (12 for both CSFs). Through the contest, players decide a bid of a whole number between 0 and 1000 tokens on a computer through an oTree program (Chen et al. 2016). The player who wins two battles first wins a prize of 1000 tokens for that contest. One token is worth $0.01 as the result of the experiment.

In phase two of the experiment, we elicited participants' preferences toward risk, ambiguity, and loss aversion with a modified survey (Appendix A) from Shupp et al. (2013). Participants chose their preferred option between A and B for each lottery. There are a set of 12 lotteries for each of the three preference elicitations. Lastly, subjects fill out a socio-economic survey before reviewing their experiment earnings.

At the conclusion of the experiment, we paid subjects in cash, which $7 as compensation, $12 as the endowment of all-pay contest plus a randomly selected result from 12 contests, another $12 as the endowment of the Tullock contest plus a randomly selected result from 12 contests, and three randomly chosen earnings from 36 risk, ambiguity, loss aversion lotteries. On average, participants earned \$25.75 within around 90 minutes of the experiment.

4 Experimental Results

4.1 General Comparative Statics

Table 3 summarizes the participants' mean effort of each battle at the given states and their expected payoff compared to the theoretical predictions. We find consistent overbidding in the Tullock lottery contest across all battles and states. However, the overbidding only exists in battle 2 in the all-pay auction treatment. This underbidding behavior is consistent with previous multi-battle all-pay auction contest experimental studies (Mago and Sheremeta, 2017). Comparing the payoff across all treatments, it suggests a persistent over-dissipation.[7]

Table 3: Summary Statistics of Prediction vs. Observation.

CSF	**All-pay Auction**				**Tullock Lottery**			
n	$n = 3$		$n = 6$		$n = 3$		$n = 6$	
Expectation	Nash	Actual	Nash	Actual	Nash	Actual	Nash	Actual
Battle 1	334	229.96	167	199.35	140	227.59	84	165.94
Battle 2 ($s = 1$)	1	344.13	1	508.68	154	300.05	162	345.23
Battle 2 ($s = 0$)	0	254.41	0	155.87	31	206.80	15	131.32
Battle 3 ($s = 1$)	500	382.58	500	472.80	250	278.13	250	295.37
Battle 3 ($s = 0$)	0	159.18	0	114.10	0	163.92	0	117.64
Payoff	-0.34	-382.46	-0.17	-453.49	95.24	-282.36	21.65	-325.72

Table 4 presents the cumulative probability of games ending in each corresponding battle. Unlike the theoretical prediction, it shows that an insufficient number of battles ends in 2 battles. We observe a general trend that when the group size increases from 3 to 6, the probability of

[7] We consider a player's behavior as over-dissipation when the total effort surpasses the contest's prize. The definition is adopted from Gneezy and Smorodinsky (2006).

games ending in early battles decreases significantly; however, the difference is not severe when the CSF switches from the all-pay auction to the Tullock lottery. Compared to the multi-battle with 2 players' experimental studies (Mago and Sheremeta, 2017; Mago and Sheremeta, 2019)[8], the decreasing probability of ending in battle 2 is consistent. Table 4 shows no significant support for the theoretical prediction that the all-pay auction contest ends in 2 battles and the Tullock lottery contest ends in 3 battles. The probability of the contest going to battle 3 increases when n increases. The difference in possibilities of games ending in a specific battle is negotiable in different CSFs when fixing the player number n.

Table 4: The Cumulative Probabilities of Games that End in Individual Battle.

Treatment	Battle 2	Battle 3	Battle 4	Battle 5	Battle 6	Battle 7
All-pay ($n = 3$)	45.76%	91.53%	100%			
Tullock ($n = 3$)	44.78%	89.26%	100%			
All-pay ($n = 6$)	35.28%	82.08%	97.92%	100%	100%	100%
Tullock ($n = 6$)	36.47%	75.34%	94.45%	99.34%	99.97%	100%

The probabilities in all-pay auction contests are adjusted for the tie bids.

Table 5 shows the probabilities of the battle 1 winner becoming the contest winner. The probabilities we find in the all-pay auction are significantly lower than the theoretical prediction 100% ($n = 3$ or 6). On the other hand, the observed probabilities in the Tullock lottery 71.11% ($n = 3$) and 68.82% ($n = 6$) are weakly less than the prediction in theory (Doğan et al., 2018). More generally, the decreasing pattern in the experimental observation when increasing the group size and the minor difference when changing CSFs in the contests is consistent with the comparison between our results and the previous research (Mago and Sheremeta 2017, 2019).[9]We also listed the probability of battle 1 winner wins the game suppose the winner of the contest is randomly assigned. We also calculate the probability of battle 1 winner wins the game suppose the winner of the contest is randomly assigned. We see the observed probability in the experiment is sufficiently higher than the randomly assigned winning probability, which indicates evidence for strategic momentum.

Table 6 lists the maximum player's effort, the maximum battle effort, and the maximum player's total effort in a contest. Among these three categories, the two total efforts increase as n increases.

[8]The probabilities of contests ending in battle are 62% and 64%, respectively, for the Mago and Sheremeta (2017 Tullock lottery with 2 players) and Mago and Sheremeta (2019 all-pay auction with 2 players).

[9]The probabilities of battle 1 winner winning the overall contest are 75% and 80%, respectively, for the Mago and Sheremeta (2017 Tullock lottery with 2 players) and Mago and Sheremeta (2019 all-pay auction with 2 players).

Table 5: **The Probabilities of Battle 1 Winner Wins the Game.**

Treatment	All-pay ($n = 3$)	Tullock ($n = 3$)	All-pay ($n = 6$)	Tullock ($n = 6$)
Theory	100.00%	65.32%	100.00%	62.95%
Random	62.96%	62.96%	46.24%	46.24%
Experiment	68.33%	69.06%	60.56%	63.50%

The probabilities in all-pay auction contests are adjusted for the tie bids.

Some size effect studies demonstrated the phenomenon of total efforts increasing as n increases (Sheremeta 2011; Lim et al., 2014; Fallucchi et al., 2021). Meanwhile, contestants exerted more effort as the r increased from 1 to ∞ consistent with previous research (Davis and Reilly 1998; Potters et al., 1998; Cason et al., 2020). Especially intriguing, the average player's best battle effort is maximized in the six-player all-pay auction contest. We will provide the potential explanation in later sections. Table 15 reports the proportions of the corresponding condition that maximize the three categories in Table Table 6. It illustrates that players' battle effort is likely to be maximized when s=1 in the all-pay auction but is likely to be maximized at battle 1 in Tullock. A sufficient amount of battle's total effort is maximized in battle 2. Lastly, more maximum players' total effort comes from the contest winner when decreasing the n and increasing the r.

Table 6: **Average Maximum Effort**

Treatment	All-pay ($n = 3$)	Tullock ($n = 3$)	All-pay ($n = 6$)	Tullock ($n = 6$)
Player's Battle	539.25 (11.20)	429.38 (12.03)	642.25 (17.00)	545.82 (17.91)
Battle's Total	1033.60 (26.20)	826.95 (23.09)	1425.67 (36.90)	1208.19 (32.33)
Player's Total	1073.56 (28.81)	870.53 (27.66)	1362.23 (45.08)	1177.02 (53.21)

Standard errors are reported in the parentheses.

4.2 Behaviors in Battles

Result 1. There is an assertive boundary behavior difference in the all-pay auction and a weak mean difference in the Tullock lottery at battle 1.

From the theoretical prediction, we expect the group size effect in the expected effort in the all-pay auction and the Tullock lottery games. Instead of observing the significant difference in the average battle 1 bids, we find more boundary bids (0 or 1000) in the all-pay auction with 6 players compared to 3 players' games (Figure 2, Figure 6 in Appendix B). In contrast, we find weak evidence of group size effect in mean effort in battle 1 of the Tullock lottery games (Table 7).

Table 7: **Random Effect Panel on Efforts in Different Battles.**

CSF		All-pay Auction			Tullock Lottery		
Effort		Battle 1	Battle 2	Post-battle 2	Battle 1	Battle 2	Post-battle 2
Constant	($n = 3$ & $s = 0$)	289.17	318.59***	223.68***	211.41***	208.88***	177.67***
		(36.44)	(21.34)	(44.99)	(28.86)	(25.67)	(27.59)
Group	($n = 6$ & $s = 0$)	-30.60	-81.88***	-28.44	-61.65*	-69.28***	-39.27
		(29.65)	(26.01)	(39.29)	(34.45)	(27.46)	(28.96)
State	($n = 3$ & $s = 1$)		49.94***	205.97***		67.69***	93.35***
			(18.00)	(29.34)		(18.02)	(12.93)
Group $\times$ **State**	($n = 6$ & $s = 1$)		123.35	65.27		57.88	31.29
			(32.00)	(48.28)		(31.06)	(22.75)
Trend		-9.11***	-7.83***	-8.51**	2.49	0.99	-0.40
		(3.35)	(2.76)	(3.50)	(1.52)	(1.53)	(2.06)
Observations		1440	1440	1071	1440	1440	1068

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level of the constant is the testing result for overbidding compared to the theoretical prediction.
Significant at 10% * Significant at 5% ** Significant at 1% ***

Another noticeable discovery from Figure 2 is that the all-pay auction leads to more dispersion in bids than the Tullock lottery in battle 1, which aligns with the theory that the all-pay auction game has a mixed strategy symmetric equilibrium. The standard deviations for all-pay auction and Tullock lottery are 257.20 and 196.92 in battle 1, respectively.

The **Result 1** of the all-pay auction challenges the theoretical prediction in **Hypothesis 1** because there is a substantial amount maximum bid in battle 1 when $n = 6$. As n increases, the theory yields a more accurate prediction when considering the entire CDF in the all-pay auction. The U-shaped distribution of bids can be obtained in an all-pay auction experiment (Gneezy and Smorodinsky, 2006; Klose and Sheremeta 2012; Dechenaux et al., 2015) but not in a multi-battle contest experiment. We notice that as the number of participants grows, the boundary behaviors on both sides get more intense (Figure 6). The increase in n increases the boundary behavior of multi-battle contests can be generalized beyond our experimental design. Compared to Sheremeta and Mago (2017, $n = 2$), the distribution of effort when $n = 3$ is more distorted in two boundaries. When $n = 6$, we observe more efforts in both boundaries compared to $n = 3$. We checked if the distribution of bids greater than 500 in $n = 6$ is shifted to the right compared to $n = 3$ with the

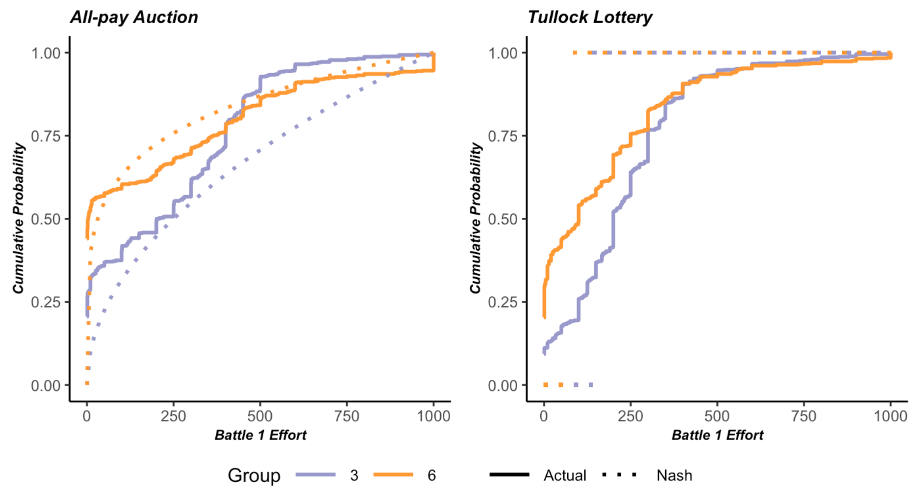

Figure 2: The Cumulative Distribution Function of Effort in Battle 1.

Mann–Whitney U test, and the p-value is sufficiently small. Ernst and Thöni (2013) suggested a loss aversion explanation for this bimodal behavior. In our experiment, the U-shaped distribution is extended from battle 1 to battle 2 and $s = 1$ in the six-player all-pay auction contest (Figure 6). One convincing explanation is that contestants bid more competitively to avoid the contests going to later battles.

Result 2. Average effort changes significantly across group size treatments in battle 2 when current state s=0.

Theory predicts no group size effect in all-pay auction contests in battle 2. On the contrary, there is a group size effect in the Tullock lottery games, and the group size effect exists in both current states ($s = 0$ or 1). Our experiment data indicate a significant difference in mean effort when the group size of the contest is different. When $s = 0$ in battle 2, the average effort in 3 player games is significantly higher than 6 players game (Table 7). We will explore the source of the group size effect in later sessions when decomposing behaviors into different strategies. **Result 2** only provides statistical evidence supporting **Hypothesis 2** in the Tullock contest.

Result 3. Sufficiently decay in group size effect after battle 2.

One advantage of our design is that players can be distinguished by their state in post-battle

2 because $s = 1$ at the battle 3 for the two-player contest (Mago and Sheremeta, 2017; Mago and Sheremeta, 2019). We find in Table 7 that all group size effect dies out for post-battle 2 as predicted, supporting **Hypothesis 3**. As the contest goes to post-battle 2, the discourage effect becomes more severe, and the high dropout rate for the player with $s = 0$ overrides the group size effect.

Result 4. Strong strategic momentum is revealed in all post-battle 1 efforts for both CSFs.

In Table 7, the regression result shows that the effort is significantly higher when the current state $s = 1$ compares to $s = 0$ for all post-battle 1 battles. This lends credence to **Hypothesis 4** Our finding is consistent across all group size treatments and compatible with previous multi-battle contest experiments (Mago et al., 2013; Gelder and Kovenock, 2017; Mago and Sheremeta, 2017; Mago and Sheremeta, 2019). Our study affirms that strategic momentum is group size invariant in multi-battle contest experiments.[10]

4.3 Contest Success Functions

Result 5. Predominantly, players bid more in the all-pay auction game than the Tullock game in most battle-state.

Comparing between CSFs in Different Battle-state, we show the regression result in Table 8 to quantify the effect of the contest success function on the effort for each given battle and state. As a supplement of Table 7, we find a consistent pattern in which participants insert more effort in all-pay than in the Tullock contest. This increase in effort generated by r vanished for post-battle 2 when the $s = 0$. The tendency of more expenditure in the all-pay than the Tullock is robust despite the group size in the contest, which rejects the **Hypothesis 5**. This result extended the discovery from the previous studies of the single-battle contests (Davis and Reilly 1998; Potters et al., 1998; Cason et al., 2020). Apart from the conduct in battle 1, our findings are in line with Mago and Sheremeta's (2017, 2019) two-player research.[11]

[10] The unreported 6-player games' strategic momentum (State) for the all-pay auction and the Tullock lottery are significant at 1%.

Table 8: Random Effect Panel on Contest Success Function

Battle		**Battle 1**	**Battle 2**	**Battle 2**	**Post-battle 2**	**Post-battle 2**
Effort		$s = 0$	$s = 0$	$s = 1$	$s = 0$	$s = 1$
Constant	Tullock & $n = 3$	211.41***	210.37***	273.60***	216.67***	249.34
		(28.86)	(24.35)	(25.87)	(32.10)	(21.66)
CDF	All-pay & $n = 3$	77.77**	107.06***	107.59**	34.92	174.15***
		(36.06)	(21.39)	(42.61)	(44.34)	(45.06)
Group	Tullock & $n = 6$	-61.65*	-69.438**	15.76	-44.23	21.57
		(34.46)	(27.13)	(36.75)	(33.66)	(34.46)
CDF $\times$ **Group**	All-pay & $n = 6$	31.05***	-22.65***	90.12**	-3.41	52.26***
		(27.94)	(25.33)	(54.25)	(37.69)	(32.36)
Trend	Tullock	2.49	0.90	3.68	-5.58**	4.67
		(1.52)	(1.14)	(4.26)	(2.21)	(3.03)
CDF $\times$ **Trend**	All-pay	-11.60***	-8.25***	-10.69	-4.64*	-10.8
		(3.63)	(2.59)	(8.09)	(2.80)	(6.94)
Observations		2,880	2,160	720	1,018	1,121

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level of the Constant is the testing result for overbidding compared to the theoretical prediction.
The reported significance level of the CSF $\times$ Group Size (All-pay & $n = 6$) is the testing result for CSF effect compared to the Group Size (Tullock & $n = 6$).
Significant at 10% * Significant at 5% ** Significant at 1% ***

4.4 Total Effort

From the game organizer's perspective, the total effort is another essential aspect to explore. In particular, we will focus on the total effort of a game, the total effort of the winner of a game, and the average total effort for all individual players across different group size treatments.

Result 6. The winners spends more total effort in 6 players games, despite the fact average individual effort is higher in 3 players games.

In Figure 7 (Appendix C), we see a pattern that the overall average individual effort exhibits an apparent group size effect. Average individual effort is higher for the game with 3 players than the game with 6 players for both CSF treatments. On the other hand, we reach a different conclusion

[11] Mago and Sheremeta (2017, 2019) found players in battle 1 bid less in the all-pay (16.7) than in the Tullock (22.1).

Table 9: Random Effect Panel on Total Efforts for Different Final States.

CSF		All-pay Auction			Tullock Lottery		
Total Effort		Final $s = 2$	Final $s = 1$	Final $s = 0$	Final $s = 2$	Final $s = 1$	Final $s = 0$
Constant	$(n = 3)$	1240.65***	1223.36***	511.12***	759.47***	887.09***	422.59***
		(96.88)	(98.71)	(60.76)	(60.69)	(61.11)	(68.05)
Group	$(n = 6)$	391.23***	383.12***	-53.11	177.24*	111.20	-103.85
		(95.94)	(122.94)	(69.11)	(104.75)	(105.72)	(68.05)
Trend		-33.15***	-32.70**	-16.02***	1.74	-0.52	1.43
		(10.99)	(15.00)	(4.81)	(7.07)	(6.59)	(3.21)
Observations		360	254	826	360	252	828

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level of the constant is the testing result for overbidding compared to the theoretical prediction.
Significant at 10% * Significant at 5% ** Significant at 1% ***

if we consider the player's final state and analyze them separately. The results from Table 9 and Figure 7 show that to win a multi-battle contest, the winner in 6 players' games exerts more effort than the winner in the contest with 3 players. The group size effect on the game-winner average effort is stronger in the all-lay auction game than in the Tullock lottery. **Result 6** provides direct evidence against **Hypothesis 6** from the theoretical prediction.

The previous section points out that most games end within three battles; moreover, the number of games that end in two battles is similar to the number of games that end in three battles. Therefore, we dig into more details about the group size effect in individual effort. In games that end in two or three battles, the winner (potential winner) spends more total effort in a three-player competition than in a six-player competition. Losses bid more in total in 6 players games than 3 players games, regardless of when the game ended (Figure 3). The result is inconsistent with the theory in either CSFs. With bigger group sizes, the rise in winner overall effort is a consequence of increased effort in individual battles.

Result 7. The total expected effort in a game is increasing as n increases.

We tested **Hypothesis 7** and present some observations in Figure 7. When n increases from 3 to 6, we observe the total expected effort increase for all-pay auctions and Tullock lottery contests. In other words, the total expected prize decreases as the player number increases in a game.

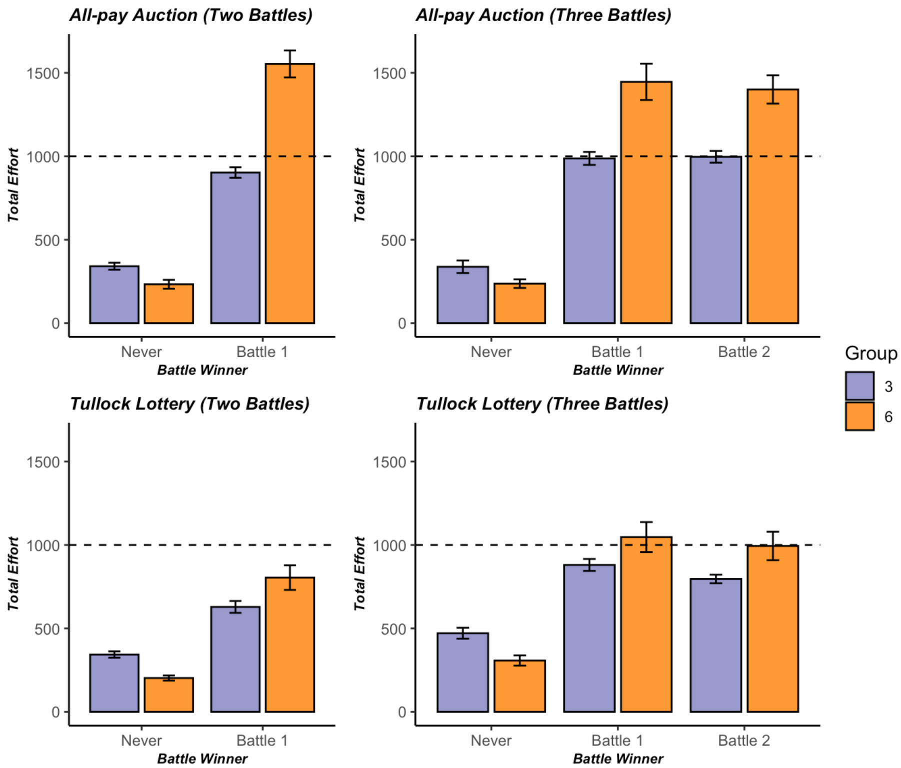

Figure 3: Total Effort with the Contests Ending in 2 and 3 Battles

The phenomena confirm the theory remarks in the Tullock lottery contest (Doğan et al., 2018); however, it does not match the prediction in the all-pay auction where the total expected prize is n-invariant. Another fact worth mentioning is that when the player number doubles from 3 to 6, we see a sufficient increase in battle total effort and contest total effort. Still, this increased total effort is far away from being doubled (Figure 7).

4.5 Strategy Types

The previous section discussed the group size effect in battle 2 when the current $s = 0$, but we did not find evidence suggesting distributional shifting among the active players considering both CSFs. We see a considerable number of zero bids in our data. This section will discover the source of the group size effect by classifying different strategy types with the difference in appearances in zero bid behaviors. Further, we will see how different treatments affect the players' types in the contest games.

Classification. A player's bids are categorized into the following six classes. Based on the six bid classes, we classify the player's strategy into three strategy types:

Entry: a player submits a positive bid in battle 1.

Stay: a player submits a positive bid for any post-battle 1, giver the previous bid is positive.

Reenter: a player submits a positive bid if the previous bid is zero.

No entry: any zero bid in a sequence of zero bids for the entire contest.

Dormant: a zero bid followed by some positive bid in the later battles.

Dropout: a zero bid followed by all zero bids in the later battles.

The three strategy types are:

Strategy 1. Entry-Stay (Dropout).

Strategy 2. Dormant-Reenter (Dropout).

Strategy 3. No Entry.

Table 10: Participation Rate.

Treatment	**All-pay** ($n = 3$)	**Tullock** ($n = 3$)	**All-pay** ($n = 6$)	**Tullock** ($n = 6$)
Proportion	89.58%	93.75%	69.02%	85.56%

The participation rates are shown in Table 10. We consider the players who are adopting **Strategy 1** and **Strategy 2** as participating in the contest and adopting **Strategy 3** are not participating. Two things stand out. First, the participation rate decreases as group size increases. Second, the participation rate decreases when the contest success function switch from the Tullock lottery to the all-pay auction.

Result 8. An increase in group size leads to an increase in zero-effort rate when the current $s = 0$ for both CSFs. This increasing treads in zero effort are consistent among all three classes.

In Figure 4, empirical evidence suggests the zero effort rate is significantly higher in 6 player's contests than 3 player contests for battle 1 and post-battle 1 given $s = 0$. When increasing the group size in a game, such increased zero effort rates are consistent with different CSFs. The increase in zero effort rate explains the observed group size effect in the battle 2 bid, given that

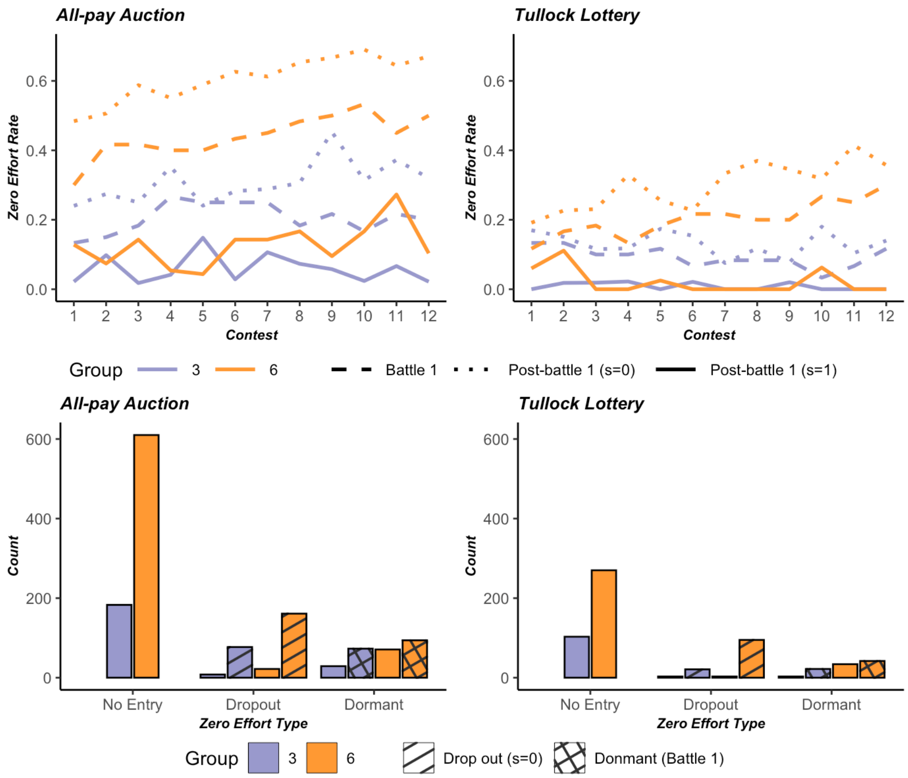

Figure 4: Different Classes of Zero Effort

there is no significant change in the positive bids in the battle (Table 17 in Appendix D). When $s = 1$, The dropout rate differences in different group sizes are not noticeable for both contest success functions. (Figure 4).

Moreover, we discovered that the rising trend in No Entry, Dropout, and Dormant is persistent, while the group size increased from 3 to 6 in auction and lottery games (Figure 4). We also persistently detect that the dropout behavior happens at $s = 0$ and the dormant behavior happens at battle 1, suggesting that both zero effort types are planned rather than random decisions.

Result 9. Increasing the number of players increases the probability of entering the contest in battle 1 but decreases their probability of staying in the game.

Comparing the **Strategy 1** and **Strategy 3** subjects in the experiment, we see that increasing the player number has significantly decreased the probability of entry for auction and lottery games in battle 1. The probability of staying in the auction is significantly higher for all post-battle 1 in

Table 11: Random Effect Probit Panel on Strategy Types.

CSF		All-pay Auction			Tullock Lottery		
Strategy (Excluded Dormant & Reenter)		Entry (Battle 1)	Stay (Battle 2)	Stay (Post-battle 2)	Entry (Battle 1)	Stay (Battle 2)	Stay (Post-battle 2)
Group	($n = 6$ & $s = 0$)	-1.71***	-1.31***	-0.16	-1.27***	-1.54***	-0.34
		(0.35)	(0.40)	(0.36)	(0.43)	(0.33)	(0.39)
State	($n = 3$ & $s = 1$)		6.96***	2.07***		7.33***	1.56***
			(0.77)	(0.31)		(0.53)	(0.44)
Group × State	($n = 6$ & $s = 1$)		-4.88***	-0.77***		0.46	0.02
			(0.82)	(0.40)		(0.39)	(0.60)
Trend		-0.07***	-0.06***	-0.05*	-0.04	-0.11***	-0.04*
		(0.02)	(0.02)	(0.03)	(0.04)	(0.03)	(0.02)
Observations		1,273	921	763	1,376	1,206	952

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level of the Group × State ($n = 6$ & $s = 1$) is the testing result for group size effect compared to the State ($n = 3$ & $s = 1$).
Significant at 10% * Significant at 5% ** Significant at 1% ***

the 6 players' auction games when $s = 1$. Then, when increasing group size in battle 2 for both CSFs, evidence suggests that the probability of staying in the game decreases. Besides the group size effect, the probability of stay is higher when $s = 1$ in all post-battle 1 for both auction and lottery contests, suggesting strategic momentum in the strategy type point of view (Table 11).

4.6 Overbidding

We previously addressed the over-dissipation behavior where the total bid surpasses the prize of a contest. This section will closely examine the origins of over-dissipation in various battles and states. We emphasize that we define overbidding as a player bid higher than the expected prediction in Nash equilibrium at the current battle and state, which is slightly different from the definition of over-dissipation that compares the total effort with the prize in a contest. Sheremeta (2013) summarized the observed over-dissipation and overbidding behavior in earlier studies. Commonly overbidding is more desired in Tullck experiments than in the all-pay auction.

Overbidding behavior is revealed differently depending on CSFs, group size, battle, and state. We summarize some observations from Figure 5. First, the dispersion of overbidding is more severe in all-pay than in the Tullock lottery contest. Second, there is a clear tendency that 6 player's

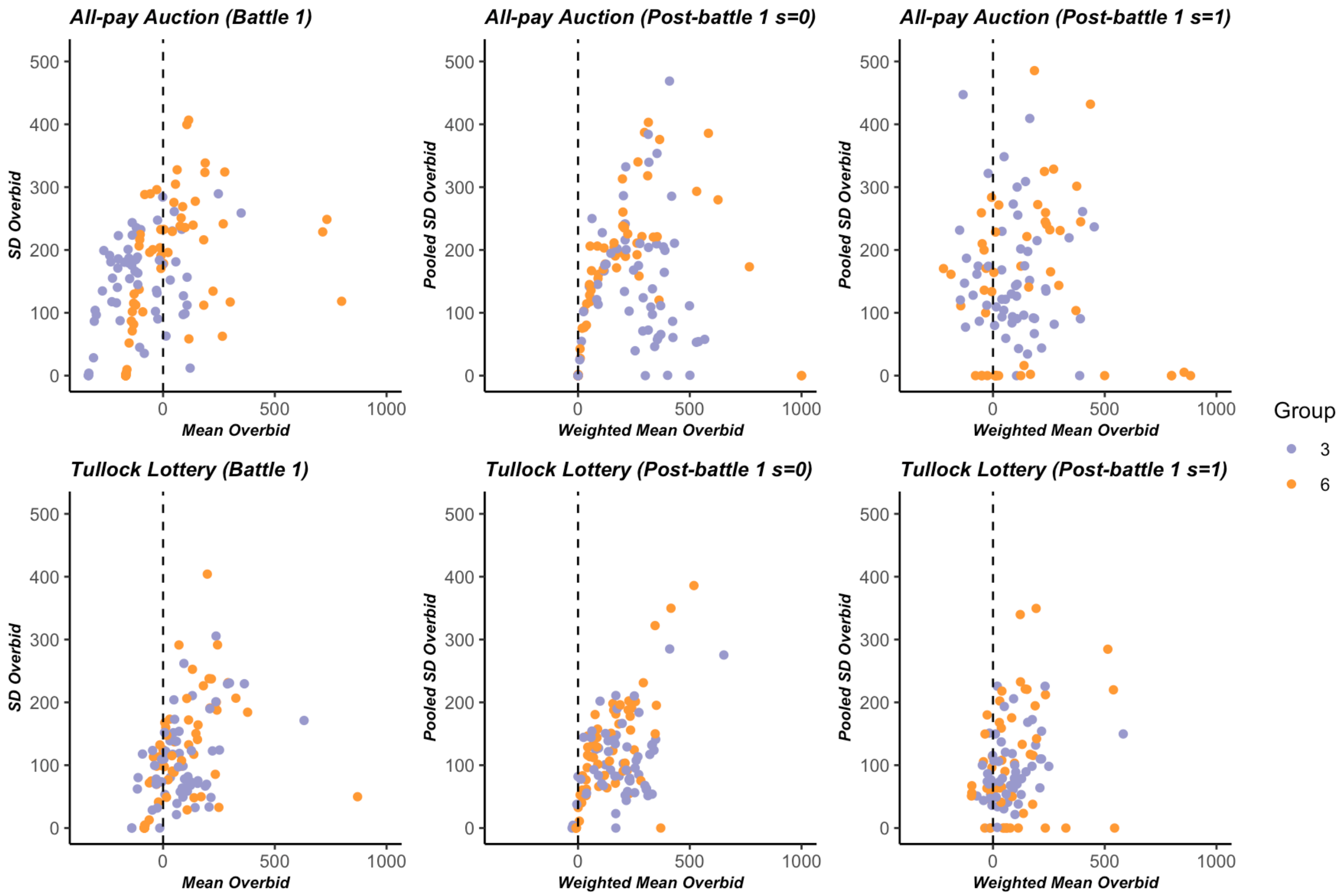

Figure 5: Overbid

auction game exhibits more overbidding than 3 player's game in battler 1. Third, when the $s = 0$ in post-battle 1, 3 players game presents more overbidding than 6 players game for both CSFs.

Result 10. Sunk cost fallacy explains the overbidding behavior in both CSFs in post-battle 1 when $s = 0$.

Mago and Sheremeta (2019) discussed that sunk cost fallacy is an explication of overbidding behavior in a lottery multi-battle contest. They conducted a second experiment to confirm that overbidding in battle 2 is directly from a false concern of the sunk cost in battle 1 rather than a player is constantly bidding high in both battles. In our experiment, we ruled out this concern for the following reasons. Firstly, we do not observe consistently overbidding as a popular strategy among the participants from Figure 5. Secondly, the contest's proportion of contest ends in 2

Table 12: **Random Effect Panel on Overbidding.**

CSF		**All-pay Auction**		**Tullock Lottery**	
Post-battle 1 Overbid		$s = 1$	$s = 0$	$s = 1$	$s = 0$
Constant	$(n = 3)$	151.49**	261.71***	42.88	148.94***
		(62.75)	(23.98)	(32.28)	(20.09)
Group	$(n = 6)$	75.76	-77.62***	11.48	-44.50***
		(52.86)	(20.26)	(35.62)	(19.67)
Previous Total Effort		-0.03	0.16***	0.03	0.20***
		(0.08)	(0.05)	(0.04)	(0.04)
Trend		-5.61	-5.55***	3.86	-1.71***
		(5.15)	(1.90)	(2.97)	(1.00)
Observations		917	1594	924	1584

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level of the constant is the testing result for overbidding compared to the theoretical prediction.
Significant at 10% * Significant at 5% ** Significant at 1% ***

battles and 3 battles are very similar. We are using the summation of the bids in all previous battles to represent the magnetite of the sunk cost. As shown in Table 12, the amount of sunk cost is positively correlated to the amount of overbidding in the current battle when $s = 0$ for both all-pay and Tullock contests. Meanwhile, the overbidding is group size sensitive as we pointed out from the summary in Figure 5.

4.7 Psychological and Demographic Heterogeneity

In Mago et al. 2013, psychological momentum exists in a 2 players' game if the winner of battle 1 and the winner of battle 2 bid differently in battle 3. Our research extended the number of players to more than 2 in the experiments, so we specify the psychological momentum into a more general case. We consider that for any post-battle 2 if the bid of the previous battle winner is different from other players whose current $s = 1$, then there exists psychological momentum. In Table 13, the panel regression shows that the previous battle winner's effort is significantly higher than the other battles' winner for post-battle 2 in the all-pay auction game, which suggests the evidence for psychological momentum defined in Mago et al. (2013).

We also explored the near-win effect as (Kassinove and Schare 2001; Daugherty and MacLin

Table 13: Random Effect Panel on Psychological Momentum.

CSF	**All-pay Auction**		**Tullock Lottery**	
Effort (State)	Post-battle 1 ($s = 1$)	Post-battle 2 ($s = 1$)	Post-battle 1 ($s = 1$)	Post-battle 2 ($s = 1$)
Constant	443.24*** (42.20)	432.62 (48.26)	277.98*** (26.12)	266.00 (23.18)
Previous Battle Winner		47.38** (21.14)		1.81 (12.34)
Near Win Previous Contest	11.32 (11.47)	19.29 (12.48)	-34.25*** (10.49)	-42.38*** (8.26)
Trend	-6.44 (4.69)	-6.20 (5.82)	3.68 (2.95)	4.38 (2.90)
Observations	917	557	924	564

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level of the constant is the testing result for overbidding compared to the theoretical prediction.
Significant at 10% * Significant at 5% ** Significant at 1% ***

2007; Wadhwa and Kimas 2015) the psychological explanation for the behavior in the experiments. We characterize $s = 1$ at the end of a contest as a near-win since the winning threshold in our design is $T = 2$. There is a strong negative near-win effect in the lottery contest. In other words, given the $s = 1$ in the current contest, the near-win player of the previous game tends to bid less (Table 13). Our finding is opposite from the psychology research that suggests that near-win motivates players in a game even though the estimated parameters are positive in the auction games. Behavior in contest indicates more evidence supporting discouragement than motivation.

In theory, we assume the players are risk-neutral; this is one of the important reasons for the discrepancy between theoretical prediction and empirical observation. With a slightly modified version of Shupp et al. (2013), the participants' preferences toward risk, ambiguity, and loss are elicited. The original definition from prospect theory (Tversky and Kahneman 1992) is used to create the dummy variables of the preference in regressions. Take the loss aversion survey as an example (Appendix A). Suppose in a subject's revealed choice, the "turning point" is before question 6, where the expected payoff is the same for opinion A and option B; we label the subject as loss aversion. The result in Table 14 demonstrates loss-averse participants' bids sufficiently less than any players when $s = 0$ in any battle for both CSFs. This partially explains the observed

Table 14: **Random Effect Panel on Gender and Loss Aversion.**

CSF	**All-pay Auction**			**Tullock Lottery**		
Effort (State)	Battle 1	Battle 2 (s=0)	Post-battle 2 (s=0)	Battle 1	Battle 2 (s=0)	Post-battle 2 (s=0)
Constant	302.30 (43.93)	328.70*** (33.26)	314.63*** (41.32)	206.73*** (31.07)	200.12*** (29.56)	254.39*** (33.12)
Woman	126.49*** (36.57)	101.05** (45.82)	80.97* (45.60)	73.27** (35.01)	55.70** (22.22)	12.33 (19.04)
Loss Aversion	-112.54*** (37.08)	-129.99*** (21.26)	-176.75*** (38.91)	-77.97** (35.71)	-65.65** (37.58)	-82.07** (34.46)
Trend	-9.11*** (3.35)	-6.88*** (2.46)	-9.35*** (2.44)	2.49 (1.52)	0.87 (1.08)	-5.56** (2.29)
Observations	1,440	1,080	514	1,440	1,080	504

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level of the constant is the testing result for overbidding compared to the theoretical prediction.
Significant at 10% * Significant at 5% ** Significant at 1% ***

underbid in battle 1 of the auction game and the sunk cost fallacy behaviors.

From previous studies, well-known results are women tended to bid more than men in contest games (Charness and Levin, 2009; Chen et al., 2009; Ham and Kagel, 2006; Price and Sheremeta; 2012, Mago et al., 2013; Lim et al., 2014). Our experiment found that women bid more when the current $s = 0$ for auction and lottery contests. Additionally, we see a more substantial effect in auction games than in lottery games, and the gender effect declines in later battles (Table 14). The gender effect in overbidding is constrained to the current state, and it is not presented when the current $s = 1$; to put it in another way, women bid as much as men when they are in a winning position.

One last important aspect in our data we would like to discuss is the observed trending of the bid through the games. In general, we find descending trends in auction games in terms of efforts in all battles, the total effort for any final state, entering and staying rates (Table 3, Table 9, and Table 11). On the other hand, we only detect the descending trends of total effort for the winners of the games and the staying rate in Tullock lottery contests (Table 9, and Table 11). The more severe trading findings in auction contests compared to the lottery contest are different from previous studies (Mago and Sheremeta 2017, 2019; Fallucchi et al., 2021).

5 Conclusion

We conducted a laboratory experiment at Virginia Tech Econ Lab to study the group size effect in sequential multi-battle contest. The theoretical framework is based on the theory in Konrad and Kovenock 2009, and Doğan et al., 2018. We generalized all-pay auction result from $n = 2$ to $n \geq 2$. We employed a two by two design, where $n = 3$ or 6, and $r = 1$ or ∞ in a multi-battle contest ($T = 2$).

We find that a group size effect exists in the sequential multi-battle contest for various aspects. The obtained group size effect is different across contest success functions. In battle 1, the observed group size effect consists of the Tullock lottery's mean effort. On the contrary, when $r = 1$ increased to $r = \infty$, the distribution of battle 1 effort is severely distorted. As we increased the group size, the proportions of boundary bids (0 or 1000) are increased significantly. Dynamically for the players who lose battle 1, their mean effort in battle 2 dropped significantly when increasing the number of players in the all-pay auction and the Tullock lottery. When we decompose the subject behavior into different strategic profiles, we discover the mean effect in battle 2 effort mainly comes from the proportion of people who drop out of the contest. Increasing the group size in a sequential contest led to more battle 1 losers to exert zero-bid in the consecutive battle. Increasing n in the contest not only increases the dropout rate in post-battle 1, but also increases the probability of adopting the strategy of no entry and dormant-reenter in both CSFs. As for the total effort, we find that the average individual effort is higher when there are fewer contestants in the game for both CSFs. On the other hand, increasing the player number generates more effort in the player's battle, battle's total, player' total, winner total, average battle total, and average contest total. Further, the group size effect in the total effort is stronger in the all-pay auction compared to the Tullock lottery. Additionally, contestants exert more effort in all-pay games than Tullock games. Decrease n and r increase the participation rate in a multi-battle contest.

Besides the obtained group size effect, we also confirmed the overbidding and over-dissipation behaviors from the previous experimental contest studies (Davis and Reilly, 1998; Potters et al. 1998, Mago et al., 2013; Mago and Sheremeta 2017, 2019). The observed overbidding is mostly concentrated in post-battle 1, which is more consistent in the Tullock lottery than in the all-pay auction. As suggested in Mago and Sheremeta (2019), we confirm that observed overbidding can be partially explained by the sunk cost fallacy. Further, the over-dissipation behavior is more likely to be observed from the contest winners when increasing the player. This trend is more robust in the all-pay auction than in the Tullock lottery.

Lastly, we discovered some psychological and demographic heterogeneity in our experiments. Strategically, previous studies (Mago et al., 2013; Mago and Sheremeta 2017, 2019) and our result reached consensus that the players expended more effort than their opponents when they gained a leading position in a sequential multi-battle contest. Besides, we find the players bid more when they gain the leading position in the directly previous battle than all rest players who have the same leading position in the all-pay auction. Meanwhile, we note that the player who almost won the previous Tullock lottery contest tended to bid less in the current game when facing the same situation. As for the heterogeneity in the subjects, we detect women exert more effort, and the loss aversion subjects bid less when the current $s = 0$ given both CSFs. Our results are similar to the previous experimental studies (Mago et al., 2013; Lim et al., 2014; Mago and Razzolini 2019).

There are some directions that we could extend our study. First, in Zizzo (2002), the strategic momentum was undetected when the winning threshold was large. One future direction of research is discovering if the result can be generalized to n player's contest. Second, we conduct the sequential multi-battle contest with the all-pay auction and the Tullock lottery. We could expand the existing tournament and elimination literature (Parco et al., 2005; Amaldoss and Rapoport, 2009; Altmann et al., 2012) to more than two players in each stage. Lastly, there is some theoretical potential to explore group contest experiments (Abbink et al. 2010) in multi-battle settings.

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Appendix A Elicitation of Risk, Ambiguity and Loss Preference

We employed three modified versions of preference surveys from Shupp et al. 2013 to elicit risk, ambiguity, and loss preferences. Here we show the instruction of the loss preference as an example given the fact that we find the loss aversion partially explained the reduction in the bid for all post-battle 1 when the current $s = 0$ (Table 14). Unlike the loss preference survey, the omitted risk and ambiguity surveys' choices are in the gain domain. In expectation, a risk-neutral subject will have an average payoff of \$5.13 from the three surveys. On average, participants' earnings from the three surveys are \$3.81. Therefore, our average representative subject has a risk-seeking preference. We present a sample of the Loss aversion survey, and for more detailed instructions, please check the Supplementary Material.

Decision

- For each question in the following table, please choose **A** or **B** as your preferred opinion.
- There are 12 questions in total, and please consider each of them as a separate decision.

Earning

- We will **select one out of 12 questions at random** (roll a 12-sided dice) to calculate your earnings at the end of the experiment.
- For example, suppose question 6 (see below) is selected by the 12-sided dice to calculate your earning:
  - If your choice is option A (for question 6), there is a 50% of chance we pay you \$3.00 and a 50% of chance we deduct \$3.00 from your participation reward (flip a coin).
  - If your choice is option B (for question 6), we will pay you \$0.00.

	Option A	**Option B**
Question 1	**\$3.00** with 50% chance and **-\$0.50** with 50% chance.	**\$0.00** for sure.
Question 2	**\$3.00** with 50% chance and **-\$1.00** with 50% chance.	**\$0.00** for sure.
Question 3	**\$3.00** with 50% chance and **-\$1.50** with 50% chance.	**\$0.00** for sure.
Question 4	**\$3.00** with 50% chance and **-\$2.00** with 50% chance.	**\$0.00** for sure.
Question 5	**\$3.00** with 50% chance and **-\$2.50** with 50% chance.	**\$0.00** for sure.
Question 6	**\$3.00** with 50% chance and **-\$3.00** with 50% chance.	**\$0.00** for sure.
Question 7	**\$3.00** with 50% chance and **-\$3.50** with 50% chance.	**\$0.00** for sure.
Question 8	**\$3.00** with 50% chance and **-\$4.00** with 50% chance.	**\$0.00** for sure.
Question 9	**\$3.00** with 50% chance and **-\$4.50** with 50% chance.	**\$0.00** for sure.
Question 10	**\$3.00** with 50% chance and **-\$5.00** with 50% chance.	**\$0.00** for sure.
Question 11	**\$3.00** with 50% chance and **-\$5.50** with 50% chance.	**\$0.00** for sure.
Question 12	**\$3.00** with 50% chance and **-\$6.00** with 50% chance.	**\$0.00** for sure.

Appendix B Distribution of Effort

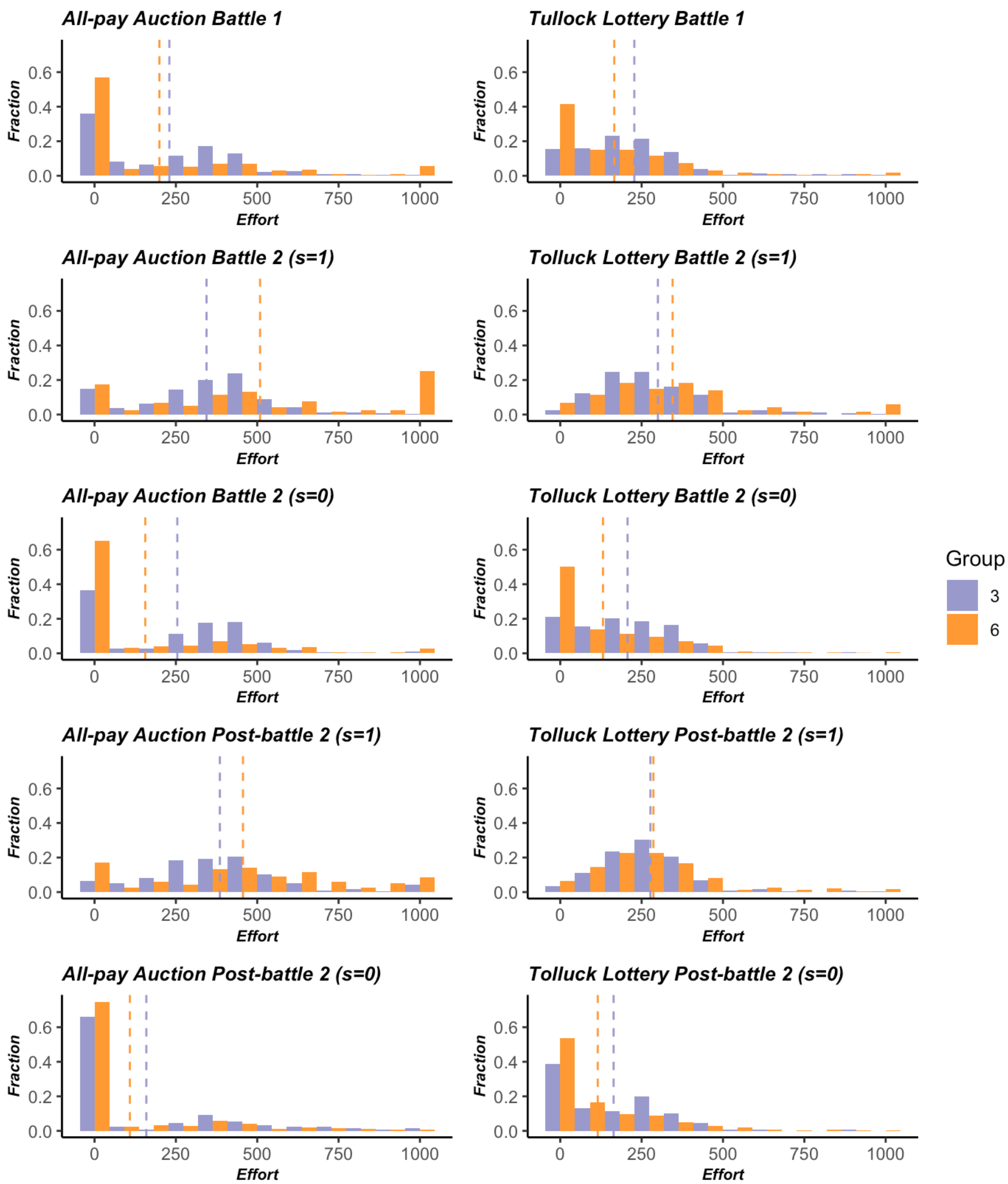

Figure 6: Distribution of Efforts for Given Battle and State.

Appendix C Optimal Mechanism Design

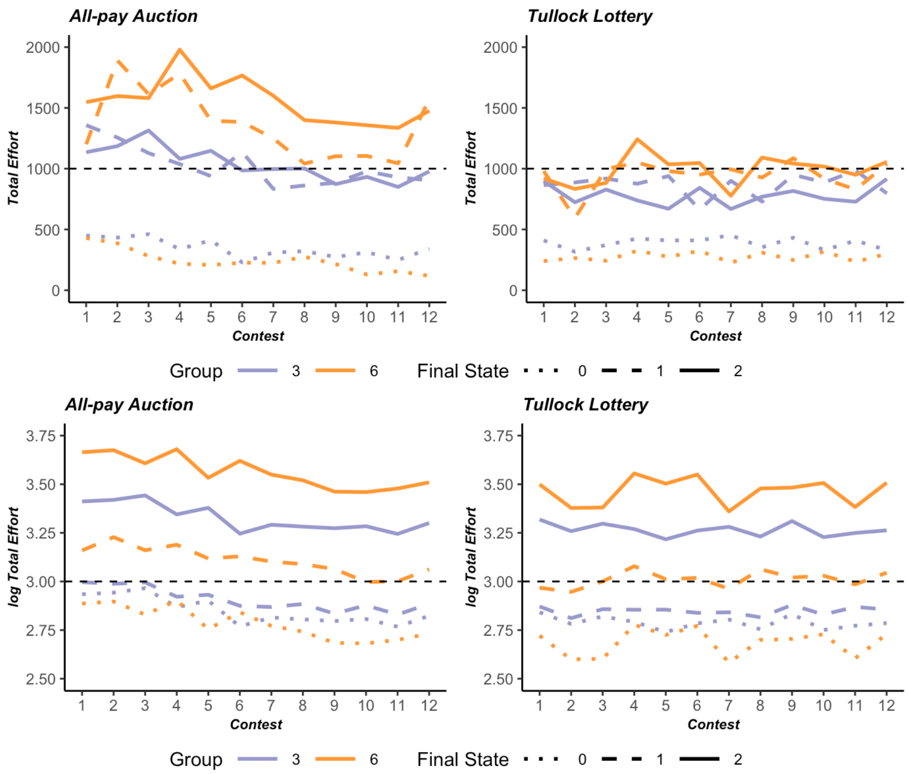

Figure 7: The Trends of Total Efforts.

In this part, we present additional plots and tables and summarize the general guidance for optimal mechanism designs according to the findings from this study. Firstly, mainly the winner spends the most total effort in a contest (Figure 7 and Table 15). Secondly, as the group size increases, the winner and the almost winner's total effort increases; however, it decreases for the player with the final $s = 0$ (Figure 7). Thirdly, as n grows, the average battle total effort and the average contest total effort rise, while the average player's effort falls.

Table 15: The Proportion of the Objective Function is Maximized in the (Condition).

Treatment		**All-pay** ($n = 3$)	**Tullock** ($n = 3$)	**All-pay** ($n = 6$)	**Tullock** ($n = 6$)
Player's Battle	($s = 1$)	53.75%	38.33%	49.17%	31.67%
Player's Battle	(Battle 1)	25.00%	43.75%	24.17%	49.16%
Battle's Total	(Battle 2)	47.08%	43.75%	46.67%	43.33%
Player's Total	(Contest Winner)	81.67%	62.08%	67.50%	58.33%

Additionally, the best-performed player's battle is likely to happen when $s = 1$ in auction game and when it is battle 1 in lottery game. Then, the best-performed battle is likely to happen in battle 2 in both CSFs (Table 15). Overall the payer in individual battle, the individual battle total, and the player in the contest have better performance (more average effort) when increasing both n and r (Table 6).

Table 16: Maximization of Each Objective Function.

Treatment	**All-pay** ($n = 3$)	**Tullock** ($n = 3$)	**All-pay** ($n = 6$)	**Tullock** ($n = 6$)
Player's Battle			✓	
Battle's Total			✓	
Player's Total			✓	
Winner Total			✓	
Participation Rate		✓		
Average Player Total	✓			
Average Battle Total			✓	
Average Contest Total			✓	

All in all, the all-pay with 6 players generates more effort from the players in the following categories: Player's Battle, Battle's Total, Player's Total, Winner Total, Average Battle Total, Average Contest Total. All-pay with 3 players and Tullock with 3 players performed the best in the perspective of Average Player Total and Participation Rate, respectively.

Appendix D Dynamic Difference in Positive Effort

We present the random effect panel regression result to check the dynamic difference in positive bids for subsequent battles Table 17. The battle 2 positive bids in the auction contest are significantly higher than battle 1. Consider that most contests end within 4 battles; we use battle 3 and battle 4 to represent post-battle 2 behavior. We find no significant difference between the post-battle positive effort.

Table 17: Random Effect Panel on Dynamic Comparison of Positive Bid.

CSF		**All-pay Auction**		**Tullock Lottery**	
Positive Effort		$n = 3$	$n = 6$	$n = 3$	$n = 6$
Battle 1 vs. Battle 2	$(s = 0)$	65.33***	43.38*	-9.22	-3.70
		(16.00)	(22.59)	10.86)	(10.64)
Battle 2 vs. Post-battle 2	$(s = 0)$	38.21	-21.17	-20.29	-4.95
		(33.93)	(21.92)	(15.58)	(7.80)
	$(s = 1)$	32.44	8.63	-18.86	-41.47**
		(21.28)	(35.40)	(16.65)	(20.16)
Battle 3 vs. Battle 4	$(s = 0)$		-4.24		-7.20
			(18.23)		(19.62)
	$(s = 1)$	24.56	-23.73	1.71	14.03
		(50.91)	49.34	(31.84)	20.48

Models include a random effect error structure with the individual subject as the random effect.
Robust standard errors are reported in the parentheses, and the standard errors are clustered at the session level.
The reported significance level reflects if the positive bid of later battle is different from the former.
Significant at 10% * Significant at 5% ** Significant at 1% ***