

The Cooper Union Department of Electrical Engineering
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ECE211 Signal Processing & Systems Analysis
Problem Set I: Signals & Spectra Part I
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1. Let $\alpha = \exp(-j3\pi/4)$. Express α^n , $0 \leq n \leq 7$, both in the form $\exp(j\theta)$ where $|\theta| \leq \pi$, and as $x + jy$ or $(x + jy)/\sqrt{2}$ where x, y are each rational. Do not do this using sin/cos expansions. **Remark:** $z^{\alpha n}$ for $0 \leq n \leq 7$ are the eight distinct complex numbers satisfying $z^8 = 1$; for this reason, α is called a *primitive 8th root of unity*. This property is significant in fast Fourier transform (FFT) algorithms.
2. Let $x(t) = 1 + t$, $-1 \leq t \leq 1$, and 0 otherwise. Sketch each of the following by hand; do not try to write formulas for the signals, instead use sketching techniques to draw the curves. Make sure values are indicated (e.g., the value at the tip of the triangular shape of x).
 - (a) $x(t)$
 - (b) $x(1 + \frac{t}{2}) + 2u(-t + 1)$
 - (c) $x(t + 2) + u(2t)$
3. The discrete-time signal $x[n]$ is given by:

$$x = \{2, \underline{3}, 5, 1, 4\}$$

where the underscore ' _ ' denotes $n = 0$.

- (a) Specify the length and support of x .
 - (b) Sketch $x[n]$ by hand.
 - (c) Express x as a superposition of impulses.
 - (d) Sketch $x[2 - n] + 3u[2n - 1]$ by hand.
 - (e) Sketch $x[2n + 1]$.
 - (f) Sketch $(\uparrow 3)x$.
4. Consider the following expression:

$$5\sqrt{2} \cos(\omega t - \pi/4) + 3\sqrt{2} \cos(\omega t + \pi/4) + 6 \cos(\omega t - 3\pi/2)$$

First, convert this directly to the phasor domain (do not do any simplification in the time domain). Then, simplify this expression via complex arithmetic to obtain the resultant phasor in both rectangular and polar form. Finally, convert this back to the time domain in the form $A \cos(\omega t + \theta)$ and $a \cos \omega t + b \sin \omega t$ (each directly from the phasor form, i.e., don't work in the time domain at all). As usual, you are NOT allowed to compute cos/sin values or use trigonometric identities.

5. Given a real signal $x(t)$ over the interval $0 \leq t \leq T$ with (exponential) Fourier series coefficients $\{c_n\}_{n \in \mathbb{Z}}$. Several coefficients are given as:

$$c_0 = 2, \quad c_1 = 3 + j4, \quad c_2 = 1 - j2$$

- (a) You have enough information to determine some other coefficients c_n . Specify the other values that are known.
- (b) Find the DC power, the power at the fundamental frequency, and the power at the second harmonic.
- (c) Assume all the other **unknown** coefficients are 0. Find the RMS value of the signal.