The Cooper Union Department of Electrical Engineering
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ECE211 Signal Processing & Systems Analysis
Problem Set VI: Partial Fractions and Modal Analysis
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1. Use the method of partial fractions to find the inverse Laplace transform of each of the following. You should start by writing the proper partial fraction expansion in terms of unknown coefficients A, B, C, etc., and your final $h(t)$ should be expressed as a real function.

$$
\begin{aligned}
H_1(s) &= \frac{3s^3+1}{(s+3)(s+5)} \\
H_2(s) &= \frac{4s+3}{(s+2)^2(2s+1)} \\
H_3(s) &= \frac{5s+2}{(s^2+4s+5)(s+4)}
\end{aligned}
$$

2. Use the method of partial fractions to find the inverse z-transform of each of the following. Again, start by writing the expansion in the proper form with unknown coefficients, and your final $h[n]$ should be expressed as a real function.

$$
\begin{aligned}
H_1(z) &= \frac{z^3+1}{(3z+1)(2z-1)} \\
H_2(z) &= \frac{z^2+z}{(2z+1)^2(3z-2)} \\
H_3(z) &= \frac{2z+1}{(2z-1)(2z^2+2z+1)}
\end{aligned}
$$

3. The input to an analog filter $H(s)$ is:

$$x(t) = 2e^{-3t}u(t)$$

The output is:

$$y(t) = (3t+1)e^{-3t}u(t) + e^{-4t}\sin(5t+\pi/3)u(t)$$

Do NOT try to find $X(s)$, $Y(s)$ or $H(s)$! Just list the poles of $H(s)$ with multiplicity.

4. The *unit step response* of an analog filter $H(s)$ is:

$$4u(t) - e^{-2t}(t\cos(5t + \pi/4) + 5\cos 5t)\,u(t) + 7e^{-3t}u(t)$$

(a) Specify the natural and forced responses.

(b) List the system poles, with multiplicity.

(c) For each mode in the natural response, specify the time constant (in seconds) and, if there is oscillation, the frequency (in Hertz).

(d) Identify the dominant pole(s).

5. A causal digital filter $H(z)$ has poles at $-\frac{1}{2} \pm j\frac{1}{2}$ (simple), $1/5$ (triple) and $-1/3$ (simple).

(a) Write the general form of the impulse response $h[n]$, expressed as a real function. [Your answer will contain several unspecified constant coefficients]

(b) For each mode, write the time constant (expressed as a function of the sample period T, so your answer would have units of seconds), and if there is oscillation, the frequency of oscillation (expressed as a function of the sampling rate f_s, so your answer would have units of Hertz).

(c) Identify the dominant pole(s).

6. Suppose we have a causal digital filter $H(z)$. We apply an input with known (simple) poles at $1/2$ and $-1/3$. We observe an output with poles at $1/2$, $-1/3$, $1/4e^{\pm j\pi/3}$ and $-1/5$, all simple.

(a) What do we know for certain about the poles of $H(z)$?

(b) There is NOT enough information to completely determine (with certainty) ALL the poles of H, and in fact H could even be unstable! BRIEFLY state what could be happening here that causes this uncertainty.