1.

$t<-1$: $\int_{t-2}^{t-1} 1\cdot 0\, dT + \int_{t-1}^{-1} 3\cdot 0\, dT = 0$

$-1<t<0$: $\int_{-1}^{t} 3\cdot 1\, dT = 3t+3$

$0<t<1$: $\int_{-1}^{t-1} 1\cdot 1\, dT + \int_{t-1}^{t} 3\cdot 1\, dT = 3+t$

$1<t<2$: $\int_{t-2}^{t-1} 1\cdot 1\, dT + \int_{t-1}^{1} 3\cdot 1\, dT + \int_{1}^{t} 3\cdot 2\, dT = 1+3t$

$2<t<3$: $\int_{t-2}^{1} 1\cdot 1\, dT + \int_{1}^{t-1} 1\cdot 2\, dT + \int_{t-1}^{2} 3\cdot 2\, dT = 17-5t$

$3<t<4$: $\int_{t-2}^{2} 1\cdot 2\, dT = 8-2t$

$t>4$: $\int_{t-2}^{t-1} 1\cdot 0\, dT + \int_{t-1}^{4} 3\cdot 0\, dT = 0$

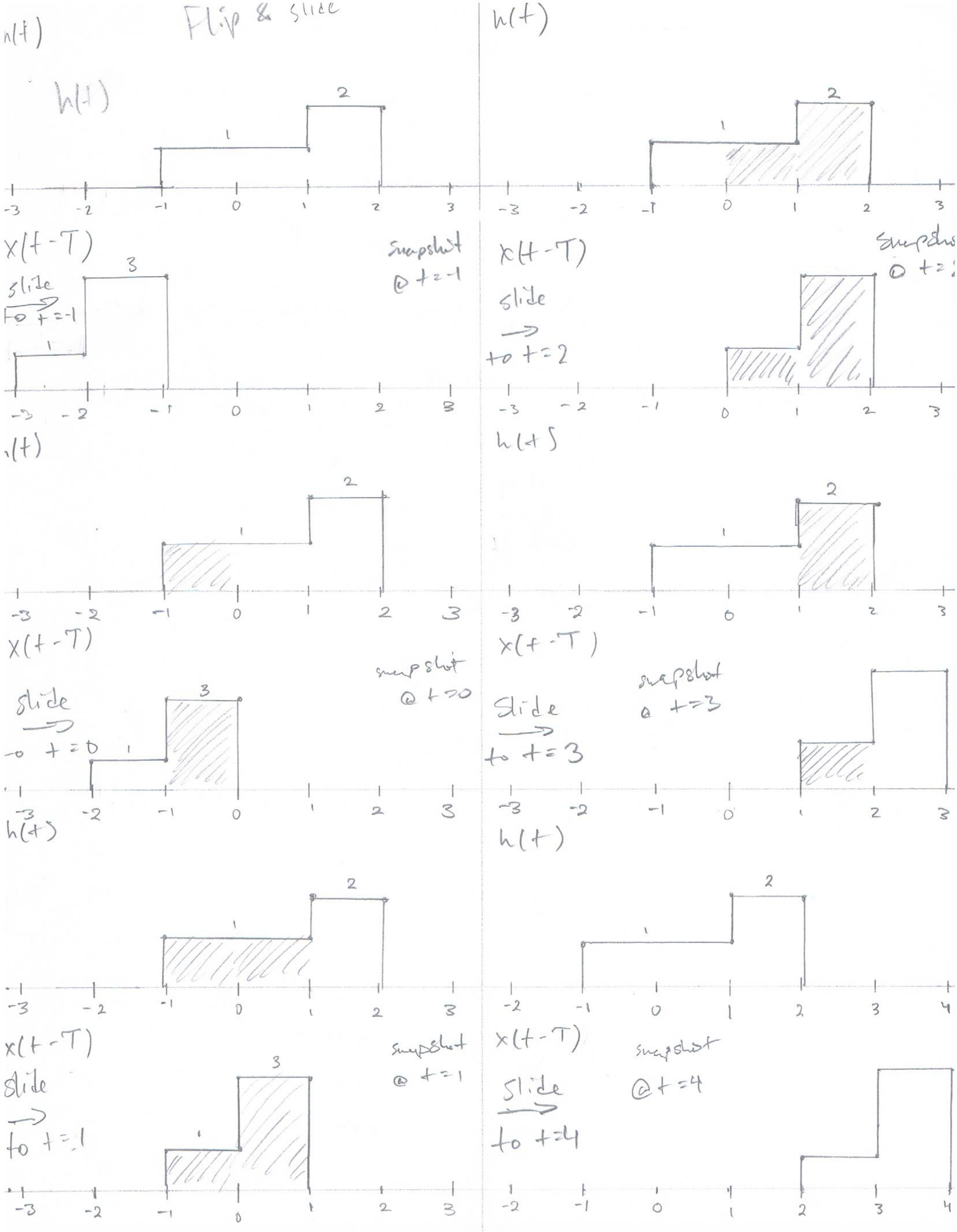
$$y(t) = \begin{cases} 0 & t<-1 \\ 3t+3 & -1<t<0 \\ 3+t & 0<t<1 \\ 1+3t & 1<t<2 \\ 17-5t & 2<t<3 \\ 8-2t & 3<t<4 \\ 0 & 4<t \end{cases}$$

graph of y

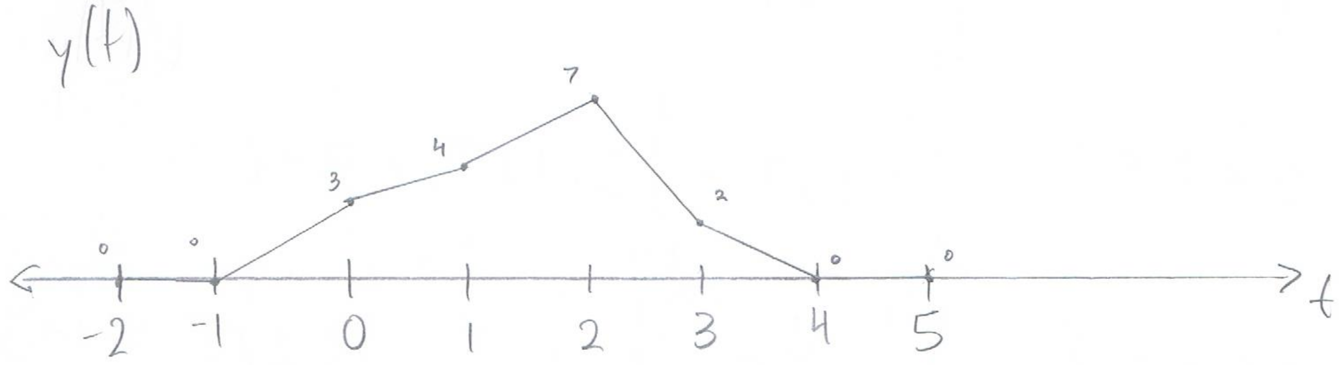
$\longrightarrow$

on back

Flip & slice



h(t)
x(t-T)
slide → to t=-1
snapshot @ t=-1
slide → to t=0
snapshot @ t=0
slide → to t=1
snapshot @ t=1
slide → to t=2
snapshot @ t=2
slide → to t=3
snapshot @ t=3
slide → to t=4
snapshot @ t=4


y(t)
0
0
3
4
7
2
0
0
-2
-1
0
1
2
3
4
5
t

2. (a) length(h) = 5

support(h) = $\{N_h \le n \le M_h\} = \{-1 \le n \le 3\}$

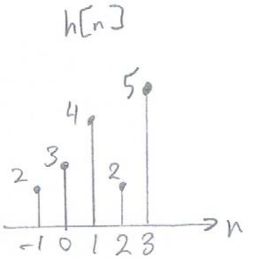
length(x) = 5

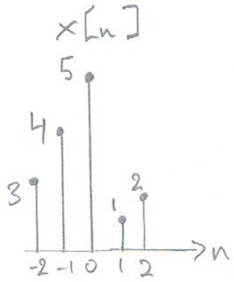
support(x) = $\{N_x \le n \le M_x\} = \{-2 \le n \le 2\}$

length(y) = length(h) + length(x) − 1 = 9

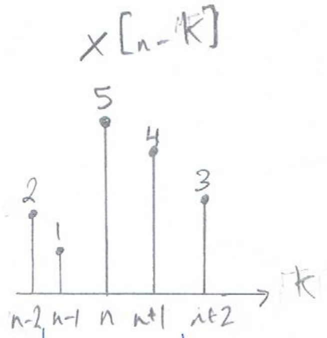
support(y) = $\{N_h + N_x \le n \le M_h + M_x\} = \{-3 \le n \le 5\}$

(d)





flip →



write the indices: $h_{-1}x_2 + h_0 x_1 + \dots$

sum to 1

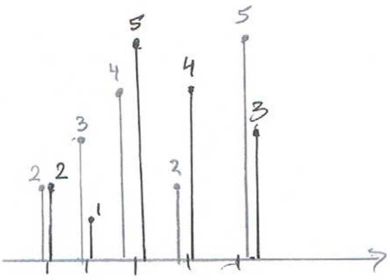
$y[1] = \sum h[k]\cdot x[1-k] = 2\cdot2 + 1\cdot3 + 5\cdot4 + 4\cdot2 + 3\cdot5$

$= 4 + 3 + 20 + 8 + 15$

$= 50$ (same as matlab result)

x[1-k]

h[k]



3(f) $h = (\uparrow M) g \Rightarrow h(Mk) = g[k]$ (1)

$y = (\downarrow M) v \Rightarrow y[k] = v[Mk]$ (2)

$u = (\downarrow M) x \Rightarrow u[k] = x[Mk]$ (3)

$$v[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Since $h = (\uparrow M) g$, $h[k] = 0$ for $k \neq Mn$, where $n \in \mathbb{Z}$.

$$= \sum_{k=-\infty}^{\infty} h[Mk]\, x[n-Mk]$$

To match indices.

$$y[n] \overset{(2)}{=} v[Mn] = \sum_{k=-\infty}^{\infty} h[Mk]\, x[Mn-Mk]$$

(1)

$$= \sum_{k=-\infty}^{\infty} g[k]\, x[Mn-Mk] \quad (A)$$

$$y'[n] = g[n] * u[n]$$

$$= \sum_{k=-\infty}^{\infty} g[k]\, u[n-k]$$

(3)

$$= \sum_{k=-\infty}^{\infty} g[k]\, x[M(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} g[k]\, x[Mn-Mk] \quad (B)$$

(A) = (B), Thus, $y = y'$ //