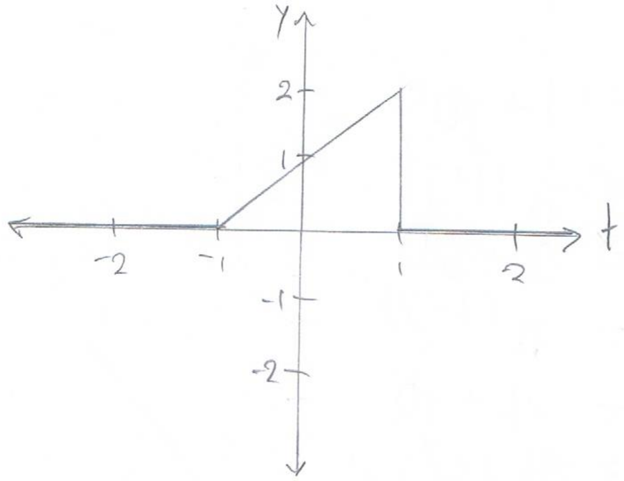
1. $\alpha = \exp(-j3\pi/4)$. α^n, $0 \le n \le 7$?

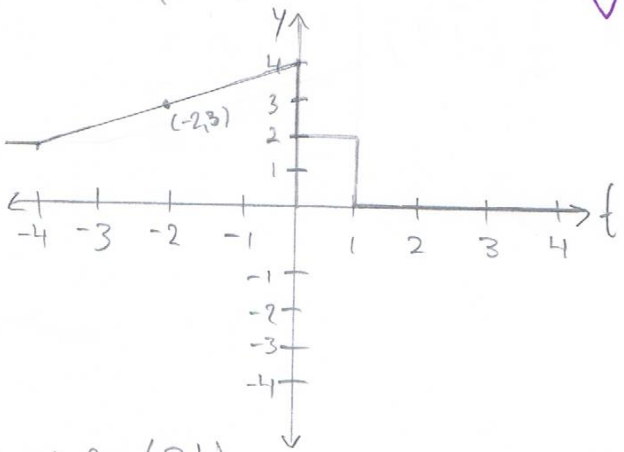
$$\alpha^0 = 1 = 1 + j0$$
$$\alpha^1 = \exp(-j3\pi/4) = -\frac{1}{\sqrt{2}}(1+j)$$
$$\alpha^2 = \exp(j\pi/2) = 0 + j(1)$$
$$\alpha^3 = \exp(-j\pi/4) = \frac{1}{\sqrt{2}}(1-j)$$
$$\alpha^4 = \exp(j\pi) = -1 + j0$$
$$\alpha^5 = \exp(j\pi/4) = \frac{1}{\sqrt{2}}(1+j)$$
$$\alpha^6 = \exp(-j\pi/2) = 0 - j(1)$$
$$\alpha^7 = \exp(3\pi/4) = \frac{1}{\sqrt{2}}(-1+j)$$

2. $x(t) = 1 + t, \quad -1 \le t \le 1$

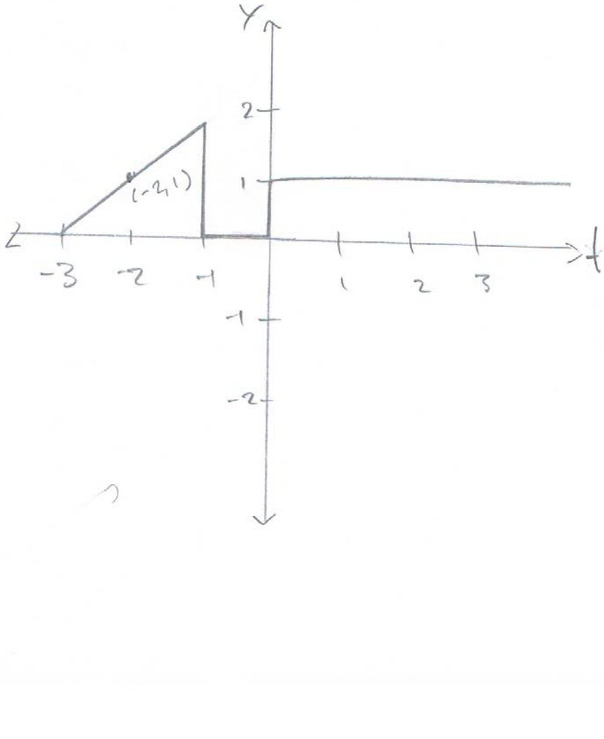
(a) $x(t)$



b) $x(1 + \frac{t}{2}) + 2u(-t+1)$



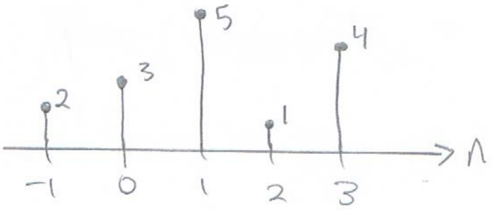
c) $x(t+2) + u(2t)$



3. $x = \{2, \underline{3}, 5, 1, 4\}$, where "_" indicates $n=0$.

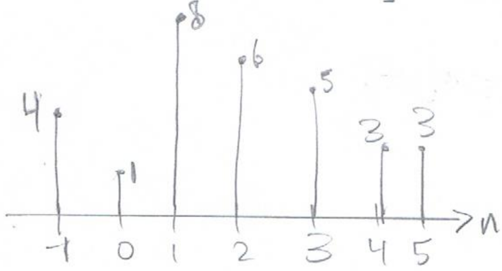
(a) $\text{length}(x) = 5$, $\text{support}(x) = \{-1 \le n \le 3\}$

b)

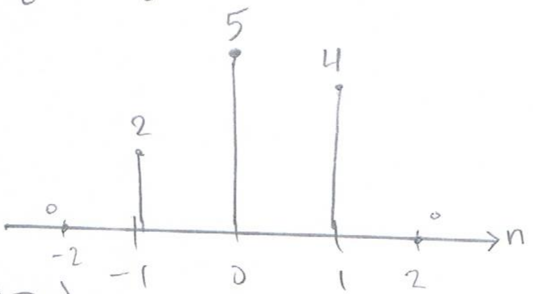


(c) $2\delta[n+1] + 3\delta[n] + 5\delta[n-1] + \delta[n-2] + 4\delta[n-3]$

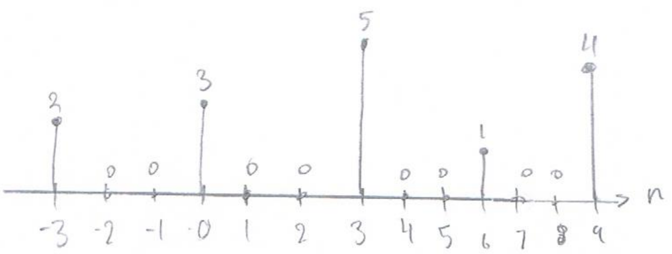
(d) $x[2-n] + 3u[2n-1]$



(e) $x[2n+1]$



f) $(\uparrow 3)x$



4. $5\sqrt{2}\cos(\omega t - \pi/4) + 3\sqrt{2}\cos(\omega t + \pi/4) + 6\cos(\omega t - 3\pi/2)$

$\downarrow$ Phasor domain.

$5\sqrt{2}e^{-j\pi/4} + 3\sqrt{2}e^{j\pi/4} + 6e^{-j3\pi/2}$

$*$ $e^{j\theta} = \cos\theta + j\sin\theta$

$*\downarrow$ Rectangular form

$5\sqrt{2}(\sqrt{2}/2 - j\sqrt{2}/2) + 3\sqrt{2}(\sqrt{2}/2 + j\sqrt{2}/2) + 6(-j)$

$= (5-5j) + (3+3j) + 6j$

$= 8 + 4j$

$\downarrow$ Polar form

$4\sqrt{5}e^{j\tan^{-1}(1/2)}$

$\downarrow$ Time domain

$4\sqrt{5}\cos(\omega t + \tan^{-1}(1/2))$

$\downarrow$ Rectangular Form

$8\cos(\omega t) - 4\sin(\omega t)$

5. (a) $c_{-2} = 1 + j2$, $c_{-1} = 3 - j4$ $\quad (c_{-m} = c_m^*)$

(b) DC Power: $|c_0|^2 = |2|^2 = 4$ W

Fundamental Frequency: $|c_1|^2 + |c_{-1}|^2 = |3+j4|^2 + |3-j4|^2$
$= 50$ W

Second Harmonic: $|c_2|^2 + |c_{-2}|^2 = |1-j2|^2 + |1+j2|$
$= 10$ W

(c) $$||x(t)||^2 = \sum_{m=-\infty}^{\infty} |c_m|^2 = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$X_{rms} = \sqrt{\sum_{m=-\infty}^{\infty} |c_m|^2} = \sqrt{\frac{1}{T}\int_0^T |x(t)|^2 dt}$$

$$X_{rms} = \sqrt{4 + 50 + 10} = \sqrt{64} = 8 \text{ W}$$