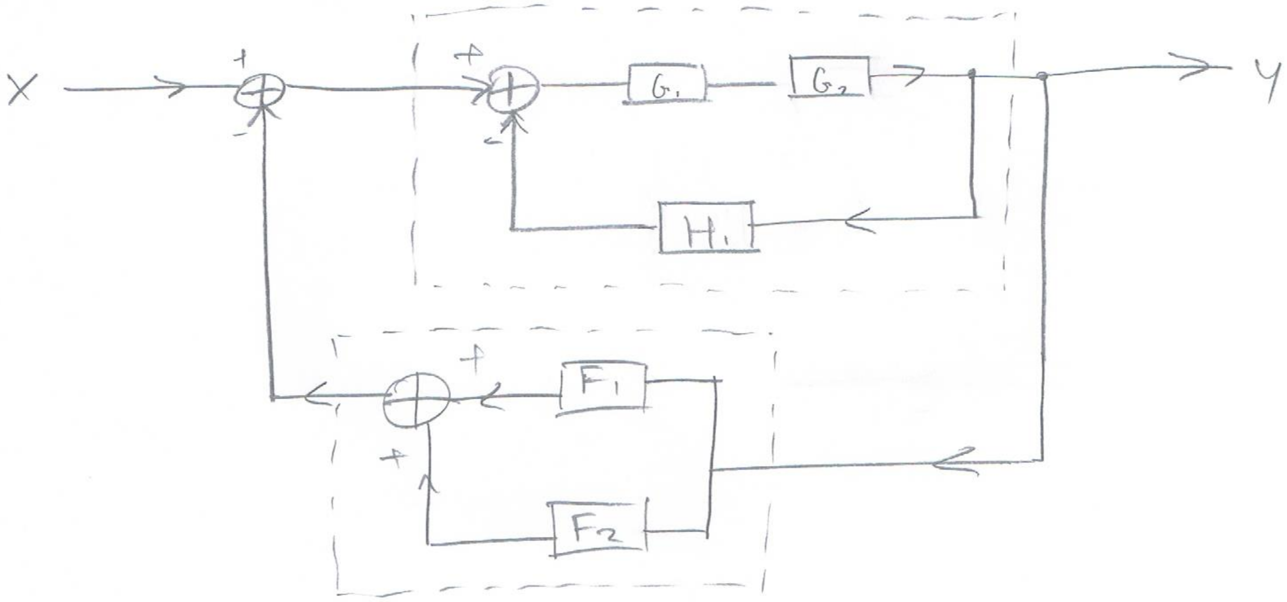
1. $H(z) = \left(\frac{G_1 G_2}{1 + G_1 G_2 H_1}\right) \cdot \frac{1}{1 + \left(\frac{G_1 G_2}{1 + G_1 G_2 H_1}\right)(F_1 + F_2)}$ ✓



4. $X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$, $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w) e^{jwt} dw$ ✓

Let $Y(w) = e^{jwt_0} X^*(w - w_0)$, $y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(w) e^{jwt} dw$

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{jwt_0} X^*(w - w_0) e^{jwt} dw$$

$$= \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jwt_0} e^{-jwt} X(w - w_0) dw \right)^*$$

$$= \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{jw(-(t+t_0))} X(w - w_0) dw \right)^*$$

$$= \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j(w' + w_0)(-(t+t_0))} X(w') dw' \right)^* \quad w' = w - w_0$$

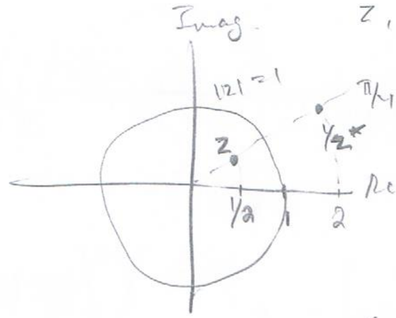
$$= \left(\frac{e^{jw_0(-(t+t_0))}}{2\pi} \int_{-\infty}^{\infty} e^{jw'(-(t+t_0))} X(w') dw' \right)^*$$

$$= \left(e^{jw_0(-(t+t_0))} x(-(t+t_0)) \right)^*$$

$$= e^{jw_0(t+t_0)} x^*(-(t+t_0))$$

5. (a) $z = |z| e^{j\theta}$

$$\frac{1}{z^*} = \frac{1}{|z|} e^{j\theta}$$

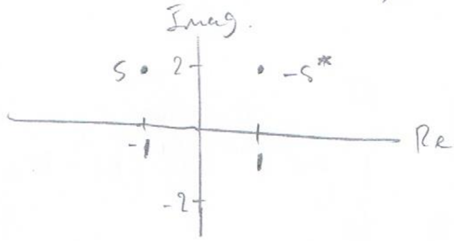


$z, \frac{1}{z^*}$, for $z = \frac{1}{2} e^{j\pi/4}$

$\frac{1}{z^*} = 2e^{j}$

(b) $s = \sigma + j\omega$

$$-s^* = -\sigma + j\omega$$



$s, -s^*$, for $s = -1+j$

$-s^* = 1+j$

(c) (i) $\underline{\tilde{H}(z)}$ (ii) $\underline{\hat{H}(s)}$

$H(z) = H(e^{j\omega}) \to H_{FT}(\omega)$ $\qquad$ $H(s) = H(j\omega) \to H_{FT}(\omega)$

$\hat{H}(z) = H^*\left(\frac{1}{z^*}\right)$ $\qquad$ $\hat{H}(s) = H^*(-s^*)$

$\hat{H}(e^{j\omega}) = H^*(e^{j\omega}) \to H^*_{FT}(\omega)$ $\qquad$ $\hat{H}(j\omega) = H^*(j\omega) \Rightarrow H^*_{FT}(\omega)$

(d) $$H(z) = \frac{b_2 z^2 + b_1 z + b_0}{a_2 z^2 + a_1 z + a_0}$$

$$\begin{aligned}
\hat{H}(z) &= H^*\left(\frac{1}{z^*}\right) \\
&= \left(\frac{b_2\left(\frac{1}{z^*}\right)^2 + b_1\left(\frac{1}{z^*}\right) + b_0}{a_2\left(\frac{1}{z^*}\right)^2 + a_1\left(\frac{1}{z^*}\right) + a_0}\right)^* \\
&= \left[\frac{b_2 + b_1 z^* + b_0 (z^*)^2}{a_2 + a_1 z^* + a_0 (z^*)^2}\right]^* \\
&= \frac{b_2^* + b_1^* z + b_0^* z^2}{a_2^* + a_1^* z + a_0^* z^2}
\end{aligned}$$

Coefficients associated w/ new powers of z.

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

$$\hat{H}(s) = H^*(-s^*)$$

$$= \left[\frac{b_2(-s^*)^2 + b_1(-s^*) + b_0}{a_2(-s^*)^2 + a_1(-s^*) + a_0}\right]^*$$

$$= \left[\frac{b_2(s^2)^* + b_1(-s^*) + b_0}{a_2(s^2)^* + a_1(-s^*) + a_0}\right]^*$$

$$= \frac{b_2^* s^2 - b_1^* s + b_0^*}{a_2^* s^2 - a_1^* s + a_0^*}$$

$$s = \sigma + j\omega$$

$$-s^* = -\sigma + j\omega$$

$$(-s^*)^2 = \sigma^2 - 2j\sigma\omega - \omega^2$$

$$(s^2)^* = \sigma^2 - 2j\sigma\omega - \omega^2$$

$$\Rightarrow (s^2)^* = (-s^*)^2$$

Coefficients associated w/ odd powers of s have a negative sign.

(e) (1) For $L > 0$:

z_L introduces a zero at zero for A & a pole at infinity for $\tilde{A}$.

For $L = 0$:

z_L does not introduce anything new.

For $L < 0$:

z_L introduces a pole at infinity for A. & a zero at zero for $\hat{A}$.

(2) Let $A(z) = \dfrac{1-\alpha^* z}{z-\alpha} \Rightarrow \hat{A}(z) = A^*\left(\dfrac{1}{z^*}\right) = \left(\dfrac{1-\frac{\alpha^*}{z^*}}{\frac{1}{z^*}-\alpha}\right)$

$$A(e^{jw}) = \frac{1-\alpha^* e^{jw}}{e^{jw}-\alpha} \qquad \hat{A}(e^{jw}) = \frac{1-\alpha e^{-jw}}{e^{-jw}-\alpha^*}$$

$$A(e^{jw})\hat{A}(e^{jw}) = \left(\frac{1-\alpha^* e^{jw}}{e^{jw}-\alpha}\right)\left(\frac{1-\alpha e^{-jw}}{e^{-jw}-\alpha^*}\right)$$

$$= \frac{1-\alpha^* e^{jw}-\alpha e^{-jw}+\alpha\alpha^*}{1-\alpha^* e^{jw}-\alpha e^{-jw}+\alpha\alpha^*} = 1 \quad //$$

(3) Let $A(s) = \dfrac{s+\alpha^*}{s-\alpha} \longrightarrow \hat{A}(s) = A^*(-s^*) = \left(\dfrac{-s^*+\alpha^*}{-s^*-\alpha}\right)$

$$A(jw) = \frac{jw+\alpha^*}{jw-\alpha} \qquad \hat{A}(jw) = \frac{-jw+\alpha}{-jw-\alpha^*}$$

$$A(jw)\hat{A}(jw) = \left(\frac{jw+\alpha^*}{jw-\alpha}\right)\left(\frac{-jw+\alpha}{-jw-\alpha^*}\right)$$

$$= \frac{w^2-\alpha^* jw+\alpha jw+\alpha\alpha^*}{w^2-\alpha^* jw+\alpha jw+\alpha\alpha^*} = 1 \quad //$$