The Cooper Union
Department of Electrical Engineering
ECE111 Signal Processing & Systems Analysis
Problem Set VIII System Models
April 18, 2019

1. Draw magnitude Bode plots by hand. In each case, you may use MATLAB (or a calculator) to compute $|H(j\omega)|$ in dB at DC (or at one point, to get you started, if there is a pole or zero at DC).

   (a) Basic case:
$$H(s) = \frac{4(s+4)(s+20)}{(s+1)(s+80)^2}$$

   (b) Look out at DC!
$$H(s) = \frac{4(s+8)^2}{s(s+2)(s+50)}$$

2. Determine the phase at DC and at $\omega = \infty$ for the following transfer function:
$$H(s) = \frac{(s+20)(s+100)}{s(s+5)(s+50)^2}$$

3. Consider the transfer function:
$$H(s) = \frac{\omega_n^2}{s^2 + (\omega_n/Q)s + \omega_n^2}$$

   For purposes of this problem, we will take $\omega_n = 1$, and will examine the effect of Q on the frequency response. We want Bode frequency response plots (magnitude and phase) from $\omega = 0.1 rad/\sec$ to $\omega = 10 rad/\sec$, i.e., one decade above and below the corner at $\omega_n = 1$. You are not allowed to use the MATLAb *bode* function; use *freqs* to compute (but not graph) the frequency response.

   (a) Write a MATLAB function to do the following, given Q. Graph the straight line asymptotes for the magnitude response, and superimpose the actual magnitude response. Also, it should compute the actual magnitude response and compare with the value predicted by the asymptotes at the following frequencies: one decade above and below the corner, and one octave above and below the corner. Finally, it should compute the magnitude response at the corner, and confirm (numerically) that it is $20\log_{10} Q$ above the value predicted by the straight-line asymptotes.

   (b) Run your above function for $Q = 4$ and $Q = 10$. (Obtain separate figures for each case).

   (c) On a separate figure, graph the actual phase responses for $Q = 4$ and $Q = 10$ (superimposed on each other; make sure the two curves are properly labeled, e.g. use *legend*).

4. The **unit step response** of an analog system is:

$$2u(t) + (3t+2)e^{-3t}u(t) + 4e^{-t}\cos(3t+\pi/3)u(t) + 6e^{-5t}u(t)$$

Identify the **dominant mode in the natural response,** and use it to establish a second order system model by specifying the damping factor ζ and natural frequency ω_n.

5. Given the following system determinant:

$$s^2 + Ks + 4K$$

where K is a real number such that the system is stable.

(a) Specify the condition on K for this system to be stable. Assume this holds for the rest of the problem.

(b) Express ω_n and ζ in terms of K.

(c) Specify conditions on K for the system to be underdamped, critically damped and overdamped.

(d) Can every possible value of ζ (subject to the stability constraint) be achieved by proper selection of K?