1. (a) $\psi_1 = s_1$

$$\|\psi_1\|^2 = \int_0^2 \psi_1^2(t)\,dt = \int_0^2 1^2\,dt = 2 \quad \Rightarrow \|\psi_1\| = \sqrt{2}$$

$$\phi_1(t) = \frac{\psi_1(t)}{\|\psi_1(t)\|} = \begin{cases} 1/\sqrt{2} & 0 \le t \le 2 \\ 0 & \text{else} \end{cases}$$

$$\psi_2 = s_2 - \langle s_2, \phi_1 \rangle \phi_1$$

$$= s_2 - (4/\sqrt{2})\,\phi_1(t)$$

$$= \begin{cases} 2 & 2 \le t \le 3 \\ 0 & \text{else} \end{cases}$$

$$\langle s_2, \phi_1 \rangle = \int_0^3 s_2(t)\,\phi_1(t)\,dt = \int_0^2 2 \cdot \frac{1}{\sqrt{2}}\,dt = 4/\sqrt{2}$$

$$\|\psi_2\|^2 = \int_2^3 \psi_2^2(t)\,dt = \int_2^3 2^2\,dt = 4$$

$$\Rightarrow \|\psi_2\| = 2$$

$$\phi_2(t) = \frac{\psi_2(t)}{\|\psi_2(t)\|} = \begin{cases} 1 & 2 \le t \le 3 \\ 0 & \text{else} \end{cases}$$

$$\psi_3 = s_3 - \langle s_3, \phi_2 \rangle \phi_2 - \langle s_3, \phi_1 \rangle \phi_1 \longrightarrow$$

$$\langle s_3, \phi_2 \rangle = \int_2^3 (1)(1)\,dt = 1$$

$$\langle s_3, \phi_1 \rangle = \int_0^1 (1)(1/\sqrt{2})\,dt = 1/\sqrt{2}$$

$$\psi_3 = s_3 - \langle s_3, \phi_2 \rangle \phi_2 - \langle s_3, \phi_1 \rangle \phi_1$$

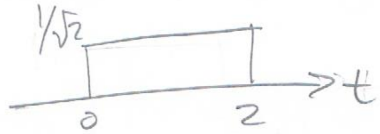
$$= s_3 - (1)\phi_2 - (1/\sqrt{2})\phi_1$$

$$= \begin{cases} 1/2 & 0 \le t \le 1 \\ -1/2 & 1 \le t \le 2 \\ 0 & \text{else} \end{cases}$$

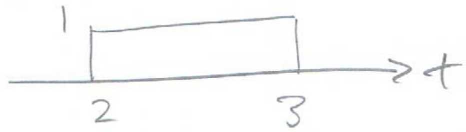
$$\|\psi_3\|^2 = \int_0^2 (1/2)^2 \, dt = 1/2 \implies \|\psi_3\| = 1/\sqrt{2}$$

$$\phi_3 = \left\{ \begin{array}{ll} 1/\sqrt{2} & 0 \le t \le 1 \\ -1/\sqrt{2} & 1 \le t \le 2 \\ 0 & \end{array} \right\}$$

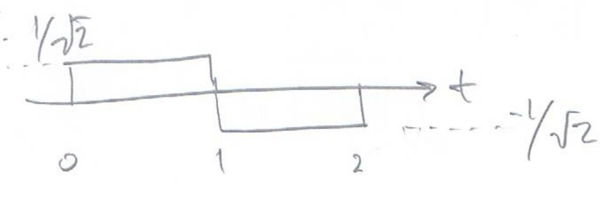
$\{\phi_i(t)\}_{i=1}^{3}$: ϕ_1 :



ϕ_2 :



ϕ_3 :



(b) $s_1(t) = \|\psi_1\| \phi_1(t)$

$s_2(t) = \langle s_2, \phi_1 \rangle \phi_1(t) + \|\psi_2\| \phi_2(t)$

$s_3(t) = \langle s_3, \phi_1 \rangle \phi_1(t) + \langle s_3, \phi_2 \rangle \phi_2(t) + \|\psi_3\| \phi_3(t)$
$= (1/\sqrt{2}) \phi_1(t) + (1) \phi_2(t) + (1/\sqrt{2}) \phi_3(t)$

$\Rightarrow \vec{s}_3 = \begin{bmatrix} 1/\sqrt{2} \\ 1 \\ 1/\sqrt{2} \end{bmatrix}$ wrt the basis $\{\phi_i\}_{i=1}^{3}$

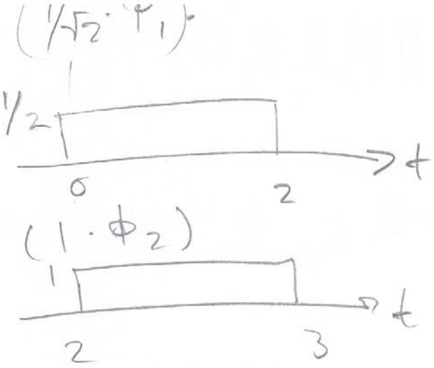
$\hat{s}_3 = \begin{bmatrix} 1/\sqrt{2} \\ 1 \\ 0 \end{bmatrix}$ (projection of $\vec{s}_3$ on span$\{\phi_1, \phi_2\}$)

(c) $\|\vec{s}_3\|^2 = (1/\sqrt{2})^2 + (1)^2 + (1/\sqrt{2})^2$
$= 1/2 + 1 + 1/2$
$= 2 \quad \Rightarrow \|\vec{s}_3\| = \sqrt{2}$

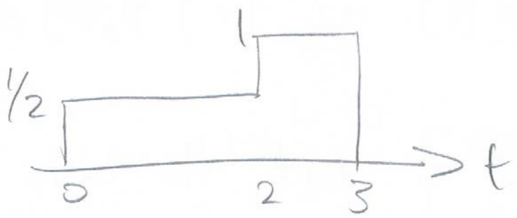
$\|s_3\|^2 = \int_0^1 1^2 dt + \int_2^3 1^2 dt = 2 \Rightarrow \|s_3\| = \sqrt{2}$

Thus, $\|\vec{s}_3\| = \|s_3\| = \sqrt{2}$

(d) $\hat{s}_3 = \begin{bmatrix} 1/\sqrt{2} \\ 1 \\ 0 \end{bmatrix}$



$\hat{s}_3$:



(e) $e(t) = s_3(t) - \hat{s}_3(t)$

$$= \begin{bmatrix} 1/\sqrt{2} \\ 1 \\ 1/\sqrt{2} \end{bmatrix} - \begin{bmatrix} 1/\sqrt{2} \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$||e||^2 = (1/\sqrt{2})^2 = 1/2$$

$$\Rightarrow ||e|| = 1/\sqrt{2}$$

* by orthogonality

Alternatively: $||e||^2 = ||s_3 - \hat{s}_3||^2 = ||s_3||^2 - ||\hat{s}_3||^2$

$$= 2 - 3/2$$

$$= 1/2$$

$$\Rightarrow ||e|| = 1/\sqrt{2}$$

2. (a) $G_{DB} = 20 \log_{10} 100$

$= 40 dB$

(b) $\text{Output, DB} = \text{Input, DB} + G_{DB}$

$= -10 \text{ dBm} + 40 \text{dBm}$

$= 30 \text{dBm}$

(c) $30 = 10 \log_{10} \frac{P_{OUT}}{10^{-3}}$

$\Rightarrow 3 = \log_{10} \frac{P_{OUT}}{10^{-3}}$

$\Rightarrow 10^3 = \frac{P_{OUT}}{10^{-3}}$

$\Rightarrow P_{OUT} = 10^3 \cdot 10^{-3}$

$= 1 W$

3. (a) 20 kHz → 70 kHz, 120 kHz, 170 Hz

-20 kHz → 30 kHz, 80 kHz, 130 kHz, 180 kHz

All these frequencies would alias into 20 kHz

(b) Cutoff frequency : $f_c = f_s/2 = 25$ kHz
(for the LPF for anti-aliasing)

↳ so that the signal can be uniquely reconstructed.

Input frequency: 20 kHz

(c) Analog radian frequency: $.4\pi \times 10^5$ rad/sec

Normalized Digital Radian Frequency: $\frac{4}{5}\pi$ rad.

Fraction of the Sampling Rate: $2/5$

Fraction of the Nyquist Bandwidth: $4/5$

A.R.F : $\omega = 2\pi f = 2\pi(20 \times 10^3) \frac{rad}{sec} = .4\pi \times 10^5 \frac{rad}{sec}$

N.D.R.F.: $\omega = \frac{2\pi f}{f_s} = \frac{2\pi(20 \times 10^3)}{(50 \times 10^3)}$ rad $= \frac{4}{5}\pi$ rad

S.R. : $f_s \Rightarrow 1$, $f = f/f_s = 20 \times 10^3 / 50 \times 10^3 = \frac{2}{5}$

N.B. : $\frac{f_s}{2} \Rightarrow 1$, $f = f/f_{s/2} = 20 \times 10^3 / 25 \times 10^3 = \frac{4}{5}$

4. (a) Valid choices for f_0: 3.7 GHz
4.3 GHz

(b) $f_0 = 3.7$ GHz
$\Rightarrow$ Must filter out 3.4 GHz before mixing

$f_0 = 4.3$ GHz
$\Rightarrow$ Must filter out 4.6 GHz before mixing

Thus, for $f_0 = 3.7$ GHz the image frequency is 3.4 GHz
& for $f_0 = 4.3$ GHz the image frequency is 4.6 GHz

~~(c)~~ $\pm f_0 \pm m f_s$, where $f_s = 30\text{kHz}$

=> Six lowest frequencies @ output are:

10 kHz, 20 kHz, 40 kHz, 50 kHz, 70 kHz, 80 kHz

(d) $\frac{2}{5} f_s = 12\text{kHz}$ -> 12kHz, 42kHz, 72kHz

~~12kHz~~, 18kHz, 48kHz, 78kHz

(e) Cutoff frequency: $f_c = f_s/2 = 15\text{kHz}$ (for anti-imaging)

(for LPF)

5. (a) $f_{inst}(t) = \frac{d}{dt}\left[2\pi f_c t + \beta \sin(2\pi f_m t) + \phi_0\right] \cdot \frac{1}{2\pi}$

$= \left[2\pi f_c + 2\pi \beta f_m \cos(2\pi f_m t)\right] \cdot \frac{1}{2\pi}$

$= f_c + \beta f_m \cos(2\pi f_m t)$

DC-Value $= \frac{1}{T}\int_0^T f_{inst}(t)\,dt = f_c$ (integral of cosine from 0 to T is 0)

$f_{inst}(t) - f_c = \beta f_m \cos(2\pi f_m t)$

Also, as $-1 \le \cos\theta \le 1$, $|f_{inst} - f_c| \le \beta f_m = B/2$

$B = f_{max} - f_{min}$
$= (f_c + \beta f_m) - (f_c - \beta f_m)$
$= 2\beta f_m$
$= 2KA_m$

$\Rightarrow B/2 = \beta f_m$; $\beta = \frac{KA_m}{f_m}$

$B = 2KA_m$

(b) $s(t) = Re\left(s_{BB}(t) \cdot e^{j2\pi f_0 t}\right)$, (where $f_0 = f_m$)

$\Rightarrow s_{BB}(t) = A_c e^{j\phi(t)}$, where $\phi(t) = \beta \sin(2\pi f_m t) + \phi_0$

Finding T: $s_{BB}(t+T) = s_{BB}(t)$ $\forall t$ ($s_{BB}(t)$ is periodic w/ period T)

$\Rightarrow A_c e^{j\phi(t+T)} = A_c e^{j\phi(t)}$

$\Rightarrow \sin(2\pi f_m t) = \sin(2\pi f_m (t+T))$

$\Rightarrow 2\pi f_m T = 2\pi$

$\Rightarrow T = 1/f_m$

given $T = 1/f_m = 1/f_0$, $S_{BB}(f)$ has spectral components at DC and $\pm f_0$, $\pm 2f_0$, ... as $f_0 = f_m$

Moreover, as $B = 2f_0 = 2f_m$, the bandwidth does not vary w/ amplitude of the signal.

(c) $$B_{SUM} = B_A + B_B$$
$$= 2kA_m + 2f_m$$
$$= 2\beta f_m + 2f_m$$
$$= 2f_m(1+\beta)$$
$$= B_{carson's}$$