1. (a) $|H(j\omega)| = \dfrac{4(s+4)(s+20)}{(s+1)(s+80)^2}$

$s+1$: Pole @-1 of multiplicity 1, Corner Frequency Breakpoint : $\omega = 1$
$-20\frac{dB}{dec}$ slope

$s+4$: Zero @-4 of multiplicity 1, Corner Frequency Breakpoint : $\omega = 4$
$+20\frac{dB}{dec}$ slope

$s+20$: Zero @-20 of multiplicity 1, Corner Frequency Breakpoint : $\omega = 20$
$+20\frac{dB}{dec}$ slope

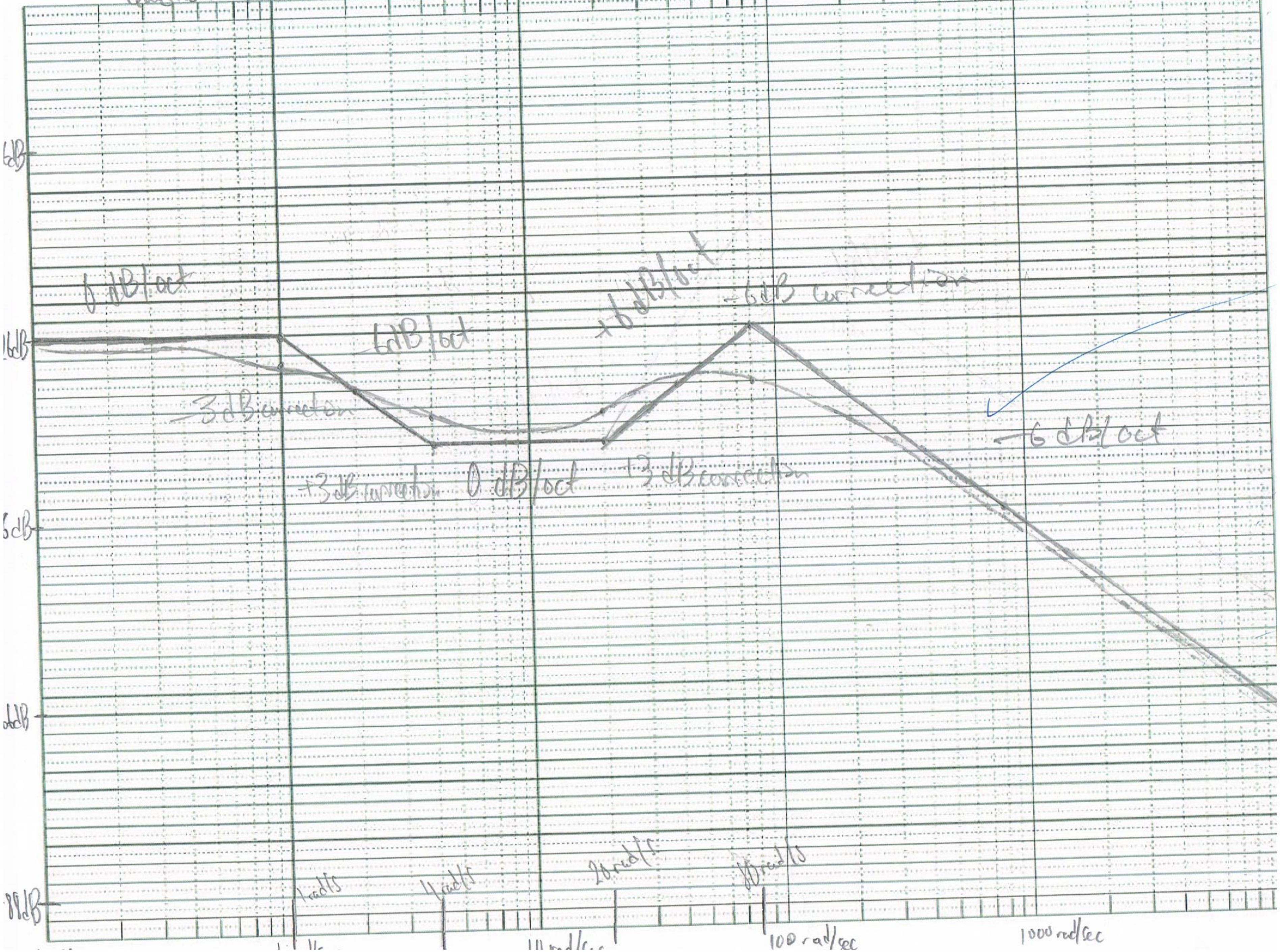
$s+80$: Pole @-80 of multiplicity 2, Corner Frequency Breakpoint : $\omega = 40$
$-40\frac{dB}{dec}$ slope

$$H(\omega) = \frac{4(j\omega+4)(j\omega+20)}{(j\omega+1)(j\omega+80)^2}$$

@ $\omega = 0$: $$|H(0)| = \left| \frac{4(4)(20)}{(1)(80)^2} \right|$$

$$20\log 10(|H(0)|) \approx -26 \text{ dB}$$

Question 1 A



0 dB/oct
-6dB/oct
-3dB correction
+3dB correction
0 dB/oct
+3dB correction
+6dB/oct
-6dB correction
-6 dB/oct
1 rad/s
4 rad/s
20 rad/s
80 rad/s
10 rad/sec
100 rad/sec
1000 rad/sec
16dB
88dB

1.(b) $H(s) = \dfrac{4(s+8)^2}{s(s+2)(s+50)}$

$s+0$: Pole @ 0, m=1, DC
-20 dB/dec slope

$s+2$: Pole @ -2, m=1, Breakpoint @ $w = 2$ rad/s
-20 dB/dec slope

$s+8$: Zero @ -8, m=2, Breakpoint @ $w = 8$ rad/s
+40 dB/dec slope

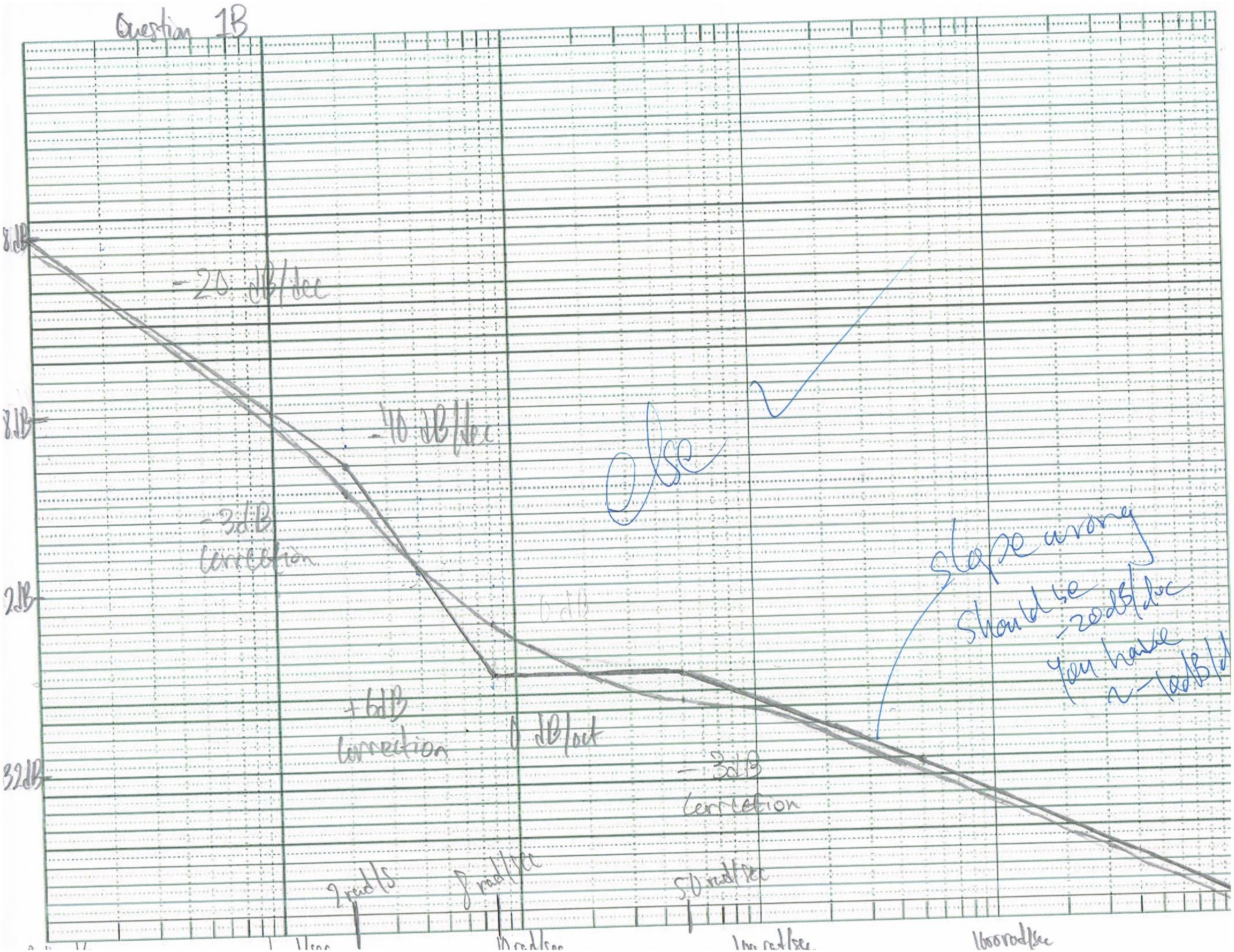
$s+50$: Pole @ -50, m=1, Breakpoint @ $w = 50$ rad/s
-20 dB/dec slope

$$H(w) = \frac{4(jw+8)^2}{(jw)(jw+2)(jw+50)}$$

@ w=0 : $|H(0.1)| = \left| \dfrac{4(j0.1+8)^2}{(j0.1)(j0.1+2)(j0.1+50)} \right|$

$20\log_{10}(|H(0.1)|) \approx 28\,dB$

Question 1B



8dB
8dB
2dB
32dB
-20 dB/dec
-40 dB/dec
-3dB correction
0 dB
+6dB correction
0 dB/oct
-3dB correction
2rad/s
8rad/sec
50rad/sec
1 r/sec
10 rad/sec
100 rad/sec
1000rad/sec
slope wrong
Should be -20dB/dec
You have ~-10dB/d

2. $H(s) = \dfrac{(s+20)(s+100)}{s(s+5)(s+50)^2}$

$H(\omega) = \dfrac{(j\omega+20)(j\omega+100)}{j\omega(j\omega+5)(j\omega+50)^2}$

DC: $-90°$ (due to pole @ zero)

∞ : $-180°$ (due to remaining two zeros and three poles ⇒ net of one pole)

4. Unit Step Response: $2u(t) + (3t+2)e^{-3t}u(t)$
$+ 4e^{-t}\cos(3t + \pi/3)u(t) + 6e^{-5t}u(t)$

Dominant Mode in the Natural Response:

$4e^{-t}\cos(3t + \pi/3)\,u(t)$ (Dominant Pole(s): $-1 \pm j3$)

$\downarrow$

$a_0 e^{-\alpha t}\cos(\beta t + \theta_0)u(t)$

$$\Rightarrow \left.\begin{aligned} \alpha &= 1 = \zeta\omega_n \\ \beta &= 3 = \sqrt{1-\zeta^2}\,\omega_n \end{aligned}\right\} \quad \begin{aligned} \zeta &= \frac{1}{\sqrt{10}} \\ \omega_n &= \sqrt{10} \end{aligned}$$

Second order system model: $s^2 + 2\zeta\omega_n s + \omega_n^2$

$= s^2 + 2s + 10$

5. system determinant: $s^2 + Ks + 4K$, K real s.t. system is stable

(a) $K = 2\zeta\omega_n \rightarrow \zeta = \frac{\sqrt{K}}{4} \Rightarrow K > 0$

+ (b) $4K = \omega_n^2 \rightarrow \omega_n = 2\sqrt{K}$

(c) $\zeta < 1$: Underdamped, $0 < K < 16$

$\zeta = 1$: critically damped, $K = 16$

$\zeta > 1$: overdamped, $K > 16$

(d) Yes, the range of $\zeta = \frac{\sqrt{K}}{4}$ is $(0, \infty)$ for $K > 0$.