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ECE211 Signal Processing & Systems Analysis
Problem Set III: Systems
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1. The waveforms $h(t)$, $x(t)$ are shown in **Figure 1.** Use the flip and slide method to compute and sketch $y = h * x$. Draw several sketches indicating the "flip and slide" action, where $x(t)$ is the one being flipped and slid. **Note:** You can just find the values at "boundary points" and connect them with piecewise linear segments, as discussed in class.

2. The discrete-time signals h, x are given as follows:

$$h = \{2, \underline{3}, 4, 2, 5\} \quad x = \{3, 4, \underline{5}, 1, 2\}$$

where the underscore denotes $n = 0$. Let $y = h * x$. DO NOT COMPUTE ALL y EXCEPT AS NOTED BELOW.

   (a) Specify the length and support of h, x and y.

   (b) In MATLAB represent a finite duration discrete time vector with a pair of variables: the data values in a row vector, and a separate variable that indicates the start time (i.e., the time index n for the first entry in the vector). Then write code to use a stem plot to graph such a function (with the time axis labeled properly). Use the MATLAB *conv* function to convolve these objects, but also to compute the start time of the result.

   (c) Use your code to sketch h, x, and y as three subplots, with time axes labeled properly.

   (d) BY HAND compute $y[1]$ as a sum of $h[?] \cdot x[?]$ terms (clearly indicate the indices), and then evaluate. Also, (by hand) draw a sketch illustrating how you would use "flip and slide" to compute $y[1]$.

3. **Noble Identities for Multirate Processing**

The two basic operations in multirate digital signal processing are *decimation (downsampling)* and *upsampling.* For an integer $M \geq 2$, define decimation by M, denoted $y = (\downarrow M)\, x$, as $y[n] = x[Mn]$. That is, we keep one out of every M samples of x, and discard the rest. Upsampling by M, denoted $y = (\uparrow M)\, x$, means $y[Mn] = x[n]$ and $y[n] = 0$ if n is not an integer multiple of M. That is, the samples x are spread apart by a factor of M, with 0 insertion. For example, $(\uparrow 2)\, x$ is nonzero only at even times. Note that M is called the "decimation factor" for both $(\uparrow M)$ and $(\downarrow M)$.

The notation $y = Hx$ is used to denote an LTI operation, i.e., $y = h * x$. In general, decimation and upsampling are time-varying, so they cannot be interchanged with each other, or LTI operations. However, there is an important result related to the idea of interchanging such operations. They are called the *Noble identities.*

The notation AB means apply B first then A, for example: $y = ABx = A(Bx)$.

Assume $h = (\uparrow M)\, g$. Specifically, this means $h[n] = 0$ if n is not an integer multiple of M. Then the Noble identities are:

$$\begin{aligned} (\downarrow M)\, H &= G\, (\downarrow M) \\ H\, (\uparrow M) &= (\uparrow M)\, G \end{aligned}$$

For simplicity in this problem, all signals are assumed to start at $n = 0$, and you can assume in your MATLAB code a signal is represented as a row vector with first entry corresponding to $n = 0$. (In particular, if say $x[0] = 0$, then the MATLAB vector $x = [0\ \cdots\,)$.

(a) Write MATLAB functions called *dec* and *ups* (note: DO NOT USE THE MATLAB FUNCTION *decimate* or others, write your own) that will decimate and upsample signals; each function should not require more than a line or two of code.

(b) Take $g = \{2, 1, 3, 4\}$ with support $\{0 \le n \le 3\}$, and $h = (\uparrow 4)\, g$. Generate x as a vector of length 10, comprised of random integers in the range -2 to 2, assuming support $\{0 \le n \le 9\}$. On three subplots, obtain stem plots of g, h, x.

(c) Compute $(\downarrow 4)\, (\uparrow 4)\, x$, and $(\uparrow 4)\, (\downarrow 4)\, x$, and plot each (separate plots, via subplot).

(d) Let $d1 = (\downarrow 4)\, Hx$, $d2 = G\, (\downarrow 4)\, x$. Check that they match. Plot the result.

(e) Let $u1 = H\, (\uparrow 4)\, x$, $u2 = (\uparrow 4)\, Gx$. Check that they match. Plot the result.

(f) Here I will ask you to prove one of the Noble identities. Assume $h = (\uparrow M)\, g$. Let $v = Hx$, and $y = (\downarrow M)\, v$. First, write the convolution sum $v = h * x$, discard the indices where h is 0, substitute in the g values, and then go to $y = (\downarrow M)\, v$. Now start again, letting first $u = (\downarrow M)\, x$ and write $y' = Gu$; write out the convolution formula $y' = g * u$ and check that $y = y'$.

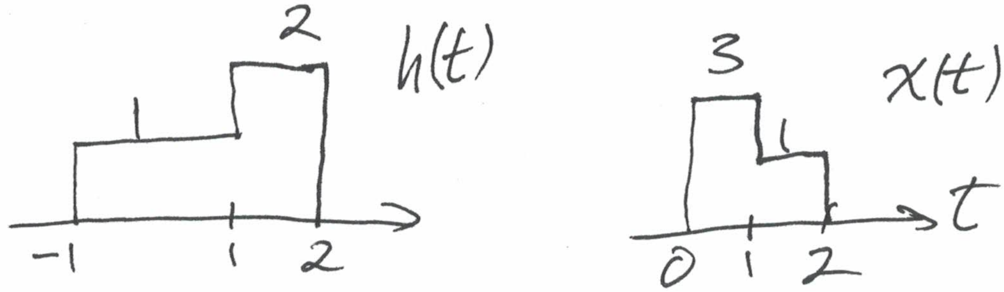

FIGURE 1: Convolving Two Signals