1. (a) $e_k[n] \xrightarrow[\text{trans.}]{z} E_k(z) = \sum_{n=-\infty}^{\infty} e_k[n] z^{-n}$

For each $n \in \mathbb{Z}$ $\exists!$ m, k, $0 \le k \le M-1$ s.t. $n = Mm+k$.

$$E_k(z) = \sum_{m=-\infty}^{\infty} e_k[m] z^{-m}$$

$$E_k(z^M) = \sum_{m=-\infty}^{\infty} e_k[m] (z^M)^{-m}$$

$$z^{-k} E_k(z^M) = z^{-k} \sum_{m=-\infty}^{\infty} e_k[m] (z^M)^{-m}$$

$$= \sum_{m=-\infty}^{\infty} h[Mm+k] z^{-Mm-k}$$

$e_k[m] = h[Mm+k]$

$$= \sum_{m=-\infty}^{\infty} h[Mm+k] z^{-(Mm+k)}$$

$n = Mm+k$

$$= \sum_{m=-\infty}^{\infty} h[n] z^{-n}$$

$\Rightarrow$ z^{-n} can only occur in the $z^{-k} E_k(z^M)$ term.

(b) $e_k[m]$ is the unique coefficient, as we are summing wrt m. (It is different for each m)

(c) $e_k[m] = h[Mm+k] = h[n]$

$e_k[n] = h[Mn+k]$ $\qquad$ $n = Mm+k$

2. (a) $H(z) = \dfrac{2(z-3)(3z-2)^2}{(z-4)^2(2z+1)^2}$

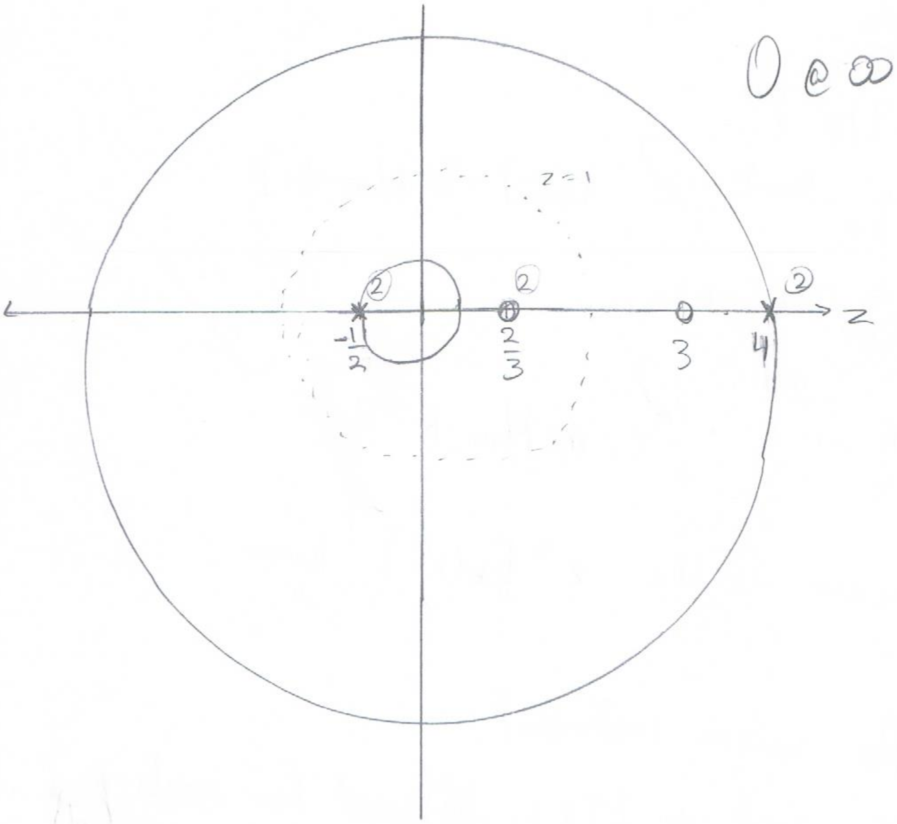
Poles: $z = -1/2$, mult = 2

$z = 4$, mult = 2

Zeros: $z = 2/3$, mult = 2

$z = 3$, mult = 1

$z = \infty$, mult = 1



(b), (c) All possible ROC: I. $|z| < 1/2$

II. $1/2 < |z| < 4$ → Stable, well-defined freq. response

III. $4 < |z| \leq \infty$ → Causal

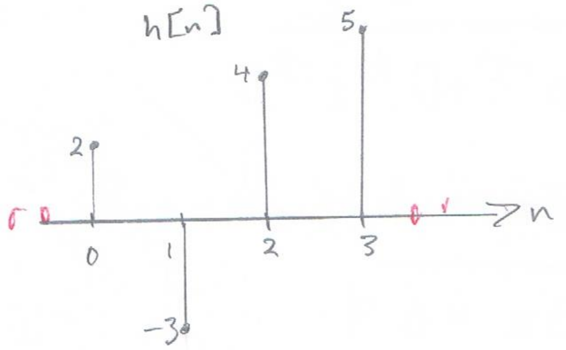
(d) $L \leq 1$

3. $H(z) = 2 - 3z^{-1} + 4z^{-2} + 5z^{-3}$

$h[n] \xrightarrow[\text{trans.}]{z} \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$

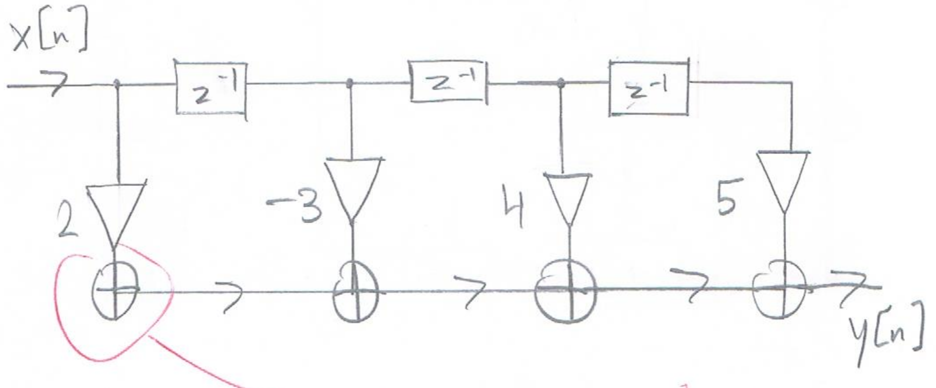
$\Rightarrow h = \{\underline{2}, -3, 4, 5\}$

(a)



(b) $h[n] = 2\delta[n] - 3\delta[n-1] + 4\delta[n-2] + 5\delta[n-3]$

(c)



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(d) Length : 4

Order : 3

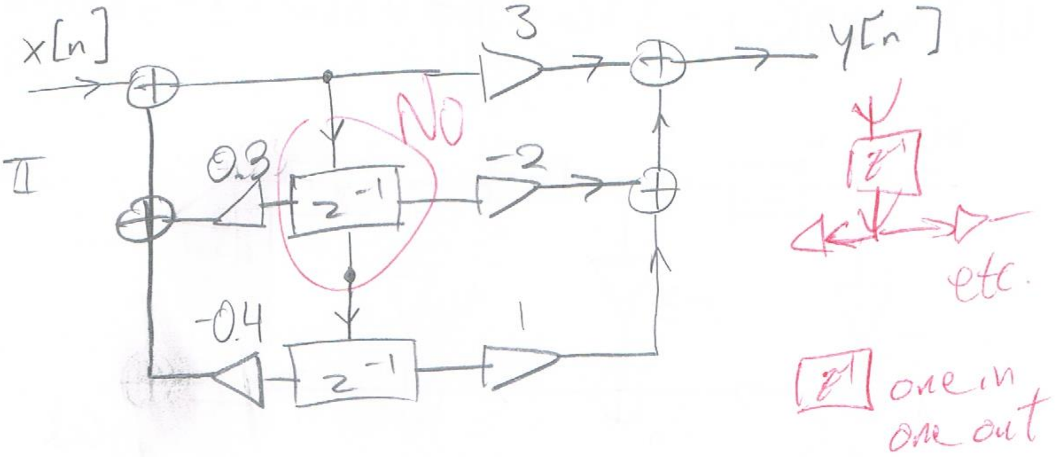
4. $y[n] = 3x[n] - 2x[n-1] + x[n-2] + 0.3y[n-1] - 0.4y[n-2]$

(a) $$H(z) = \frac{3 - 2z^{-1} + z^{-2}}{1 - 0.3z^{-1} + 0.4z^{-2}}$$

$$= \frac{3z^2 - 2z + 1}{z^2 - 0.3z + 0.4}$$

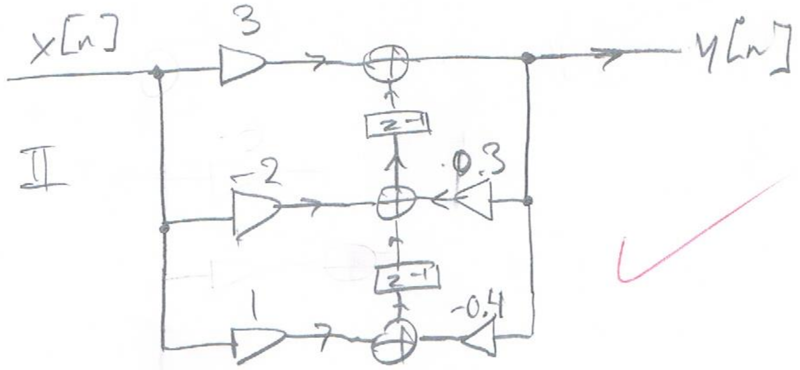
(b)

Direct Form II



(c)

Direct Form II Transposed



5. $W_M = e^{-j2\pi/M}$

$$X(z) \longrightarrow X_{DTFT}(\omega)$$

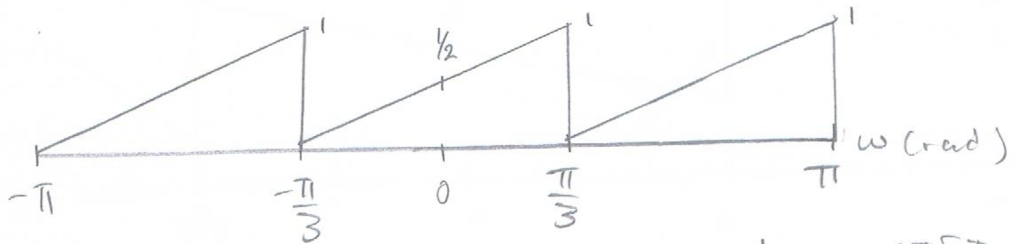
$$Y(z) = X(zW_M^k) = X\left(z\left(e^{-j2\pi/M}\right)^k\right) = X\left(ze^{-j2\pi k/M}\right)$$

$$\downarrow$$

$$Y_{DTFT}(\omega) = X_{\cancel{DTFT}}\left(e^{j\omega} e^{-j2\pi k/M}\right) = X_{DTFT}(\omega - 2\pi k/M)$$

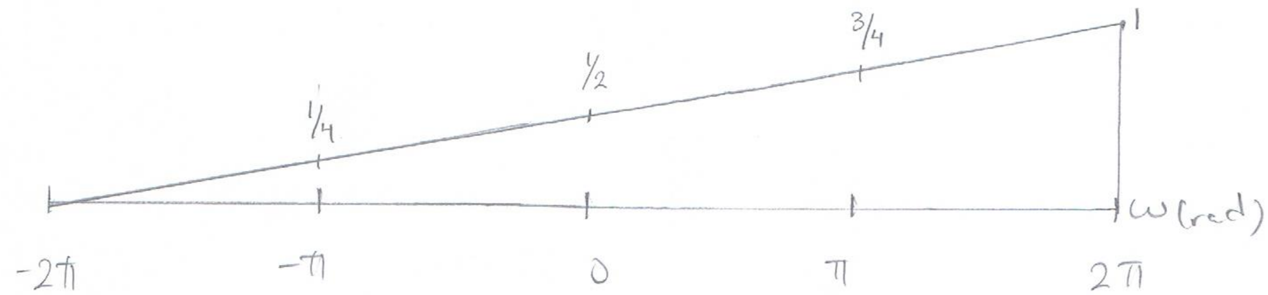
$$= X_{DTFT}\left(e^{j\left(\omega - \frac{2\pi k}{M}\right)}\right)$$

6. (a) $X(3\omega)$, $-\pi \le \omega \le \pi$

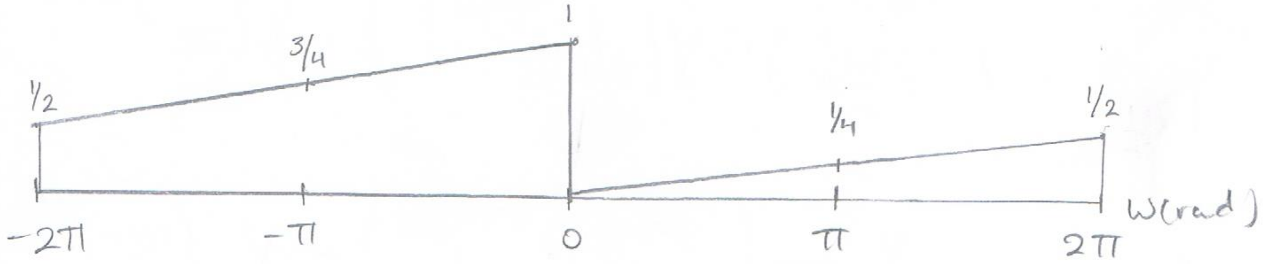


There is a discrete-time signal whose DTFT is $X(3\omega)$ as $X(3\omega)$ is 2π periodic.

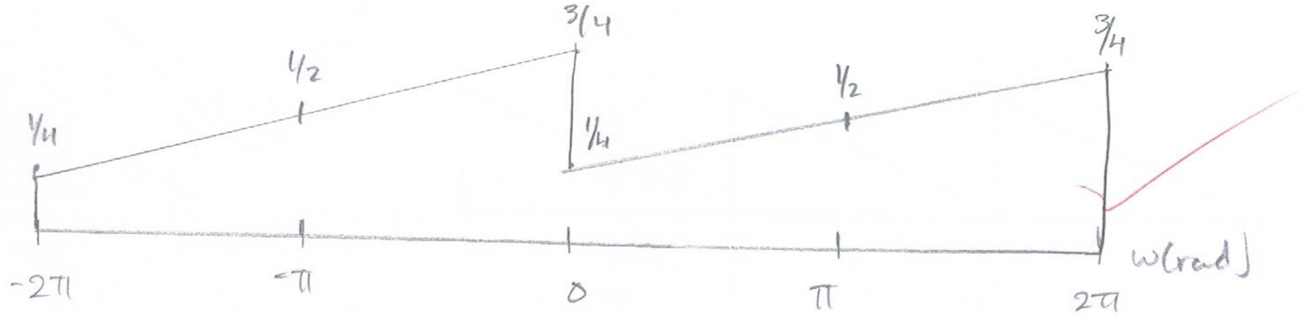
(b) $X\left(\frac{\omega}{2}\right)$, $-2\pi \le \omega \le 2\pi$



$$X\left(\frac{w}{2}+\pi\right), \quad -2\pi \le w \le 2\pi$$



$$\frac{1}{2}\left[X\left(\frac{W}{2}\right) + X\left(\frac{W}{2}+\pi\right)\right], \quad -2\pi \le W \le 2\pi$$



The last digital spectrum corresponds to a valid DTFT of a discrete-time signal as it is 2π periodic.