1. $H_1(s) = \dfrac{3s^3+1}{(s+3)(s+5)} = \dfrac{3s^3+1}{s^2+8s+15} = 3s - 24 + \dfrac{147s+361}{s^2+8s+15}$

$$\begin{array}{r} 3s-24 \\ s^2+8s+15 \overline{\smash{)}\,3s^3+0s^2+0s+1} \\ -(3s^3+24s^2+45s+0) \\ \hline -24s^2-45s+1 \\ -(-24s^2-192s-360) \\ \hline 147s+361 \end{array}$$

$= 3s - 24 + \dfrac{A}{(s+3)} + \dfrac{B}{(s+5)}$

$= 3s - 24 - \dfrac{40}{(s+3)} + \dfrac{187}{(s+5)}$

$A = \left.\dfrac{\cancel{147s+361}\ 3s^5+1}{(s+5)}\right|_{s=-3} = -40$

$B = \left.\dfrac{\cancel{147s+361}\ 3s^3+1}{(s+3)}\right|_{s=-5} = 187$

$H_1(s) = 3s - 24 - \dfrac{40}{(s+3)} + \dfrac{187}{(s+5)}$

Inverse laplace Transform.

$h_1(t) = \cancel{3u(t)} - 24\delta(t) - 40e^{-3t}u(t) + 187e^{-5t}u(t)$

$3\delta'(t)$

oops!

DO NOT USE THE REMAINDER POLYNOMIAL

$A = \left.\dfrac{3s^3+1}{s+5}\right|_{s=-3} = \dfrac{-80}{2} = -40$

$B = \left.\dfrac{3s^3+1}{s+3}\right|_{s=-5} = \dfrac{-374}{-2} = 187$

$$H_2(s) = \frac{4s+3}{(s+2)^2(2s+1)} = \frac{4s+3}{2(s+2)^2(s+1/2)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(s+1/2)}$$

$$= \frac{-2/9}{(s+2)} + \frac{5/3}{(s+2)^2} + \frac{2/9}{(s+1/2)}$$

$$A = \left[\frac{4s+3}{2(s+1/2)}\right]' \Big|_{s=-2} = \frac{1}{2}\left(\frac{4(s+1/2) - (4s+3)(s)}{[s+1/2]^2}\right)\Big|_{s=-2} = -2/9$$

$$B = \frac{4s+3}{2(s+1/2)}\Big|_{s=-2} = 5/3$$

$$C = \frac{4s+3}{2(s+2)^2}\Big|_{s=-1/2} = 2/9$$

$$H_2(s) = \frac{-2/9}{(s+2)} + \frac{5/3}{(s+2)^2} + \frac{2/9}{(s+1/2)}$$

Inverse laplace Transform

$$h_2(t) = -\frac{2}{9}e^{-2t}u(t) + \frac{5}{3}te^{-2t}u(t) + \frac{2}{9}e^{-1/2t}u(t)$$

$$H_3(s) = \frac{5s+2}{(s^2+4s+5)(s+4)}$$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{4^2-4(1)(5)}}{2(1)} = -2 \pm j$$

$$= \frac{5s+2}{[s-(-2+j)][s-(-2-j)](s+4)}$$

$$= \frac{A}{[s-(-2+j)]} + \frac{B}{[s-(-2-j)]} + \frac{C}{(s+4)}$$

$$= \frac{\left(\frac{18+j11}{10}\right)}{[s-(-2+j)]} + \frac{\left(\frac{18-j4}{10}\right)}{[s-(-2-j)]} + \frac{\left(\frac{-18}{5}\right)}{(s+4)}$$

$$A = \left.\frac{5s+2}{[s-(-2-j)](s+4)}\right|_{s=-2+j} = \frac{5(-2+j)+2}{[(-2+j)-(-2-j)](-2+j+4)}$$

$$= \frac{-10+5j+2}{2j(j+2)}$$

$$= \frac{-8+5j}{-2+4j}\left(\frac{-2-4j}{-2-4j}\right)$$

$$= \frac{18+j11}{10}$$

$$B = A^*$$

$$C = \left.\frac{5s+2}{[s-(-2+j)][s-(-2-j)]}\right|_{s=-4} = \frac{-18}{5}$$

$$h_3(t) = -\frac{18}{5}e^{-4t}u(t) + \left[\left(\frac{18+j11}{10}\right)e^{-2t}e^{jt} + \left(\frac{18-j11}{10}\right)e^{-2t}e^{jt}\right]u(t)$$

$$= \frac{-18}{5}e^{-4t}u(t) + 2\,\mathrm{Re}\left(\left[\frac{18+j11}{10}\right]e^{jt}\right)u(t)$$

$$= -\frac{18}{5}e^{-4t}u(t) + \frac{36}{10}e^{-2t}\cos(t)u(t) - \frac{11}{5}e^{-2t}\sin(t)u(t)$$

Oops – forgot

2. $$H_1(z) = \frac{z^3+1}{(3z+1)(2z-1)}$$

$$z^{-1}H_1(z) = \frac{z^3+1}{6z(z+1/3)(z-1/2)} = \frac{1}{6} + \frac{A}{z} + \frac{B}{z+1/3} + \frac{C}{z-1/2}$$

$$A = \left.\frac{z^3+1}{6(z+1/3)(z-1/2)}\right|_{z=0} = \frac{1}{6(\frac{1}{3})(-\frac{1}{2})} = -1$$

$$B = \left.\frac{z^3+1}{6z(z-1/2)}\right|_{z=-1/3} = \frac{\frac{-1}{27}+1}{-2(\frac{-5}{6})} = \frac{\frac{26}{27}}{\frac{5}{3}} = \frac{26}{27}\cdot\frac{3}{5} = \frac{26}{9\cdot 5} = \frac{26}{45}$$

$$C = \left.\frac{z^3+1}{6z(z+1/3)}\right|_{z=1/2} = \frac{\frac{1}{8}+1}{3(\frac{1}{2}+\frac{1}{3})} = \frac{\frac{9}{8}}{3(\frac{5}{6})} = \frac{\frac{9}{8}}{\frac{5}{2}} = \frac{9}{8}\cdot\frac{2}{5} = \frac{9}{20}$$

$$H_1(z) = \frac{1}{6}z^1 + (-1)z^0 + \left(\frac{26}{45}\right)\frac{z}{z+1/3} + \left(\frac{9}{20}\right)\frac{z}{z-1/2}$$

$$\boxed{h_1[n] = \frac{1}{6}\delta[n+1] - \delta[n] + \left(\frac{26}{45}\right)\left(-\frac{1}{3}\right)^n u[n] + \left(\frac{9}{20}\right)\left(\frac{1}{2}\right)^n u[n]}$$

$$H_2(z) = \frac{z^2+z}{(2z+1)^2(3z-2)}$$

$$z^{-1}H_2(z) = \frac{z+1}{12(z+1/2)^2(z-2/3)} = \frac{A}{z+1/2} + \frac{B}{(z+1/2)^2} + \frac{C}{z-2/3}$$

$$A = \left.\left[\frac{z+1}{12(z-2/3)}\right]'\right|_{z=-1/2} = \left.\frac{1}{12}\left(\frac{(z-2/3)-(z+1)}{(z-2/3)^2}\right)\right|_{z=-1/2} = \left.\frac{1}{12}\left(\frac{-5/3}{(z-2/3)^2}\right)\right|_{z=-1/2}$$

$$= \frac{-5}{36}\cdot\frac{1}{49/36} = -5/49$$

$$B = \left.\frac{z+1}{12(z-2/3)}\right|_{z=-1/2} = \frac{1/2}{12(-7/6)} = \frac{1/2}{-14} = -1/28$$

$$C = \left.\frac{z+1}{12(z+1/2)^2}\right|_{z=2/3} = \frac{5/3}{49/3} = 5/49$$

$$H_2(z) = \left(\frac{-5}{49}\right)\frac{z}{z+1/2} + \left(-1/28\right)\frac{z}{(z+1/2)^2} + \left(5/49\right)\frac{z}{z-2/3}$$

$$\boxed{h_2[n] = \frac{-5}{49}\left(-\frac{1}{2}\right)^n u[n] - \frac{1}{28}n\left(-\frac{1}{2}\right)^{n-1}u[n] + \frac{5}{49}\left(\frac{2}{3}\right)^n u[n]}$$

$$H_3(z) = \frac{2z+1}{(2z-1)(2z^2+2z+1)}$$

$$z = \frac{-2 \pm \sqrt{2^2 - 4(2)(1)}}{2(2)} = -\frac{1}{2} \pm j\frac{1}{2}$$

$$z^{-1}H_3(z) = \frac{2z+1}{4z(z-1/2)(z-(-\frac{1}{2}+j\frac{1}{2}))(z-(-\frac{1}{2}-j\frac{1}{2}))}$$

$$= \frac{A}{z} + \frac{B}{z-1/2} + \frac{C}{z-(-\frac{1}{2}+j\frac{1}{2})} + \frac{D}{z-(-\frac{1}{2}-j\frac{1}{2})}$$

$$A = \left.\frac{2z+1}{4(z-1/2)(z^2+z+1/2)}\right|_{z=0} = \frac{1}{4(-\frac{1}{2})(\frac{1}{2})} = -1$$

$$B = \left.\frac{2z+1}{4z(z^2+z+1/2)}\right|_{z=1/2} = \frac{2}{2(5/4)} = 4/5$$

$$C = \left.\frac{2z+1}{4z(z-1/2)(z-(-\frac{1}{2}-j\frac{1}{2}))}\right|_{z=-\frac{1}{2}+j\frac{1}{2}} = \frac{j}{2(-1+j)(-1+j1/2)j} = \frac{1}{2}\cdot\frac{1}{(\frac{1}{2}-\frac{3}{2}j)}\left(\frac{\frac{1}{2}+\frac{3}{2}j}{\frac{1}{2}+\frac{3}{2}j}\right)$$

$$= \frac{1}{2}\left(\frac{\frac{1}{2}+\frac{3}{2}j}{\frac{1}{4}+\frac{9}{4}}\right) = \frac{4}{40}(1+3j) = \frac{1}{10}+\frac{3}{10}j$$

$$D = C^*$$

$$H_3(z) = (-1) + (4/5)\frac{z}{z-1/2} + \left(\frac{1}{10}+\frac{3}{10}j\right)\frac{z}{z-(-\frac{1}{2}+j\frac{1}{2})} + \left(\frac{1}{10}-\frac{3}{10}j\right)\frac{z}{z-(-\frac{1}{2}-j\frac{1}{2})}$$

$$h_3[n] = -\delta[n] + \frac{4}{5}\left(\frac{1}{2}\right)^n u[n] + \left(\frac{\sqrt{10}}{10}e^{j\tan^{-1}(3)}\right)\left(\frac{1}{\sqrt{2}}e^{j3\pi/4}\right)^n + \left(\frac{\sqrt{10}}{10}e^{-j\tan^{-1}(3)}\right)\left(\frac{1}{\sqrt{2}}e^{-j3\pi/4}\right)^n$$

$$\boxed{h_3[n] = -\delta[n] + \frac{4}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{\sqrt{10}}{5}\left(\frac{1}{\sqrt{2}}\right)^n \cos\left(\frac{3\pi}{4}n + \tan^{-1}(3)\right)u[n]}$$

* $\left|\frac{1}{10}+\frac{3}{10}j\right| = \sqrt{(\frac{1}{10})^2+(\frac{3}{10})^2} = \frac{\sqrt{10}}{10}$

$\measuredangle\left(\frac{1}{10}+\frac{3}{10}j\right) = \tan^{-1}\left(\frac{3/10}{1/10}\right) = \tan^{-1}(3)$

* $\left|-\frac{1}{2}+j\frac{1}{2}\right| = \sqrt{(\frac{1}{2})^2+(\frac{1}{2})^2} = \frac{1}{\sqrt{2}}$

$\measuredangle\left(-\frac{1}{2}+j\frac{1}{2}\right) = \tan^{-1}\left(\frac{1/2}{-1/2}\right) = \tan^{-1}(-1) = 3\pi/4$

3. Input: $x(t) = 2e^{-3t} u(t)$

Output: $y(t) = (3t+1)e^{-3t}u(t) + e^{-4t}\sin(5t + \pi/3)u(t)$

Poles of $H(s)$: $s = -3$, $m = 1$

$s = -4 + j5$, $m = 1$

$s = -4 - j5$, $m = 1$

4. Unit step response of $H(s)$: $4u(t) - e^{-2t}[t\cos(5t + \pi/4) + 5\cos(5t)]u(t) + 7e^{-3t}ut$

(a) Natural response(s):

$-e^{-2t}t\cos(5t + \frac{\pi}{4})u(t)$

$5e^{-2t}\cos(5t)u(t)$

$7e^{-3t}u(t)$

Forced response(s):

$4u(t)$

(b) System Poles:

$s = -3$, $m = 1$

$s = -2 + j5$, $m = 2$

$s = -2 - j5$, $m = 2$

(c)

Mode	T (seconds)	f (Hertz)
$-e^{-2t}t\cos(5t + \frac{\pi}{4})u(t)$	$\frac{1}{2}$	$\frac{5}{2\pi}$
$5e^{-2t}\cos(5t)u(t)$	$\frac{1}{2}$	$\frac{5}{2\pi}$
$7e^{-3t}u(t)$	$\frac{1}{3}$	0

Considered one mode

(d) Dominant Pole(s): $s = -2 \pm j5$

5. $H(z)$ has poles @ $-\frac{1}{2} \pm j\frac{1}{2}$ $(m=1)$

$\frac{1}{5}$ $(m=3)$

$-\frac{1}{3}$ $(m=1)$

(a) general form of $H(z)$:

$$H(z) = A\frac{z}{z-1/5} + B\frac{z}{(z-1/5)^2} + C\frac{z}{(z-1/5)^3}$$

$$+ D\frac{z}{z-(-1/3)} + E\frac{z}{z-(-\frac{1}{2}+j\frac{1}{2})} + E^*\frac{z}{z-(-\frac{1}{2}-j\frac{1}{2})}$$

$\alpha = -\frac{1}{2}+j\frac{1}{2}$

$\rightarrow |\alpha| = \sqrt{(\frac{1}{2})^2+(\frac{1}{2})^2} = \frac{1}{\sqrt{2}}$

$\rightarrow \measuredangle\alpha = \omega = \tan^{-1}\left(\frac{1/2}{-1/2}\right) = \frac{3\pi}{4}$

$E = e_1 + je_2$

$\rightarrow |E| = \sqrt{e_1^2+e_2^2}$

$\rightarrow \measuredangle|E| = \theta = \tan^{-1}\left(\frac{e_2}{e_1}\right)$

general form of $h[n]$:

$$h[n] = A\left(\frac{1}{5}\right)^n u[n] + Bn\left(\frac{1}{5}\right)^{n-1}u[n] + Cn(n-1)\left(\frac{1}{5}\right)^{n-2}u[n]$$

$$+D\left(-\frac{1}{3}\right)^n u[n] + \left(|E|e^{j\theta}\right)\left(|\alpha|e^{j\omega}\right)^n + \left(|E|e^{-j\theta}\right)\left(|\alpha|e^{-j\omega}\right)^n$$

$$= A\left(\frac{1}{5}\right)^n u[n] + Bn\left(\frac{1}{5}\right)^{n-1}u[n] + Cn(n-1)\left(\frac{1}{5}\right)^{n-2}u[n]$$

$$+D\left(-\frac{1}{3}\right)^n u[n] + 2|E||\alpha|^n\cos(\omega n+\theta)\,u[n]$$

$$= A\left(\frac{1}{5}\right)^n u[n] + Bn\left(\frac{1}{5}\right)^{n-1}u[n] + Cn(n-1)\left(\frac{1}{5}\right)^{n-2}u[n]$$

$$+D\left(-\frac{1}{3}\right)^n u[n] + 2|E|\left(\frac{1}{\sqrt{2}}\right)^n\cos\left(\frac{3\pi}{4}n+\theta\right)u[n]$$

ok

* Possibly a $\delta[n]$ term, depending on num.

More directly: $(A_2n^2+A_1n+A_0)\left(\frac{1}{5}\right)^n u[n] + B\left(\frac{1}{\sqrt{2}}\right)^n\cos\left(\frac{3\pi n}{4}+\theta\right)u[n] + C\left(-\frac{1}{3}\right)^n u[n]$ $+D\delta[n]$

(b) Mode $\longrightarrow$ $\mathcal{T}$ (seconds) $\longrightarrow$ f (Hertz)

$A\left(\frac{1}{5}\right)^n \longrightarrow T/|\ln(1/5)| = T/\ln(5) \longrightarrow 0$

$Bn\left(\frac{1}{5}\right)^n \longrightarrow T/\ln(5) \longrightarrow 0$

$Cn(n-1)\left(\frac{1}{5}\right)^n \longrightarrow T/\ln(5) \longrightarrow 0$

$D\left(-\frac{1}{3}\right)^n \longrightarrow T/|\ln(1/3)| = T/\ln(3) \longrightarrow \pi\left(\frac{f_s}{2\pi}\right) = \frac{1}{2}f_s$

$2|E|\left(\frac{1}{\sqrt{2}}\right)^n\cos\left(\frac{3\pi}{4}n+\theta\right) \longrightarrow T/|\ln(1/\sqrt{2})| = T/\ln(\sqrt{2}) \longrightarrow \frac{3\pi}{4}\left(\frac{f_s}{2\pi}\right) = \frac{3}{8}f_s$

(c) Dominant Pole(s): $z = -\frac{1}{2} \pm j\frac{1}{2}$

6. Input Poles: $\frac{1}{2}$, $-\frac{1}{3}$ (All simple)

Output Poles: $\frac{1}{2}$, $-\frac{1}{3}$, $\frac{1}{4}e^{\pm j\pi/3}$, $-\frac{1}{5}$ (All simple)

(a) Poles of $H(z)$: (for certain): $z = -\frac{1}{5}$
(System poles)

$z = \frac{1}{4}e^{\pm j\pi/3}$ indeterminate multiplicity

(b) Hidden Poles! (Due to ~~internal~~ cancellation) Not internal

These are not "hidden poles" input-system canc.

For example: $X(z) = \dfrac{(z-\alpha)}{(z-\frac{1}{2})(z+\frac{1}{3})}$ ← $X(z)$ has a zero @ α

$$H(z) = \frac{1}{(z-\alpha)(z+\frac{1}{5})(z-\frac{1}{4}e^{j\pi/3})(z-\frac{1}{4}e^{-j\pi/3})}$$ ← $H(z)$ has a pole @ α

$Y(z) = H(z)X(z)$ => $(z-\alpha)$ would not appear @ output!

✓