5.二叉树

（e1）先序遍历

真君曰：“昔吕洞宾居庐山而成仙，鬼谷子居云梦而得道，今或无此吉地么？”
璞曰：“有，但当遍历耳。”

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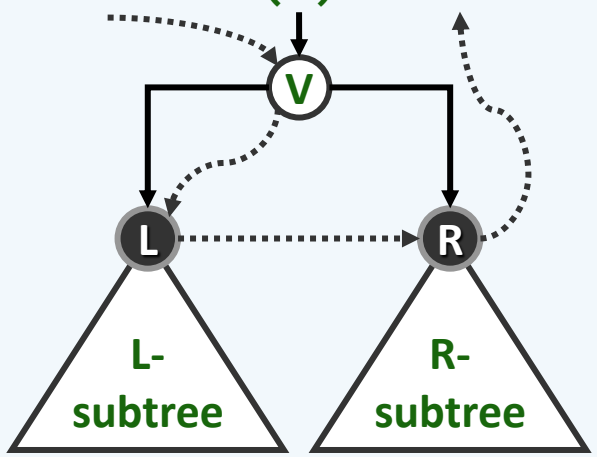
遍历

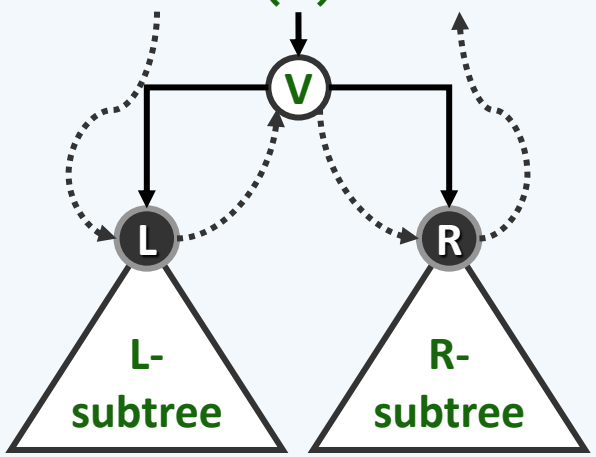
❖ 按照某种次序访问树中各节点，每个节点被访问恰好一次

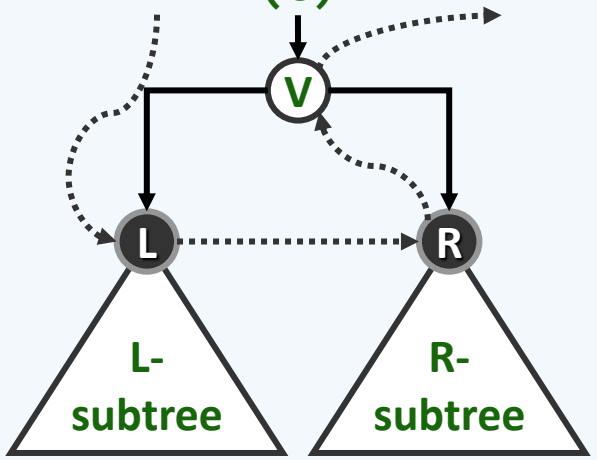
$$T = L \cup V \cup R$$

❖ 遍历结果 ~ 遍历过程 ~ 遍历次序 ~ 遍历策略

先序	中序	后序	层次（广度）
V | L | R	L | V | R	L | R | V	自上而下，先左后右
preorder	inorder	postorder	
(a)	(b)	(c)	



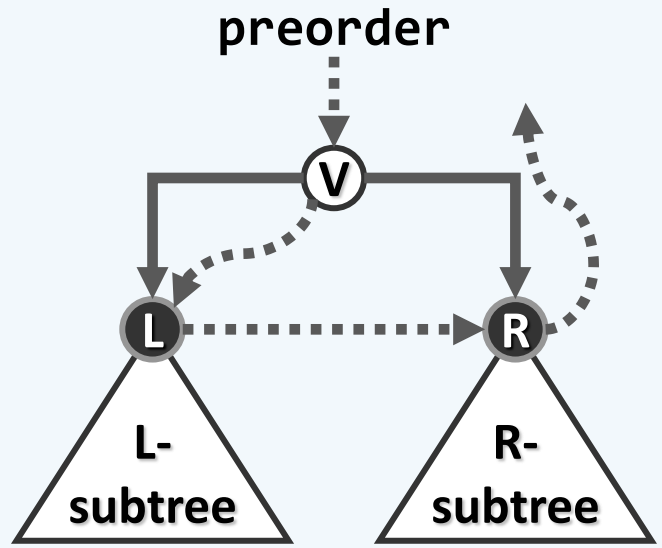


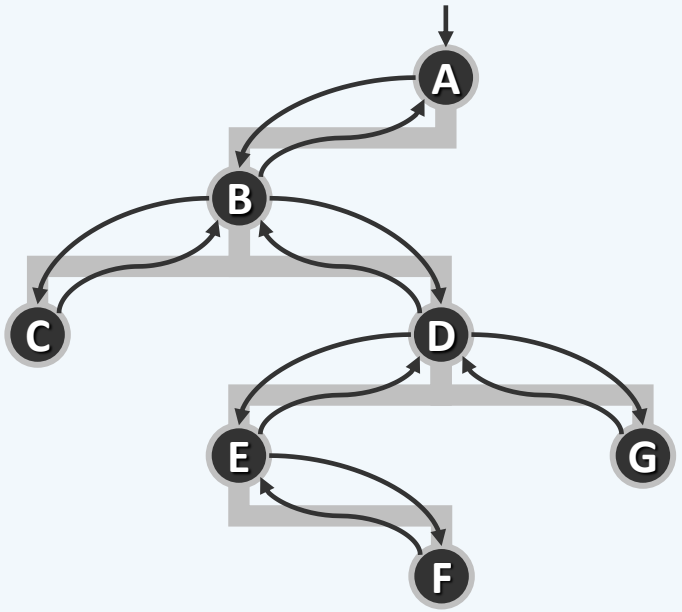


递归

```
template <typename T, typename VST>
void traverse( BinNodePosi(T) x, VST & visit ) {
    if ( ! x ) return;
    visit( x->data );
    traverse( x->lc, visit );
    traverse( x->rc, visit );
} //T(n) = O(1) + T(a) + T(n - a - 1) = O(n)
```

- 先序输出文件树结构：`c:\> tree.com c:\windows`
- 挑战：不依赖递归机制，能否实现先序遍历？
  如何实现？效率如何？





迭代1：思路

❖ 先序遍历任一二叉树T的过程，无非是

先访问根节点r

再先后递归地遍历T_L和T_R

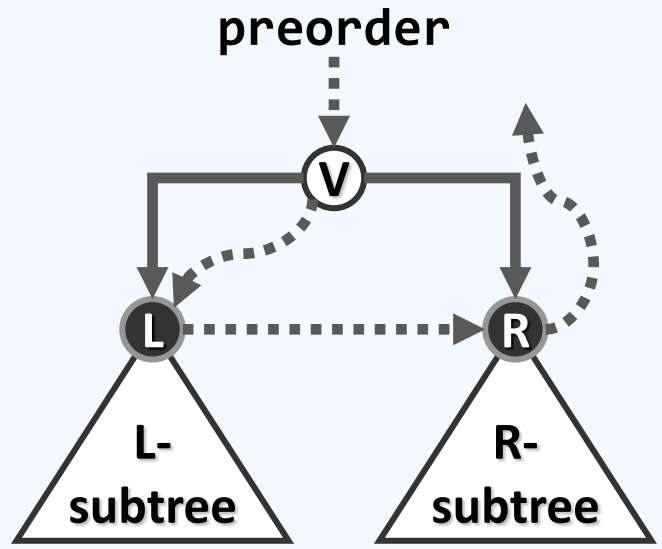
❖ 递归实现中，对左、右子树的递归遍历

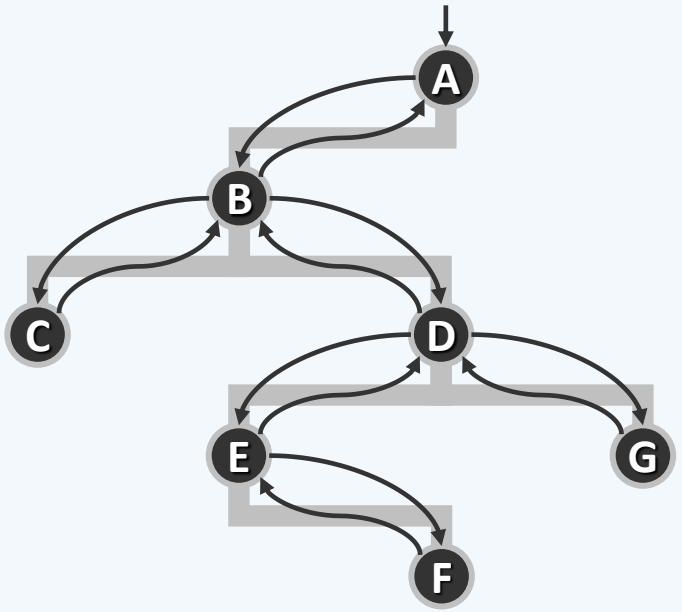
都类似于尾递归

故不难直接消除

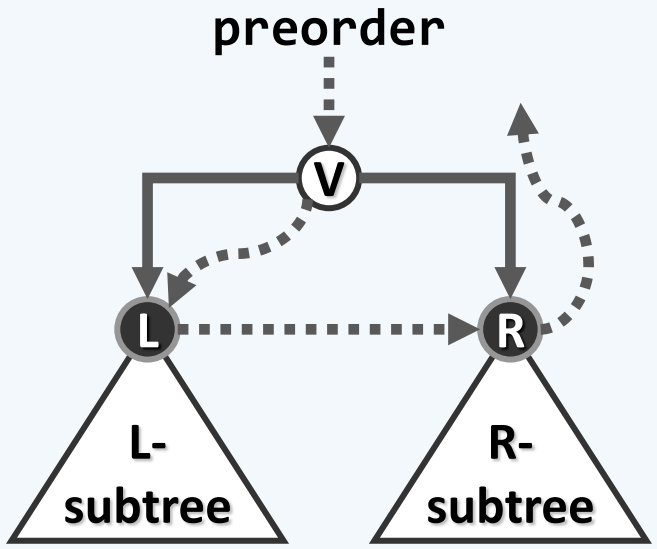
❖ 思路：

二分递归 → 迭代 + 单递归 → 迭代 + 栈



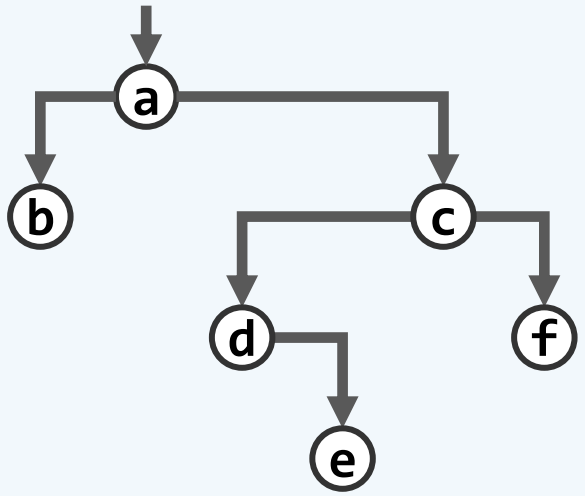


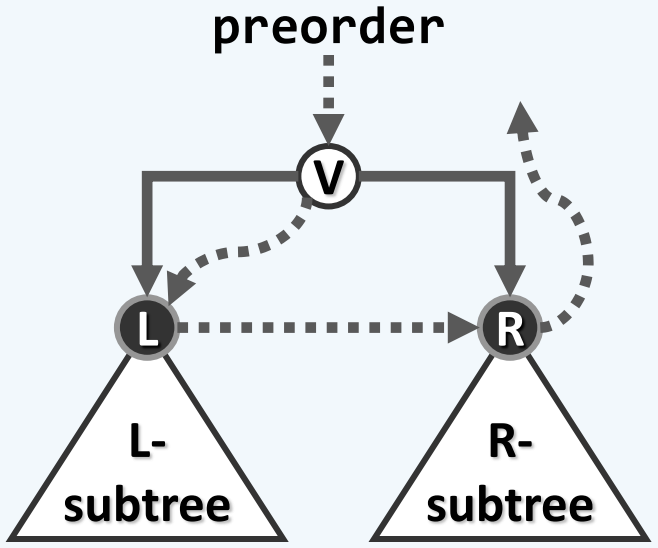
迭代1：实现

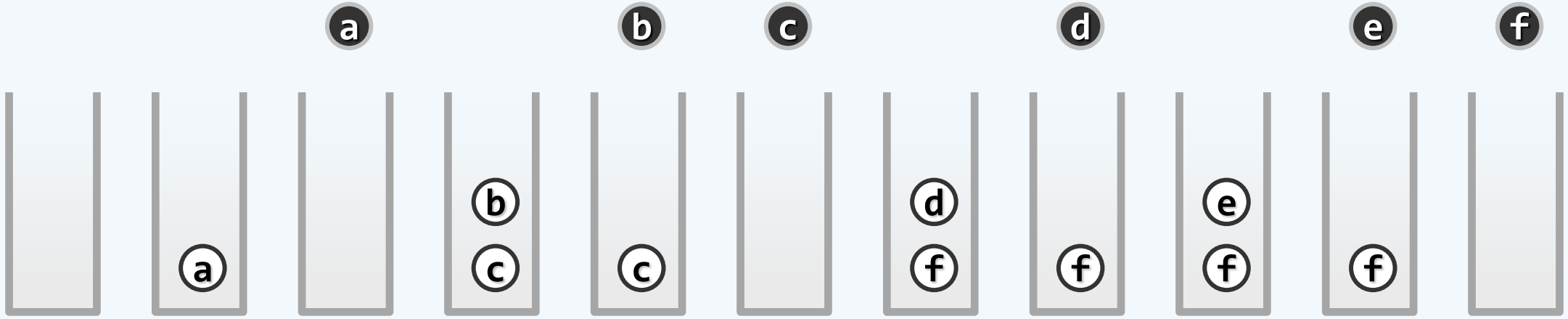



```
❖template <typename T, typename VST>
  void travPre_I1( BinNodePosi(T) x, VST & visit ) {
     Stack < BinNodePosi(T) > S; //辅助栈
     if (x) S.push( x ); //根节点入栈
     while ( ! S.empty() ) { //在栈变空之前反复循环
        x = S.pop(); visit( x->data ); //弹出并访问当前节点
        if ( HasRChild( *x ) ) S.push( x->rc ); //右孩子先入后出
        if ( HasLChild( *x ) ) S.push( x->lc  ); //左孩子后入先出
     } //体会以上两句的次序
  }
```

迭代1：实例







迭代1：分析

- 正确性

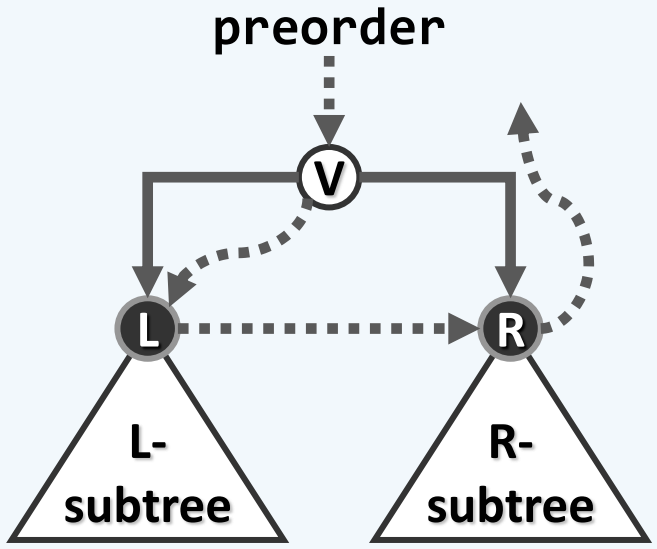
  无遗落：每个节点都会被访问到

  归纳假设：若深度为d的节点都能被正确访问到，则深度为d+1的也是

  根先：对于任一子树，根被访问后才会访问其它节点

  只需注意到：若u是v的真祖先，则u必先于v被访问到

  左先右后：同一节点的左子树，先于右子树被访问



- 效率：$O(n)$

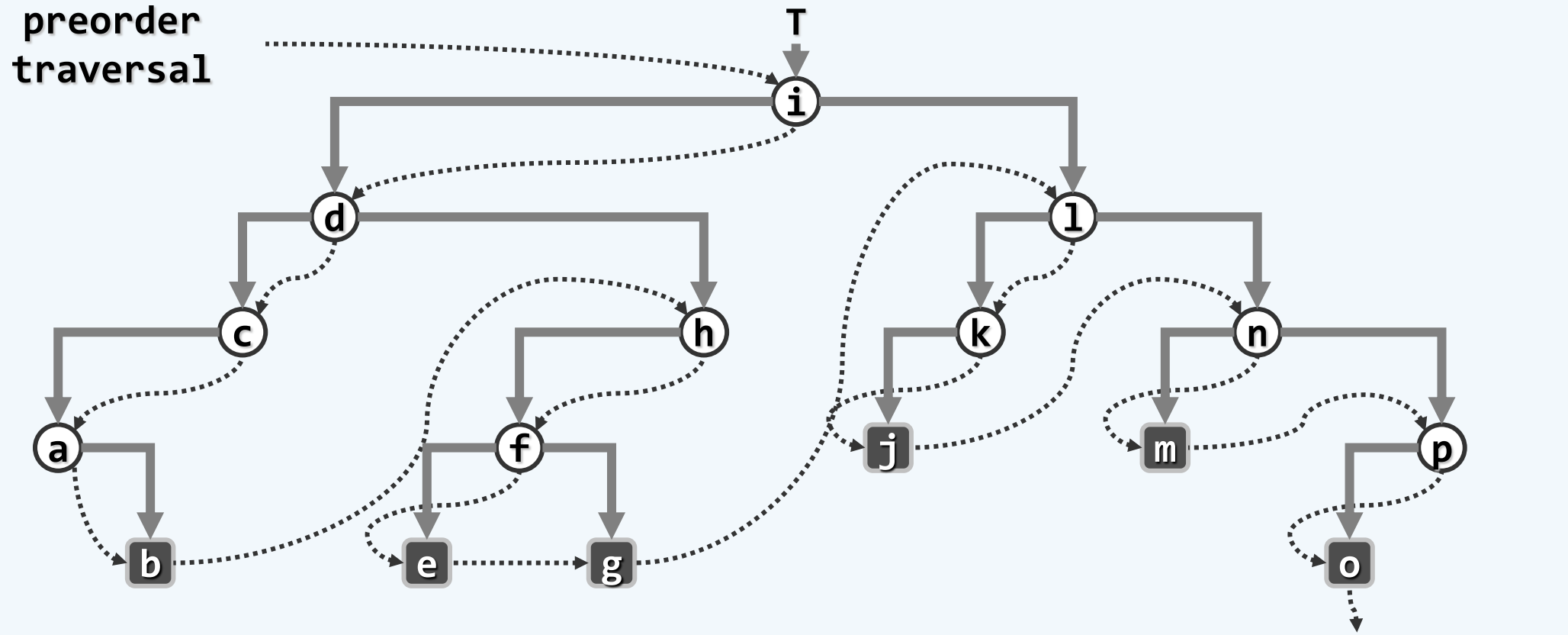
  每步迭代，都有一个节点出栈并被访问

  每个节点入/出栈一次且仅一次

  每步迭代只需$O(1)$时间

- 以上消除尾递归的思路不易推广，需要另寻他法...

迭代2：思路



→ i → d → c → a → b → h → f → e → g → l → k → j → n → m → p → o →

迭代2：思路

- 沿着左侧分支

  各节点与其右孩子（可能为空）一一对应

- 从宏观上，整个遍历过程可划分为

  自上而下对**左侧分支**的**访问**，及随后

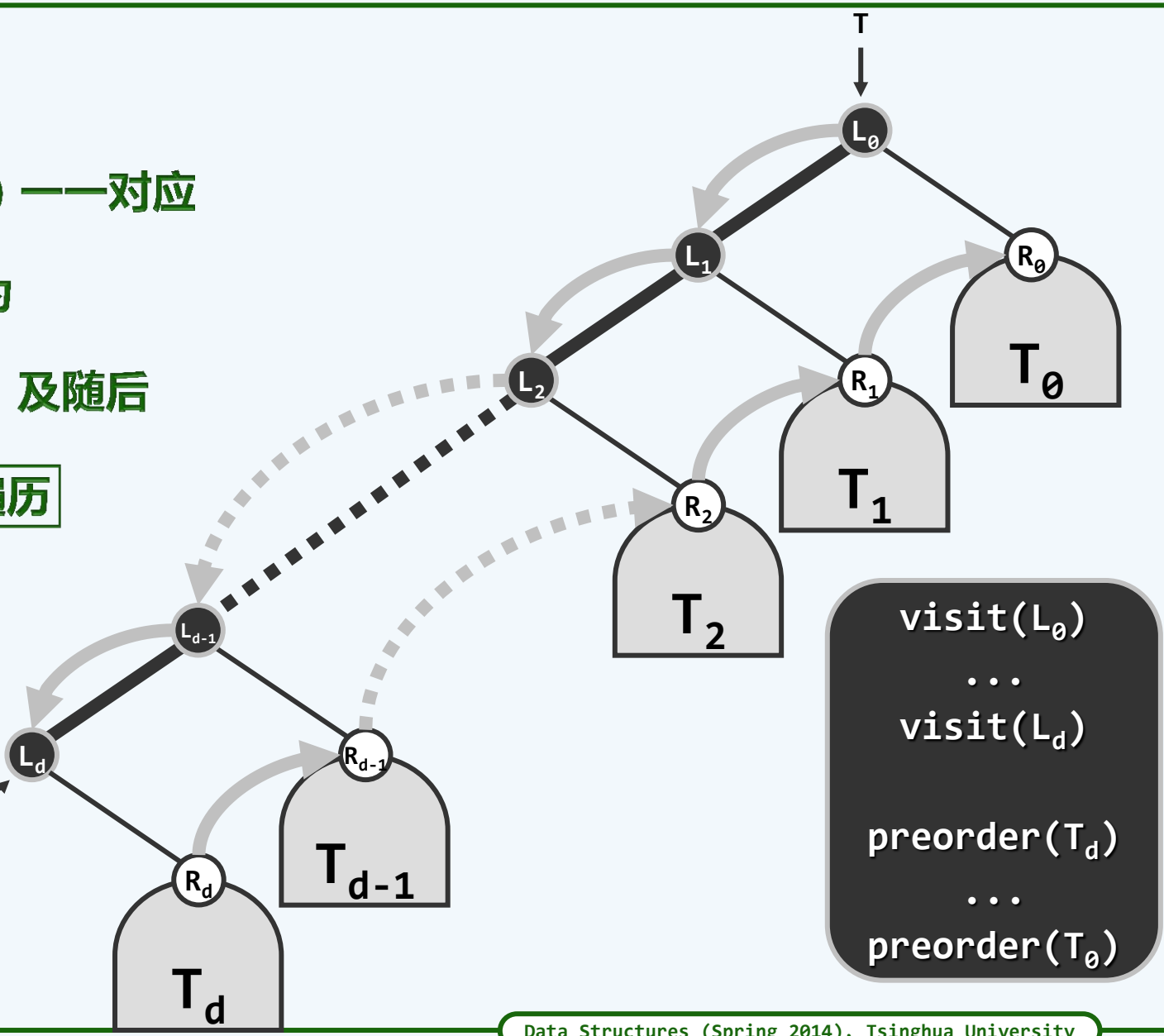
  自下而上对一系列**右子树**的**遍历**

- 不同右子树的遍历

  相互独立

  自成一个子任务

deepest node
along left branch



迭代2：实现

```
❖template <typename T, typename VST> //分摊O(1)
  static void visitAlongLeftBranch(
     BinNodePosi(T) x,
     VST & visit,
     Stack < BinNodePosi(T) > & S )
  {
     while ( x ) { //反复地
        visit( x->data ); //访问当前节点
        S.push( x->rc ); //右孩子（右子树）入栈（将来逆序出栈）
        x = x->lc; //沿左侧链下行
     } //只有右孩子、NULL可能入栈——增加判断以剔除后者，是否值得？
  }
```

迭代2：实现

```
❖ template <typename T, typename VST>
  void travPre_I2( BinNodePosi(T) x, VST & visit ) {
     Stack < BinNodePosi(T) > S; //辅助栈
     while ( true ) { //以（右）子树为单位，逐批访问节点
        visitAlongLeftBranch( x, visit, S ); //访问子树x的左侧链，右子树入栈缓冲
        if ( S.empty() ) break; //栈空即退出
        x = S.pop(); //弹出下一子树的根
     } //#pop = #push = #visit = O(n) = 分摊O(1)
  }
```

迭代2：实例