6.图

（a）概述

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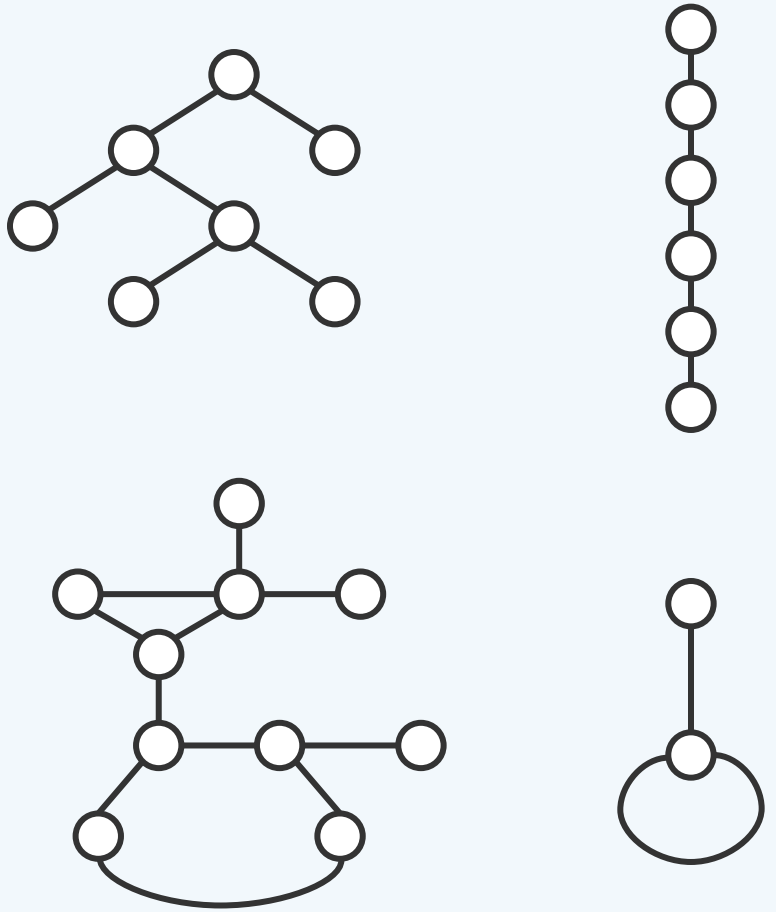
基本术语

- G = (V; E)
  - vertex: n = |V|
  - edge|arc: e = |E|
- 同一条边的两个顶点，彼此邻接 adjacency
  - 同一顶点自我邻接，构成自环 self-loop
  - 不含自环，即为简单图 simple graph
  - 非简单图 non-simple，暂不讨论
- 顶点与其所属的边，彼此关联 incidence
  - 度（degree）：与同一顶点关联的边数



无向图/有向图

- 若邻接顶点u和v的次序无所谓

  则(u，v)为无向边undirected edge

- 所有边均无方向的图，即无向图undigraph

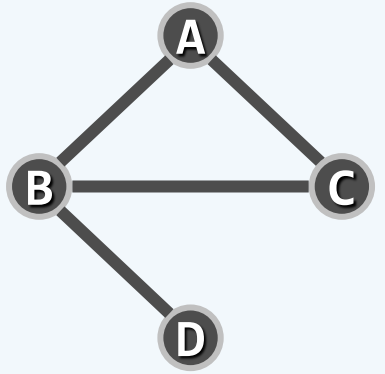
- 反之，有向图digraph中均为有向边directed edge

  u、v分别称作边(u，v)的尾（tail）、头（head）

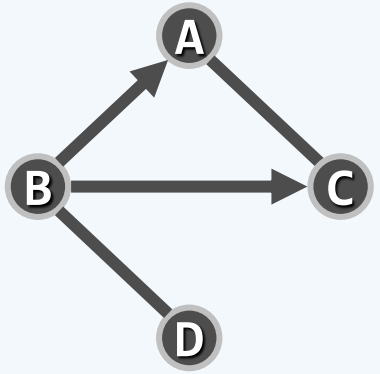
- 无向边、有向边并存的图，称作混合图mixed graph

- 有向图通用性更强

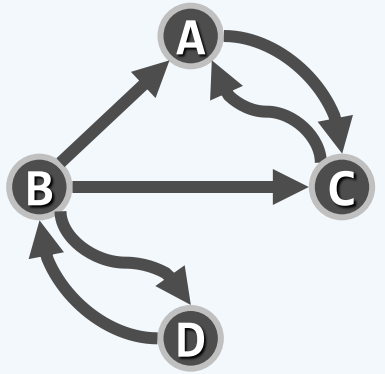
  故本章主要针对有向图介绍相关结构及算法



(a) undigraph



(b) mixed graph



(c) digraph

路径/环路

- 路径$\pi = \langle v_0, v_1, \ldots, v_k \rangle$

  长度$|\pi| = k$

- 简单路径：

  $v_i = v_j$除非$i = j$

- 环/环路：$v_0 = v_k$

- 有向无环图（DAG）

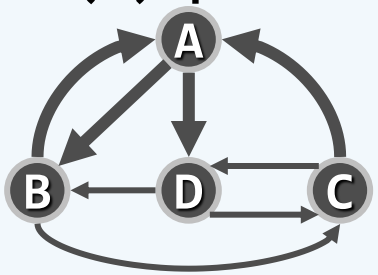
- 欧拉环路：$|\pi| = |E|$

  各边恰好出现一次

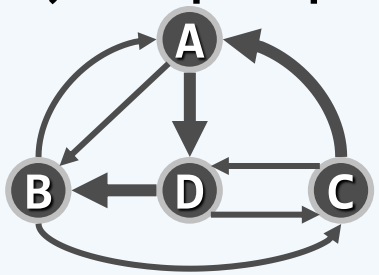
- 哈密尔顿环路：$|\pi| = |V|$

  各顶点恰好出现一次

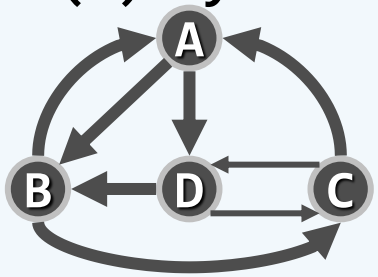
(i) path



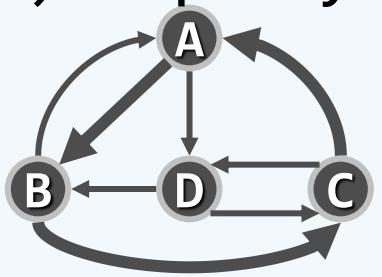
(ii) simple path



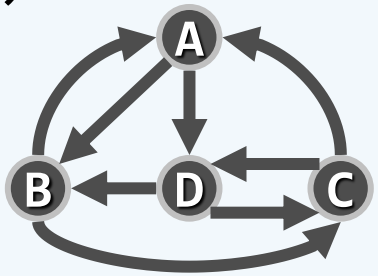
(i) cycle



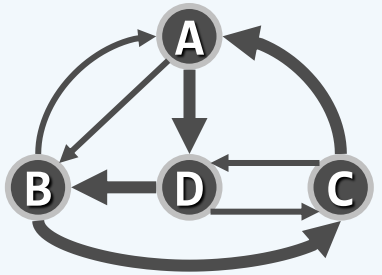
(ii) simple cycle



(i) Eulerian tour



(ii) Hamiltonian tour



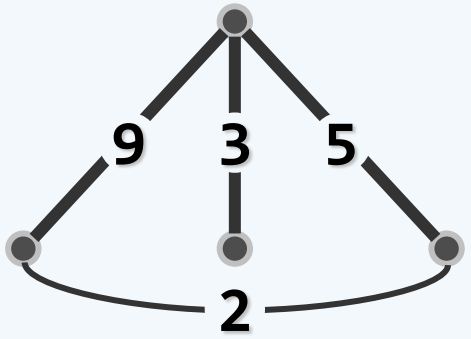
支撑树/带权网络/最小支撑树

- 图G = (V; E)的子图T = (V; F)若是树，即为其支撑树 spanning tree

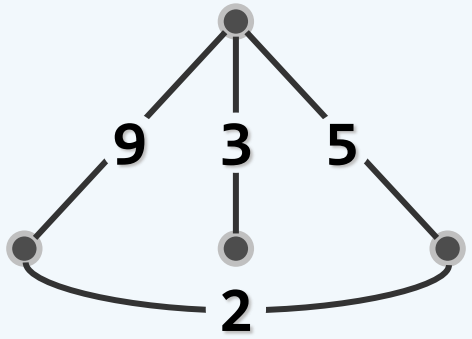
  同一图的支撑树，通常并不唯一

- 各边e均有对应的权值wt(e)，则为带权网络 weighted network

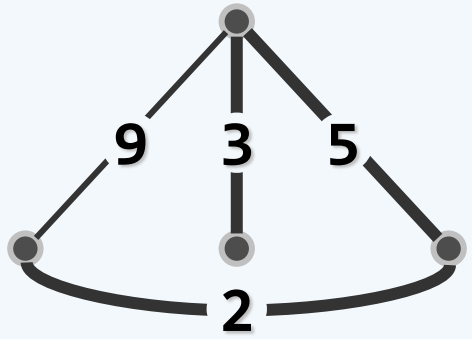
- 同一网络的支撑树中，总权重最小者为最小支撑树 MST



spanning tree



weighted network
(triangle inequality?)



minimum spanning tree