12.排序

（c2）希尔排序：逆序对

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Postage Problem

- The postage for a letter is 50F, and a postcard 35F

  But there are only stamps of 4F and 13F available

- Possible to stamp the letter and the postcard EXACTLY?

- How about other postages?

- Is it possible to

  represent the numbers 50 and 35 as linear combinations:

  $4m + 13n$, where $m, n \in N = \{ 0, 1, 2, \ldots \}$

Linear Combination

- Let g, h ∈ N

  Then for any m, n ∈ N

  f = mg + nh is called a linear combination of g and h

- Let g, h ∈ N be relatively prime

  N(g, h) = { numbers that are NOT combinations of g and h }

- Let x(g, h) = max(N(g, h))

- Theorem: x(g, h) = (g – 1)*(h - 1) - 1 = gh - g - h

  e.g. x(3, 7) = 11, x(4, 9) = 23, x(4, 13) = 35, x(5, 14) = 51

h-sorting & h-ordered

- Let h ∈ N. A sequence S[0, n) is called h-ordered if

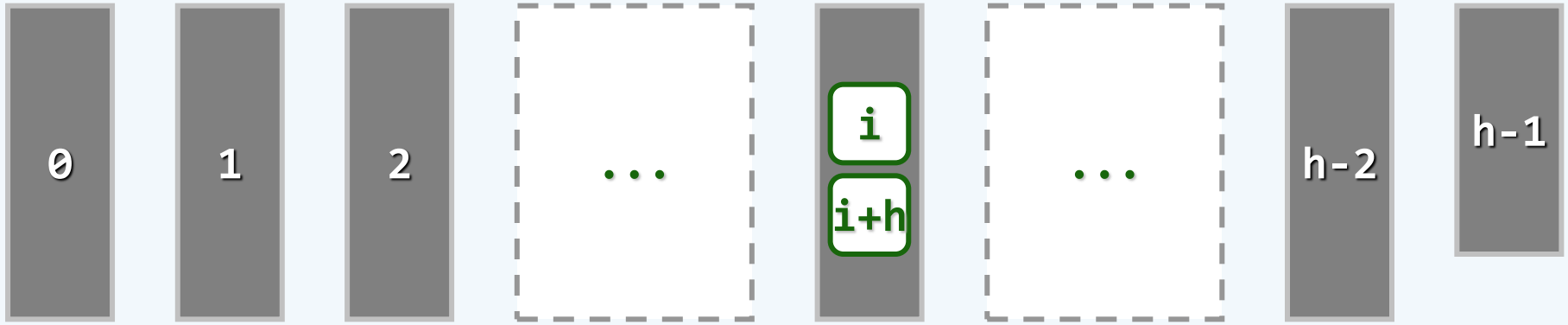
  S[i] ≤ S[i + h] holds for 0 <= i < n - h

- A 1-ordered sequence is sorted

- h-sorting: an h-ordered sequence is obtained by

  1) arranging S into a 2D matrix with h columns and

  2) sorting each column respectively

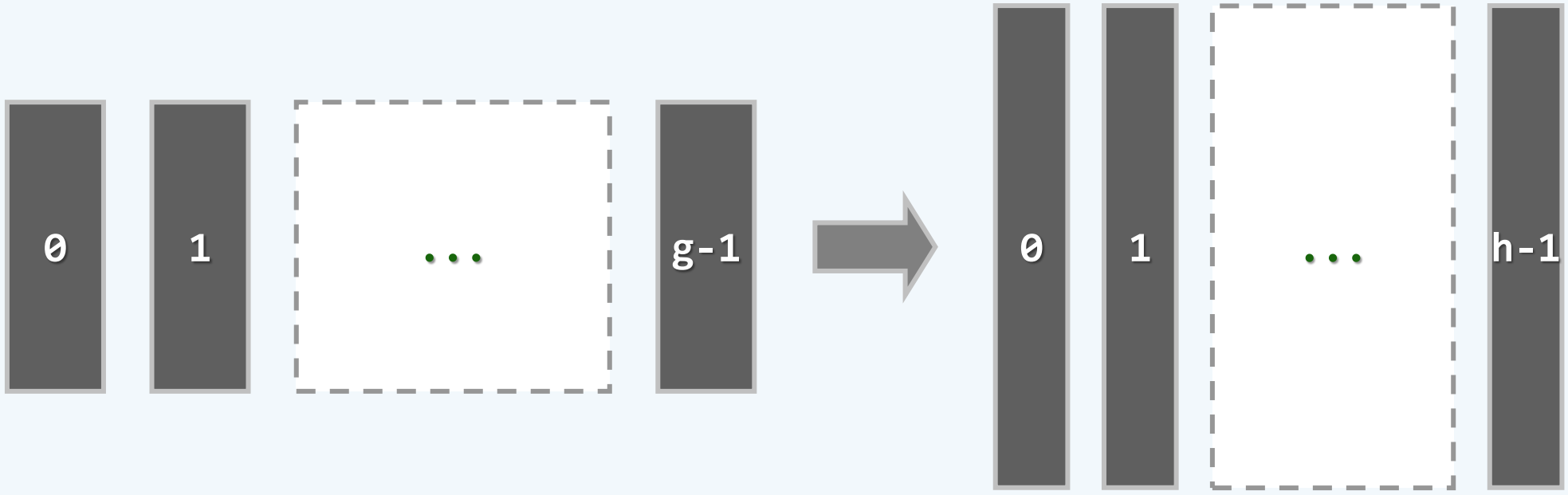


Theorem K

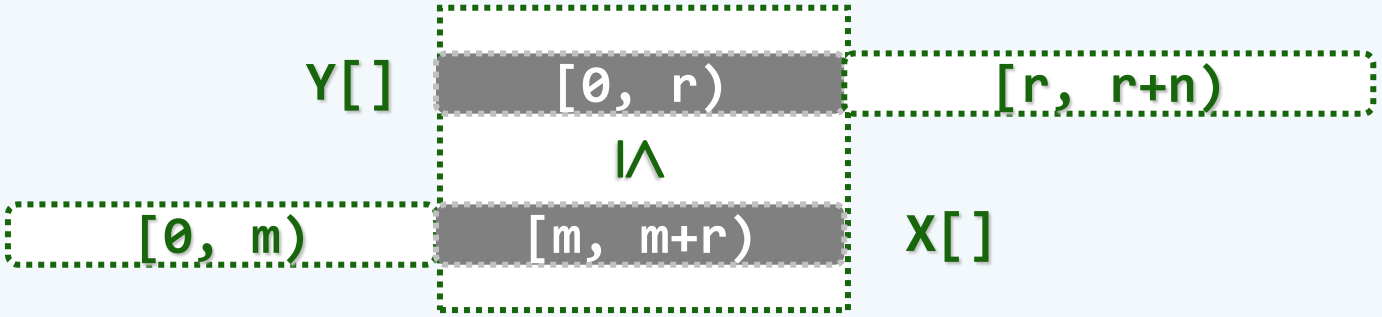
❖ [Knuth, ACP Vol.3 p.90]

//习题解析[12-12, 12-13]

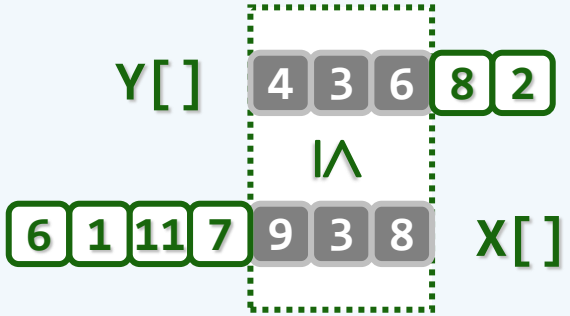
A g-ordered sequence REMAINS g-ordered after being h-sorted



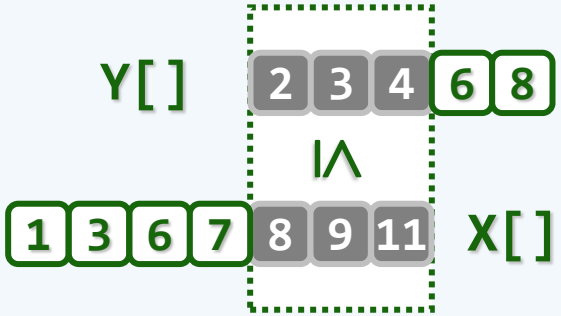
Lemma L



(a)



(b)

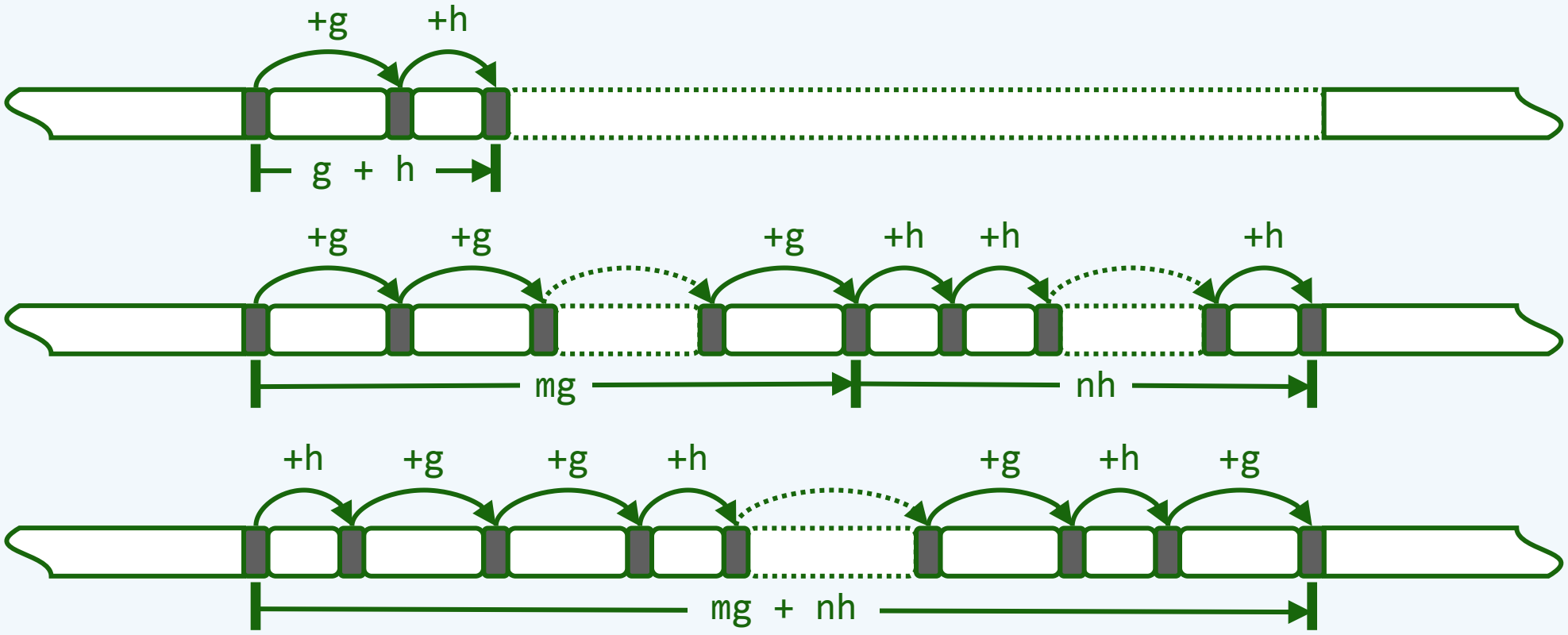


Linear Combination

❖ A sequence that is both g-ordered and h-ordered

is called (g, h)-ordered, which must be both

(g + h)-ordered and (mg + nh)-ordered for any m, n ∈ N



Inversion

- Let S[0, n) be a (g, h)-ordered sequence, where g & h are relatively prime
- Then for all elements S[i] and S[j], we have

  $j - i \geq x(g, h) + 1 = (g - 1)*(h - 1)$

  only if S[i] <= S[j]

- This implies that to the RIGHT of each element,

  only the next $(g - 1)*(h - 1) - 1$ elements could be smaller

- There would be no more than $n * x(g, h)$ inversions altogether