ERROR Relativo de la división

Sea $a \in \mathbb{R}$ y $b \in \mathbb{R}$. Dadas las aproximaciones
$$\tilde{a} = a \pm \delta a \quad y \quad \tilde{b} = b \pm \delta b$$
Estimamos el error del cociente $z := a/b$

Llamamos
$$\left.\begin{aligned}\tilde{z} = \tilde{a}/\tilde{b} &= \frac{a/a}{b/b} \cdot \frac{a \mp \delta a}{b \pm \delta b} \\ &= \underbrace{a/b}_{z} \cdot \left[\frac{1 \pm \delta a/a}{1 \pm \delta b/b}\right]\end{aligned}\right\} \circledast$$

Además
$$\frac{1}{1 \pm \delta b/b} = 1 \pm \frac{\delta b}{b} \pm \left(\frac{\delta b}{b}\right)^2 + \ldots$$

como δb es pequeño $\left(\frac{\delta b}{b}\right)^2 \to 0$, entonces
$$\frac{1}{1 \pm \delta b/b} \cong 1 \pm \frac{\delta b}{b}$$

Reemplazando en $\circledast$ Tenemos
$$\tilde{z} = z \cdot \underbrace{\left[1 \pm \delta a/a\right] \cdot \left[1 \pm \delta b/b\right]}_{\text{Aplicando el error de la multiplicación}}$$

$$\tilde{z} = z \cdot \left[1 \pm \left(\frac{\delta a}{a} + \frac{\delta b}{b}\right)\right]$$

$$= z \pm \frac{a}{b}\left[\frac{\delta a}{a} \pm \frac{\delta b}{b}\right]$$

$$= z \pm \underbrace{\left(\frac{\delta a}{b} \pm \frac{a\,\delta b}{b^2}\right)}_{\text{error} = \delta z}$$

El error relativo es

$$\frac{\delta z}{z} = \delta z \cdot \frac{b}{a} = \frac{\delta a}{a} \pm \frac{\delta b}{b}$$

Error relativo - cifras significativas - Redondeo

Sea $p = \pm(0.d_1 d_2 \ldots d_k d_{k+1} \ldots) \times \beta^{\alpha}$ → exponente

$d_1 \neq 0$

β Base

Nosotros pensaremos específicamente $\beta = 10$

Entonces $d_i \in \{0, 1, 2, \ldots, 9\}$

$$p = \pm\, 0.d_1 d_2 \ldots d_k d_{k+1} \ldots \times 10^{\alpha}$$

Supongamos que p^* es una aproximación a p (redondeo)

$$p^* = \begin{cases} \overset{\bar{p}^*}{\pm\, 0.d_1 d_2 \ldots d_k \times 10^{\alpha}}, & \text{si } \underbrace{d_{k+1} < 5} \\ \underset{\underline{p}^*}{\pm [0.d_1 d_2 \ldots d_k + 10^{-k}] \times 10^{\alpha}}, & \text{si } \underbrace{d_{k+1} > 5} \end{cases}$$

Ahora:

$$|p-p^*| = 0.00\ldots d_{k+1}\,d_{k+2}\ldots \times 10^{\alpha}$$

$$= 0.\underbrace{d_{k+1}}\,d_{k+2}\ldots \times 10^{\alpha-k}$$

$$d_{k+1} < 5 \Rightarrow 0.d_{k+1}\,d_{k+2} \times 10^{\alpha-k} < (0.5)\,10^{\alpha-k}$$

$$< 0.5 * 10^{\alpha-k}$$

$$|p-\underline{p}^*| = 0.d_{k+1}\,d_{k+2}\ldots \times 10^{\alpha-k} - 10^{\alpha-k}$$

$$= (0.\underbrace{d_{k+1}}\,d_{k+2}\ldots - 1) \times 10^{\alpha-k}$$

$$d_{k+1} > 5 \Rightarrow 0.d_{k+1}\,d_{k+2}\ldots \times 10^{\alpha-k} > (0.5)\,10^{\alpha-k}$$

$$< 0.5 * 10^{\alpha-k}$$

Además $|p| = 0.d_1\ldots\ldots d_{k+1}\,d_{k+2}\ldots \times 10^{\alpha}$

$$> 0.1 \times 10^{\alpha} = 10^{\alpha-1}$$

$$\frac{|p-p^*|}{|p|} < \frac{0.5 \times 10^{\alpha-k}}{|p|} < \frac{0.5 \times 10^{\alpha-k}}{10^{\alpha-1}}$$

$$= \boxed{0.5 * 10^{1-k}}$$

$$= 5 \times 10^{-k}$$

Cifras significativas