Proof of Correctness for Prim's Algorithm

Claim: *Prim's algorithm correctly finds a minimum spanning tree of the given graph G.*

Proof. (By Induction)

Let T_i be the tree produced after step i of Prim's Algorithm. We show that for $1 \leq i \leq n$, T_i is a subtree of some minimum spanning tree of G.

Base Case ($i = 1$)

T_i is a single vertex, which is a subtree of every MST.

Inductive Hypothesis ($i = k$)

Assume T_k is a subtree of some MST M of G.

Induction Step ($i = k + 1$)

Let e be the edge selected by Prim's algorithm at step $k + 1$. We need to show $T_k \cup \{e\}$ is a subtree of a MST M' of G.

There are two cases to consider:

Case 1: $e \in M$.

Then $M' = M$, and we're done.

Case 2: $e \notin M$. In this case we describe how to construct a different MST M' that contains e.

Start with M and separate the edges in T_k from the edges not in T_k.

Add e to M, with one end vertex of e in T_k and the other not in T_k.

This creates a cycle in M. So there must be some other edge e' that also has one end vertex in T_k and the other not in T_k.

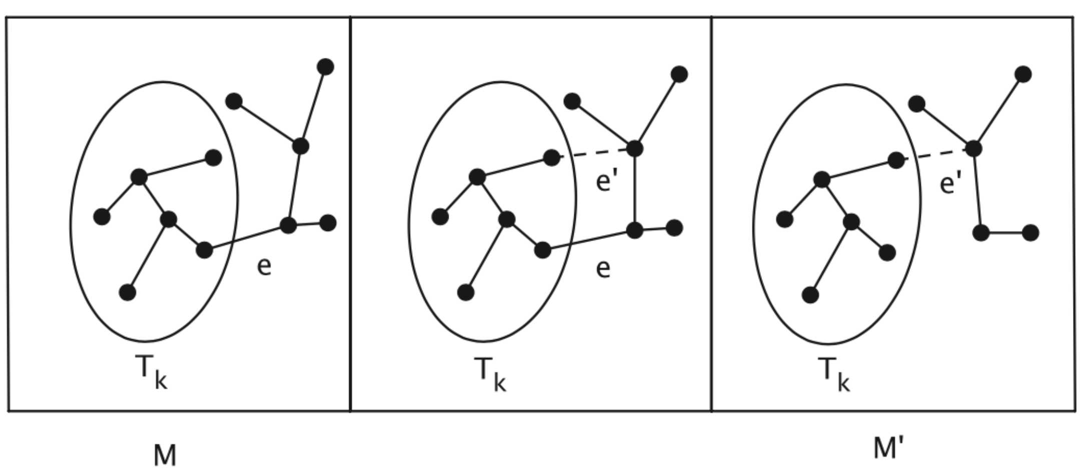
By the way the algorithms works, $w(e') \geq w(e)$

Remove edge e' to get a new spanning tree M' that contains edge e.

Since we started with M and then added edge e and removed edge e' to get M', and since $w(e') \geq w(e)$, we have $w(M') \leq w(M)$.

But since M is a MST, $w(M) \leq w(M')$. This implies $w(M) = w(M')$.

Therefore M' is a MST containing edge e.



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