Example Proof by Induction

1. Belcastro Example 4.2.1

   Claim: For all $n > 0, 2^n > n$

   Proof. (By Induction)

   Base Case (n=1)

   $$2^n = 2^1 = 2$$

   Also

   $$n = 1$$

   Since $2 > 1$, the claim holds for $n = 1$.

   Inductive Hypothesis $(n = k)$

   Assume for some $k > 1$

   $$2^k > k$$

   Induction step $(n = k + 1)$

   To Show:

   $$2^{k+1} > k + 1$$

   $$\begin{aligned} 2^{k+1} &= 2 \cdot 2^k \\ &> 2 \cdot k \quad \textit{By I.H.} \\ &= k + k \\ &> k + 1 \quad \textit{Since } k > 1 \end{aligned}$$

   By the principle of mathematical induction, for all $n > 0, 2^n > n$ □