CSCI-393 – Spring 2020

Name: _____

Quiz 2 (30 points) – This is a closed book, open notes quiz. Solutions

1. This question is about the brute-force string matching algorithm.

   (a) (8 points) How many comparisons (both successful and unsuccessful) will be made by the brute-force string matching algorithm to search for the pattern `001` in the string consisting of six zeros (`000000`)?

   Solution: The search string has length $n = 6$. The pattern has length $m = 3$.

   - There are 2 successful and 1 unsuccessful comparisons for each starting position.
   - The number of starting positions is $n - m + 1 = 6 - 3 + 1 = 4$.
   - Since there are 3 comparisons made for each starting location, and there are 4 starting locations, the total number of comparisons is $3 \cdot 4 = 12$.

```
0 0 0 0 0 0
0 0 1
  0 0 1
    0 0 1
      0 0 1
```

   (b) (8 points) How many comparisons (both successful and unsuccessful) will be made by the brute-force string matching algorithm to search for the pattern `00001` in the string consisting of one thousand zeros?

   Solution: The search string has length $n = 1000$. The pattern has length $m = 5$.

   - There are 4 successful and 1 unsuccessful comparisons for each starting position.
   - The number of starting positions is $n - m + 1 = 1000 - 5 + 1 = 996$.
   - Since there are 5 comparisons made for each starting location, and there are 996 starting locations, the total number of comparisons is $5 \cdot 996 = 4980$.

2. There are n stack of n identical-looking coins. All of the coins in one of these stacks are counterfeit, while all the coins in the other stacks are genuine. Every genuine coin weighs 10 grams; every fake weights 11 grams. You have a scale that can determine the exact weight of any number of coins.

   (a) (7 points) Devise a brute-force algorithm to identify the stack with the fake coins.

Solution:

```
for(i from 1 to n-1)
    Take a coin from stack i
    Weigh the coin
    If (coin weighs 11 grams)
        This is the stack of fake coins
        STOP
The last stack contains the fakes
```

   (b) (7 points) What is the worst-case efficiency of your algorithm

Solution: $\Theta(n)$

   (c) (5 points) **Extra Credit** — Only work on this question after you've answered the other questions. What is the minimum number of weighings needed to identify the stack with the fake coins?

Solution:

1. Number the stacks 1 through n
2. Take i coins from each stack i
3. Weigh all of the coins together at once, call this weight w.
   (a) If there were no fakes, the total weight would be $10 \cdot \frac{n(n+1)}{2} = 5n(n+1)$
   (b) So $w - 5n(n+1) = i$, where Stack i contains the fake coins.

Example, with $n = 4$ the weight with no fakes is $10 + 20 + 30 + 40 = 100$, which equals $5n(n+1) = 5 \cdot 4(4+1) = 20 \cdot 5 = 100$.
Now consider each possible fake stack:

1. Stack 1 is fake: $w = 11 + 20 + 30 + 40 = 101$. $w - 5n(n+1) = 101 - 100 = 1$.
2. Stack 2 is fake: $w = 10 + 22 + 30 + 40 = 102$. $w - 5n(n+1) = 102 - 100 = 2$.
3. Stack 3 is fake: $w = 10 + 20 + 33 + 40 = 103$. $w - 5n(n+1) = 103 - 100 = 3$.
4. Stack 4 is fake: $w = 10 + 20 + 30 + 44 = 104$. $w - 5n(n+1) = 104 - 100 = 4$.

So the minimum number of weighings is one.