CSCI-393 – Spring 2020

Name: ______________________________

Quiz 1 (30 points) – This is a closed book, open notes quiz. Solutions

1. (8 points) Use the definition of Big-O to show $n^2 + 4n + 8 \in O(n^2)$

Solution:

$$\begin{aligned}
n^2 + 4n + 8 &\leq cn^2 && \text{Definition of Big-O} \\
8 &\leq cn^2 - n^2 - 4n && \text{Subtract } (n^2 + 4n) \\
8 &\leq 5n^2 - n^2 - 4n && \text{Let } c = 5 \\
8 &\leq 4n^2 - 4n && \text{Combine } n^2 \text{ terms} \\
2 &\leq n^2 - n && \text{Divide by 4}
\end{aligned}$$

The above inequality is true for $n = 2$, and as n grows from there, the RHS grows while the LHS remains constant. So we can let $n_0 = 2$.

2. (8 points) Use the definition of Ω to show $n^2 + 4n + 8 \in \Omega(n^2)$

Solution:

$$\begin{aligned}
n^2 + 4n + 8 &\geq cn^2 && \text{Definition of } \Omega \\
n^2 + 4n + 8 - cn^2 &\geq 0 && \text{Subtract } cn^2 \\
n^2 + 4n - cn^2 &\geq -8 && \text{Subtract 8} \\
n^2 - cn^2 + 4n &\geq -8 && \text{Rearrange terms} \\
n^2 - n^2 + 4n &\geq -8 && \text{Let } c = 1 \\
4n &\geq -8 && \text{Combine } n^2 \text{ terms} \\
n &\geq -4 && \text{Divide by 4}
\end{aligned}$$

The above inequality is true for all $n \geq 0$, so we can let $n_0 = 0$.

3. (5 points) Show $n^2 + 4n + 8 \in \Theta(n^2)$

Solution:

Since $n^2 + 4n + 8 \in O(n^2)$ and $n^2 + 4n + 8 \in \Omega(n^2)$, $n^2 + 4n + 8 \in \Theta(n^2)$

4. (9 points) Show that if $g(n) \in O(f(n))$, then $a \cdot g(n) \in O(f(n))$, for any constant $a > 0$

Solution:

Since $g(n) \in O(f(n))$, then there exists a positive real constant c and a non-negative integer n_0, such that for all $n \geq n_0$

$$
\begin{aligned}
g(n) &\leq c \cdot f(n) && \text{By definition of Big-}O \\
a \cdot g(n) &\leq a \cdot c \cdot f(n) && \text{Multiply by the constant } a \\
a \cdot g(n) &\leq c' f(n) && \text{Let } c' = a \cdot c
\end{aligned}
$$

Since $a > 0$ and $c > 0$, $c' = a \cdot c > 0$. So by the definition of Big-O, $a \cdot g(n) \in O(f(n))$.