CSCI 310 – Data Structures – Spring 2019
HW 06 – Complexity Functions (12 points) – Solutions

1. (3 points) Use the definition of Big-O to show that if $g(n) \in O(f(n))$, then $a \cdot g(n) \in O(f(n))$, for any constant $a > 0$

Solution:

Since $g(n) \in O(f(n))$, then there exists a positive real constant c and a non-negative integer n_0, such that for all $n \geq n_0$

$$\begin{aligned} g(n) &\leq c \cdot f(n) \\ a \cdot g(n) &\leq a \cdot c \cdot f(n) \\ \textit{Letting } c' = a \cdot c & \\ a \cdot g(n) &\leq c' f(n) \end{aligned}$$

Since $a > 0$ and $c > 0$, $c' = a \cdot c > 0$. So by the definition of Big-O, $a \cdot g(n) \in O(f(n))$.

2. (3 points) Use the definition of Big-O to show that $2^{n+1} \in O(2^n)$

Solution:

Need to find a positive real constant c and a non-negative integer n_0, such that for all $n \geq n_0$

$$2^{n+1} \leq c \cdot 2^n$$

Letting $c = 2$

$$\begin{aligned} 2^{n+1} &\leq 2 \cdot 2^n \\ &= 2^{n+1} \end{aligned}$$

By the definition of Big-O, with $c = 2$ and $n_0 = 0$, $2^{n+1} \in O(2^n)$.

3. (3 points) Show that $n^2 \in \Omega(n \lg n)$

Solution:

Need to find a positive real constant c and a non-negative integer n_0, such that for all $n \geq n_0$

$$n^2 \geq c \cdot n \lg n$$

$$\begin{aligned} & n^2 \geq c \cdot n \lg n & \\ & n \geq c \cdot \lg n & \text{Divide by } n \\ & n \geq \lg n & \text{Let } c = 1 \end{aligned}$$

Since $n > \lg n$ for all n, by the definition of Big-Ω, with $c = 1$ and $n_0 = 0$, $n^2 \in \Omega(n \lg n)$.

4. (3 points) Show that $n^3 \notin O(n^2)$

Solution:

By Contradiction

Suppose the statement is false. Then $n^3 \in O(n^2)$, which means there is a positive real constant c and a non-negative integer n_0, such that for all $n \geq n_0$

$$n^3 \leq c \cdot n^2$$

$$\begin{aligned} n^3 &\leq c \cdot n^2 \\ n &\leq c && \text{Divide by } n^2 \end{aligned}$$

This is a contradiction since we can make the left hand side can be arbitrarily big.

What to turn in: This assignment is to be turned in through Blackboard. You can type up your solution using a computer program or you can prepare your solution by hand and scan it.