Order Examples

1. Show that $2n^2 + 9n \in O(n^2)$

   Need find a positive real constant c and a nonnegative integer n_0 such that for all $n \geq n_0$

   $$2n^2 + 9n \leq cn^2$$

   $$\begin{aligned}
   2n^2 + 9n &\leq cn^2 && \text{Definition of Big-O} \\
   9n &\leq cn^2 - 2n^2 && \text{Subtract } 2n^2 \\
   9n &\leq n^2(c-2) && \text{Factor out } n^2 \\
   9 &\leq n(c-2) && \text{Divide by } n \\
   9 &\leq n(3-2) && \text{Let } c = 3 \\
   9 &\leq n && \text{Simple subtraction}
   \end{aligned}$$

   This inequality holds when $n \geq 9$, so let $n_0 = 9$

   With $c = 3$ and $n_0 = 9$, the inequality holds for all $n \geq n_0$.

   $\therefore n^2 + 9n \in O(n^2)$

2. Show that $6n^3 - 12n \in \Omega(n^3)$

   Need to find a positive real constant c and a non-negative integer n_0, such that for all $n \geq n_0$

$$6n^3 - 12n \geq n^3$$

$6n^3 - 12n \geq cn^3$	Definition of Ω
$6n^2 - 12 \geq cn^2$	Divide by n
$6n^2 \geq cn^2 + 12$	Add 12
$6n^2 - cn^2 \geq 12$	Subtract cn^2
$n^2(6 - c) \geq 12$	Factor out n^2
$n^2 \geq \frac{12}{6-c}$	Divide by $6 - c$
$n^2 \geq \frac{12}{6-3}$	Let $c = 3$
$n^2 \geq \frac{12}{3}$	Simple subtraction
$n^2 \geq 4$	Simple division
$\sqrt{n^2} \geq \sqrt{4}$	Take square root of both sides
$n \geq 2$	Compute square roots

   This inequality holds when $n \geq 2$, so let $n_0 = 2$

   With $c = 3$ and $n_0 = 2$, the inequality holds for all $n \geq n_0$.

   $\therefore 6n^3 - 12n \in \Omega(n^3)$