Example of a proof using Logical Equivalences

1. Prove the following is a tautology:

$$((p \vee q) \wedge \neg p) \implies q$$

Proof.

$$\begin{aligned}
((p \vee q) \wedge \neg p) \implies q &\equiv \neg((p \vee q) \wedge \neg p) \vee q && \text{Implication as or} \\
&\equiv (\neg(p \vee q) \vee \neg\neg p) \vee q && \text{De Morgan} \\
&\equiv (\neg(p \vee q) \vee p) \vee q && \text{Double Negative} \\
&\equiv \neg(p \vee q) \vee (p \vee q) && \text{Associative} \\
&\equiv \mathbf{t} && \text{Negation}
\end{aligned}$$

□