

Example of Showing a Repeating Decimal is Rational

1. Show that $2.5\overline{4} \in \mathbb{Q}$ (Notice that only the 4 is repeating).

Let $x = 2.5\overline{4}$

Then $10x = 25.\overline{4}$

And $100x = 254.\overline{4}$

Which means

$$\begin{aligned} 100x - 10x &= 254.\overline{4} - 25.\overline{4} \\ &= 229 \end{aligned}$$

Also, $100x - 10x = 90x$.

Then

$$100x - 10x = 100x - 10x$$

$$90x = 229$$

$$x = \frac{229}{90}$$

Since 229 and 90 are both integers and $90 \neq 0$, by the definition of Rational, $x \in \mathbb{Q}$.