

Logical Equivalences

Given any statement variables p , q , and r , a tautology \mathbf{t} , and a contradiction \mathbf{c} , the following logical equivalences hold:

1. Commutative laws: $p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
2. Associative laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws: $p \wedge \mathbf{t} \equiv p$ $p \vee \mathbf{c} \equiv p$
5. Negation laws: $p \vee \neg p \equiv \mathbf{t}$ $p \wedge \neg p \equiv \mathbf{c}$
6. Double negative law: $\neg(\neg p) \equiv p$
7. Idempotent laws: $p \wedge p \equiv p$ $p \vee p \equiv p$
8. Universal bound law: $p \vee \mathbf{t} \equiv \mathbf{t}$ $p \wedge \mathbf{c} \equiv \mathbf{c}$
9. De Morgan's laws: $\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
10. Absorption law: $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
11. Negations of \mathbf{t} and \mathbf{c} : $\neg \mathbf{t} \equiv \mathbf{c}$ $\neg \mathbf{c} \equiv \mathbf{t}$
12. Division into cases: $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$
13. Implication as or: $p \rightarrow q \equiv \neg p \vee q$
14. Negating a conditional: $\neg(p \rightarrow q) \equiv p \wedge \neg q$
15. Contrapositive: $p \rightarrow q \equiv \neg q \rightarrow \neg p$
16. iff as implications: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$