Master Method (see handout on Class Website)

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$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

a = # of sc-problems

b = size of each subproblem

$f(n)$ = Time to create subproblems & Combine solutions

$a \geq 1$, $b > 1$

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

if $f(n) \in \Theta(n^d)$ $d \geq 0$

so we would know what d is

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \lg n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

Example #1

$$T(n) = 9T\left(\frac{n}{3}\right) + n$$

- 9 smaller instances.
- each of size $\frac{n}{3}$

$T(n) = 9T\left(\frac{n}{3}\right) + n$

each of size $\frac{n}{3}$

- n work to create subproblems & combine solutions

$f(n) \in \Theta(n^1)$

$\boxed{n \in \Theta(n^1)} \Rightarrow d = 1$

Case 1?

$a < b^d$

$9 < 3^1$

$9 < 3$ NO

Case 2?

$a = b^d$

$9 = 3$ NO

Case 3?

$a > b^d$

$9 > 3$ Yes!

Case 3 applies

$\Rightarrow T(n) \in \Theta(n^{\log_b a})$

$= \Theta(n^{\log_3 9})$

$= \Theta(n^2)$

$a = 9$

$b = 3$

Master Method Example # 2

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$$T(n) = T\left(\frac{2n}{3}\right) + 1$$

$\boxed{a = 1}$ $\boxed{b = \frac{3}{2}}$

$f(n) \in \Theta(n^0)$

$1 \in \Theta(1) \implies \boxed{d = 0}$

Case # 1?

$a < b^d$

$1 < \left(\frac{3}{2}\right)^0$

$1 < 1$ NO - CAN'T use case 1

Case # 2?

$a = b^d$

$1 = \left(\frac{3}{2}\right)^0$

$1 = 1$ $\boxed{\text{YES! we can use case 2}}$

Case # 3?

$a > b^d$

$1 > 1$ NO - CAN'T use case 3

By mm Case 2:

$$T(n) \in \Theta(n^d \lg n)$$

$$= \Theta(n^0 \lg n)$$

$$= \Theta(1 \cdot \lg n)$$

$$= \Theta(1 \cdot \lg n)$$
$$= \Theta(\lg n)$$

Levitin's problem form the beginning of the divide-and-conquer chapter.

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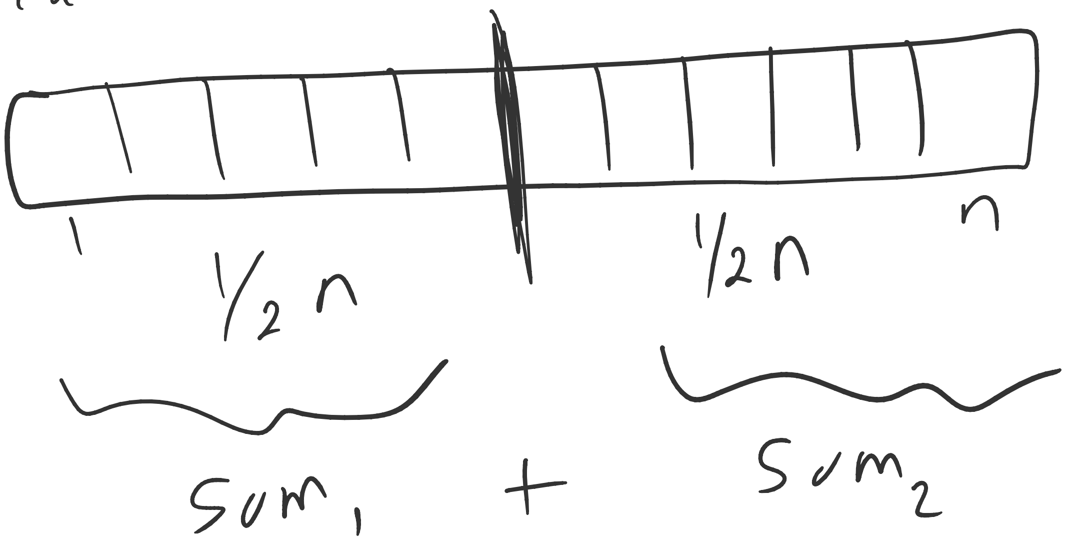
$a_0 + a_1 + \dots a_n$

Recursion is BAD for this problem.



$$A(n) = 2A\left(\frac{n}{2}\right) + 1$$

$a = 2 \quad b = 2 \quad f(n) = 1$

$f(n) \in \Theta(n^0) \Rightarrow d = 0$

$1 \in \Theta(1)$

$a = 2 \quad b = 2 \quad d = 0 \quad b^d = 2^0 = 1$

Case 1

$a < b^d$

Case 2

$a = b^d$

Case 3

$\boxed{a > b^d}$

$= 1$

$a < b^d$ $\qquad$ $a = b^d$

$2 < 1$ $\qquad$ $2 = 1$ $\qquad$ $2 > 1$

yes

$$A(n) \in \Theta\left(n^{\log_b a}\right)$$

$$= \Theta\left(n^{\log_2 2}\right)$$

$$= \Theta\left(n^1\right)$$

$$= \Theta(n)$$