CSCI 310 – Data Structures – Spring 2019
HW 05 – Big-O Problems (10 points) – Solutions

1. (4 points) Use the definition of Big-O to show $5n^2 + n \in O(n^2)$.

Solution: Find positive real constant c and non-negative integer n_0, such that for all $n \geq n_0$

$$5n^2 + n \leq cn^2$$

$$\begin{aligned}
5n^2 + n &\leq cn^2 & \\
5n + 1 &\leq cn & \text{Divide by } n \\
1 &\leq cn - 5n & \text{Subtract } 5n \\
1 &\leq 6n - 5n & \text{Let } \mathbf{c} = 6 \\
1 &\leq n &
\end{aligned}$$

Since the above is true for all $n \geq 1$, let $n_0 = 1$.

2. (2 points) Use the definition of Big-O to show $d \in O(1)$ for any constant $d > 0$. Note: Because of this result, we refer to all Constant-time complexity functions simply as $O(1)$.

Solution: Find positive real constant c and non-negative integer n_0, such that for all $n \geq n_0$

$$d \leq c \times 1$$

$$\begin{aligned}
d &\leq c \times 1 & \\
d &\leq d & \text{Let c = d}
\end{aligned}$$

This inequality holds for all values of n, so let $n_0 = 0$

3. (4 points) Use the definition of Big-O to show $6n^2 + 12 \in O(n^3)$.

Solution: Find positive real constant c and non-negative integer n_0, such that for all $n \geq n_0$

$$6n^2 + 12 \leq cn^3$$

$$\begin{aligned}
6n^2 + 12 &\leq cn^3 & \\
12 &\leq cn^3 - 6n^2 & \text{Subtract } 6n^2 \\
12 &\leq 6n^3 - 6n^2 & \text{Let } c = 6 \\
2 &\leq n^3 - n^2 & \text{Divide by } 6
\end{aligned}$$

The above inequality is true for $n = 2$, and as n grows from there, the RHS grows while the LHS remains constant. So we can let $n_0 = 2$.

What to turn in: This assignment is to be turned in through Blackboard. You can type up your solution using a computer program or you can prepare your solution by hand and scan it.