CSCI 393, Algorithm Design and Analysis, Spring 2020
HW 02 – Order of Growth   (37 points)

1. For each of the following functions, indicate how much the function's value will change if its argument is increased eightfold. Use either (i) The difference between, or (ii) The ratio of, $f(8n)$ and $f(n)$, whichever is more convenient for getting a compact answer. If possible, try to get an answer that does not depend on n.

   (a) (2 points) $f(n) = \lg n$ (Recall: $\lg n$ is another of writing $\log_2 n$)

   (b) (2 points) $f(n) = \sqrt[3]{n}$

   (c) (2 points) $f(n) = n$

   (d) (2 points) $f(n) = n^2$

   (e) (2 points) $f(n) = n^3$

   (f) (2 points) $f(n) = 2^n$

2. Indicate whether the first function of each of the following pairs has a smaller, same, or larger order of growth (to within a constant multiple) than the second function. For this question you *do not* need to use the formal definitions for $Big - O$, Ω, or Θ.

   Hint: If necessary, simplify the functions to single out terms defining their orders of growth.

   (a) (2 points) $n(n+1)$ *and* $2000n^2$

   (b) (2 points) $100n^2 + 12n + 4$ *and* $.006n^3$

   (c) (2 points) $\lg n$ *and* $\ln n$

   (d) (2 points) 2^{n-1} *and* 2^n

   (e) (2 points) $(n-1)!$ *and* $n!$

3. (5 points) Show directly, using the definition of $Big - O$, that $6n^2 + 12n \in O(n^2)$.

4. (5 points) Show directly, using the definition of Ω, that $6n^2 + 12n \in \Omega(n^2)$.

5. (5 points) Show that $6n^2 + 12n \in \Theta(n^2)$.