CSCI 310 – Data Structures – Spring 2020
HW 11 – Hashing (15 points) – Solutions

1. For the given Table Sizes, Hash Functions, Keys, and Probing Strategies, give the *first five* table locations probed.

   (a) (3 points) Table Size = 23; Key = 2297; $hash(x) = x \ \% \ Table\ Size$; Linear Probing function.

Solution:
Need to calculate $h_i(key)$ for $0 \leq i \leq 4$. For each value of i we use

$$h_i(key) = [hash(key) + f(i)] \ \% \ Table_Size$$

And since this is Linear Probing, $f(i) = i$.

i = 0

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_0(2297) &= [hash(2297) + f(0)] \ \% \ 23 \\ &= [2297 \ \% \ 23 + 0] \ \% \ 23 \\ &= [20 + 0] \ \% \ 23 \\ &= [20] \ \% \ 23 \\ &= 20 \end{aligned}$$

i = 1

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_1(2297) &= [hash(2297) + f(1)] \ \% \ 23 \\ &= [2297 \ \% \ 23 + 1] \ \% \ 23 \\ &= [20 + 1] \ \% \ 23 \\ &= [21] \ \% \ 23 \\ &= 21 \end{aligned}$$

i = 2

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_2(2297) &= [hash(2297) + f(2)] \ \% \ 23 \\ &= [2297 \ \% \ 23 + 2] \ \% \ 23 \\ &= [20 + 2] \ \% \ 23 \\ &= [22] \ \% \ 23 \\ &= 22 \end{aligned}$$

i = 3

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_3(2297) &= [hash(2297) + f(3)] \ \% \ 23 \\ &= [2297 \ \% \ 23 + 3] \ \% \ 23 \\ &= [20 + 3] \ \% \ 23 \\ &= [23] \ \% \ 23 \\ &= 0 \end{aligned}$$

i = 4

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_4(2297) &= [hash(2297) + f(4)] \ \% \ 23 \\ &= [2297 \ \% \ 23 + 4] \ \% \ 23 \\ &= [20 + 4] \ \% \ 23 \\ &= [24] \ \% \ 23 \\ &= 1 \end{aligned}$$

First five locations probed are: 20, 21, 22, 0, 1

(b) (3 points) Table Size = 29; Key = 512; $hash(x) = x \ \% \ Table\ Size$; Quadratic Probing function.

Solution:
Need to calculate $h_i(key)$ for $0 \leq i \leq 4$. For each value of i we use

$$h_i(key) = [hash(key) + f(i)] \ \% \ Table_Size$$

And since this is Quadratic Probing, $f(i) = i^2$.

i = 0

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_0(512) &= [hash(512) + f(0)] \ \% \ 29 \\ &= [512 \ \% \ 29 + 0^2] \ \% \ 29 \\ &= [19 + 0] \ \% \ 29 \\ &= [19] \ \% \ 29 \\ &= 19 \end{aligned}$$

i = 1

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_1(512) &= [hash(512) + f(1)] \ \% \ 29 \\ &= [512 \ \% \ 29 + 1^2] \ \% \ 29 \\ &= [19 + 1] \ \% \ 29 \\ &= [20] \ \% \ 29 \\ &= 20 \end{aligned}$$

i = 2

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_2(512) &= [hash(512) + f(2)] \ \% \ 29 \\ &= [512 \ \% \ 29 + 2^2] \ \% \ 29 \\ &= [19 + 4] \ \% \ 29 \\ &= [23] \ \% \ 29 \\ &= 23 \end{aligned}$$

i = 3

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_3(512) &= [hash(512) + f(3)] \ \% \ 29 \\ &= [512 \ \% \ 29 + 3^2] \ \% \ 29 \\ &= [19 + 9] \ \% \ 29 \\ &= [28] \ \% \ 29 \\ &= 28 \end{aligned}$$

i = 4

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_4(512) &= [hash(512) + f(4)] \ \% \ 29 \\ &= [512 \ \% \ 29 + 4^2] \ \% \ 29 \\ &= [19 + 16] \ \% \ 29 \\ &= [35] \ \% \ 29 \\ &= 6 \end{aligned}$$

First five locations probed are: $\mathbf{19, 20, 23, 28, 6}$

(c) (3 points) Table Size = 73; Key = 5697; $hash(x) = x \ \% \ Table\ Size$; Probing function $f(i) = i(i+7)(-1)^i$.

Solution:
Need to calculate $h_i(key)$ for $0 \le i \le 4$. For each value of i we use

$$h_i(key) = [hash(key) + f(i)] \ \% \ Table_Size$$

Here the probing function is $f(i) = i(i+7)(-1)^i$.

i = 0

$$\begin{aligned}
h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\
h_0(5697) &= [hash(5697) + f(0)] \ \% \ 73 \\
&= [5697 \ \% \ 73 + f(0)] \ \% \ 73 \\
&= [3 + f(0)] \ \% \ 73 \\
&= [3 + 0 \times (0+7)(-1)^0] \ \% \ 73 \\
&= [3 + 0] \ \% \ 73 \\
&= [3] \ \% \ 73 \\
&= 3
\end{aligned}$$

i = 1

$$\begin{aligned}
h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\
h_1(5697) &= [hash(5697) + f(1)] \ \% \ 73 \\
&= [5697 \ \% \ 73 + f(1)] \ \% \ 73 \\
&= [3 + f(1)] \ \% \ 73 \\
&= [3 + 1 \times (1+7)(-1)^1] \ \% \ 73 \\
&= [3 + 1 \times (8)(-1)] \ \% \ 73 \\
&= [3 - 8] \ \% \ 73 \\
&= [-5] \ \% \ 73 \\
&= 68
\end{aligned}$$

$i = 2$

$$\begin{aligned}
h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\
h_2(5697) &= [hash(5697) + f(2)] \ \% \ 73 \\
&= [5697 \ \% \ 73 + f(2)] \ \% \ 73 \\
&= [3 + f(2)] \ \% \ 73 \\
&= [3 + 2 \times (2+7)(-1)^2] \ \% \ 73 \\
&= [3 + 2 \times (9)(1)] \ \% \ 73 \\
&= [3 + 18] \ \% \ 73 \\
&= [21] \ \% \ 73 \\
&= 21
\end{aligned}$$

i = 3

$$\begin{aligned}
h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\
h_3(5697) &= [hash(5697) + f(3)] \ \% \ 73 \\
&= [5697 \ \% \ 73 + f(3)] \ \% \ 73 \\
&= [3 + f(3)] \ \% \ 73 \\
&= [3 + 3 \times (3+7)(-1)^3] \ \% \ 73 \\
&= [3 + 3 \times (10)(-1)] \ \% \ 73 \\
&= [3 - 30] \ \% \ 73 \\
&= [-27] \ \% \ 73 \\
&= 46
\end{aligned}$$

i = 4

$$\begin{aligned}
h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\
h_4(5697) &= [hash(5697) + f(4)] \ \% \ 73 \\
&= [5697 \ \% \ 73 + f(4)] \ \% \ 73 \\
&= [3 + f(4)] \ \% \ 73 \\
&= [3 + 4 \times (4+7)(-1)^4] \ \% \ 73 \\
&= [3 + 4 \times (11)(1)] \ \% \ 73 \\
&= [3 + 44] \ \% \ 73 \\
&= [47] \ \% \ 73 \\
&= 47
\end{aligned}$$

First five locations probed are: **First five locations probed are: 3, 68, 21, 46, 47**

(d) (3 points) Table Size = 67; Key = 1812; $hash_1(x) = x \ \% \ Table \ Size$; $hash_2(x) = R - (x \ \% \ R)$; Probing function is Double Hashing.

Solution:
Need to calculate $h_i(key)$ for $0 \leq i \leq 4$. For each value of i we use

$$h_i(key) = [hash(key) + f(i)] \ \% \ Table_Size$$

With double hashing, $f(i) = i \times hash_2(key)$

$hash_2(key) = R - (key \ \% \ R)$, where R is the largest prime less than the table size. Thus $R = 61$

Then: $hash_2(key) = 61 - (key \ \% \ 61)$, which means

$$\begin{aligned} f(i) &= i \times hash_2(key) \\ &= i(61 - (key \ \% \ 61)) \\ &= i(61 - (1812 \ \% \ 61)) \\ &= i(61 - 43) \\ &= i \times 18 \end{aligned}$$

i = 0

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_0(1812) &= [hash(1812) + f(0)] \ \% \ 67 \\ &= [1812 \ \% \ 67 + f(0)] \ \% \ 67 \\ &= [3 + f(0)] \ \% \ 67 \\ &= [3 + 0 \times 18] \ \% \ 67 \\ &= [3 + 0] \ \% \ 67 \\ &= [3] \ \% \ 67 \\ &= 3 \end{aligned}$$

i = 1

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_1(1812) &= [hash(1812) + f(1)] \ \% \ 67 \\ &= [1812 \ \% \ 67 + f(1)] \ \% \ 67 \\ &= [3 + f(1)] \ \% \ 67 \\ &= [3 + 1 \times 18] \ \% \ 67 \\ &= [3 + 18] \ \% \ 67 \\ &= [21] \ \% \ 67 \\ &= 21 \end{aligned}$$

i = 2

$$\begin{aligned} h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\ h_2(1812) &= [hash(1812) + f(2)] \ \% \ 67 \\ &= [1812 \ \% \ 67 + f(2)] \ \% \ 67 \\ &= [3 + f(2)] \ \% \ 67 \\ &= [3 + 2 \times 18] \ \% \ 67 \\ &= [3 + 36] \ \% \ 67 \\ &= [39] \ \% \ 67 \\ &= 39 \end{aligned}$$

i = 3

$$\begin{aligned}
h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\
h_3(1812) &= [hash(1812) + f(3)] \ \% \ 67 \\
&= [1812 \ \% \ 67 + f(3)] \ \% \ 67 \\
&= [3 + f(3)] \ \% \ 67 \\
&= [3 + 3 \times 18] \ \% \ 67 \\
&= [3 + 54] \ \% \ 67 \\
&= [57] \ \% \ 67 \\
&= 57
\end{aligned}$$

i = 4

$$\begin{aligned}
h_i(key) &= [hash(key) + f(i)] \ \% \ Table_Size \\
h_4(1812) &= [hash(1812) + f(4)] \ \% \ 67 \\
&= [1812 \ \% \ 67 + f(4)] \ \% \ 67 \\
&= [3 + f(4)] \ \% \ 67 \\
&= [3 + 4 \times 18] \ \% \ 67 \\
&= [3 + 72] \ \% \ 67 \\
&= [75] \ \% \ 67 \\
&= 8
\end{aligned}$$

First five locations probed are: **First five locations probed are: $\mathbf{3, 21, 39, 57, 8}$**

2. (3 points) Consider hashing with:

   - $hash(x) = x \ \% \ TableSize$
   - A table size of 10
   - Quadratic probing

   Show the contents of the hash table after inserting the keys $612, 9278, 4, 212, 613, 815, 1694$.

0	**1694**
1	
2	**612**
3	**212**
4	**4**
5	**815**
6	
7	**613**
8	**9278**
9	

Solution: $key = 612$

$$\begin{aligned} h_0(key) &= [hash(key) + 0^2] \ \% \ TableSize \\ &= [key \ \% \ TableSize + 0] \ \% \ TableSize \\ &= [612 \ \% \ 10 + 0] \ \% \ 10 \\ &= [2 + 0] \ \% \ 10 \\ &= [2] \ \% \ 10 \\ &= 2 \end{aligned}$$

Table location 2 is free so store 612 at 2.

0	
1	
2	612
3	
4	
5	
6	
7	
8	
9	

Solution: $key = 9278$

$$\begin{aligned} h_0(key) &= [hash(key) + 0^2] \ \% \ TableSize \\ &= [key \ \% \ TableSize + 0] \ \% \ TableSize \\ &= [9278 \ \% \ 10 + 0] \ \% \ 10 \\ &= [8 + 0] \ \% \ 10 \\ &= [8] \ \% \ 10 \\ &= 8 \end{aligned}$$

Table location 8 is free so store 9278 at 8.

0	
1	
2	612
3	
4	
5	
6	
7	
8	9278
9	

Solution: $key = 4$

$$\begin{aligned} h_0(key) &= [hash(key) + 0^2] \;\%\; TableSize \\ &= [key \;\%\; TableSize + 0] \;\%\; TableSize \\ &= [4 \;\%\; 10 + 0] \;\%\; 10 \\ &= [4 + 0] \;\%\; 10 \\ &= [4] \;\%\; 10 \\ &= 4 \end{aligned}$$

Table location 4 is free so store 4 at 4.

0	
1	
2	612
3	
4	4
5	
6	
7	
8	9278
9	

Solution: $key = 212$

$$\begin{aligned} h_0(key) &= [hash(key) + 0^2] \ \% \ TableSize \\ &= [key \ \% \ TableSize + 0] \ \% \ TableSize \\ &= [212 \ \% \ 10 + 0] \ \% \ 10 \\ &= [2 + 0] \ \% \ 10 \\ &= [2] \ \% \ 10 \\ &= 2 \end{aligned}$$

Table location 2 already contains 612, so probe using Quadratic Probing.

$$\begin{aligned} h_1(key) &= [hash(key) + 1^2] \ \% \ TableSize \\ &= [key \ \% \ TableSize + 1] \ \% \ TableSize \\ &= [212 \ \% \ 10 + 1] \ \% \ 10 \\ &= [2 + 1] \ \% \ 10 \\ &= [3] \ \% \ 10 \\ &= 3 \end{aligned}$$

Table location 3 is free so store 212 at 3.

0	
1	
2	612
3	212
4	4
5	
6	
7	
8	9278
9	

Solution: $key = 613$

$$\begin{aligned}
h_0(key) &= [hash(key) + 0^2] \;\%\; TableSize \\
&= [key \;\%\; TableSize + 0] \;\%\; TableSize \\
&= [613 \;\%\; 10 + 0] \;\%\; 10 \\
&= [3 + 0] \;\%\; 10 \\
&= [3] \;\%\; 10 \\
&= 3
\end{aligned}$$

Table location 3 already contains 212, so probe using Quadratic Probing.

$$\begin{aligned}
h_1(key) &= [hash(key) + 1^2] \;\%\; TableSize \\
&= [key \;\%\; TableSize + 1] \;\%\; TableSize \\
&= [613 \;\%\; 10 + 1] \;\%\; 10 \\
&= [3 + 1] \;\%\; 10 \\
&= [4] \;\%\; 10 \\
&= 4
\end{aligned}$$

Table location 4 already contains 4, so continue probing using Quadratic Probing.

$$\begin{aligned}
h_2(key) &= [hash(key) + 2^2] \;\%\; TableSize \\
&= [key \;\%\; TableSize + 4] \;\%\; TableSize \\
&= [613 \;\%\; 10 + 4] \;\%\; 10 \\
&= [3 + 4] \;\%\; 10 \\
&= [7] \;\%\; 10 \\
&= 7
\end{aligned}$$

Table location 7 is free so store 613 at 7.

0	
1	
2	612
3	212
4	4
5	
6	
7	613
8	9278
9	

Solution: $key = 815$

$$
\begin{aligned}
h_0(key) &= [hash(key) + 0^2] \ \% \ TableSize \\
&= [key \ \% \ TableSize + 0] \ \% \ TableSize \\
&= [815 \ \% \ 10 + 0] \ \% \ 10 \\
&= [5 + 0] \ \% \ 10 \\
&= [5] \ \% \ 10 \\
&= 5
\end{aligned}
$$

Table location 5 is free so store 815 at 5.

0	
1	
2	612
3	212
4	4
5	815
6	
7	613
8	9278
9	

Solution: $key = 1964$

$$\begin{aligned}
h_0(key) &= [hash(key) + 0^2] \ \% \ TableSize \\
&= [key \ \% \ TableSize + 0] \ \% \ TableSize \\
&= [1964 \ \% \ 10 + 0] \ \% \ 10 \\
&= [4 + 0] \ \% \ 10 \\
&= [4] \ \% \ 10 \\
&= 4
\end{aligned}$$

Table location 4 already contains 4, so continue probing using Quadratic Probing.

$$\begin{aligned}
h_1(key) &= [hash(key) + 1^2] \ \% \ TableSize \\
&= [key \ \% \ TableSize + 1] \ \% \ TableSize \\
&= [1964 \ \% \ 10 + 1] \ \% \ 10 \\
&= [4 + 1] \ \% \ 10 \\
&= [5] \ \% \ 10 \\
&= 5
\end{aligned}$$

Table location 5 already contains 815, so continue probing using Quadratic Probing.

$$\begin{aligned}
h_2(key) &= [hash(key) + 2^2] \ \% \ TableSize \\
&= [key \ \% \ TableSize + 4] \ \% \ TableSize \\
&= [1964 \ \% \ 10 + 4] \ \% \ 10 \\
&= [4 + 4] \ \% \ 10 \\
&= [8] \ \% \ 10 \\
&= 8
\end{aligned}$$

Table location 8 already contains 9278, so continue probing using Quadratic Probing.

$$\begin{aligned}
h_3(key) &= [hash(key) + 3^2] \ \% \ TableSize \\
&= [key \ \% \ TableSize + 9] \ \% \ TableSize \\
&= [1964 \ \% \ 10 + 9] \ \% \ 10 \\
&= [4 + 9] \ \% \ 10 \\
&= [13] \ \% \ 10 \\
&= 3
\end{aligned}$$

Table location 3 already contains 212, so continue probing using Quadratic Probing.

$$\begin{aligned}
h_4(key) &= [hash(key) + 4^2] \ \% \ TableSize \\
&= [key \ \% \ TableSize + 16] \ \% \ TableSize \\
&= [1964 \ \% \ 10 + 16] \ \% \ 10 \\
&= [4 + 16] \ \% \ 10 \\
&= [20] \ \% \ 10 \\
&= 0
\end{aligned}$$

Table location 0 is free so store 1964 at 0.

0	1964
1	
2	612
3	212
4	4
5	815
6	
7	613
8	9278
9	

What to turn in: This assignment is to be turned in through Blackboard. You can type up your solution using a computer or you can prepare your solution by hand and scan it. The file that gets uploaded to Blackboard ***Must Be A PDF File***.