Example of a Direct Proof

1. Prove the following:

   Claim:

   If w is even, then $6w$ is even.

   Proof.
   Let w be an even number. By the definition of even, $w = 2k$, for $k \in \mathbb{Z}$. Then

   $$\begin{aligned} 6w &= 6(2k) && \text{substitute } 2k \text{ for } w \\ &= 2(6k) \end{aligned}$$

   Since $6, k \in \mathbb{Z}$ and $\mathbb{Z}$ is closed under multiplication, $6k \in \mathbb{Z}$.
   $\therefore$ by the definition of even, $6w$ is even.

   $\square$