Example of Proof by Contradiction

1. Use proof by contradiction to prove the following statement:
   If A and B are sets, then $A \cap (B - A) \equiv \emptyset$

   Proof. Suppose not.
   That is, suppose A and B are sets but $A \cap (B - A) \not\equiv \emptyset$.
   Then there is an element $e \in A \cap (B - A)$.

   By the definition of Intersection, $A \cap (B - A) \equiv \{x \mid x \in A \text{ and } x \in B - A\}$
   By the definition of Set Difference, $B - A \equiv \{x \mid x \in B \text{ and } x \notin A\}$

   Combining these two definitions, we have $A \cap (B - A) \equiv \{x \mid x \in A \text{ and } x \in B \text{ and } x \notin A\}$

   Then $e \in \{x \mid x \in A \text{ and } x \in B \text{ and } x \notin A\}$
   Which means $e \in A$ and $e \notin A$
   This is a contradiction.

   $\therefore$ If A and B are sets, then $A \cap (B - A) \equiv \emptyset$ $\square$