

Indicator Random Variable Example

Problem

Use indicator random variables to solve the following problem:

A pair of fair dice are rolled n times. What is the expected number of times they come up 7?

Solution

Define a random variable C that equals the number of rolls that come up 7. We are asked to compute $E[C]$.

For $i = 1, 2, \dots, n$, define the indicator random variable

$$X_i = \begin{cases} 1 & \text{if roll } i \text{ comes up 7} \\ 0 & \text{if roll } i \text{ does not come up 7} \end{cases}$$

Then $C = \sum_{i=1}^n X_i$

Note that $Pr[X_i = 1] = Pr[\text{roll } i \text{ comes up 7}] = \frac{1}{6}$.

This means

$$\begin{aligned} E[X_i] &= 1 \cdot Pr[X_i = 1] + 0 \cdot Pr[X_i = 0] \\ &= Pr[X_i = 1] \\ &= \frac{1}{6} \end{aligned}$$

We now have

$$\begin{aligned} E[C] &= E\left[\sum_{i=1}^n X_i\right] \\ &= \sum_{i=1}^n E[X_i] \quad (\text{by linearity of expectation}) \\ &= \sum_{i=1}^n \frac{1}{6} \\ &= \frac{n}{6} \end{aligned}$$

Thus we expect that $\frac{n}{6}$ of the rolls will come up 7.