Example of a Direct Proof

1. Prove the following:

   Claim:

   If n is odd, then $5n + 2$ is odd.

   Proof.
   Let n be an odd number. By the definition of odd, $n = 2k + 1$, for $k \in \mathbb{Z}$. Then

$$\begin{aligned} 5n + 2 &= 5(2k + 1) + 2 \quad \textit{substitute } 2k + 1 \textit{ for } n \\ &= 10k + 5 + 2 \\ &= 10k + 6 + 1 \\ &= (10k + 6) + 1 \\ &= 2(5k + 3) + 1 \end{aligned}$$

   Since $5, k, 3 \in \mathbb{Z}$ and $\mathbb{Z}$ is closed under multiplication and addition, $5k + 3 \in \mathbb{Z}$.
   $\therefore$ by the definition of odd, $5n + 2$ is odd.

   □