Sets

A set is a collection of objects, called elements. The elements must be distinct (each element can appear in the set only once). A set may have a finite or an infinite number of elements. The order of the elements doesn't matter.

Symbols and Terms

$a \in A$	a is an element of set A.
$a \notin A$	a is not an element of set A.
$\{a_1, a_2, \ldots, a_n\}$	The set with elements $a_1, a_2, \ldots, a_n$.
$\{x \in D \mid \textit{condition}\}$	The set of all elements x in D that satisfy the *condition*.
$\mathbb{R}, \mathbb{R}^-, \mathbb{R}^+, \mathbb{R}^{nonneg}$	Set of all real numbers, negative reals, positive reals, and nonnegative reals.
$\mathbb{Z}, \mathbb{Z}^-, \mathbb{Z}^+, \mathbb{Z}^{nonneg}$	Set of all integers, negative integers, positive integers, and nonnegative integers.
$\mathbb{Q}, \mathbb{Q}^-, \mathbb{Q}^+, \mathbb{Q}^{nonneg}$	Set of rational numbers, negative rationals, positive rationals, and nonnegative rationals.
$\mathbb{N}$	Set of natural numbers. $\mathbb{N} = \{1, 2, \ldots\}$.
$\mathbb{W}$	Set of whole numbers. $\mathbb{W} = \{0, 1, \ldots\}$.
$\mathbb{Z}_2$	Set of binary digits. $\mathbb{Z}_2 = \{0, 1\}$.
$\lvert A \rvert$	The number of elements in A; The *cardinalty* of A.
$A \subset B$	A is a proper subset of B. Equivalently $B \supset A$.
$A \subseteq B$	A is a subset of B. Equivalently $B \supseteq A$.
$A \not\subset B$	A is a not a proper subset of B. Equivalently $B \not\supset A$.
$A \not\subseteq B$	A is not a subset of B. Equivalently $B \not\supseteq A$.
$A \supset B$	A is a proper superset of B. Equivalently $A \subset B$.
$A \supseteq B$	A is a superset of B. Equivalently $B \subseteq A$.
$A \not\supset B$	A is a not a proper superset of B. Equivalently $B \not\subset A$.
$A \not\supseteq B$	A is not a superset of B. Equivalently $B \not\subseteq A$.
$A = B$	A equals B.
$A \neq B$	A does not equal B.
$A \cup B$	A union B.
$A \cap B$	A intersect B.
$A - B$	The difference of A minus B. An alternate notation for difference is $A \setminus B$.
A^c	The complement of A. An alternate notation is $\overline{A}$ or A'.
(x, y)	Ordered pair.
$(x_1, x_2, \ldots, x_n)$	n-tuple.
$A \times B$	The Cartesian product of A and B.
$A_1 \times A_2 \times \ldots \times A_n$	The Cartesian product of $A_1, A_2, \ldots A_n$.
$\emptyset$	The empty set. An alternate notation for the empty set is $\varnothing$ or $\{\}$.
$\mathscr{P}(A)$	The power set of A.
$Disjoint$	A and B are disjoint if $A \cap B = \emptyset$.
$Pairwise\ Disjoint$	$A_1, A_2, \ldots A_n$ are pairwise disjoint if, for every pair A_i and A_j, $A_i \cap A_j = \emptyset$.

Selected Examples

Set Definitions and Cardinality

We can define a set simply by listing its elements:
$A_1 = \{2, 3, 4, 5, 6\}$.
$A_2 = \{2, 4, 6\}$.
$A_3 = \{3, 5, 7\}$.
$A_4 = \{4, 5, 6, 7, 8, 9\}$.

Sets can containing anything, not just numbers:
$A_5 = \{Dalek,\ Tardis,\ Sonic\ Screwdriver,\ Cybarman\}$.

Instead of listing every element, we can use notation with a condition:
$A_6 = \{x \in \mathbb{Z}^+ \mid x \leq 6\ and\ x\ is\ even\}$ is the set {2,4,6}.
$A_7 = \{x \in \mathbb{Z} \mid x\ is\ even\}$ is the set of all even integers.
$A_8 = \{x \in \mathbb{N} \mid x <= 1,000,000,000\}$ is the set of all counting numbers up to one billion.

Since the number 2 is an element of set A_1, we say $2 \in A_1$.
Since *Bow Tie* is not a member of set A_5, we say $Bow\ Tie \notin A_5$.

We can see that there are 5 elements in A_1. We write this as $|A_1| = 5$.
Similarly $|A_2| = 3$, $|A_3| = 3$, $|A_4| = 6$, $|A_5| = 4$, $|A_6| = 3$, $|A_7| = \infty$, and $|A_8| = 1,000,000,000$. Also, since the empty set contains no elements, $|\emptyset| = 0$.

Comparing Two Sets

Proper Subset
$A \subset B$ means every element in A is also in B, and there is at least one element in B that is not in A. Using the above sets, we have $A_2 \subset A_1$. However, $A_2 \not\subset A_4$ because $2 \in A_2$ but $2 \notin A_4$. Also, $A_2 \not\subset A_6$ because, while every element in A_2 is in A_6, there is no element in A_6 that is not in A_2.

Subset
$A \subseteq B$ means every element in A is also in B. We still have $A_2 \subseteq A_1$, but now we also have $A_2 \subseteq A_6$. Also, the empty set is a subset of every set. In other words, for any set A, $\emptyset \subseteq A$.

Set Equality
$A = B$ means A and B have exactly the same elements. This is often expressed as $A \subseteq B$ and $B \subseteq A$. Using the above sets we have:
$A_2 = A_6$
$\mathbb{N} = \mathbb{Z}^+$
$\mathbb{W} = \mathbb{Z}^{nonneg}$

Creating New Sets from Old

Union
$A \cup B$ is the set containing every element that is in A or in B. So:
$A_1 \cup A_2 = \{2, 3, 4, 5, 6\}$.
$A_1 \cup A_3 = \{2, 3, 4, 5, 6, 7\}$.

Inersection
$A \cap B$ is the set containing every element that is in A and in B. So:
$A_1 \cap A_2 = \{2, 4, 6\}$.
$A_1 \cap A_3 = \{3\}$.
$A_2 \cap A_3 = \emptyset$.

Set Difference
$A - B$ is the set containing all elements that are in A but not in B. You can think of this as starting out with the elements of A and removing everything that is also in B.
$A_1 - A_2 = \{3, 5\}$.
$A_2 - A_1 = \emptyset$.
$A_4 - A_7 = \{5, 7, 9\}$.
$\mathbb{Z}^{nonneg} - \mathbb{Z}^+ = \{0\}$.

Complement
If set A is a subset of a universal set U, then A^c is the set of all elements in U that are not in A. So if the universal set for A_1 is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then $A_2^c = \{1, 3, 5, 7, 8, 9, 10\}$.

Cartesian Product
$A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.
$A_2 \times A_3 = \{(2, 3), (2, 5), (2, 7), (4, 3), (4, 5), (4, 7), (6, 3), (6, 5), (6, 7)\}$
Notice that $|A \times B|$=$|A|\cdot|B|$.

Power Sets
$\mathscr{P}(A)$ is the set containing all subsets of A.
$\mathscr{P}(A_2) = \{\emptyset, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{2, 6\}, \{4, 6\}, \{2, 4, 6\}\}$
Note, $|\mathscr{P}(A)|$= $2^{|A|}$

Set Identities

Let all sets referred to below be subsets of a universal set U.

1.	Commutative laws:	$A \cap B = B \cap A$	$A \cup B = B \cup A$
2.	Associative laws:	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
3.	Distributive laws:	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4.	Identity laws:	$A \cap U = A$	$A \cup \emptyset = A$
5.	Complementation laws:	$A \cup A^c = U$	$A \cap A^c = \emptyset$
6.	Double complement law:	$(A^c)^c = A$	
7.	Idempotent laws:	$A \cap A = A$	$A \cup A = A$
8.	De Morgan's laws:	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
9.	Universal bound law:	$A \cup U = U$	$A \cap \emptyset = \emptyset$
10.	Absorption law:	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
11.	Complements of U and $\emptyset$:	$U^c = \emptyset$	$\emptyset^c = U$
12.	Set Difference	$A - B = A \cap B^c$	