(Local) In[3]:=

```
AppendTo[$Path, NotebookDirectory[]];
<< Matte2D`
```

```
Matte package loaded. © 2017-2018 Jonas Einarsson (me@jonaseinarsson.se)
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```

Tensor Angular Harmonics are made up of *irreducible* rank-n tensors

(Local) In[ ]:=

```
On[Assert];
Assert[trace[delta] == 2]
```

These are extremely hard to find. This code was put together through blood sweat and tears.
It is simple in concept, you can express an angular function in terms of sines and cosines.

Cos[n θ] , Sin[n θ]

The most info I’ve found is in a *footnote* in this paper. https://doi.org/10.1209/0295-5075/101/20010

Describing how the order n harmonics are related to eigenfunctions of the rotational laplacian operator with eigenvalue -n^2.

(Local) In[7]:=

```
RotOpSqrd[ex_] :=
  Module[{j1, j2, j3}, {j1 = getNewIndex[], j2 = getNewIndex[], j3 = getNewIndex[]};
   (delta[j1, j2] - invr[2] * r[j1] * r[j2]) *
     partialr[(delta[j3, j2] - invr[2] * r[j3] * r[j2]) * (partialr[ex, j3]) // Expand //
       contract , j1] // contract] ;
```

Now use that small tidbit of information to get this.

Essentially take the angular laplacian of the product of r_i ... r_n and then divide that by it’s eigenvalues. Somehow that seems to get the correct information.

(Local) In[8]:=

```
IrreducibleRanknTensor2D[0] := 1
IrreducibleRanknTensor2D[n_] := ReplaceAll[invr[x_] :→ 1][
    -1/(n^2) RotOpSqrd[Table[r[idx[indx]], {indx, 1, n}] /. List → Times] // contract ];
```

(Local) In[ ]:=

```
RotOpSqrd[r[idx[1]] * r[idx[2]]] // prettyprint
```

(Local) Out[ ]=

$$-\frac{4\, r_i\, r_j}{r^2} + 2\, \delta_{i,j}$$

So for *any* vector r, the irreducible rank-n tensor that describes it follows that formula

(Local) In[ ]:=

```
unitsphereIntegral[
    (IrreducibleRanknTensor2D[2] /. idx[1] → idx[3] /. idx[2] → idx[4]) *
       IrreducibleRanknTensor2D[2] // Expand //
     contract] * Q[idx[1], idx[2]] // Expand // contract
```

(Local) Out[ ]=

$$\frac{1}{2}\, \pi\, Q[idx[3], idx[4]]$$

(Local) In[12]:=

```
IrreducibleRanknTensor2D[0] // prettyprint
IrreducibleRanknTensor2D[1] // prettyprint
IrreducibleRanknTensor2D[2] // prettyprint
IrreducibleRanknTensor2D[3] // prettyprint
IrreducibleRanknTensor2D[4] // prettyprint
IrreducibleRanknTensor2D[5] // prettyprint
```

(Local) Out[ ]=

$$1$$

(Local) Out[ ]=

$$r_i$$

(Local) Out[ ]=

$$r_i\, r_j - \frac{\delta_{i,j}}{2}$$

(Local) Out[ ]=

$$r_i\, r_j\, r_k - \frac{2}{9}\, r_k\, \delta_{i,j} - \frac{2}{9}\, r_j\, \delta_{i,k} - \frac{2}{9}\, r_i\, \delta_{j,k}$$

(Local) Out[ ]=

$$r_i\, r_j\, r_k\, r_l - \frac{1}{8}\, r_k\, r_l\, \delta_{i,j} - \frac{1}{8}\, r_j\, r_l\, \delta_{i,k} - \frac{1}{8}\, r_j\, r_k\, \delta_{i,l} - \frac{1}{8}\, r_i\, r_l\, \delta_{j,k} - \frac{1}{8}\, r_i\, r_k\, \delta_{j,l} - \frac{1}{8}\, r_i\, r_j\, \delta_{k,l}$$

(Local) Out[ ]=

$$r_i\, r_j\, r_k\, r_l\, r_p - \frac{2}{25}\, r_k\, r_l\, r_p\, \delta_{i,j} - \frac{2}{25}\, r_j\, r_l\, r_p\, \delta_{i,k} - \frac{2}{25}\, r_j\, r_k\, r_p\, \delta_{i,l} - \frac{2}{25}\, r_j\, r_k\, r_l\, \delta_{i,p} - \frac{2}{25}\, r_i\, r_l\, r_p\, \delta_{j,k} - \frac{2}{25}\, r_i\, r_k\, r_p\, \delta_{j,l} - \frac{2}{25}\, r_i\, r_k\, r_l\, \delta_{j,p} - \frac{2}{25}\, r_i\, r_j\, r_p\, \delta_{k,l} - \frac{2}{25}\, r_i\, r_j\, r_l\, \delta_{k,p} - \frac{2}{25}\, r_i\, r_j\, r_k\, \delta_{l,p}$$

Go through the same steps as described in the other TensorHarmonics File to utilize.