



Figure 1: Photograph of a fully built and functioning compressed air train.

## Objectives

Although the method of trial and error is certainly a viable approach for some engineering design problems. The process of build-test-break-repeat can be quite expensive (both in terms of time and money) and thus prohibitive in many situations. Safety can also be a concern if designs are constructed haphazardly or without due consideration of forces involved. Whenever possible, design solutions derived from numerical simulations based on sound engineering theory are preferred. This project will guide you through the steps in such a design process. At the conclusion of the project, you will be able to specify the materials and dimensions of a model train that should be competitive in the hypothetical race.

The specific objectives are:

- i. Follow best-practices in the process of obtaining an engineering design solution.
- ii. Demonstrate proficiency in writing computer programs in Matlab.
- iii. Implement a fourth-order Runge-Kutta technique to solve the differential equations describing the motion of the train.
- iv. Implement a simple structured or unstructured search algorithm to perform multidimensional optimization with constraints.
- v. Interpret numerical results to select a final design solution.

## Work Due

The work due for this design project consists of a single pdf file in memo format. A template for the memo using Word is provided in CANVAS (and also included in the appendix of this document). The template outlines the requirements for each section of the memo. The easiest strategy is to download the template, modify it in Word based on your own work, print it to a pdf file, and then upload the pdf file to CANVAS. Be sure to modify the “To:” and “From:” lines in the memo heading.

## Train Design Competition

The project involves the design of a model train to compete in a race as described below. Note, you will NOT be building anything in this class, but rather specifying the physical dimensions of the train and locomotion system in order to be competitive in the race.

### Race Track

The race track consists of a 10-m section of straight track with a single tunnel, as shown in Figure 2. There is a 1.5 m length of “set-up” track before the start line, and a 2 m length of “run-out” track after the finish line. The tunnel has a minimum internal width of 0.2 m, a length of 1 m, and a maximum internal height of 0.26 m above the tracks. The track used in the competition is standard model G-scale railroad tracks (rails spaced 45 mm apart). Steel wheels (diameter of 40 mm) and axles will be supplied.

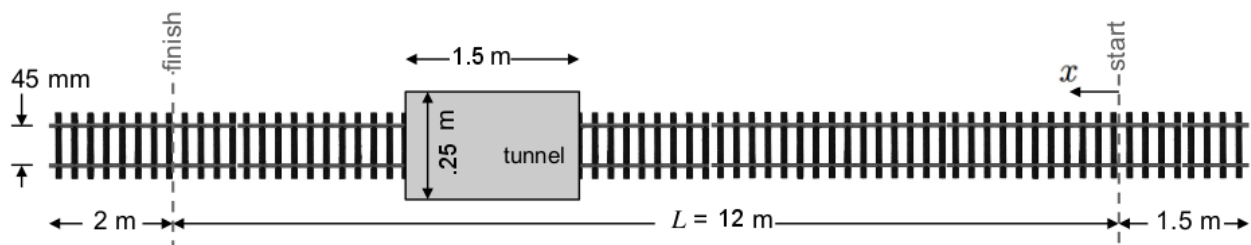


Figure 2: Layout of the race track. The distance traveled by the train is denoted by  $x(t)$ , with  $x(t=0)=0$  located at the start line.

### Goal of the Race

The goal of the competition is to design a train that completes the race from start to finish in the least amount of time without damaging the tunnel and without running off the end of the track.

### Power Source

The train is to be powered by compressed air stored on-board in a pressurized tank having a cylindrical geometry. The tank will be filled prior to the start of the race using a manual bicycle pump with an inline dial-type pressure gage. According to the rules of the competition, the initial tank pressure must not exceed 100 psi.

### Locomotion System

In order to simplify the modeling equations, we will restrict the locomotion system to one consisting of a pneumatic piston connected to a rack and pinion gear that drives the rear axle of the train. Furthermore, we will restrict the actuation of the piston to a single stroke. The locomotion system will provide an “initial push” to accelerate the train up to its maximum velocity; after which time, the train simply coasts (decelerates) to the finish line. In this manner, the propulsion system is only active during a portion of the race, as illustrated in Figure 3. We will denote the acceleration stage from  $0 \leq x \leq L_a$  and the deceleration stage from  $L_a < x \leq L$ , where  $L$  is the total length traveled. Here,  $x(t)$  represents the distance traveled by the model train as a function of time, where  $x=0$  represents the starting line. We will see in the next section that the propulsive force acting on the pneumatic piston decreases with  $x$  (i.e., the driving force is non-constant over the acceleration stage). Contrast this case to that of a gravitational force (as one would experience rolling down a hill, say), which is constant and simply proportional to the weight of the object.

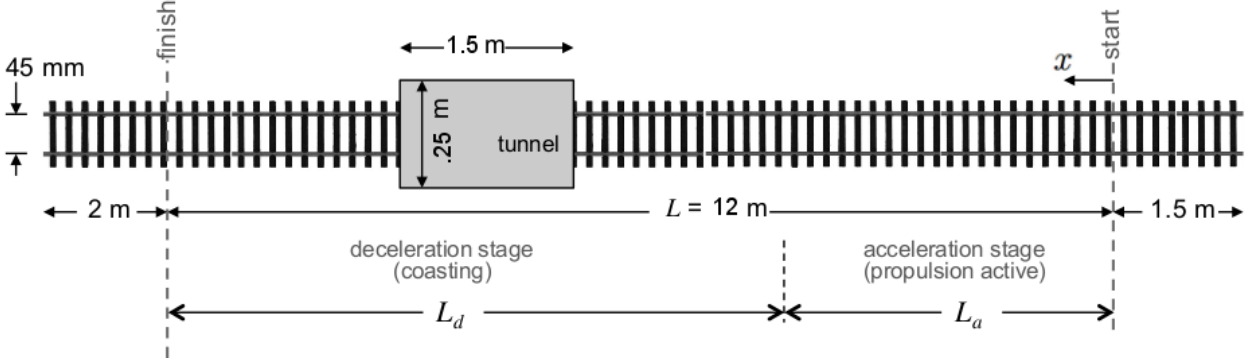


Figure 3: Schematic highlighting the locomotion strategy whereby the train is initially accelerated over a distance  $0 \leq x \leq L_a$ , and then allowed to coast (decelerate) to the finish line over the distance  $L_a < x \leq L$ .

## Train Dynamic Model

In order to obtain a design solution for a winning train, we need to be able to faithfully simulate the dynamics of the train. We start with a free-body diagram of the train (Figure 4), from which Newton’s 2nd law can be written. The forces acting on the train include the



Figure 4: Free body diagram of the forces acting on the train during the race.

rolling friction,  $F_r$ , the aerodynamic drag force,  $F_d$ , and the traction force,  $F_t$ . Note, the traction force depends on the type of locomotion system used and is only applicable during the *acceleration stage*. The engineering theory needed to derive a mathematical model of the system is presented below.

For the case of a train moving down a flat track, Newton's 2nd law in the  $x$ -direction parallel to the track is

$$\text{acceleration:} \quad m \frac{d^2x}{dt^2} = F_t - F_d - F_r, \quad (1)$$

$$\text{deceleration:} \quad m \frac{d^2x}{dt^2} = -F_d - F_r. \quad (2)$$

where  $m$  denotes the total mass of the train and  $a (=d^2x/dt^2)$  denotes the acceleration. The aerodynamic drag force can be written in terms of a drag coefficient as

$$F_d = \frac{1}{2} C_d \rho A V^2, \quad (3)$$

where  $\rho$  denotes the density of the air,  $A$  is the frontal area of the train, and  $V$  is the velocity of the train ( $V = dx/dt$ ). The drag coefficient  $C_d$  depends on the shape of the object and the surface roughness. The model train to be used in the race will consist of a pressurized cylindrical tank on wheels. One option for the pressurized tank is to use PVC pipe with end caps, as illustrated in Figure 5. A good estimate for the drag coefficient of a circular cylinder oriented axially to the flow is  $C_d \approx 0.8$ . The rolling friction force between the wheel of the train and the rail of the train track is parameterized by the rolling resistance coefficient  $C_r$  according to the expression

$$F_r = C_r m g. \quad (4)$$

One can empirically (using experiments) determine a value for the rolling resistance. The value should be  $C_r \approx 0.03$ . We will assume that  $C_d$  and  $C_r$  are constant, which is a very good assumption under steady state conditions (i.e., the train is traveling at a constant speed). During the race, the model train never reaches a steady state condition; therefore, some error is expected by assuming a constant  $C_d$ . Substituting the expressions for  $F_d$  and  $F_r$  into (1) and (2) yields

$$\text{acceleration:} \quad m \frac{d^2x}{dt^2} = F_t - \frac{1}{2} C_d \rho A \left( \frac{dx}{dt} \right)^2 - C_r m g, \quad (5)$$

$$\text{deceleration:} \quad m \frac{d^2x}{dt^2} = -\frac{1}{2} C_d \rho A \left( \frac{dx}{dt} \right)^2 - C_r m g. \quad (6)$$

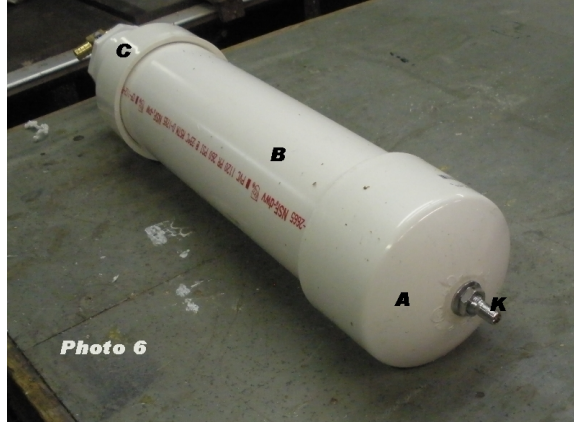


Figure 5: Example of a pressurized tank using PVC pipe with end caps.

### Traction Force

The traction force  $F_t$  is the frictional force that the track rail exerts on the rotating train wheel. The traction force is applied at the point of contact between the wheel and the rail (see Figure 6). The traction force can be related to the applied torque on the wheel through conservation of angular momentum, which states that the sum of the torque is equal to the moment of inertia of the wheel ( $I$ ) times the angular acceleration ( $\alpha$ ) of the wheel. Applying conservation of momentum about the axis of the wheel gives

$$T - r_w F_t = I \alpha , \quad (7)$$

where  $r_w$  is the radius of the train wheel. Approximating the train wheel as a solid disc, we can write the moment of inertia of the wheel as  $I = \frac{1}{2} m_w r_w^2$ , where  $m_w$  denotes the mass of the wheel. Note, since there are two wheels per axle, we need to multiply  $I$  by two. Therefore, the traction force is

$$F_t = \frac{T}{r_w} - m_w r_w \alpha. \quad (8)$$

Note, wheel slip will occur if the traction force is greater than the static friction force,

$$\text{wheel-slip criterion: } F_t > \mu_s \frac{m}{2} g , \quad (9)$$

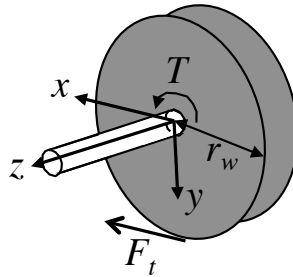


Figure 6: Schematic of the train wheel illustrating the traction force,  $F_t$ , between the wheel and rail (not shown).

where  $\mu_s$  is the coefficient of static friction between the wheel and the rail. The factor of  $\frac{1}{2}$  is used because we assume the propulsion force only drives one of the axles, and that the total mass of the train is distributed equally between the rear and front axles. The value of  $\mu_s$  depends on the type of materials in contact. According to equation (9), one can increase traction in two ways: (i) increase  $\mu_s$  by putting a rubber sleeve around the wheels that are driven by the propulsion system, and (ii) increase the total mass  $m$  of the train. Your code will need to check for the wheel-slip criterion in equation (9).

### Applied Torque

The applied torque driving the wheel,  $T$  in equation (8), is produced by the locomotion system, which in this case includes a rack and pinion gear as shown in Figure 7. Therefore, the applied torque is equal to the driving force of the rack,  $F_p$ , multiplied by the radius of the pinion gear,  $r_g$ , i.e.,

$$T = r_g \cdot F_p . \quad (10)$$

The rack is to be connected to a pneumatic piston as shown in Figure 8. A photograph of an actual pneumatic piston that can be purchased off-the-shelf is given in Figure 9.

The force  $F_p$  driving the rack and pinion is equal to the *net pressure* acting on the piston times the area of the piston head  $A_p$ ,

$$F_p = P_{\text{net}} \cdot A_p . \quad (11)$$

The net pressure is equal to the pressure inside the piston chamber,  $P$ , minus the pressure acting on the outside of the piston which is assumed to be atmospheric pressure, i.e.,  $P_{\text{net}} = P - P_{\text{atm}}$ . Therefore, we can write

$$F_p = (P - P_{\text{atm}}) A_p . \quad (12)$$

Note, the pressure inside the piston chamber is decreasing with time as the piston expands.

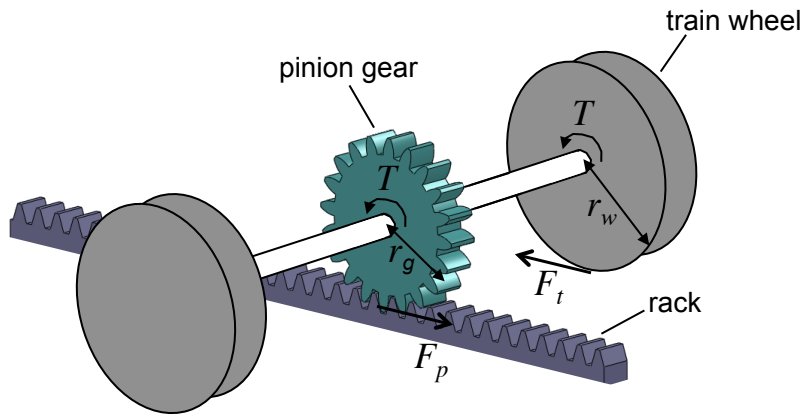


Figure 7: Schematic of the train axle showing one possible configuration of the rack/pinion.

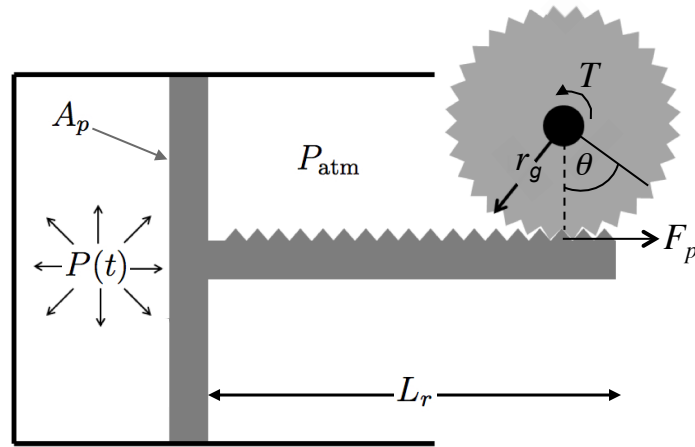


Figure 8: Schematic of the piston pushing against the rack thereby causing an applied torque  $T$  on the pinion gear.

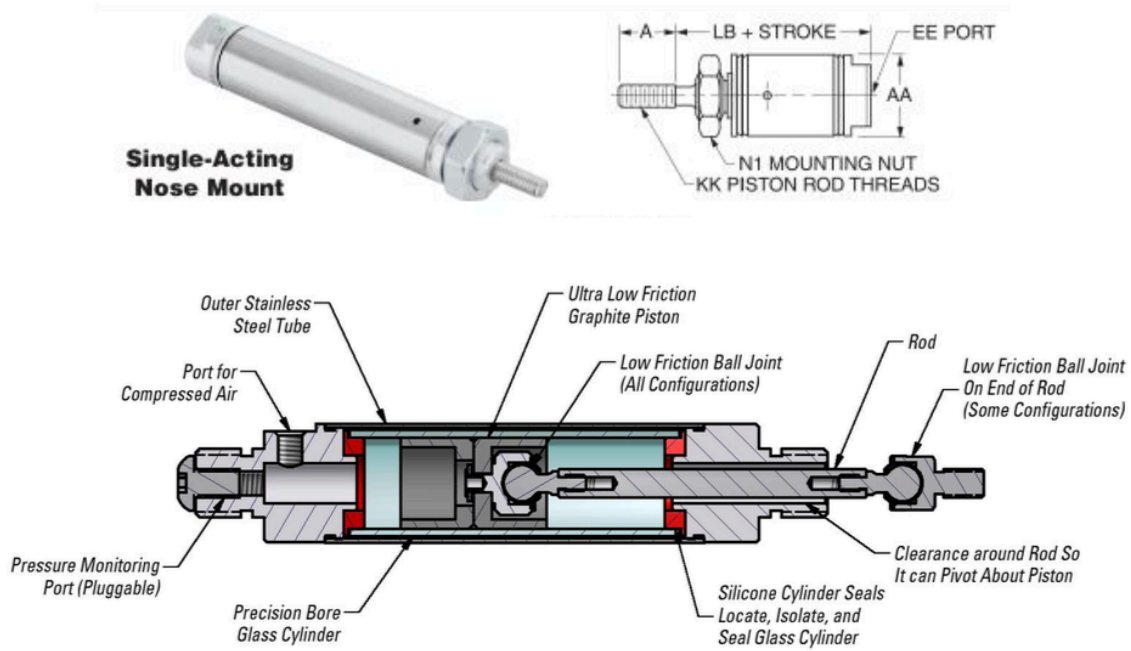


Figure 9: Example of a pneumatic piston that can be purchased off-the-shelf.

## Piston Pressure

The pressure inside the piston chamber is governed by the ideal gas law,

$$P \mathcal{V} = n R T, \quad (13)$$

where  $\mathcal{V}$  is the volume of the gas in the tank,  $n$  is the amount of gas in moles,  $R$  is the universal gas constant, and  $T$  is the temperature of the gas. Note,  $P$  in equation (13) must be expressed in terms of an absolute pressure and NOT a gauge pressure. Since no gas is exhausted to the surroundings during locomotion, the mass of gas, and hence  $n$ , remains constant. If we assume that the expansion of the piston occurs isothermally, i.e., temperature remains constant, then the left hand side of (13) must be constant and equal to its initial value. This means (13) can be rewritten as

$$P \mathcal{V} = P_0 \mathcal{V}_0, \quad (14)$$

where  $P_0$  and  $\mathcal{V}_0$  denote the initial tank pressure and volume, respectively. At a given time  $t$ , the total volume of the gas is equal to the initial tank volume plus the volume of the piston chamber,  $\mathcal{V} = \mathcal{V}_0 + \Delta \mathcal{V}$ . Solving (14) for  $P$  gives

$$P = \frac{P_0 \mathcal{V}_0}{\mathcal{V}_0 + \Delta \mathcal{V}}. \quad (15)$$

As the piston expands, the volume increases by an amount  $\Delta \mathcal{V} = A_p \ell$  as shown in Figure 10. Since the piston is connected to the rack and pinion gear,  $\ell$  is determined by the angular rotation of the pinion gear,  $\ell = r_g \theta$ , as illustrated in Figure 10. Therefore, (15) becomes

$$P = \frac{P_0 \mathcal{V}_0}{\mathcal{V}_0 + A_p r_g \theta}. \quad (16)$$

## Final System of ODEs

Finally, we can substitute (16), (12), and (10) into (8) to obtain an expression for the traction force in terms of wheel rotation  $\theta$ ,

$$F_t = \frac{r_g A_p}{r_w} \left[ \frac{P_0 \mathcal{V}_0}{\mathcal{V}_0 + A_p r_g \theta} - P_{\text{atm}} \right] - m_w r_w \alpha. \quad (17)$$

We now recognize that, since the pinion gear and train wheel are connected to the same axle, the wheel/gear rotation is determined by the distance traveled,

$$x = r_w \theta. \quad (18)$$

This means we can write both the angular rotation of the wheel,  $\theta$ , and the angular acceleration of the wheel,  $\alpha$ , in terms of  $x$  as

$$\theta = \frac{x}{r_w} \quad \text{and} \quad \alpha = \frac{d^2 \theta}{dt^2} = \frac{1}{r_w} \frac{d^2 x}{dt^2}. \quad (19)$$



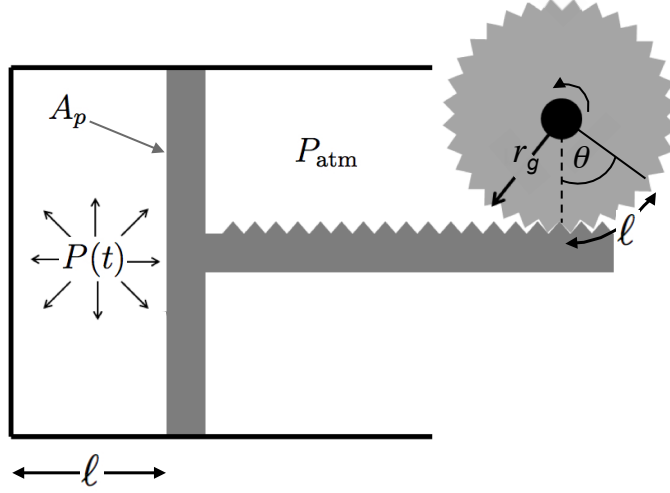


Figure 10: Schematic of the piston head displaced by a distance  $\ell$ . The displaced volume is equal to  $A_p \ell$ .

Substituting (19) into (17) yields

$$F_t = \frac{r_g A_p}{r_w} \left[ \frac{P_0 \mathcal{V}_0}{\mathcal{V}_0 + A_p \frac{r_g}{r_w} x} - P_{\text{atm}} \right] - m_w \frac{d^2 x}{dt^2} . \quad (20)$$

Note, the traction force is inversely proportional to  $x$ .

Finally, we can substitute (20) back into the original equations of motion

$$\text{acceleration: } m \frac{d^2 x}{dt^2} = \frac{r_g A_p}{r_w} \left[ \frac{P_0 \mathcal{V}_0}{\mathcal{V}_0 + A_p \frac{r_g}{r_w} x} - P_{\text{atm}} \right] - m_w \frac{d^2 x}{dt^2} - \frac{1}{2} C_d \rho A \left( \frac{dx}{dt} \right)^2 - C_r m g , \quad (21)$$

$$\text{deceleration: } m \frac{d^2 x}{dt^2} = -\frac{1}{2} C_d \rho A \left( \frac{dx}{dt} \right)^2 - C_r m g . \quad (22)$$

Dividing through by  $m$  and rearranging the top equation to get the derivative term on the left hand side produces the final set of governing equations

$$\text{acceleration: } \frac{d^2 x}{dt^2} = \frac{1}{m + m_w} \left[ \frac{r_g A_p}{r_w} \left( \frac{P_0 \mathcal{V}_0}{\mathcal{V}_0 + A_p \frac{r_g}{r_w} x} - P_{\text{atm}} \right) - \frac{1}{2} C_d \rho A \left( \frac{dx}{dt} \right)^2 - C_r m g \right] , \quad (23)$$

$$\text{deceleration: } \frac{d^2 x}{dt^2} = -\frac{C_d \rho A}{2m} \left( \frac{dx}{dt} \right)^2 - C_r g . \quad (24)$$

You will solve these ODEs in one of your lab sessions using the 4th order Runge-Kutta method. Note, your code will need to test when the acceleration stage ends and the deceleration stage begins. This is set by the length of the rack  $L_r$  as shown in Figure 8. Accordingly,

the propulsion system will continue to apply a traction force as long as  $r_g \theta < L$ , or substituting (19) for  $\theta$ , gives the following conditions

$$\text{acceleration stage: } x \leq L_r \frac{r_w}{r_g} \quad (25)$$

$$\text{deceleration stage: } x > L_r \frac{r_w}{r_g} \quad (26)$$

Solution of (23) and (24) for a competitive train might look like the plot in Figure 11, which shows the distance traveled  $x$  versus time  $t$ . The train crosses the finish line at a time  $t_f \approx 8.5$  s, and comes to a full stop at  $x_s \approx 11.6$  m before reaching the end of the run-out track.

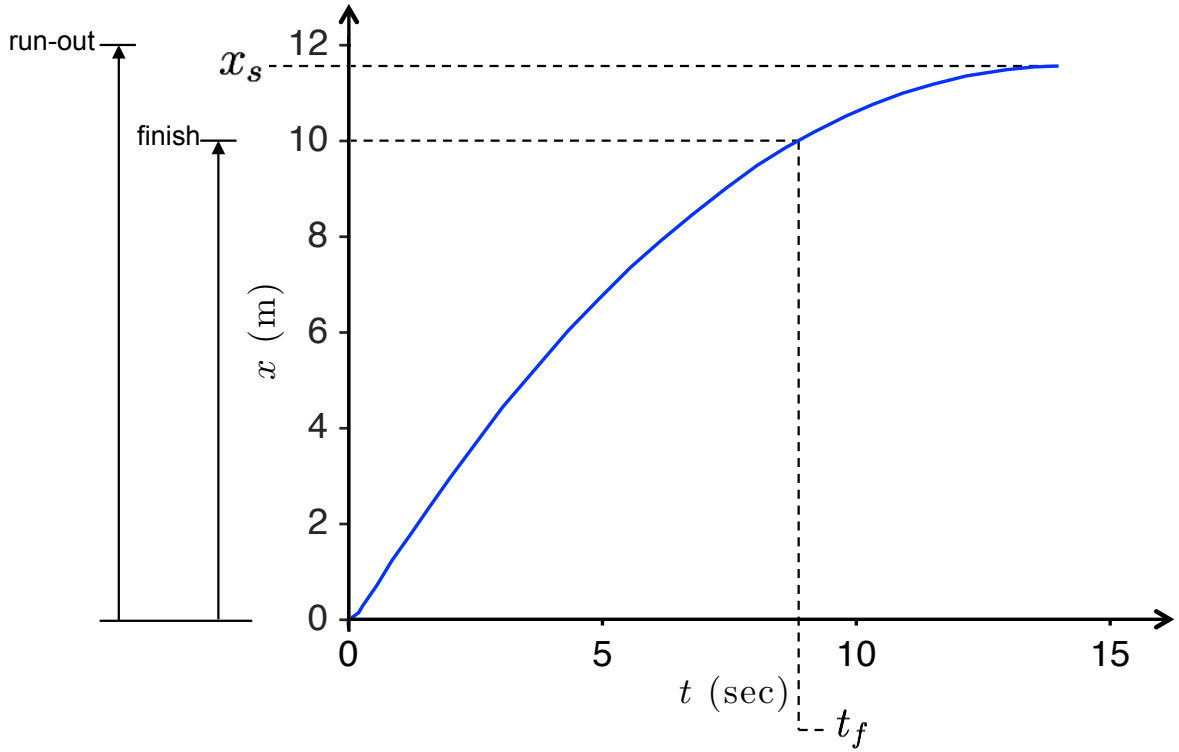


Figure 11: Behavior of a competitive train, where  $x(t)$  represents the distance traveled as a function of time  $t$ .

## Multidimensional Optimization Problem

Table 1 lists relevant physical parameters that appear in equations (23) and (24) and therefore can affect the performance of the train,  $x(t)$ . Note, the first 7 parameters ( $L_t$ ,  $D_o$ ,  $\rho_t$ ,  $P_{0\text{gage}}$ ,  $r_g$ ,  $L_r$ ,  $D_p$ ) are the ones that need to be selected as part of the design solution; whereas, the last 7 parameters ( $\rho_a$ ,  $P_{\text{atm}}$ ,  $C_d$ ,  $C_r$ ,  $\mu_s$ ,  $D_w$ ,  $m_w$ ) will be considered fixed at preset values. Recall that the value for the initial tank pressure used in equation (23) is the absolute pressure calculated using

$$P_0 = P_{0\text{gage}} + P_{\text{atm}}. \quad (27)$$

You will need to determine realistic values for each of the parameters in Table 1.

The model train is assumed to be made in the shape of a cylinder, consisting of a pipe/tube with end caps, and can be made out of PVC, steel, stainless steel, aluminum, or any other material that is readily purchased in tube/pipe form. Your code should consider at least three different materials. The densities of various tube/pipe materials is given in Table 2.

Table 1: Relevant Physical Parameters

| parameter                      | symbol             | range of values | units             |
|--------------------------------|--------------------|-----------------|-------------------|
| length of train                | $L_t$              |                 | ft                |
| outer diameter of train        | $D_o$              |                 | in                |
| density of train material      | $\rho_t$           |                 | kg/m <sup>3</sup> |
| initial tank gage pressure     | $P_{0\text{gage}}$ |                 | psig              |
| pinion gear radius             | $r_g$              |                 | in                |
| length of piston stroke        | $L_r$              |                 | in                |
| diameter of piston             | $D_p$              |                 | in                |
| air density                    | $\rho_a$           |                 | kg/m <sup>3</sup> |
| atmospheric pressure           | $P_{\text{atm}}$   |                 | psi               |
| drag coefficient               | $C_d$              |                 | —                 |
| rolling friction coefficient   | $C_r$              |                 | —                 |
| coefficient of static friction | $\mu_s$            |                 | —                 |
| wheel diameter                 | $D_w$              |                 | mm                |
| mass of wheels and axles       | $m_w$              |                 | kg                |

Table 2: Density of Various Materials used for Pipes/Tubes

| material         | density (kg/m <sup>3</sup> ) |
|------------------|------------------------------|
| PVC              | 1400                         |
| acrylic          | 1200                         |
| galvanized steel | 7700                         |
| stainless steel  | 8000                         |
| titanium         | 4500                         |
| copper           | 8940                         |
| aluminum – 6061  | 2700                         |

Your Matlab script file should include the following features, not necessarily in the order listed.

### 1. Define Ranges for Physical Parameters

Select appropriate values for the *fixed* parameters in Table 1. Then, create an array for each of the *design* parameters in Table 1 that contains an appropriate range of values for that parameter. You should consult information from manufacturers in order to determine realistic values for off-the-shelf parts that need to be purchased. Your structured/unstructured search algorithm should eliminate any combination of parameters that violates the design constraints (see Item 3).

To simplify things, we will assume the following:

- The inside diameter  $D_i$  of the pipe tube is proportional to the outer diameter ( $D_o = 1.3 D_i$ ).
- The mass of the pneumatic piston is proportional to its volume, i.e.,  $M_p = \rho_p (\frac{\pi}{4} D_p^2 L_p)$ , with a proportionality constant of  $\rho_p = 1250 \text{ kg/m}^3$ .
- The total length of the piston is proportional to its stroke ( $L_p = 2 L_r$ ).
- The length of the rack is equal to the length of the stroke of the pneumatic piston.

Note, you need to consider where the piston will be placed. Options are underneath the train or behind the train like a caboose. If the piston is placed underneath the train, then enough clearance must be ensured, which will affect the total height of the train. If the piston is placed behind the train, then the total length of the train is the length of the tank plus the length of the piston. There are design constraints on both the total length of the train and the height of the train as discussed below.

### 2. Create Parameter Matrix for Structured or Unstructured Search

Include lines of code (or write a separate user-defined function) that automatically creates the parameter matrix for the structured or unstructured search given the allowable parameter ranges that were defined above. Each row of the matrix represents a different set of parameters; and, each column represents the values for a different parameter. The matrix needs to include enough different combinations of the parameters to cover the entire function space. This means that your parameter matrix will likely have hundreds to thousands of rows, if not more. Note, the search algorithm will solve the system of ODEs in (23) and (24) for each row of the parameter matrix.

### 3. Check Design Constraints

Your code should eliminate any set of parameters that violates one or more of the design constraints:

- train height/width: the train must be capable of passing through the tunnel. Therefore, the height cannot exceed 0.26 m; and, the width cannot exceed 0.25 m.
- train length: the entire train must fit on the track at the starting line. Since the length of “set-up” track is only 1.5 m, the total length of the train plus propulsion system cannot exceed 1.5 m.

- gear radius: the diameter of the pinion gear must be less than the diameter of the train wheel (i.e.,  $r_g/r_w < 1$ ).
- wheel slippage: the wheels will slip if the maximum traction force exceeds the force due to static friction, as described by equation (9). The maximum traction force will be experienced right at start up when the pressure inside the tank is a maximum. Using the relation for the traction force in equation (17), one can derive a condition on the maximum allowable initial tank pressure to avoid wheel slippage.

Note, in order to find a design solution, you may need to add dead-weight to the train. You can use the wheel slippage condition that you derived above to solve for the total mass required to prevent wheel slippage. If you apply dead-weight to your train, then you can operate at a higher initial pressure  $P_0$  without slipping. However, there is a cost trade-off because a heavier train will not accelerate as quickly and therefore may be slower to cross the finish line. Your optimization code should be able to find the optimum solution to this dilemma.

#### 4. Determine Best Step Size

Based on your results from one of your labs, define an appropriate step size  $h$  to use in your 4th order Runge-Kutta method.

#### 5. Perform Search

Include lines of code (or write a separate user-defined function) that performs the structured or unstructured search. This basically consists of solving the set of ODEs for every set of physical parameters in your parameter matrix. Note, you will have to calculate quantities such as the frontal area of the train  $A$ , the initial tank volume  $\mathcal{V}_0$ , the total mass of the train  $m$ , etc., based on the values provided for the design parameters. For each case, you want to save the final distance traveled  $x_s$  and the time  $t_f$  to reach the finish line. The optimum solution is determined by finding the set of parameters with the lowest  $t_f$  that also satisfies the criterion  $10\text{ m} < x_s < 12\text{ m}$ .

#### 6. Display Output from Structured Search

Include lines in your script that use the `disp` command to display relevant information about the search, such as:

- computational time to run the search (via the `tic` and `toc` commands)
- number of iterations (this should be equal to the number of different parameter sets evaluated)
- final distance traveled by the train using the optimum set of parameters
- time to reach the finish line using the optimum set of parameters
- a list of the values associated with the optimum set of parameters ( $L_t$ ,  $D_o$ ,  $\rho_t$ ,  $m$ ,  $P_{0\text{gage}}$ ,  $r_g$ ,  $L_r$ ,  $L_p$ ,  $D_p$ ,  $M_p$ )

[Paste the screenshot of the Matlab command window showing this output to your memo]

## 7. Plot Distance Traveled and Train Velocity versus Elapsed Time (Optimum Design Solution)

For the optimum set of parameters obtained, plot  $x$  versus  $t$  and  $V$  versus  $t$  based on your numerical simulation. The plots should be on separate axes. For distance, make a plot similar to Figure 11 above. It should include two horizontal dashed lines that denote the finish line at  $x = 10$  m and the end of the “run-out” track at  $x = 12$  m. Also, include a vertical dashed line that marks the time  $t_f$  when the train passed the finish line. [Paste these plots into your memo]

## 8. Display Optimum Design Solution

Once you have found the optimum set of design parameters, your code should calculate the following quantities, which are to be included in your memo in tabular format as shown below. [Paste your table into your memo]

| parameter                  | symbol                | optimum value | units          |
|----------------------------|-----------------------|---------------|----------------|
| length of train            | $L_t$                 |               | m              |
| outer diameter of train    | $D_o$                 |               | m              |
| height of train            | $H_t$                 |               | m              |
| material of train          | —                     |               | —              |
| total mass of train        | $m$                   |               | kg             |
| train frontal area         | $A$                   |               | m <sup>2</sup> |
| initial tank gage pressure | $P_{0_{\text{gage}}}$ |               | psig           |
| tank volume                | $\mathcal{V}_0$       |               | m <sup>3</sup> |
| pinion gear radius         | $r_g$                 |               | m              |
| length of piston stroke    | $L_r$                 |               | m              |
| total length of piston     | $L_p$                 |               | m              |
| diameter of piston         | $D_p$                 |               | m              |
| mass of piston             | $M_p$                 |               | kg             |

## 9. Select *Realistic* Components

Note, you may not be able to build an actual model train having the exact specifications given by your optimum solution, because materials and components will typically only be available in standard sizes. You are now tasked with selecting realistic components for your train that match as close as possible to the parameter values given by your optimum design solution. In order to do this, you will need to search the internet to identify manufacturers that make pipe/tubing, pneumatic pistons, and rack/pinion gears. Based on your research put together a parts list for your train including the size, model number, vendor, and estimated price for each item. [Paste your parts lists into your memo]

Note, if your *realistic* train has different parameter values than your optimum numerical solution, then you will need to rerun your simulation on the *realistic* train in order to verify that the *realistic* train still completes the race without running off the track. In your memo, you should state the time to cross the finish line for your *realistic* train.