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Project #4

Analysis-model write-up: due Nov 20, 2020
Report: due Dec 11, 2020

Background

In this project we are going to revisit the wing-spar design problem from Project #2, but this time the loading will be uncertain. Most of the design parameters and specifications will remain the same. They are repeated here for reference.

wing semi-span: 7.5 m, which is also the length of the spar.

spar cross-section shape: circular annulus.

material: Carbon fiber composite, with density 1600 $\mathrm{km/m}^3$, Young's modulus 70 GPa, and ultimate tensile/compressive strength 600 MPa.

manufacturing constraints: The inner and outer radii of the annulus cannot be less than 2.5 mm apart, the inner radius cannot be smaller than 1 cm, and the outer radius cannot be larger than 5 cm.

aircraft operational weight: The total mass of the aircraft will be 500 kg, including the spar.

The nominal loading at the 2.5 g maneuver remains as in Project #2, i.e. the force distribution in the spanwise direction has a linear distribution with maximum load at the root and zero load at the tip. Recall, the integral of the nominal loading must equal 2.5 times the half the aircraft's weight, so the nominal loading is given by

$$f_{\text{nom}}(x) = \frac{2.5W}{L}\left(1 - \frac{x}{L}\right).$$

However, for this project, we assume there is uncertainty in the loading. In particular, we assume the loading is modeled by

$$f(x,\xi) = f_{\text{nom}}(x) + \delta_f(x,\xi)$$

where the probabilistic perturbation has the form

$$\delta_f(x,\xi) = \sum_{n=1}^{4} \xi_n \cos\left(\frac{(2n-1)\pi x}{2L}\right)$$

with $\xi_n \sim \mathcal{N}\left(0, \frac{f_{\text{nom}}(0)}{10n}\right)$.

Your objective is to minimize the weight of the spar, while ensuring that the mean plus 6 standard deviations of the stress remains below the ultimate strength of the carbon fiber. That is,

$$\mathsf{E}[s(x,\xi)] + 6\sqrt{\mathsf{Var}[s(x,\xi)]} \leq s_{\text{yield}},$$

where $s(x,\xi)$ denotes the spanwise stress distribution in the spar.

Analysis Model

Write a concise (less than one page) description of how you will account for the load uncertainty in the analysis of the wing spar. Discuss how you will compute the mean and standard deviation of the constraints.

Project Report

Your report should summarize the finite-element analysis and other commonalities with Project #2, but *this summary should be brief.* Instead, **your report should focus on the uncertainty analysis and optimization under uncertainty**. Be sure to compare the results from this project with the deterministic results from Project #2 and provide some discussion explaining the differences.

As always, your report should concisely describe your approach and results, and should contain

- an executive summary;
- a description of the analysis method, including any assumptions and limitations inherent in the method;
- a description of the geometry parameterization, if any;
- the optimization problem statement and optimization method(s), including the objective and any constraints;
- results, including the final geometry(ies) and convergence history(ies);
- conclusions and/or discussion of the results, and;
- an appendix with the source code.

The length of the report is not to exceed 10 pages (excluding the code appendix). Please refer to the corresponding rubric for how the report will be assessed.

Collaboration

You are permitted and encouraged to discuss the project with each other, provided each of you writes your own code and report. A good policy to follow in order to avoid academic misconduct is to not take project notes or exchange project files with one another; i.e. exchange information verbally and you should be fine.