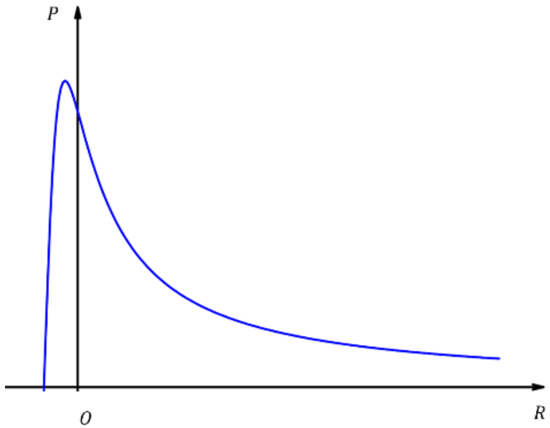
MẠCH RLC NỐI TIẾP (CUỘN DÂY THUẦN CẢM) CÓ R THAY ĐỔI

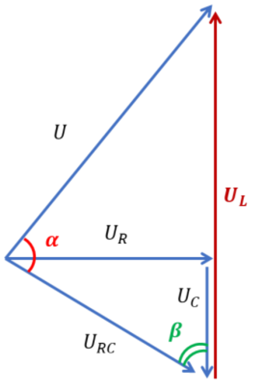
Dạng bài	Chứng minh	Công thức cần nhớ
Dạng 1. *R thay đổi để* $I_{\max}$; $U_{L\max}$; $U_{C\max}$.	$I=\dfrac{U}{\sqrt{R^2+\left(Z_L-Z_C\right)^2}}\Rightarrow I_{\max}\Leftrightarrow R=0$.	$R=0$; $I_{\max}=\dfrac{U}{\left\|Z_L-Z_C\right\|}$
Dạng 2. *R thay đổi để công suất mạch đạt cực đại.*	$P=I^2R=\dfrac{U^2R}{R^2+\left(Z_L-Z_C\right)^2}=\dfrac{U^2}{R+\dfrac{\left(Z_L-Z_C\right)^2}{R}}$ Áp dụng BĐT Cauchy: $R+\dfrac{\left(Z_L-Z_C\right)^2}{R}\geq 2\sqrt{R\cdot\dfrac{\left(Z_L-Z_C\right)^2}{R}}=2\left\|Z_L-Z_C\right\|$ $\Rightarrow P\leq\dfrac{U^2}{2\left\|Z_L-Z_C\right\|}\rightarrow P_{\max}=\dfrac{U^2}{2\left\|Z_L-Z_C\right\|}$. Khi đó $R=\dfrac{\left(Z_L-Z_C\right)^2}{R}\Rightarrow R=\left\|Z_L-Z_C\right\|$.	$R=\left\|Z_L-Z_C\right\|$; $P_{\max}=\dfrac{U^2}{2\left\|Z_L-Z_C\right\|}$
Dạng 3. *Có hai giá trị $R_1;R_2$ để mạch có cùng công suất P.*	$P=\dfrac{U^2R}{R^2+\left(Z_L-Z_C\right)^2}\Rightarrow P\cdot R^2-U^2\cdot R+P\left(Z_L-Z_C\right)^2=0$ Theo Viète: $R_1+R_2=\dfrac{-b}{a}=\dfrac{U^2}{P}$ và $R_1R_2=\dfrac{c}{a}=\left(Z_L-Z_C\right)^2$. • Gọi $\varphi_1;\varphi_2$ lần lượt là độ lệch pha u; i khi $R=R_1$ và $R=R_2$, ta có: $\tan\varphi_1\cdot\tan\varphi_2=\dfrac{\left(Z_L-Z_C\right)}{R_1}\cdot\dfrac{\left(Z_L-Z_C\right)}{R_2}=\dfrac{\left(Z_L-Z_C\right)^2}{R_1R_2}=1$ $\Rightarrow\tan\varphi_1=\dfrac{1}{\tan\varphi_2}=\cot\varphi_2$ $\quad\left(Z_L\neq Z_C\right)\Rightarrow\varphi_1+\varphi_2=\pm\dfrac{\pi}{2}$. • Chứng minh từ Dạng 2 ta có $P_{\max}=\dfrac{U^2}{2\left\|Z_L-Z_C\right\|}=\dfrac{U^2}{2\sqrt{R_1R_2}}$ khi $R=\left\|Z_L-Z_C\right\|=\sqrt{R_1R_2}$	• $\begin{cases}R_1+R_2=\dfrac{U^2}{P}\\R_1R_2=\left(Z_L-Z_C\right)^2\end{cases}$ • $\tan\varphi_1\cdot\tan\varphi_2=1$ hay $\varphi_1+\varphi_2=\pm\dfrac{\pi}{2}$. • $P_{\max}=\dfrac{U^2}{2\sqrt{R_1R_2}}$ khi $R_0=\sqrt{R_1R_2}$.

MẠCH RLC NỐI TIẾP (CUỘN DÂY KHÔNG THUẦN CẢM) CÓ R THAY ĐỔI

Dạng bài	Chứng minh	Công thức cần nhớ
Dạng 1. *R thay đổi để I_{max}; $U_{L\max}$; $U_{C\max}$.*	$I=\dfrac{U}{\sqrt{(R+r)^2+(Z_L-Z_C)^2}}\Rightarrow I_{\max}\Leftrightarrow R=0$.	$R=0$ $I_{\max}=\dfrac{U}{\sqrt{r^2+(Z_L-Z_C)^2}}$
Dạng 2. *R thay đổi để công suất mạch đạt cực đại.*	$P=I^2(R+r)=\dfrac{U^2(R+r)}{(R+r)^2+(Z_L-Z_C)^2}=\dfrac{U^2}{(R+r)+\dfrac{(Z_L-Z_C)^2}{(R+r)}}$ Áp dụng BĐT Cauchy: $(R+r)+\dfrac{(Z_L-Z_C)^2}{(R+r)}\geq 2\sqrt{(R+r)\cdot\dfrac{(Z_L-Z_C)^2}{(R+r)}}=2\lvert Z_L-Z_C\rvert$ $\Rightarrow P\leq\dfrac{U^2}{2\lvert Z_L-Z_C\rvert}\rightarrow P_{\max}=\dfrac{U^2}{2\lvert Z_L-Z_C\rvert}$ Khi đó $(R+r)=\dfrac{(Z_L-Z_C)^2}{(R+r)}\Rightarrow R+r=\lvert Z_L-Z_C\rvert\Rightarrow R=\lvert Z_L-Z_C\rvert-r$ * Nếu $r>\lvert Z_L-Z_C\rvert$ thì $P_{\max}=\dfrac{U^2r}{r^2+(Z_L-Z_C)^2}$ khi $R=0$.	$R=\lvert Z_L-Z_C\rvert-r$; $P_{\max}=\dfrac{U^2}{2\lvert Z_L-Z_C\rvert}$ * Nếu $r>\lvert Z_L-Z_C\rvert$ thì $P_{\max}=\dfrac{U^2r}{r^2+(Z_L-Z_C)^2}$ khi $R=0$. 

Dạng 3. *Có hai giá trị $R_1;R_2$ để mạch có cùng công suất P.*	$P=\dfrac{U^2(R+r)}{(R+r)^2+(Z_L-Z_C)^2}\Rightarrow P\cdot(R+r)^2-U^2\cdot(R+r)+P(Z_L-Z_C)^2=0$. Theo Viète: $\begin{cases} R_1+r+R_2+r=\dfrac{-b}{a}=\dfrac{U^2}{P}\\ (R_1+r)\cdot(R_2+r)=\dfrac{c}{a}=(Z_L-Z_C)^2\end{cases}$. • Gọi $\varphi_1;\varphi_2$ lần lượt là độ lệch pha u; i khi $R=R_1$ và $R=R_2$, ta có: $\tan\varphi_1\cdot\tan\varphi_2=\left(\dfrac{Z_L-Z_C}{R_1+r}\right)\cdot\left(\dfrac{Z_L-Z_C}{R_2+r}\right)=\dfrac{(Z_L-Z_C)^2}{(R_1+r)\cdot(R_2+r)}=1$ $\Rightarrow\tan\varphi_1=\dfrac{1}{\tan\varphi_2}=\cot\varphi_2\qquad(Z_L\neq Z_C)\Rightarrow\varphi_1+\varphi_2=\pm\dfrac{\pi}{2}$. • Chứng minh từ Dạng 2 ta có $P_{\max}=\dfrac{U^2}{2\lvert Z_L-Z_C\rvert}=\dfrac{U^2}{2\sqrt{(R_1+r)\cdot(R_2+r)}}$ khi $R=\lvert Z_L-Z_C\rvert-r=\sqrt{(R_1+r)\cdot(R_2+r)}-r$.	• $\begin{cases} R_1+r+R_2+r=\dfrac{U^2}{P}\\ (R_1+r)\cdot(R_2+r)=(Z_L-Z_C)^2\end{cases}$ • $\tan\varphi_1\cdot\tan\varphi_2=1$ hay $\varphi_1+\varphi_2=\pm\dfrac{\pi}{2}$. • $P_{\max}=\dfrac{U^2}{2\sqrt{(R_1+r)\cdot(R_2+r)}}$ khi $R_0=\sqrt{(R_1+r)\cdot(R_2+r)}-r$.
Dạng 4. *R thay đổi để công suất tiêu thụ trên R cực đại.*	$P_R=I^2R=\dfrac{U^2R}{(R+r)^2+(Z_L-Z_C)^2}=\dfrac{U^2R}{R^2+r^2+(Z_L-Z_C)^2+2rR}=\dfrac{U^2}{R+\dfrac{r^2+(Z_L-Z_C)^2}{R}+2r}$ Áp dụng BĐT Cauchy: $R+\dfrac{r^2+(Z_L-Z_C)^2}{R}\ge 2\sqrt{r^2+(Z_L-Z_C)^2}$ $\Rightarrow P_R\le\dfrac{U^2}{2(R_0+r)}$ với $R_0=\sqrt{r^2+(Z_L-Z_C)^2}$ $\Rightarrow P_{R\max}=\dfrac{U^2}{2(R_0+r)}$.	$P_{R\max}=\dfrac{U^2}{2(R_0+r)}$ khi $R=R_0=\sqrt{r^2+(Z_L-Z_C)^2}$.

MẠCH RLC NỐI TIẾP CÓ L THAY ĐỔI

Dạng bài	Chứng minh	Công thức cần nhớ
Dạng 1. *L thay đổi để* $I_{\max}$; $P_{\max}$; $U_{R\max}$; $U_{C\max}$.	$I=\frac{U}{\sqrt{R^2+(Z_L-Z_C)^2}}\Rightarrow I_{\max}=\frac{U}{R}$ khi $Z_L=Z_C$.	Hiện tượng cộng hưởng $\begin{cases}Z_L=Z_C\\ I_{\max}=\frac{U}{R}\end{cases}$.
Dạng 2. *L thay đổi để* $U_{L\max}$.	 $\frac{U_L}{\sin\alpha}=\frac{U}{\sin\beta}\Rightarrow U_L=\frac{U\cdot\sin\alpha}{\sin\beta}$ với $\sin\beta=\frac{R}{\sqrt{R^2+Z_C^2}}=const$ $\Rightarrow U_{L\max}=\frac{U\sqrt{R^2+Z_C^2}}{R}$ khi $\sin\alpha=1\Leftrightarrow\alpha=\frac{\pi}{2}$. * Hệ quả: $U_C\cdot U_{L\max}=U_R^2+U_C^2\Rightarrow U_{L\max}=\frac{U_R^2+U_C^2}{U_C}\Rightarrow Z_L=\frac{R^2+Z_C^2}{Z_C}$.	$U_{L\max}=\frac{U\sqrt{R^2+Z_C^2}}{R}=\frac{U_R^2+U_C^2}{U_C}$ Khi $Z_L=\frac{R^2+Z_C^2}{Z_C}$.
Dạng 3. *L thay đổi để* U_{RL} *cực đại hoặc cực tiểu.*	$U_{RL}=I\cdot Z_{RL}=\frac{U\sqrt{R^2+Z_L^2}}{\sqrt{R^2+(Z_L-Z_C)^2}}=\frac{U}{\sqrt{1+\frac{Z_C(Z_C-2Z_L)}{R^2+Z_L^2}}}$ với $Z_L\in[0;+\infty)$ • Khi $Z_L=0\Rightarrow\begin{cases}Z_C(Z_C-2Z_L)_{\max}=Z_C^2\\ (R^2+Z_L^2)_{\min}=R^2\end{cases}\Rightarrow\left[1+\frac{Z_C(Z_C-2Z_L)}{R^2+Z_L^2}\right]_{\max}=\frac{Z_C^2+R^2}{R^2}$ $\Rightarrow U_{RL\min}=\frac{U\cdot R}{\sqrt{R^2+Z_C^2}}$. • Khi $Z_L\to+\infty$	• $U_{RL\min}=\frac{U\cdot R}{\sqrt{R^2+Z_C^2}}$ khi $Z_L=0$ • $U_{RL\max}=\frac{U\cdot Z_L}{R}$ với $Z_L=\frac{Z_C+\sqrt{Z_C^2+4R^2}}{2}$.

$$\lim_{Z_L\to+\infty}\left(\frac{Z_C^2-2Z_LZ_C}{R^2+Z_L^2}\right)\overset{l'Hopital}{=}\lim_{Z_L\to+\infty}\left[\frac{\left(Z_C^2-2Z_LZ_C\right)'}{\left(R^2+Z_L^2\right)'}\right]=\lim_{Z_L\to+\infty}\left(\frac{-Z_C}{Z_L}\right)=0\ \Rightarrow U_{RL}\to U$$

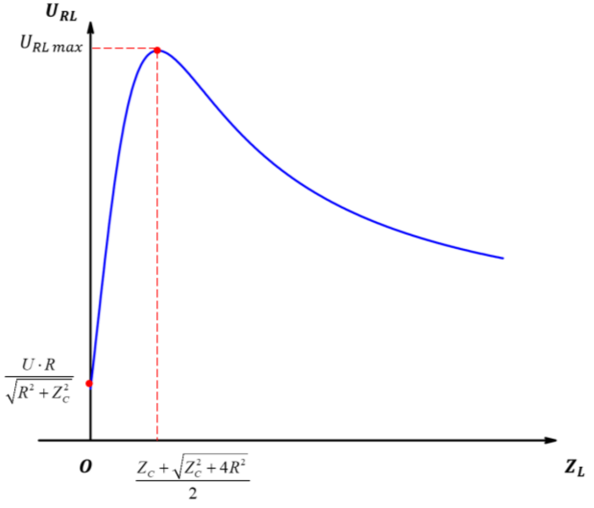
- Xét hàm số $f\left(Z_L\right)=\dfrac{Z_C\left(Z_C-2Z_L\right)}{R^2+Z_L^2}$, ta có $f'\left(Z_L\right)=\dfrac{2Z_C\left(Z_L^2-Z_LZ_C-R^2\right)}{\left(R^2+Z_L^2\right)^2}$

$$f'=0\Leftrightarrow Z_L^2-Z_LZ_C-R^2=0\quad(*)\Leftrightarrow\left[\begin{array}{ll}Z_L=\dfrac{Z_C+\sqrt{Z_C^2+4R^2}}{2} & (n)\\ Z_L=\dfrac{Z_C-\sqrt{Z_C^2+4R^2}}{2}<0 & (\ell)\end{array}\right.$$

Từ (*) ta có: $Z_L-Z_C=\dfrac{R^2}{Z_L}\Rightarrow U_{RL}=\dfrac{U\sqrt{R^2+Z_L^2}}{\sqrt{R^2+\left(\dfrac{R^2}{Z_L}\right)^2}}=\dfrac{U\cdot Z_L}{R}$

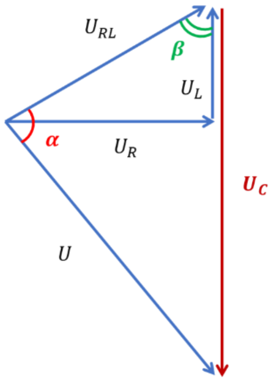
- Vì $Z_L=\dfrac{Z_C+\sqrt{Z_C^2+4R^2}}{2}>R$ nên ta có $\Rightarrow U_{RL}=\dfrac{U\cdot Z_L}{R}>U$

Z_L	0		$\dfrac{Z_C+\sqrt{Z_C^2+4R^2}}{2}$		$+\infty$
U_{RL}	$\dfrac{U\cdot R}{\sqrt{R^2+Z_C^2}}$	$\nearrow$	$\dfrac{U\cdot\left(Z_C+\sqrt{Z_C^2+4R^2}\right)}{2R}$	$\searrow$	U



Dạng 4. *Có hai giá trị* $L_1; L_2$ *để mạch có cùng cường độ dòng điện* I *hoặc có cùng công suất* P. *Tìm* L_0 *để* $I_{\max}$ *hoặc* $P_{\max}$.	$I=\dfrac{U}{\sqrt{R^2+\left(Z_L-Z_C\right)^2}}$ Có $I_1=I_2\Rightarrow Z_1=Z_2\Leftrightarrow \left\|Z_{L_1}-Z_C\right\|=\left\|Z_{L_2}-Z_C\right\|$ $\Rightarrow\left[\begin{array}{ll} Z_{L_1}-Z_C=Z_{L_2}-Z_C & (\ell) \\ Z_{L_1}-Z_C=Z_C-Z_{L_2} & (n)\end{array}\right.\Rightarrow Z_C=\dfrac{Z_{L_1}+Z_{L_2}}{2}$. Để $I_{\max}$ thì $Z_{L_0}=Z_C=\dfrac{Z_{L_1}+Z_{L_2}}{2}\Rightarrow L_0=\dfrac{L_1+L_2}{2}$. * *Hệ quả:* $\tan\varphi_1=-\tan\varphi_2\Rightarrow\varphi_1=-\varphi_2$.	$Z_C=\dfrac{Z_{L_1}+Z_{L_2}}{2}, \varphi_1=-\varphi_2$. Để $I_{\max}$ hoặc $P_{\max}$ thì $L_0=\dfrac{L_1+L_2}{2}$.
Dạng 5. *Có hai giá trị* $L_1; L_2$ *để có cùng* U_L. *Tìm* L_0 *để* $U_{L\max}$.	$U_L=I\cdot Z_L=\dfrac{U\cdot Z_L}{\sqrt{R^2+\left(Z_L-Z_C\right)^2}}\Rightarrow\left(R^2+Z_C^2\right)\cdot\dfrac{1}{Z_L^2}-2Z_C\cdot\dfrac{1}{Z_L}+1-\dfrac{U^2}{U_L^2}=0$ Theo Viète: $\dfrac{1}{Z_{L_1}}+\dfrac{1}{Z_{L_2}}=\dfrac{-b}{a}=\dfrac{2Z_C}{R^2+Z_C^2}=\dfrac{2}{Z_{L_0}}$ Với $Z_{L_0}=\dfrac{R^2+Z_C^2}{Z_C}$ là giá trị của Z_L để $U_{L\max}$. $\Rightarrow\dfrac{2}{L_0}=\dfrac{1}{L_1}+\dfrac{1}{L_2}$.	$\dfrac{2}{L_0}=\dfrac{1}{L_1}+\dfrac{1}{L_2}$.

MẠCH RLC NỐI TIẾP CÓ C THAY ĐỔI *(chứng minh tương tự L thay đổi)*

Dạng bài	Chứng minh	Công thức cần nhớ
Dạng 1. *C thay đổi để* $I_{\max}$; $P_{\max}$; $U_{R\max}$; $U_{L\max}$.	$I=\dfrac{U}{\sqrt{R^2+(Z_L-Z_C)^2}}$ $I_{\max}=\dfrac{U}{R}$ khi $Z_C=Z_L$.	Hiện tượng cộng hưởng $\begin{cases}Z_C=Z_L\\ I_{\max}=\dfrac{U}{R}\end{cases}$.
Dạng 2. *C thay đổi để* $U_{C\max}$.	 $\dfrac{U_C}{\sin\alpha}=\dfrac{U}{\sin\beta}\Rightarrow U_C=\dfrac{U\cdot\sin\alpha}{\sin\beta}$ với $\sin\beta=\dfrac{R}{\sqrt{R^2+Z_L^2}}=const$ $\Rightarrow U_{C\ \max}=\dfrac{U\sqrt{R^2+Z_L^2}}{R}$ khi $\sin\alpha=1\Leftrightarrow\alpha=\dfrac{\pi}{2}$. * Hệ quả: $U_L\cdot U_{C\max}=U_R^2+U_L^2\Rightarrow U_{C\max}=\dfrac{U_R^2+U_L^2}{U_L}\Rightarrow Z_C=\dfrac{R^2+Z_L^2}{Z_L}$.	$U_{C\ \max}=\dfrac{U\sqrt{R^2+Z_L^2}}{R}=\dfrac{U_R^2+U_L^2}{U_L}$ Khi $Z_C=\dfrac{R^2+Z_L^2}{Z_L}$.
Dạng 3. *C thay đổi để* U_{RC} *cực đại hoặc cực tiểu.*	$U_{RC}=I\cdot Z_{RC}=\dfrac{U\sqrt{R^2+Z_C^2}}{\sqrt{R^2+(Z_L-Z_C)^2}}=\dfrac{U}{\sqrt{1+\dfrac{Z_L(Z_L-2Z_C)}{R^2+Z_C^2}}}$ với $Z_C\in[0;+\infty)$ • Khi $Z_C=0\Rightarrow\begin{cases}Z_L(Z_L-2Z_C)_{\max}=Z_L^2\\ (R^2+Z_C^2)_{\min}=R^2\end{cases}\Rightarrow\left[1+\dfrac{Z_L(Z_L-2Z_C)}{R^2+Z_C^2}\right]_{\max}=\dfrac{Z_L^2+R^2}{R^2}$ $\Rightarrow U_{RC\min}=\dfrac{U\cdot R}{\sqrt{R^2+Z_L^2}}$. • Khi $Z_C\to+\infty$	• $U_{RC\min}=\dfrac{U\cdot R}{\sqrt{R^2+Z_L^2}}$ khi $Z_C=0$ • $U_{RC\max}=\dfrac{U\cdot Z_C}{R}$ với $Z_C=\dfrac{Z_L+\sqrt{Z_L^2+4R^2}}{2}$.

$$\lim_{Z_C\to+\infty}\left(\frac{Z_L^2-2Z_CZ_L}{R^2+Z_C^2}\right)\overset{l'Hopital}{=}\lim_{Z_C\to+\infty}\left[\frac{\left(Z_L^2-2Z_LZ_C\right)'}{\left(R^2+Z_C^2\right)'}\right]=\lim_{Z_C\to+\infty}\left(\frac{-Z_L}{Z_C}\right)=0\Rightarrow U_{RC}\to U$$

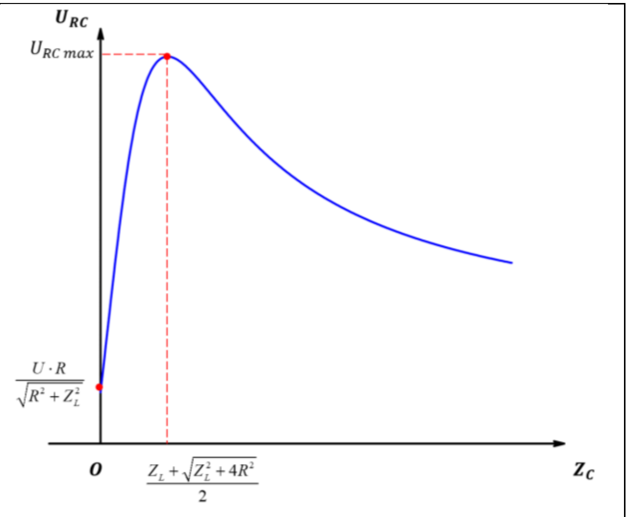
- Xét hàm số $f\left(Z_C\right)=\dfrac{Z_L\left(Z_L-2Z_C\right)}{R^2+Z_C^2}$, ta có $f'\left(Z_C\right)=\dfrac{2Z_L\left(Z_C^2-Z_LZ_C-R^2\right)}{\left(R^2+Z_C^2\right)^2}$

$$f'=0\Leftrightarrow Z_C^2-Z_LZ_C-R^2=0\quad(*)\Leftrightarrow\left[\begin{array}{ll}Z_C=\dfrac{Z_L+\sqrt{Z_L^2+4R^2}}{2} & (n)\\ Z_C=\dfrac{Z_L-\sqrt{Z_L^2+4R^2}}{2}<0 & (\ell)\end{array}\right.$$

Từ (*) ta có: $Z_C-Z_L=\dfrac{R^2}{Z_C}\Rightarrow U_{RC}=\dfrac{U\sqrt{R^2+Z_C^2}}{\sqrt{R^2+\left(\dfrac{R^2}{Z_C}\right)^2}}=\dfrac{U\cdot Z_C}{R}$

- Vì $Z_C=\dfrac{Z_L+\sqrt{Z_L^2+4R^2}}{2}>R$ nên ta có $\Rightarrow U_{RC}=\dfrac{U\cdot Z_C}{R}>U$

Z_C	0	$\dfrac{Z_L+\sqrt{Z_L^2+4R^2}}{2}$	$+\infty$
U_{RC}	$\dfrac{U\cdot R}{\sqrt{R^2+Z_L^2}}$ $\nearrow$	$\dfrac{U\cdot\left(Z_L+\sqrt{Z_L^2+4R^2}\right)}{2R}$	$\searrow$ U



Dạng 4. *Có hai giá trị* $C_1;C_2$ *để mạch có cùng cường độ dòng điện* I *hoặc có cùng công suất* P*. Tìm* C_0 *để* $I_{\max}$ *hoặc* $P_{\max}$.	$I=\dfrac{U}{\sqrt{R^2+\left(Z_L-Z_C\right)^2}}$ Có $I_1=I_2\Rightarrow Z_1=Z_2\Leftrightarrow \left\|Z_{C_1}-Z_L\right\|=\left\|Z_{C_2}-Z_L\right\|$ $\Rightarrow\left[\begin{array}{ll} Z_{C_1}-Z_L=Z_{C_2}-Z_L & (\ell) \\ Z_{C_1}-Z_L=Z_L-Z_{C_2} & (n)\end{array}\right.\Rightarrow Z_L=\dfrac{Z_{C_1}+Z_{C_2}}{2}$. Để $I_{\max}$ thì $Z_{C_0}=Z_L=\dfrac{Z_{C_1}+Z_{C_2}}{2}\Rightarrow\dfrac{1}{C_0}=\dfrac{1}{2}\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}\right)$. * *Hệ quả:* $\tan\varphi_1=-\tan\varphi_2\Rightarrow\varphi_1=-\varphi_2$.	$Z_L=\dfrac{Z_{C_1}+Z_{C_2}}{2}$, $\varphi_1=-\varphi_2$. Để $I_{\max}$ hoặc $P_{\max}$ thì $\dfrac{1}{C_0}=\dfrac{1}{2}\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}\right)$.
Dạng 5. *Có hai giá trị* $C_1;C_2$ *để có cùng* U_C*. Tìm* C_0 *để* $U_{C\max}$.	$U_C=I\cdot Z_C=\dfrac{U\cdot Z_C}{\sqrt{R^2+\left(Z_L-Z_C\right)^2}}\Rightarrow\left(R^2+Z_L^2\right)\cdot\dfrac{1}{Z_C^2}-2Z_L\cdot\dfrac{1}{Z_C}+1-\dfrac{U^2}{U_C^2}=0$ Theo Viète: $\dfrac{1}{Z_{C_1}}+\dfrac{1}{Z_{C_2}}=\dfrac{-b}{a}=\dfrac{2Z_L}{R^2+Z_L^2}=\dfrac{2}{Z_{C_0}}$ Với $Z_{C_0}=\dfrac{R^2+Z_L^2}{Z_L}$ là giá trị của Z_C để $U_{C\max}$. $\Rightarrow C_0=\dfrac{C_1+C_2}{2}$.	$C_0=\dfrac{C_1+C_2}{2}$.