# Problem outline

June 11, 2021

Original NMF factorization:

$$D = XY \tag{1}$$

Diffusion Kernel:

$$K = (I + \beta L)^{-1} \tag{2}$$

Now assume that Y can be approximated by a diffusion process that began at a number of initiator locations. Let V be the sparse k x n matrix of initiators.

$$Y = VK = V(I + \beta L)^{-1} \tag{3}$$

Factorizing:

$$D = XVK \tag{4}$$

Alternatively:

$$K^{-1}D = (I + \beta L)D = XV \tag{5}$$

Now, our problem becomes the minimization of the following:

$$||K^{-1}D - XV||_F^2 \tag{6}$$

Constrained by the fact that X must be non-negative and V is sparse. K is assumed to be known from prior calculation given the graph Laplacian and the parameter $\beta$ which is measures the extent of diffusion.