EL9343 Homework 5

(Due Nov 27th, 2020)

No late assignments accepted

All problem/exercise numbers are for the third edition of CLRS text book

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1. Show the ordering of vertices produced by TOPOLOGICAL-SORT when it is run on the DAG below. Assume that **for** loop of lines 5—7 of the DFS procedure (page 604 in CLRS) considers the vertices in alphabetical order, and assume the adjacency list is ordered alphabetically.
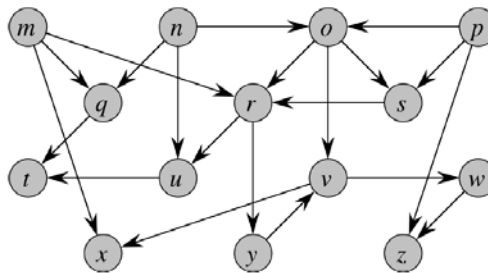
2. Run the procedure STRONGLY-CONNECTED-COMPONENTS on the graph below. Show the:

   (a) The finishing times for each node after running DFS in line 1

   (b) The DFS forest produced by line 3

   (c) The nodes of each tree in the DFS forest produced in line 3 as a separate strongly connected component.

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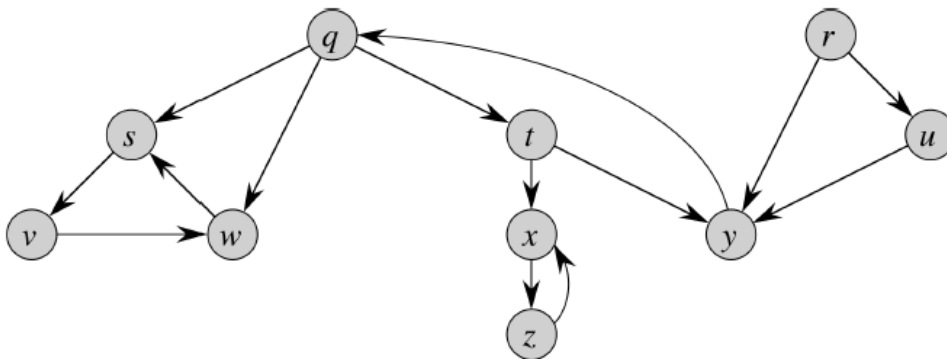
3. Define the edit distance between two string X and Y of length m and n, respectively, to be the number of edits that it takes to change X into Y. An edit consists of a character insertion, a character deletion, or a character replacement. For example, the strings "algorithm" and "rhythm" have edit distance 6. Design an $O(mn)$-time algorithm for computing the edit distance between X and Y.

4. The holidays are here! You decide to travel to see your friends in California.

   Since you are on a budget, so you decide to drive to visit them. You choose your route so you pass by the most spectacular vistas on your route. Along your route are n hotels, at miles $m_1 < m_2 < \cdots < m_n$ from your starting location. The cost of the ith hotel is h_i. Your friends live at mile marker m_{n+1} where $m_n < m_{n+1}$. The starting location is m_0.

   You won't drive more than 400 miles per day for safety concerns. (The hotels on your route are all within 400 miles of each other)

   You want to minimize the cost of your travel.

   Provide a recurrence formula that would calculate the answer.

   Using your recurrence formula, design an algorithm to find the minimum cost.

5. A native Australian named Alice wishes to cross a desert carrying only a single water bottle. She has a map that marks all the watering holes along the way. Assuming she can walk k miles on one bottle of water, design an efficient algorithm for determining where Alice should refill her bottle in order to make as few stops as possible. Argue why your algorithm is correct. Justify the running time of your algorithm.

6. Suppose you've been sent back in time and have arrived at the scene of an ancient Roman battle. Moreover, suppose you have just learned that it is your job to assign n spears to n Roman soldiers, so that each man has a spear. You observe that the men and spears are of various heights, and you have been told that the army is at its best if you can minimize the total difference in heights between each man and his spear. That is, if the ith man has height m_i and his spear has height s_i, then you want to minimize the

$$\sum_{i}^{n} |m_i - s_i|$$

Consider a greedy strategy of repeatedly matching the man and the spear that minimizes the difference in heights between these two. Prove or disprove that this greedy strategy results in the optimal assignment of spears to men.