

1.

h(k)	keys
0	
1	19->10
2	2->20
3	39
4	22
5	5
6	
7	
8	8->35->26

2.

Under the assumption of simple uniform hashing, we will use linearity of expectation to compute this. Suppose that all the keys are totally ordered $\{k_1, \dots, k_n\}$. Let X_i be the number of $l > k_i$ so that $h(l) = h(k_i)$. Note, that this is the same thing as $\sum_{j>i} \Pr(h(k_j) = h(k_i)) = \sum_{j>i} \frac{1}{m} = \frac{(n-i)}{m}$. Then,

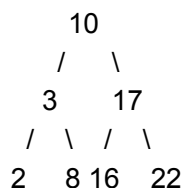
by linearity of expectation, the number of collisions is the sum of the number of collisions for each possible smallest element in the collision.

The expected number of collisions is $\sum_{i=1}^n \frac{n-i}{m} = \frac{n^2 - \frac{n(n+1)}{2}}{m} = \frac{n^2 - n}{2m}$.

3.

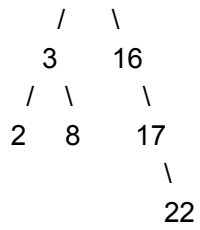
(correct answer not unique)

height=2:

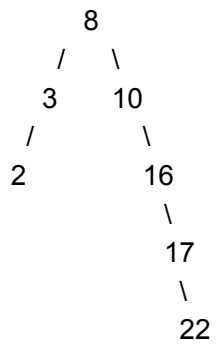


Height = 3:

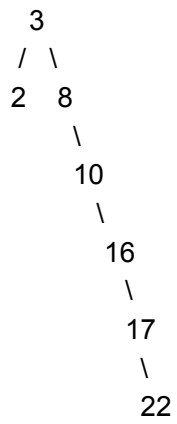
10



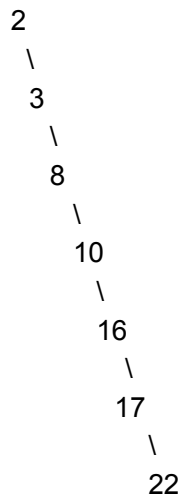
Height = 4:



Height = 5:

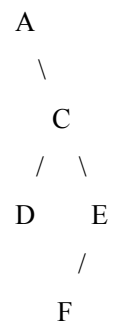


Height = 6:

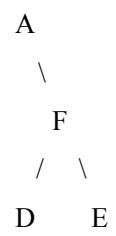


4.

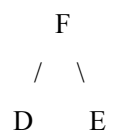
a)



b)



c)



d)

