Seat No.________ Major/Class________ School________ Student ID □□□□□□□□□□□□ Name________

(DONNOT WRITE YOUR ANSWER IN THIS AREA)

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WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

Mathematical Analysis I Exam Paper A（2020-2021-1）

Notice:
1. **Make sure that you have filled the form on the left side of seal line.**
2. **Write your answers on the exam paper.**
3. **This is a close-book exam.**
4. **The exam with full score of 100 points lasts 120 minutes.**

Question No.	I	II	III	IV	V	Sum
Score						

Score

I. Please fill the correct answers in the following blanks. $(4'\times 6=24')$

1. If $y=x^{\sqrt{x}}\ (x>0)$, then $\mathrm{d}y=$ ______________.

2. If $\lim\limits_{n\to\infty} a_n = a$, then $\lim\limits_{n\to\infty}\dfrac{[na_n]}{n}=$ ______________________where the symbol $[\cdot]$ denotes the greatest integer function.

3. The slant asymptotes of the curve $y=\sqrt{x^2+4x}$ are________________________.

4. If $f(0)=0,\ f'(0)=1$, then $\lim\limits_{x\to 0}\dfrac{f(1-\cos x)}{\ln(1-x^2)}=$ ______________.

5. The improper integral $\int_1^{+\infty} e^{-x^2}\mathrm{d}x$ is (convergent or divergent)____________.

6. $\int_{-1}^{1}\left(x+\sqrt{4-x^2}\right)\mathrm{d}x=$ ____________.

II. **Finish the following calculations.** (7-11: $6' \times 5 = 30'$)

Score

7. $\lim\limits_{x \to 2} \dfrac{\sqrt{6-x}-2}{\sqrt[3]{3-x}-1}$.

8. $\lim\limits_{x \to 0} \dfrac{\frac{x^2}{2}+1-\sqrt{1+x^2}}{(\cos x - e^{x^2})\sin x^2}$.

9. $\lim\limits_{n\to+\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right)$

10. If a curve is given by the parametric equation $\begin{cases} x=\ln(t+\sqrt{t^2+1}) \\ y=\int_0^t \sin u^2 \mathrm{d}u \end{cases}$, $t\in R$, find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

11. Evaluate the indefinite integral $\int x \arcsin x \, \mathrm{d}x$.

III. Applications of calculations. (12-13: $8' \times 2 = 16'$)

Score

12. Find the local maximum and minimum values, the intervals of concavity, and inflection points of the function $f(x) = x^{2/3}(6-x)^{1/3}$.

13. Find length of one arch of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ ($a > 0$ is a constant).

IV. Prove the following conclusions. (14-16: $7'\times 3=21'$**)**

Score

14. Prove the inequality $\frac{x}{1+x}<\ln(1+x)<x,(x>0)$.

15. Suppose the function $f(x)$ is continuous on $[0,+\infty)$, $f(0)<0$, $\lim\limits_{x\to+\infty} f(x)=A>0$, show that the equation $f(x)=0$ has at least one root on $[0,+\infty)$.

16. If $f(x)$ has continuous derivatives up to order 2 on $[a,b]$, and $f'(a)=f'(b)=0$, show that

there exists $\xi \in [a,b]$ such that

$$|f''(\xi)| \geq \frac{4}{(b-a)^2}|f(b)-f(a)|.$$

Score

V. Finish the following questions. ($17: 9'\times 1 = 9'$**)**

17. Show that the sequence defined by $0 < x_1 < \sqrt{3}$, $x_{n+1} = \dfrac{3(1+x_n)}{3+x_n}(n = 1, 2, 3\cdots)$ is convergent and find its limit.