Q18

a) $p(\text{inlier}) = \frac{4}{10} = \frac{2}{5}$

$p(2 \text{ inliers}) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

different points

$\frac{4}{10} \times \frac{3}{10} = \frac{3}{25}$

b) $p(\text{incorrect sample}) = 1 - \frac{4}{25} = \frac{21}{25}$

$p(\text{all incorrect in } k) = \frac{21}{25}^k$

$\frac{21}{25}^k < 0.05$

$k \log \frac{21}{25} < \log 0.05$

$k > \frac{\log 0.05}{\log \frac{21}{25}}$ (note div by -ve, reverse sign)

Q19

	t		t+1		
	0	7	4	10	3	5
3	3	3	3	5	1
0	10	8	6	7	1

$I_x = 3 - 3 = 0$

$I_y = 10 - 3 = 7$

$I_t = 5 - 3 = 2$

$0u + 7v + 2 = 0$

$v = -2/7$

u is undefined → could be any value

Q21

$\hat{y}_0 = \text{sign}(w_0 \cdot x_0)$

$= \text{sign}\left(\begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}-1\\-1\end{bmatrix}\right) = -1, \quad y_0 = -1, \quad w_1 = \begin{bmatrix}1\\0\end{bmatrix}$ correct class, do nothing

$\hat{y}_1 = \text{sign}(w_1 \cdot x_1)$

$= \text{sign}\left(\begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}-1\\1\end{bmatrix}\right) = -1, \quad y_1 = +1, \quad w_2 = w_1 + y_1 x_1$

$= \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$

$\hat{y}_2 = \text{sign}(w_2 \cdot x_2)$

$= \text{sign}\left(\begin{bmatrix}0\\1\end{bmatrix} \cdot \begin{bmatrix}1\\1\end{bmatrix}\right) = 1, \quad y_2 = +1, \quad w_3 = \begin{bmatrix}0\\1\end{bmatrix}$ correct class, do nothing

$\hat{y}_3 = \text{sign}(w_3 \cdot x_3)$

$= \text{sign}\left(\begin{bmatrix}0\\1\end{bmatrix} \cdot \begin{bmatrix}-0.5\\0.1\end{bmatrix}\right) \overset{\text{sign}(0.1)}{=} 1, \quad y_3 = -1 \quad w_4 = w_3 + y_3 x_3$

$= \begin{bmatrix}0\\1\end{bmatrix} - \begin{bmatrix}-0.5\\0.1\end{bmatrix}$

$= \begin{bmatrix}0.5\\0.9\end{bmatrix}$

check $\text{sign}(w_4^T x_i) = y_i$?

Perceptron rule would fail if data are not linearly separable from the origin.