# MATH 307 Practice Midterm Exam 2

*November 2021*

- No calculators, cellphones, laptops or notes
- Time allowed: 45 minutes
- 35 total marks
- Write your name and student number in the space below

**Name:**

**Student Number:**

1. Short answer questions. Each part is independent of the others.

   (a) (3 marks) **True** or **False**: If $A$ is diagonalizable, then $A^3$ is also diagonalizable. Justify your answer.

   (b) (3 marks) Let $\lambda_1, \lambda_2 \in \mathbb{R}$ such that $\lambda_1 \neq \lambda_2$, and let $\boldsymbol{v}_1, \boldsymbol{v}_2 \in \mathbb{R}^2$ be linearly independent. How many different $2 \times 2$ matrices exist which have eigenvalues $\lambda_1$ and $\lambda_2$ with corresponding eigenvectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$? Justify your answer.

(c) (3 marks) Find the QR decomposition of $A$ given the *thin* QR decomposition

$$A = Q_1 R_1 = \begin{bmatrix} 1/\sqrt{2} & -2/3 \\ 0 & 1/3 \\ 1/\sqrt{2} & 2/3 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & -\sqrt{2} \\ 0 & 3 \end{bmatrix}$$

(d) (3 marks) Suppose $A$ is a $3 \times 3$ symmetric matrix with eigenvalues $\lambda_1 = 3$ (multiplicity 2) and $\lambda_2 = -1$. Let $\boldsymbol{v}_1 = (1, 2, 1)^T$ and $\boldsymbol{v}_2 = (2, 3, -1)^T$ be two eigenvectors corresponding to $\lambda_1$. Find an eigenvector $\boldsymbol{v}_3$ for $\lambda_2$.

2. (6 marks) Find the shortest distance from the point $P = (2, -6, -4)$ to a point on the line given by $(x, y, z) = (-4t, 5t, -2t)$.

3. (5 marks) Find the orthogonal projection matrix $P$ that projects a vector in $\mathbb{R}^3$ onto the plane spanned by the vectors

$$\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \boldsymbol{u}_2 = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

4. (6 marks) Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 2 & 2 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

(a) (4 marks) Find the singular values of $A$.

(b) (2 marks) Compute $\|A\|$.

5. (6 marks) Find the linear function of the form $f(t) = c_0 + c_1 t$ that best fits the data points $(-6, 23), (0, -1), (6, -37)$ using least squares.

*Extra workspace. Do not write in the table below.*

| | |
|---|---|
| Q1 | /12 |
| Q2 | /6 |
| Q3 | /5 |
| Q4 | /6 |
| Q5 | /6 |
| Total | /35 |