# Advanced Image Analysis
# Image restoration by Wiener and Constrained Least Square Filtering

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***Abstract*—This project is to study a technique to restore noised images (gaussian, pepper, etc..) by applying two types of filters : Wiener and Constrained Least Squares Filtering (CLSF). The Wiener technique is more popular thant the second one but the second is a little bit more complex. The final work of this project is to compare this two techniques and try to implement them one first a user-noised image (Lena with gaussian noise) and another one which is naturally noised.**

***Index Terms*—Advanced image analysis, image restoration, Wiener filter, Constrained Least Squares Filtering.**

## I. Introduction

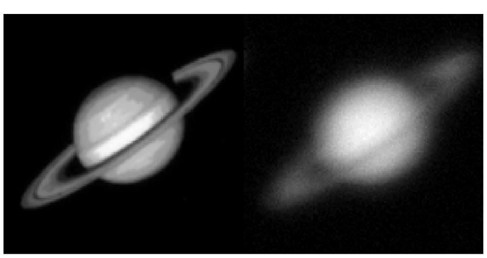
Image Restoration is the operation of taking a corrupt/noisy image and estimating the clean, original image. Image restoration is performed by reversing the process that blurred the image and such is performed by imaging a point source and use the point source image, which is called the Point Spread Function (PSF) to restore the image information lost to the blurring process.

From the beginning of the photography we had a lot of problems for keeping the positives readable for many years. This problems are due to moisture, climate changing, weather, temperature, etc.. And rework on negatives to reproduce this photos may damage them each time. Some noises was considered as 'normal' because of the paper technology, dust on the lens.

Now with digital camera and numerical systems for stocking and scanning this photos we can repair and improve it without using chemicals or touching there surface.

We have a lot of solutions to change the colours, repair surface destruction by adding estimate data thanks to numerical datasets and remove the noise, the blur, the bad contrast thanks to the techniques we will see.

Nowadays, this techniques of image restoration are used on the road traffic. First to see perfectly the licence plates of faster cars but almost for road sign recognition and in astronomy to correct the deflection our atmosphere.



(a) (b)

Fig. 1. Saturn photography (a) after and (b) before restoration

The processus of the digital restoration of an image is always on the same way. and generally the complete inverse of noising.

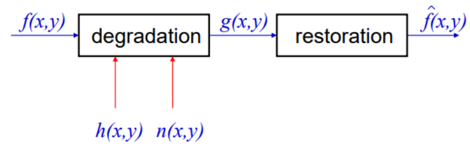



Fig. 2. Restoration schematic

Where :

- $f(x, y)$ is the original image
- $g(x, y)$ is the image after noising
- $h(x, y)$ is the noise filter
- $\hat{f}(x, y)$ restored image
- $n(x, y)$ is the random additive noise

$$g(x, y) = h(x, y) * f(x, y) + n(x, y)$$

On restoration we have to work on Fourier domain.

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

So in the Fourier domain, we don't have convolution products, we can estimate the mathematical result of the restored image:

$$\hat{F}(u, v) = \frac{G(u,v)}{H(u,v)}$$

This is the Inverse filter function.

## II. Explanation

### A. The Wiener filter

The Wiener filter is the MSE-optimal stationary linear filter for images degraded $x(n, m)$ by additive noise and blurring. Calculation of the Wiener filter requires the assumption that the signal and noise processes are second-order stationary.

The Wiener filter is usually applied in the frequency domain. which gives on the Discrete Fourier Transform the degraded image $X(u, v)$

The original image spectrum $\hat{S}(u, v)$ is estimated by computing the product of $X(u, v)$ with the Wiener filter $G(u, v)$

$$\hat{S} = G(u, v)X(u, v)$$

To obtain the image estimated $s(u, v)$ we have to apply the Inverse Discrete Fourier Transform (IDFT)

The Wiener filter is :

$$G(u,v) = \frac{H*(u,v)P_s(u,v)}{|H(u,v)|^2 + \frac{P_n(u,v)}{P_s(u,v)}}$$

Where :

- $H(u,v)$ is the Fourier transform of the point-spread function (PSF) (Figure 1). The point spread function describes the response of an imaging system to a point source or point object. A more general term for the PSF is a system's impulse response. Our Point-spread function will be a Gaussian function (Figure 2,3). The Spread-point define the noise we want to remove. Here, with a Gaussian SP we can only remove Gaussian noises.
$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{1\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$
- $P_s$ is the power spectrum of the signal process obtained by taking the Fourier transform of the signal autocorrelation.
- $P_n$ is the power spectrum of the noise process, obtained by taking the Fourier transform of the noise autocorrelation.

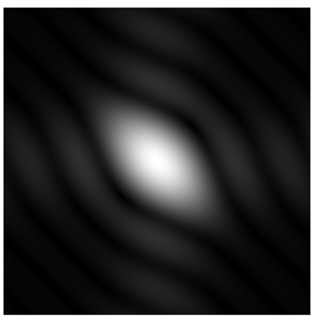


Fig. 3. Point-spread of 6x6 Gaussian function

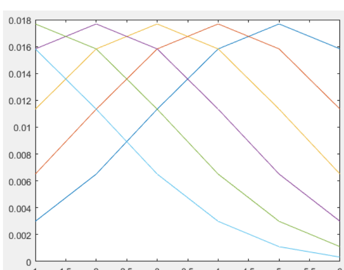



Fig. 4. Point-spread of 6x6 Gaussian function 2D curve

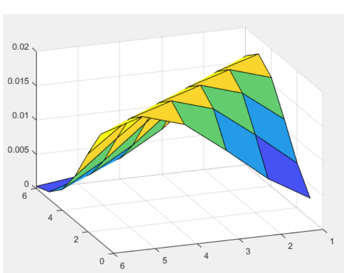



Fig. 5. Point-spread of 6x6 Gaussian function 3D curve

The term $\frac{P_n}{P_s}$ is the signal to noise ratio.

For our approach, we chosen to replace the signal-noise ratio by a constant 'K(u,v) which changes by our-self the noise level of the filter. Then we decided to use a variation of the Wiener filter : The geometric mean filter:

$$G(u,v) = \left[\frac{H*(u,v)}{|H(u,v)|^2}\right]^{\alpha} \left[\frac{H*(u,v)}{|H(u,v)|^2 + K}\right]^{1-\alpha}$$

Where $\alpha$ is a positive real constant.
This technique is a generalization of inverse and Wiener filtering.

### *B. Constrained Least Squares Filter*

The Constrained least-squares image restoration, is a linear image restoration technique in which the smoothness of the restored image is maximized subject to a constraint on the fidelity of the restored image. It is similar to the Wiener filter unless it uses a Fourier transform of 3x3 Laplacian filter instead of $\frac{P_s(u,v)}{P_n(u,v)}$ or 'K'.

$$G(u,v) = \frac{H*(u,v)P_s(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)^2|}$$

$\gamma$ is a parameter that can be adjusted interactively or automatically to meet the minimization constraint using mean and variance of the noise.
When $\gamma$ is equal to 0, we have an inverse filtering.

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & -4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

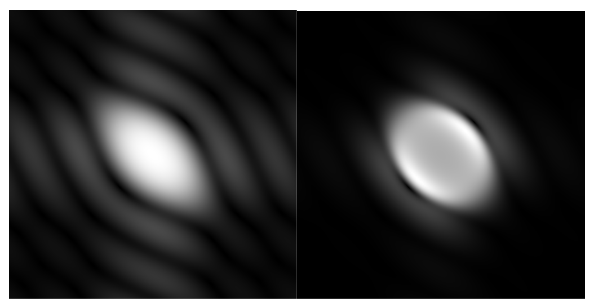


Fig. 6. The FFT of the two techniques : (a) for Wiener, K = 3, Alpha = 0.01 (b) for CLSF, Sigma = 0.2

Here is the general schematic for the three techniques enunciated :

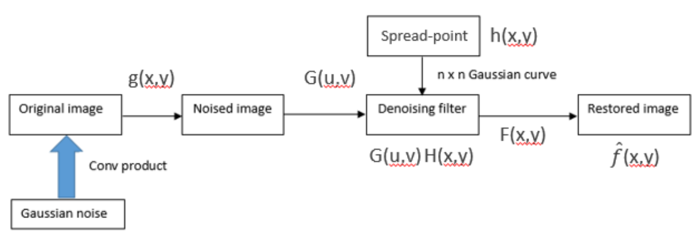



Fig. 7. General schematic of iamge restoration

## III. Results

Fig. 8. (a) : Original image of Lena in Gray scale and (b) Image "damaged" with Gaussian noise

Fig. 9. (a) : Image restored by inverse filter, (b) Image restored by Wiener filter and (c) Image restored by Constraint Least Square filter


Fig. 10. (a) : Original image non user-damaged (b) Image restored by Wiener filter

## IV. Conclusion

We can see there's no a big difference between the three techniques presented. But in the details, the Wiener technique is better than Inverse and Constraint Least Square, as much in the pixel resolution as the time cost. And after computing CLSF creates more artefacts.

For a further work it could be interesting to investigate on the computing of color images, because here we had to work only on grayscale, maybe by creating a 3 dimension PSF. Another work could be to implement this filters on a Convolutional Neural Network.

## References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