UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2018 – 1st Year Examination – Semester 2

IT2105 - Mathematics for Computing I

22nd September 2018

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **41** questions and **8** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 (*All the incorrect choices are marked & no correct choices are marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers — N – set of positive integers

R – set of real numbers — ∅ - (null) empty set

S – Universal set — R^+- set of positive real numbers

1) Which of the following is/are correct ?

(a) $\log_a 1 = 1$, for all $a \in$ N.
(b) $\log_a a = 1$, for all $a \in$ N.
(c) $log_a\left(\frac{u}{v}\right) = \frac{log_a u}{log_a v}$, for all u, v(≠0) ∈ R^+, $a \in$ N.
(d) $log_a a = a$, for all $a \in$ N.
(e) $log_a 1 = 0$, for all $a \in$ N.

2) $\frac{9^{1/2} \times 2^2}{9^{3/2} \times 2^0}$ is equal to

(a) 2 (b) $\frac{4}{9}$ (c) 1. (d) $\frac{2}{27}$ (e) 4.

3) $3 + \log_4\left(\frac{1}{64}\right)$ is equal to

(a) $\log_4 16$. (b) $2 \log_4 4$. (c) $4 \log_4 2$.
(d) 0. (e) 1.

4) Suppose A = {(m, n) | m, n ∈ Z and n < m < n + 1}. Which of the following is/are true?

(a) A={(0,0)}. (b) A=∅. (c) A={∅}. (d) A={0}. (e) A={ (n+1, n) | n ∈ Z }.

5) Let A = {(x, y) | x, y ∈ Z and 2x + 3y = 13} and B = {(x, y) | x, y ∈ Z and 3x - 2y = 0}.

What is A ∩ B?

(a) {(2,3)} (b) {(3,2)} (c) ∅ (d) { ∅ } (e) (3,2)

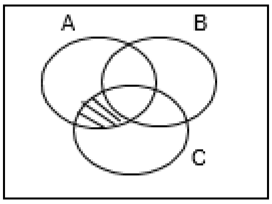
6) Let A and B be non empty sets. Which of the following is/are correct?

(a) $A \cap (A\backslash B) \cap B^c = A\backslash B$	(b) $(A \cap (A\backslash B) \cap B^c = \varnothing$	(c) $(A\backslash B) \cap B = \varnothing$
(d) $A\backslash B \subseteq B$	(e) $A\backslash B \subseteq A$	

7) Let A and B be two distinct non-empty sets. Which of the following is/are **false**?

(a) $A \subseteq A \cup B$	(b) $A \subset A$	(c) $A \subseteq A \cap B$	(d) $U^c = \varnothing$	(e) $A \subseteq \varnothing^c$

8) Consider the following Venn diagram.



Which of the following set(s) represent(s) by the shaded portion?

(a) $(A \cap B)^c \backslash (A \cap B \cap C)$.	(b) $A^c \cap (B \cup A)$.	(c) $(B \cup C)^c \cap A$.
(d) $(A \cap B \cap C)^c \backslash (A \cap C)$.	(e) $(A \cap C) \backslash (A \cap B \cap C)$.	

9) The sets A, B, C are such that $B \neq C$ and $A \backslash B = A \backslash C$. Which of the following is/are true?

(a) $A=B$.	(b) $A \cap B = A \cap C$.	(c) $A \cap C \subseteq B$.
(d) $A \cap B \subseteq C$.	(e) $A=C$.	

10) Let *p* and *q* be two atomic propositions. Which of the following is/are a contradiction(s) ?

(a) $p \vee \sim q$.	(b) $p \wedge q \rightarrow p \vee q$.	(c) $p \wedge (q \wedge \sim q)$.
(d) $p \vee (q \vee \sim q)$.	(e) $(p \wedge q \rightarrow p \vee q) \vee \sim q$.	

11) Which of the following pairs of propositions is/are equivalent?

(a) $(p \rightarrow q)$, $\sim(\sim p \wedge q)$.	(b) $(p \rightarrow q)$, $(\sim q \rightarrow \sim p)$.	(c) $(p \wedge q)$, $\sim(\sim p \wedge q)$.
(d) $(p \rightarrow q)$, $(\sim p \rightarrow \sim q)$..	(e) $(p \wedge q)$, $(\sim p \vee \sim q)$.	

12) Consider the following truth table of the proposition Q with three propositional variables *p*, *q* and *r*

p	*q*	*r*	Q
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

Which of the following gives Q?

(a) $(\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$.
(b) $(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee r)$.
(c) $(p \wedge q \wedge \sim r) \vee (p \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r)$.
(d) $(p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee \sim r)$.
(e) $(p \vee q) \rightarrow r$.

13) Which of the following proposition(s) is/are equivalent to $(p \wedge q) \Rightarrow r$?

(a) $\sim r \rightarrow (p \wedge q)$.
(b) $\sim r \rightarrow \sim(p \wedge q)$.
(c) $\sim r \rightarrow (p \vee q)$.
(d) $\sim r \rightarrow \sim(p \vee q)$.
(e) $\sim r \rightarrow \sim p \vee \sim q$.

14) Suppose A and B are two sets and $A \subseteq B$. Which of the following is/are true?

(a) $\forall x\, (\sim(x \in A) \vee (x \in B))$.
(b) $\forall x\, (x \in B \rightarrow x \in A)$.
(c) $\forall x\, (x \in A \rightarrow x \in B)$.
(d) $\exists x\, (x \in A \rightarrow x \in B)$.
(e) $\exists x\, (\sim(x \in A) \vee (x \in B))$.

15) Suppose A and B are two sets and A=B. Which of the following is/are true?

(a) $(A \subseteq B) \wedge \sim(\forall x\, (x \in B \Rightarrow x \in A))$.
(b) $(A \subseteq B) \wedge (\forall x\, (x \in B \Rightarrow x \in A))$.
(c). $(A \subseteq B) \wedge \sim(\forall x\, (x \in A \Rightarrow x \in B))$.
(d) $(A \subseteq B) \wedge (B \subseteq A)$.
(e) $\forall x\, (x \in B \Leftrightarrow x \in A)$.

16) Which of the following arguments is/are **invalid**?

(a) $p \wedge q \vdash q$
(b) $p \Rightarrow q,\ q \vdash \sim p$
(c) $p \Rightarrow q,\ \sim q \vdash p$
(d) $p \vee \sim q,\ p \vdash \sim q$
(e) $p \Rightarrow q,\ p \vdash q$

17) If p(x,y) is a predicate defined on the set D and $(\forall x \ \forall y \ p(x,y))$ is false, which of the following **must** be true?

(a) $(\exists x \exists y \ p(x,y))$ is true.
(b) $(\exists x \exists y \ p(x,y))$ is false .
(c) $\sim(\exists x \exists y \ p(x,y))$ is true.
(d) $\sim(\exists x \exists y \ p(x,y))$ is false.
(e) $\sim(\forall x \ \forall y \ p(x,y))$ is true.

18) Let $D=\{x_1, x_2, x_3, \ldots\ldots, x_n\}$. If p(x) is a predicate defined on the set D and $\exists x \ p(x)$ is false, which of the following **must** be true?

(a) $(\forall x \sim p(x))$ is true..
(b) $\sim(\exists x \ p(x))$ is false.
(c) $\sim(\exists x \ p(x))$ is true.
(d) $(\forall x \ p(x))$ is false.
(e) $(\forall x \ p(x))$ is true.

19) Let the two predicates p(x) and q(x) be defined as p(x): $x < -2$ and q(x): $x > +2$ where $x \in R$.

Which of the following propositions is/are **false**?

(a) $\exists x \sim(p(x) \vee q(x))$	(b) $(\forall x \ p(x)) \vee (\forall x \ q(x))$	(c) $\forall x \ (p(x) \vee q(x))$
(d) $(\exists x \ p(x)) \wedge (\exists x \ q(x))$	(e) $(\exists x \ p(x)) \vee (\exists x \ q(x))$	

20) Which set(s) of the following statements is/are consistent?

(a) $p \vee \sim q, \sim p, \sim q$.	(b) $p \vee q, \sim p, \sim q$.	(c) $\sim p \vee \sim q, p, q$.
(d) $p \wedge q, \sim p, q$.	(e) $\sim p \wedge \sim q, \sim p, \sim q$.	

21) If $x \in \mathbf{N}$, which of the following is/are true?

(a) $\forall x \ (x^2 + x \geq 2 \wedge x^2 \geq 1)$.	(b) $\forall x \ (x^2 + x > 2 \vee x^2 \geq 1)$.
(c) $\forall x \ (x^2 + x > 2 \vee x^2 > 1)$.	(d) $(\forall x \ x^2 + x > 2) \vee (\forall x \ x^2 > 1)$.
(e) $(\forall x \ x^2 + x > 2) \vee (\forall x \ x^2 \geq 1)$.	

22) Let A={a,b} and B={3,8}. Which of the following is/are true?.

(a) $A \times B = \{(3,a),(8,a),(3,b),(8,b)\}$.	(b) $A \times B = \{(a,3),(b,8),(a,8),(b,3)\}$.
(c) $A \times B = \{(a,3),(a,8),(b,3),(b,8)\}$.	(d) $A \times B = \{(x,y) \mid x \in A, y \in B\}$.
(e) $A \times B = B \times A$.	

Question 23 – 27 are based on the following relations.

$\mu = \{ (a, b) | a \leq b \wedge a, b \in Z \}$
$\pi = \{ (a, b) | a \geq b \wedge a, b \in Z \}$
$\theta = \{ (a, b) | a^2 = b^2 \wedge a, b \in Z \}$
$\sigma = (a, b) | b = a + 2 \wedge a, b \in Z \}$

23) Which of the following define the relation $\mu \cap \pi$?

(a) $\{(a, b) | a, b \in Z\}$
(b) $\{(a, b) | a, b \in Z \wedge a = b \}$
(c) $\{(a, a) | a \in Z \}$
(d) $\{(a, b) | a, b \in Z \wedge (a < b) \vee (a > b) \}$
(e) Z x Z

24) Which of the following define the relation θ o σ?

(a) $\{(a,b) | a,b \in Z, b=a^2+2 \}$.
(b) $\{(a,b) | a,b \in Z, b=(a+2)^2 \}$.
(c) $\{(a,b) | a,b \in Z, b=(a-2)^2 \}$.
(d) $\{(a,b) | a,b \in Z, b=a^2-2 \}$.
(e) $\{(a,b) | a,b \in Z, b^2=a^2-2 \}$.

25) Which of the following relation(s) is/are Reflexive?

(a) μ (b) π (c) θ (d) σ (e) $\mu \cap \pi$

26) Which of the following relation(s) is/are Symmetric?

(a) μ (b) π (c) θ (d) σ (e) $\mu \cap \pi$

27) $|5|_\sigma$ is equal to

(a) { 25 }. (b) $\{(5,b) | b=5+2 \wedge b \in Z \}$. (c) { 7 }. (d) { 3 }. (e) { (5,7) }.

28) Suppose ρ is a relation, and D(ρ) and R(ρ) are domain and range of ρ respectively. Which of the following is/are true?

(a) $D(\rho)=\{ x | \exists y \in R(\rho) , (y,x) \in \rho\}$.
(b) $D(\rho)=\{ y | \exists x \in R(\rho) \ (x,y) \in \rho\}$.
(c) $D(\rho)=\{ x | \exists y \in R(\rho) \ (x,y) \in \rho\}$.
(d) $D(\rho)=\{ y | \exists x \in R(\rho) \ (y,x) \in \rho\}$.
(e) $D(\rho)=\{ x | \forall y \in R(\rho) \ (x,y) \in \rho\}$.

29) Let f be a 1-1 function and $x_1,x_2 \in D(f)$. Which of the following is/are true?.

(a) $\forall x_1, \forall x_2 (f(x_1) = f(x_2) \Rightarrow x_1 = x_2)$.
(b) $\forall x_1, \forall x_2 (x_1 = x_2 \Rightarrow f(x_1) = f(x_2))$.
(c) $\forall x_1, \forall x_2 (x_1 \neq x_2 \vee f(x_1) = f(x_2))$.
(d) $\forall x_1, \forall x_2 (x_1 \neq x_2 \vee f(x_1) \neq f(x_2))$.
(e) $\forall x_1, \forall x_2 (f(x_1) \neq f(x_2) \vee x_1 = x_2)$.

30) Let A={1,2,3}. Which of the following represents a function *f* defined on A?

(a) $f(1)=10, f(2)=10, f(3)=10$.	(b) $f(1)=8, f(2)=8, f(3)=10$.
(c) $f(1)=7, f(1)=8, f(2)=9, f(3)=10$.	(d) $f(1)=8, f(2)=9$.
(e) $f(1)=1, f(2)=2, f(3)=3$.	

31) Suppose *f* is a 1-1 function. Which of the following is/are true about its inverse function f^{-1}?

(a) f^{-1} is 1-1.	(b) $D(f^{-1})=R(f)$.
(c) $R(f^{-1})=D(f)$.	(d) $D(f^{-1})=D(f)$. .
(e) $R(f^{-1})=R(f)$..	

32) Let the functions *f* and *g* be defined by $f(x) = 2x$ and $g(x) = 2x+1$where $x \in R$. Then $(f \circ g)(x)$ is equal to

(a) $4x+1$	(b) $4x+2$	(c) $4(x+1)$	(d) $2(2x+1)$
(e) $4(x-1)$			

33) How many words can be made from the letters of the word COMMITTEE such that all vowels in it are together?

(a) $\frac{9!\times4!}{2!\times2!\times2!}$.	(b) $\frac{9!}{2!\times2!\times2!}$.	(c) $\frac{6!\times4!}{2!\times2!\times2!}$
(d) $\frac{9!}{6!}$	(e) $6!$	

34) Give the number of ways of making a necklace using ten beads with different colours.

(a) ${}^{10}C_9$.	(b) ${}^{10}P_9$.	(c) $\frac{10!}{2}$
(d) ${}^{9}P_2$.	(e) $\frac{9!}{2}$.	

35) Let <S, +, *, c, 0, 1 > be a Boolean algebra, where S is a set, + and * are the sum and the product operators respectively and 0 and 1 be the zero and the unit element respectively, and c the compliment operator. Suppose $A,B,C \in S$.

Which of the following is/are true?

(a) $A + A^c = 0$	(b) $A + 1 = A$
(c) $A+1 = 1$	(d) $A + AB = A$
(e) $A + AB = B$	

36) A box contains 5 black and 4 white socks. A man pulls out 2 socks. The probability that they are of the same colour is:

(a) 5/108	(b) 2/9	(c) 4/9	(d) 41/81	(e) 5/9

37) Six boys and six girls sit in a row at random. Then the probability that six boys sit together is:

(a) $1/32$	(b) $1/36$	(c) $1/132$	(d) $1/360$	(e) $1/95040$

38) A bag contains 40 tickets numbered 1, 2, 3 ... 40, of which five are drawn at random and arranged in ascending order of magnitude ($y_1 < y_2 < y_3 < y_4 < y_5$), the probability that $y_3 = 30$ is:

(a) ${}^{29}C_2 . {}^{10}C_2 / {}^{40}C_5$.	(b) ${}^{30}C_2 . {}^{10}C_2 / {}^{40}C_5$.	(c) ${}^{10}C_2 / {}^{40}C_5$.
(d) ${}^{29}C_2 / {}^{40}C_5$.	(e) ${}^{30}C_3 . {}^{10}C_2 / {}^{40}C_5$.	

39) If A and B are two events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\overline{A}) = 2/3$, then $P(\overline{A} \cap B)$ is:

(a) $1/2$	(b) $3/8$	(c) $5/8$	(d) $5/12$	(e) $1/12$

40) Let A and B be two event such that $P(\overline{A \cup B}) = 1/6$, $P(A \cap B) = 1/4$, $P(\overline{A}) = 1/4$. Then events A and B are:

(a) mutually exclusive and independent.
(b) equally likely but not independent.
(c) equally likely and mutually exclusive.
(d) independent and mutually exclusive.
(e) independent but not equally likely.

41) For k = 1, 2, 3, the bag B_k contains k red marbles and (k + 1) white marbles. Let the probability of selecting bag B_1, B_2, B_3 respectively be $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$. A bag is selected at random and a marble is drawn from it. If a red marble is drawn, then probability that it has come from bag B_2 is:

(a) $35/78$	(b) $14/39$	(c) $10/13$	(d) $12/13$	(e) $35/39$

****