



UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2020 – 1st Year Examination – Semester 2

IT2106 - Mathematics for Computing I

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **40** questions and **7** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 (All the incorrect choices are marked & no correct choices are marked) to +1 (All the correct choices are marked & no incorrect choices are marked).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

\mathbb{Z} - set of integers \mathbb{N} - set of positive integers
 \mathbb{R} - set of real numbers \emptyset - (null) empty set
 \mathbb{R}^+ - set of non-negative real numbers

- 1) Which of the following is/are equal to $\sqrt[3]{x^2}$?

(a) $x^{\frac{3}{2}}$ (b) $x^{\frac{2}{3}}$ (c) $\left(x^{\frac{1}{3}}\right)^2$ (d) $\left(x^{\frac{1}{2}}\right)^3$ (e) $\left(x^{\frac{1}{3}}\right)^{\frac{1}{2}}$

- 2) $x^{-1}y^{-1}z^2$ is equal to which of the following?

(a) $\frac{1}{xyz^{-2}}$ (b) $\frac{1}{xyz^2}$ (c) $\frac{x^2yz^3}{x^3y^2z}$ (d) $\frac{z^2}{xy}$ (e) $(x^{-1}y^{-1}z^2)^{-1}$

- 3) $\log_9 27$ is equal to which of the following?

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{\log_3 9}{\log_3 27}$ (d) $\frac{\log_3 27}{\log_3 9}$ (e) $\log_3 27 - \log_3 9$

- 4) Which of the following is/are correct?

(a) $\forall a, u, v \in \mathbb{N}$ and $a \neq 1, \log_a uv = \log_a u + \log_a v$
 (b) $\forall a, u, v \in \mathbb{N}$ and $a \neq 1 \log_a uv = \log_a u - \log_a v$
 (c) $\forall a \in \mathbb{N} \setminus \{1\}, \log_a 1 = 0$.
 (d) $\forall a \in \mathbb{N} \setminus \{1\}, \log_a 1 = 1$
 (e) $\forall a, u, v \in \mathbb{N}$ and $a \neq 1 \log_a uv = (\log_a u) (\log_a v)$

- 5) Let $X = \{x \mid x \in \mathbb{R} \text{ and } x^2 - 2x + 1 = 0\}$ and $Y = \{y \mid y \in \mathbb{R} \text{ and } y^2 - 1 = 0\}$. What is $X \cap Y$?

(a) $\{-1\}$ (b) $\{1\}$ (c) \emptyset (d) $\{1, -1\}$ (e) 1

- 6) Let \mathbb{Z} be the set of integers, \mathbb{N} be the set of positive integers and $A = \{-n \mid n \in \mathbb{N}\}$. What is $\mathbb{Z} \cap \mathbb{N}$?

(a) \mathbb{Z} . (b) $\{0\}$. (c) $\{0\} \cup A$. (d) A . (e) \mathbb{N} .

- 7) Let A and B be two non-empty sets. If $A \subseteq B$, which of the following **must** be false?

(a) $A \setminus B = \emptyset$ (b) $B \setminus A = \emptyset$ (c) $A \cap B = \emptyset$ (d) $A \cap B \neq \emptyset$ (e) $A = B$

- 8) Let A and B be two non-empty **disjoint** sets. Which of the following **must** be false?
- (a) $A \setminus B = \emptyset$ (b) $B \setminus A = B$ (c) $A \cap B = \emptyset$ (d) $(A \setminus B) \cap (B \setminus A) \neq \emptyset$ (e) $A = B$
- 9) Let U be the universal set and A be a non-empty subset of it. Which of the following **must** be false?
- (a) $\emptyset \subseteq A$. (b) $A^c = U$. (c) $A^c \neq U$. (d) $A^c \subseteq U$ (e) $A^c \subset U$
- 10) Let X and Y be any two non-empty sets. If $X \subset Y$, which of the following **must** be true?
- (a) $X \subseteq Y$. (b) $Y \subseteq X$. (c) $Y \setminus X \neq \emptyset$ (d) $Y \subset X$ (e) $X \neq Y$
- 11) Let A, B and C be three non-empty sets. Which of the following is/are **false**?
- (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (b) $A \cup (B \cap C) = (B \cap C) \cup A$.
(c) $A \cup (B \cap C) = (A \cup C) \cap (A \cup B)$. (d) $A \cup (B \cap C) = (A \cap C) \cup (A \cap B)$.
(e) $A \cup (B \cap C) = A \cup (C \cap B)$.
- 12) Let A, B and C be any three non-empty sets. Which of the following is/are true about A?
- (a) $A = (B^c \cap C) \cup (C \cap B)$. (b) $A = (B^c \cap A) \cup (A \cap B)$.
(c) $A = (C^c \cap B) \cup (C \cap B)$. (d) $A = (A^c \cap B) \cup (A \cap B)$.
(e) $A = (C^c \cap A) \cup (C \cap A)$.
- 13) Let A and B be two sets. Which of the following is/are correct?
- (a) $A \cup B = \{x \mid x \in A \wedge x \in B\}$. (b) $A \cup B = \{x \mid x \in A \vee x \in B\}$.
(c) $A \cap B = \{x \mid x \in A \wedge x \in B\}$. (d) $A \cap B = \{x \mid x \in A \vee x \in B\}$.
(e) $A \cup B \subseteq A \cap B$.
- 14) Let p and q be two propositions. Which of the following propositions is/are logically equivalent to $(p \leftrightarrow q)$?
- (a) $(p \rightarrow q) \vee (q \rightarrow p)$. (b) $(\sim p \vee q) \wedge (\sim q \vee p)$. (c) $(p \vee \sim q) \wedge (q \vee \sim p)$.
(d) $\sim(p \rightarrow q) \vee \sim(q \rightarrow p)$. (e) $(p \rightarrow q) \wedge (q \rightarrow p)$.

- 15) Let p and q be two propositions. Which of the following is/are **contradictions**?

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|---|---|
| (a) $(\sim p \vee q) \leftrightarrow \sim(p \wedge \sim q)$. | (b) $(p \rightarrow q) \leftrightarrow \sim(p \wedge \sim q)$. |
| (c) $(\sim p \vee q) \leftrightarrow (p \rightarrow q)$. | (d) $(\sim p \wedge q) \leftrightarrow \sim(\sim p \wedge q)$. |
| (e) $(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$ | |

- 16) Let p and q be two propositions. Which of the following arguments is/are **invalid**?

- | | | |
|--------------------------------------|--------------------------------------|--|
| (a) $p \vee q, \sim p \vdash q$ | (b) $p \vee q, \sim p \vdash \sim q$ | (c) $p \rightarrow q, \sim q \vdash p$ |
| (d) $\sim p \vee q, p \vdash \sim q$ | (e) $p \rightarrow q, p \vdash q$ | |

- 17) Let p and q be two propositions. Which of the following arguments is/are **valid**?

- | | | |
|---|-----------------------------------|--|
| (a) $\sim p \rightarrow q, \sim p \vdash q$ | (b) $\sim p \vee q, p \vdash q$ | (c) $p \vee q, p \wedge \sim q \vdash q$ |
| (d) $\sim(p \rightarrow q), p \wedge \sim q \vdash q$ | (e) $p, p \wedge \sim q \vdash q$ | |

- 18) Which of the following sets of statements is/are inconsistent?

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|-----------------------------------|---|------------------------------------|
| (a) $p \vee q, \sim p, q$ | (b) $\sim(q \rightarrow p), q, \sim p$ | (c) $p \wedge q, p \vee q, \sim p$ |
| (d) $\sim(q \rightarrow p), q, p$ | (e) $\sim(q \rightarrow p), \sim q, \sim p$ | |

- 19) Let $D = \{x_1, x_2, x_3, \dots, x_n\}$ and the predicate $p(x)$ is defined on D . If $\exists x p(x)$ is true, which of the following **must** be true?

- | | | |
|-----------------------------|-----------------------------|------------------------|
| (a) $\exists x p(x)$. | (b) $p(x_1)$. | (c) $\forall x p(x)$. |
| (d) $\exists x \sim p(x)$. | (e) $\forall x \sim p(x)$. | |

- 20) Let $D = \{x_1, x_2, x_3, \dots, x_n\}$ and the predicate $p(x)$ is defined on D . If $\exists x p(x)$ is **false**, which of the following **must** be true?

- | | | |
|----------------------------------|-----------------------------|---------------------|
| (a) $\sim \exists x \sim p(x)$. | (b) $\forall x p(x)$. | (c) $\sim p(x_1)$. |
| (d) $\sim \exists x p(x)$. | (e) $\forall x \sim p(x)$. | |

- 21) Let $p(x): x \leq 0$ and $q(x): x \geq 0$ be two predicates of the variable x defined on \mathbb{Z} . Which of the following propositions is/are true?

- | | | |
|------------------------------------|--------------------------------------|--------------------------|
| (a) $\forall x [p(x) \vee q(x)]$. | (b) $\forall x [p(x) \wedge q(x)]$. | (c) $p(0) \wedge q(0)$. |
| (d) $\sim p(0) \vee \sim q(0)$. | (e) $\exists x [p(x) \wedge q(x)]$. | |

- 22) Let $p(x)$ be a predicate defined on a domain D . Which of the following is/are equivalent to $\sim \forall x p(x)$?
- | | | |
|----------------------------------|-----------------------------|----------------------------------|
| (a) $\exists x \sim p(x)$. | (b) $\forall x \sim p(x)$. | (c) $\sim \forall x \sim p(x)$. |
| (d) $\sim \exists x \sim p(x)$. | (e) $\sim \exists x p(x)$. | |
- 23) Let $X = \{3, 4, 6\}$, $Y = \{1, 2, 8, 9\}$, $\alpha = \{(a, b) \mid a \leq b \wedge a \in X \wedge b \in Y\}$. Which of the following belong to α ?
- | | | | | |
|------------|------------|------------|-----------|-----------|
| (a) (3,4). | (b) (9,9). | (c) (4,2). | (d) (2,3) | (e) (6,8) |
|------------|------------|------------|-----------|-----------|
- 24) Let α and β be two relations defined by $\alpha = \{(a, b) \mid a \geq b \wedge a, b \in Z\}$ and $\beta = \{(a, b) \mid a^2 = b^2 \wedge a, b \in Z\}$. Which of the following is/are true?
- | | |
|--|---|
| (a) both α and β are symmetric. | (b) both α and β are reflexive. |
| (c) α is not symmetric and β is symmetric | (d) α is reflexive and β is not reflexive. |
| (e) both α and β are transitive. | |
- 25) Let α be a relation defined on Z by $\alpha = \{(a, b) \mid b = a + 2 \wedge a, b \in Z\}$. Find α^{-1} .
- | | |
|---|---|
| (a) $\alpha^{-1} = \{(x, y) \mid y = x + 2 \wedge x, y \in Z\}$. | (b) $\alpha^{-1} = \{(x, y) \mid (y, x) \in \alpha\}$. |
| (c) $\alpha^{-1} = \{(x, y) \mid x = y - 2 \wedge x, y \in Z\}$. | (d) $\alpha^{-1} = \{(x, y) \mid y = x - 2 \wedge x, y \in Z\}$. |
| (e) $\alpha^{-1} = \{(x, y) \mid x = y + 2 \wedge x, y \in Z\}$. | |
- 26) Let α be a relation defined on a non-empty set D such that it is reflexive. Which of the following is/are true?
- | | |
|--|---|
| (a) $\forall x \forall y \forall z [(x, y) \in \alpha \wedge (z, y) \in \alpha \rightarrow (x, z) \in \alpha]$. | (b) $\forall x \forall y [(x, y) \in \alpha \rightarrow (y, x) \in \alpha]$. |
| (c) $\forall x (x, x) \in \alpha$. | (d) $\exists x, [x \in D(\alpha) \wedge (x, x) \in \alpha]$. |
| (e) $\forall x, [x \in D(\alpha) \wedge (x, x) \in \alpha]$. | |
- 27) Let α be the relation defined on $A = \{x, y\}$ by $\alpha = \{(x, x), (y, y), (x, y), (y, x)\}$. Find $[y]_{\alpha}$.
- | | | | | |
|---------------|---------------|------------------|--------------|-----------|
| (a) $\{x\}$. | (b) $\{y\}$. | (c) $\{x, y\}$. | (d) $\{\}$. | (e) A . |
|---------------|---------------|------------------|--------------|-----------|
- 28) Which of the following is/are equivalence relation(s)?
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|---|---|
| (a) $\{(a, b) \mid a \leq b \wedge a, b \in Z\}$. | (b) $\{(a, b) \mid a \geq b \wedge a, b \in Z\}$. |
| (c) $\{(a, b) \mid a^2 = b^2 \wedge a, b \in Z\}$. | (d) $\{(a, b) \mid b = a + 2 \wedge a, b \in Z\}$. |
| (e) $\{(a, b) \mid b = a - 2 \wedge a, b \in Z\}$. | |
- 29) Let $X = \{2, 4, 6\}$, $Y = \{3, 6, 9\}$ and f be a relation from X to Y . Which of the following define(s) a function?
- | | |
|--|--------------------------------|
| (a) $f(2)=9, f(4)=9, f(6)=9$. | (b) $f(2)=9, f(4)=2, f(6)=6$. |
| (c) $f(2)=9, f(2)=3, f(4)=9, f(6)=6$. | (d) $f(2)=9, f(4)=6$. |
| (e) $f(2)=3, f(4)=3, f(6)=3$. | |

- 30) Suppose f is a function defined on A . Which of the following **must be false**?

| | | |
|--|--|--|
| (a) $D(f) \subseteq A, R(f) \subseteq A$. | (b) $D(f) \subset A, R(f) \subseteq A$. | (c) $D(f) \subseteq A, A \subset R(f)$. |
| (d) $D(f) = A, R(f) \subseteq A$. | (e) $D(f) = A, R(f) = A$. | |

- 31) Suppose f is a 1-1 function. Which of the following is/are true?

| | |
|---|--|
| (a) $\forall x \forall y \ x \neq y \Rightarrow f(x) = f(y)$. | (b) $\forall x \forall y \ f(x) = f(y) \Rightarrow x = y$. |
| (c) $\forall x \forall y \ \sim(f(x) = f(y)) \Rightarrow x \neq y$. | (d) $\sim(\exists x \exists y \ \sim(x = y) \vee f(x) = f(y))$. |
| (e) $\forall x \forall y \ \sim(x = y) \Rightarrow \sim(f(x) = f(y))$. | |

- 32) Let f and g be functions defined by $f(x) = 2x-1$ and $g(x) = 3x$ where $x \in \mathbb{R}$. Then $(f \circ g)(1)$ is equal to which of the following?

| | | | | |
|--------|--------|--------|--------|--------|
| (a) 3. | (b) 5. | (c) 6. | (d) 1. | (e) 0. |
|--------|--------|--------|--------|--------|

- 33) Let f be a function defined on \mathbb{R} by $f(x) = 2(x+1)$. Which of the following is/are true?

| | | |
|--|--|------------------------------|
| (a) f is 1-1. | (b) $D(f^{-1}) = \mathbb{R}, f^{-1}(x) = \frac{1}{2}x + 1$. | (c) f^{-1} does not exist. |
| (d) $D(f^{-1}) = \mathbb{R}, f^{-1}(x) = \frac{1}{2}(x-1)$. | (e) $D(f^{-1}) = \mathbb{R}, f^{-1}(x) = \frac{1}{2}x - 1$. | |

- 34) What is the number of arrangements that can be made by taking all the letters in the word "COMPOSITION" if the three letters "O" are together?

| | | | | |
|-------------------------|-----------------------------|----------------------|-----------------------|------------------------------|
| (a) $\frac{9!}{(2!)}$. | (b) $\frac{9!}{(2!)(3!)}$. | (c) $\frac{9!}{2}$. | (d) $\frac{9!}{26}$. | (e) $\frac{11!}{(2!)(3!)}$. |
|-------------------------|-----------------------------|----------------------|-----------------------|------------------------------|

- 35) When the occurrence of one event has no effect on the probability of the occurrence of another event, the events are called:

| | | |
|-------------------------|--------------------------------|----------------------------|
| (a) Independent events. | (b) Dependent events. | (c) Equally likely events. |
| (d) Exhaustive events. | (e) Mutually exclusive events. | |

- 36) Two fair coins are tossed simultaneously. If one of them turned head, what is the probability that the other one turn tail?

| | | | | |
|-----------|-----------|-----------|----------|--------|
| (a) 0.01. | (b) 0.05. | (c) 0.25. | (d) 0.5. | (e) 1. |
|-----------|-----------|-----------|----------|--------|

- 37) Two six-sided fair dice are rolled simultaneously. The probability of getting a total of 4 or less given that both-faces are similar is:

| | | | | |
|----------------------|----------------------|----------------------|---------------------|---------------------|
| (a) $\frac{2}{36}$. | (b) $\frac{3}{36}$. | (c) $\frac{6}{36}$. | (d) $\frac{1}{6}$. | (e) $\frac{2}{6}$. |
|----------------------|----------------------|----------------------|---------------------|---------------------|

- 38) If A and B are two events such that $P(\bar{A} \cap B) = 5/12$, $P(A) = 1/3$, then $P(\bar{A} \cap \bar{B})$ is:

| | | | | |
|-------------------|-------------------|-------------------|--------------------|--------------------|
| (a) $\frac{1}{2}$ | (b) $\frac{1}{4}$ | (c) $\frac{3}{4}$ | (d) $\frac{1}{12}$ | (e) $\frac{7}{12}$ |
|-------------------|-------------------|-------------------|--------------------|--------------------|

- 39) A box contains 10 marbles, of which 7 are green and 3 are white. A marble is picked at random from the box and its colour is noted. Without replacing the first marble a second marble is then picked out. What is the probability that the first marble is green and the second marble is white?

| | | | | |
|----------------------|--------------------|--------------------|-------------------|--------|
| (a) $\frac{21}{100}$ | (b) $\frac{7}{30}$ | (c) $\frac{3}{10}$ | (d) $\frac{1}{3}$ | (e) 1. |
|----------------------|--------------------|--------------------|-------------------|--------|

- 40) What is the probability that in a random arrangement of the letters of the word "UNIVERSITY", the two I's do not come together?

| | | | | |
|--------------------|-------------------|--------------------|-------------------|--------------------|
| (a) $\frac{1}{10}$ | (b) $\frac{1}{5}$ | (c) $\frac{3}{10}$ | (d) $\frac{4}{5}$ | (e) $\frac{9}{10}$ |
|--------------------|-------------------|--------------------|-------------------|--------------------|
