UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2022 – 3rd Year Examination – Semester 5

IT5506 – Mathematics for Computing II

Structured Question Paper

(TWO HOURS)

To be completed by the candidate
BIT Examination Index No: ....

Important Instructions:

- The duration of the paper is **Two (2) hours**.
- The medium of instruction and questions is English.
- This paper has **4 questions** and **19 pages**.
- **Answer all questions.** All questions carry **equal** marks.
- **Write your answers** in English using the space provided **in this question paper**.
- Note that **The Standard Normal Distribution Table** is attached with the paper
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.
- All kinds of electronic devices including calculators are **not** allowed.
-

Questions Answered

Indicate by a cross (×), (e.g. ✗) the numbers of the questions answered.

	Question numbers				
To be completed by the candidate by marking a cross (×).	1	2	3	4	
To be completed by the examiners:					

Index No ………………………….

1) (a) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & k \end{bmatrix}$. Find a value for k such that $AB = BA$.

(4 marks)

ANSWER IN THIS BOX

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & k \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & k \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\begin{bmatrix} 7 & 2+2k \\ 15 & 6+4k \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 3+3k & 6+4k \end{bmatrix}$

$2(1+k) = 10$

$k = 4$

K is consistent

(b) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find values for a, b, c and d such that $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A \neq \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

(4 marks)

ANSWER IN THIS BOX

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Then $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A \neq \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

Ans. $a = 1, b = 0, c = 0$, and $d = -1$.

(c) Consider the following system of linear equations:

$$\begin{aligned} 3x - y + 5z &= 8 \\ y - 10z &= 1 \\ 6x - y &= 17. \end{aligned}$$

(i) Transform this system of linear equations into matrix form and identify the coefficient matrix.

(2 marks)

ANSWER IN THIS BOX

$$\begin{bmatrix} 3 & -1 & 5 \\ 0 & 1 & -10 \\ 6 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 17 \end{bmatrix}.$$

Hence, the coefficient matrix is $\begin{bmatrix} 3 & -1 & 5 \\ 0 & 1 & -10 \\ 6 & -1 & 0 \end{bmatrix}$

(ii) Apply elementary row operations to solve the given system of linear equations.

(10 marks)

ANSWER IN THIS BOX

$$\begin{bmatrix} 3 & -1 & 5 & : & 8 \\ 0 & 1 & -10 & : & 1 \\ 6 & -1 & 0 & : & 17 \end{bmatrix}$$

$$\Downarrow \ \boldsymbol{R_3} \leftarrow \boldsymbol{R_3 - 2R_1}$$

$$\begin{bmatrix} 3 & -1 & 5 & : & 8 \\ 0 & 1 & -10 & : & 1 \\ 0 & 1 & -10 & : & 1 \end{bmatrix}$$

$$\Downarrow \ \boldsymbol{R_3} \leftarrow \boldsymbol{R_3 - 2R_1}$$

$$\begin{bmatrix} 3 & -1 & 5 & : & 8 \\ 0 & 1 & -10 & : & 1 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$ This matrix is now in echelon form. Pivot variables are x, y

and z is a free variable. The equations corresponding to this echelon form are

$$\begin{aligned} 3x - y + 5z &= 8 \\ y - 10z &= 1\,. \end{aligned}$$

Let $z = t$, where t is a parameter. By back substitution, we get

$y = 1 + 10z = 1 + 10t$

$x = \frac{1}{3}(8 + y - 5z) = \frac{1}{3}(8 + 1 + 10t - 5t) = 3 + \frac{5}{3}\,t.$

Therefore, the general solution of this system is

$x = 3 + \frac{5}{3}\,t$

$y = 1 + 10t$

$z = t$, where $t \in \mathbb{R}$ is arbitrary.

(d) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 9 \\ 2 & 4 & 6 \end{bmatrix}$. Find the rank of A by reducing A to its echelon form.

(5 marks)

ANSWER IN THIS BOX

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 9 \\ 2 & 4 & 6 \end{bmatrix} \xRightarrow{R_2 \leftarrow R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 2 & 4 & 6 \end{bmatrix} \xRightarrow{R_3 \leftarrow R_3 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence the Rank of A is 2.

2) (a) Find the redundant vectors in the following sequence of vectors in $\mathbb{R}^3$ and write each redundant vector as a linear combination of previous non-redundant vectors:

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, u_3 = \begin{bmatrix} 2 \\ 7 \\ -4 \end{bmatrix}, u_4 = \begin{bmatrix} 5 \\ 7 \\ -10 \end{bmatrix}, and\ u_5 = \begin{bmatrix} 12 \\ 17 \\ -24 \end{bmatrix}.$$

(12 marks)

ANSWER IN THIS BOX

$$\begin{bmatrix} 1 & 1 & 2 & 5 & 12 \\ 1 & 2 & 7 & 7 & 17 \\ -2 & -4 & -4 & -10 & -24 \end{bmatrix} \xRightarrow[R_3 \leftarrow R_3 + (2)R_1]{R_2 \leftarrow R_2 + (-1)R_1} \begin{bmatrix} 1 & 1 & 2 & 5 & 12 \\ 0 & 0 & 5 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Downarrow R_2 \leftarrow {}^{1}/_{5}\, R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & {}^{21}/_{5} & 10 \\ 0 & 0 & 1 & {}^{2}/_{5} & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xLeftarrow{R_1 \leftarrow R_1 + (-2)R_2} \begin{bmatrix} 1 & 1 & 2 & 5 & 12 \\ 0 & 0 & 1 & {}^{2}/_{5} & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Redundant vectors are u_2, u_4, and u_5.

$u_2 = 2u_1,$

$u_4 = \left({}^{21}/_{5}\right)u_1 + \left({}^{2}/_{5}\right)u_3$, and

$u_5 = 10u_1 + u_3.$

(b) Let X and Y be two vector spaces over the same field $\mathcal{F}$. When do we say that a function $T: X \longrightarrow Y$ is a Linear Transformation?

(2 marks)

ANSWER IN THIS BOX

$T: X \longrightarrow Y$ is called a Linear Transformation if it satisfies the following conditions:

$\mathrm{T}(x + y) = \mathrm{T}x + \mathrm{T}y$ for all $x, y \in X$, and

$\mathrm{T}(\alpha x) = \alpha \mathrm{T}x$ for all $\alpha \in \mathcal{F}$, $x \in X$.

(c) Let X and Y be two vector spaces over the same field $\mathcal{F}$, and let $\mathbf{0}_X$, and $\mathbf{0}_Y$ be the zero vectors of X and Y respectively. let $T: X \longrightarrow Y$ be a Linear Transformation. Show that

i. $\mathrm{T}(\mathbf{0}_X) = \mathbf{0}_Y$,
ii. $T(-v) = -Tv$, for all $v \in X$.
iii. $\mathrm{T}(\alpha \mathrm{x} + \beta y) = \alpha \mathrm{T}(\mathrm{x}) + \beta T(y)$ for all $\alpha, \beta \in \mathrm{F}$, for all $x, y \in V$.

(6 marks)

ANSWER IN THIS BOX

Since T is linear, $\mathrm{T}(\mathbf{0}_X) = \mathrm{T}(\mathbf{0}_X + \mathbf{0}_X) = \mathrm{T}(\mathbf{0}_X) + \mathrm{T}(\mathbf{0}_X)$.

Hence, by the additive identity law in Y, $\mathrm{T}(\mathbf{0}_X) = \mathbf{0}_Y$.

Since T is linear, $\mathrm{T}(\alpha v) = \alpha \mathrm{T}(\mathrm{v})$. By taking $\alpha = -1$,

we get $T(-v) = -Tv$.

$$\mathrm{T}(\alpha \mathrm{x} + \beta y) = \mathrm{T}(\alpha \mathrm{x}) + \mathrm{T}(\beta y) \quad (\because \text{T is linear})$$

$$= \alpha T(x) + \beta T(y) \quad (\because \text{T is linear}).$$

Since T is linear, $\mathrm{T}(\mathbf{0}_X) = \mathrm{T}(\mathbf{0}_X + \mathbf{0}_X) = \mathrm{T}(\mathbf{0}_X) + \mathrm{T}(\mathbf{0}_X)$.

(d) Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ be a linear transformation such that $T\left(\begin{bmatrix}1\\3\\1\end{bmatrix}\right) = \begin{bmatrix}4\\4\\0\\-2\end{bmatrix}$ and

$T\left(\begin{bmatrix}4\\0\\5\end{bmatrix}\right) = \begin{bmatrix}4\\5\\-1\\5\end{bmatrix}$. Find $T\left(\begin{bmatrix}-7\\3\\-9\end{bmatrix}\right)$.

(5 marks)

ANSWER IN THIS BOX

Since $\begin{bmatrix}-7\\3\\-9\end{bmatrix} = \begin{bmatrix}1\\3\\1\end{bmatrix} - 2\begin{bmatrix}4\\0\\5\end{bmatrix}$, from part (c), we have

$$T\left(\begin{bmatrix}4\\0\\5\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\3\\1\end{bmatrix}\right) - 2T\left(\begin{bmatrix}4\\0\\5\end{bmatrix}\right)$$

$$= \begin{bmatrix}4\\4\\0\\-2\end{bmatrix} - 2\begin{bmatrix}4\\5\\-1\\5\end{bmatrix} = \begin{bmatrix}-4\\-6\\2\\-12\end{bmatrix}.$$

3) A company produces two types of boats X and Y. The following information is given.

	Boat X requires	Boat Y requires	Resource availability per month
Inputs			
Aluminium	24 kg	6 kg	2400 kg
Machine Time (minutes)	6 min	4 min	12 hours
Labour (hours)	2 hours	8 hours	1000 hours

(a) The profit per unit of X and Y are \$600 and \$500 respectively. Assume that all assumptions of a linear programming problem hold.

(i) Formulate a linear programming problem to find the optimal production quantities.

(07 marks)

ANSWER IN THIS BOX

Decision Variables : Let x and y the monthly production quantities of X and Y.

Objective Function : Maximize profits $Z = 600x + 500y$

Constraints : $24x + 6y \leq 2400,$

$6x + 4y \leq 720$

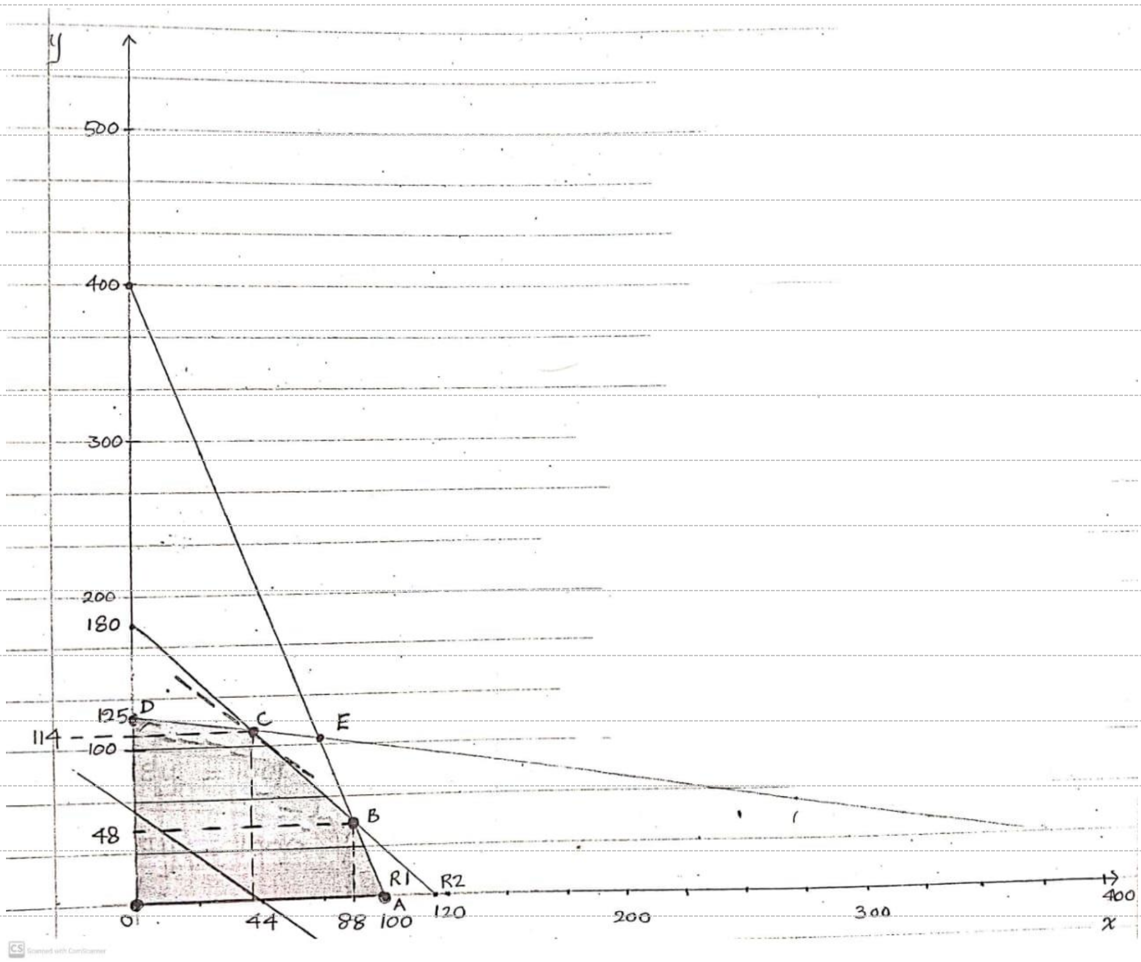
$2x + 8y \leq 1000$

$x \geq 0, \ y \geq 0.$

(ii) Solve the problem graphically and find the optimal production quantities and the maximum profit achievable.

(08 marks)

ANSWER IN THIS BOX



Since the feasible region is a convex set the maximum will occur at the corners of the feasible region

Corner Points	Profit $Z = 600x + 500y$ $
O (0,0)	0
A (100,0)	60,000
B (88,48)	76,800
C (44,114)	**83,400**
D (0,125)	62,500

The optimal solution is $(x, y) = (44, 114)$ with maximum profit achievable $ 83,400.

(b) If the unit profit of X and Y are \$600 and \$400 respectively find all possible optimal production quantities and the maximum profit achievable

(10 marks)

ANSWER IN THIS BOX

The problem reads

Maximize profits $Z = 600x + 500y$

Subject to the constraints : $24x + 6y \leq 2400,$

$6x + 4y \leq 720$

$2x + 8y \leq 1000$

$x \geq 0,\ y \geq 0$

Here the feasible region will be the same as in part (a).

Corner Points	Profit $Z = 600x + 400y$ \$
O (0,0)	0
A (100,0)	60,000
B (88,48)	**72,000**
C (44,114)	**72,000**
D (0,125)	50,000

This problem has multiple solutions. All possible solutions are the points on the line segment BC. That is the optimal possible production quantities are

$(x, y) = (44 + 44k,\ 114 - 66k)$ where $0 \leq k \leq 1.$

with the maximum profit achievable \$ 72,000.

This problem has multiple solutions. All possible solutions are the points on the line segment BC. Those are the optimal possible production quantities.

4) (a) The lifetime of a certain brand of a power bank is normally distributed with a mean 48 months and standard deviation 8 months. Calculate the probability that the lifetime of that brand of power bank is,

(i) less than 50 months.

(ii) lies between 38 and 52 months.

(09 marks)

<u>ANSWER IN THIS BOX</u>

(i) $P(X < 50) = P\left(\frac{X-\mu}{\sigma} < \frac{50-48}{8}\right) = P(Z < 0.25) = 0.5987$

(ii) $P(38 < X < 52) = P\left(\frac{38-48}{8} < \frac{X-\mu}{\sigma} < \frac{52-48}{8}\right) = P(-1.25 < Z < 0.5) =$
$P(Z < 0.5) - P(Z < -1.25) = 0.6915 - 0.1056 = 0.5859$

(b) It was found that the probability of having a damage in a certain brand of mobile phone is 0.1. A researcher randomly selects 10 mobile phones from that brand.

(i) Calculate the probability that there will be at least one damaged mobile phone in that selected 10 mobile phones.

(ii) Calculate the mean and the standard deviation of the number of damaged mobile phones in the sample of 10 mobile phones.

(09 marks)

ANSWER IN THIS BOX

(i) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - {}^{10}C_0 0.1^0 0.9^{10}$

$= 1 - 0.3487 = 0.5613$

(ii) $E(X) = np = 10 \times 0.1 = 1$

$V(X) = npq = 10 \times 0.1 \times 0.9 = 0.9$

$SD(X) = \sqrt{V(X)} = \sqrt{0.9} = 0.9487$

(c) The average of the number of miscalls for an undergraduate student is 2 per day. Using Poisson distribution,

i. Calculate the probability that there are exactly three miscalls per day.
ii. Calculate the standard deviation of the number of miscalls per day.

(07 marks)

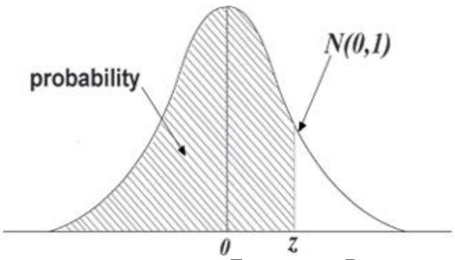
ANSWER IN THIS BOX

(i) $P(X=3)=\frac{e^{-2}2^3}{3!}=\frac{0.1353\times 8}{6}=0.1804$

(ii) $SD(X)=\sqrt{V(X)}=\sqrt{2}=1.4142$

Index No ………………………….

The Standard Normal Distribution Table



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	***P***	***Z***	***P***	***Z***	***P***
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997

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