**UNIVERSITY OF COLOMBO, SRI LANKA**

**UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING**

**DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)**

**Academic Year 2019 – 1st Year Examination – Semester 2**

# *IT2105 - Mathematics for Computing I*

***2nd November 2019***

***(TWO HOURS)***

**Important Instructions :**

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **42** questions and **7** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

**Notations:**

Z – set of integers N – set of positive integers

R – set of real numbers ∅ - (null) empty set

$R^+$- set of non-negative real numbers

1) Find $x$ such that $\left(\frac{1}{100}\right)^{3x+2} = 1$.

| | | |
|---|---|---|
| (a) $x = -\frac{1}{3}$ | (b) $x = \frac{2}{3}$ | (c) $x = -\frac{2}{3}$ |
| (d) $x = -\frac{1}{\sqrt{3}}$ | (e) $x = \frac{1}{3}$ | |

2) $\dfrac{x^{\frac{3}{7}} \times y^{\frac{2}{3}}}{x^{\frac{1}{7}} \times y^{\frac{1}{2}}}$ is equal to

| | | |
|---|---|---|
| (a) $x^{\frac{2}{7}} \times y^{\frac{1}{6}}$ | (b) $x^{\frac{2}{7}} \times y^{-\frac{1}{6}}$ | (c) $x^{-\frac{2}{7}} \times y^{\frac{1}{6}}$ |
| (d) $xy^{-\frac{5}{21}} \times y^{-\frac{5}{14}}$ | (e) $xy^{-\frac{5}{21}} \times y^{\frac{5}{14}}$ | |

3) Let $x$ be a real number and $2\log_6(x+3) = 1 + \log_6(x+2)$. Find $x$.

| | | |
|---|---|---|
| (a) $x = \sqrt{2}$. | (b) $x = -\sqrt{2}$ . | (c) $x = \sqrt{3}$ . |
| (d) $x = -\sqrt{3}$. | (e) 3. | |

4) Let A = {2,4,6,8} and B= {2,3,4, 6, 8,9}. Find A ∪ B, A ∩ B and A \ B.

(a) A ∪ B = A, A ∩ B = B, A \ B = {3,9}.
(b) A ∪ B = A, A ∩ B = B, A \ B = ∅.
(c) A ∪ B = B, A ∩ B = A, A \ B = A.
(d) A ∪ B = B, A ∩ B = A, A \ B = ∅.
(e) A ∪ B = B, A ∩ B = A, A \ B = {3,9}.

5) Suppose the universal set U={1,2,3,4,5,6,7,8,9,10}. Let X = {1,4,8,9} and Y= {3, 4, 6, 9}.
Find $X^c \cap Y$ .

| | | | | |
|---|---|---|---|---|
| (a) {3,6,} | (b) {2,3,4,5,6,7,9,10} | (c) ∅ | (d) U | (e) {4,9} |

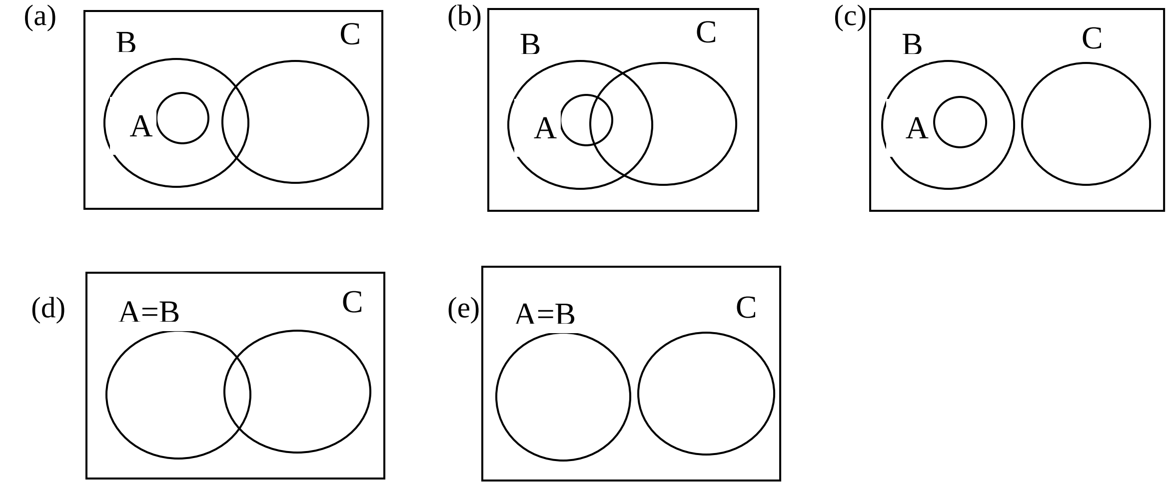
6) Let S = {(x, y) |x, y∈Z, x – y = 2} and T = {(b, a) | a, b∈Z, a + b = 8}. Find S ∩ T.

| | | | | |
|---|---|---|---|---|
| (a) {(4,2)} | (b) {(5,3)} | (c) {(2,4)} | (d) {(3,5)} | (e) {(10,8)} |

7) Let A and B be two different non-empty sets such that $B \setminus A = \varnothing$. Which of the following must be **false**?

| | | | | |
|---|---|---|---|---|
| (a) $A \subseteq B$ | (b) $B \subseteq A$ | (c) $A \cap B = \varnothing$ | (d) $A \cap B \neq \varnothing$ | (e) $A \setminus B = \varnothing$ |

8) Let A,B and C be three non-empty sets such that $A \subset B$, $A \cap C = \varnothing$ and C is not a proper subset of B. Which of the following Venn diagrams reflect these properties ?.




9) Let A be a non-empty subset of the universal set U . Which of the following **must** be **false**?

| | | |
|---|---|---|
| (a) $\varnothing \subseteq A$. | (b) $A \subseteq U$. | (c) $A \subseteq A$. |
| (d) . $A^c \subseteq U$ | (e) $A \subset A$. | |

10) Let X and Y be two non-empty sets and $X \cap Y \neq \varnothing$. If P(X) and P(Y) are power sets of X and Y respectively, which of the following is/are true?

| | | |
|---|---|---|
| (a) $P(X) \subseteq X$. | (b) $X \subseteq P(X)$. | (c) $\varnothing \subseteq P(X) \cap P(Y)$. |
| (d) $\varnothing \in P(X) \cap P(Y)$. | (e) $P(X) \cup P(Y) = X \cup Y$. | |

11) Let A and B be any two non-empty sets. Which of the following is/are a proposition/ propositions?

| | | |
|---|---|---|
| (a) Human has four legs. | (b) Sign your attendance. | (c) $A \subseteq B$. |
| (d) $A \subset A$. | (e) it is raining. | |

12) Consider the following truth tables for three different propositions, P,Q, R of a propositional variable *p*.

| p | P | Q | R |
|---|---|---|---|
| F | T | F | T |
| T | T | T | F |

Which of the following gives P,Q and R respectively.?

(a) $p \vee \sim p$, $p \vee p$, $\sim p \wedge \sim p$.
(b) $p \vee \sim p$, $p$, $\sim p$,.
(c) $p \vee p$, $p$, $\sim p$.
(d) $p \Rightarrow p$, $p$, $\sim p$.
(e) $\sim(p \wedge \sim p)$, $p$, $\sim p$.

13) Let p and q be two propositions. Which of the following proposition is/are (a) **contradiction(s)**?

(a) $(p \wedge \sim p) \vee \sim q$.
(b) $(q \vee \sim q) \Rightarrow (p \wedge \sim p)$
(c) $(p \wedge q) \Rightarrow (\sim p \vee \sim q)$
(d) $(p \wedge q) \Rightarrow (\sim p \vee q)$
(e) $(p \wedge q) \Rightarrow (p \wedge \sim p)$

14) Let *p* and *q* be two propositions. Which of the following is/are **tautologies**?

(a) $(p \wedge q \Rightarrow \sim p \vee q) \vee \sim q$.
(b) $(p \wedge q \Rightarrow \sim p \vee q) \vee q$.
(c) $p \vee \sim p$.
(d) $(p \vee \sim p) \wedge \sim q$.
(e) $p \wedge q \Rightarrow p \vee q$.

15) Let p and q be two propositions. Which of the following pairs of propositions are logically equivalent?

(a) $p \Rightarrow q$, $\sim p \vee \sim q$.
(b) $p \Rightarrow q$, $q \vee \sim p$.
(c) $p \Leftrightarrow q$, $(p \Rightarrow q) \vee (q \Rightarrow p)$.
(d) $p \Leftrightarrow q$, $(\sim p \vee q) \wedge (p \vee \sim q)$.
(e) $p \Rightarrow q$, $\sim p \Rightarrow \sim q$.

16) Let p and q be two propositions. Which of the following arguments is/are **valid**?

(a) $p \Rightarrow q, p \vdash q$
(b) $p \Rightarrow q, \sim p \vdash \sim q$
(c) $\sim p \Leftrightarrow \sim q, p \vdash \sim q$
(d) $p \Leftrightarrow q, p \vdash q$
(e) $p \Leftrightarrow \sim q, p \vdash q$

17) Let p and q be two propositions. Which of the following arguments is/are **invalid**?

(a) $p \Rightarrow q, p \vdash q$
(b) $p \Rightarrow q, \sim p \vdash \sim q$
(c) $\sim p \Leftrightarrow \sim q, p \vdash \sim q$
(d) $p \Leftrightarrow q, p \vdash q$
(e) $p \Leftrightarrow \sim q, p \vdash q$

18) Which of the following sets of statements is/are consistent?

(a) $\{ p \wedge q, p \vee q, \sim p \}$
(b) $\{ p \vee q, \sim p, \sim q \}$
(c) $\{ q \Rightarrow p, p \Rightarrow \sim r, q, r \}$
(d) $\{ q \Leftrightarrow p, \sim p, \sim q \}$
(e) $\{ q \Rightarrow p, p \Rightarrow \sim r, r \}$

19) Let the two predicates p(x) and q(x) be defined as p(x): $x \leq 4$ and q(x): $x > 4$ where $x \in R$.

Which of the following propositions is/are **true**?

| | | |
|---|---|---|
| (a) $\exists x\ (p(x) \vee q(x))$ | (b) $(\forall x\ p(x)) \vee (\forall x\ q(x))$ | (c) $\forall x\ (p(x) \vee q(x))$ |
| (d) $(\exists x\ p(x)) \wedge (\exists x\ q(x))$ | (e) $(\exists x\ p(x)) \vee (\exists x\ q(x))$ | |

20) Let A={4,6,,8,10} and x, y∈A. Which of the following is/are true?

| | | |
|---|---|---|
| (a) $\forall x\ \exists y\ x + y < 14$ | (b) $\forall x\ \forall y\ x + y < 14$ | (c) $\exists x\ \forall y\ x + y < 14$ |
| (d) $\exists x\ \exists y\ x + y < 14$ | (e) $\forall x\ \exists y\ x < y$ | |

21) Let the two predicates p(x) and q(x) be defined on R and suppose $\forall x\ (p(x) \vee q(x))$ is **true**. Which of the following **must** be true?

| | | |
|---|---|---|
| (a) $\exists x\ p(x)$. | (b) $\sim\exists x\ p(x)$. | (c) $\sim\forall x\ p(x)$. |
| (d) $\forall x\ p(x)$. | (e) $\exists x\ (p(x) \vee q(x))$. | |

22) Which of the following pairs of propositions is/are equivalent?

| | | |
|---|---|---|
| (a) $\forall x\ p(x)$, $\sim(\exists x\ p(x))$. | (b) $\forall x\ p(x)$, $\sim\exists x \sim p(x)$. | (c) $\forall x\ p(x)$, $\exists x\ p(x)$. |
| (d) $\exists x\ p(x)$, $\sim\forall x \sim p(x)$. | (e) $\exists x\ p(x)$, $\sim(\forall x\ p(x))$. | |

**Question 23 – 28 are based on the following relations.**

$\mu$ = { (a, b)|a ≤ b ∧ a, b ∈ Z }
$\pi$ = { (a, b)| a ≥ b ∧ a, b ∈ Z }
$\theta$ = { (a, b)| $a^2 = b^2$ ∧ a, b ∈ Z }
$\sigma$ = (a, b)| b = a + 2 ∧ a, b ∈ Z }
$\alpha$ = (a, b)| b = a - 2 ∧ a, b ∈ Z }

23) Which of the above relation is/are Reflexive?

| | | | | |
|---|---|---|---|---|
| (a) $\mu$ | (b) $\pi$ | (c) $\theta$ | (d) $\sigma$ | (e) $\alpha$ |

24) Which of the above relation is/are Symmetric?

| | | | | |
|---|---|---|---|---|
| (a) $\mu$ | (b) $\pi$ | (c) $\theta$ | (d) $\sigma$ | (e) $\alpha$ |

25) Which of the above relation is/are equivalence?

| | | | | |
|---|---|---|---|---|
| (a) $\mu$ | (b) $\pi$ | (c) $\theta$ | (d) $\sigma$ | (e) $\alpha$ |

26) Which of the following is/are true?

| | | | | |
|---|---|---|---|---|
| (a) $\mu^{-1} = \pi$ | (b) $\pi^{-1} = \mu$ | (c) $\alpha^{-1} = \sigma$ | (d) $\sigma^{-1} = \alpha$ | (e) $\theta^{-1}$ |

27) Which of the following is/are true?

(a) $\mu \mathbf{o} \pi = \mu$. (b) $\pi \mathbf{o} \mu = \pi$. (c) $\mu \mathbf{o} \pi \neq \mu$.
(d) $\pi \mathbf{o} \mu \neq \pi$. (e) $\mu \mathbf{o} \pi = \{(a,a) \mid a \in Z\}$.

28) Let $x \in$ Z. Which of the following is/are true?

(a) $|x|_\theta = \{y|(x,y) \in \theta\}$. (b) $|x|_\theta = \{y|y = \pm x, y \in Z\}$. (c) $|x|_\theta = Z$.
(d) $|1|_\theta = N$. (e) $|x|_\theta = \{y|y \geq\ y \in Z\}$.

29) Suppose β is a relation defined on A. Which of the following is/are true?

(a) $D(\beta)=\{x|\exists y\in A,(x,y)\in \beta\}$. (b) $D(\beta)=\{y|\exists x\in A,(y,x)\in\beta\}$.
(c) $R(\beta)=\{x|\exists y\in D(\beta),(x,y)\in\beta\}$. (d) $R(\beta)=\{y|\exists x\in D(\beta),(x,y)\in\beta\}$.
(e) $D(\beta) = R(\beta)$.

30) Suppose $f$ is a function defined on A. Which of the following must be true?

(a) $D(f) \subseteq A$, $R(f) \subseteq A$. (b) $D(f) \subset A$, $R(f) \subseteq A$. (c) $D(f) \subset A$, $R(f) \subset A$.
(d) $D(f) = A$, $R(f) \subseteq A$. (e) $D(f) = A$, $R(f) = A$.

31) Suppose $f$ is a 1-1 function. Which of the following is/are true?

(a) $\forall x_1, \forall x_2,\ x_1 \in D(f), x_2 \in D(f),\ f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
(b) $\forall x_1, \forall x_2,\ x_1 \in D(f), x_2 \in D(f),\ f(x_1) \neq f(x_2) \Rightarrow x_1 = x_2$.
(c) $\exists x_1, \exists x_2,\ x_1 \in D(f), x_2 \in D(f), x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$.
(d) $\exists x_1, \exists x_2,\ x_1 \in D(f), x_2 \in D(f), x_1 = x_2 \Rightarrow f(x_1) \neq f(x_2)$.
(e) $\exists x_1, \exists x_2,\ x_1 \in D(f), x_2 \in D(f), x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$.

32) Let the functions $f$ and $g$ be defined on R by $f(x)=2x$ and $g(x)=x+2$. Then $(g \mathbf{o} f)(x)$ is equal to

(a) $2(x+2)$. (b) $2(x+1)$. (c) $4(x+1)$. (d) $2x+1$. (e) $g(2x)$.

33) Let $f$ be a function defined on R by $f(x) = 2x+1$. Find $f^{-1}$.

(a) $D(f^{-1})$=R, $f^{-1}(x) = \frac{1}{2}(x+1)$. (b) $D(f^{-1})$=R, $f^{-1}(x) = 2(x+1)$.. (c) $f^{-1}$ does not exit.
(d) $D(f^{-1})$=R, $f^{-1}(x) = 2(x-1)$. (e) $D(f^{-1})$=R, $f^{-1}(x) = \frac{1}{2}(x-1)$.

34) In a group of 4 boys and 2 girls, two children are to be selected. In how many different ways can they be selected such that at least one boy is there.

(a) ${}^4C_2$. (b) 8. (c) 14.
(d) ${}^4C_1 \times {}^2C_1$. (e) $({}^4C_1 \times {}^2C_1)+{}^4C_2$.

35) In how many ways can the letters of the word BALL be arranged?

(a) 4!. (b) $\frac{4!}{2!}$. (c) 12. (d) 24. (e) 6.

36) Let <B, "+", ".", 0, 1 > be a Boolean algebra, where B is a set, "+" and " **.** " are the sum and the product operators respectively and, 0 and 1 be the zero and the unit element respectively. Suppose X,Y $\in$ B and $\overline{X}$ is the compliment of X.

Which of the following is/are true?

| | | | | |
|---|---|---|---|---|
| (a) X+XY=X. | (b) X**.**X=X. | (c) X+ $\overline{X}$=X. | (d) X+ $\overline{X}$=0. | (e) X**.**X=1. |

37) The events having no experimental outcomes in common are called:

| | | |
|---|---|---|
| (a) Equally likely events. | (b) Exhaustive events. | (c) Mutually exclusive even |
| (d) Independent events. | (e) Dependent events. | |

38) If $A$ and $B$ are two mutually exclusive and exhaustive events and $P(A) = 2P(B)$, then $P(B)$ is equal to:

| | | | | |
|---|---|---|---|---|
| (a) $\frac{1}{2}$ | (b) $\frac{1}{3}$ | (c) $\frac{2}{3}$ | (d) $\frac{1}{4}$ | (e) $\frac{2}{4}$ |

39) Given that $A$ and $B$ are not mutually exclusive and $P(\bar{A}\cap\bar{B}) = 2/5$, then a possible value for $P(\bar{A}\cup\bar{B})$ is:

| | | | | |
|---|---|---|---|---|
| (a) 0. | (b) $\frac{1}{10}$ | (c) $\frac{3}{10}$ | (d) $\frac{7}{10}$ | (e) 1. |

40) A part of an exam contains two multiple-choice questions, each with three answer choices (listed "A", "B", and "C") out of which only one is correct. Assuming the outcomes to be equally likely, find the probability that at least one answer is "C".

| | | | | |
|---|---|---|---|---|
| (a) $\frac{7}{9}$ | (b) $\frac{5}{9}$ | (c) $\frac{4}{9}$ | (d) $\frac{1}{3}$ | (e) $\frac{2}{3}$ |

41) The independent events $A$ and $B$ are such that $P(A|B) = 0.5$ and $P(B|A) = 0.2$. The probability $P(A \cup B)$ is:

| | | | | |
|---|---|---|---|---|
| (a) 0.1. | (b) 0.2. | (c) 0.4. | (d) 0.5. | (e) 0.6. |

42) Plant A of a company produces 6% defective products, Plant B produces 10% defective products and Plant C produces 20% defective products. If a product is chosen so that the chance it is coming from any of the plants is equally likely, then find the probability that the product chosen is defective.

| | | | | |
|---|---|---|---|---|
| (a) 0.12. | (b) 0.21. | (c) 0.22. | (d) 0.35. | (e) 0.53. |

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