

CS 6041

Theory of Computation

Deterministic finite automata

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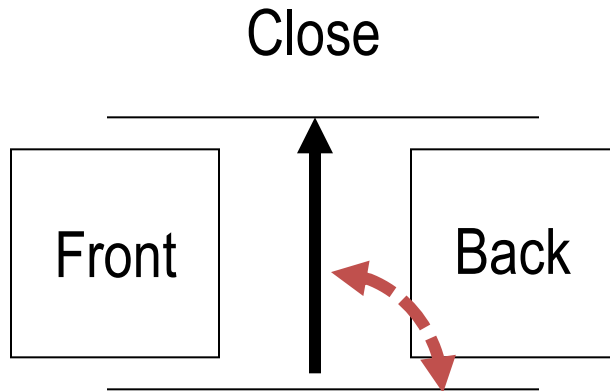
<https://kevinsuo.github.io/>

Outline

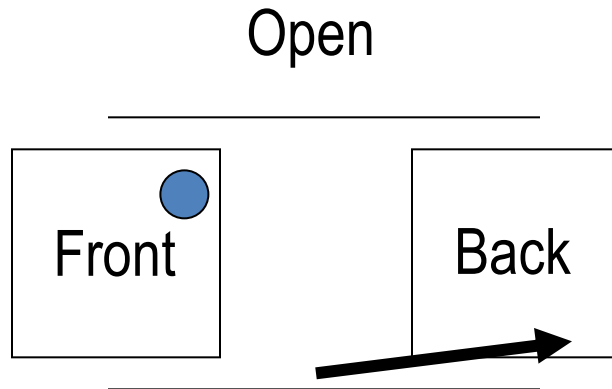
- Finite Automata
 - Definition
 - Example
 - Language of DFA
 - Computation for DFAs
- Design DFAs
 - Example
 - Regular language
 - Regular operation



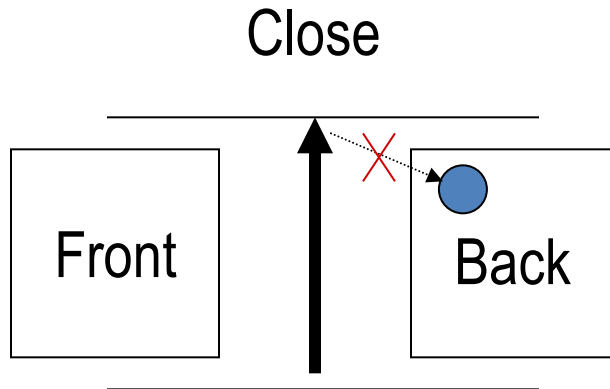
Automatic door



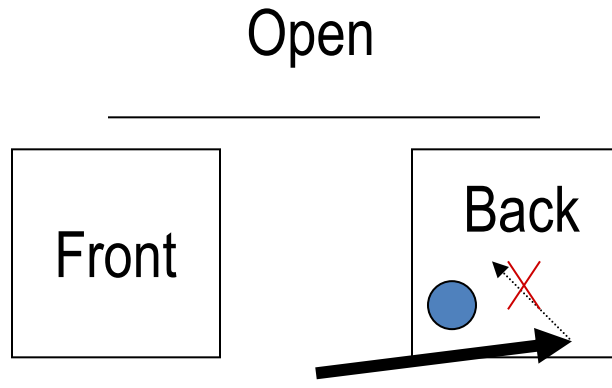
Automatic door (2)



Automatic door (3)



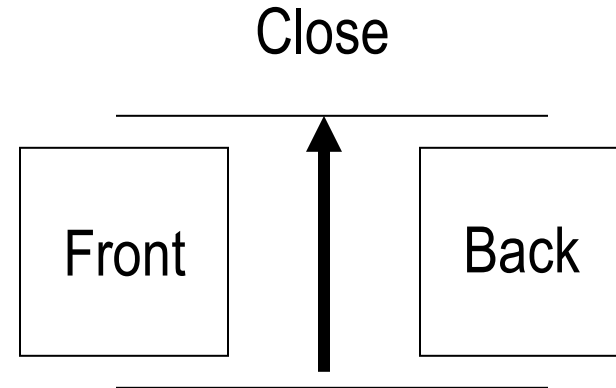
Automatic door (4)



Finite automata: automatic door

- Two states

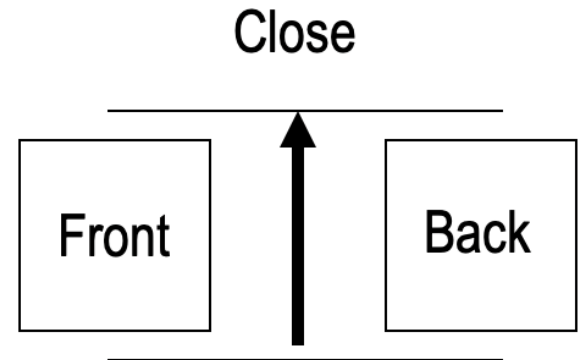
- Open
- Close



- Four inputs (4 possible cases)

- Front: a person is standing on the pad in front of the doorway
- Rear: a person is standing on the pad to the rear of the doorway
- Both: people are standing on both pads
- Neither: no one is standing on either pad

State table and diagram



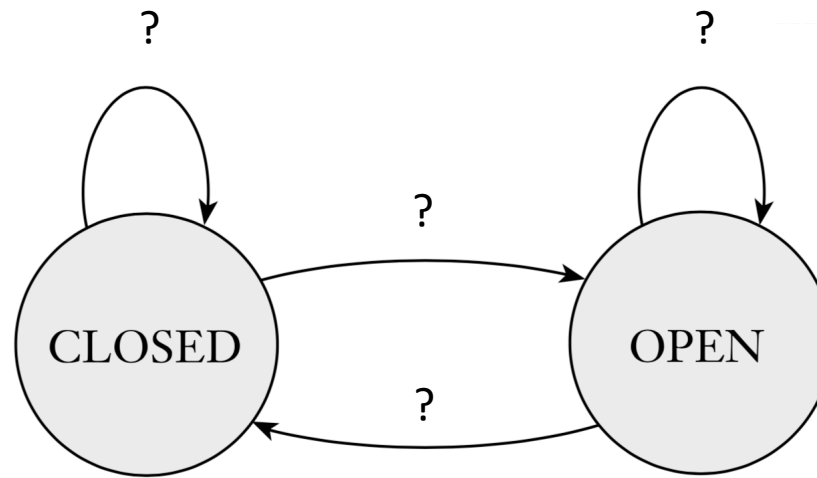
	Neither	Front	Rear	Both
Closed	Closed	Open	Closed	Closed
Open	Closed	Open	Open	Open



State table and diagram

input signal

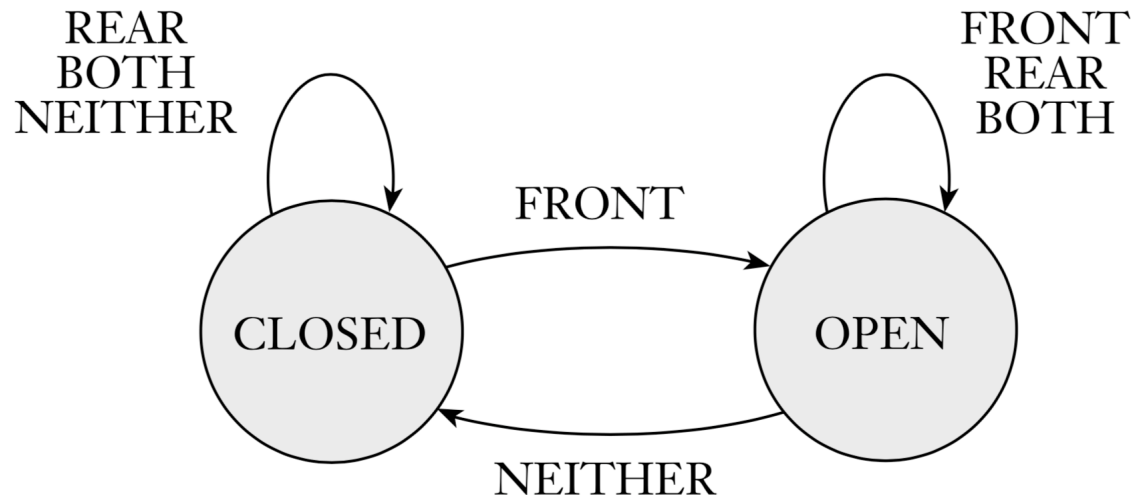
	NEITHER	FRONT	REAR	BOTH
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN



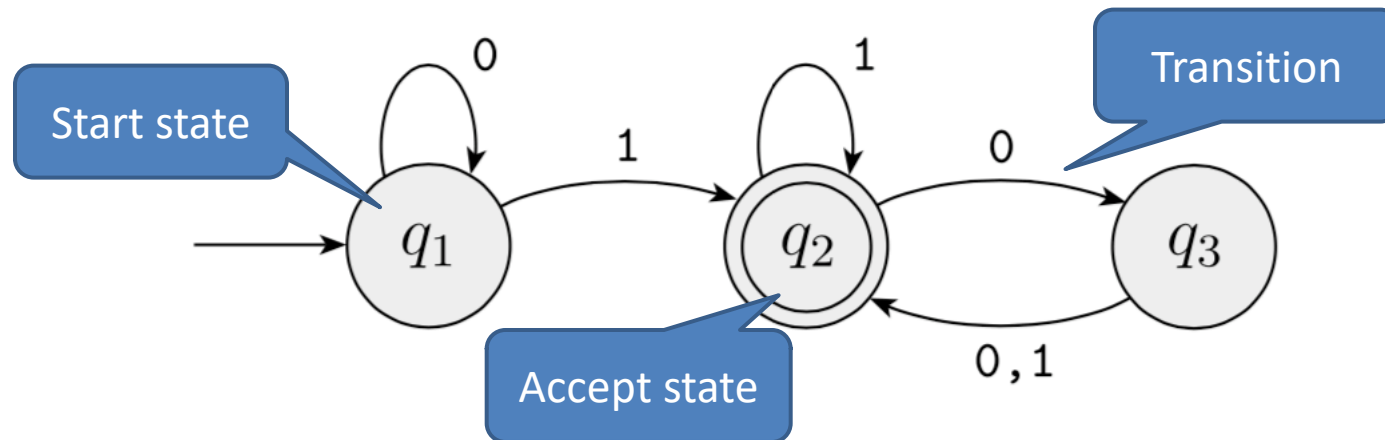
State table and diagram

input signal

	NEITHER	FRONT	REAR	BOTH
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN



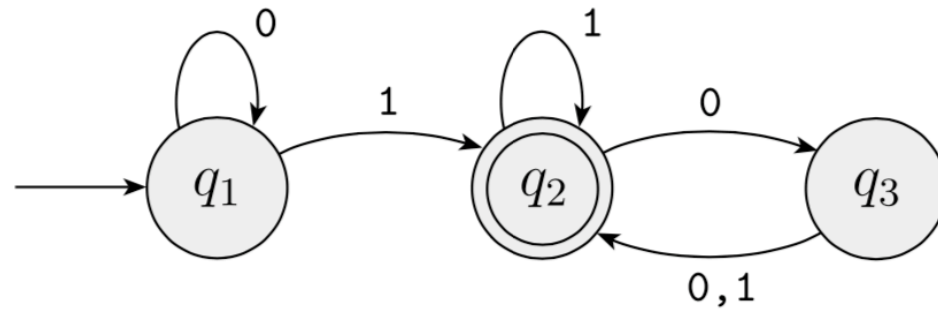
State diagram of finite automata



- **Start state**: indicated by the arrow pointing at it from nowhere
- **Accept state**: is the one with a double circle
- **Transitions**: the arrows going from one state to another with input
- **Output**: is either **accept** or **reject**. If input is 1101, output is accept.

Automata M_1

State diagram



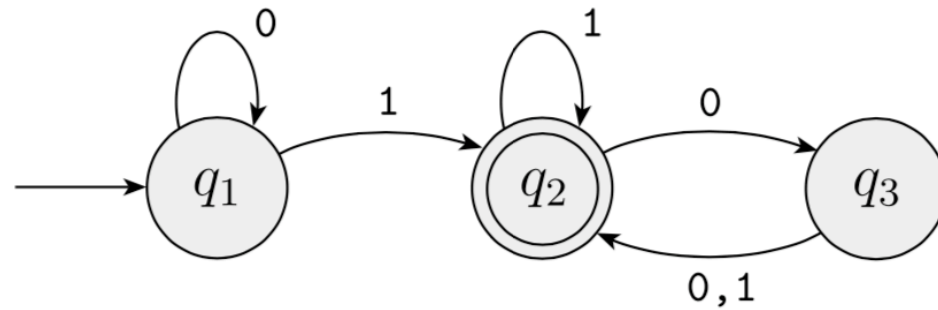
State transition table?

	0	1
q_1		
q_2		
q_3		



Automata M_1

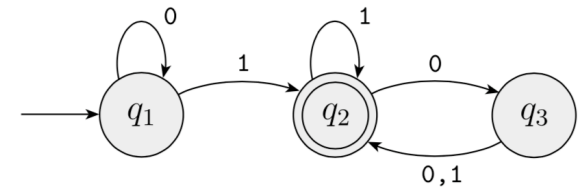
State diagram



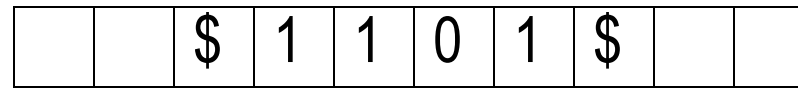
State transition table

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

M_1 on input 1101



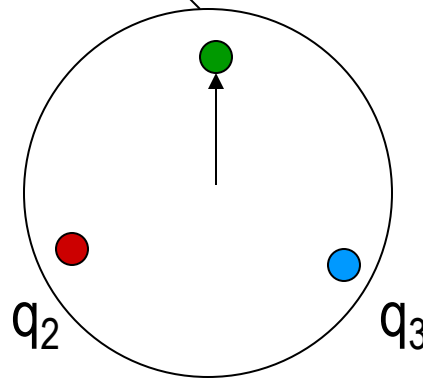
Single read-only input tape



head

q_1

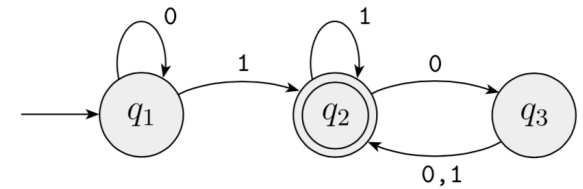
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2



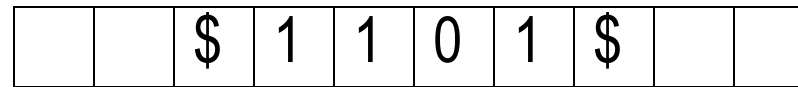
Finite state controller



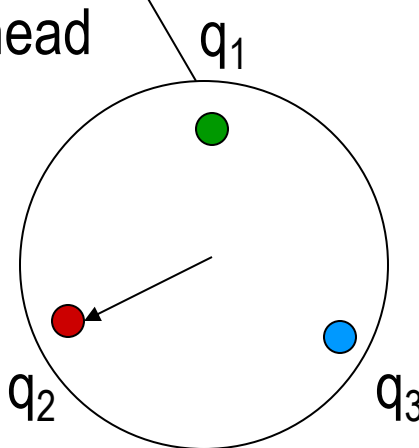
M_1 on input 1101



Single read-only input tape



head

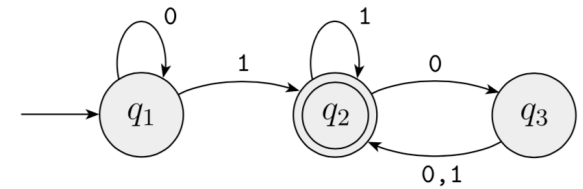


Finite state controller

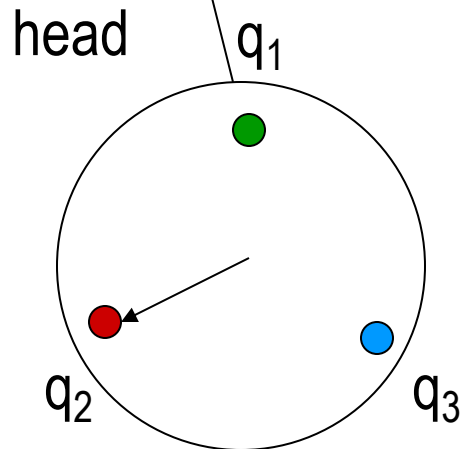
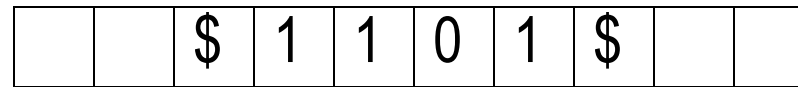
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2



M_1 on input 1101



Single read-only input tape

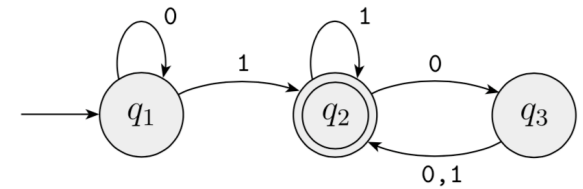


Finite state controller

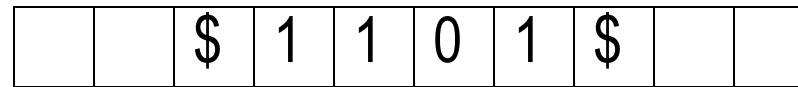
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2



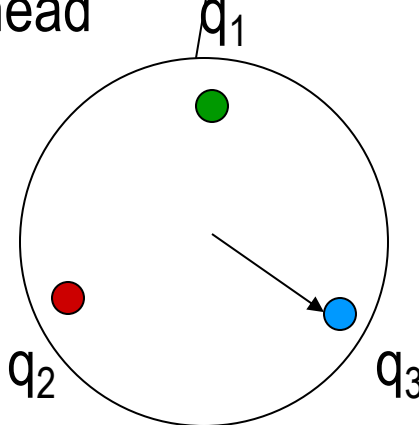
M_1 on input 1101



Single read-only input tape



head

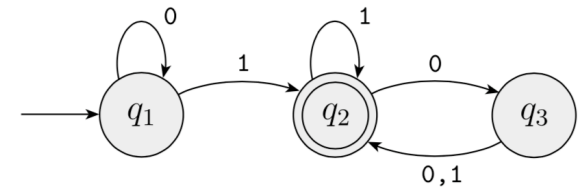


Finite state controller

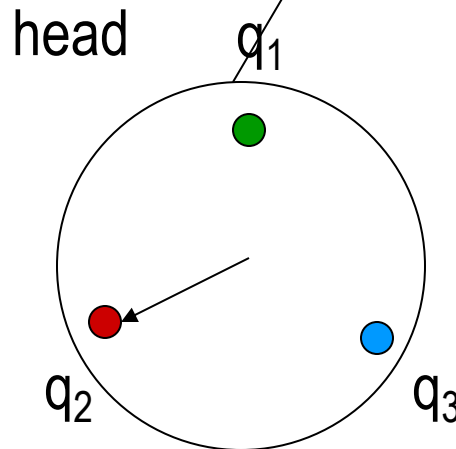
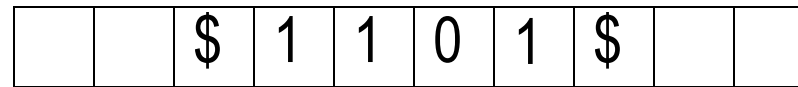
	0	1
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_2	q_2



M_1 on input 1101



Single read-only input tape

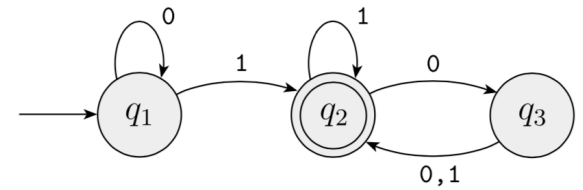


	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

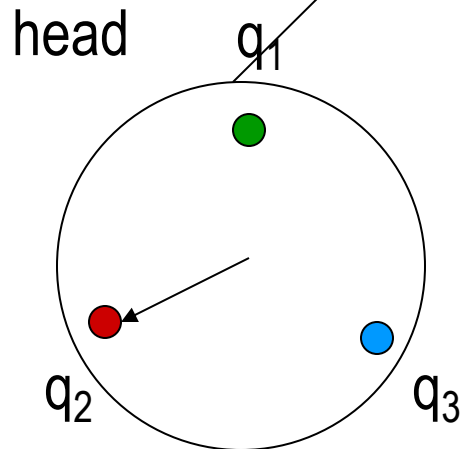
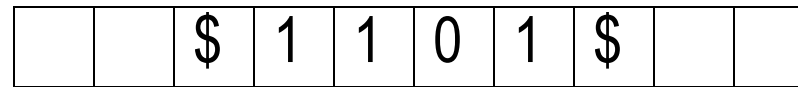
Finite state controller



M_1 on input 1101



Single read-only input tape



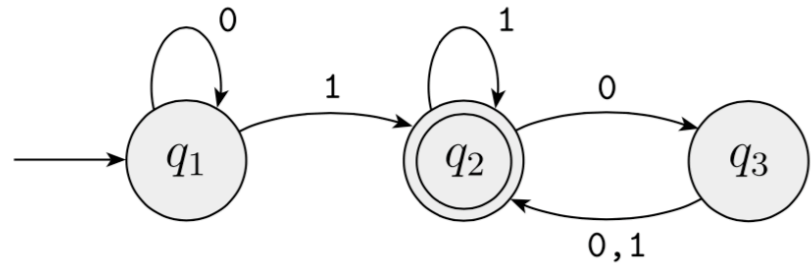
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

Finite state controller



What strings will M_1 accept

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2



1 0^*1 $0^*1 1^*$ $0^*1 1^*00$ $0^*1 1^*(00)^*$
 $0^*1 1^*(01)^*$



Definition of finite automaton

- Finite automaton is a 5-tuple $M=(Q,\Sigma,\delta,q_0,F)$
 - Q : finite set called states
 - Σ : finite set called the alphabet
 - $\delta: Q \times \Sigma \rightarrow Q$, transition function
 - $q_0 \in Q$: start state
 - $F \subseteq Q$: accept states
- Language on M : $L(M) = \{w \in \Sigma^* \mid \delta(q_0, w) \in F\}$



DFA example: definition - -> graph/language

- $M_2 = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_2\})$

Can anyone draw the DFA?

δ

	0	1
q_1	q_1	q_2
q_2	q_1	q_2

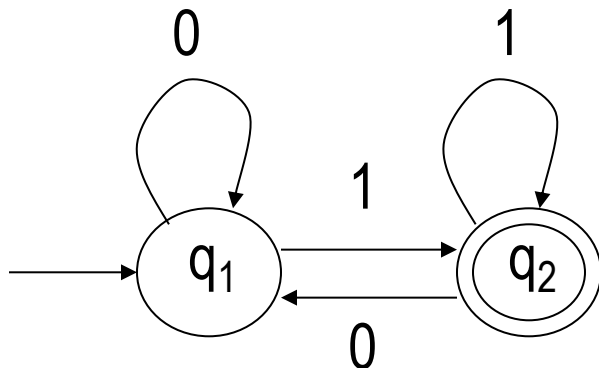
What is the language of M_2



DFA example: definition - -> graph/language

- $M_2 = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_2\})$

Can anyone draw the DFA?



δ

	0	1
q_1	q_1	q_2
q_2	q_1	q_2

What is the language of M_2

$$L(M_2) = \{ w \mid w \text{ ends with 1s} \}$$



DFA example: definition - -> graph/language

- $M_3 = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\})$

Can anyone draw the DFA?

	0	1
q_1	q_1	q_2
q_2	q_1	q_2

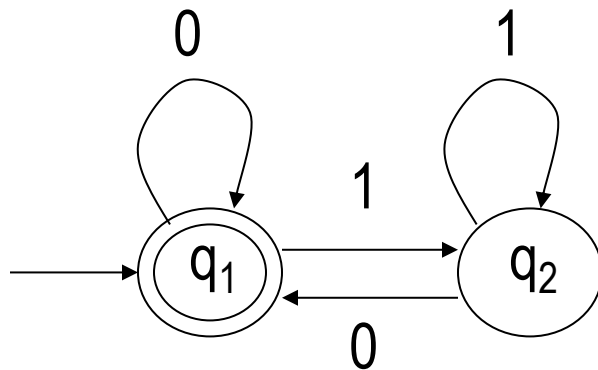
What is the language of M_3



DFA example: definition - -> graph/language

- $M_3 = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\})$

Can anyone draw the DFA?



	0	1
q_1	q_1	q_2
q_2	q_1	q_2

What is the language of M_3

$$L(M_3) = \{ w \mid w = \varepsilon \text{ or } w \text{ ends with } 0s \}$$



DFA example: definition - -> graph/language

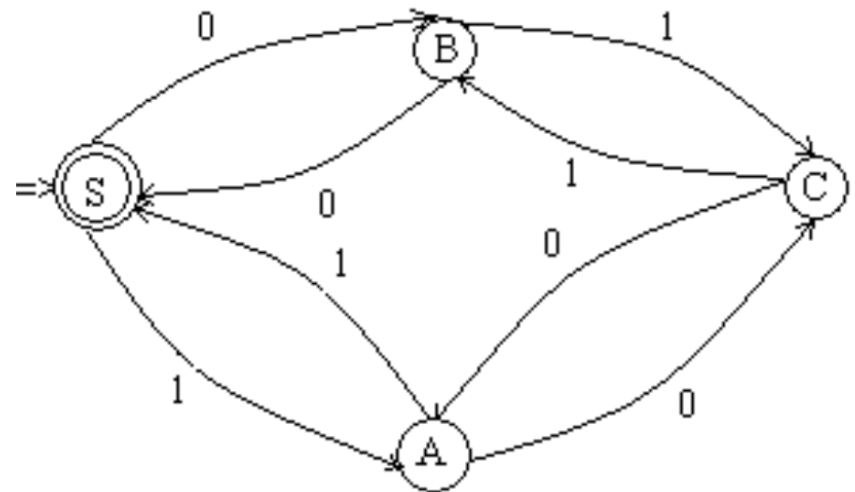
- $M = (\{S, A, B, C\}, \{0, 1\}, f, S, \{S\})$
 - $f(S, 0) = B, f(B, 0) = S$
 - $f(S, 1) = A, f(B, 1) = C$
 - $f(A, 0) = C, f(C, 0) = A$
 - $f(A, 1) = S, f(C, 1) = B$

Can anyone draw the DFA?



DFA example: definition - -> graph/language

- $M = (\{S, A, B, C\}, \{0, 1\}, f, S, \{S\})$
 - $f(S, 0) = B, f(B, 0) = S$
 - $f(S, 1) = A, f(B, 1) = C$
 - $f(A, 0) = C, f(C, 0) = A$
 - $f(A, 1) = S, f(C, 1) = B$



Can anyone draw the DFA?

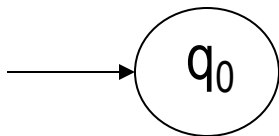
DFA example: definition - -> graph/language

- \emptyset
- $M = (\{q_0\}, \{\}, f, q_0, \{\})$



DFA example: definition - -> graph/language

- \emptyset
- $M = (\{q_0\}, \{\}, f, q_0, \{\})$



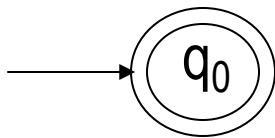
DFA example: definition - -> graph/language

- $\{\varepsilon\}$
- $M = (\{q_0\}, \{\}, f, q_0, \{q_0\})$



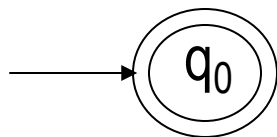
DFA example: definition - -> graph/language

- $\{\varepsilon\}$
- $M = (\{q_0\}, \{\}, f, q_0, \{q_0\})$



DFA example: definition - -> graph/language

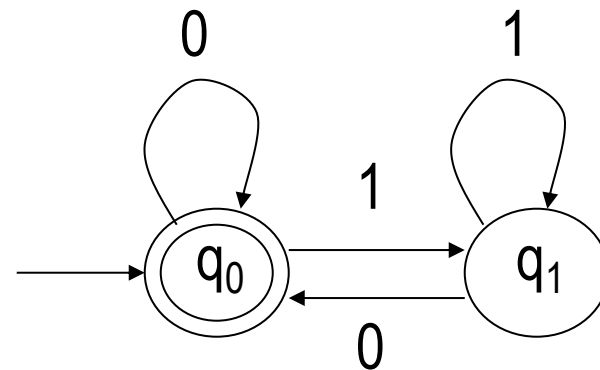
- $\{\varepsilon\}$
- $M = (\{q_0\}, \{\}, f, q_0, \{q_0\})$



$\{\varepsilon\}$

subgraph

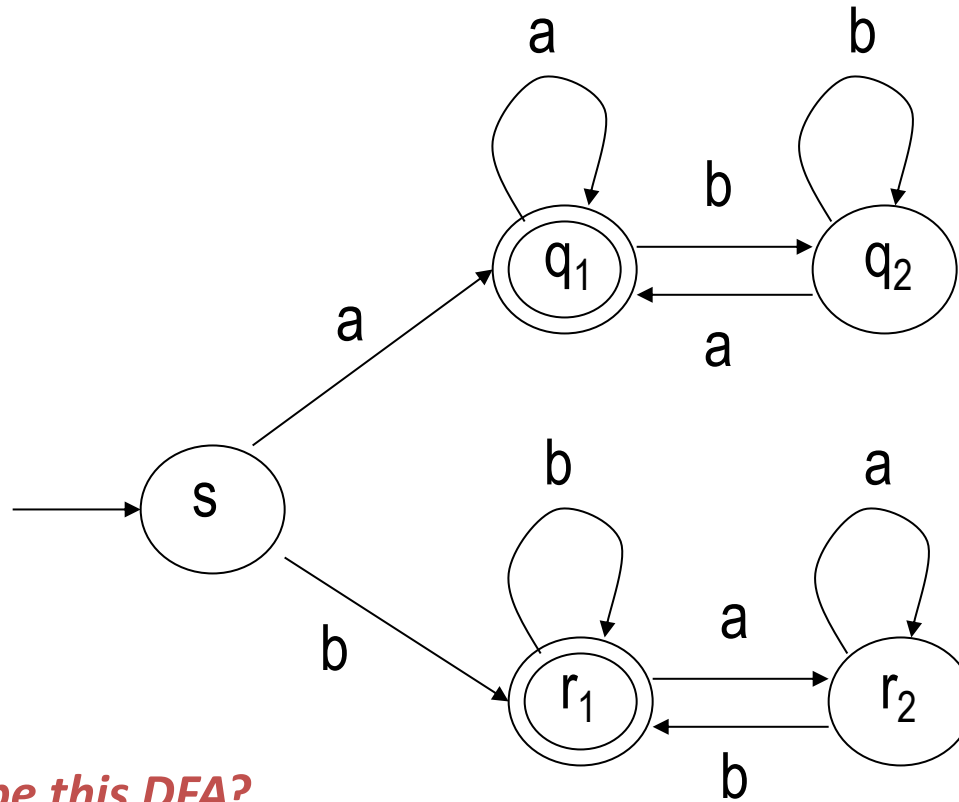
subset



$L(M_3) = \{ w \mid w = \varepsilon \text{ or } w \text{ ends with } 0s \}$



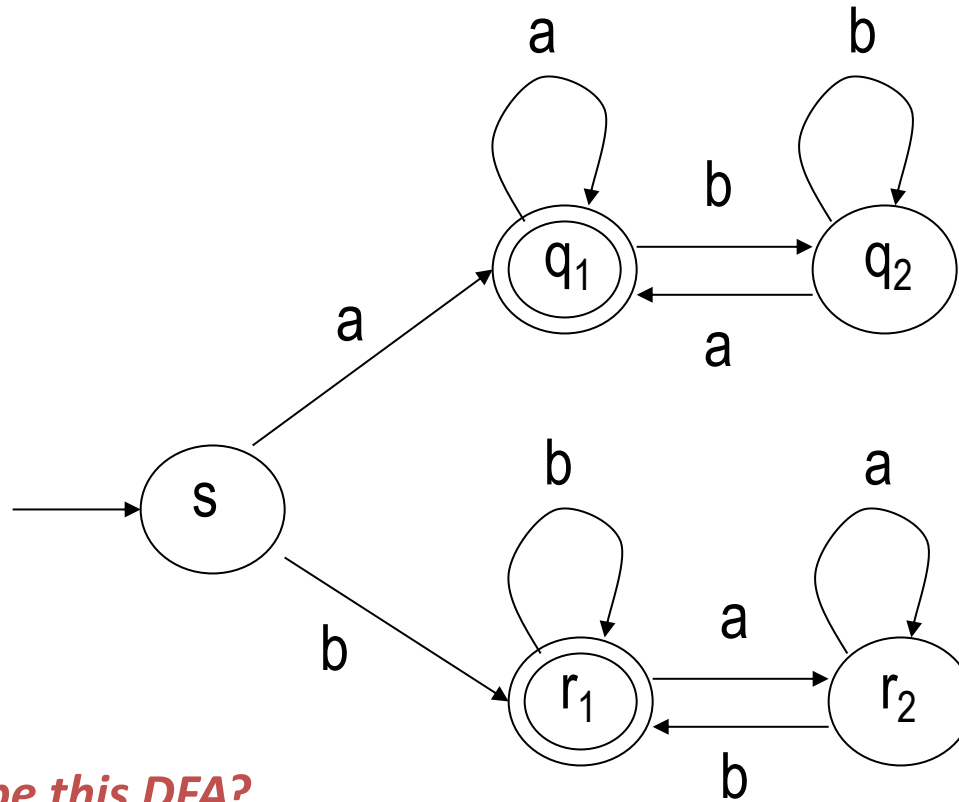
DFA example: graph- -> language



How to describe this DFA?



DFA example: graph- -> language

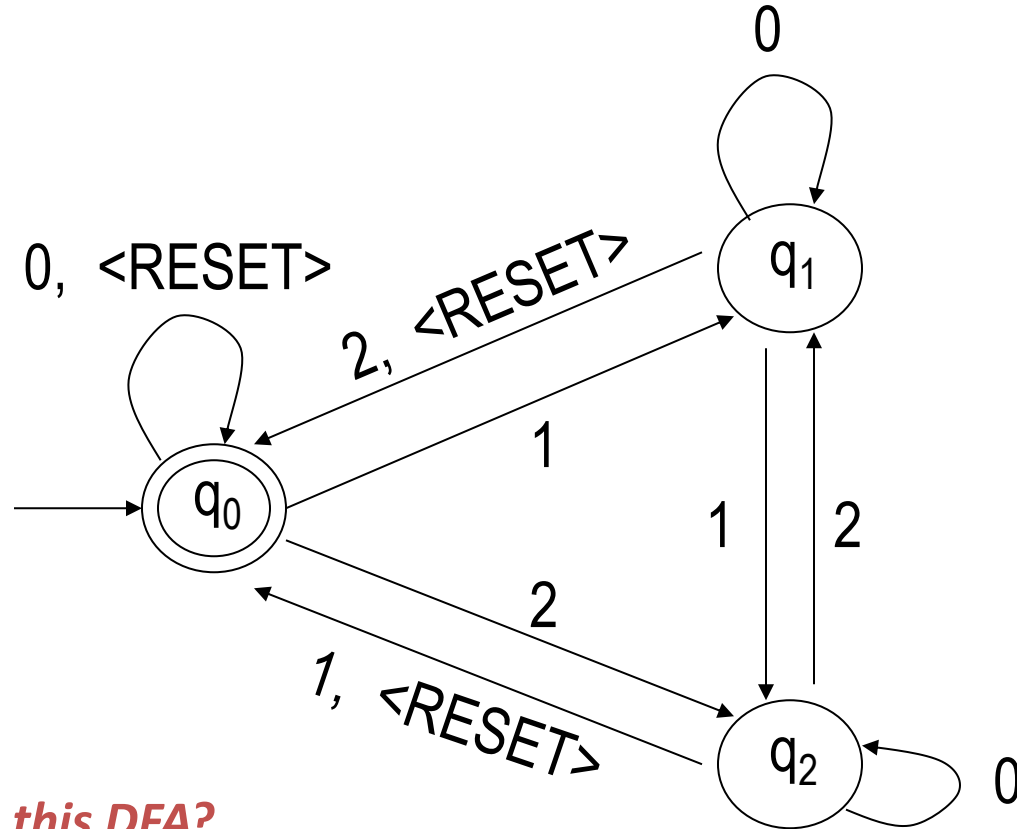


How to describe this DFA?

$$L(M_4) = \{ w \mid w \text{ starts and ends with the same letter} \}$$



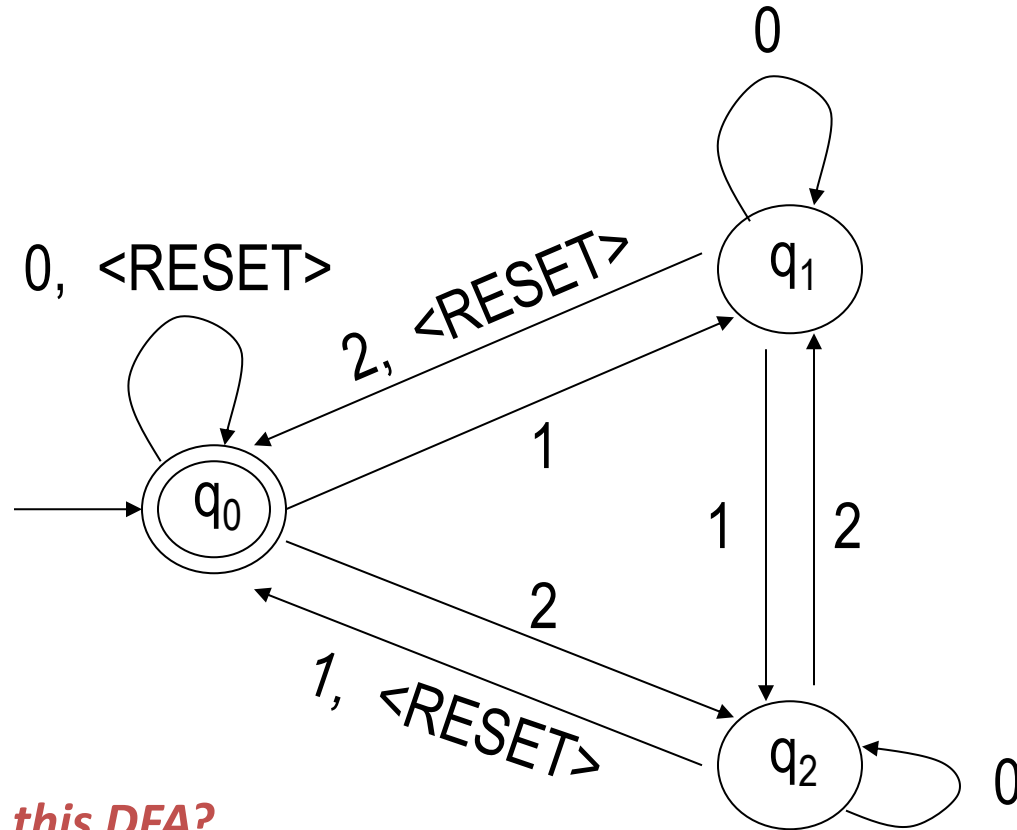
DFA example: graph- -> language



How to describe this DFA?



DFA example: graph- -> language



How to describe this DFA?

$L(M_5) = \{ w \mid \text{after the last } \langle \text{RESET} \rangle, \text{ the sum of } w \text{ is } 0 \text{ modulo } 3 \}$

Formal definition of computation for DFAs

- Finite automaton $M=(Q,\Sigma,\delta,q_0,F)$;
 - Input string $w=w_1w_2...w_n$;
- Computation: a sequence of states $r_0,r_1,..., r_n$;
 - $r_0=q_0$;
 - $\delta(r_i,w_{i+1})=r_{i+1}$; ($i=0,1,...,n-1$)
- Accept computation: M accepts w
 - $r_n \in F$;
- M recognizes language A :
 - $L(M)=\{ w \mid M \text{ accepts } w \}$

