CS 6041
Theory of Computation

Reducibility
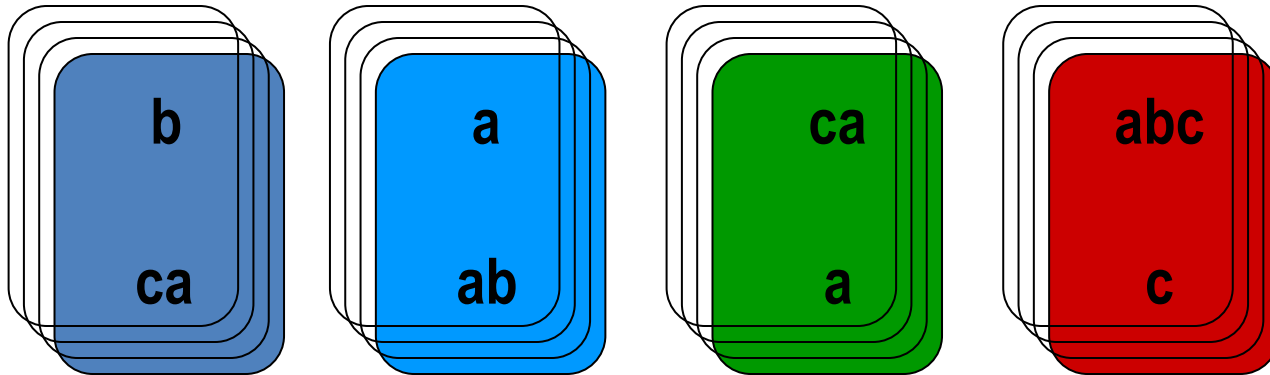
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Post Correspondence Problem (PCP)

b	a	ca	abc
ca	ab	a	c

We have limited categories of dominos

Post Correspondence Problem (PCP)

The number of domino in each category is unlimited
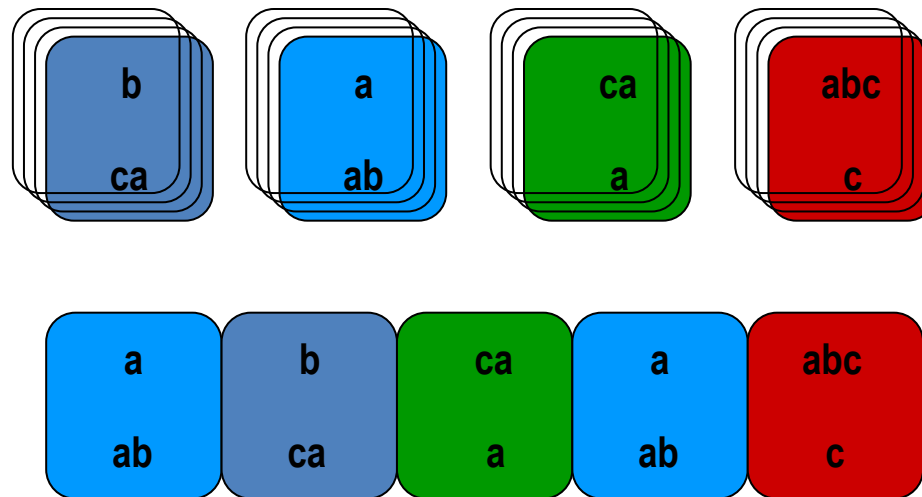
Post Correspondence Problem (PCP)

Match:
Upper string: abcaaabc
Bottom string: abcaaabc

Post Correspondence Problem (PCP)
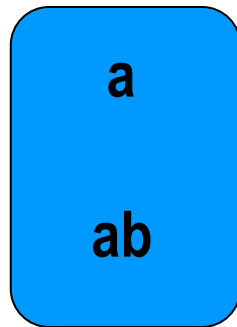
- Whether a collection of dominos has a match

- PCP = {⟨P⟩ | P is an instance of the Post Correspondence Problem with a match}.

Description of PCP

An individual domino

$$\left[\frac{a}{ab}\right]$$

Description of PCP

A collection of dominos

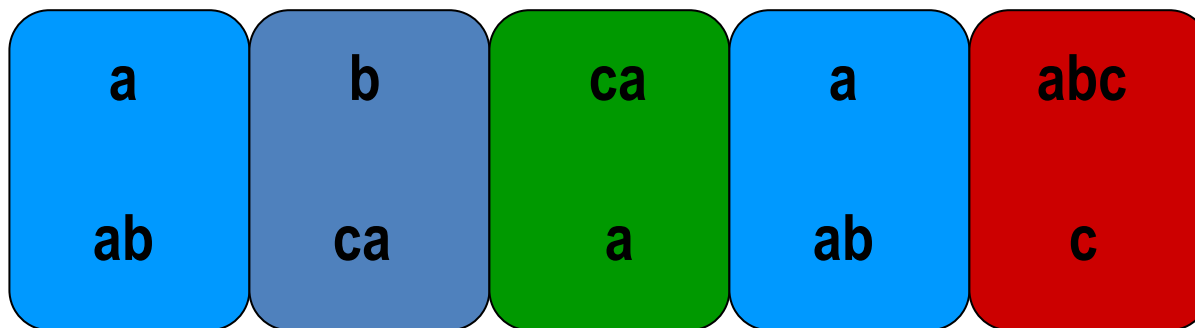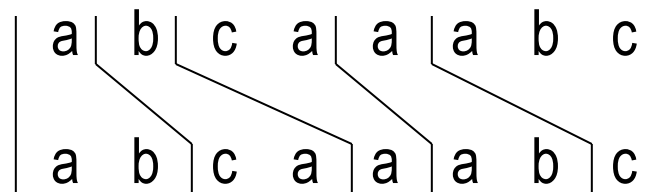$$\{\left[\frac{b}{ca}\right], \left[\frac{a}{ab}\right], \left[\frac{ca}{a}\right], \left[\frac{abc}{c}\right]\}$$

Description of PCP

A match

$$\left[\frac{a}{ab}\right] \left[\frac{b}{ca}\right] \left[\frac{ca}{a}\right] \left[\frac{a}{ab}\right] \left[\frac{abc}{c}\right]$$

a b c a a a b c
a b c a a a b c

a	b	ca	a	abc
ab	ca	a	ab	c

A collection without a match

- For a given collection

$$\left\{\left[\frac{abc}{ab}\right], \left[\frac{ca}{a}\right], \left[\frac{acc}{ba}\right]\right\}$$

- It cannot contain a match because every top string is longer than the corresponding bottom string
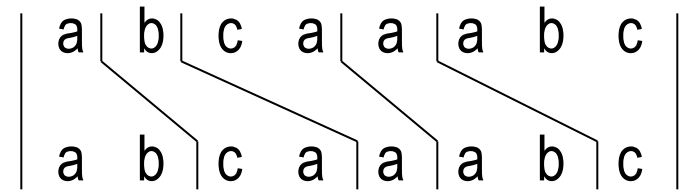
Theorem 5.15

- PCP is undecidable

- Proof:

To simplify the problem, we create ***Modified Post Correspondence Problem (MPCP)*** ,

MPCP = {⟨P⟩| P is an instance of the PCP with a match that starts with the first domino}.
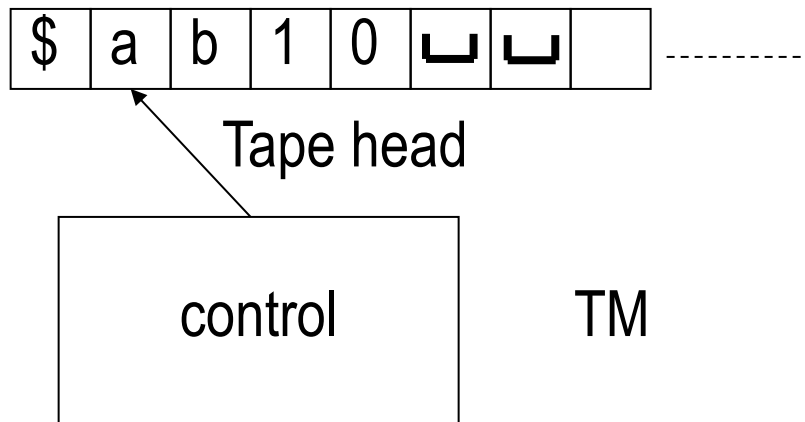
Theorem 5.15

$$\left[\frac{a}{ab}\right] \left[\frac{b}{ca}\right] \left[\frac{ca}{a}\right] \left[\frac{a}{ab}\right] \left[\frac{abc}{c}\right]$$

a	b	c	a	a	a	b	c
a	b	c	a	a	a	b	c

Input

$	a	b	1	0	⎵	⎵	

Tape head

control   TM

- The PCP program is very similar with the A_{TM} program

Theorem 5.15 proof

- Proof:

Suppose PCP is decidable

We construct TM S to decide A_{TM} (Theorem 4.11: A_{TM} is undecidable)

PCP

$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$

S constructs an instance of the PCP P that has a match iff

M accepts w: A_{TM}

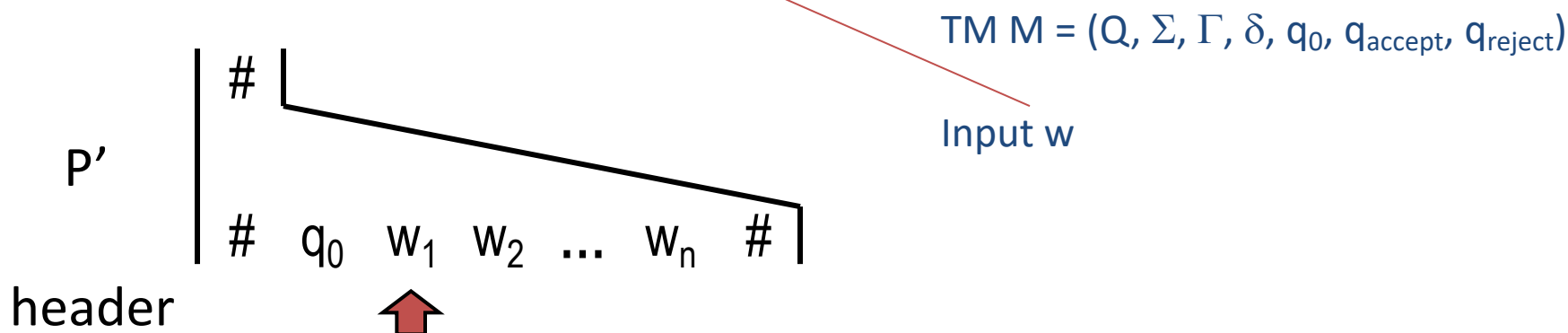- (1) S first constructs an instance P' of the MPCP;

- (2) Transfer P' into P;

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.1) Generating beginning configuration

    Put $\left[\dfrac{\#}{\# q_0 w_1 w_2 \dots w_n \#} \right]$ into P' as $\left[\dfrac{t_1}{b_1} \right]$
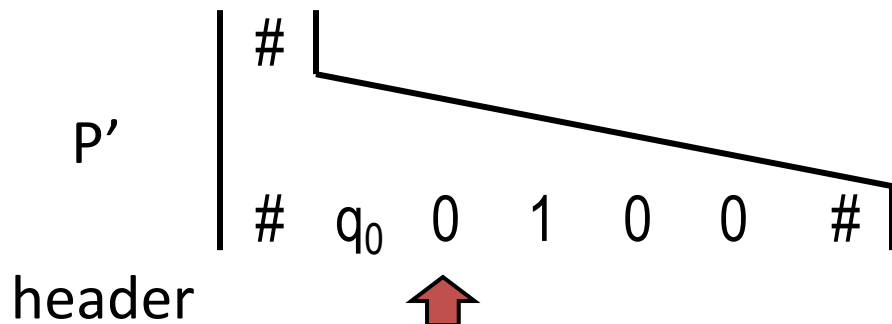
TM M = (Q, Σ, Γ, δ, q_0, q_{accept}, q_{reject})

Input w

P'

$\#$

header $\#$ q_0 w_1 w_2 ... w_n $\#$

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.1) Generating beginning configuration

    Suppose $\Gamma = \{0, 1, 2, \_\}$, w=0100,

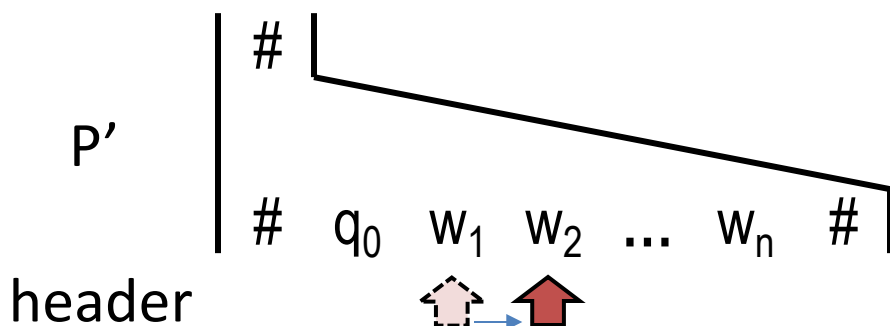   Put the following into P' : $\left[\dfrac{\#}{\#q_0 0100\#}\right] = \left[\dfrac{t_1}{b_1}\right]$

P'

#

q_0 0 1 0 0

header

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.2) The head move to the right

    For each a, b$\in \Gamma$ and q, r$\in$Q, where q $\neq$ q$_{reject}$,

    if δ**(q, a) = (r, b, R),** then put $\left[\dfrac{qa}{br}\right]$ into P'

P'

\#

\#   q$_0$ w$_1$ w$_2$ ...   w$_n$ \#

header

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.2) The head moves to the right

  For each a, b$\in\Gamma$ and q, r$\in$Q, where q $\neq$ q$_{reject}$,

  if δ(q$_0$, 0) = (q$_7$, 2, R), then put $\left[\dfrac{q_0 0}{2 q_7}\right]$ into P'

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.3) The head moves to the left

    Discuss it later on.

$$\text{P'} \quad \begin{array}{|c|c c|}\hline \# & q_0 \; 0 \\ \hline \# \; q_0 & 0 \; 1 \; 0 \; 0 \; \# & 2 \; q_7 \\ \hline \end{array}$$

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.4) For each a$\in\Gamma$,

  put $\left[\dfrac{a}{a}\right]$ into P'

P'

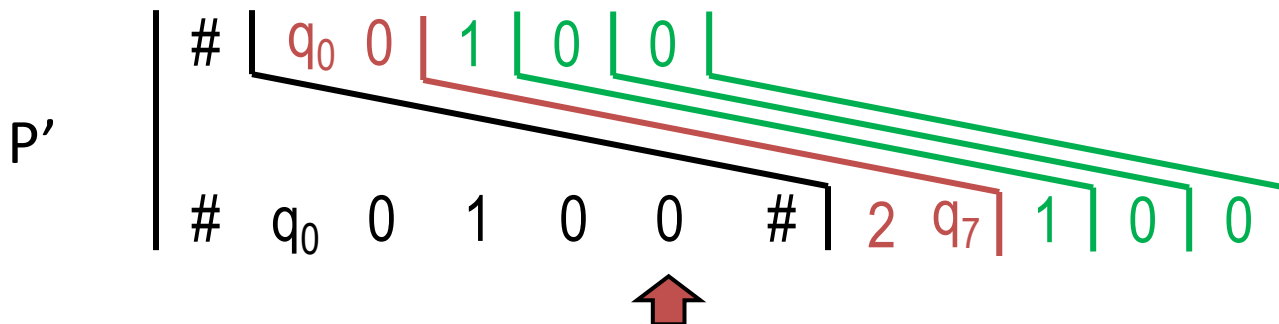| #	q_0 0
# q_0 0 1 0 0 #	2 q_7

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.4) Γ = {0, 1, 2, ␣},

  put $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} ␣ \\ ␣ \end{bmatrix}$ into P'
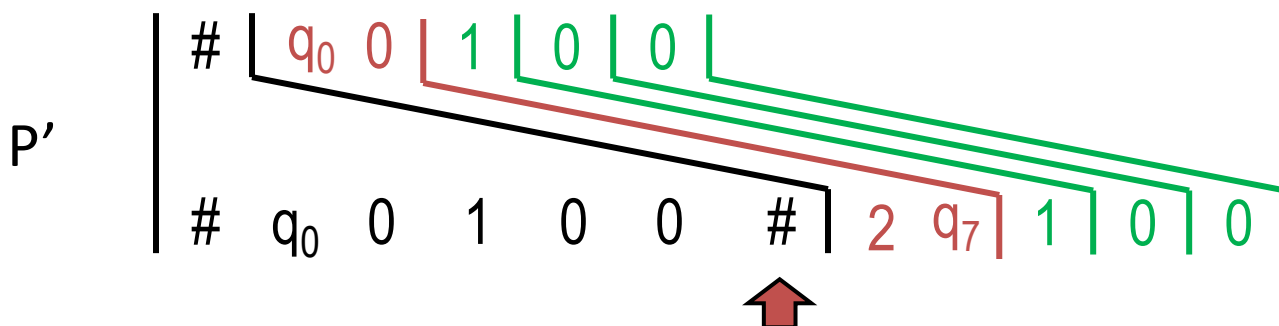
P'

#	q_0 0

| # | q_0 | 0 | 1 | 0 | 0 | # | 2 q_7 |

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.4) $\Gamma = \{0, 1, 2, \text{\textvisiblespace}\}$,

  put $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} \text{\textvisiblespace} \\ \text{\textvisiblespace} \end{bmatrix}$ into P'

P'

#	q_0 0	1

| # | q_0 | 0 | 1 | 0 | 0 | # | 2 | q_7 | 1 |

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.4) Γ = {0, 1, 2, ␣},

    put $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} ␣ \\ ␣ \end{bmatrix}$ into P'

Theorem 5.15 proof

- Proof:

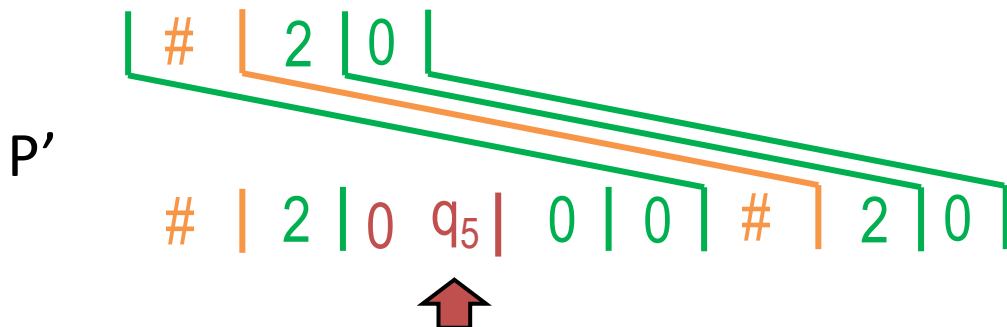(1)   S first constructs an instance P' of the MPCP;

(1.4) $\Gamma = \{0, 1, 2, \_\}$,

put $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} \_ \\ \_ \end{bmatrix}$ into P'

P'

#	q_0 0	1	0	0

| # | q_0 | 0 | 1 | 0 | 0 | # | 2 q_7 | 1 | 0 | 0 |

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.5) Copy # and Put ␣ at the end of configuration

  put $\left[\dfrac{\#}{\#}\right]$ and $\left[\dfrac{\#}{␣\,\#}\right]$ into P'

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.5) Copy # and Put ␣ at the end of configuration

  put $\left[\dfrac{\#}{\#}\right]$ and $\left[\dfrac{\#}{␣\#}\right]$ into P'

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.5) Copy # and Put ⌣ at the end of configuration

  put $\begin{bmatrix} \# \\ \# \end{bmatrix}$ and $\begin{bmatrix} \# \\ \cup\# \end{bmatrix}$ into P'

Theorem 5.15 proof

- Proof:

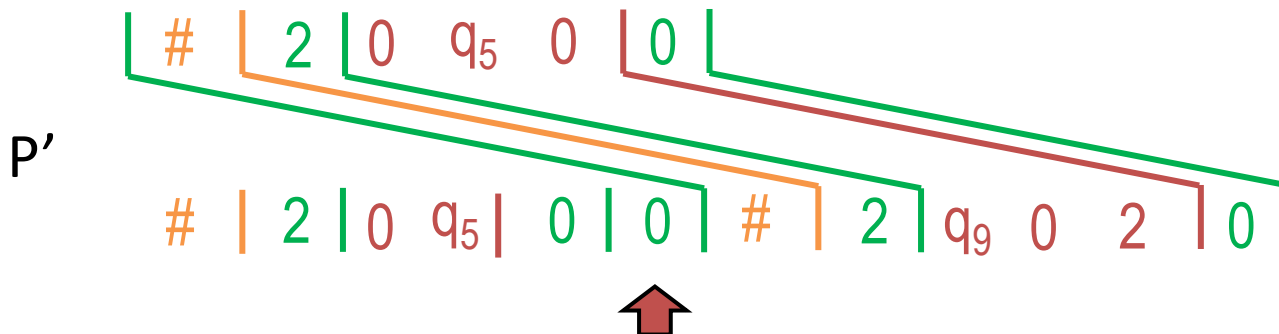 (1)   S first constructs an instance P' of the MPCP;

  (1.5) Copy # and Put ␣ at the end of configuration

  put $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{␣\#}\right]$ into P'

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.5) Copy # and Put ⌣ at the end of configuration

    suppose $\delta(q_7, 1) = (q_5, 0, R)$, then put $\left[\frac{q_7 1}{0 q_5}\right]$ into P'

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.5) Copy # and Put ␣ at the end of configuration

    keep putting $\left[\frac{0}{0}\right]$, $\left[\frac{1}{1}\right]$, $\left[\frac{2}{2}\right]$, $\left[\frac{␣}{␣}\right]$, $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{␣\#}\right]$ into P'

P'
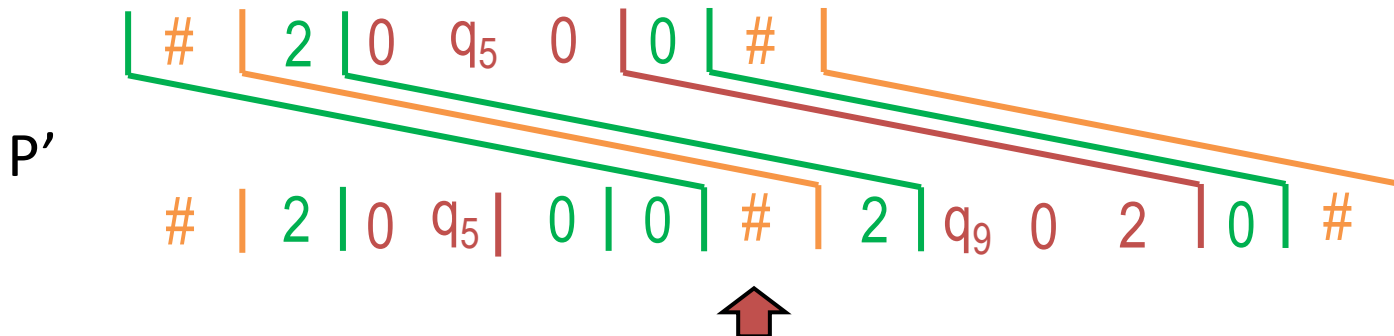
Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.3) The head moves to the left

  For each a, b, c$\in\Gamma$ and q, r$\in$Q, where q $\neq$ q$_{reject}$,

  if δ**(q, a) = (r, b, L),** then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

P'

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.3) The head moves to the left

    Suppose $\delta(q_5, 0) = (q_9, 2, L)$, remove the old 0

    if $\delta(q, a) = (r, b, L)$, then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

P'

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.3) The head moves to the left

  Suppose $\delta(q_5, 0) = (q_9, 2, L)$, remove the old 0

  if $\delta(q, a) = (r, b, L)$, then put $\left[\frac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

  P'

  $\#\ |\ 2\ |\ 0\ q_5|\ 0\ |\ 0\ |\ \#\ |\ 2\ |$

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.3) The head moves to the left

  Suppose $\delta(q_5, 0) = (q_9, 2, L)$, put $\left[\dfrac{0q_50}{q_902}\right]$ into P'

  if $\delta(q, a) = (r, b, L)$, then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

P'

$$\# \quad 2 \quad 0 \quad q_5 \quad 0$$

$$\# \mid 2 \mid 0 \; q_5 \mid 0 \mid 0 \mid \# \mid 2 \mid q_9 \; 0 \quad 2$$

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.3) The head moves to the left

  keep putting $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} \sqcup \\ \sqcup \end{bmatrix}$, $\begin{bmatrix} \# \\ \# \end{bmatrix}$ and $\begin{bmatrix} \# \\ \sqcup \# \end{bmatrix}$ into P'

  # | 2 | 0 q_5 0

P'

  # | 2 | 0 q_5 | 0 | 0 | # | 2 | q_9 0 2

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.3) The head moves to the left

  keep putting $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} \_ \\ \_ \end{bmatrix}$, $\begin{bmatrix} \# \\ \# \end{bmatrix}$ and $\begin{bmatrix} \# \\ \_\# \end{bmatrix}$ into P'

P'

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.3) The head moves to the left

  keep putting $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} \sqcup \\ \sqcup \end{bmatrix}$, $\begin{bmatrix} \# \\ \# \end{bmatrix}$ and $\begin{bmatrix} \# \\ \sqcup \# \end{bmatrix}$ into P'

P'

Theorem 5.15 proof

- Proof:

(1)   S first constructs an instance P' of the MPCP;

keep putting something into P' until M halts:

  ▸ if M rejects, S also rejects, means no match in PCP;

  ▸ if M accepts, add something to the top to match the bottom.

P'   ......   # | 2 | 1

2 1 q$_{accept}$ 0 2 # | 2 | 1

Connect PCP match program with whether M accepts w

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.6) For a $\in \Gamma$,

$$\text{put} \left[\frac{aq_{accept}}{q_{accept}}\right] \text{ and } \left[\frac{q_{accept}a}{q_{accept}}\right] \text{ into P'}$$

P'   ......

2 1

2 1 q_{accept} 0 2 # 2 1

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.6) For $a \in \Gamma$,

$$\text{put } \left[\frac{a q_{accept}}{q_{accept}} \right] \text{ and } \left[\frac{q_{accept} a}{q_{accept}} \right] \text{ into P'}$$

P'   ......

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.6) For a$\in\Gamma$,

  put $\left[\dfrac{aq_{accept}}{q_{accept}}\right]$ and $\left[\dfrac{q_{accept}a}{q_{accept}}\right]$ into P'

P'   ......

$$\# \mid 2 \mid 1 \mid q_{accept} \quad 0$$

$$\# \ 2 \ 1 \ q_{accept} \quad 0 \ 2 \ \# \mid 2 \mid 1 \mid q_{accept}$$

Difference between top and bottom is reduce by 1

Theorem 5.15 proof

- Proof:

    (1)   S first constructs an instance P' of the MPCP;

    (1.6) For a$\in\Gamma$,

$$\text{put} \left[\frac{aq_{accept}}{q_{accept}}\right] \text{ and } \left[\frac{q_{accept}a}{q_{accept}}\right] \text{ into P'}$$

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.6) For a$\in \Gamma$,

  put $\left[\dfrac{a q_{accept}}{q_{accept}} \right]$ and $\left[\dfrac{q_{accept}\, a}{q_{accept}} \right]$ into P'

P'   ......

Theorem 5.15 proof

- Proof:

   (1)   S first constructs an instance P' of the MPCP;

   (1.6) For a∈Γ,

   Keep doing until the top and bottom difference is 1.

P'

Theorem 5.15 proof

- Proof:

  (1)   S first constructs an instance P' of the MPCP;

  (1.7) Finish the match

  put $\left[\dfrac{q_{accept}\#\#}{\#}\right]$ into P'

P'

Theorem 5.15 proof

- Proof:

(2) Transfer P' into P

P' is MPCP = {⟨P⟩| P is an instance of the PCP with a match that starts with the first domino}.

Suppose $u = u_1 u_2 \ldots u_n$ to be any string of length n, define

  ▸ $\star u = *u1*u2 *\ldots*u3$

  ▸ $u\star = u_1*u_2 *\ldots*u_3*$

  ▸ $\star u\star = *u_1*u_2 *\ldots*u_3*$

Theorem 5.15 proof

$$\star u = *u1*u2 *\ldots*u3$$
$$u\star = u_1*u_2 *\ldots*u_3*$$
$$\star u\star = *u_1*u_2 *\ldots*u_3*$$

- Proof:

(2) Transfer P' into P

If P' $= \left\{ \left[\dfrac{t_1}{b_1}\right], \left[\dfrac{t_2}{b_2}\right], \ldots, \left[\dfrac{t_k}{b_k}\right] \right\}$, then let

P $= \left\{ \left[\dfrac{\star t_1}{\star b_1 \star}\right], \left[\dfrac{\star t_2}{b_2 \star}\right], \left[\dfrac{\star t_3}{b_3 \star}\right], \ldots, \left[\dfrac{\star t_k}{b_k \star}\right], \left[\dfrac{*\Delta}{\Delta}\right] \right\}$

> First domino: The only element as beginning

The match in P must be in shape as $\left[\dfrac{\star t_1}{\star b_1 \star}\right] \ldots \left[\dfrac{*\Delta}{\Delta}\right]$

Theorem 5.15 proof

- Proof:

Suppose PCP is decidable

We construct TM S using PCP match or not to decide A_{TM}

- (1) S first constructs an instance P' of the MPCP;

- (2) Transfer P' into P;

Theorem 4.11: A_{TM} is undecidable. Contradiction!

The suppose is wrong. Thus PCP is undecidable.

Example of TM: B={ w#w | w∈{0,1}* }

Example of PCP to A$_{TM}$

- ## M = (Q,Σ,Γ,δ,q$_0$,q$_{accept}$,q$_{reject}$)

  - Q= {q0, q1, q2}

  - Σ = {0, 1, x, #},

  - Γ = {0, 1, x, #, B}

  - q0={q1}

  - q$_{accept}$ = q$_{accept}$

  - q$_{reject}$ = all not q$_{accept}$

Example of PCP to A_TM , input is 1#1

Example of PCP to A_{TM} , input is 1#1

Put $\left[\dfrac{\#}{\#q_0 w_1 w_2 ... w_n \#}\right]$ into P' as $\left[\dfrac{t_1}{b_1}\right]$
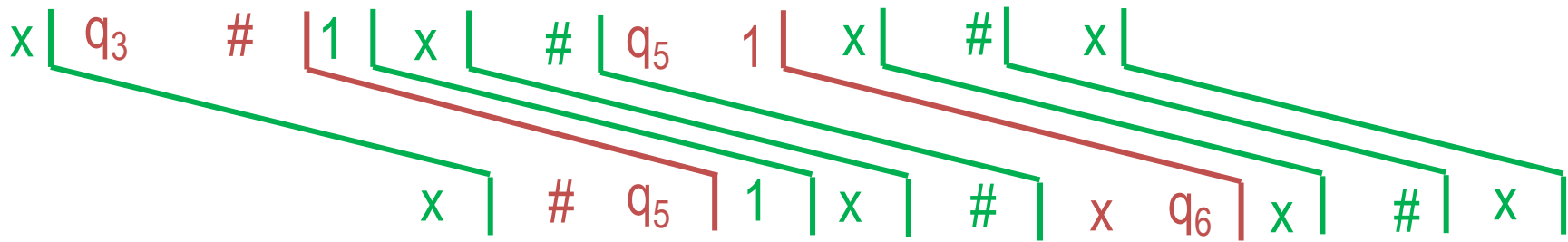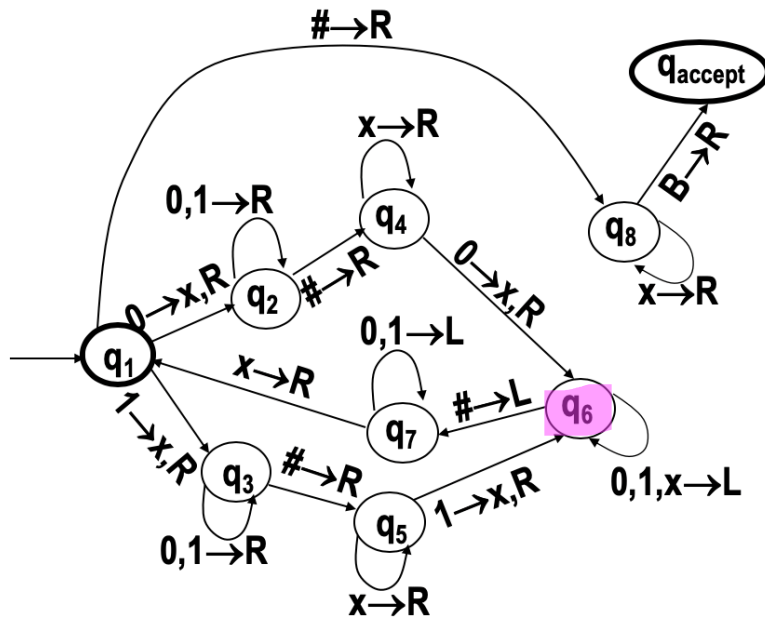
P'

header
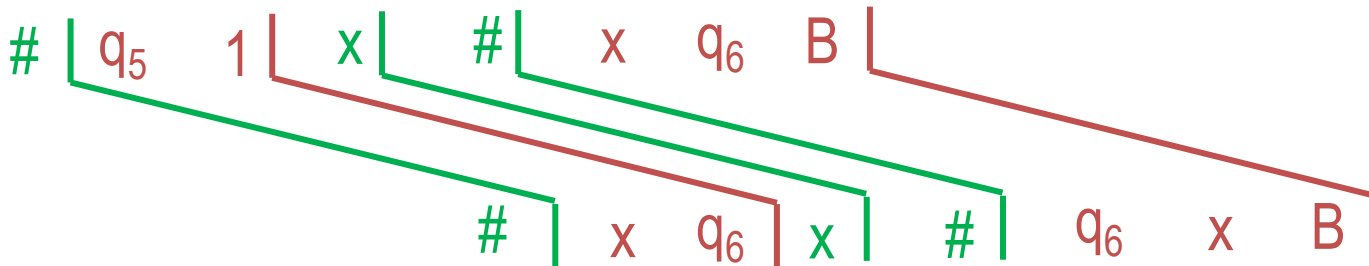
Example of PCP to A$_{TM}$, input is x#1

if δ(q, a) = (r, b, R),
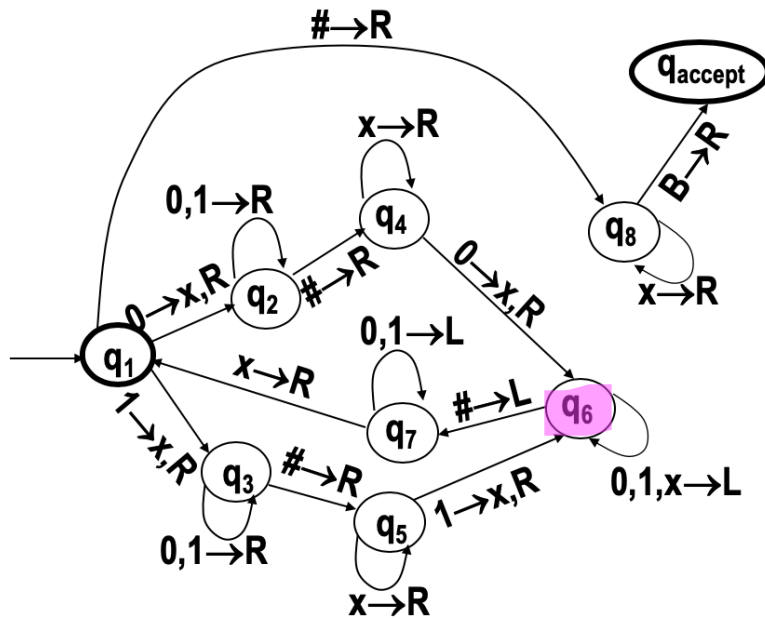
then put $\left[\dfrac{qa}{br}\right]$ into P'

P'

header

Example of PCP to A$_{TM}$, input is x#1

if δ(q, a) = (r, b, R),
then put $\left[\dfrac{qa}{br}\right]$ into P'
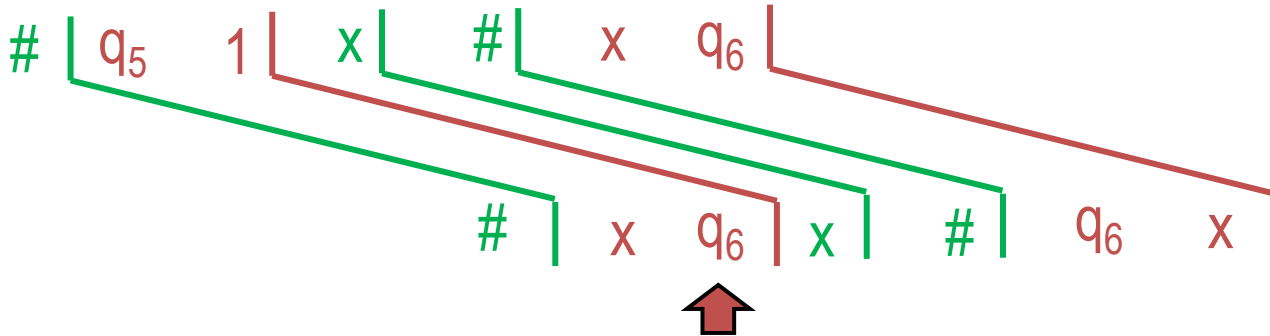
Example of PCP to A_{TM} , input is x#1

#→R

q_{accept}

x→R

B→R

0,1→R

q_4

q_8

x→R

0→x,R

0→x,R

q_2

#→R

0,1→L

q_1

x→R

#→L

q_6

1→x,R

q_7

q_3

#→R

0,1,x→L

0,1→R

q_5

1→x,R

x→R

if δ(q, a) = (r, b, R),

then put $\left[\dfrac{qa}{br}\right]$ into P'

P'

| # | q_0 1 | # | 1 |

| # q_1 1 # 1 | x q_3 | # | 1 |

header

Example of PCP to A$_{TM}$, input is x#1

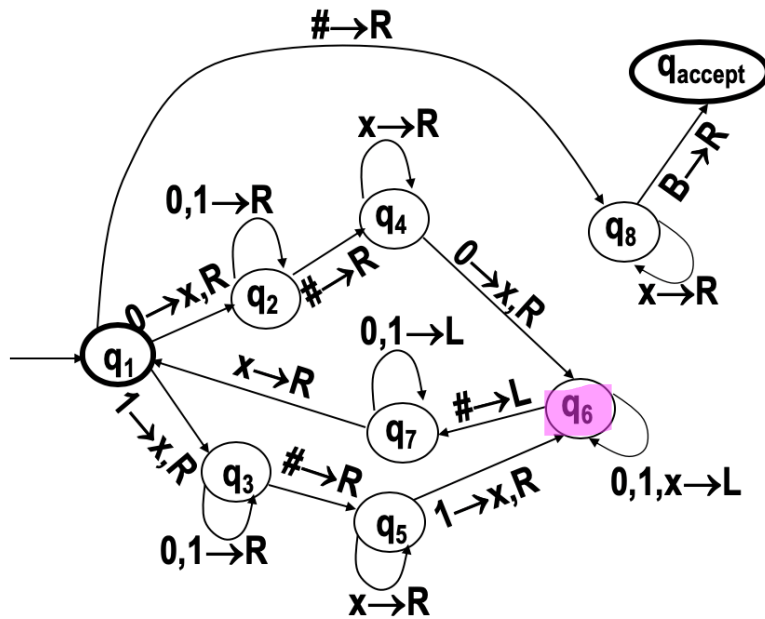if δ(q, a) = (r, b, R),

then put $\left[\frac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#1

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#1

if δ(q, a) = (r, b, R),

then put $\left[\dfrac{qa}{br}\right]$ into P'

P'

header

if δ(q, a) = (r, b, R),

then put $\left[\dfrac{qa}{br}\right]$ into P'

P'

header

Example of PCP to A$_{TM}$, input is x#1

if δ(q, a) = (r, b, R),
then put $\left[\dfrac{qa}{br}\right]$ into P'

P'

header

Example of PCP to A_{TM} , input is x#1

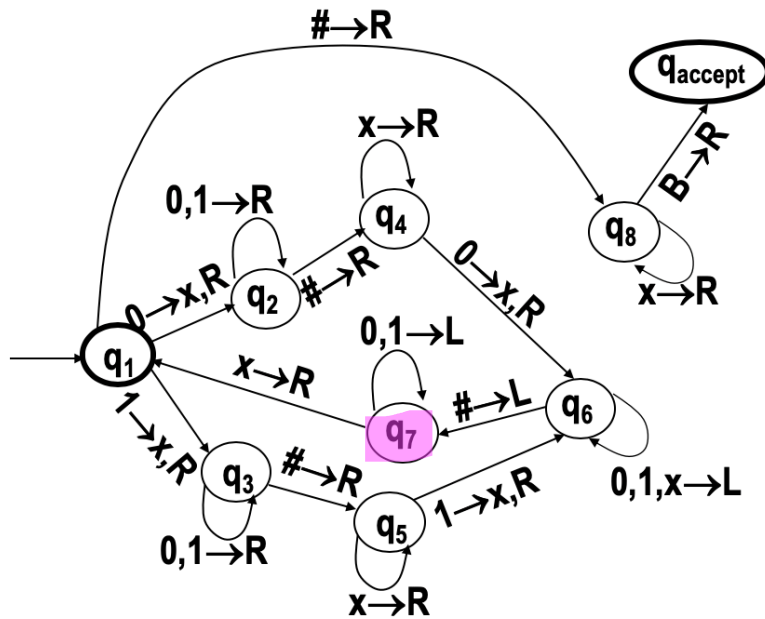if δ(q, a) = (r, b, R),
then put $\left[\frac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#1

if δ(q, a) = (r, b, R),

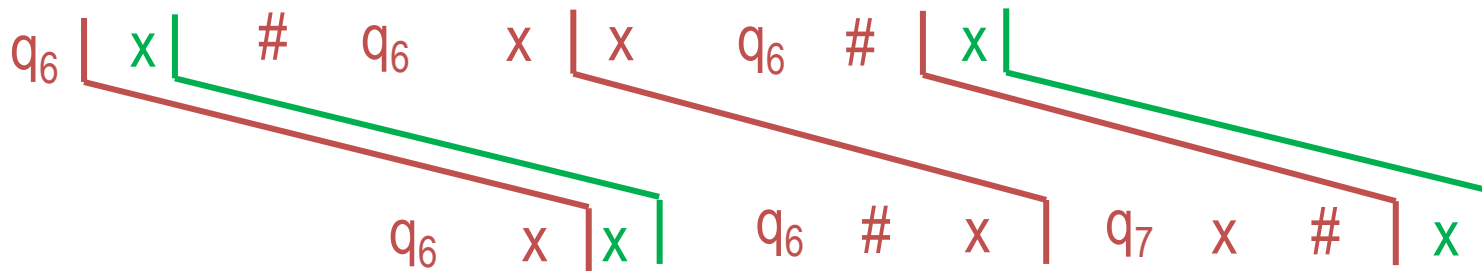then put $\left[\dfrac{qa}{br}\right]$ into P'

P'

header

Example of PCP to A_TM , input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'
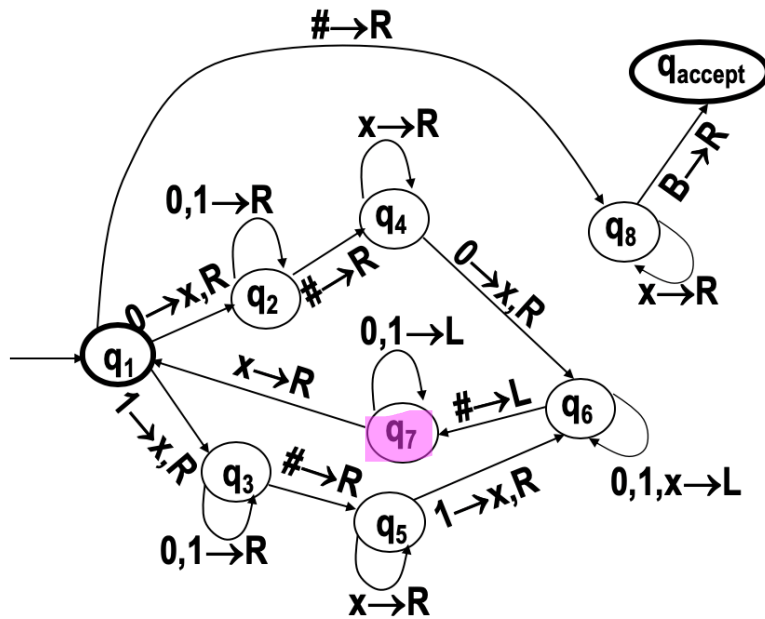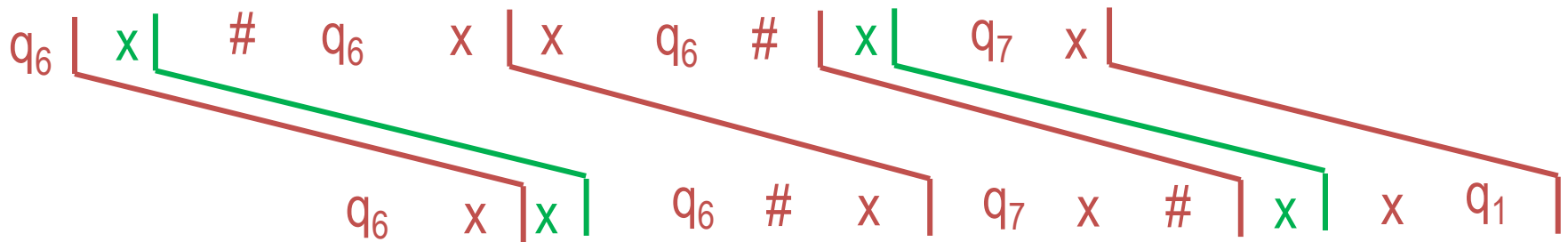
Example of PCP to A_{TM} , input is x#x

if δ(q, a) = (r, b, R),
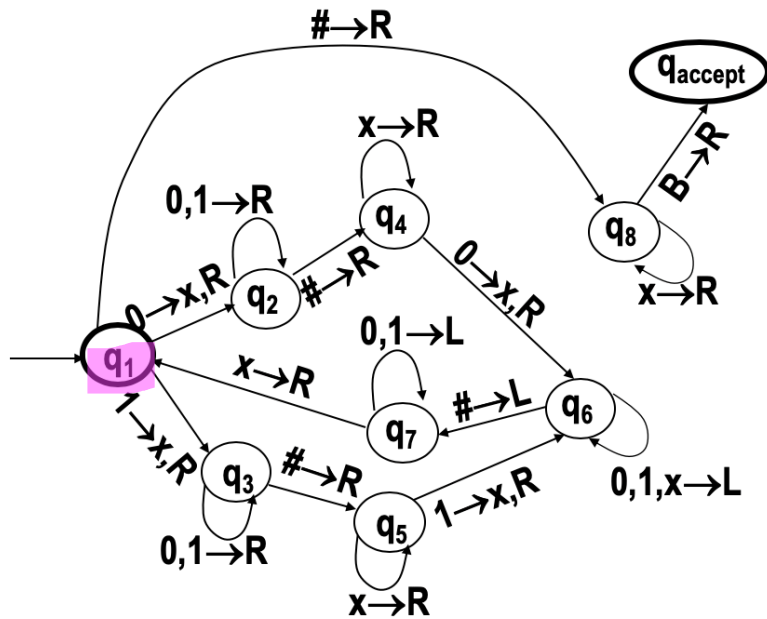
then put $\left[\frac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

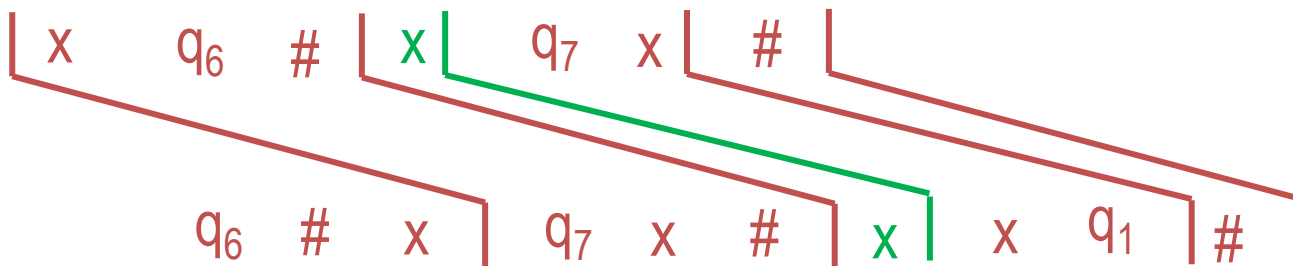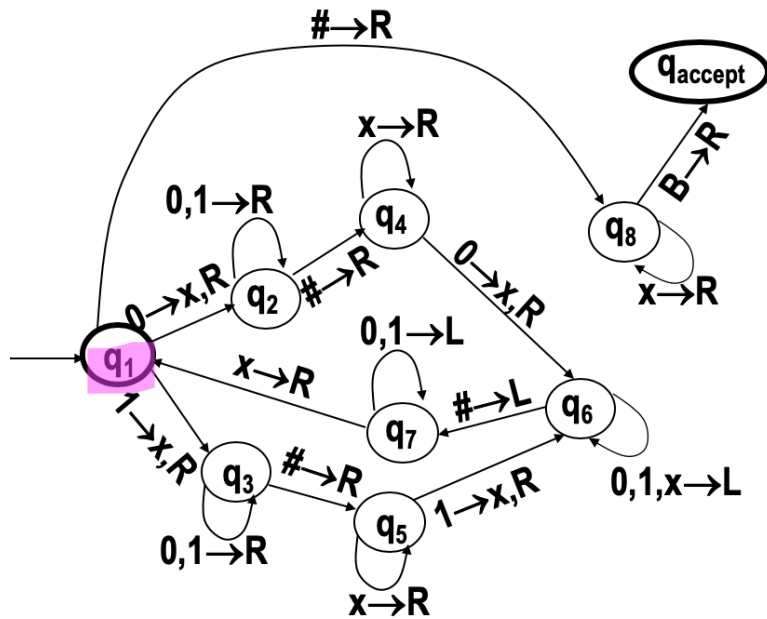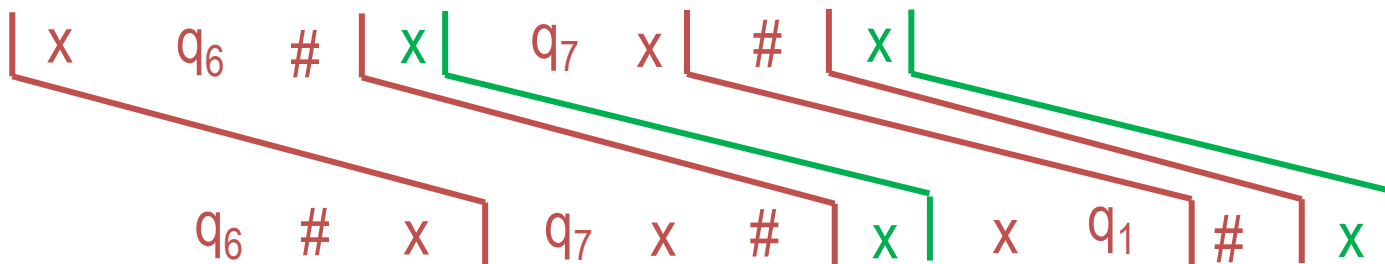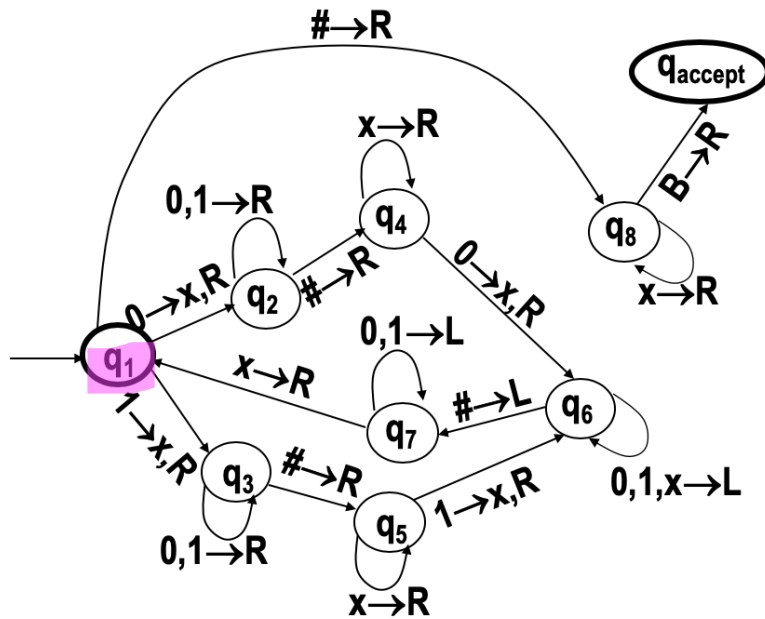

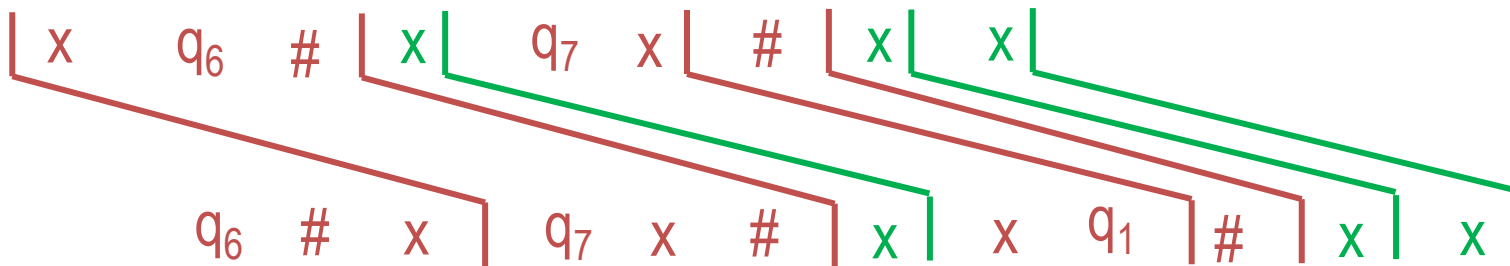if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_TM , input is x#x

if $\delta(q, a) = (r, b, L)$, then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

Example of PCP to A$_{TM}$, input is x#x

if δ(q, a) = (r, b, L), then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

Example of PCP to A$_{TM}$, input is x#x

if δ(q, a) = (r, b, L), then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

if $\delta(q, a) = (r, b, L)$, then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

Example of PCP to A$_{TM}$, input is x#x

if $\delta(q, a) = (r, b, L)$, then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q
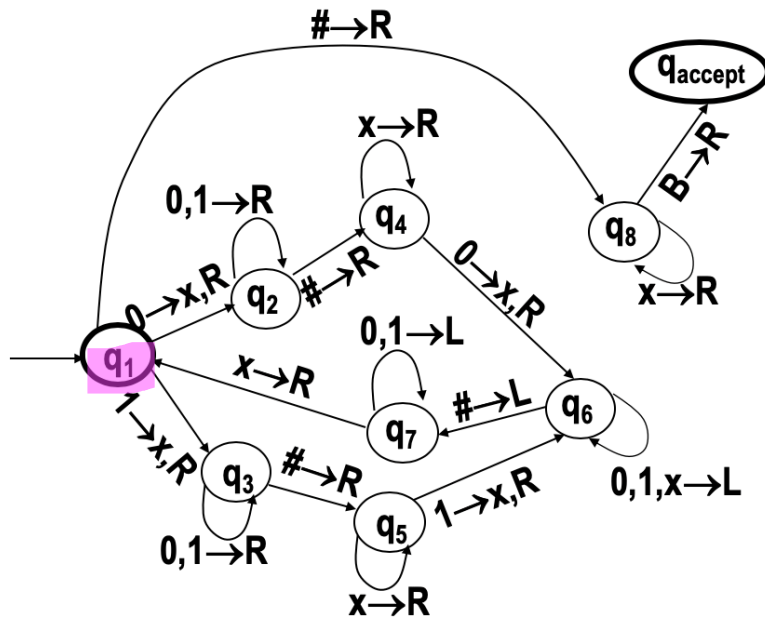
Example of PCP to A$_{TM}$, input is x#x

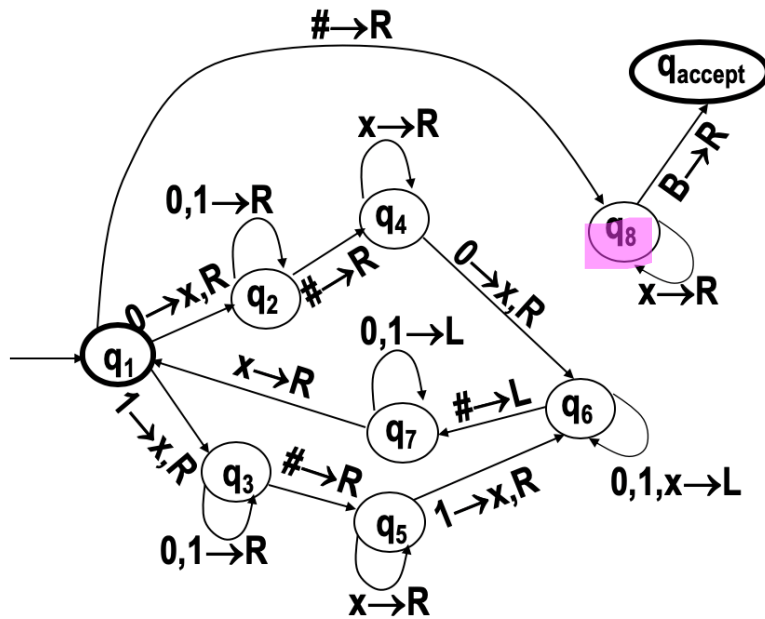if $\delta(q, a) = (r, b, L)$, then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, L)$, then put $\left[\dfrac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

q_4 $x\to R$

$0,1\to R$

q_{accept}

$B\to R$

q_8

$x\to R$

$\#\to R$

$0\to x,R$

$0\to x,R$

q_2 $\#\to R$

$0,1\to L$

q_1 $x\to R$

q_7 $\#\to L$ q_6

$1\to x,R$

q_3 $\#\to R$

$0,1\to R$

q_5 $1\to x,R$

$0,1,x\to L$

$x\to R$

q_6 | x | # q_6 x | x q_6 #

q_6 x | x q_6 # x | q_7 x #

Example of PCP to A_{TM} , input is x#x

if δ(q, a) = (r, b, R),

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,
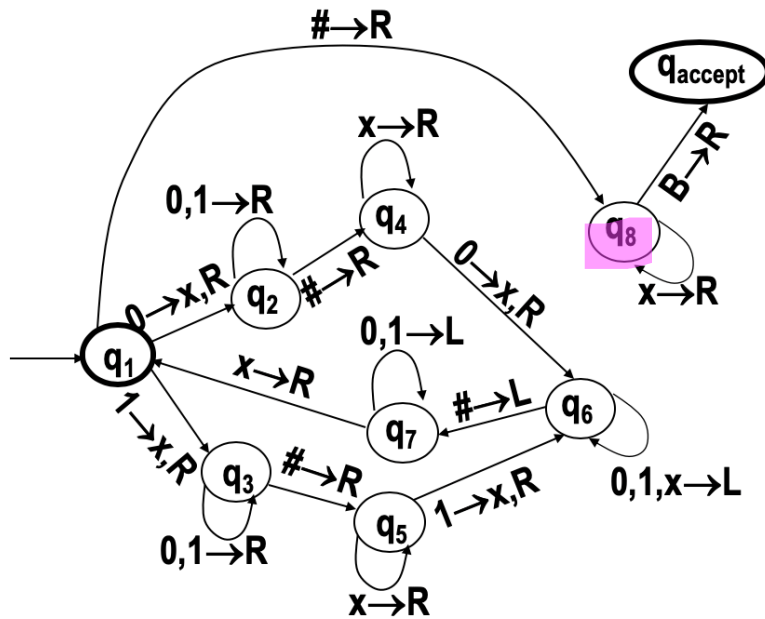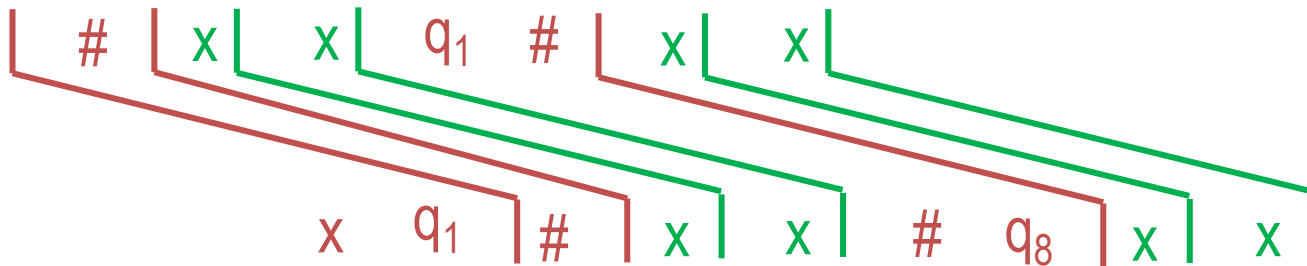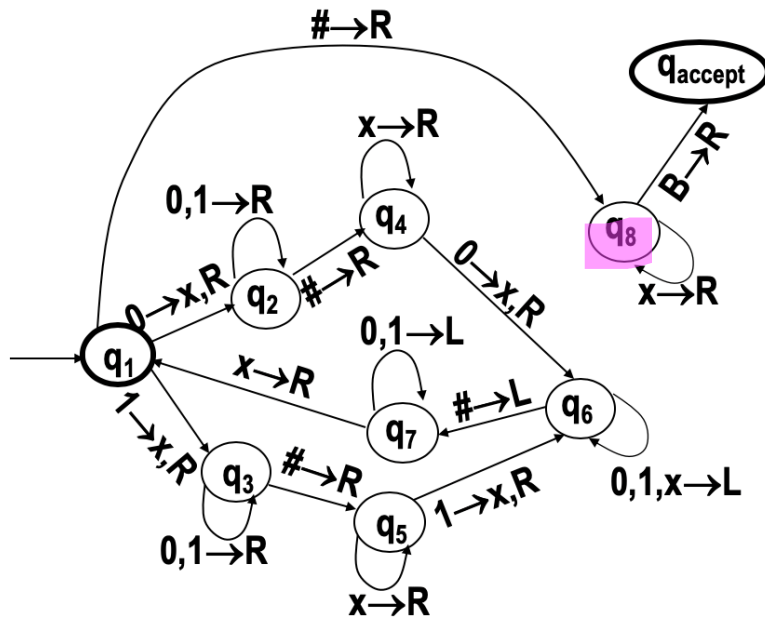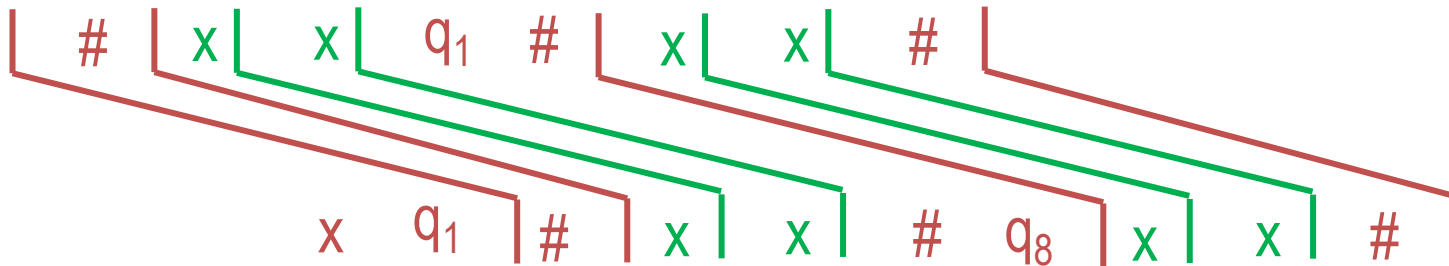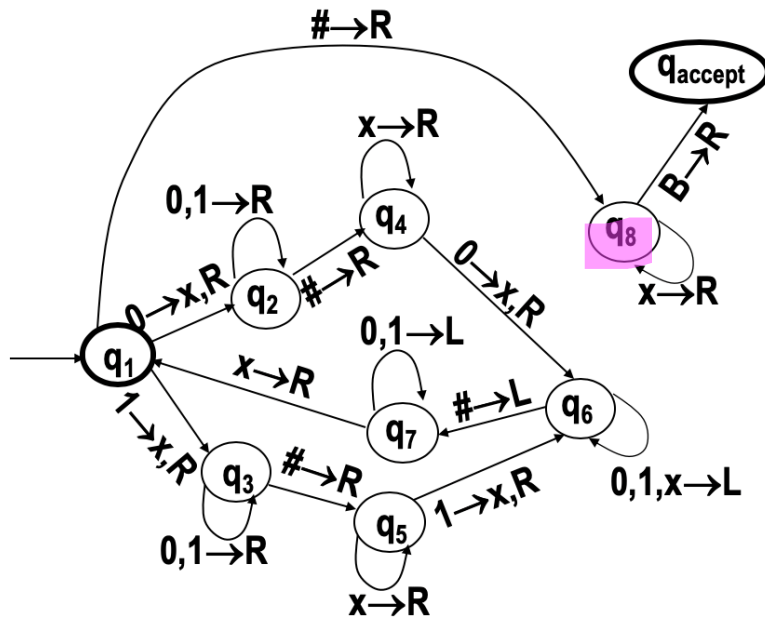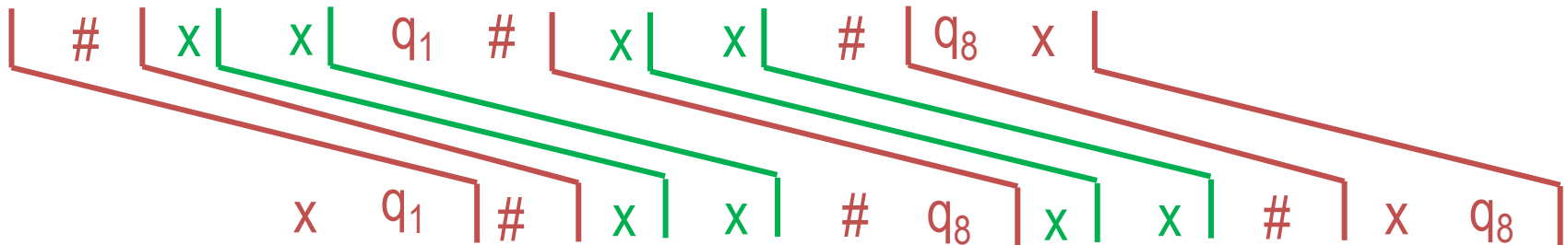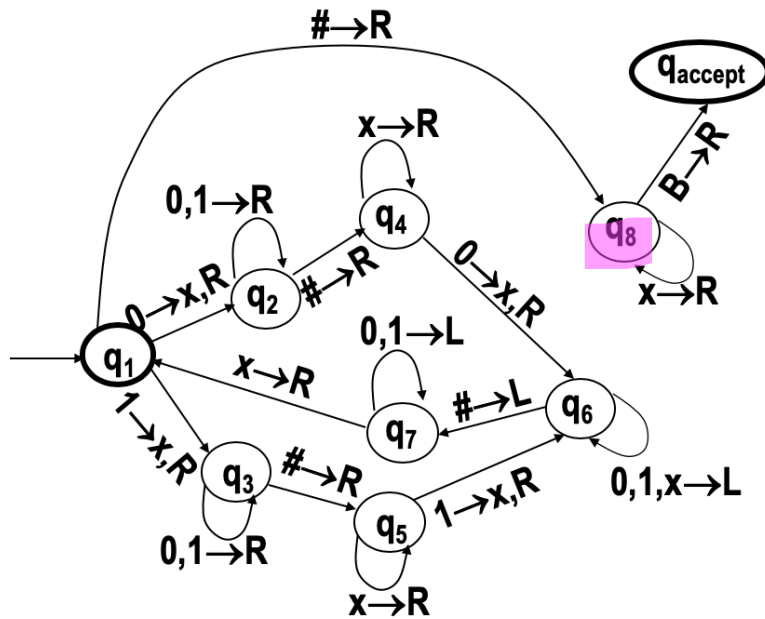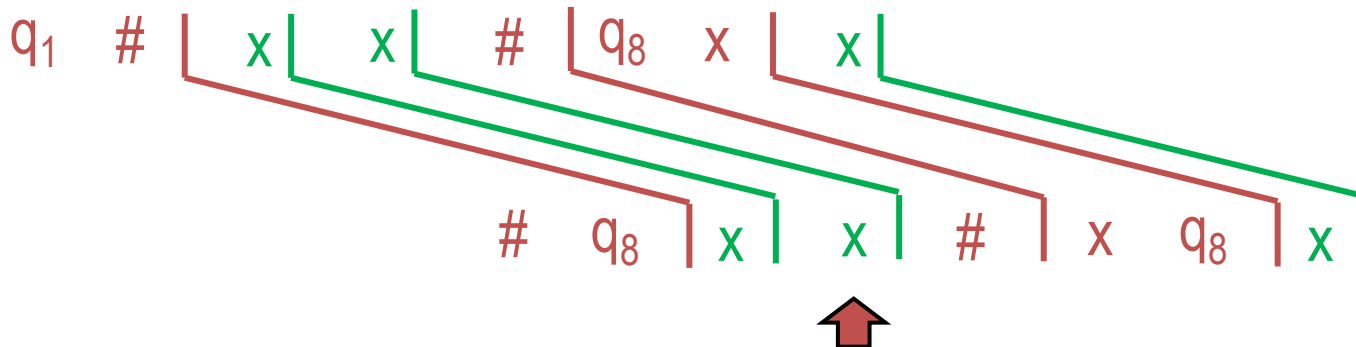
then put $\left[\dfrac{qa}{br}\right]$ into P'

q_6 | x | # q_6 x | x q_6 # | x q_7 x |

q_6 x | x q_6 # x | q_7 x # | x | x q_1 |

Example of PCP to A_{TM} , input is x#x

$$\text{if } \delta(q, a) = (r, b, R),$$

$$\text{then put } \left[\frac{qa}{br}\right] \text{ into P'}$$

x	q₆	#	x	q₇	x	#

| | q₆ | # | x | q₇ | x | # | x | x | q₁ | # |

Example of PCP to A_{TM} , input is x#x

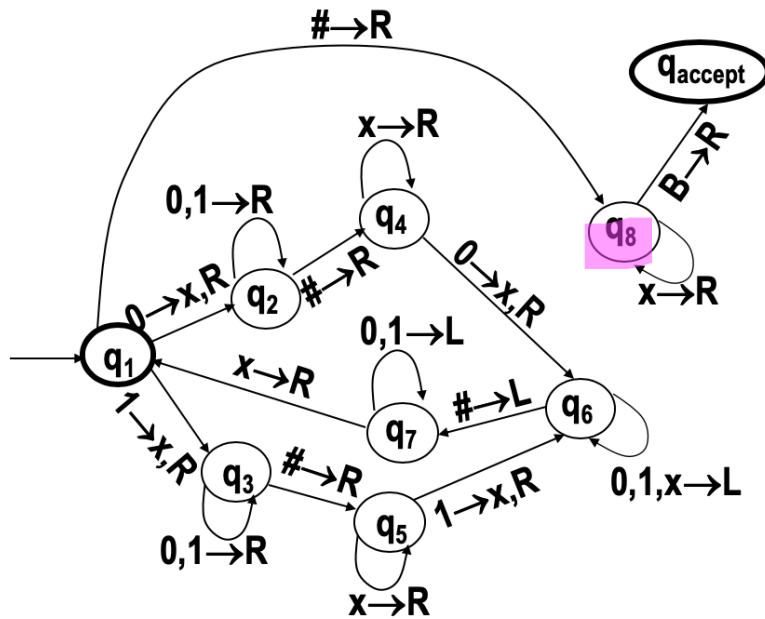if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'
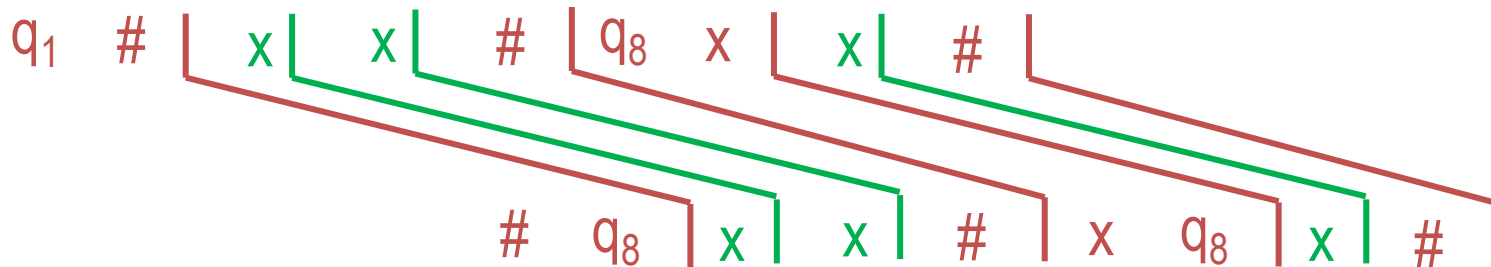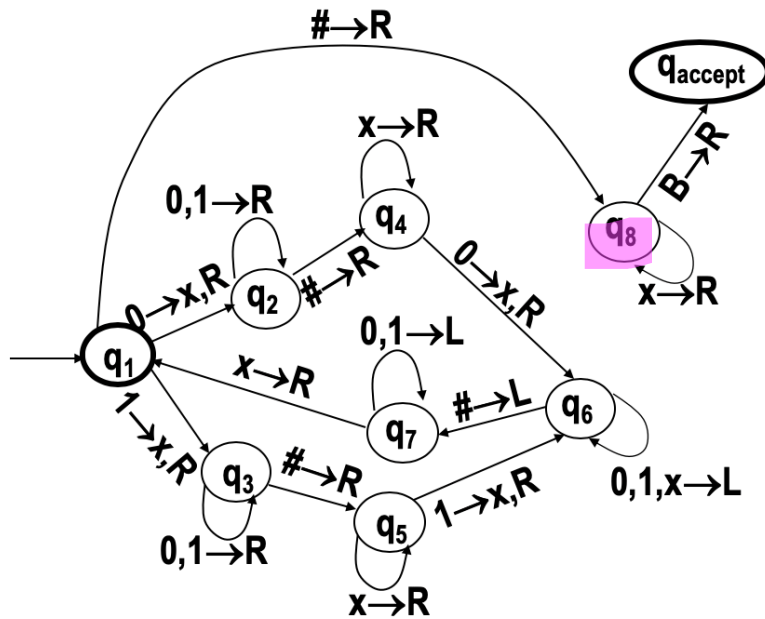
Example of PCP to A_TM , input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x
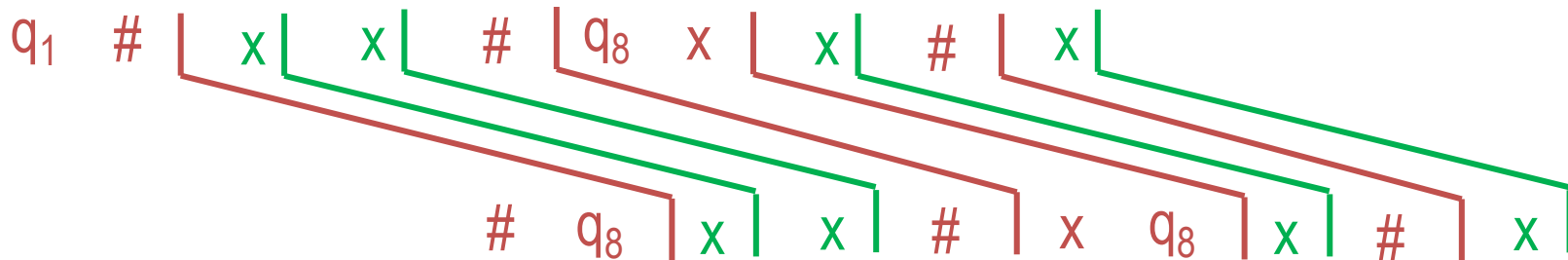

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,
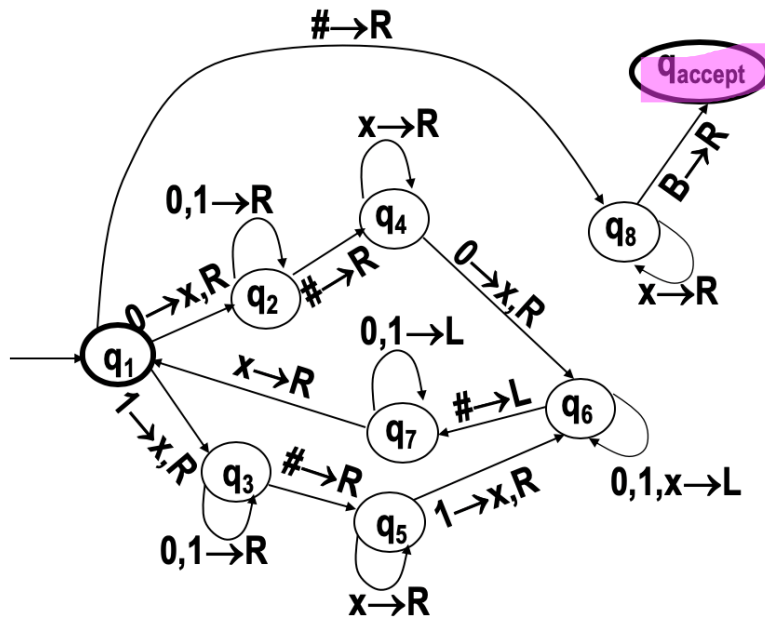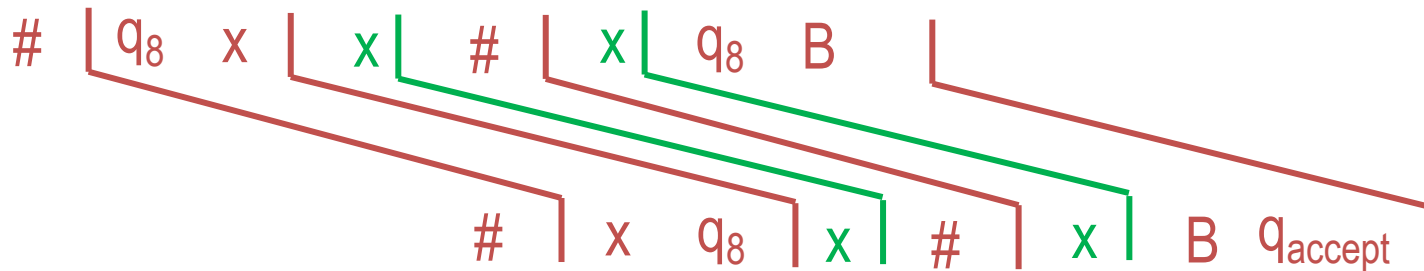
then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'
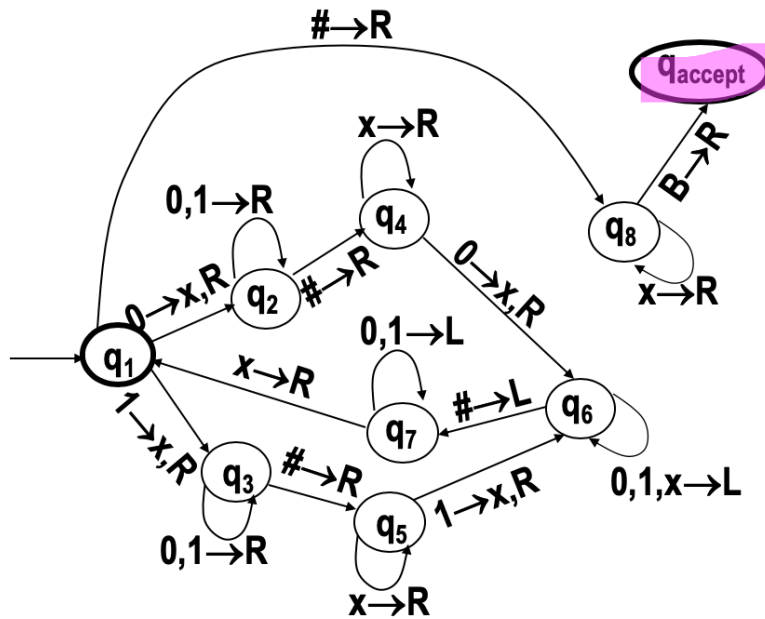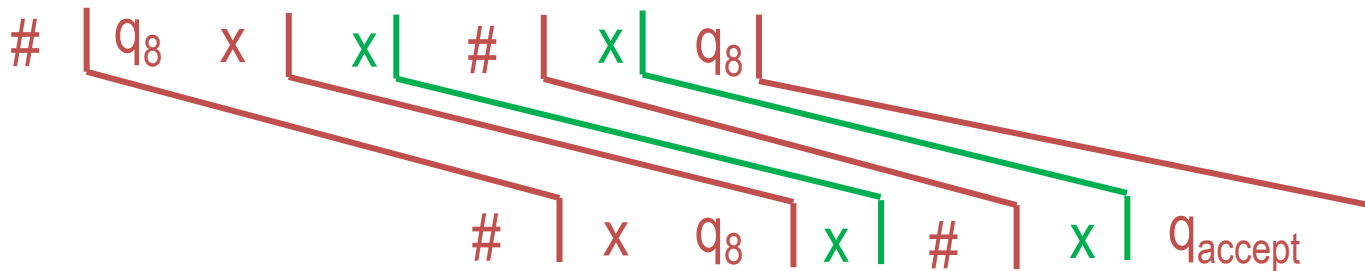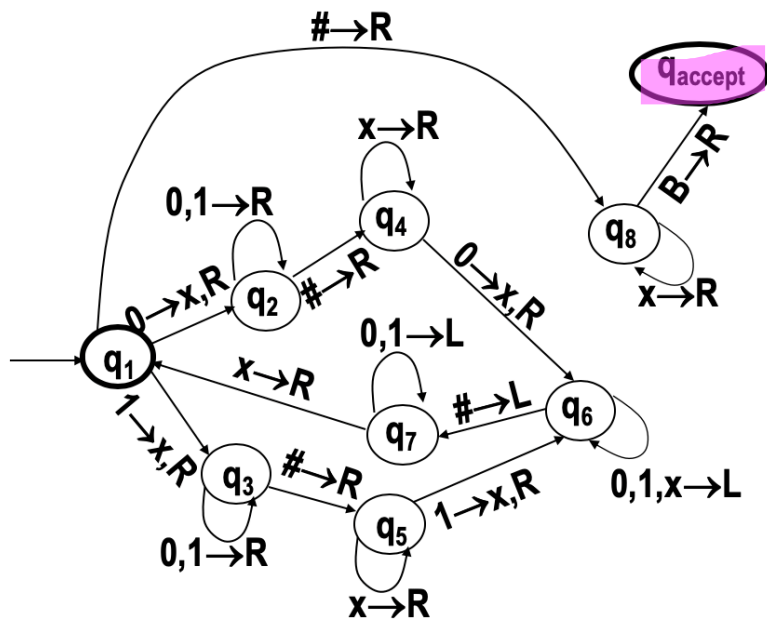
Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A$_{TM}$, input is x#x

if $\delta(q, a) = (r, b, R)$,

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Example of PCP to A_{TM} , input is x#x

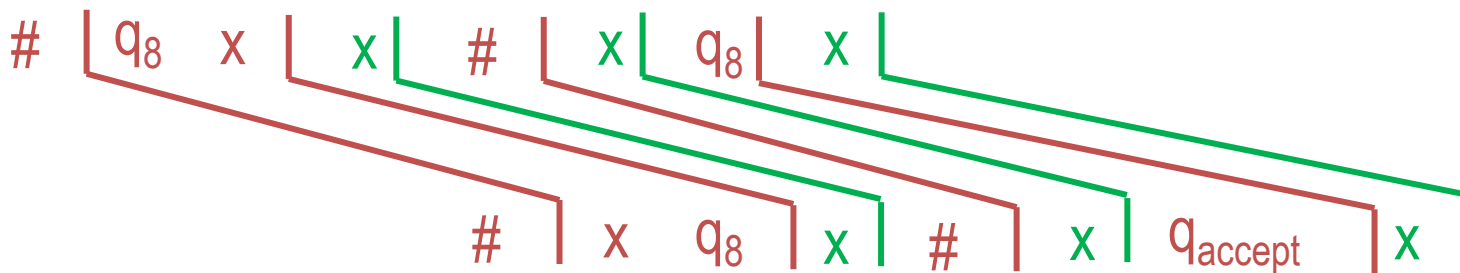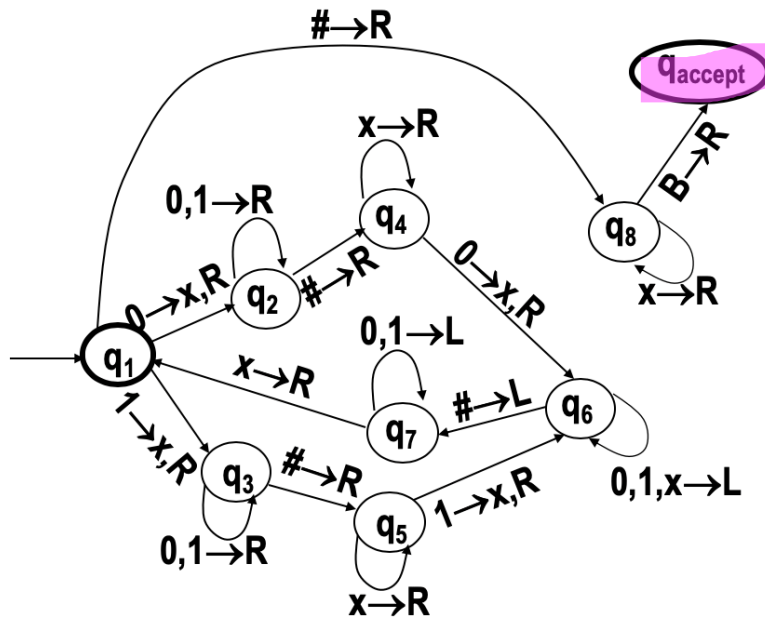
if $\delta(q, a) = (r, b, R)$,
then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,
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Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,
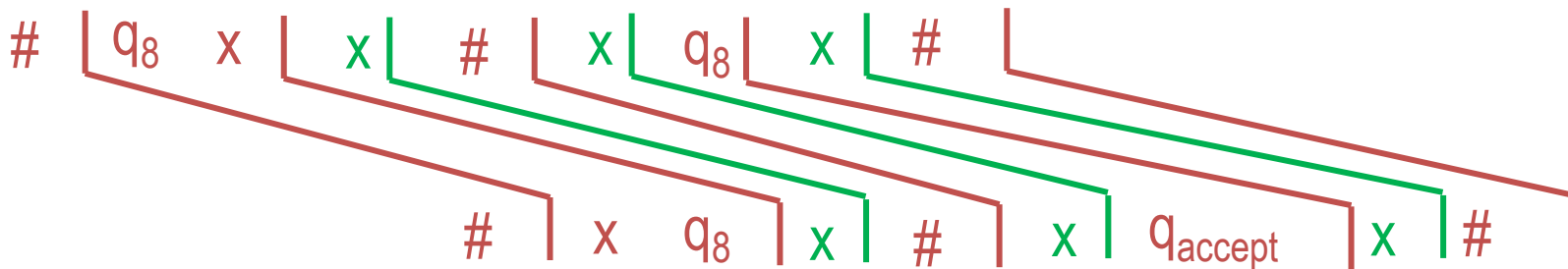
then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A$_{TM}$, input is x#x

if δ(q, a) = (r, b, R),
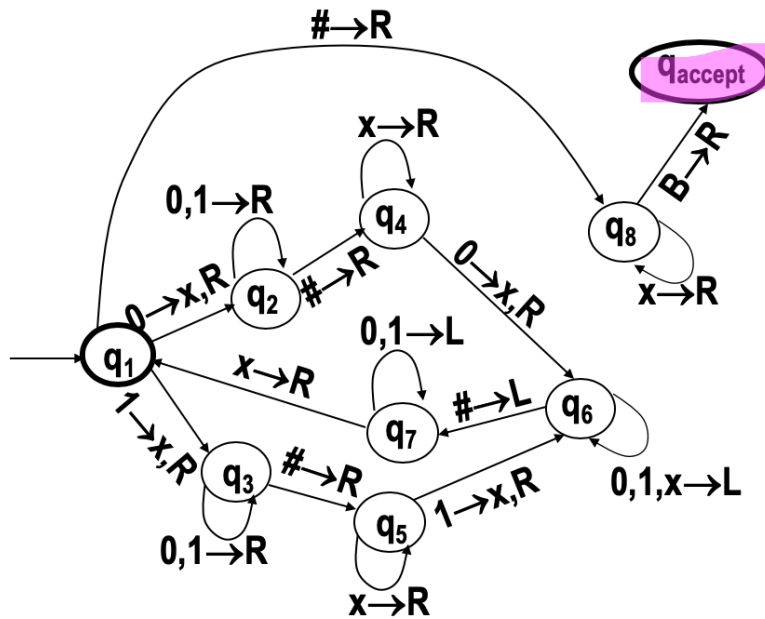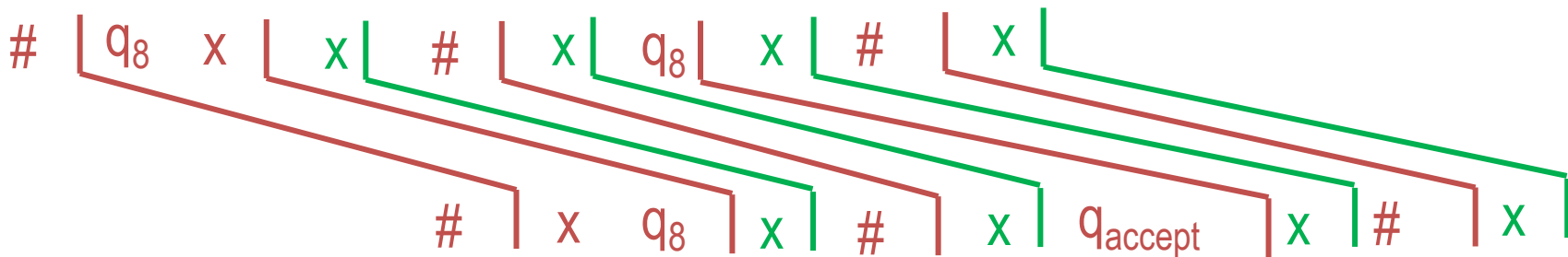
then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,
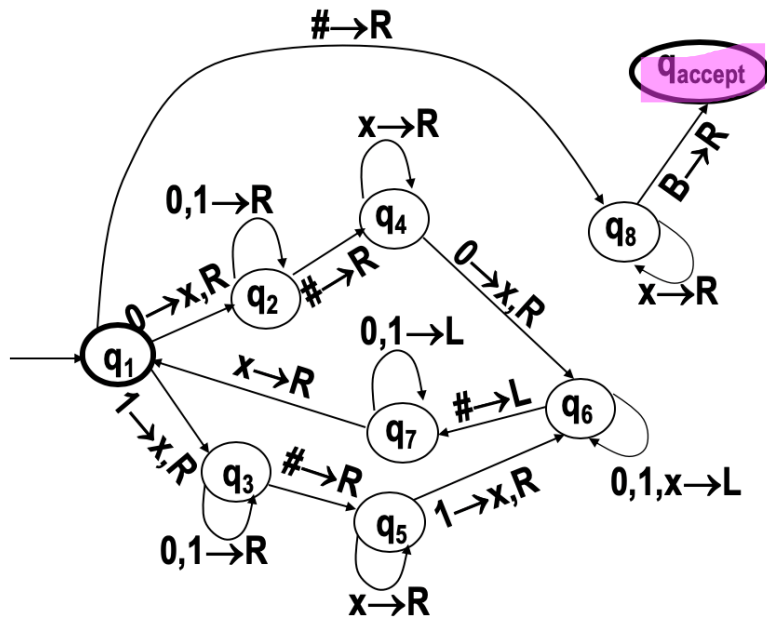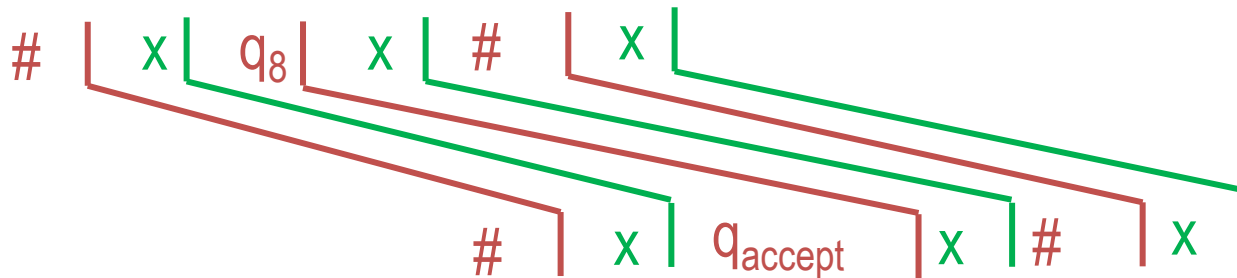
then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A$_{TM}$, input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

Example of PCP to A_{TM}, input is x#x

if $\delta(q, a) = (r, b, R)$,

then put $\left[\dfrac{qa}{br}\right]$ into P'

\#→R

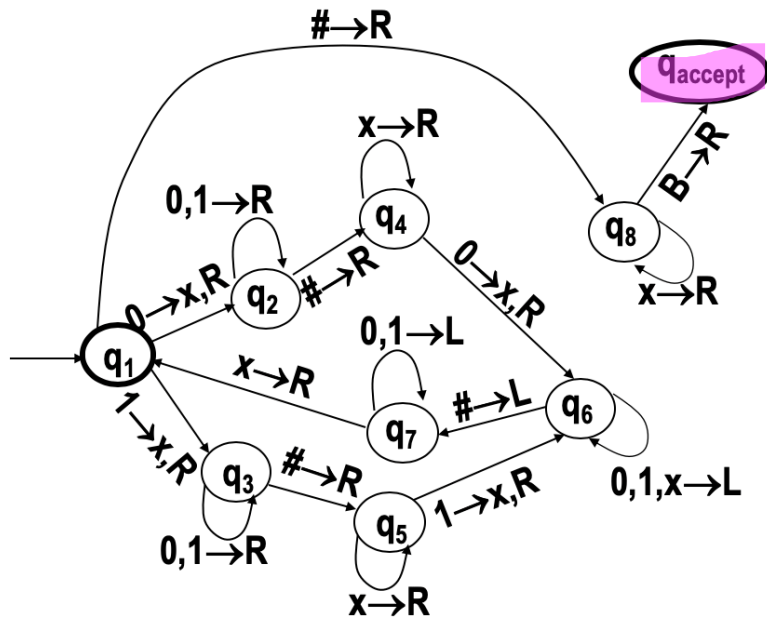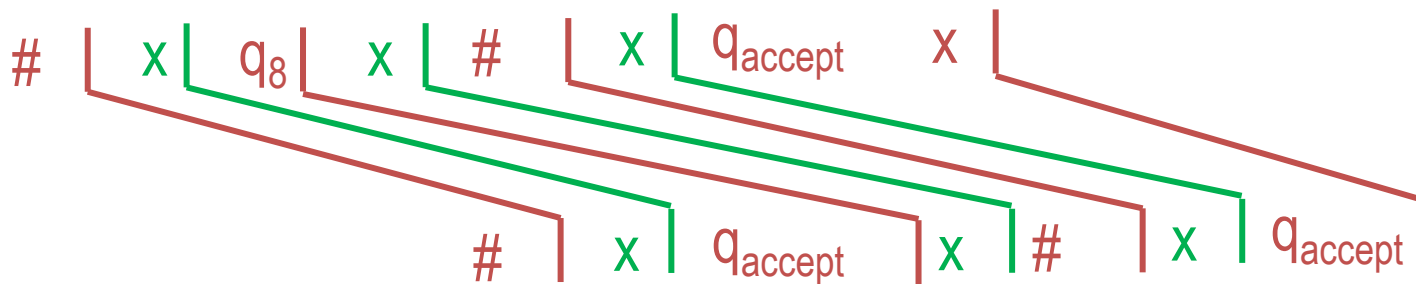q_{accept}

x→R

q_8

B→R

x→R

0,1→R

q_4

0,1→R

q_2 \#→R

0→x,R

x→R

0,1→L

q_7 \#→L q_6

q_1 x→R

0→x,R

1→x,R

q_3 \#→R

q_5 1→x,R

0,1,x→L

0,1→R

x→R

\# q_8 x x \# x q_8 x \# x

\# x q_8 x \# x q_{accept} x \# x

Example of PCP to A_{TM} , input is x#x

if $\delta(q, a) = (r, b, R)$,
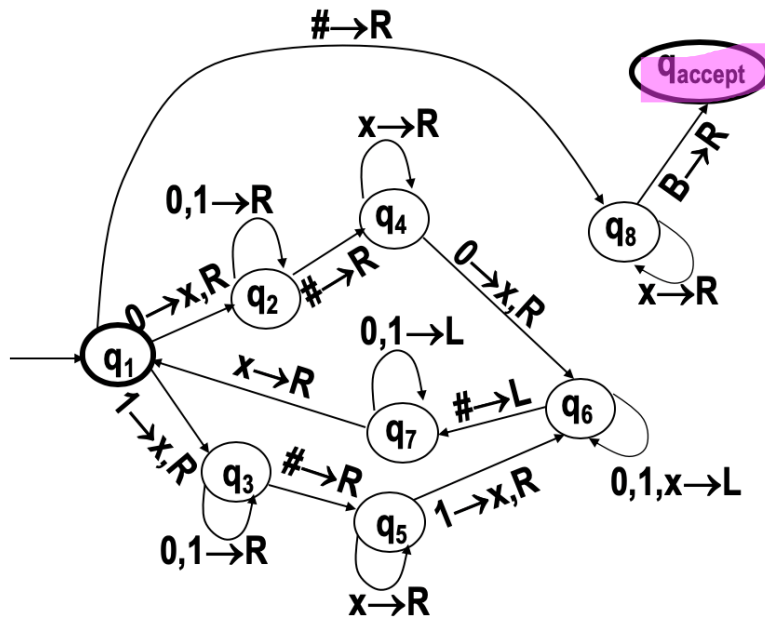
then put $\left[\dfrac{qa}{br}\right]$ into P'
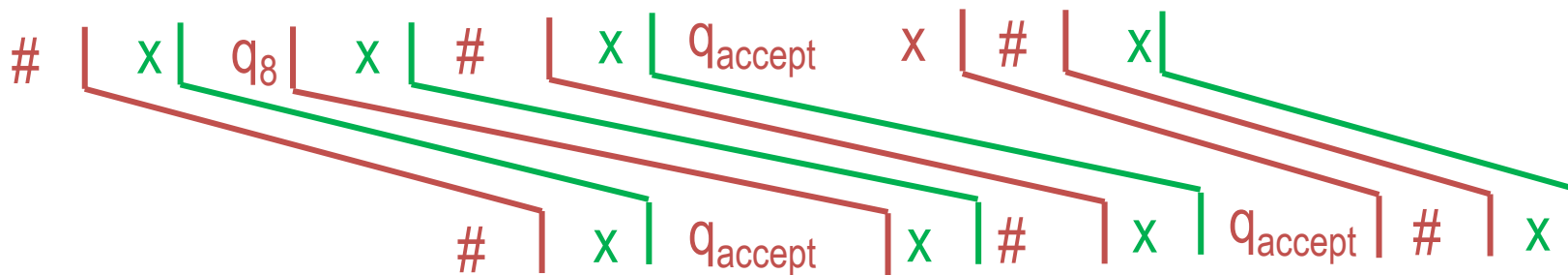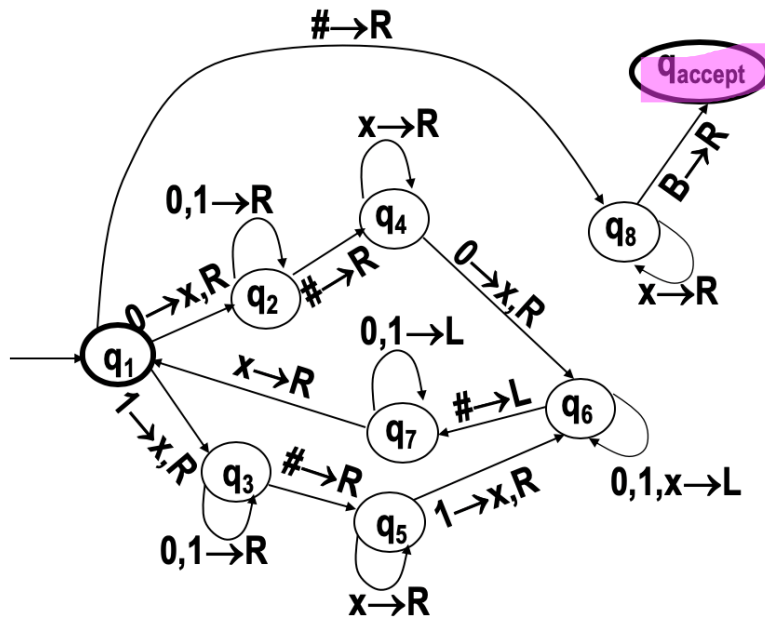
Example of PCP to A_{TM} , input is x#x

put $\left[\dfrac{aq_{accept}}{q_{accept}}\right]$ and $\left[\dfrac{q_{accept}a}{q_{accept}}\right]$ into P'
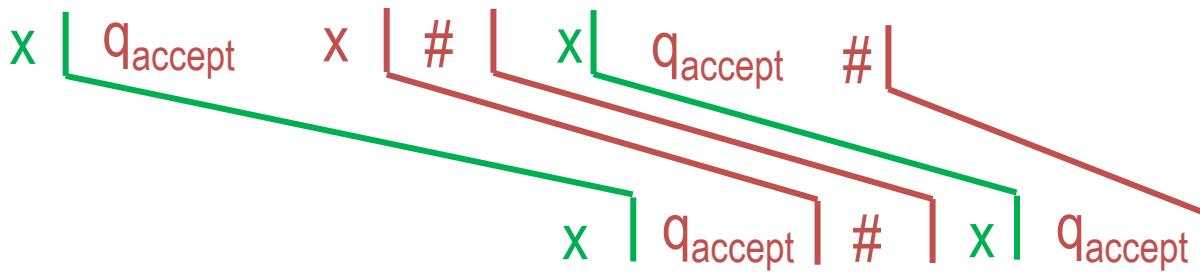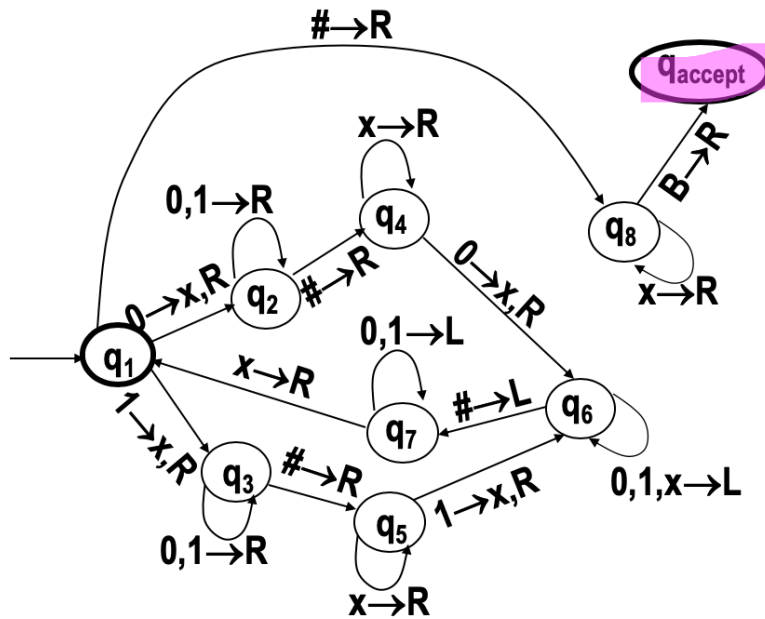
Example of PCP to A~TM~ , input is x#x
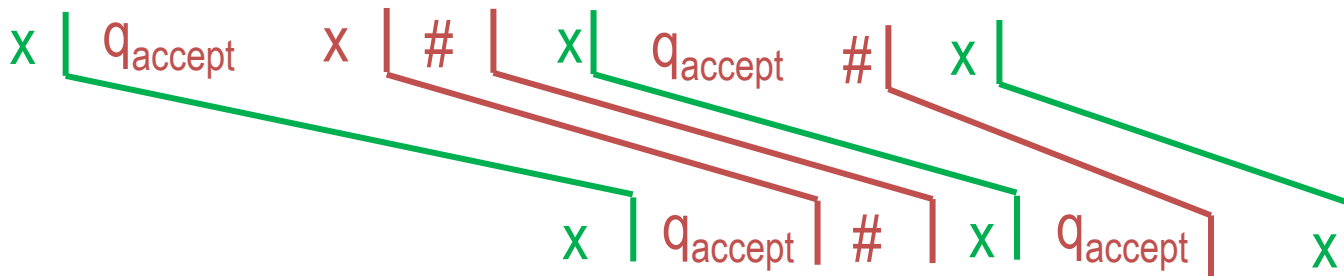

put $\left[\dfrac{aq_{accept}}{q_{accept}}\right]$ and $\left[\dfrac{q_{accept}a}{q_{accept}}\right]$ into P'

Example of PCP to A_TM , input is x#x

$$\text{put } \left[\frac{aq_{accept}}{q_{accept}}\right] \text{ and } \left[\frac{q_{accept}a}{q_{accept}}\right]$$
into P'

Example of PCP to A_TM , input is x#x

put $\left[\dfrac{a q_{accept}}{q_{accept}}\right]$ and $\left[\dfrac{q_{accept} a}{q_{accept}}\right]$ into P'
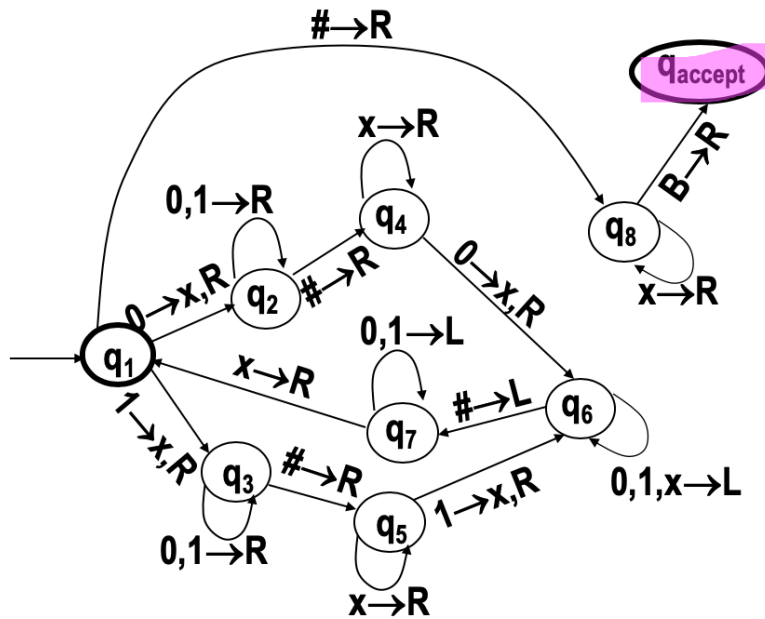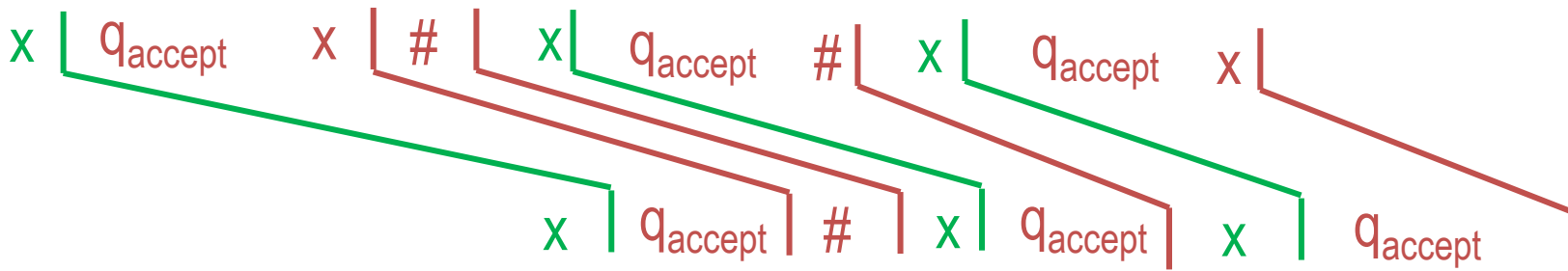
Example of PCP to A_{TM} , input is x#x

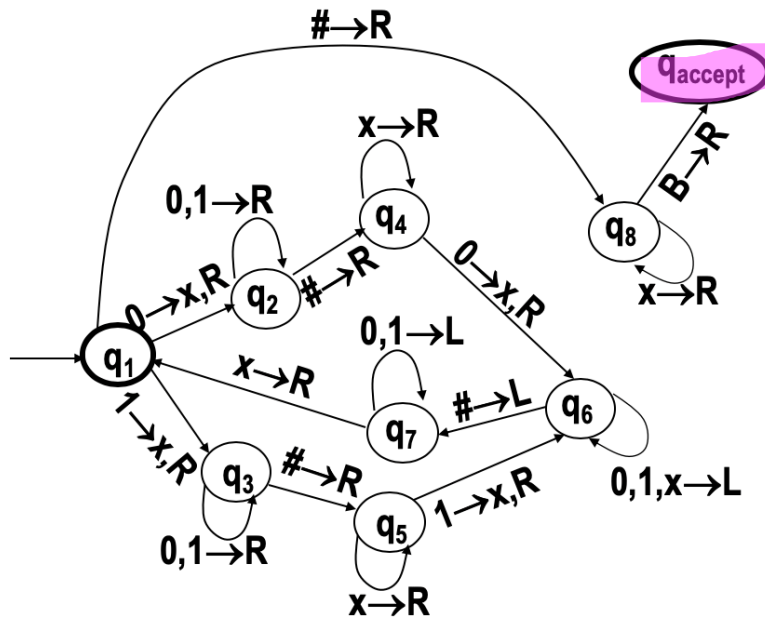put $\left[\dfrac{aq_{accept}}{q_{accept}}\right]$ and $\left[\dfrac{q_{accept}a}{q_{accept}}\right]$ into P'
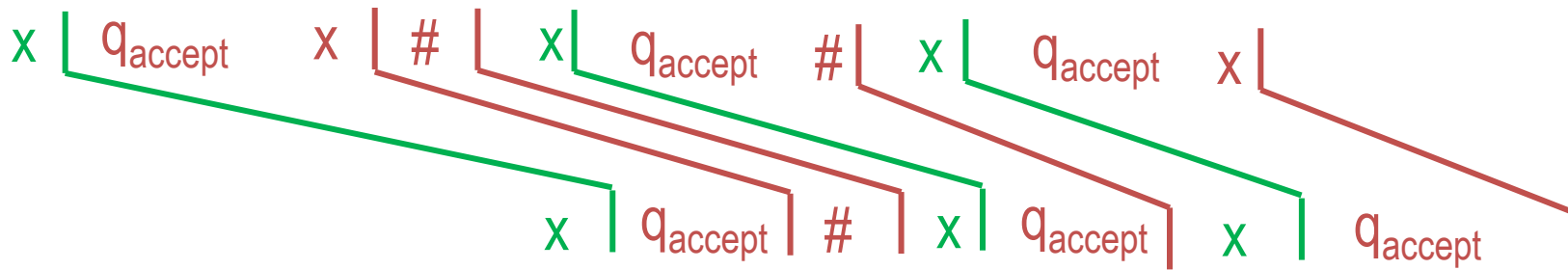
Example of PCP to A$_{TM}$, input is x#x

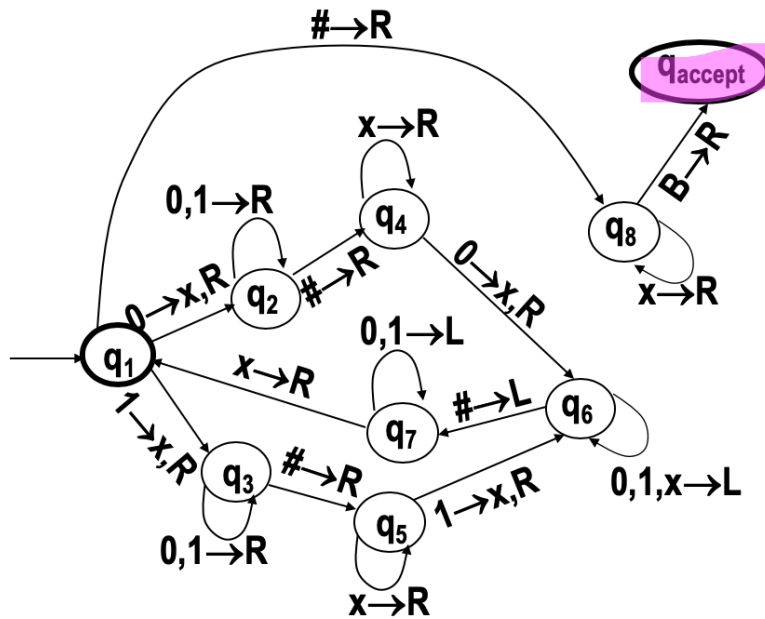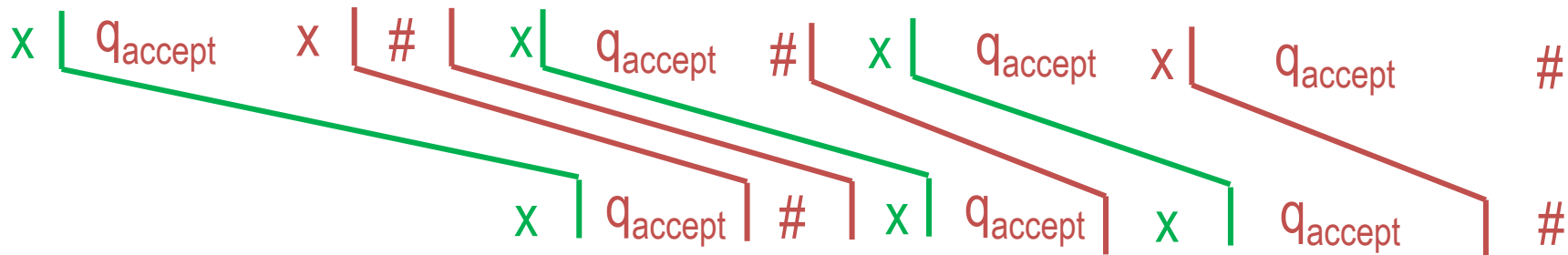Finish the match, put

$$\left[\frac{q_{accept}\#\#}{\#}\right] \text{ into P'}$$

Example of PCP to A_{TM}, input is x#x

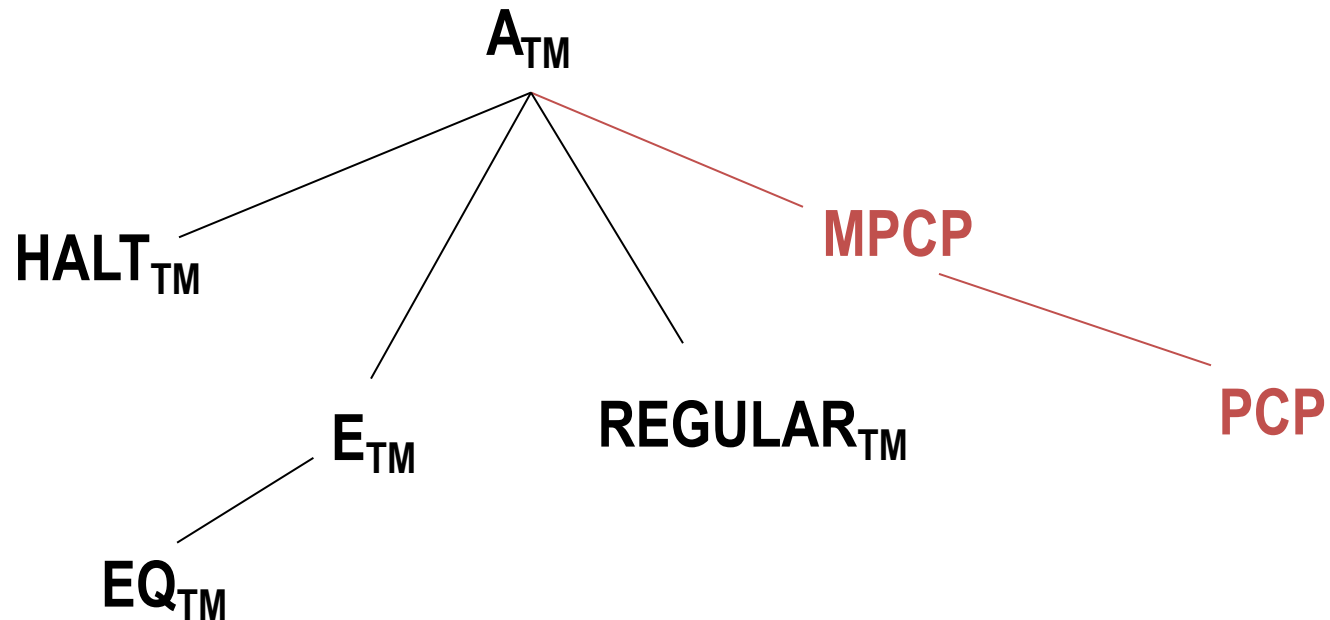Finish the match, put $\left[\dfrac{q_{accept}\#}{\#}\right]$ into P'

Conclusion

- Relationship of languages on reducibility

Conclusion

- Closure on operations

	Complement $\overline{A}$	Intersection $\cap$	Union $\cup$	Star A^*
Regular/DFA/ NFA	√	√	√	√
CFL/ PDA	×	×	√	√
Turing- decidable TM	√	√	√	√
Turing- recognizable TM	×	√	√	√