CS 6041
Theory of Computation

Reducibility

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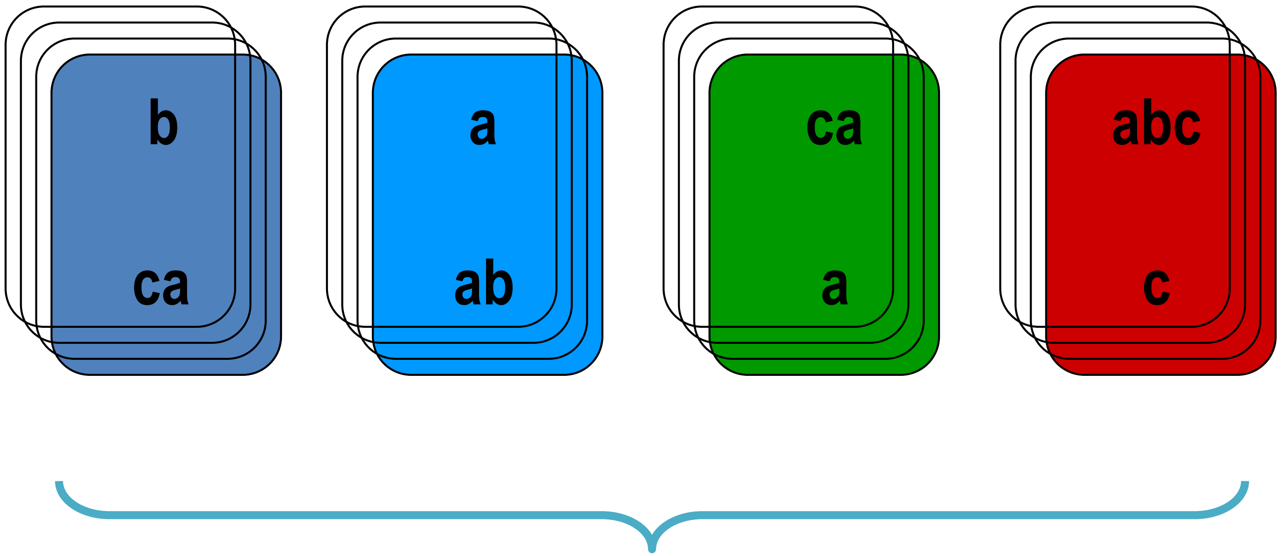
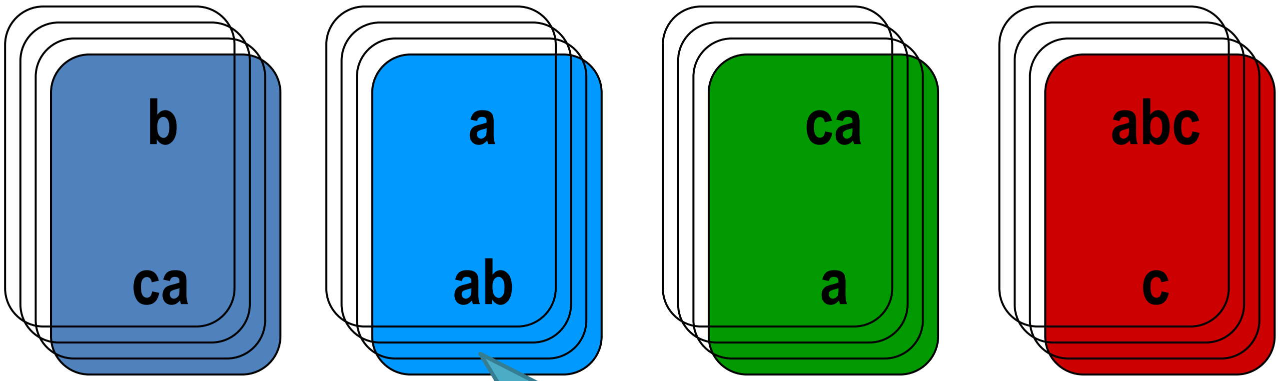
https://kevinsuo.github.io/

Post Correspondence Problem (PCP)



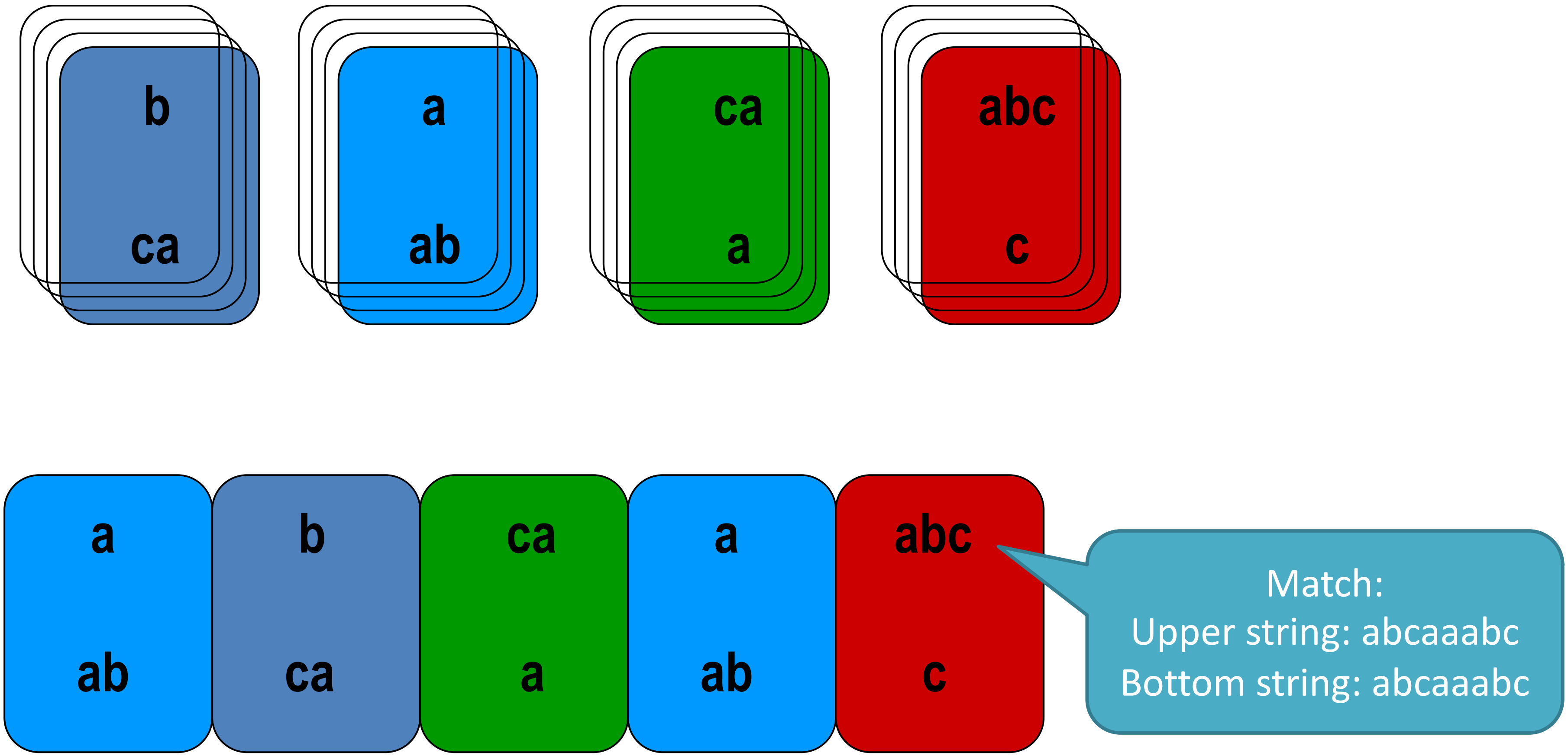
We have limited categories of dominos

Post Correspondence Problem (PCP)



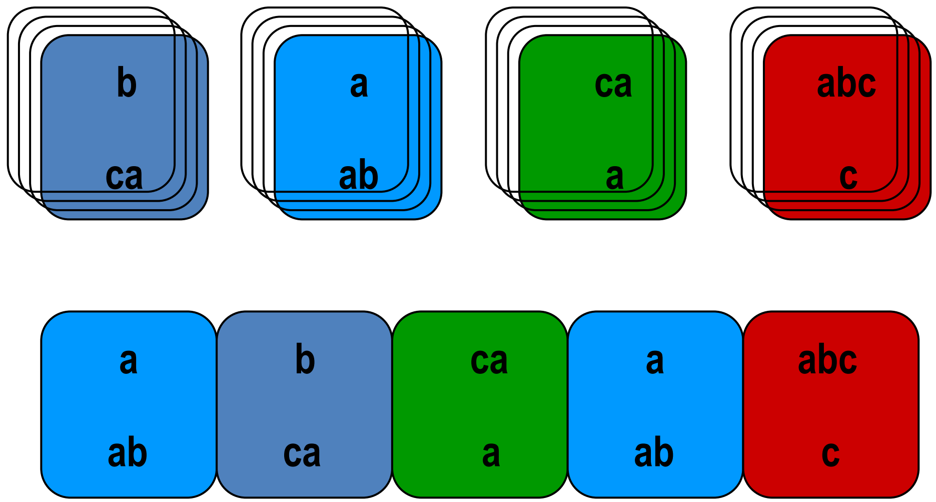
The number of domino in each category is unlimited

Post Correspondence Problem (PCP)



Post Correspondence Problem (PCP)

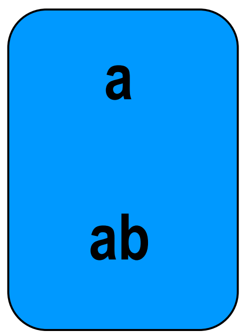
- Whether a collection of dominos has a match



- PCP = {⟨P⟩ | P is an instance of the Post Correspondence Problem with a match}.

Description of PCP

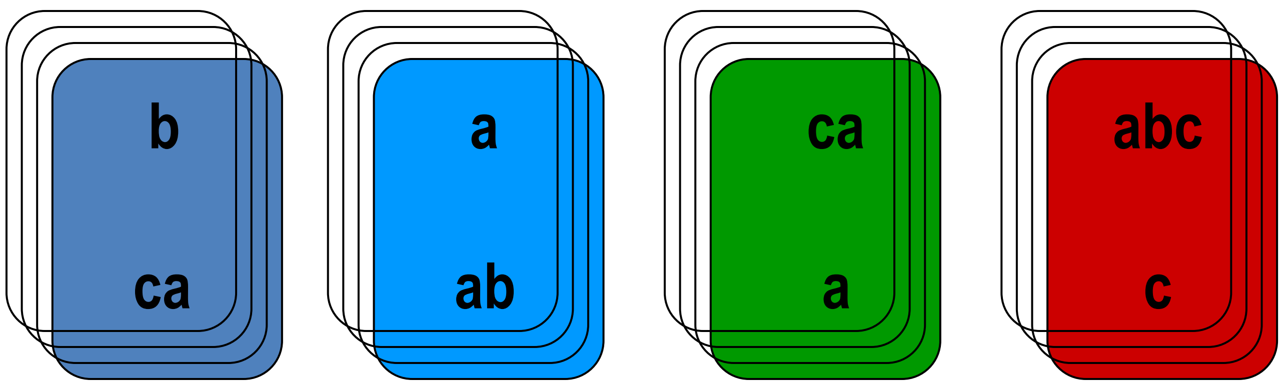
An individual domino $\left[\frac{a}{ab}\right]$



Description of PCP

A collection of dominos

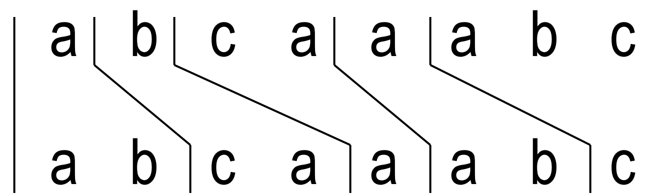
$$\left\{\left[\frac{b}{ca}\right], \left[\frac{a}{ab}\right], \left[\frac{ca}{a}\right], \left[\frac{abc}{c}\right]\right\}$$

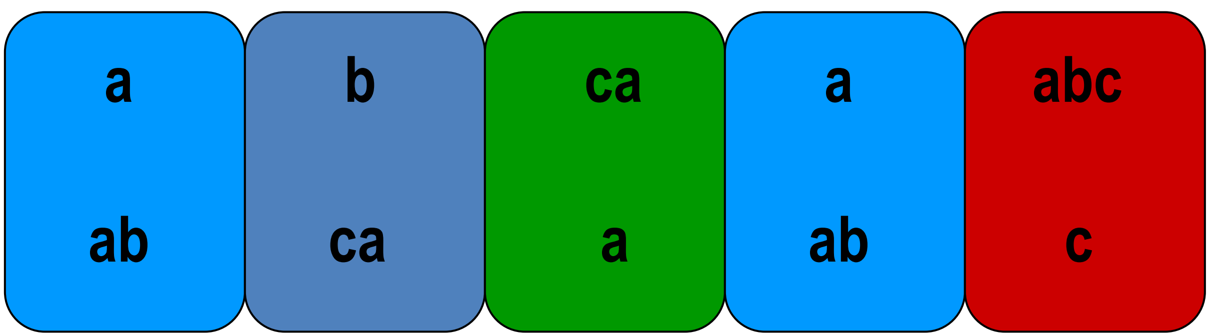


Description of PCP

A match

$$\left[\frac{a}{ab}\right]\left[\frac{b}{ca}\right]\left[\frac{ca}{a}\right]\left[\frac{a}{ab}\right]\left[\frac{abc}{c}\right]$$





A collection without a match

- For a given collection

$$\left\{\left[\frac{abc}{ab}\right], \left[\frac{ca}{a}\right], \left[\frac{acc}{ba}\right]\right\}$$

- It cannot contain a match because every top string is longer than the corresponding bottom string

Theorem 5.15

- PCP is undecidable

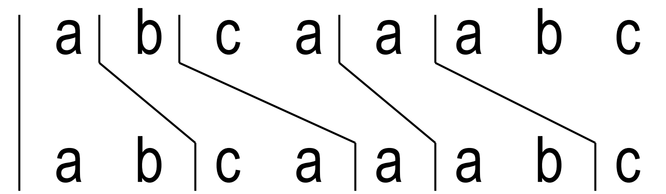
- Proof:

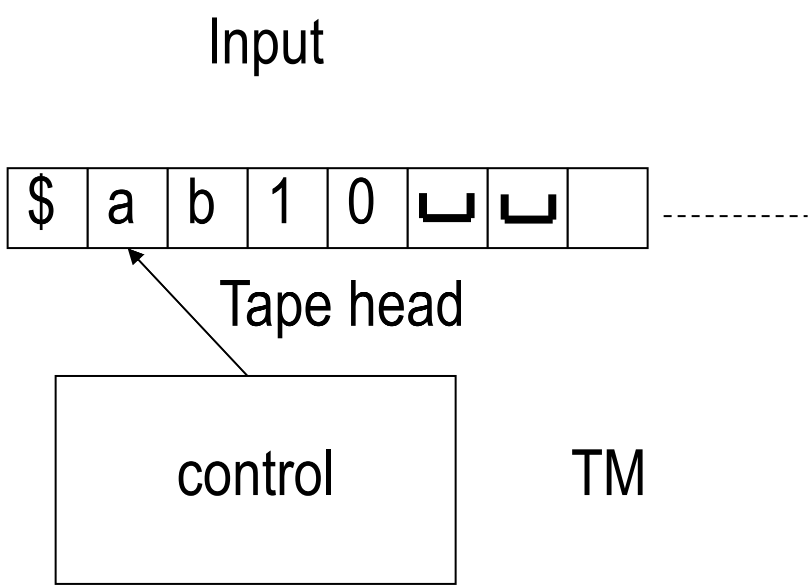
  To simplify the problem, we create ***Modified Post Correspondence Problem (MPCP)*** ,

  MPCP = {⟨P⟩| P is an instance of the PCP with a match that starts with the first domino}.

Theorem 5.15

$$\left[\frac{a}{ab}\right]\left[\frac{b}{ca}\right]\left[\frac{ca}{a}\right]\left[\frac{a}{ab}\right]\left[\frac{abc}{c}\right]$$





- The PCP program is very similar with the A_{TM} program

Theorem 5.15 proof

- Proof:

  Suppose PCP is decidable

  We construct TM S to decide A_{TM} (Theorem 4.11: A_{TM} is undecidable)

  PCP

  $\mathbf{M = (Q,\Sigma,\Gamma,\delta,q_0,q_{accept},q_{reject})}$

  S constructs an instance of the PCP P that has a match iff M accepts w: A_{TM}

  - (1) S first constructs an instance P' of the MPCP;
  - (2) Transfer P' into P;

Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P' of the MPCP;

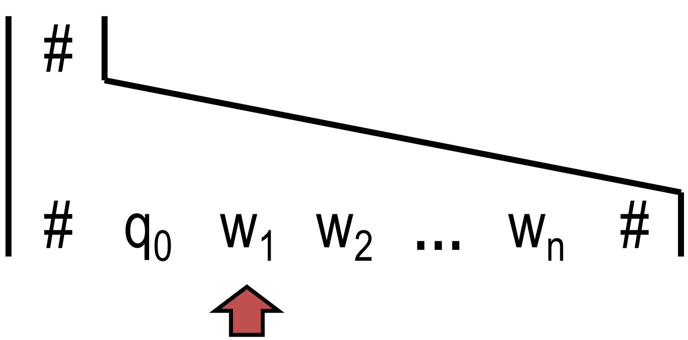
  (1.1) Generating beginning configuration

  Put $\left[\frac{\#}{\#q_0 w_1 w_2 \ldots w_n \#}\right]$ into P' as $\left[\frac{t_1}{b_1}\right]$

TM M = (Q, Σ, Γ, δ, q_0, q_{accept}, q_{reject})

Input w

P'



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Theorem 5.15 proof

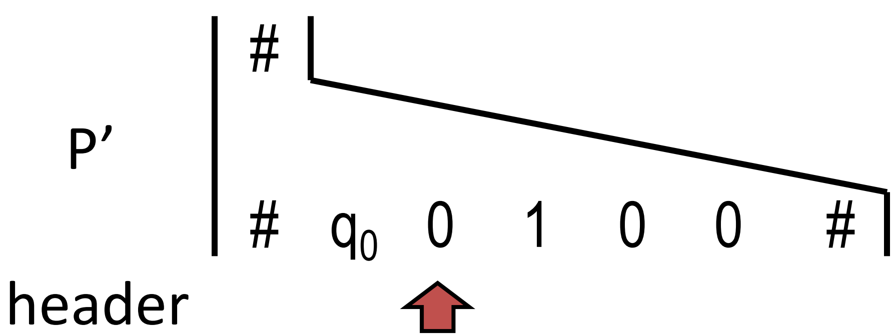
- Proof:

  (1) S first constructs an instance P' of the MPCP;

  (1.1) Generating beginning configuration

  Suppose Γ={0,1,2, ␣}, w=0100,

  Put the following into P' : $\left[\frac{\#}{\# q_0 0100 \#}\right] = \left[\frac{t_1}{b_1}\right]$



Theorem 5.15 proof

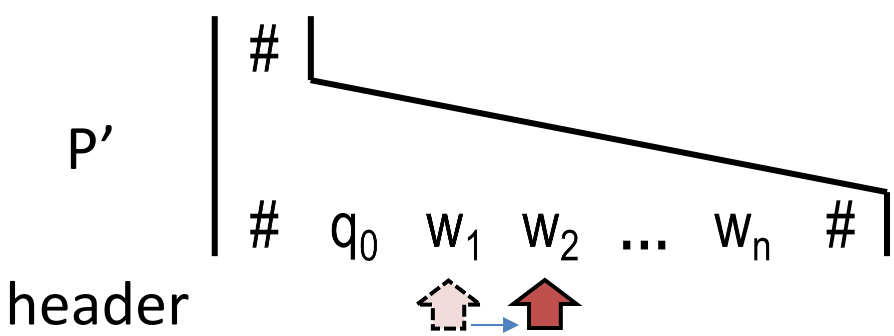
- Proof:

  (1) S first constructs an instance P′ of the MPCP;

  (1.2) The head move to the right

  For each a, b$\in\Gamma$ and q, r$\in$Q, where q $\neq$ q_{reject},

  if **δ(q, a) = (r, b, R),** then put $\left[\frac{qa}{br}\right]$ into P′



Theorem 5.15 proof

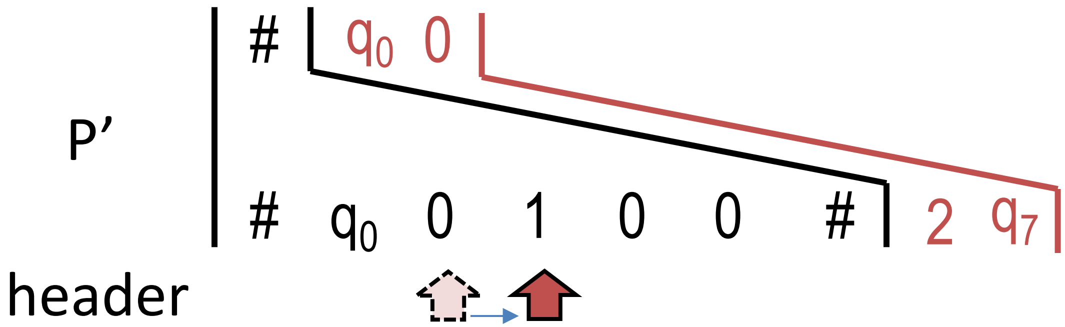
- Proof:

(1) S first constructs an instance P' of the MPCP;

(1.2) The head moves to the right

For each a, b $\in \Gamma$ and q, r $\in$ Q, where q $\neq q_{reject}$,

if $\delta(q_0, 0) = (q_7, 2, R)$, then put $\left[\frac{q_0 0}{2 q_7}\right]$ into P'



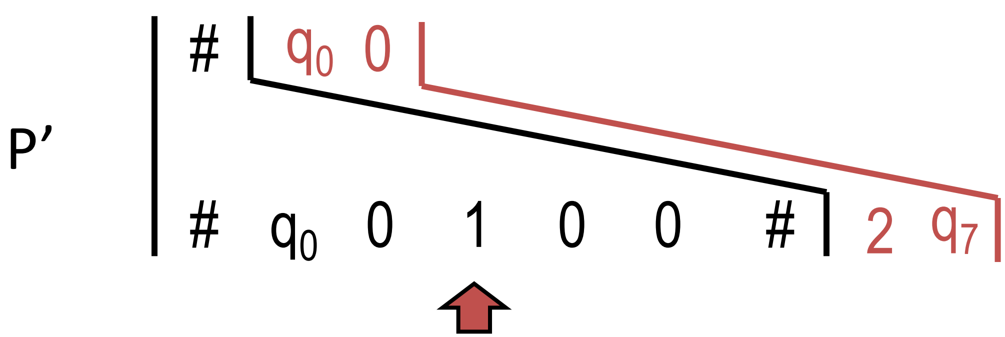
Theorem 5.15 proof

- Proof:

(1) S first constructs an instance P' of the MPCP;

(1.3) The head moves to the left

Discuss it later on.



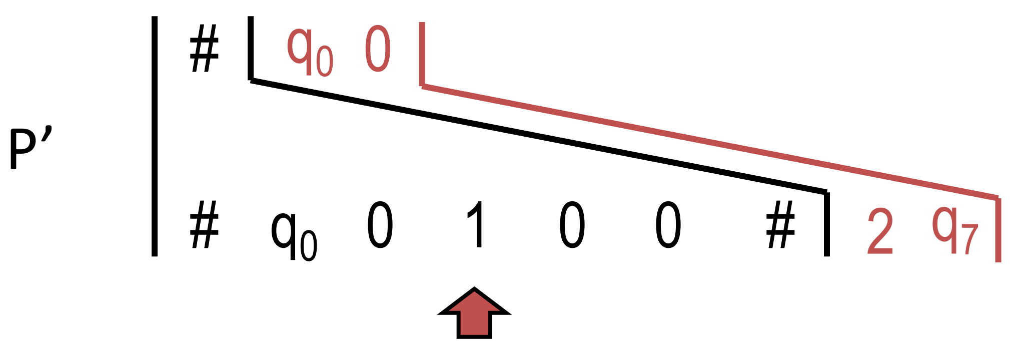
Theorem 5.15 proof

- Proof:

(1) S first constructs an instance P' of the MPCP;

(1.4) For each a∈Γ,

put $\left[\frac{a}{a}\right]$ into P'



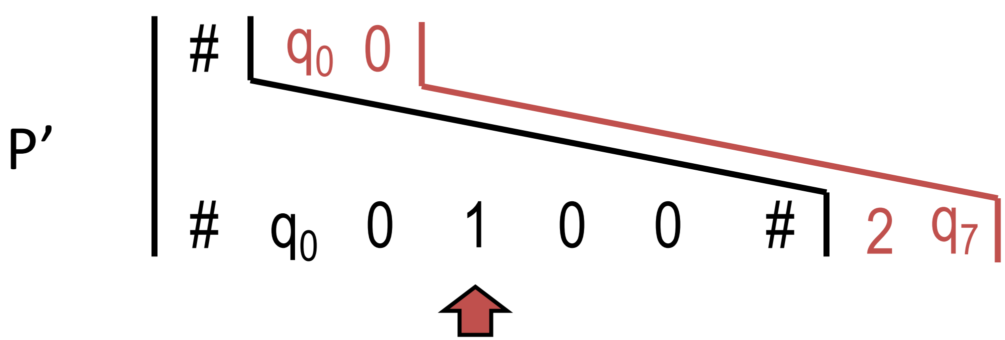
Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P' of the MPCP;

  (1.4) $\Gamma = \{0, 1, 2, \sqcup\}$,

  put $\left[\frac{0}{0}\right]$, $\left[\frac{1}{1}\right]$, $\left[\frac{2}{2}\right]$ and $\left[\frac{\sqcup}{\sqcup}\right]$ into P'



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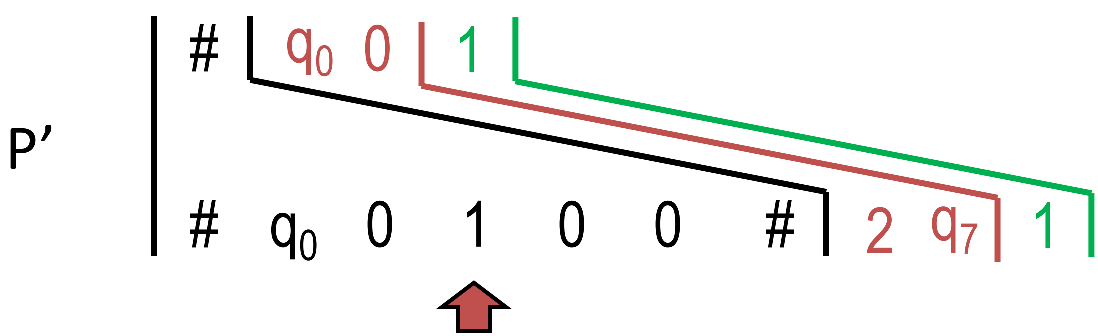

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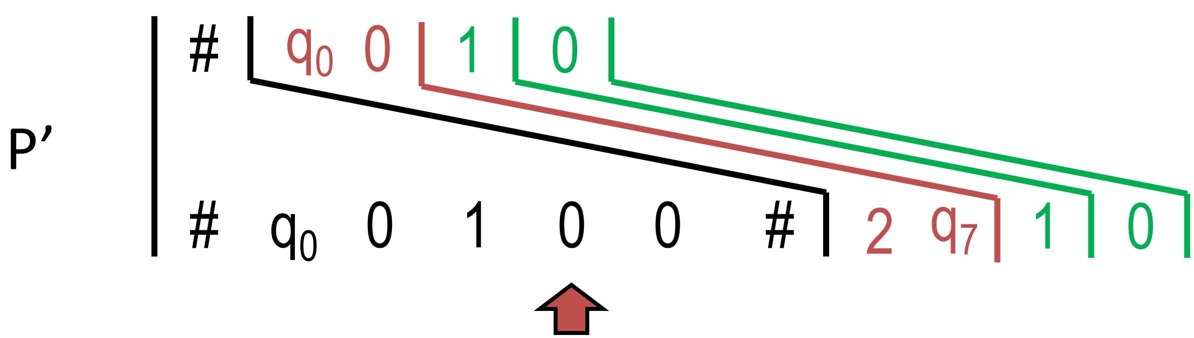

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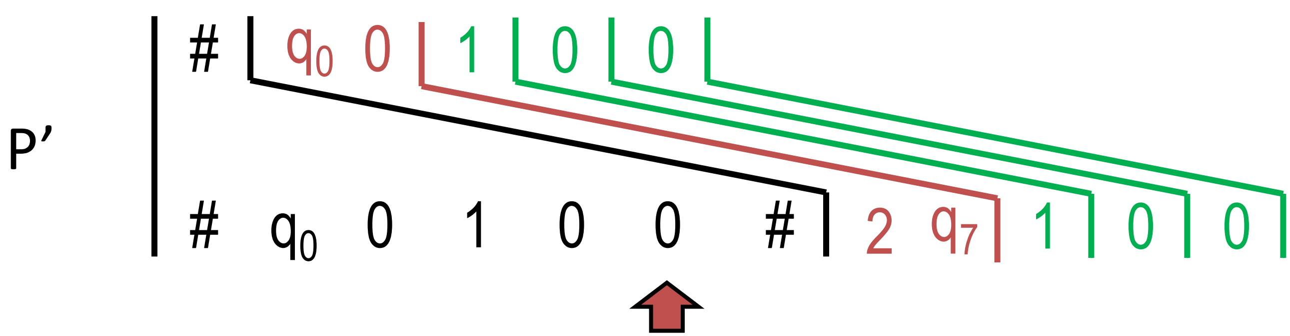
(1) S first constructs an instance P' of the MPCP;

(1.4) $\Gamma = \{0, 1, 2, \sqcup\}$,

put $\left[\frac{0}{0}\right]$, $\left[\frac{1}{1}\right]$, $\left[\frac{2}{2}\right]$ and $\left[\frac{\sqcup}{\sqcup}\right]$ into P'



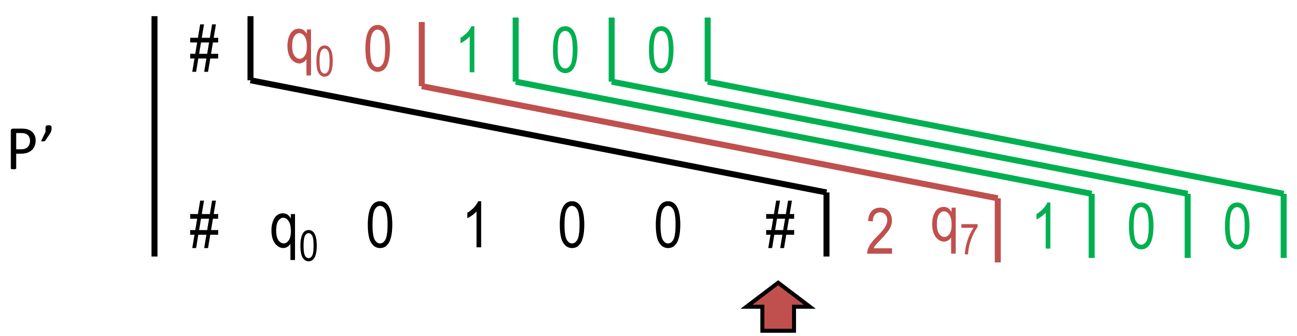
Theorem 5.15 proof

- Proof:

(1) S first constructs an instance P' of the MPCP;

(1.5) Copy # and Put ␣ at the end of configuration

put $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{\sqcup\#}\right]$ into P'



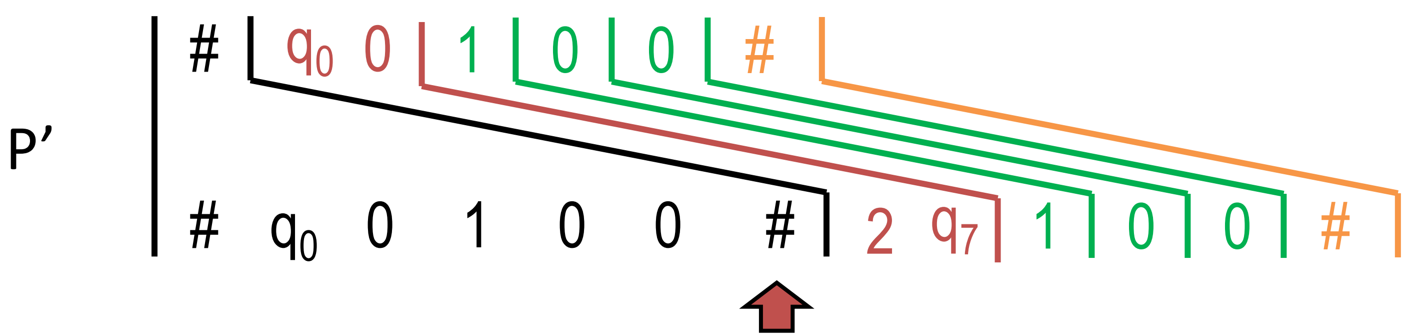
Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P' of the MPCP;

  (1.5) Copy # and Put ␣ at the end of configuration

  put $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{\sqcup\#}\right]$ into P'



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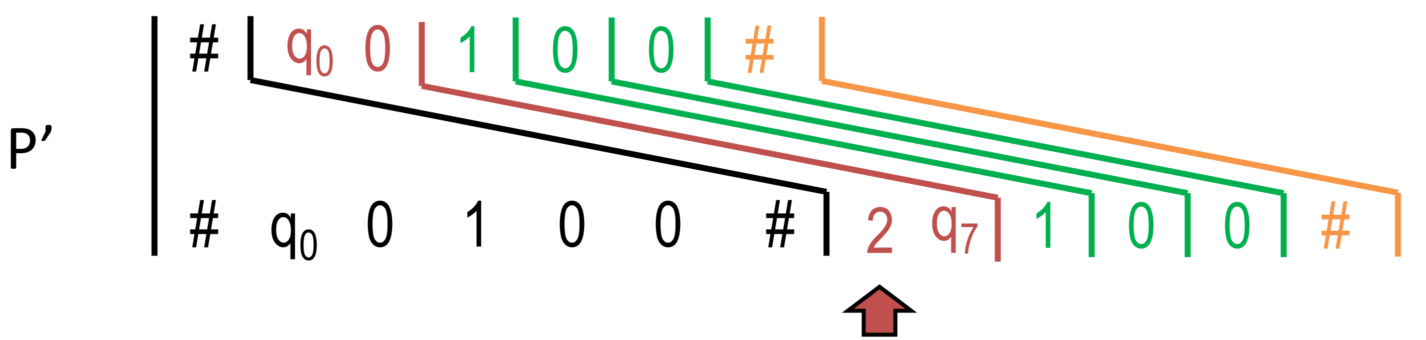
  (1) S first constructs an instance P’ of the MPCP;

  (1.5) Copy # and Put ␣ at the end of configuration

  put $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{\sqcup\#}\right]$ into P’



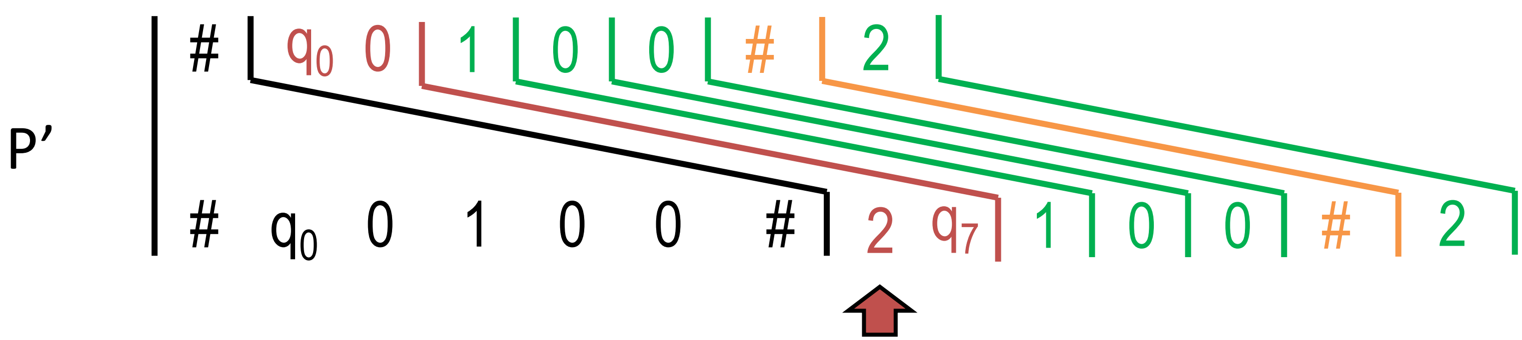
Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P' of the MPCP;

  (1.5) Copy # and Put ␣ at the end of configuration

  put $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{␣\#}\right]$ into P'



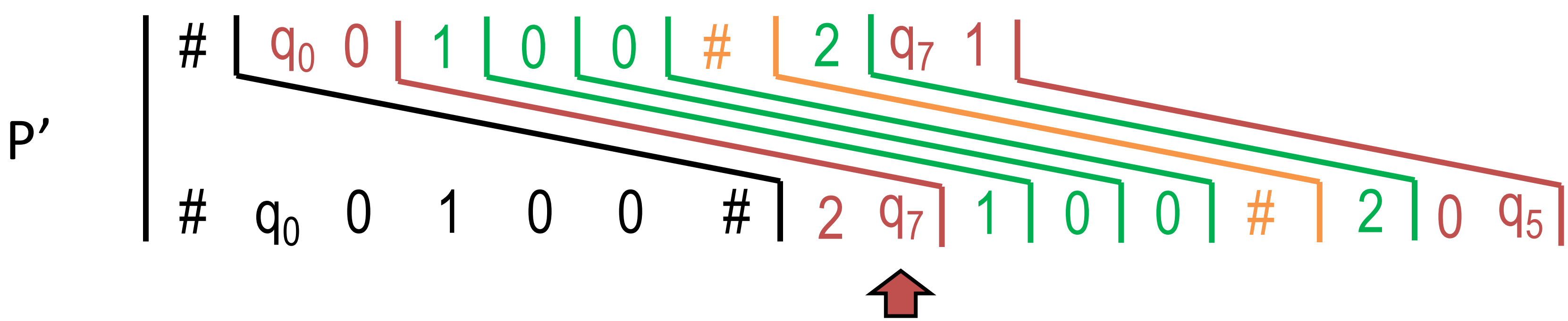
Theorem 5.15 proof

- Proof:

(1) S first constructs an instance P' of the MPCP;

(1.5) Copy # and Put ␣ at the end of configuration

suppose $\delta(q_7, 1) = (q_5, 0, R)$, then put $\left[\frac{q_7 1}{0 q_5}\right]$ into P'



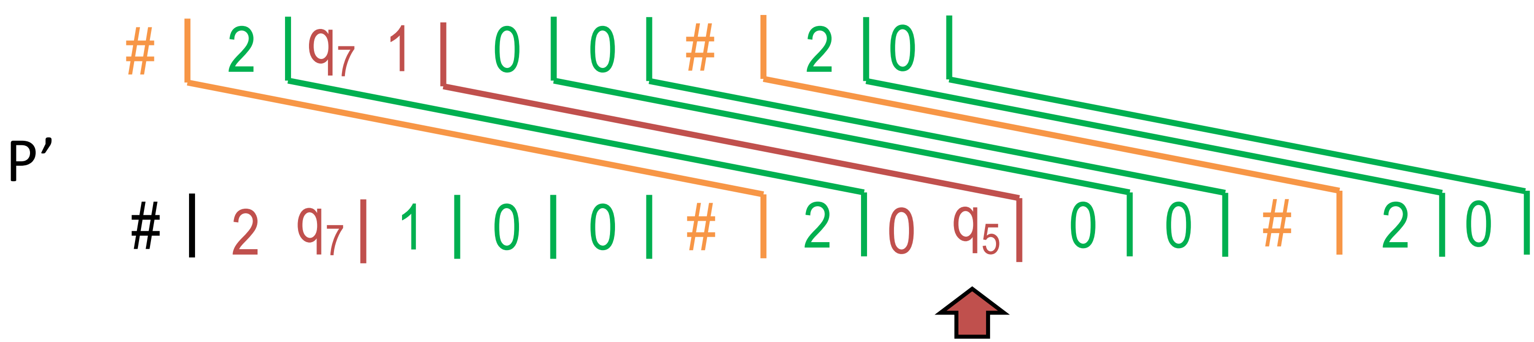
Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P’ of the MPCP;

  (1.5) Copy # and Put ␣ at the end of configuration

  keep putting $\left[\frac{0}{0}\right]$, $\left[\frac{1}{1}\right]$, $\left[\frac{2}{2}\right]$, $\left[\frac{\sqcup}{\sqcup}\right]$, $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{\sqcup\#}\right]$ into P’



Theorem 5.15 proof

- Proof:

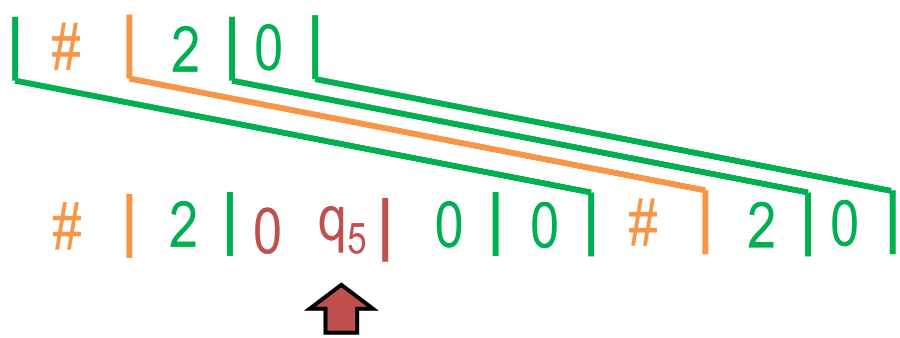
(1) S first constructs an instance P' of the MPCP;

(1.3) The head moves to the left

For each a, b, c $\in \Gamma$ and q, r $\in$ Q, where q $\neq$ q_{reject},

if δ**(q, a) = (r, b, L),** then put $\left[\frac{cqa}{rcb}\right]$ into P' , c is the element on the left of q

P'



Theorem 5.15 proof

- Proof:

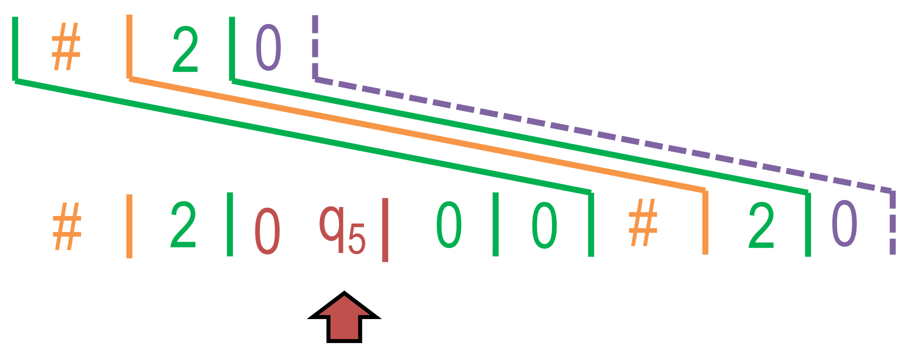
  (1) S first constructs an instance P′ of the MPCP;

  (1.3) The head moves to the left

  Suppose $\delta(q_5, 0) = (q_9, 2, L)$, remove the old 0

  if $\delta(q, a) = (r, b, L)$, then put $\left[\frac{cqa}{rcb}\right]$ into P′ , c is the element on the left of q

P′



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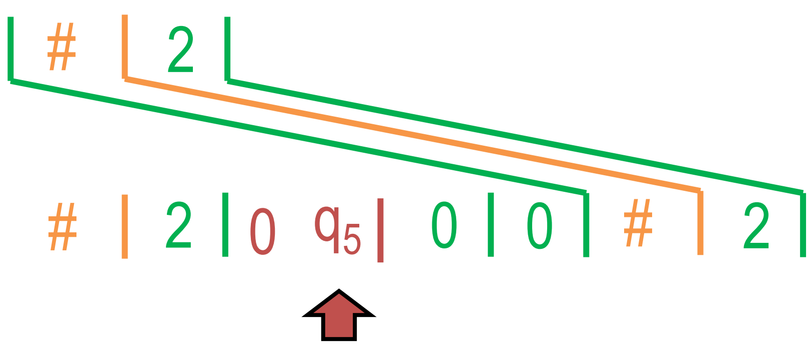
P′



Theorem 5.15 proof

- Proof:

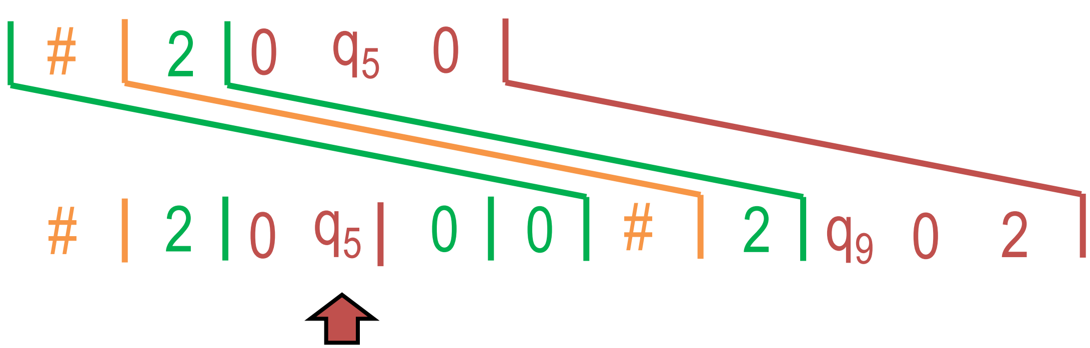
(1) S first constructs an instance P’ of the MPCP;

(1.3) The head moves to the left

Suppose $\delta(q_5, 0) = (q_9, 2, L)$, put $\left[\frac{0q_50}{q_902}\right]$ into P’

if $\delta(q, a) = (r, b, L)$, then put $\left[\frac{cqa}{rcb}\right]$ into P’ , c is the element on the left of q

P’



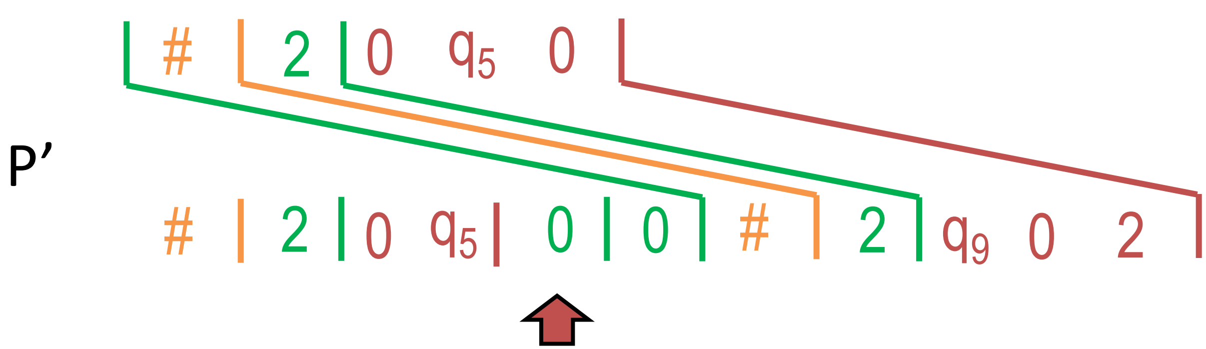
Theorem 5.15 proof

- Proof:

(1) S first constructs an instance P' of the MPCP;

(1.3) The head moves to the left

keep putting $\left[\frac{0}{0}\right]$, $\left[\frac{1}{1}\right]$, $\left[\frac{2}{2}\right]$, $\left[\frac{\sqcup}{\sqcup}\right]$, $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{\sqcup\#}\right]$ into P'



Theorem 5.15 proof

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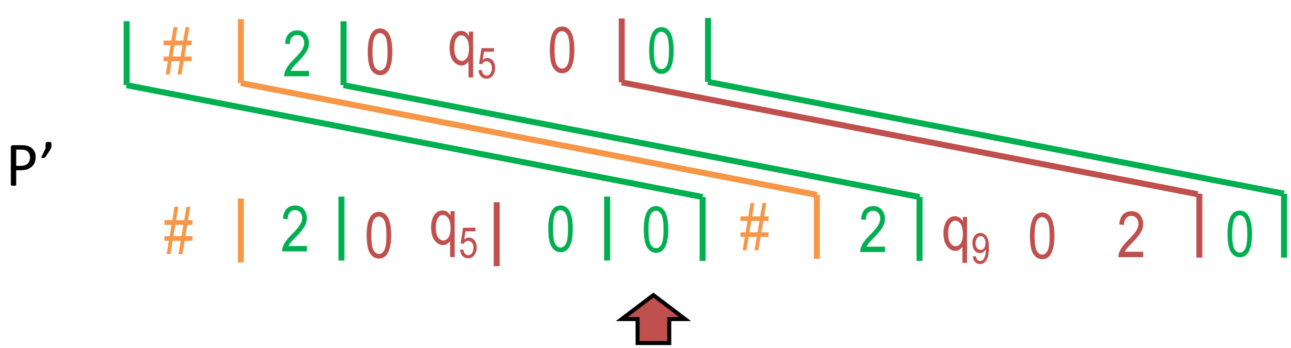

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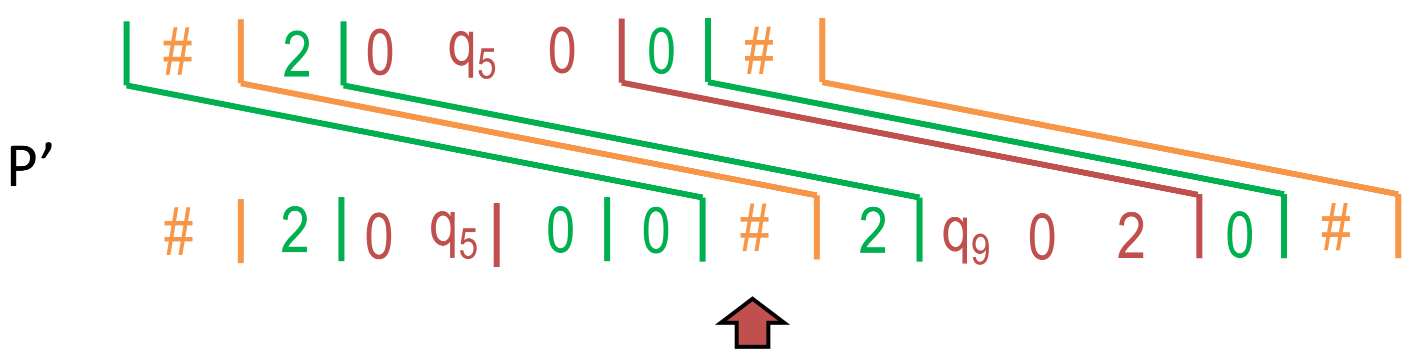


Theorem 5.15 proof

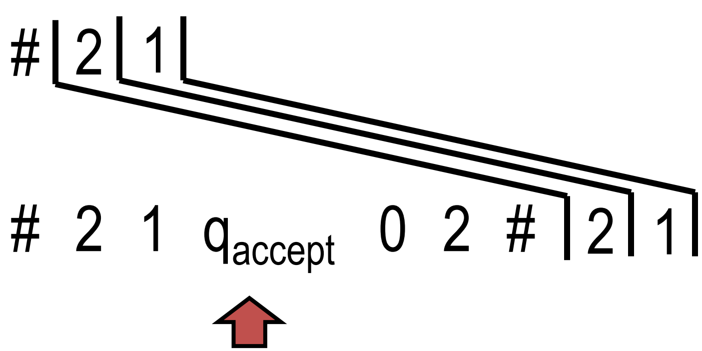
- Proof:

  (1) S first constructs an instance P' of the MPCP;

  keep putting something into P' until M halts:

  - if M rejects, S also rejects, means no match in PCP;
  - if M accepts, add something to the top to match the bottom.

P' ......



Connect PCP match program with whether M accepts w

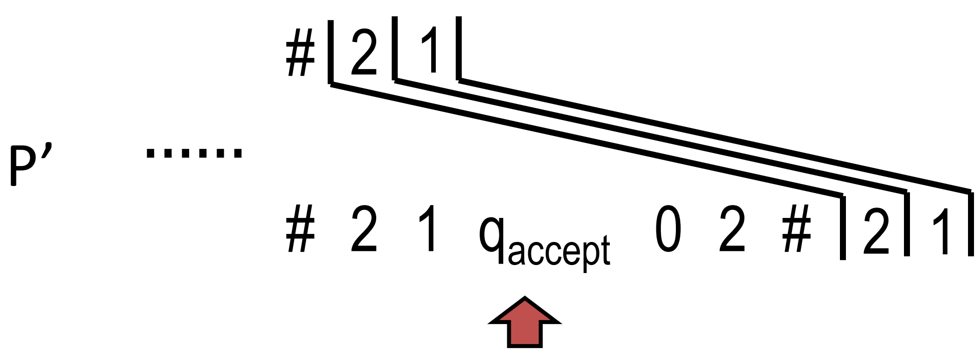
Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P’ of the MPCP;

  (1.6) For a$\in\Gamma$,

  put $\left[\frac{aq_{accept}}{q_{accept}}\right]$ and $\left[\frac{q_{accept}a}{q_{accept}}\right]$ into P’



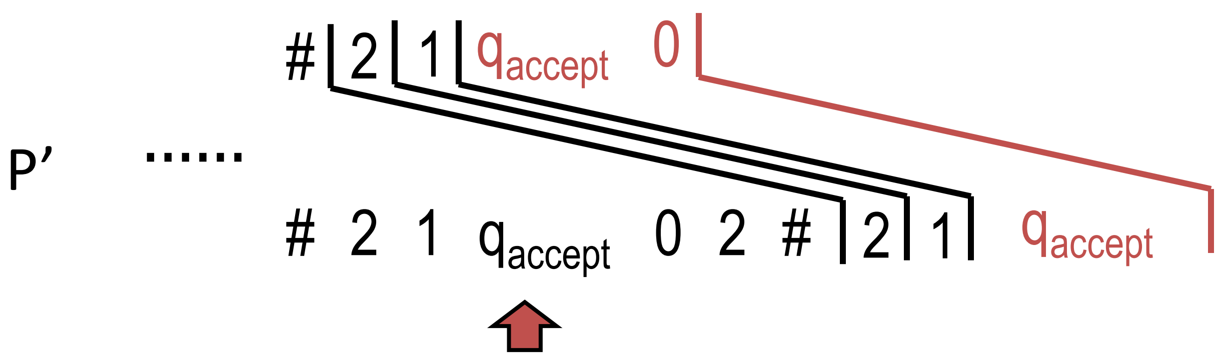
Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P' of the MPCP;

  (1.6) For a $\in \Gamma$,

  put $\left[\frac{a q_{accept}}{q_{accept}}\right]$ and $\left[\frac{q_{accept} a}{q_{accept}}\right]$ into P'



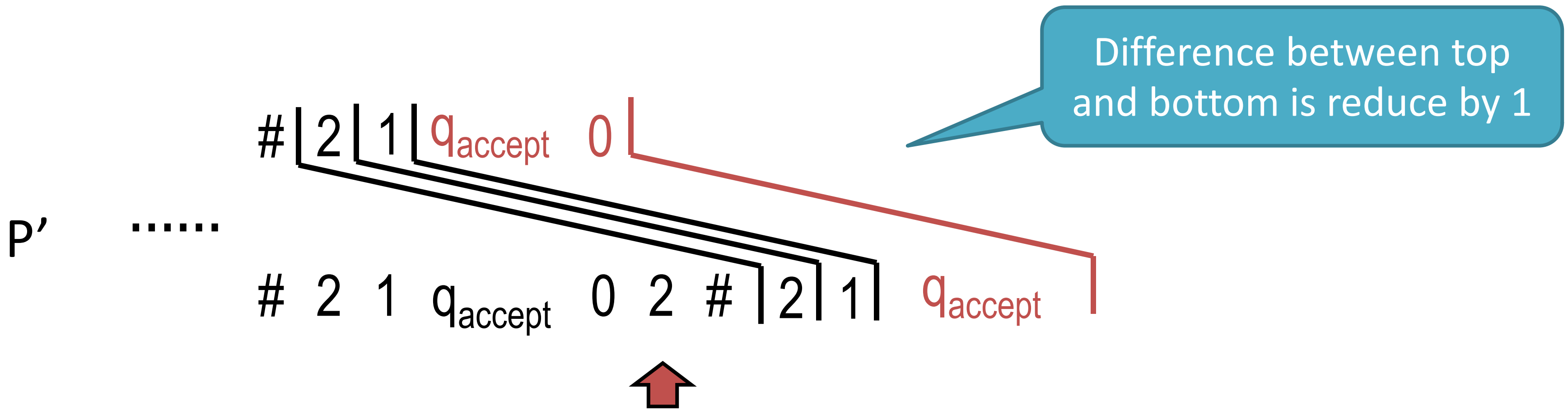
Theorem 5.15 proof

- Proof:

  (1) S first constructs an instance P′ of the MPCP;

  (1.6) For a $\in \Gamma$,

  put $\left[\dfrac{aq_{accept}}{q_{accept}}\right]$ and $\left[\dfrac{q_{accept}a}{q_{accept}}\right]$ into P′



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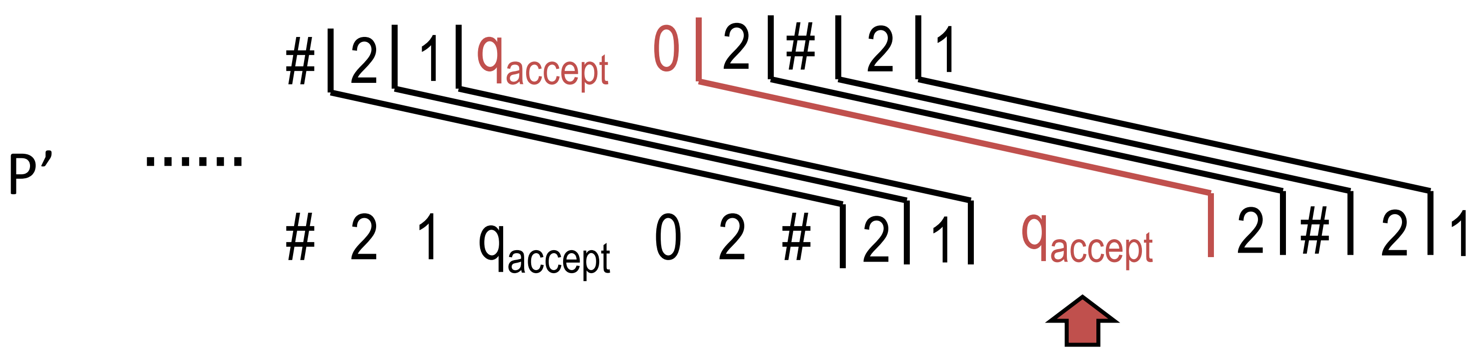

Theorem 5.15 proof

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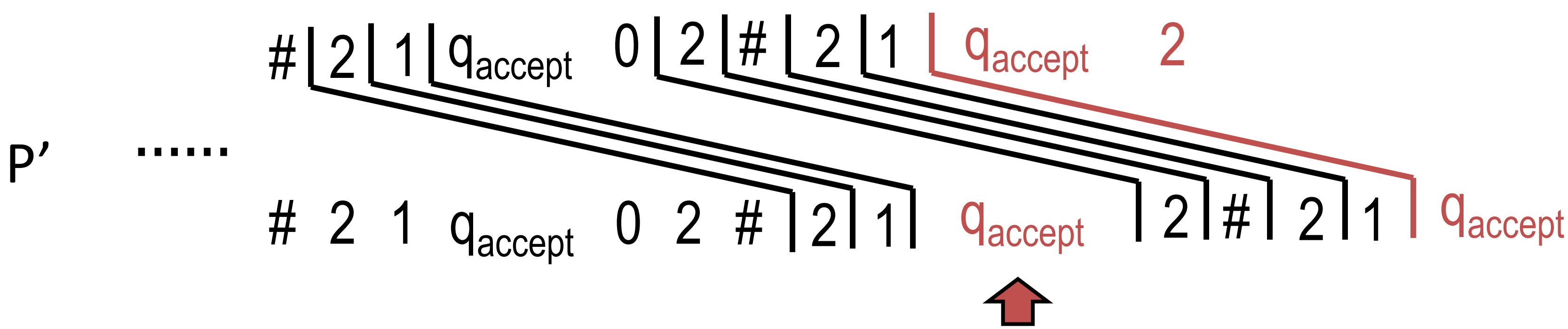


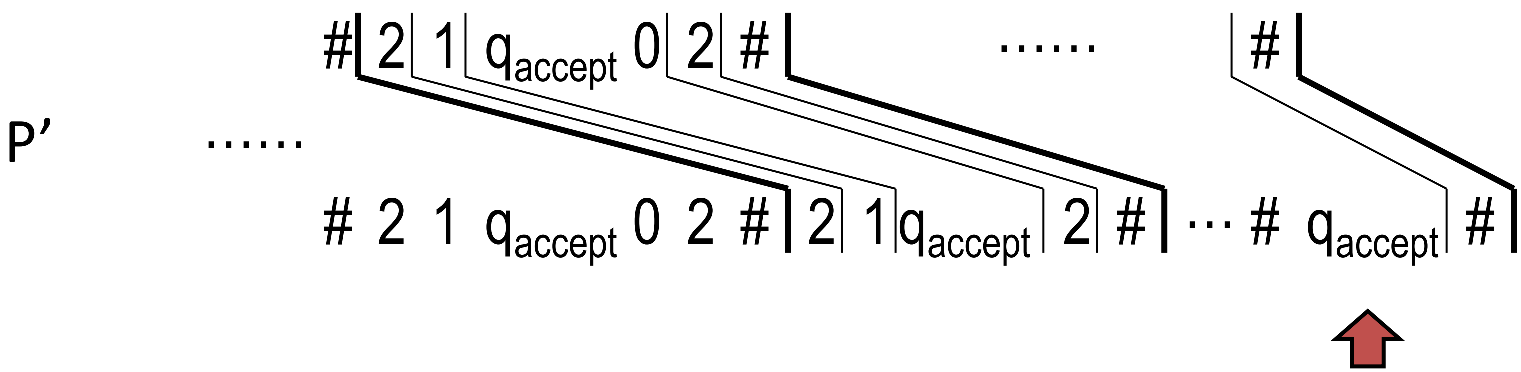
Theorem 5.15 proof

- Proof:

(1) S first constructs an instance P′ of the MPCP;

(1.6) For $a \in \Gamma$,

Keep doing until the top and bottom difference is 1.



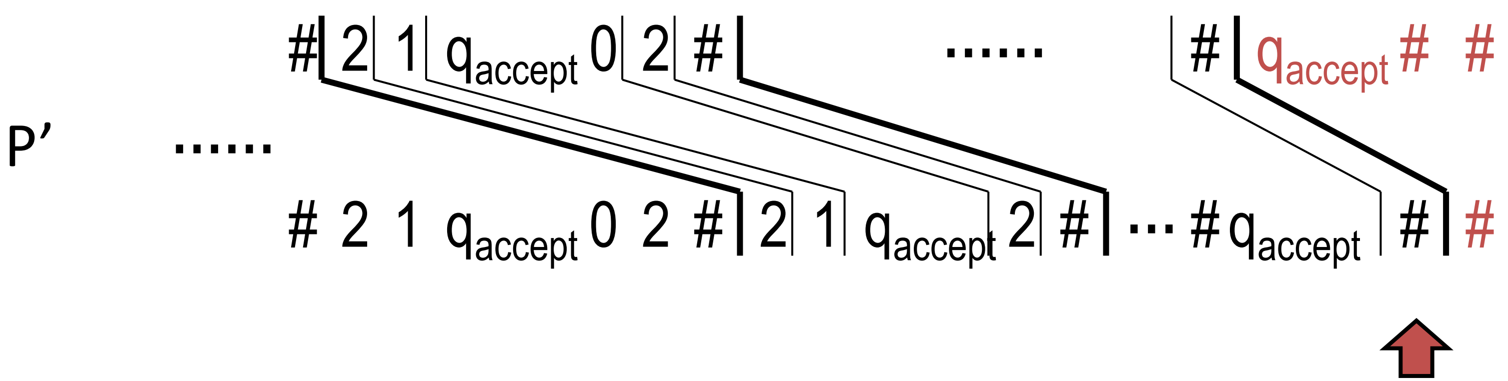
Theorem 5.15 proof

- Proof:

(1) S first constructs an instance P’ of the MPCP;

(1.7) Finish the match

put $\left[\frac{q_{accept}\#\#}{\#}\right]$ into P’



Theorem 5.15 proof

- Proof:

  (2) Transfer P' into P

  P' is MPCP = {⟨P⟩| P is an instance of the PCP with a match that starts with the first domino}.

  Suppose $u=u_1u_2\ldots u_n$ to be any string of length n, define

  - $\star u \ = *u1*u2 *\ldots*u3$
  - $u\star \ = u_1*u_2 *\ldots*u_3*$
  - $\star u\star = *u_1*u_2 *\ldots*u_3*$

Theorem 5.15 proof

$\star u\ \ = *u1*u2 *\ldots*u3$

$u\star\ \ = u_1*u_2 *\ldots*u_3*$

$\star u\star = *u_1*u_2 *\ldots*u_3*$

- Proof:

(2) Transfer P' into P

If P' = $\left\{\left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right]\right\}$, then let

P = $\left\{\left[\frac{\star t_1}{\star b_1 \star}\right], \left[\frac{\star t_2}{b_2 \star}\right], \left[\frac{\star t_3}{b_3 \star}\right], \ldots, \left[\frac{\star t_k}{b_k \star}\right], \left[\frac{*\Delta}{\Delta}\right]\right\}$

First domino: The only element as beginning

The match in P must be in shape as $\left[\frac{\star t_1}{\star b_1 \star}\right] \ldots \left[\frac{*\Delta}{\Delta}\right]$

Theorem 5.15 proof

- Proof:

Suppose PCP is decidable

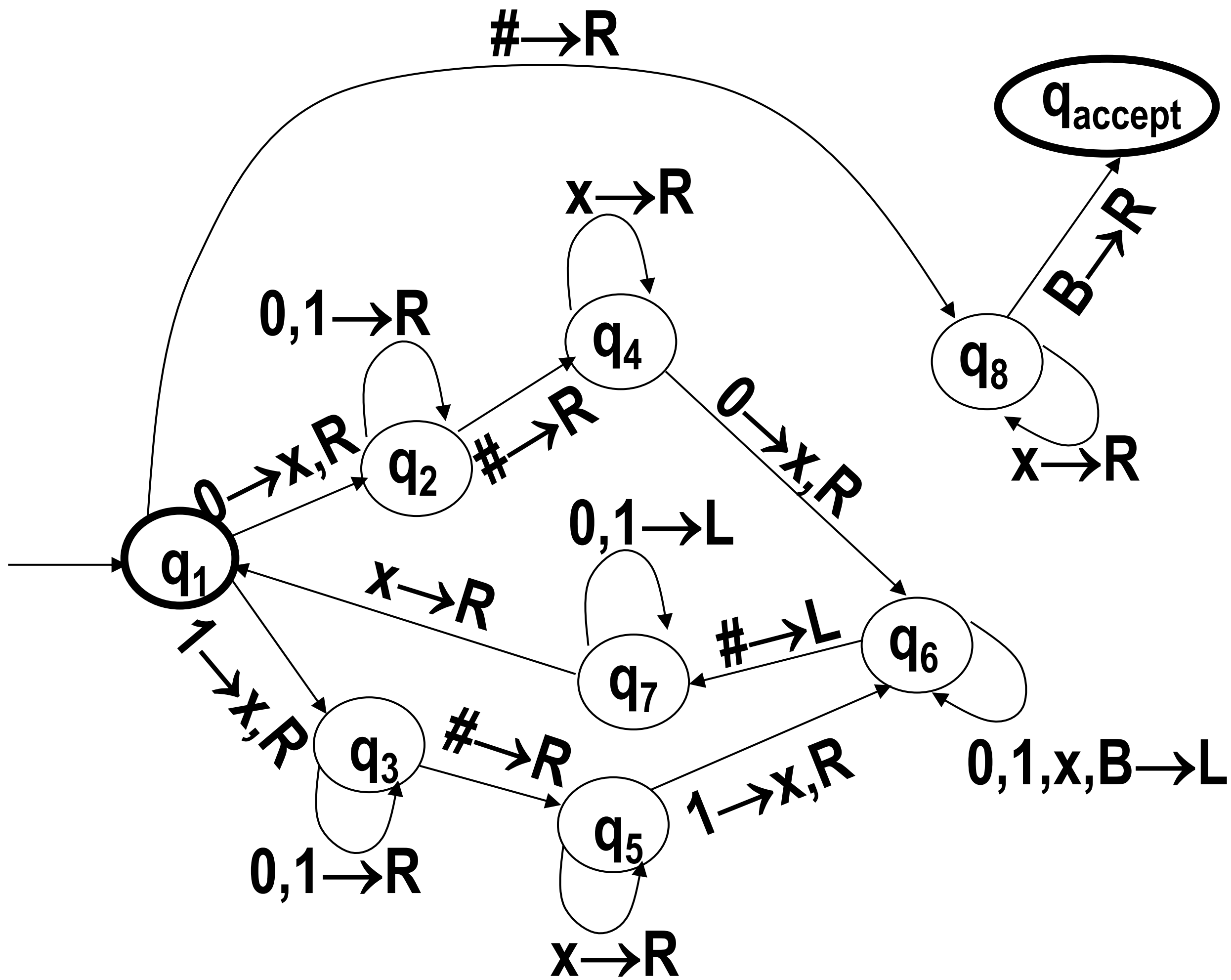
We construct TM S using PCP match or not to decide A_{TM}

- (1) S first constructs an instance P' of the MPCP;
- (2) Transfer P' into P;

Theorem 4.11: A_{TM} is undecidable. Contradiction!

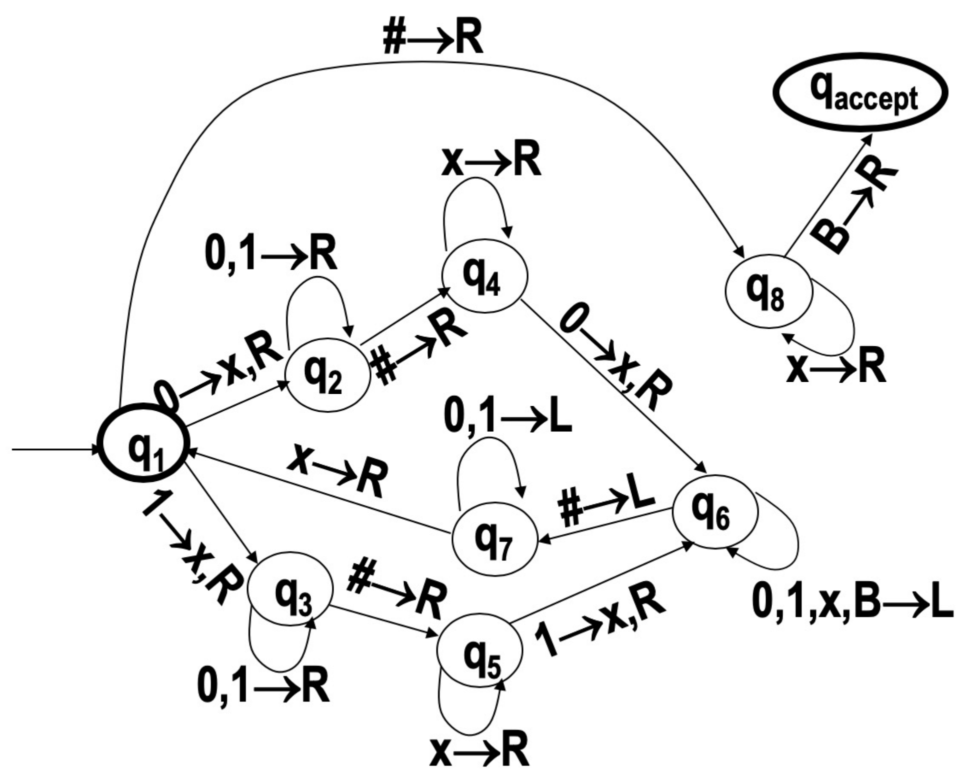
The suppose is wrong. Thus PCP is undecidable.

Example of TM: B={ w#w | w∈{0,1}* }

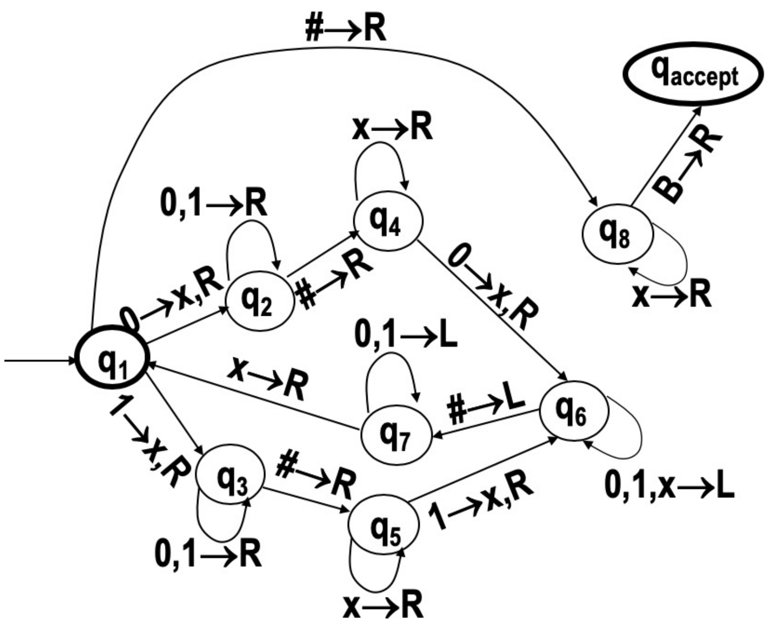


Example of PCP to A_{TM}

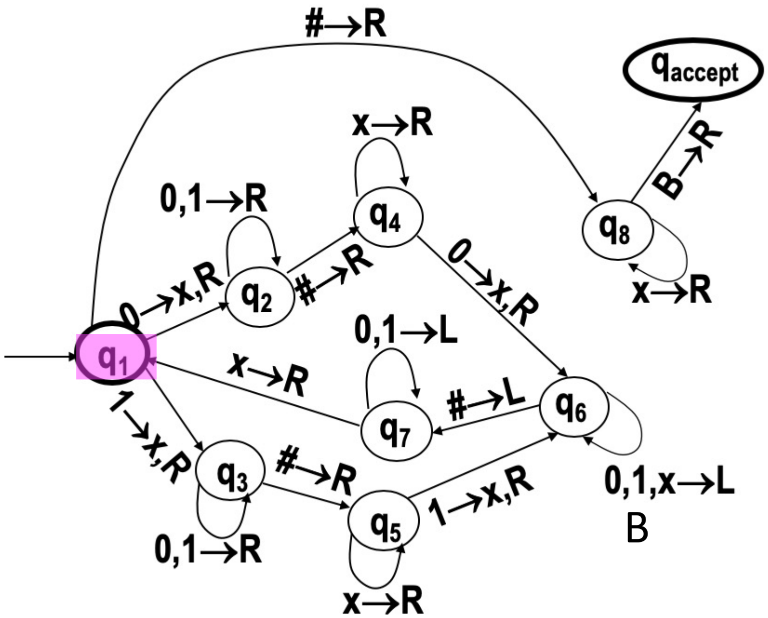
- $M = (Q,\Sigma,\Gamma,\delta,q_0,q_{accept},q_{reject})$
  - Q= {q0, q1, q2}
  - Σ = {0, 1, x, #},
  - Γ = {0, 1, x, #, B}
  - q0={q1}
  - $q_{accept} = q_{accept}$
  - q_{reject} = all not q_{accept}



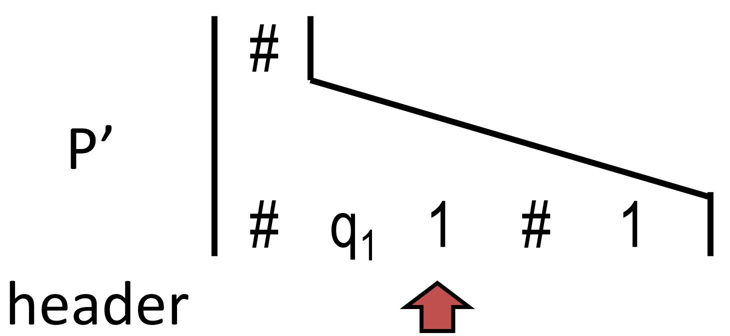
Example of PCP to A_{TM} , input is 1#1



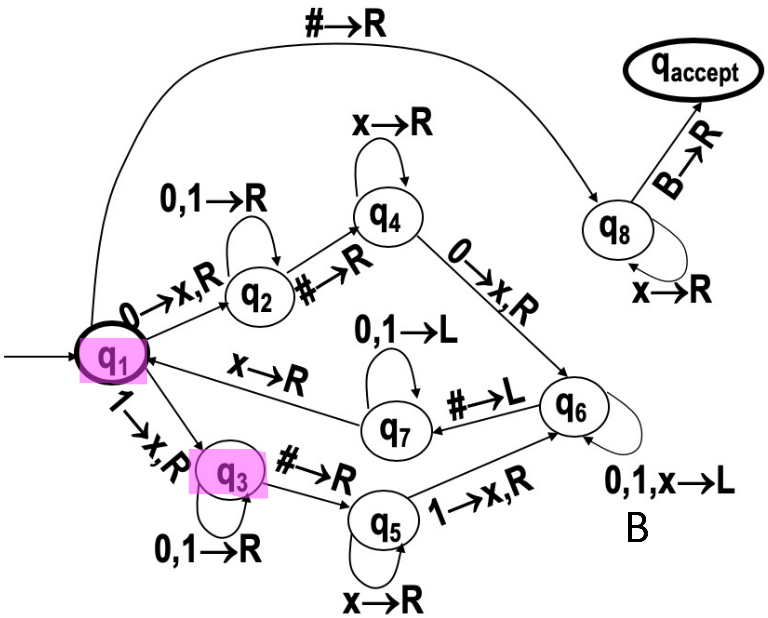
Example of PCP to A_{TM} , input is 1#1



Put $\left[\frac{\#}{\#q_0w_1w_2\ldots w_n\#}\right]$ into P' as $\left[\frac{t_1}{b_1}\right]$



Example of PCP to A_{TM} , input is x#1



if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P’

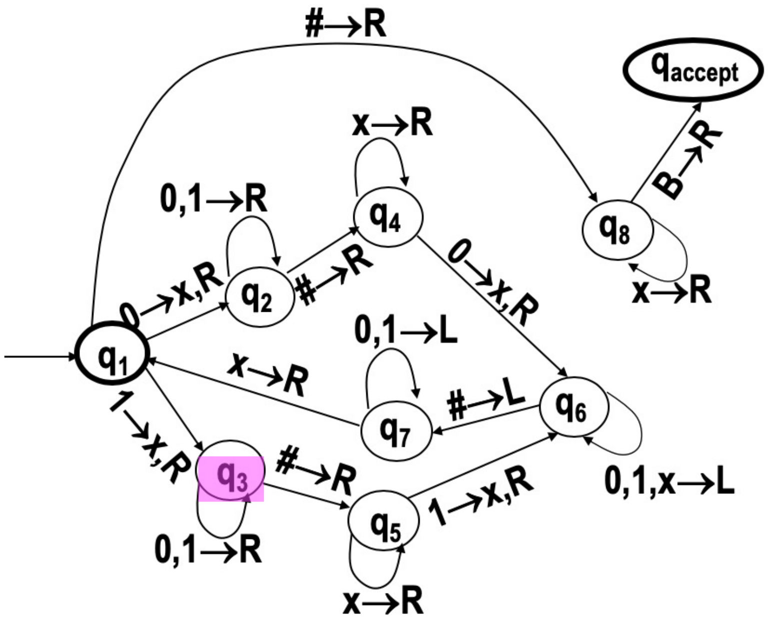
P’

\# q_0 1

\# q_1 1 # 1 x q_3

header

Example of PCP to A_{TM} , input is x#1



if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

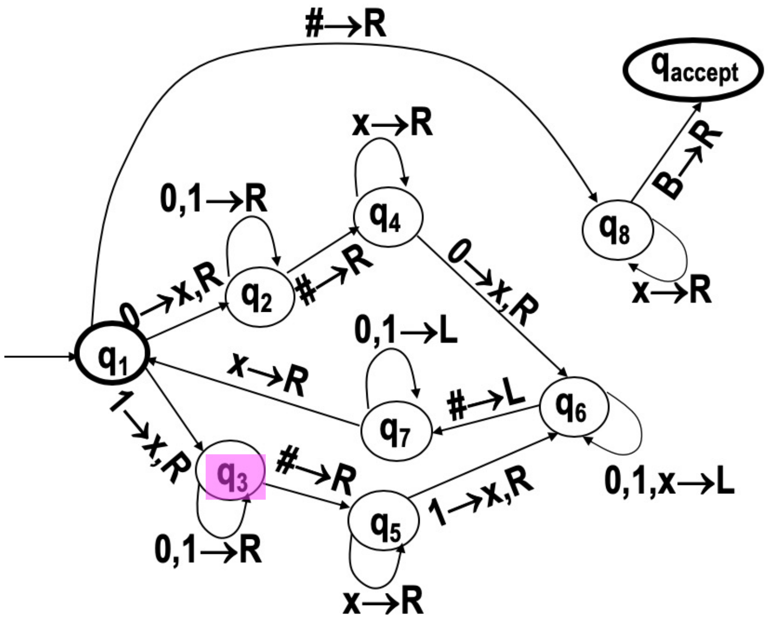
P'

| q_0 1 |

q_1 1 # 1 | x q_3 |

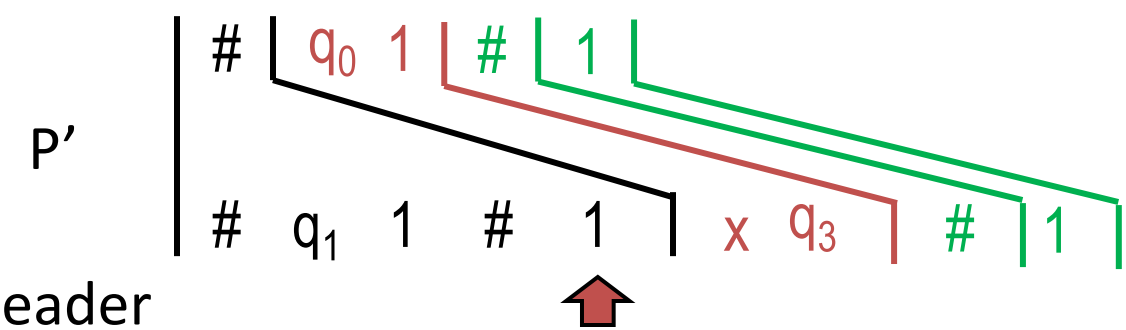
header

Example of PCP to A_{TM} , input is x#1



if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'



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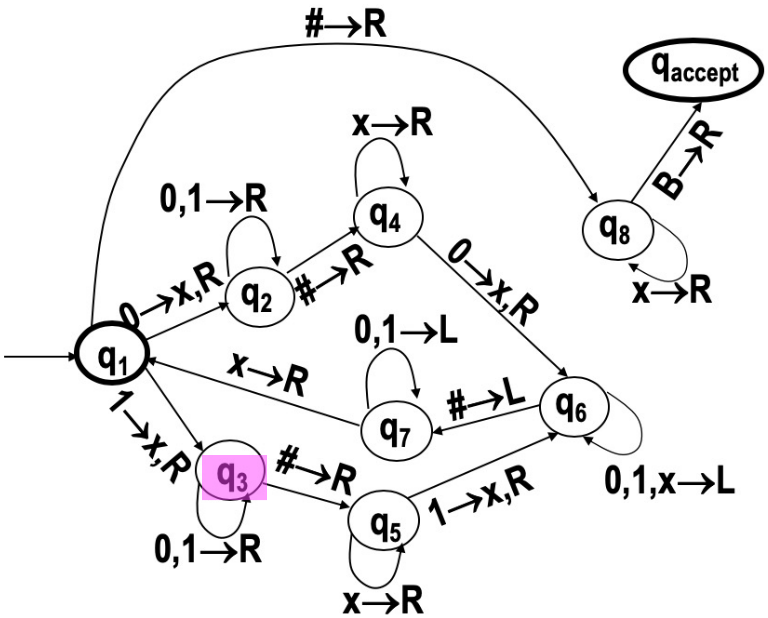
Example of PCP to A_{TM} , input is x#1



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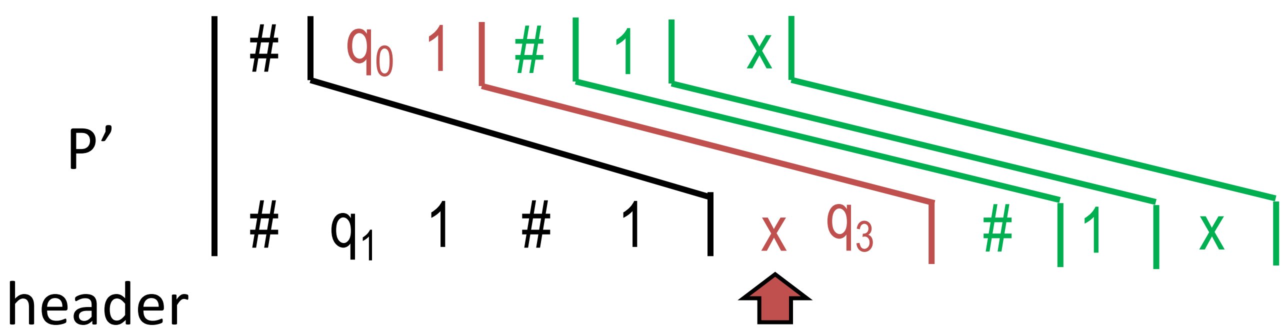
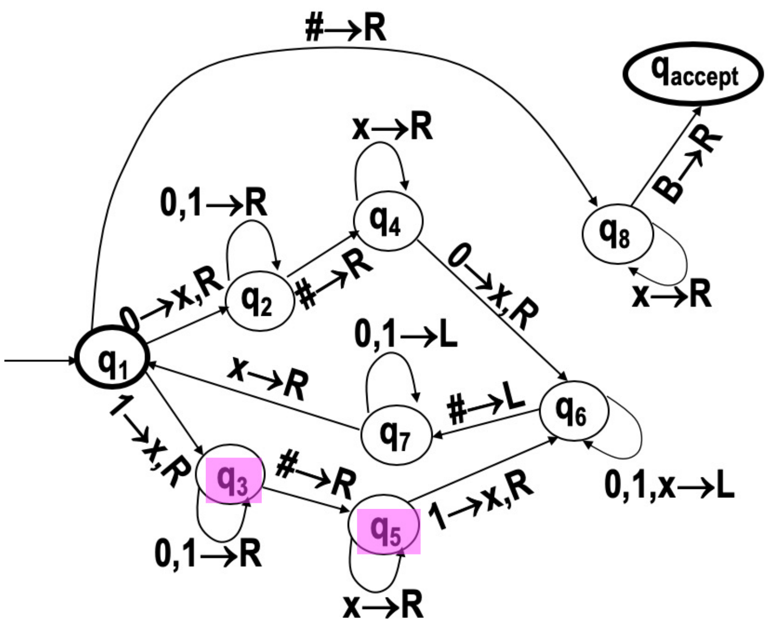


Example of PCP to A_{TM} , input is x#1



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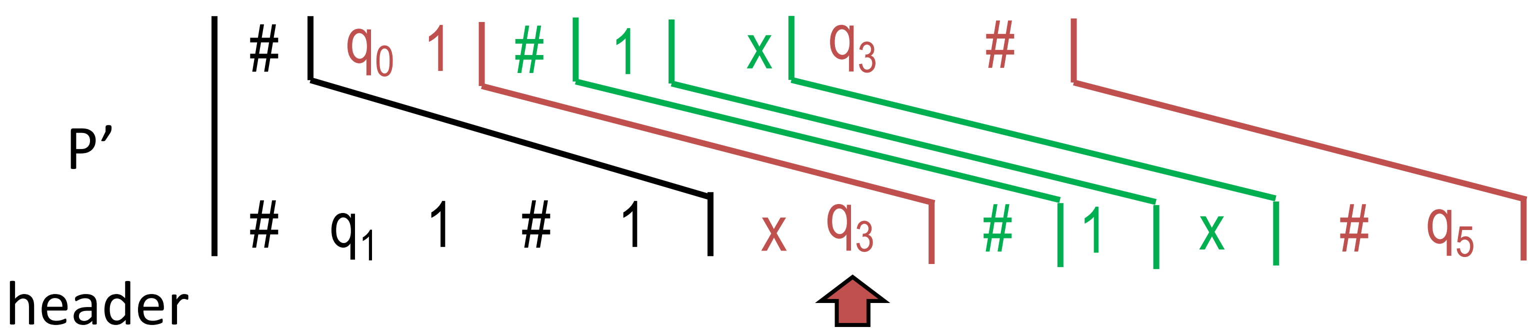
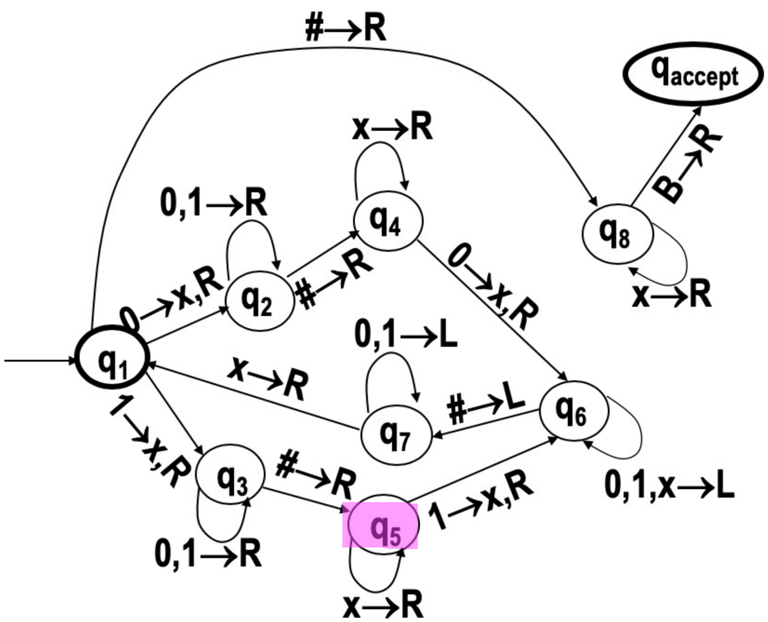


Example of PCP to A_{TM} , input is x#1



if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

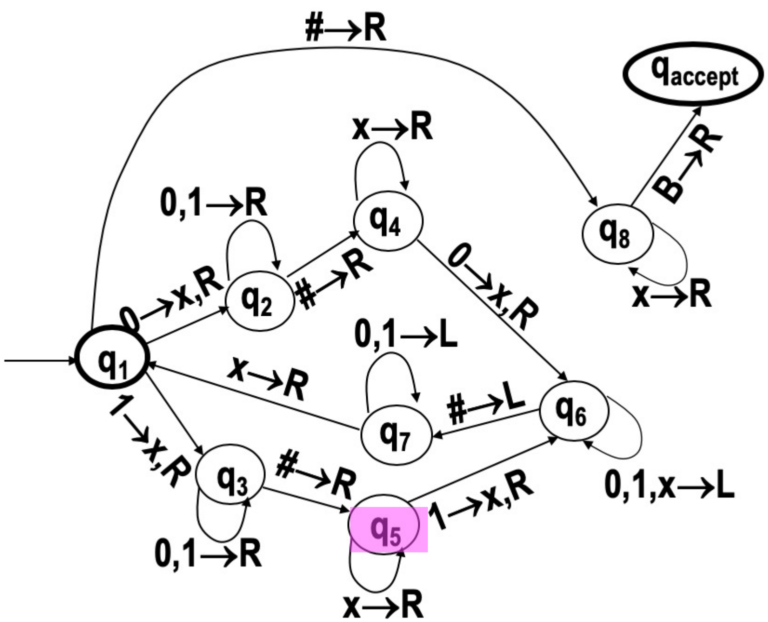
P'

$\#$ | q_0 1 | # | 1 | x | q_3 # |

$\#$ q_1 1 # 1 | x q_3 | # | 1 | x | # q_5 |

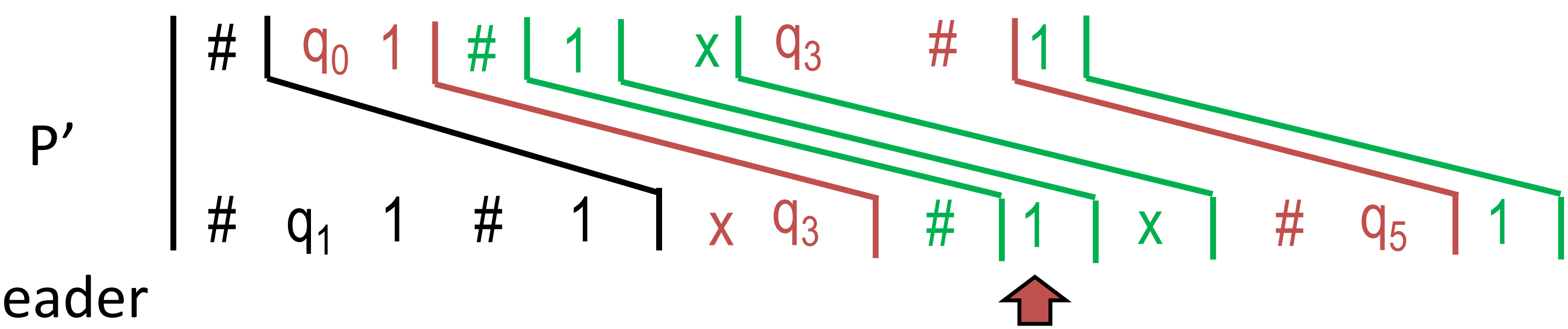
header

Example of PCP to A_{TM}, input is x#1

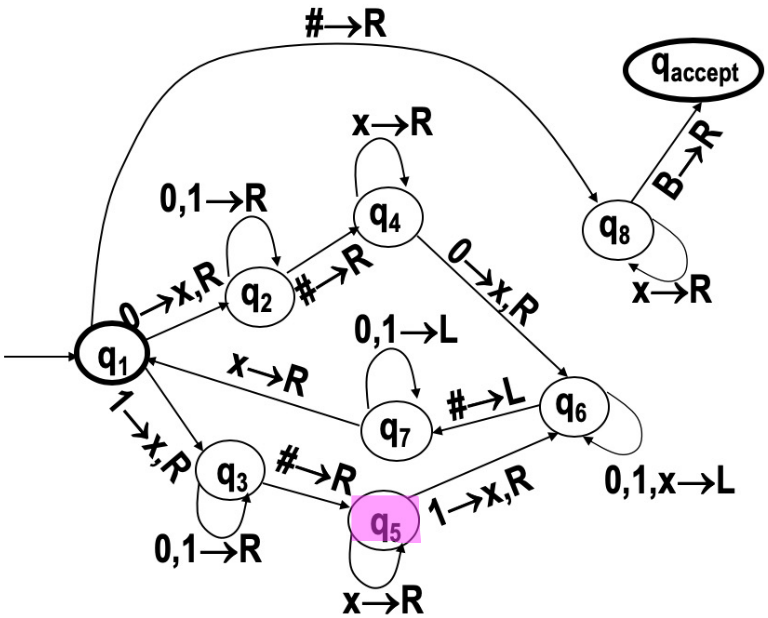


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'



Example of PCP to A_{TM} , input is x#1



if $\delta(q, a) = (r, b, R)$,

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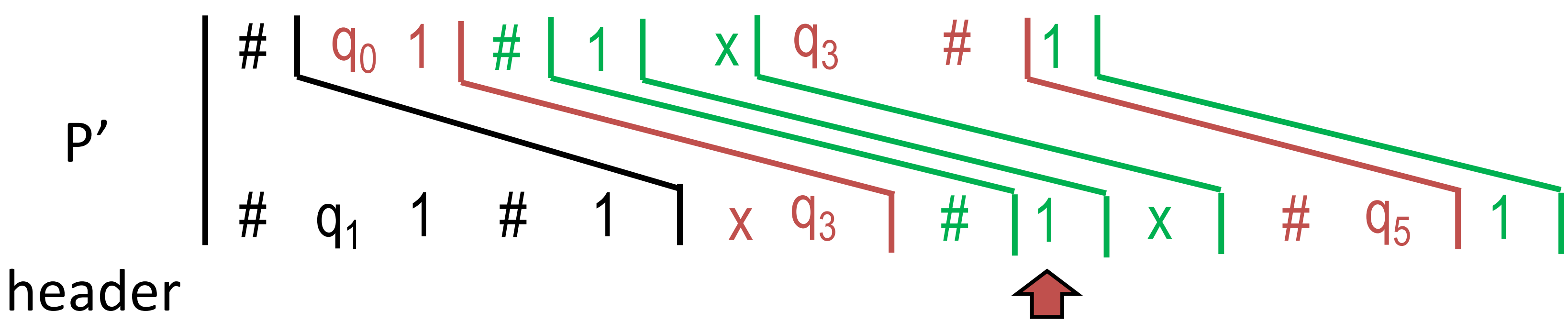
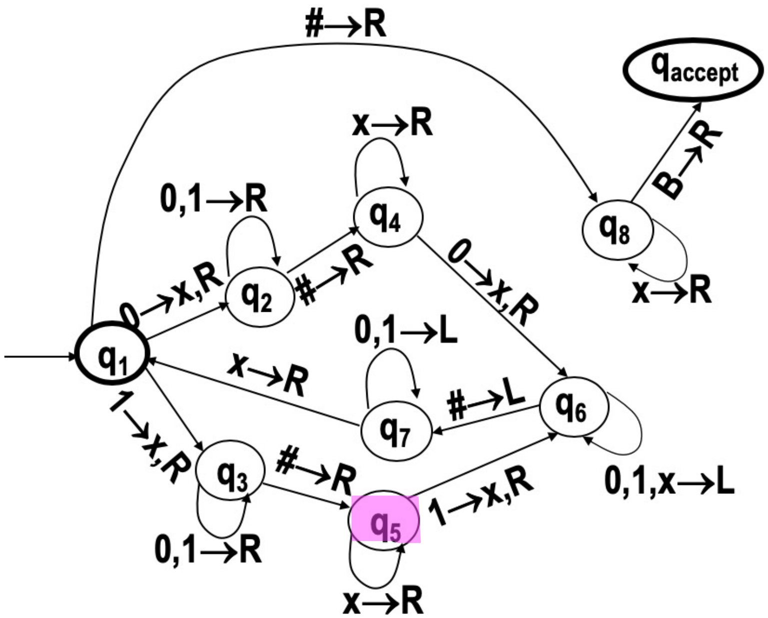


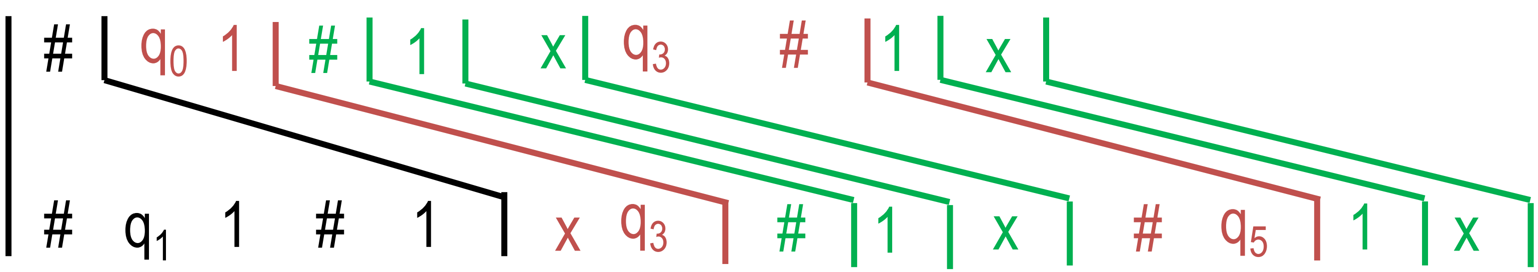
Example of PCP to A_{TM} , input is x#1



if $\delta(q, a) = (r, b, R)$,

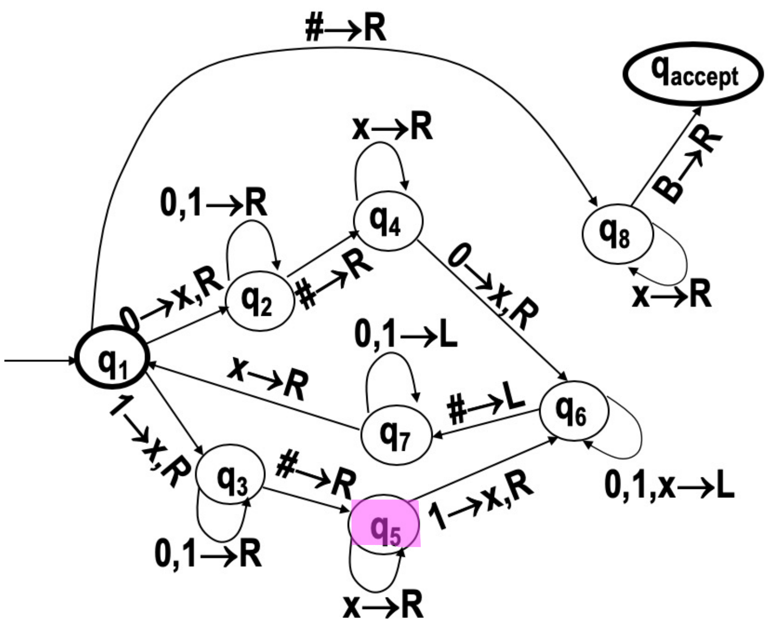
then put $\left[\frac{qa}{br}\right]$ into P’

P’



header

Example of PCP to A_{TM} , input is x#1



if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

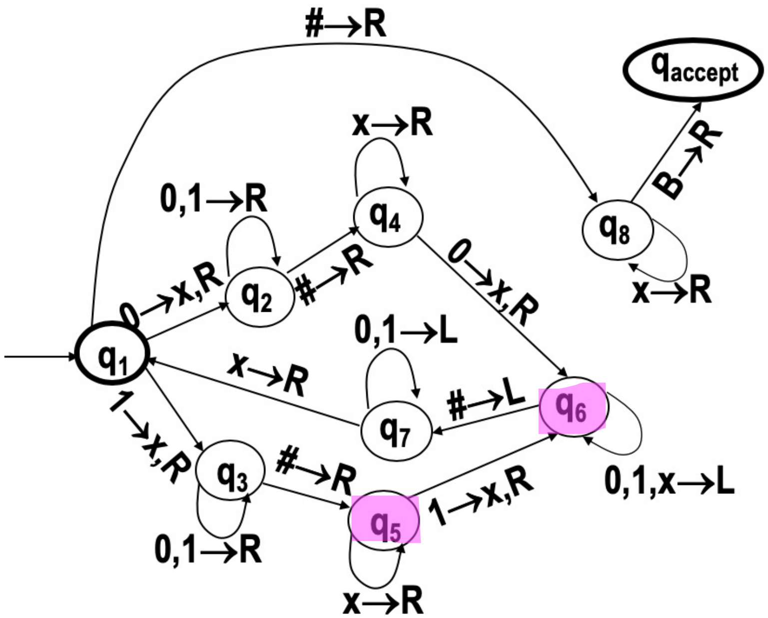
P'

q_0 1 # 1 x q_3 # 1 x

q_1 1 # 1 x q_3 # 1 x # q_5 1 x

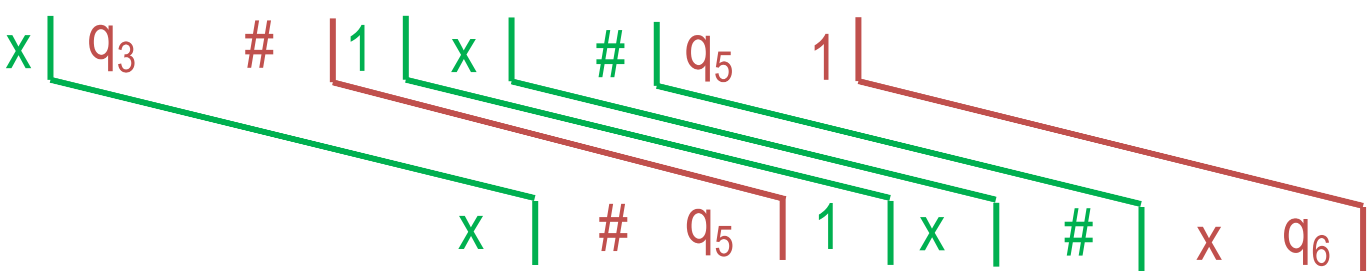
header

Example of PCP to A_{TM} , input is x#x

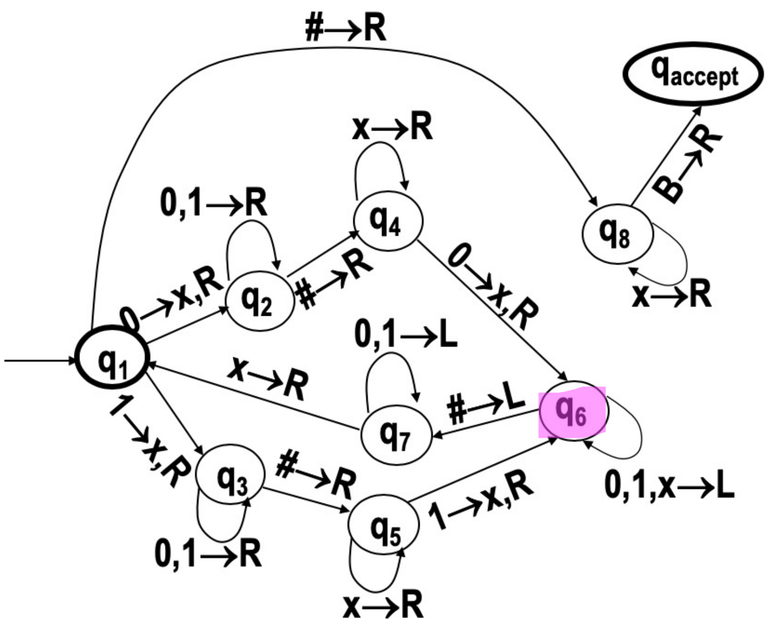


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

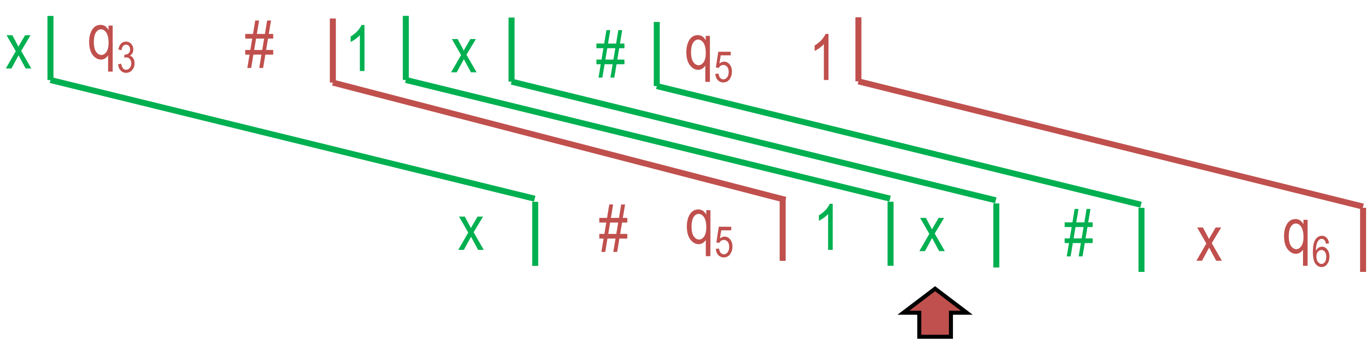


Example of PCP to A_{TM} , input is x#x

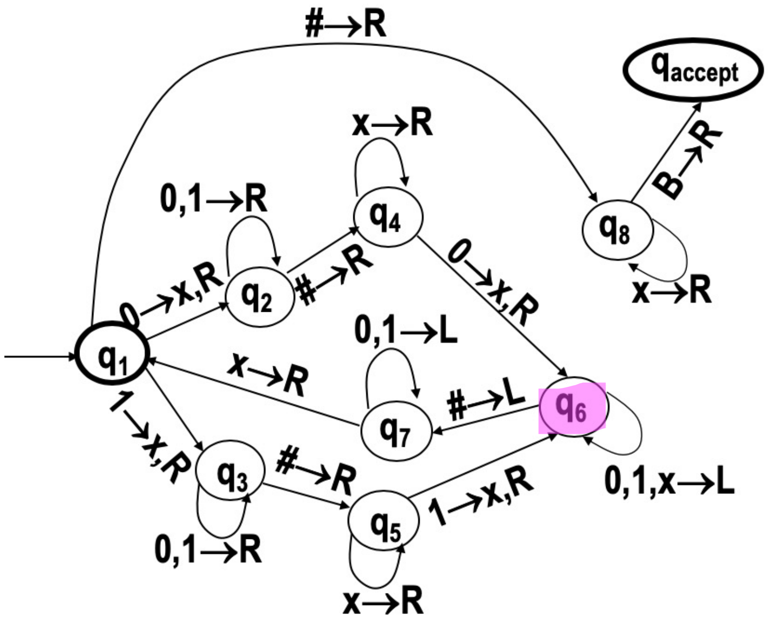


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P’



Example of PCP to A_{TM} , input is x#x



if $\delta(q, a) = (r, b, R)$,
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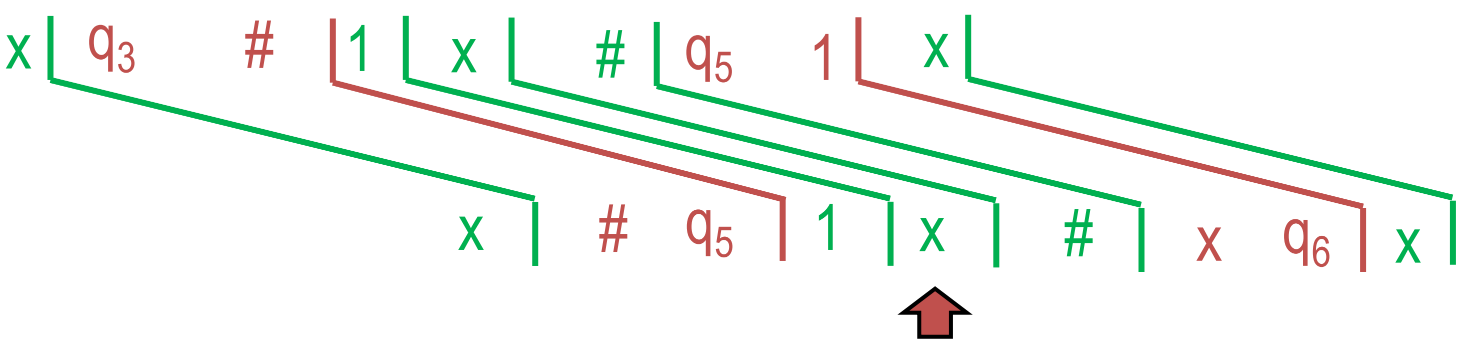
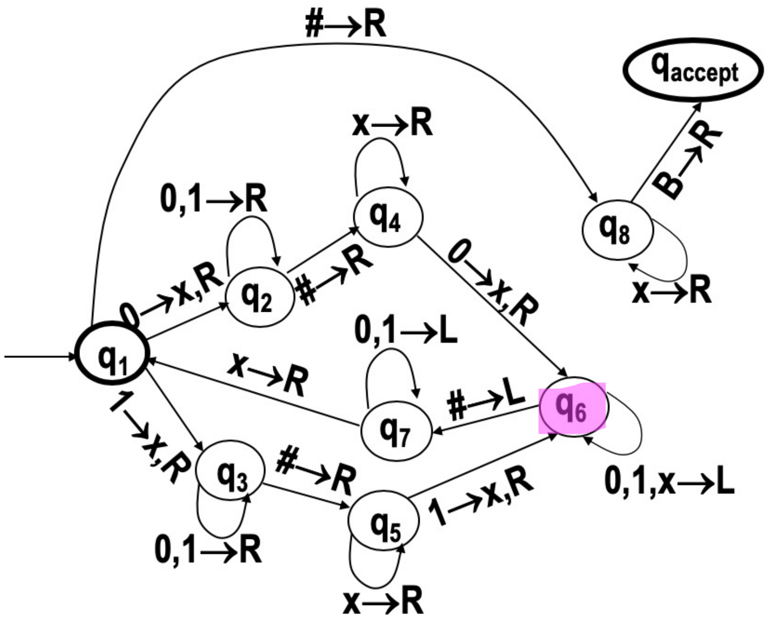


Example of PCP to A_{TM} , input is x#x



if $\delta(q, a) = (r, b, R)$,

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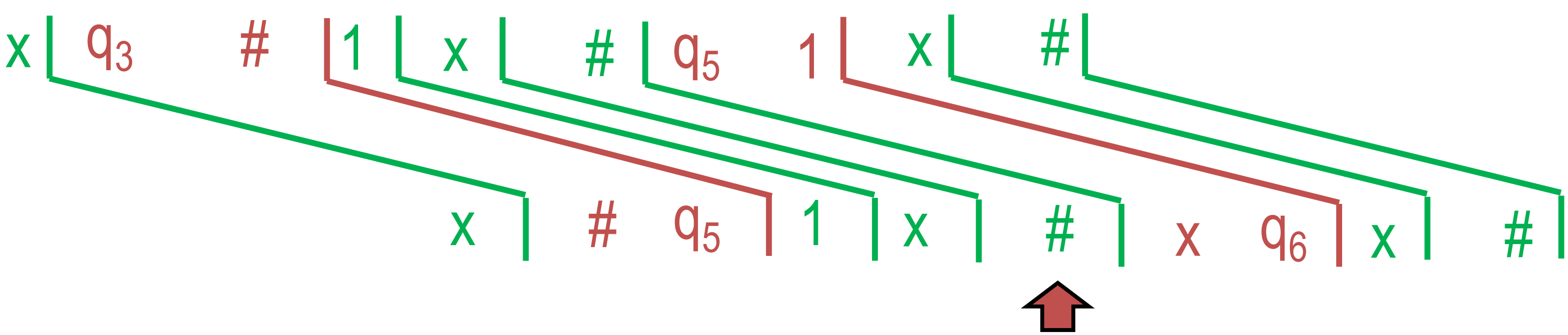
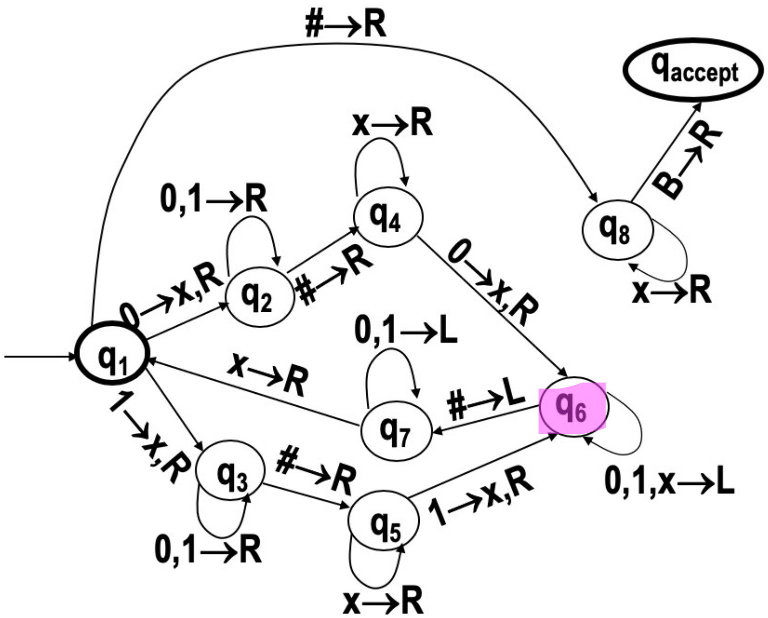


Example of PCP to A_{TM} , input is x#x



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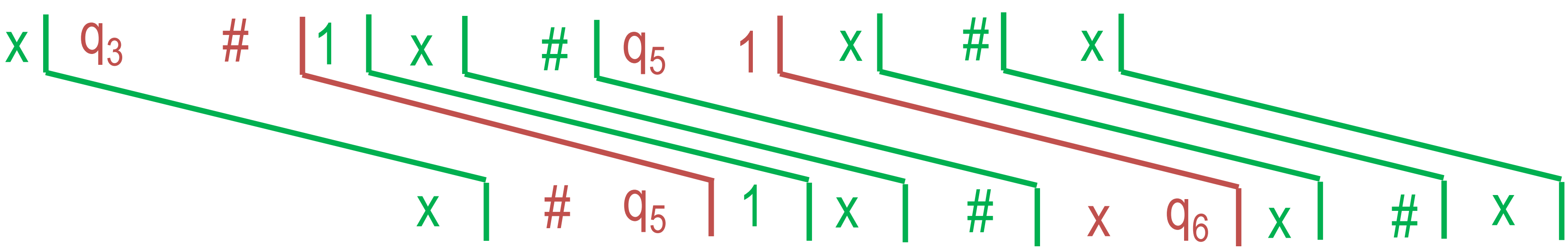
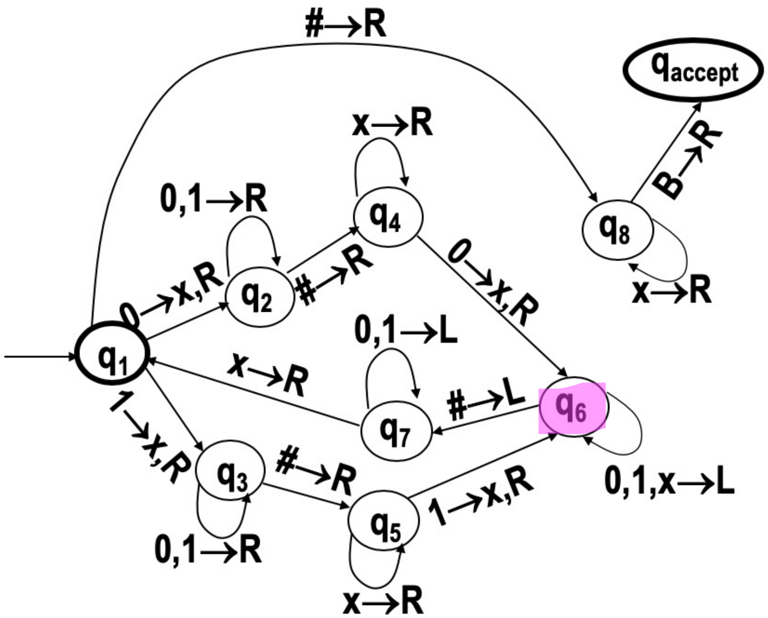
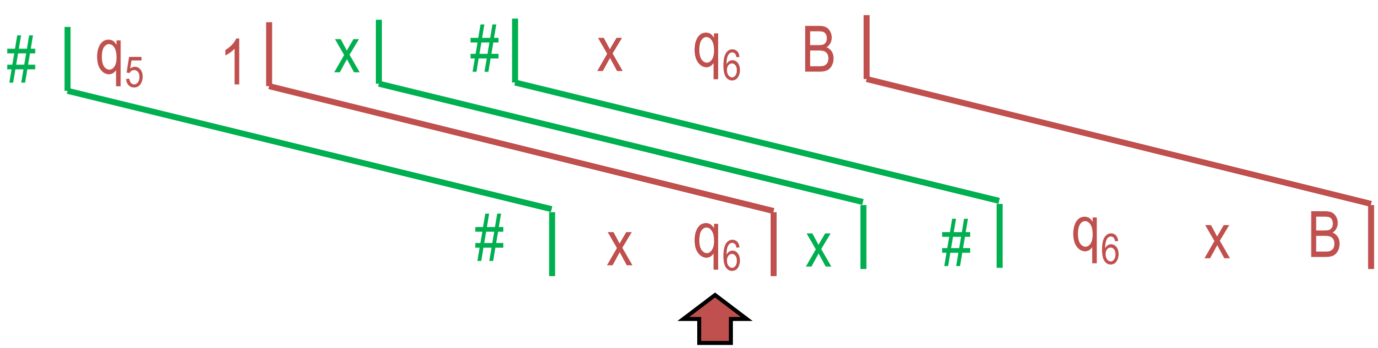


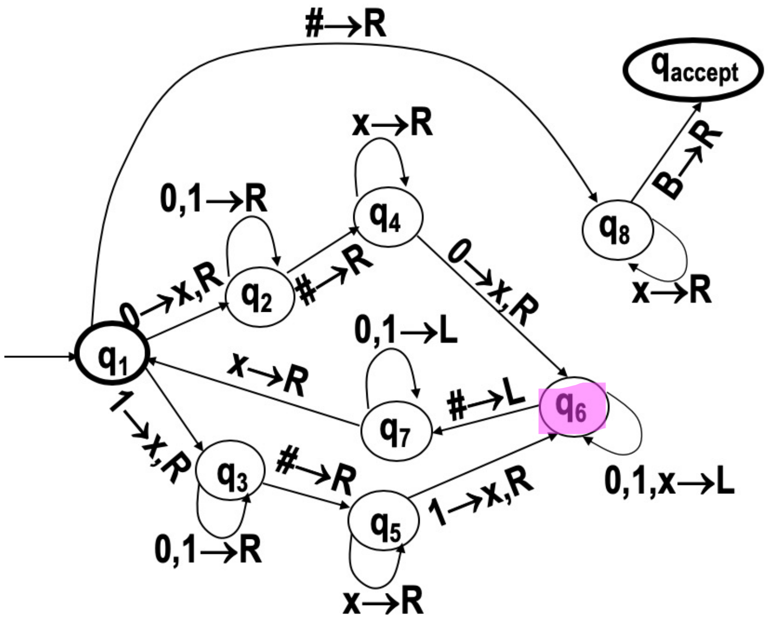
Example of PCP to A_{TM} , input is x#x



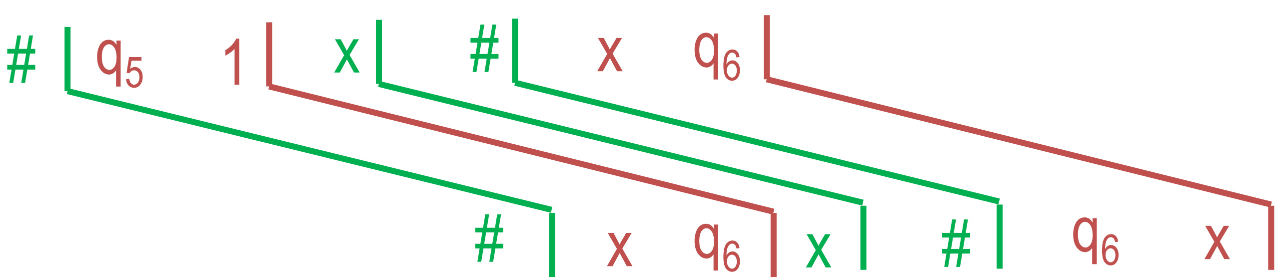
if **δ(q, a) = (r, b, L),** then put $\left[\frac{cqa}{rcb}\right]$ into P' , c is the element on the left of q



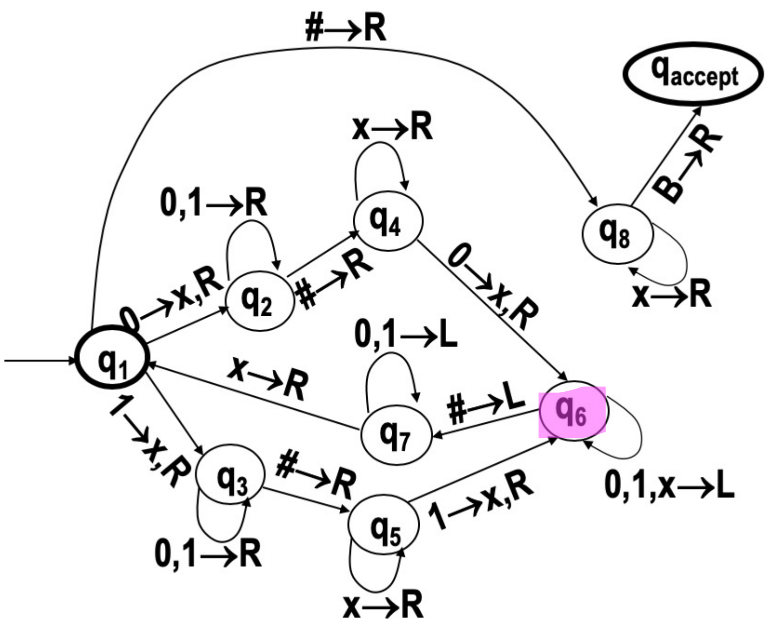
Example of PCP to A_{TM} , input is x#x



if δ**(q, a) = (r, b, L),** then put $\left[\frac{cqa}{rcb}\right]$ into P' , c is the element on the left of q



Example of PCP to A_{TM} , input is x#x



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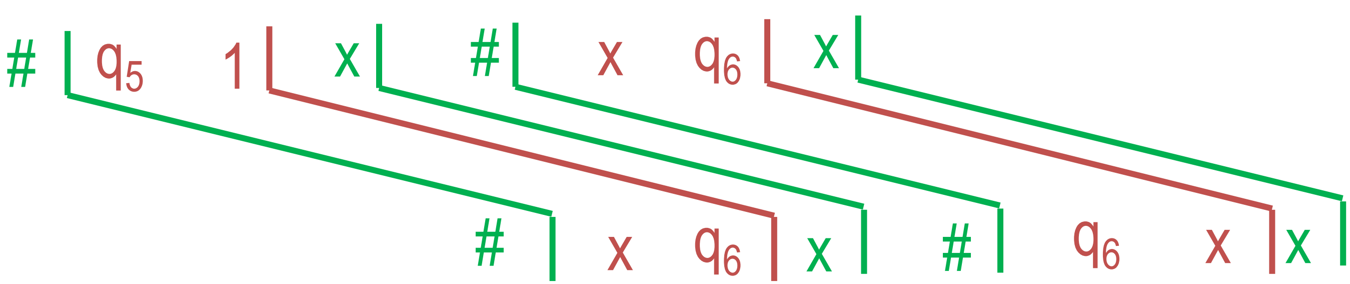
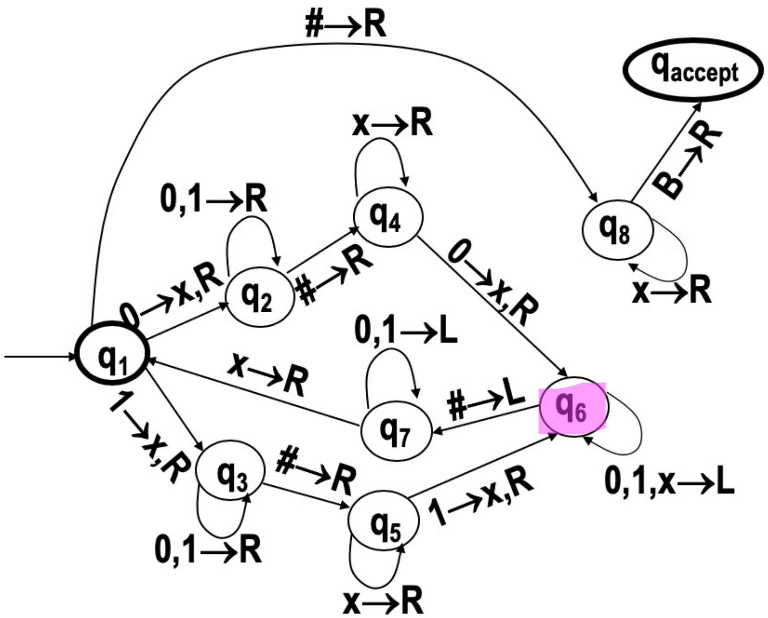


Example of PCP to A_{TM} , input is x#x



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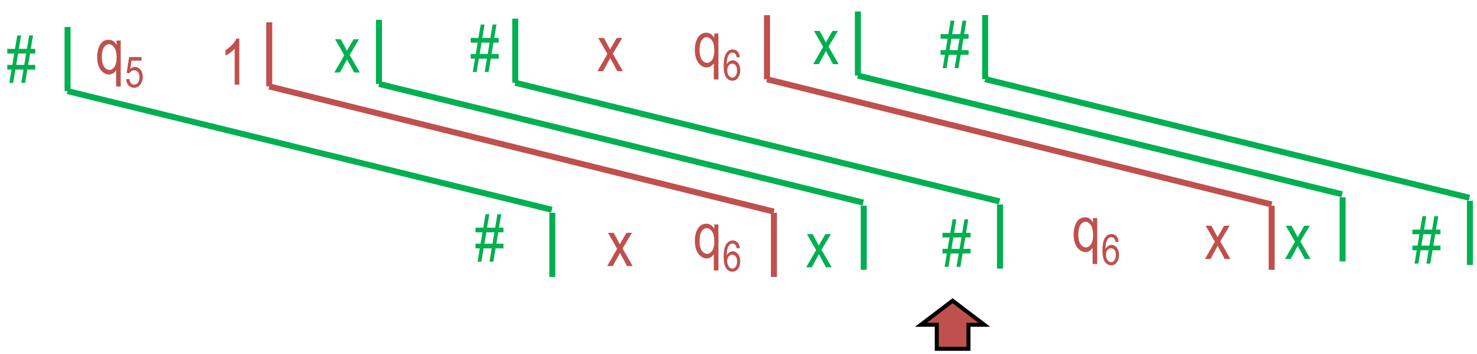
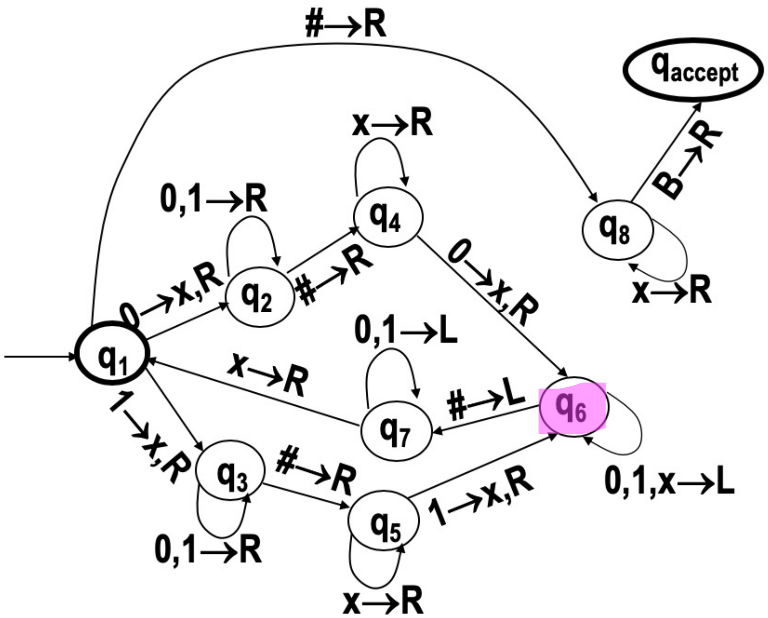


Example of PCP to A_{TM} , input is x#x



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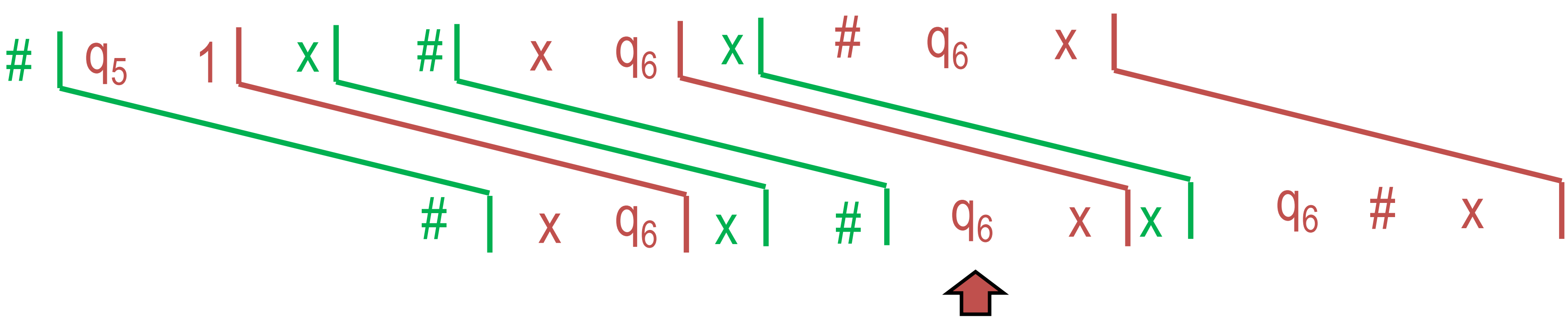
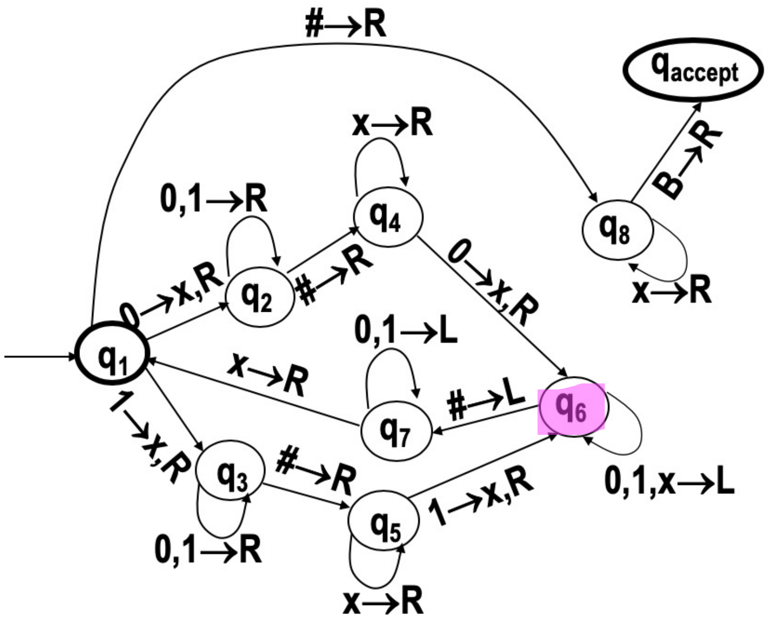


Example of PCP to A_{TM} , input is x#x



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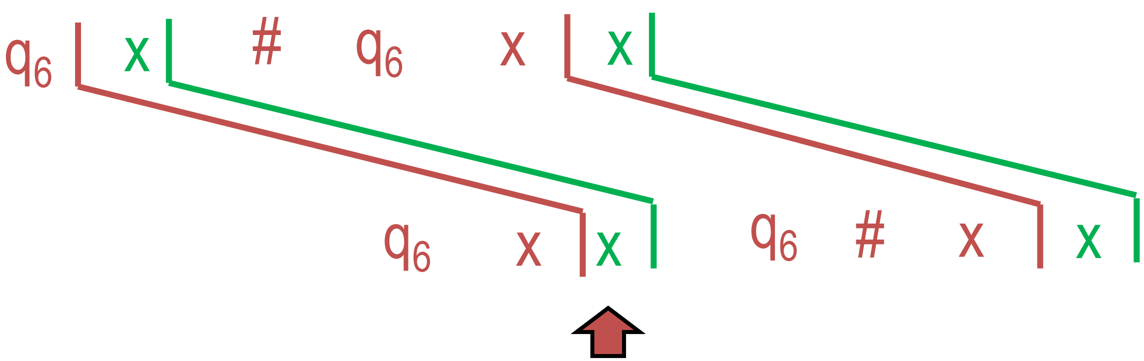
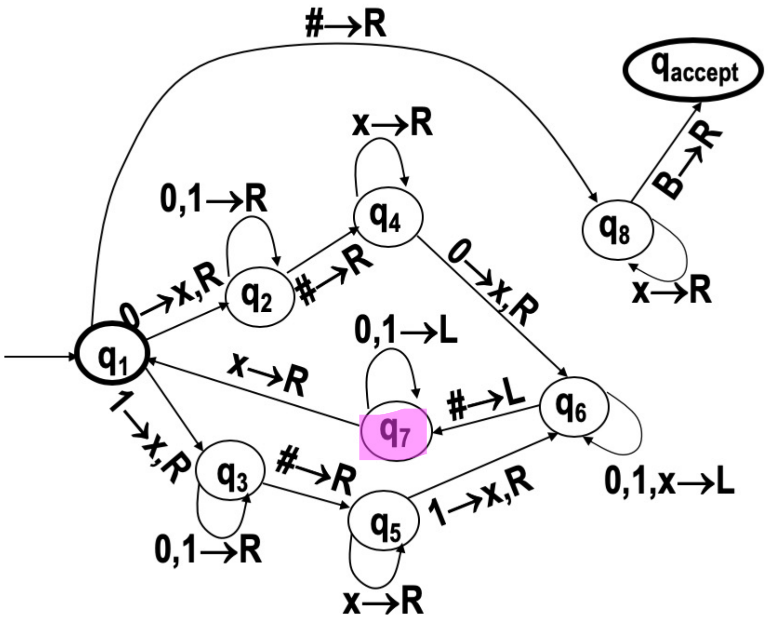


Example of PCP to A_{TM} , input is x#x



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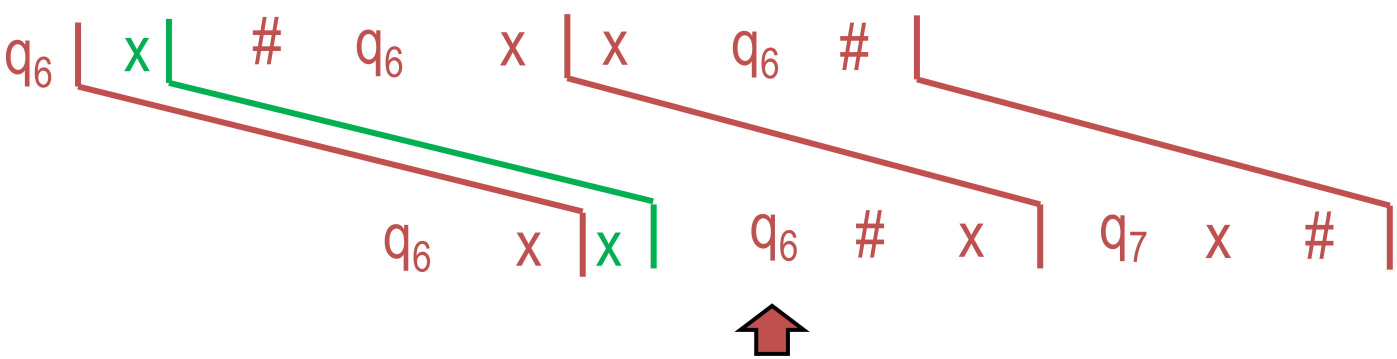
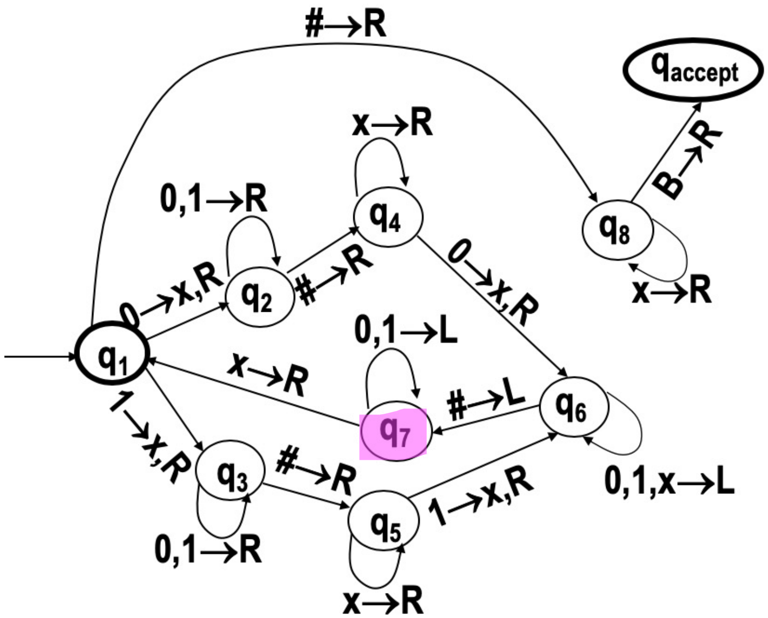


Example of PCP to A_{TM} , input is x#x



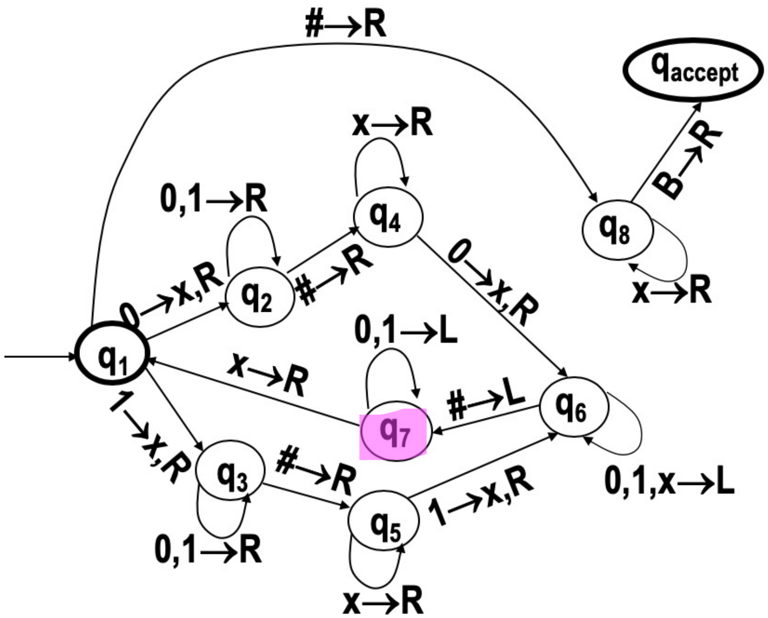
if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

q_6 | x | # q_6 x | x q_6 # | x

q_6 x | x | q_6 # x | q_7 x # | x

Example of PCP to A_{TM} , input is x#x



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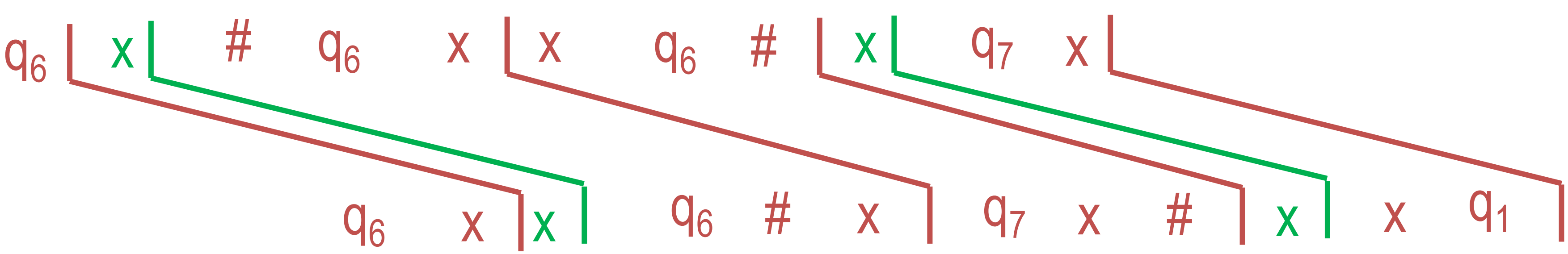
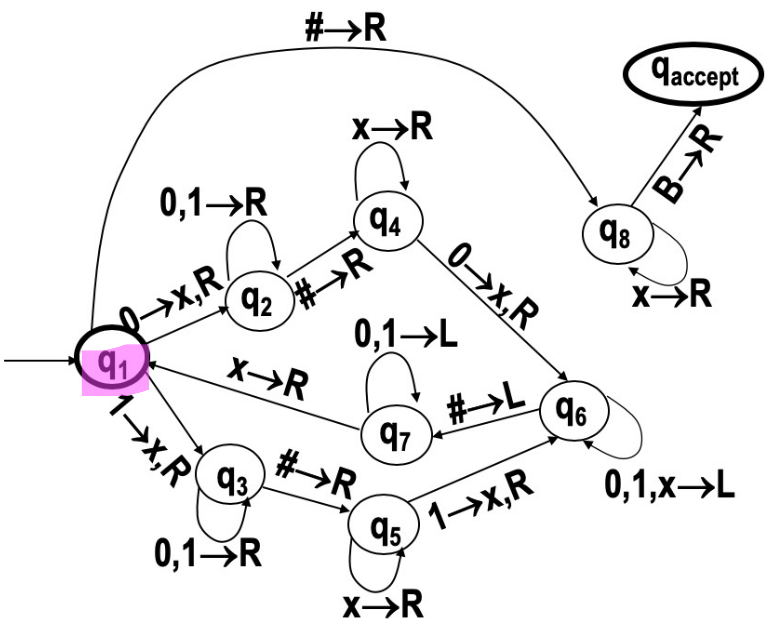


Example of PCP to A_{TM} , input is x#x



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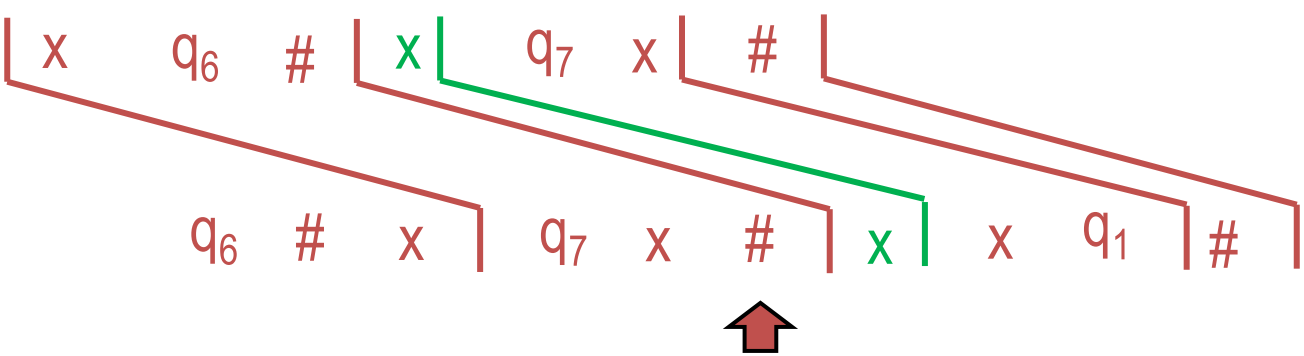
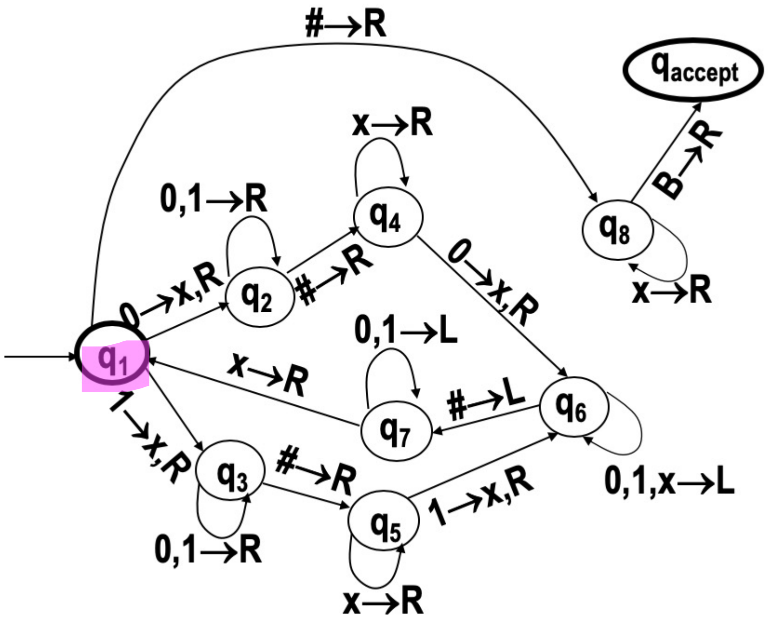


Example of PCP to A_{TM} , input is x#x



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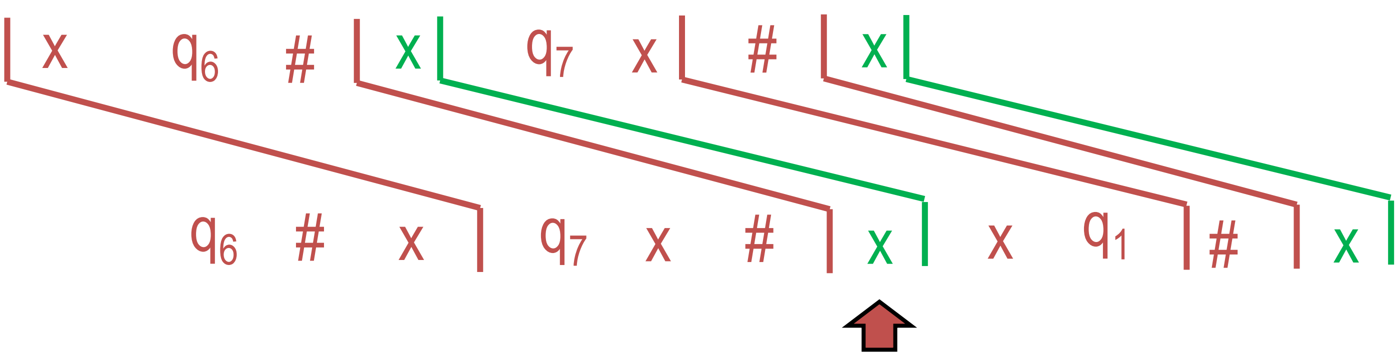
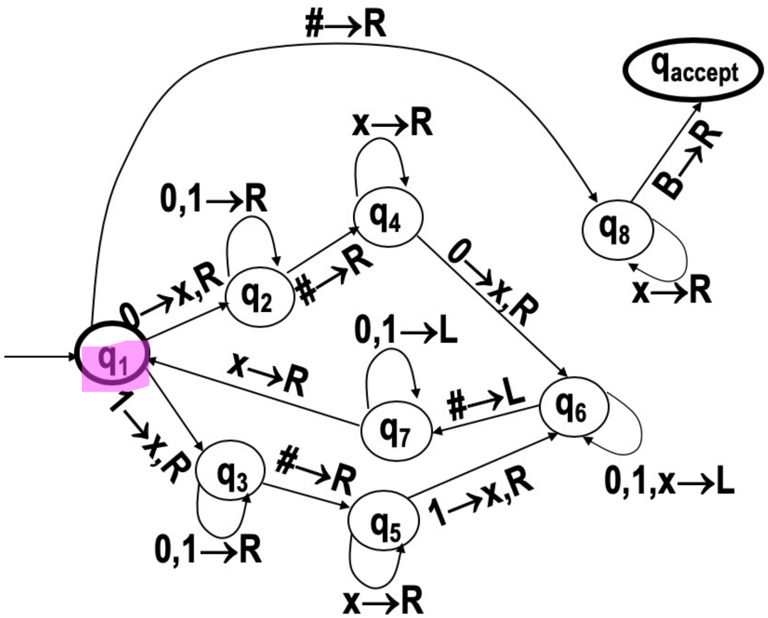


Example of PCP to A_{TM} , input is x#x



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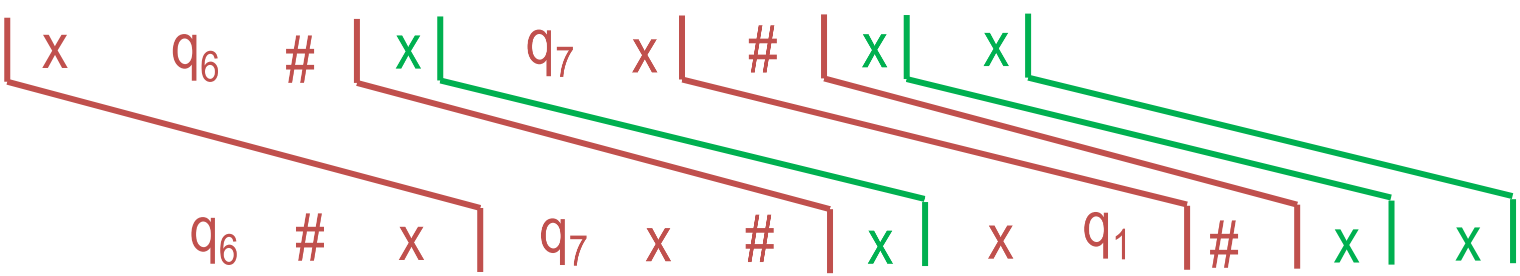
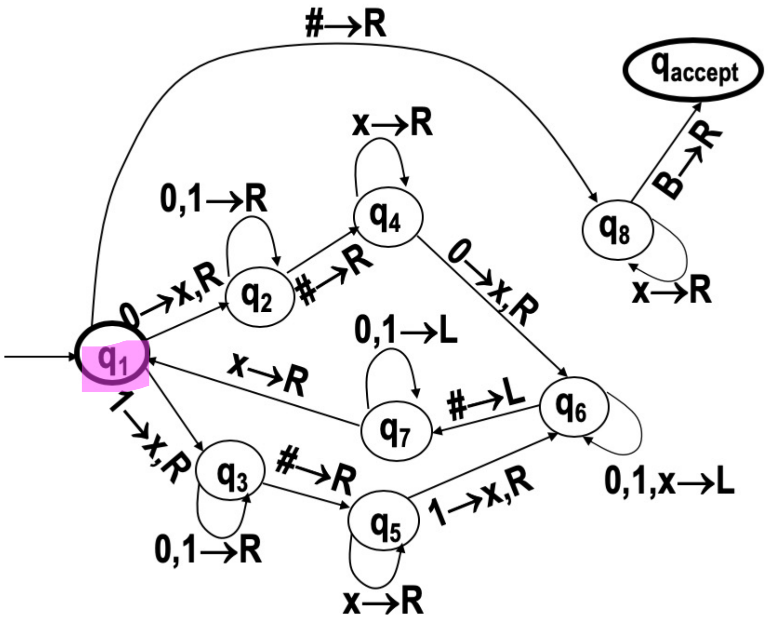
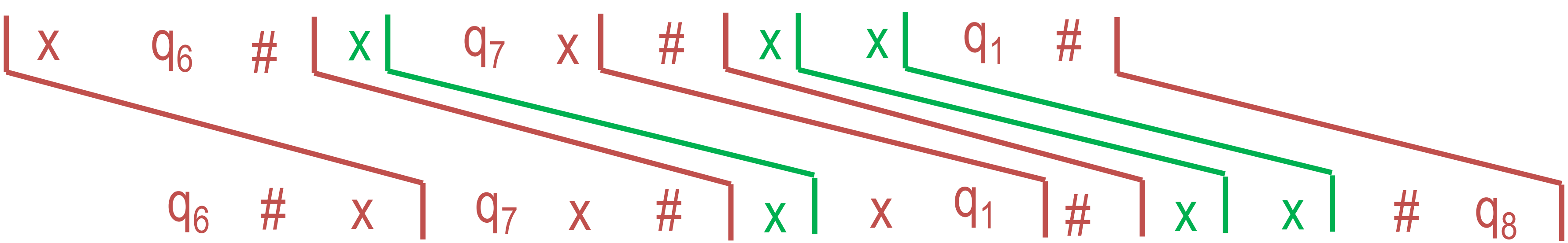


Example of PCP to A_{TM} , input is x#x

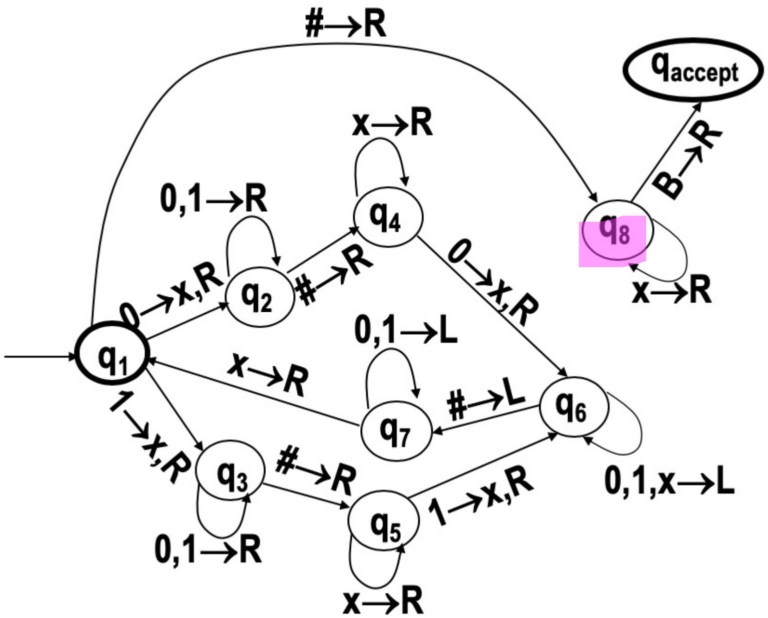


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

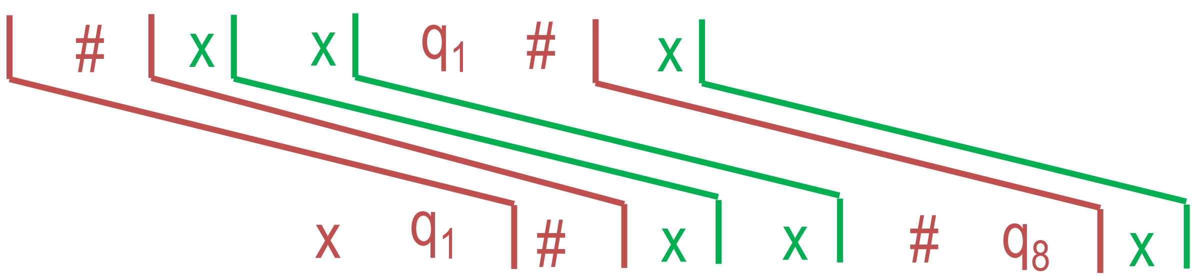


Example of PCP to A_{TM} , input is x#x

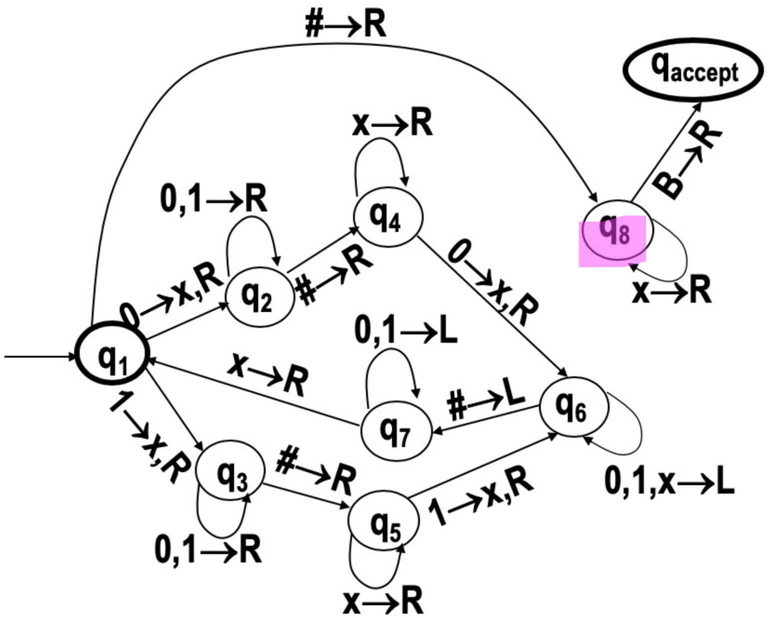


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

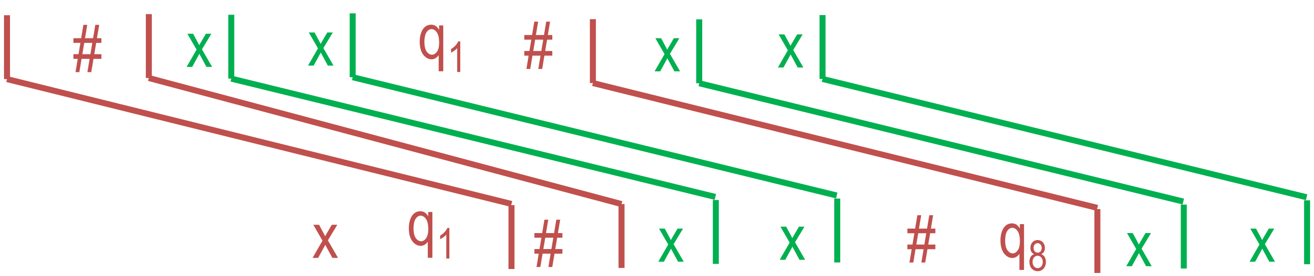


Example of PCP to A_{TM} , input is x#x

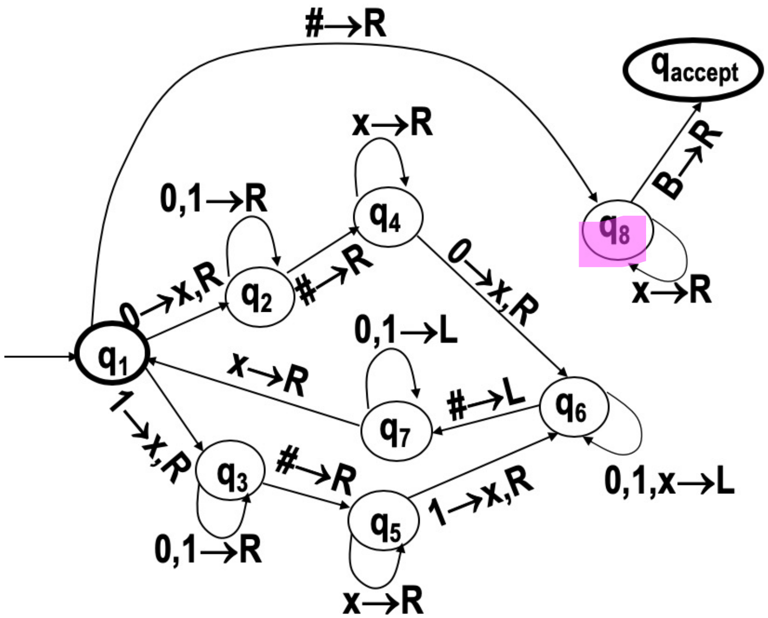


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'



Example of PCP to A_{TM} , input is x#x



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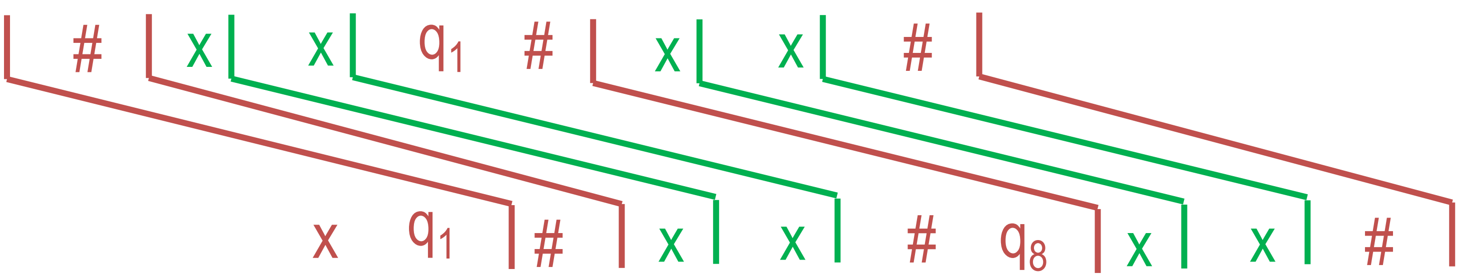
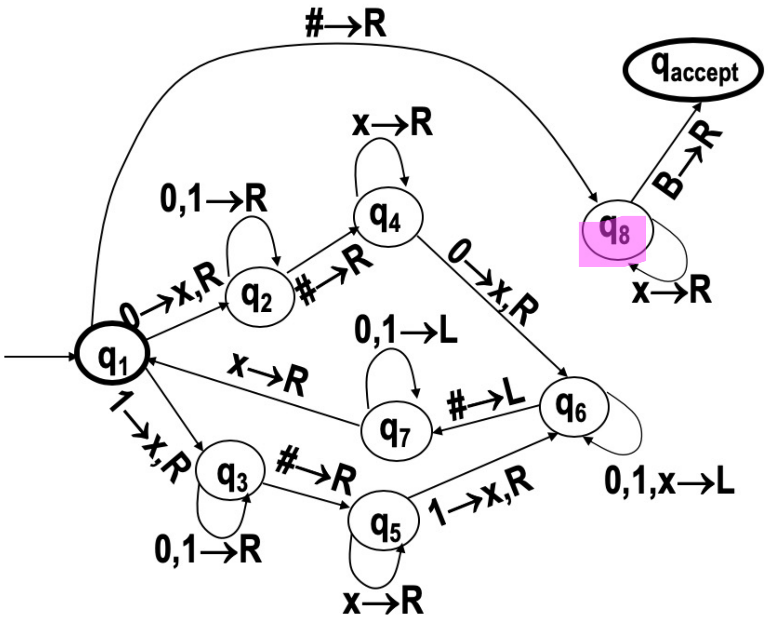


Example of PCP to A_{TM} , input is x#x



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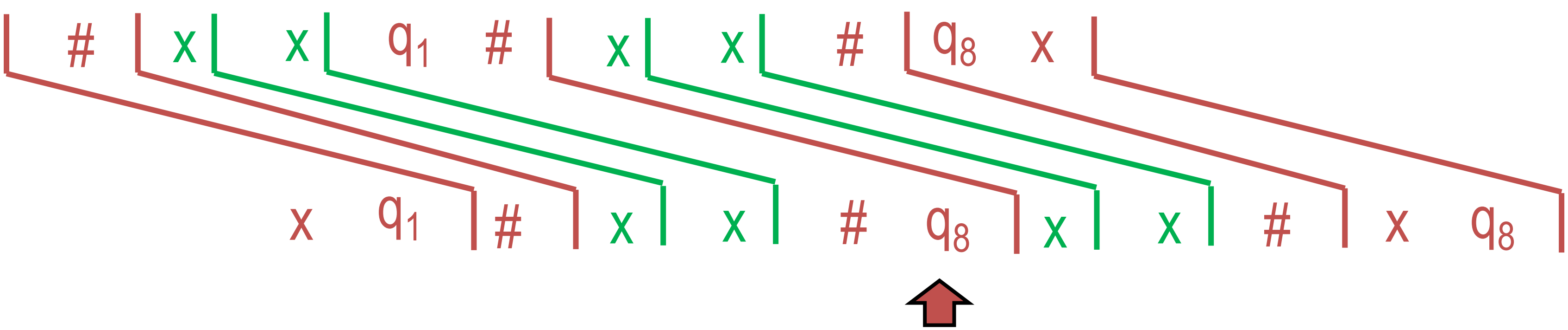
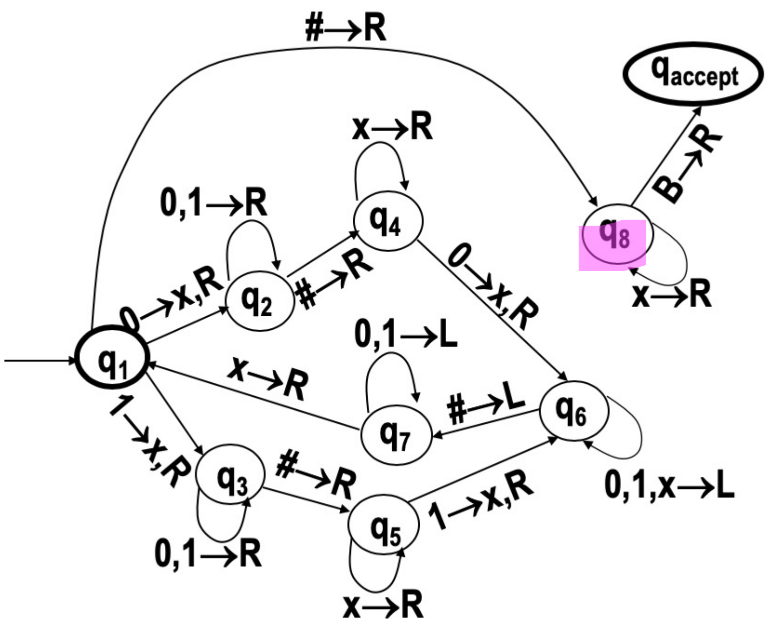
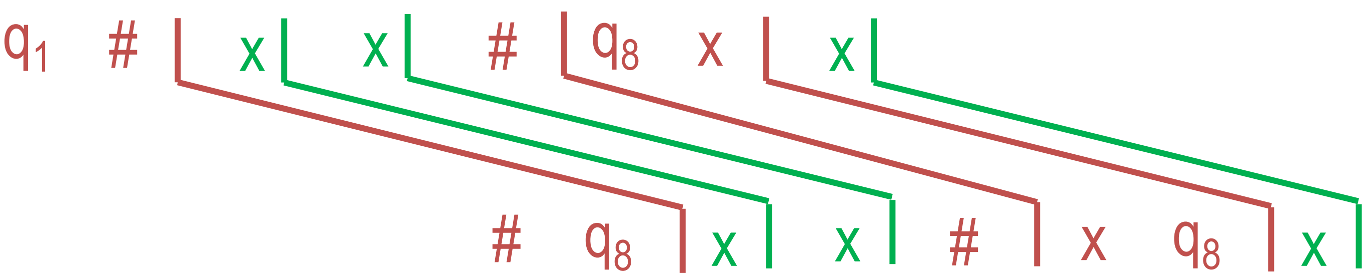


Example of PCP to A_{TM} , input is x#x

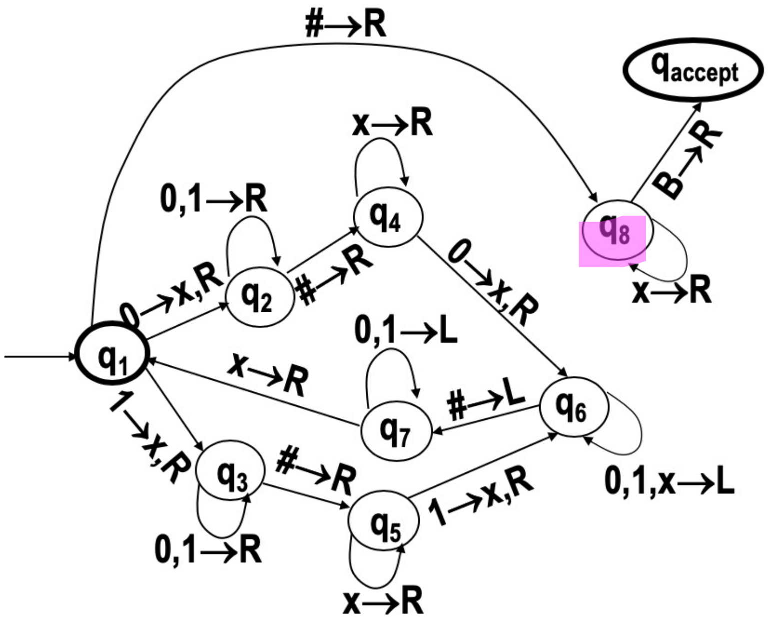


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'



Example of PCP to A_{TM} , input is x#x



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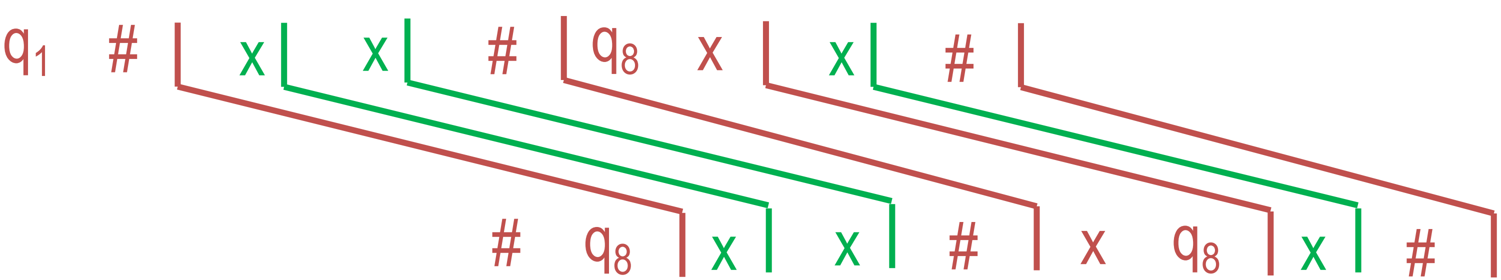
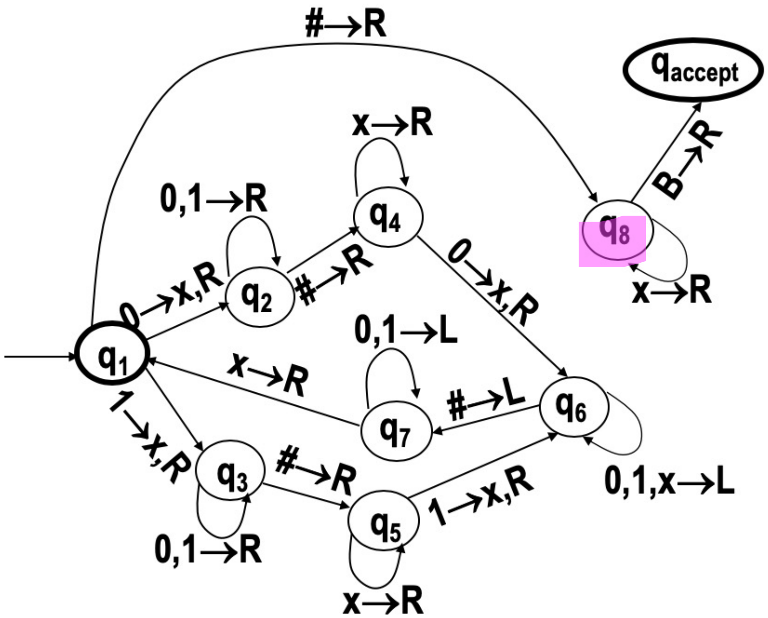


Example of PCP to A_{TM} , input is x#x



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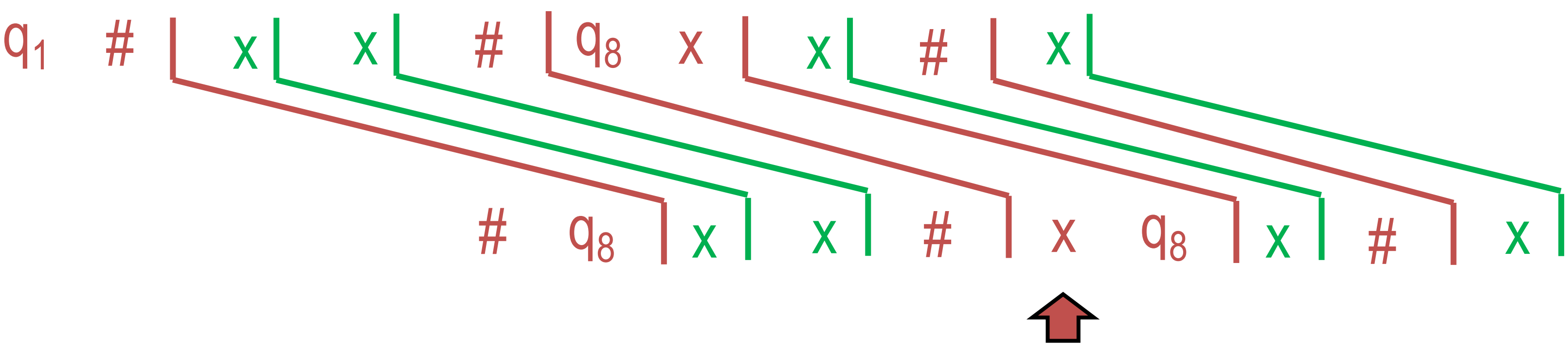
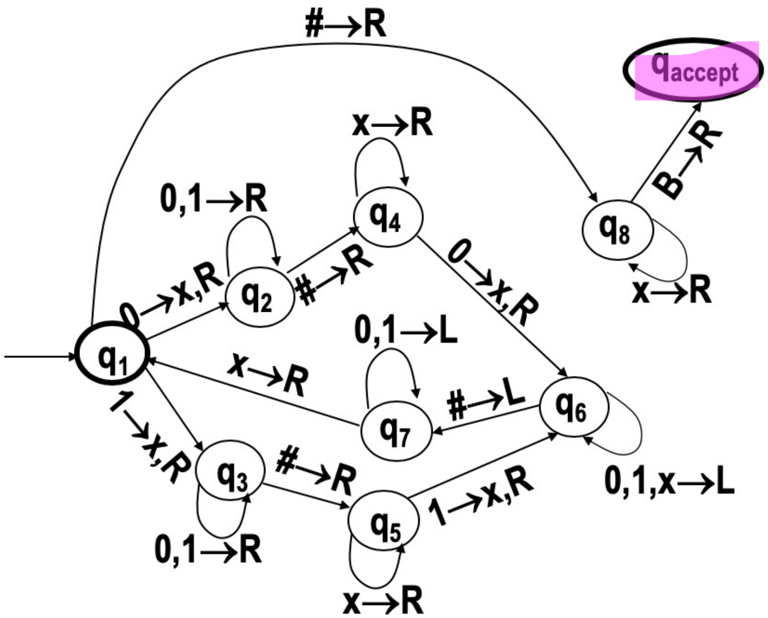
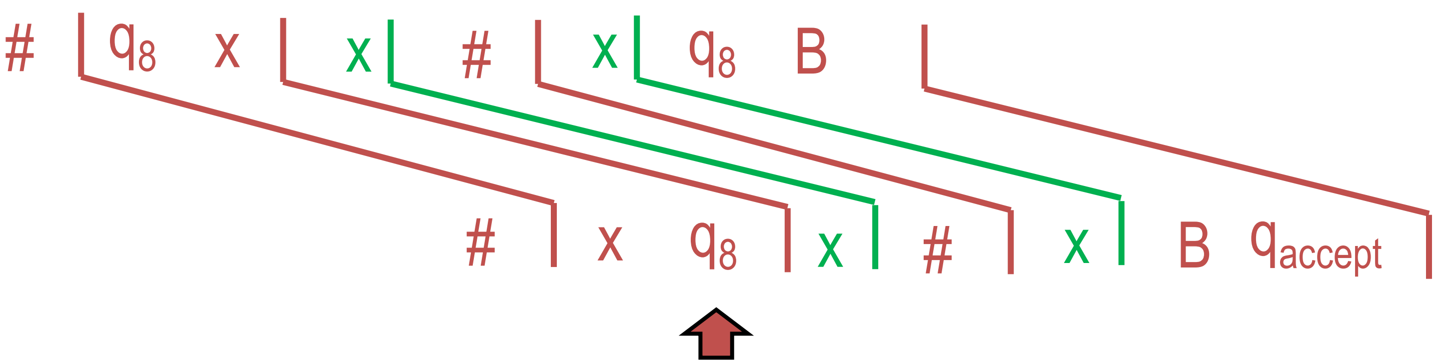


Example of PCP to A_{TM} , input is x#x

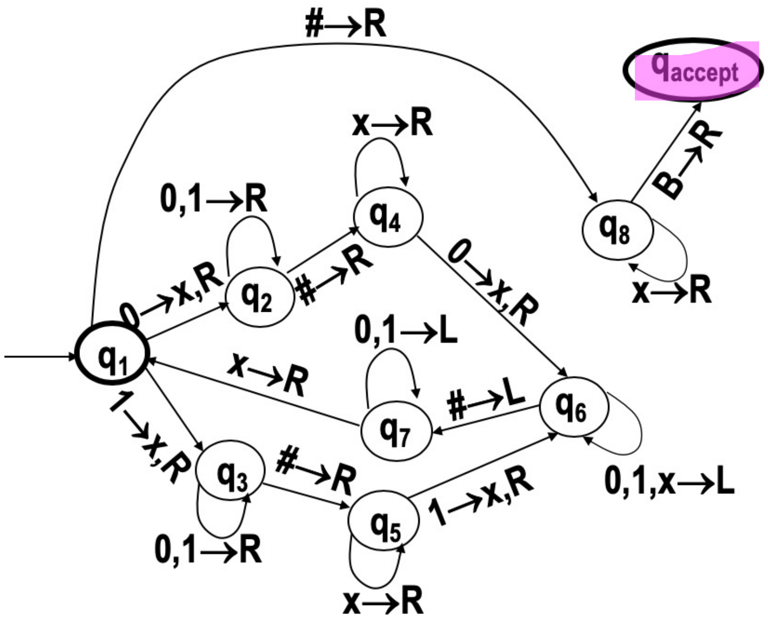


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'



Example of PCP to A_{TM} , input is x#x



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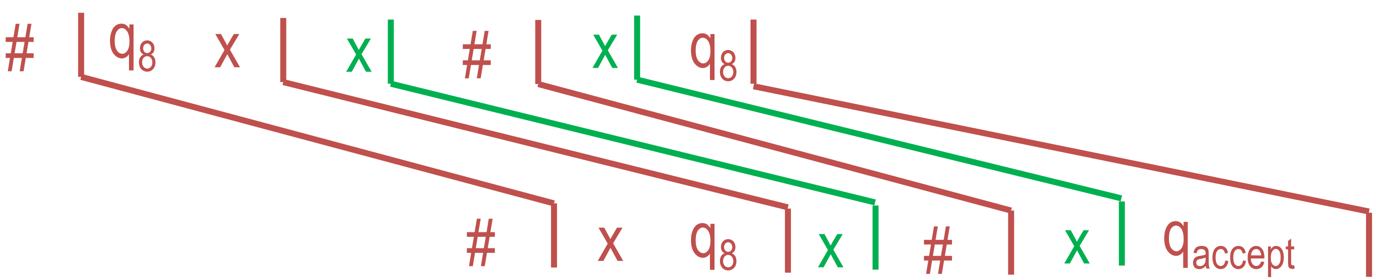
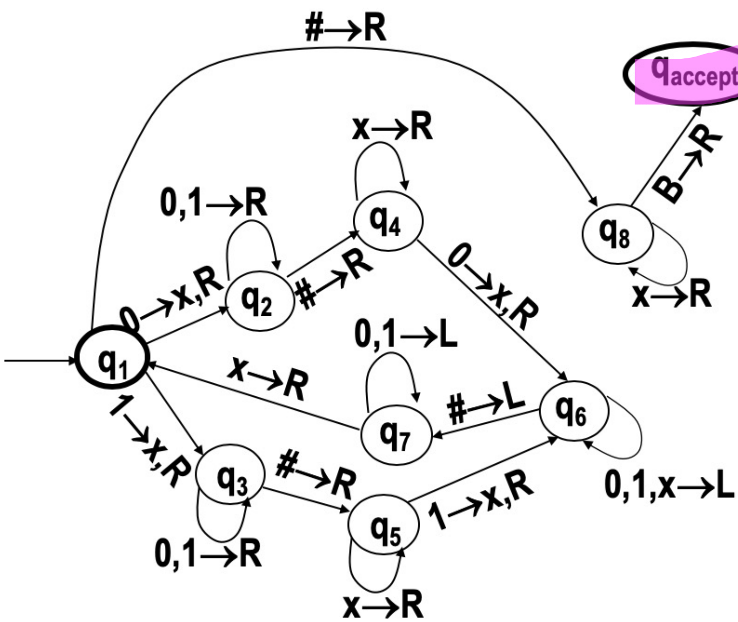


Example of PCP to A_{TM} , input is x#x



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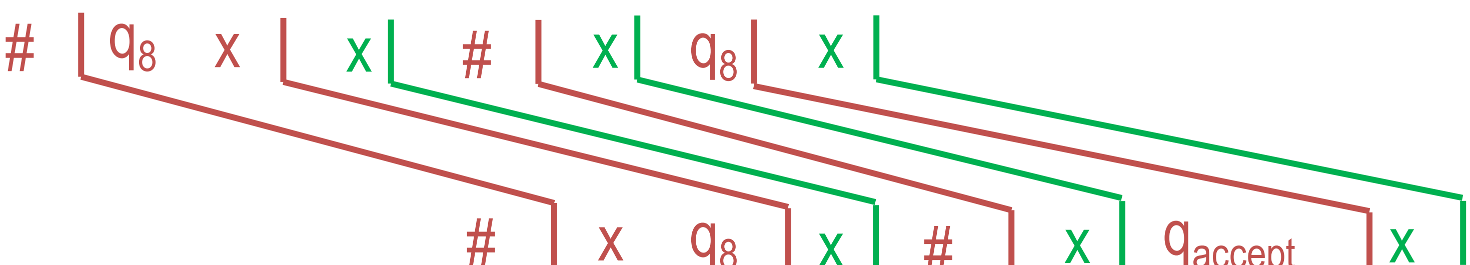

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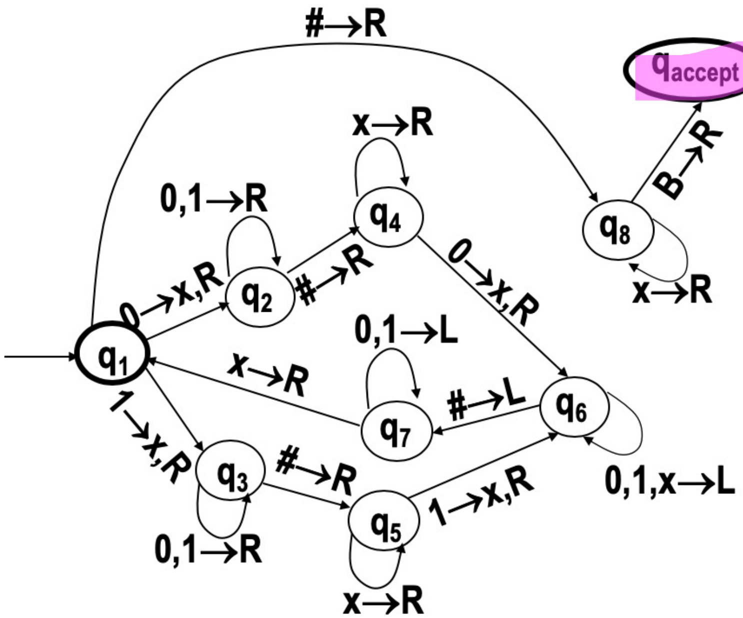
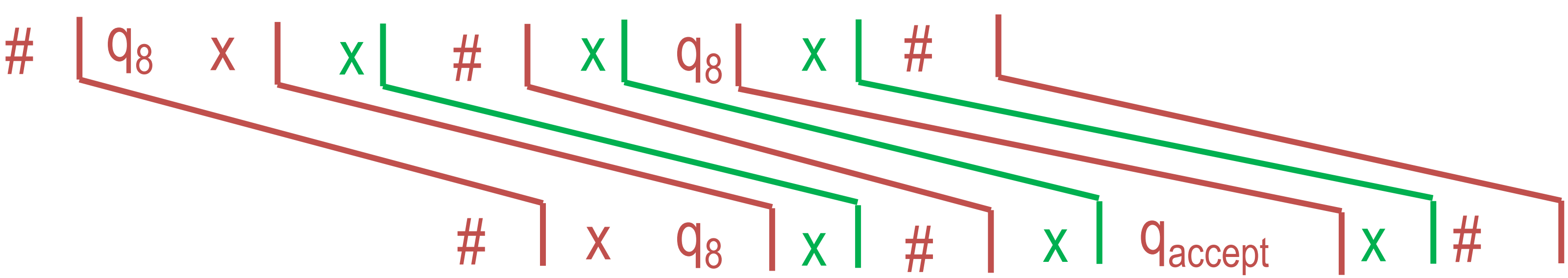
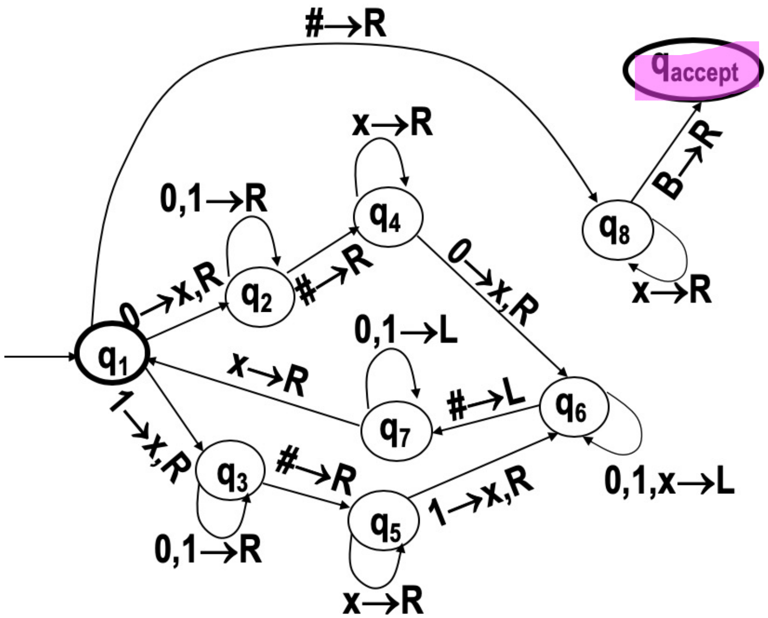


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P’

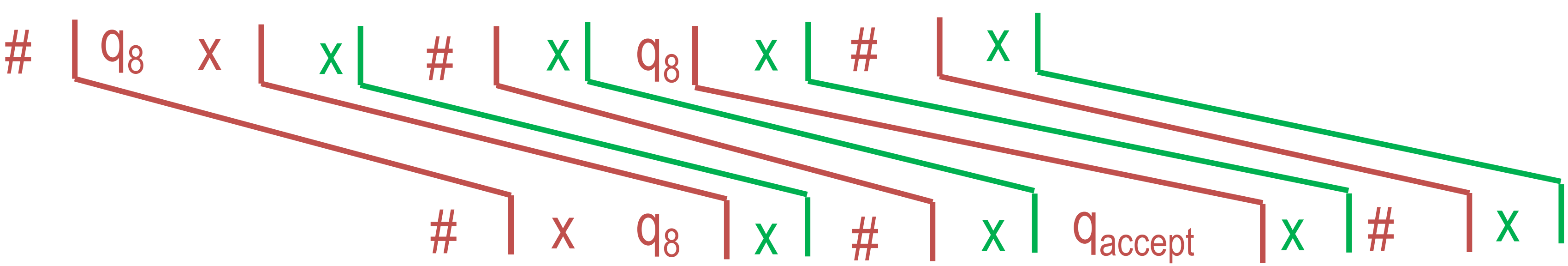


Example of PCP to A_{TM} , input is x#x

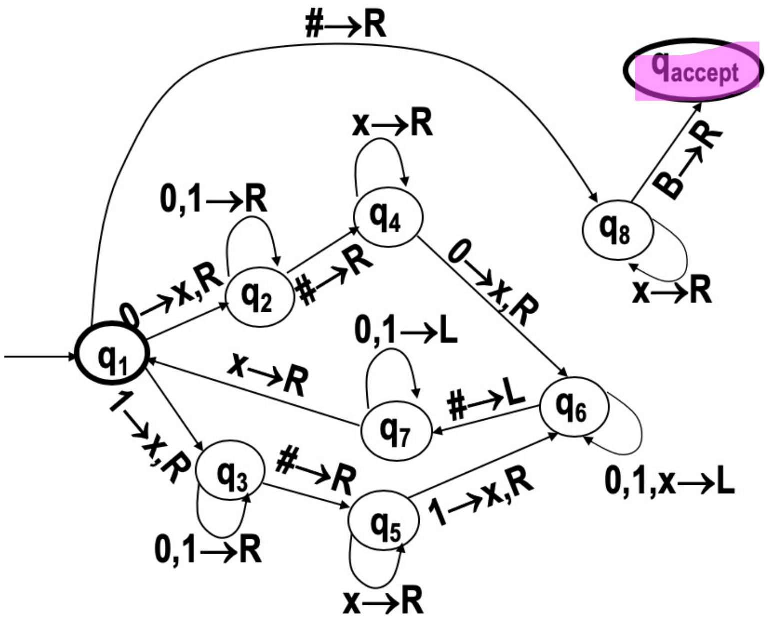


if $\delta(q, a) = (r, b, R)$,

then put $\left[\frac{qa}{br}\right]$ into P'

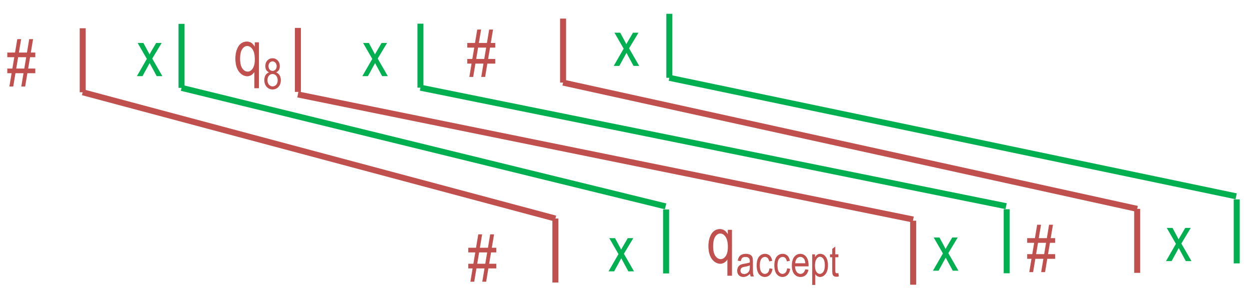


Example of PCP to A_{TM} , input is x#x

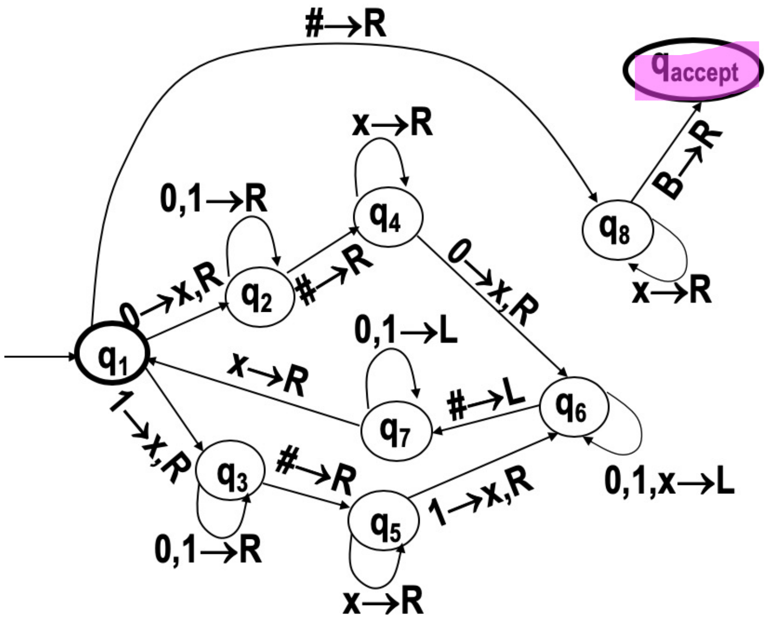


if $\delta(q, a) = (r, b, R)$,

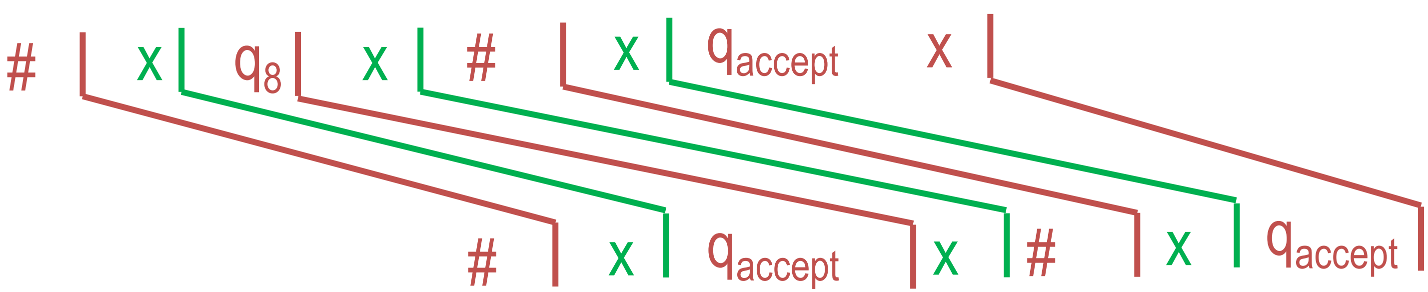
then put $\left[\frac{qa}{br}\right]$ into P'



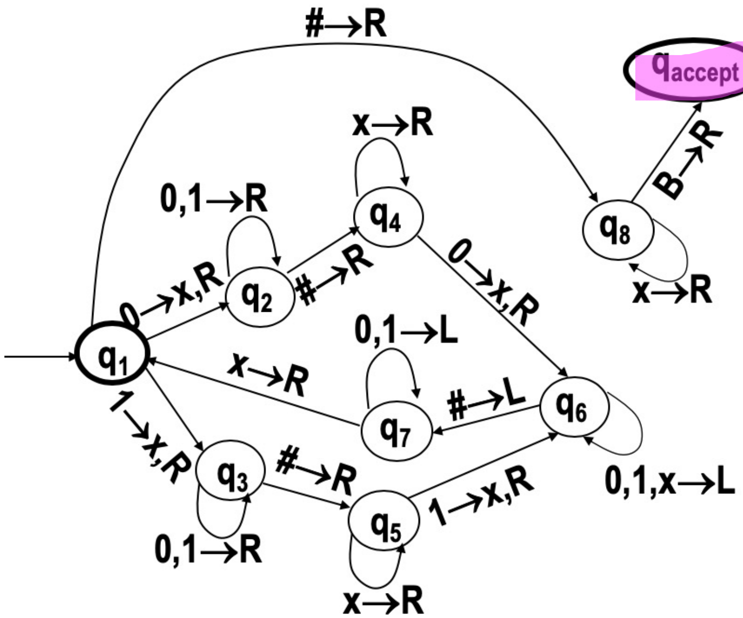
Example of PCP to A_{TM} , input is x#x



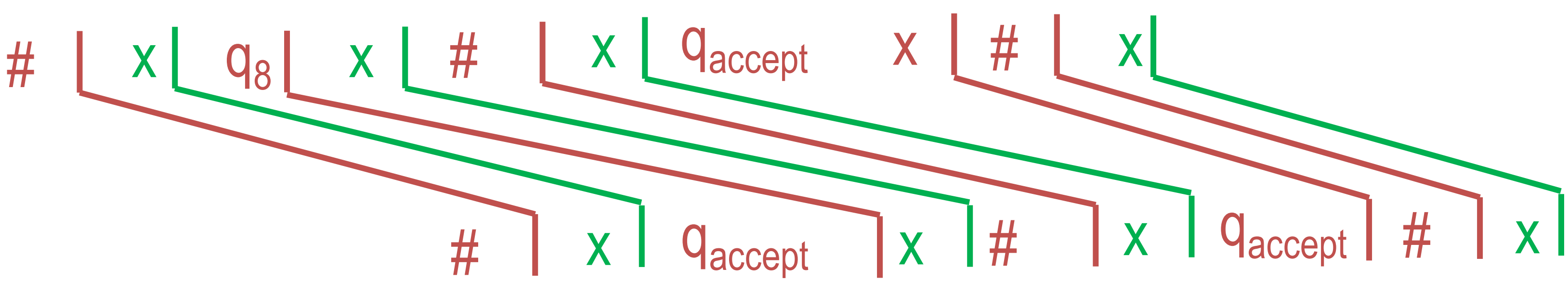
put $\left[\frac{aq_{accept}}{q_{accept}}\right]$ and $\left[\frac{q_{accept}a}{q_{accept}}\right]$ into P'



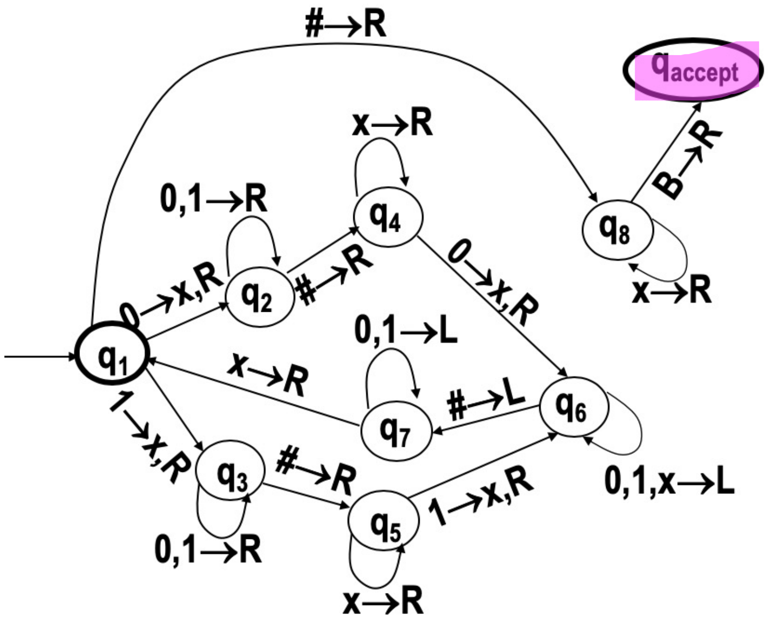
Example of PCP to A_{TM} , input is x#x



put $\left[\frac{a q_{accept}}{q_{accept}}\right]$ and $\left[\frac{q_{accept} a}{q_{accept}}\right]$ into P'

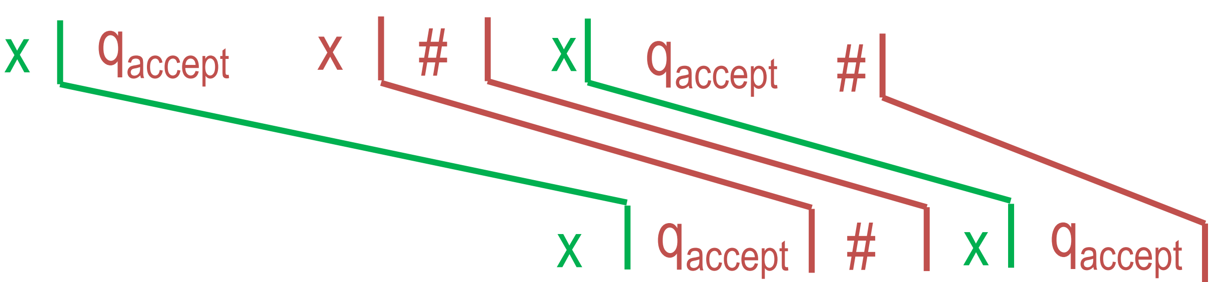


Example of PCP to A_{TM} , input is x#x

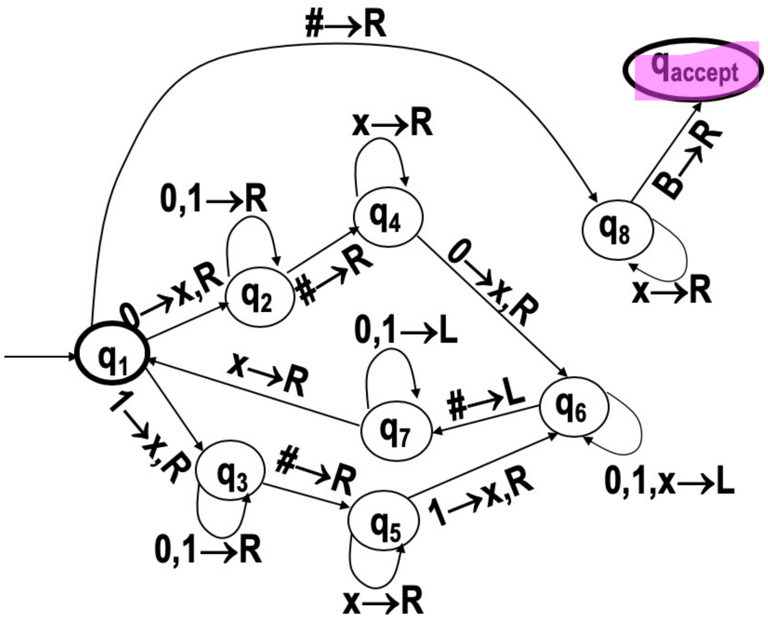


put $\left[\dfrac{a q_{accept}}{q_{accept}}\right]$ and $\left[\dfrac{q_{accept} a}{q_{accept}}\right]$

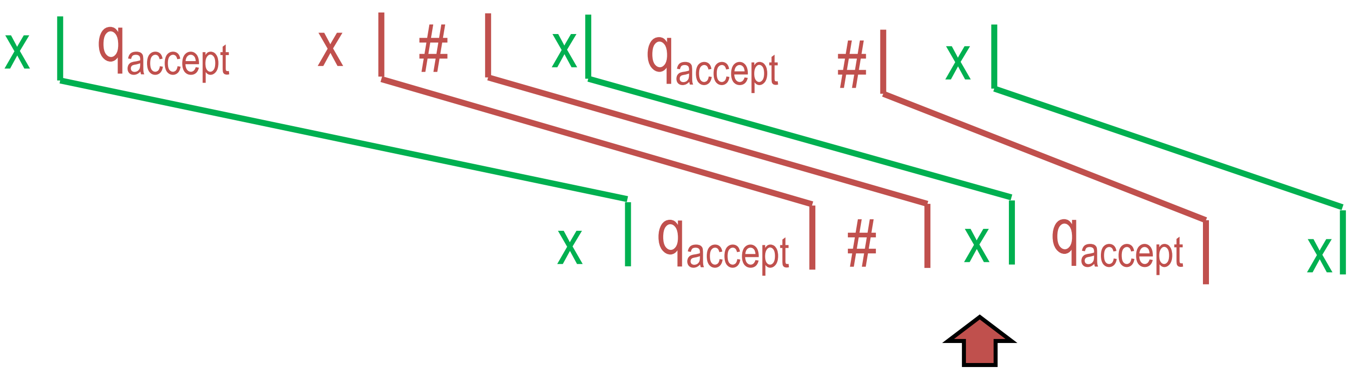
into P'



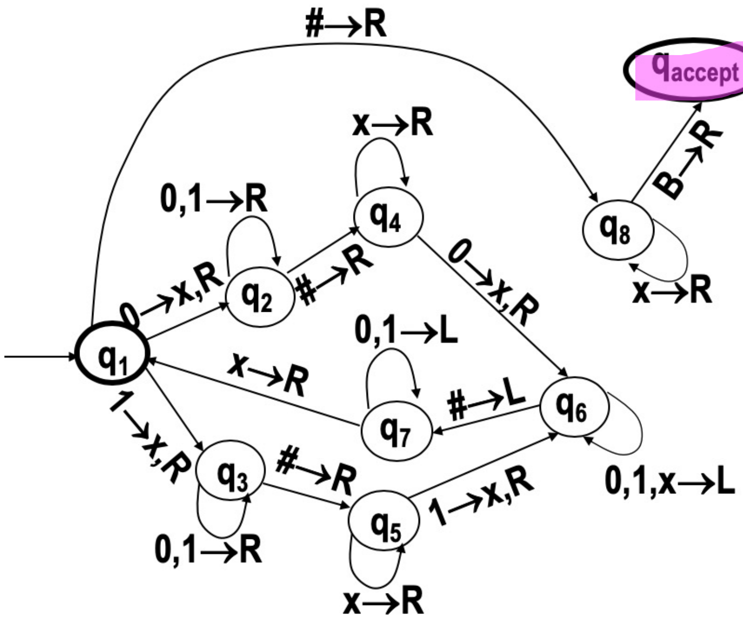
Example of PCP to A_{TM} , input is x#x



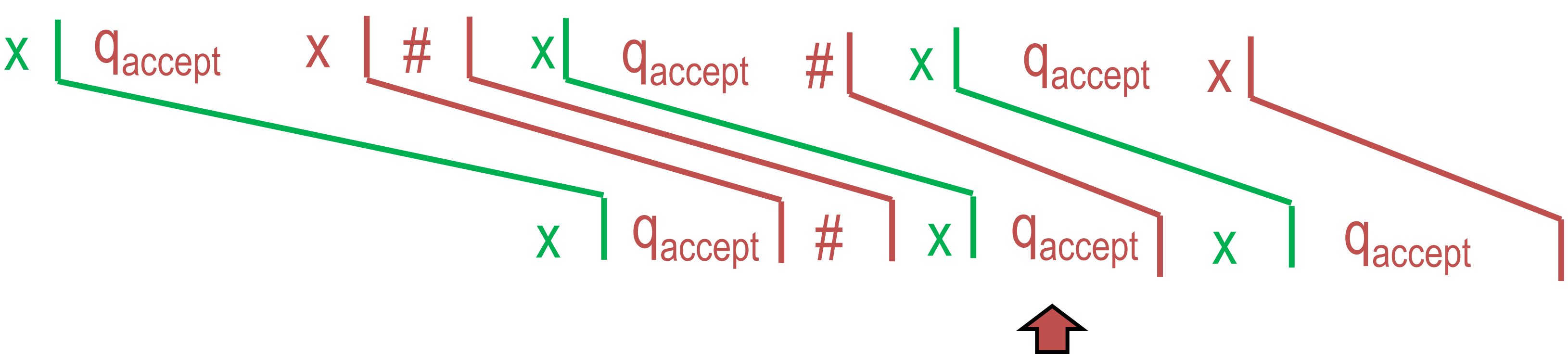
put $\left[\frac{a q_{accept}}{q_{accept}}\right]$ and $\left[\frac{q_{accept} a}{q_{accept}}\right]$ into P'



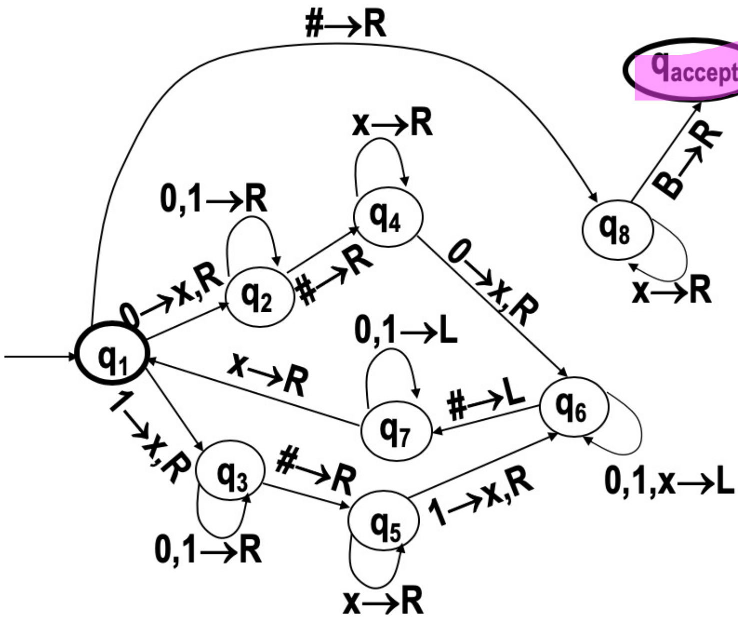
Example of PCP to A_{TM} , input is x#x



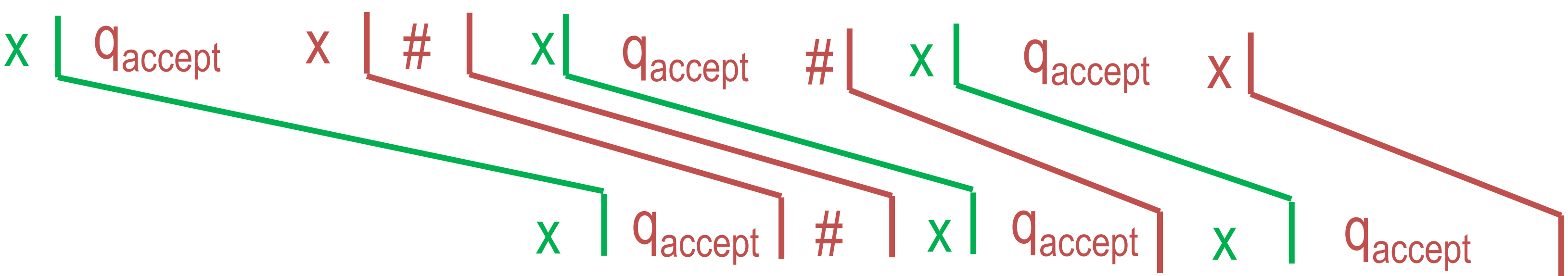
put $\left[\frac{aq_{accept}}{q_{accept}}\right]$ and $\left[\frac{q_{accept}a}{q_{accept}}\right]$ into P'



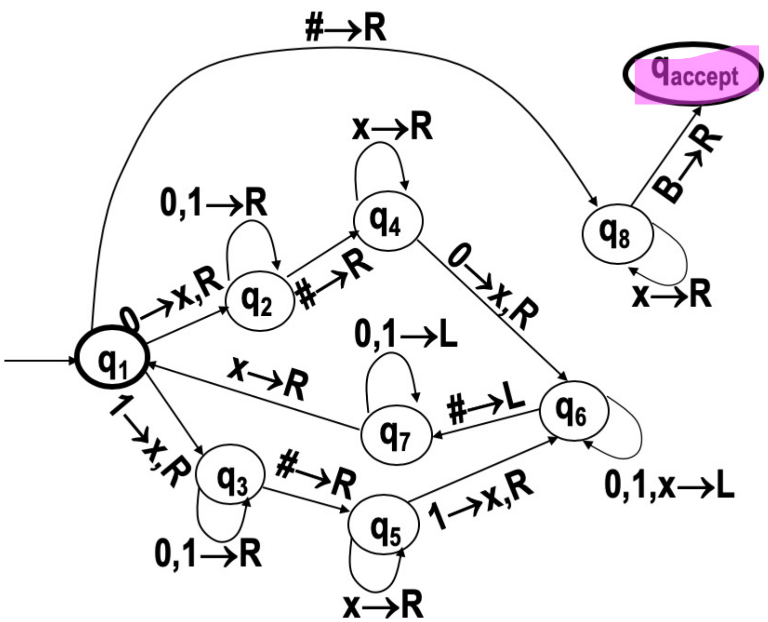
Example of PCP to A_{TM} , input is x#x



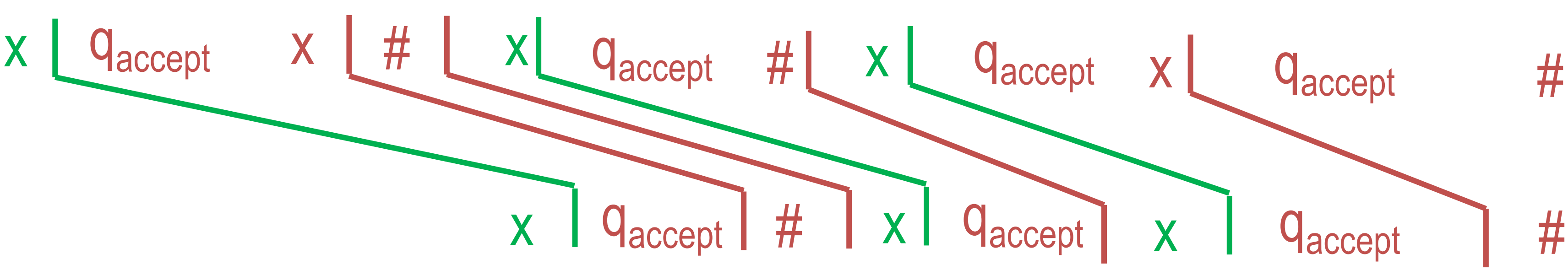
Finish the match, put $\left[\frac{q_{accept}\#\#}{\#}\right]$ into P'



Example of PCP to A_{TM} , input is x#x

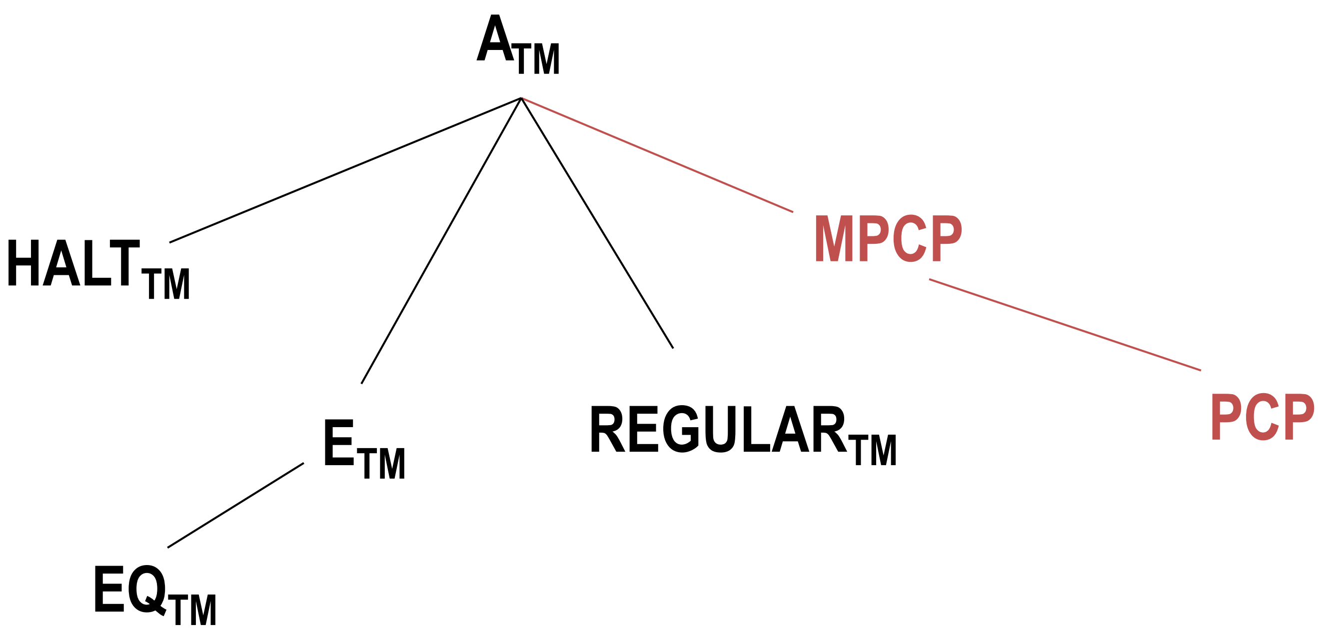


Finish the match, put $\left[\frac{q_{accept}\#}{\#}\right]$ into P’



Conclusion

- Relationship of languages on reducibility



Conclusion

- Closure on operations

	Complement $\overline{A}$	Intersection $\cap$	Union $\cup$	Star A^*
Regular/DFA/ NFA	√	√	√	√
CFL/ PDA	×	×	√	√
Turing-decidable TM	√	√	√	√
Turing-recognizable TM	×	√	√	√