CS 6041
Theory of Computation

Deterministic finite automata

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https://kevinsuo.github.io/

Outline

- Finite Automata
  - Definition
  - Example
  - Language of DFA
  - Computation for DFAs
- Design DFAs
  - Example
  - Regular language
  - Regular operation

Designing finite automata

- State:
  - Start state, accept state, etc.
- Transition:
  - from one state to another state based on the input

Example 1: language - - > figure

- $L(E_1)$={ w | w has odd amount of 1s }, Σ={0,1}

Step 1: define states

Example 1: language - - > figure

- $L(E_1)=\{ w \mid w \text{ has odd amount of 1s} \}, \Sigma=\{0,1\}$

q_{even}: even amount of 1s
q_{odd} : odd amount of 1s

Example 1: language - - > figure

- $L(E_1)$={ w | w has odd amount of 1s }, Σ={0,1}

q_{even}: even amount of 1s
q_{odd} : odd amount of 1s

Step 2: define transitions

Example 1: language - - > figure

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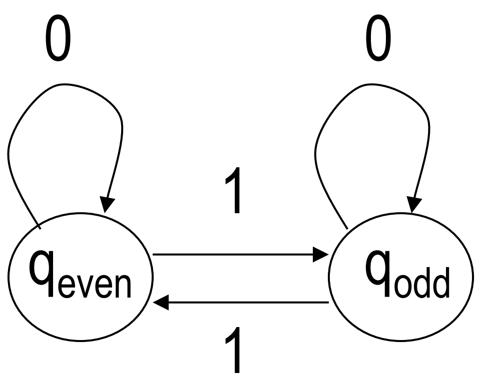

Step 3: define start state and accept states

Example 1: language - - > figure

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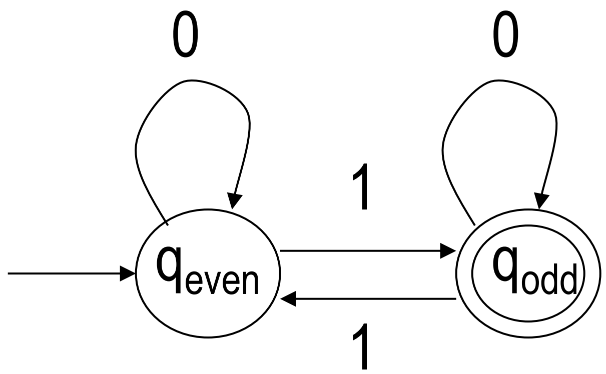

Example 2: language - - > figure

- $L(E_2)$={ w | w has substring 001 }, Σ={0,1}

Example 2: language - - > figure

- $L(E_2)=\{ w \mid w \text{ has substring } 001 \}$, $\Sigma=\{0,1\}$

q: empty string
q_0: has substring 0
q_{00}: has substring 00
q_{001}: has substring 001

Example 2: language - - > figure

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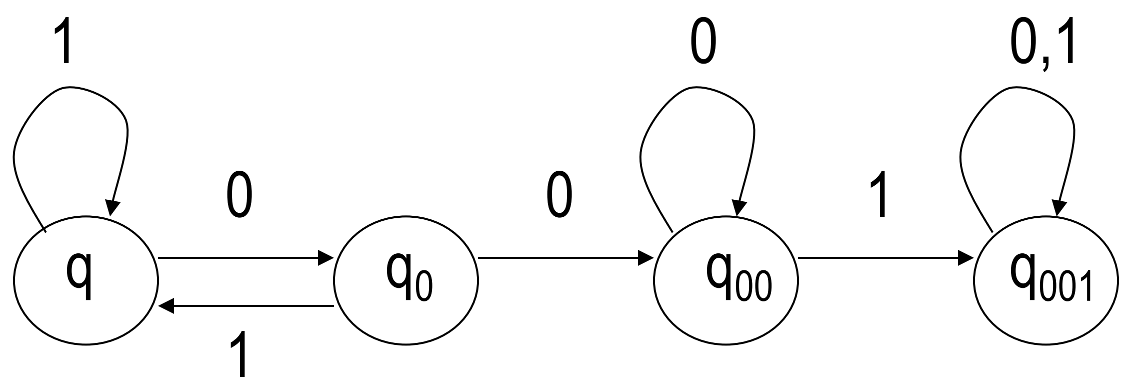

Example 2: language - - > figure

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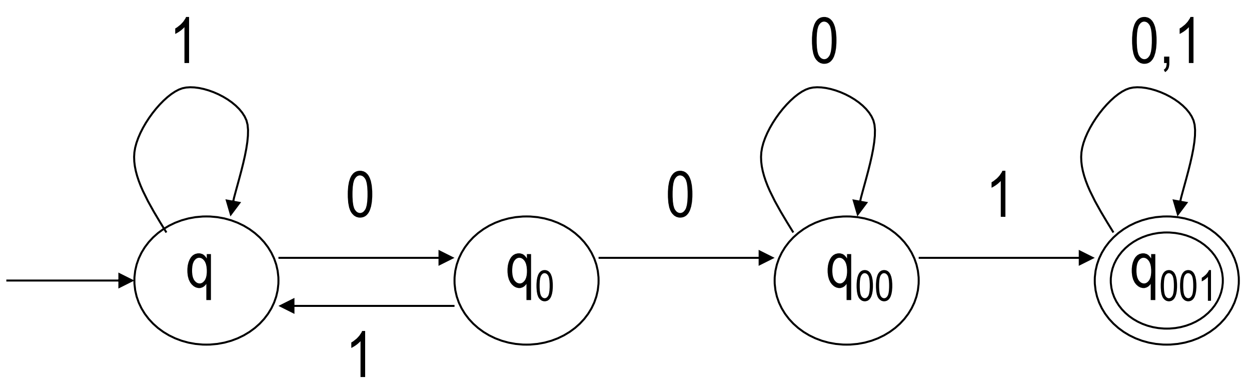


Example 3: language - - > figure

- L = Set of all strings that start with 0

  = {0, 00, 01, 000, 010, ...}

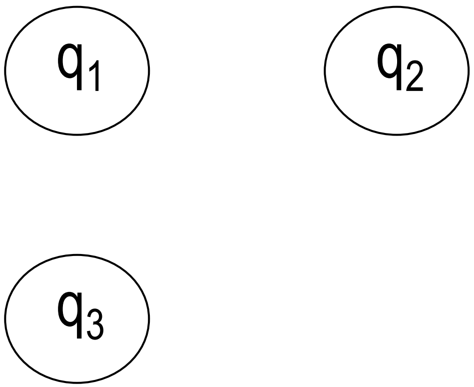
Can anyone draw the DFA?

Example 3: language - - > figure

- L = Set of all strings that start with 0

  = {0, 00, 01, 000, 010, ...}

q_1: ε
q_2: start with 0
q_3: start with 1



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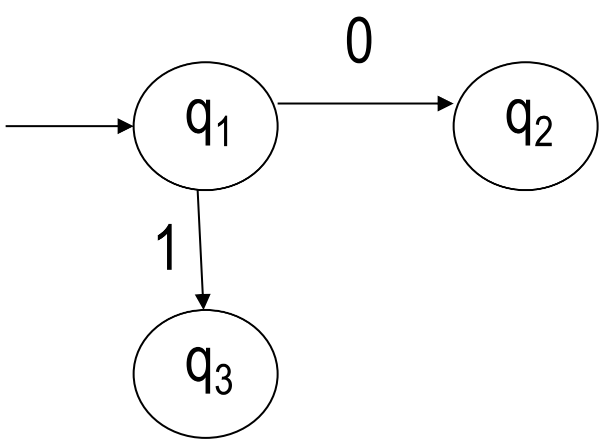

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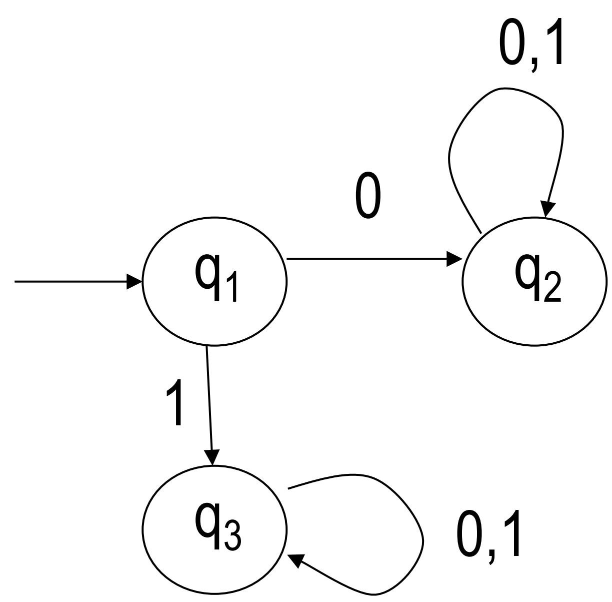

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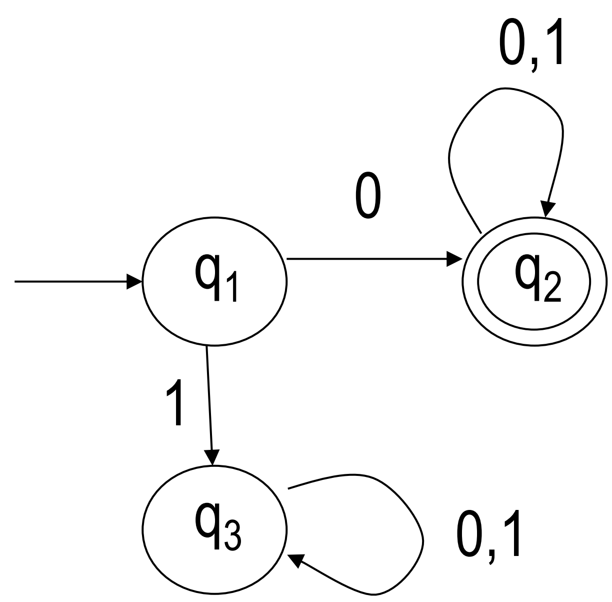

Example 4: language - - > figure

- L = Set of all strings over {0,1} that of length is 2

  = {00, 01, 10, 11}

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Example 4: language - - > figure

- L = Set of all strings over {0,1} that of length is 2

  = {00, 01, 10, 11}

q_1: ε
q_2: length is 1
q_3: length is 2
q_4: length is 3 or more

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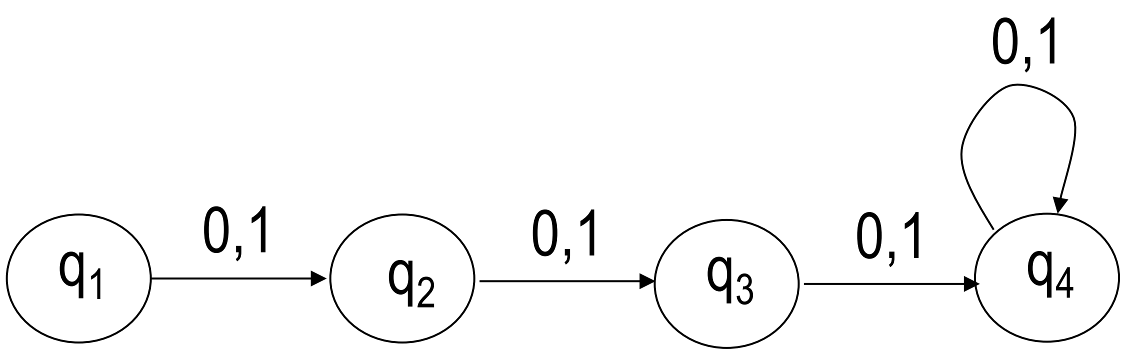

Example 4: language - - > figure

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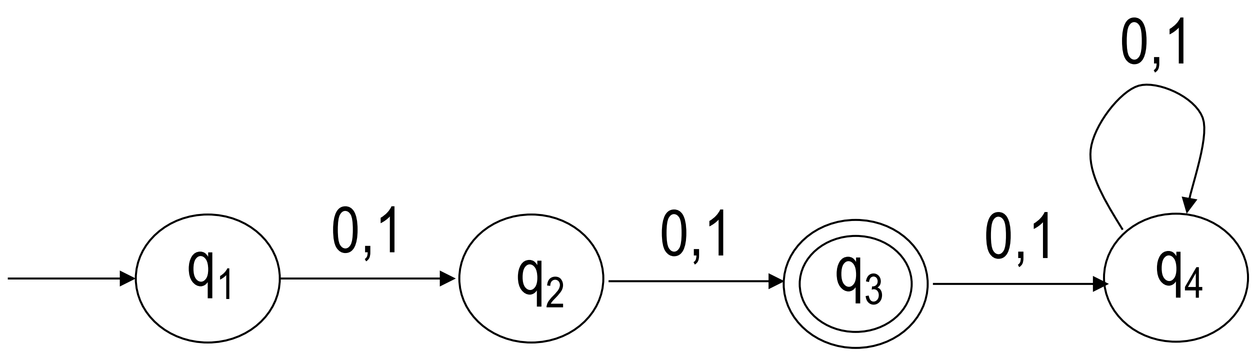

Example 5: language - - > figure

- L = Set of strings over {a,b} that contains string aabb in it

Can anyone draw the DFA?

Example 5: language - - > figure

- L = Set of strings over {a,b} that contains string aabb in it

q_1: contains nothing
q_2: contains a
q_3: contains aa
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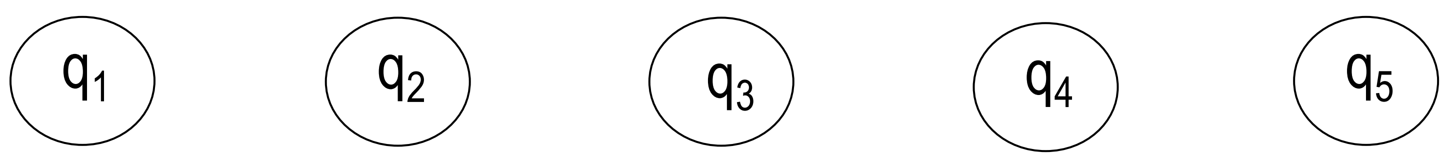

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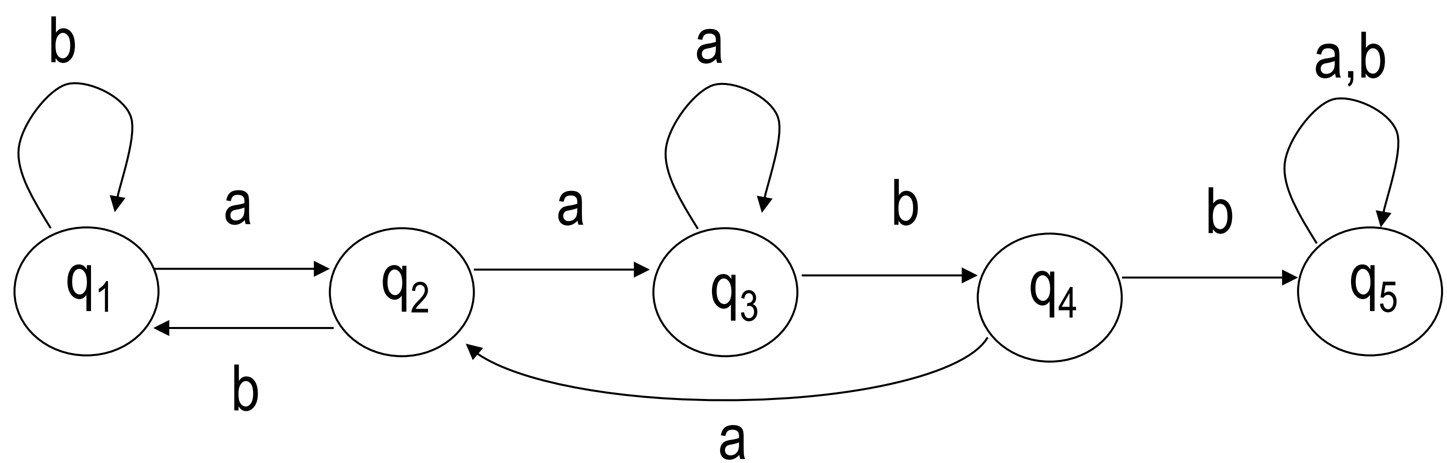

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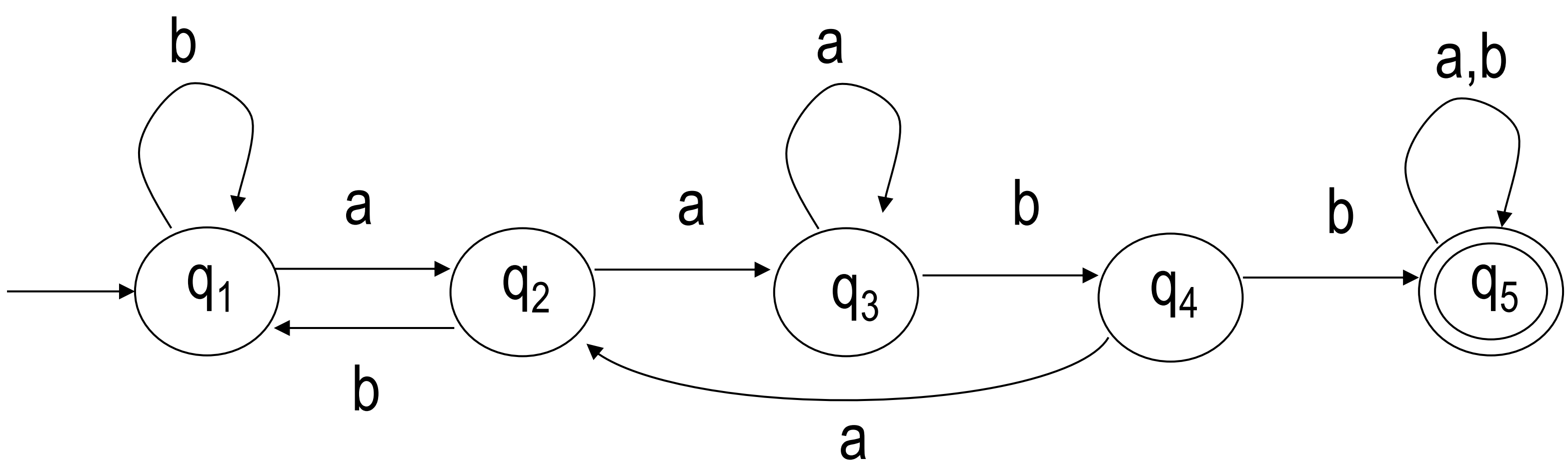

Example 6: language - - > figure

- L = Set of strings over {a,b} that does not contain string aabb in it

Can anyone draw the DFA?

Example 6: language - - > figure

- L = Set of strings over {a,b} that does not contain string aabb in it

q_1: contains nothing
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Example 6: language - - > figure

- L = Set of strings over {a,b} that does not contain string aabb in it

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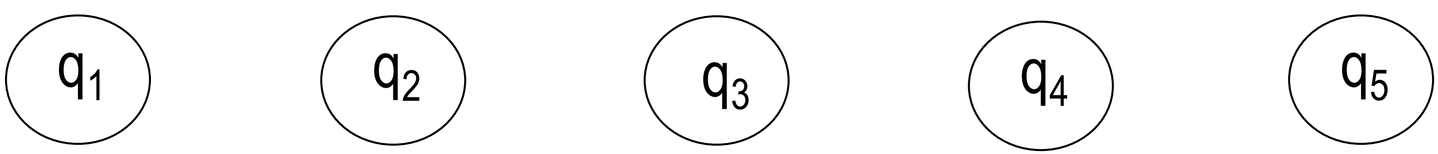

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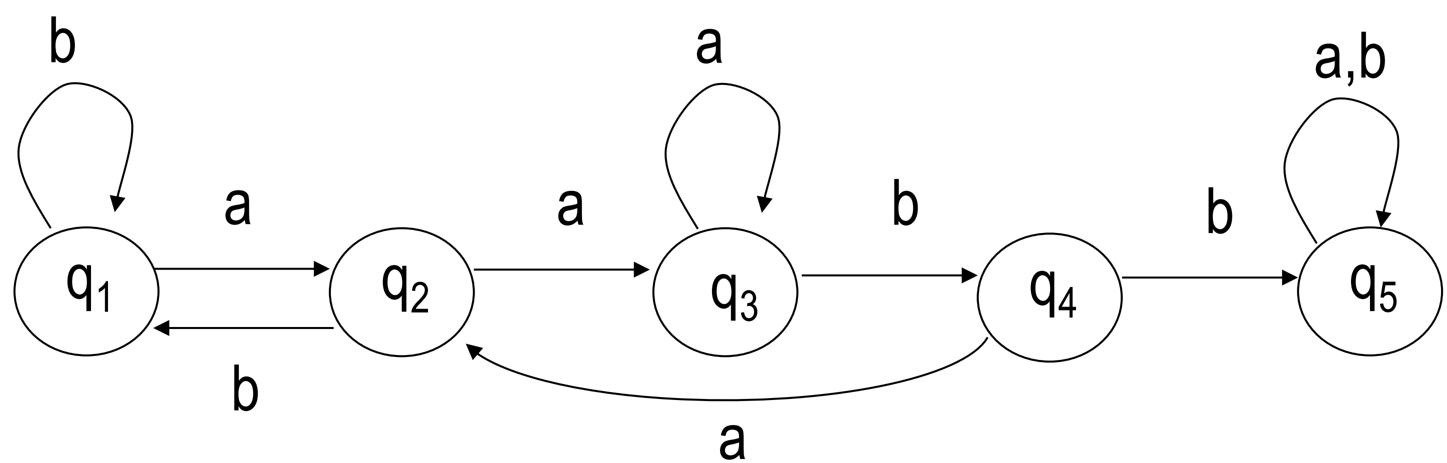

Example 6: language - - > figure

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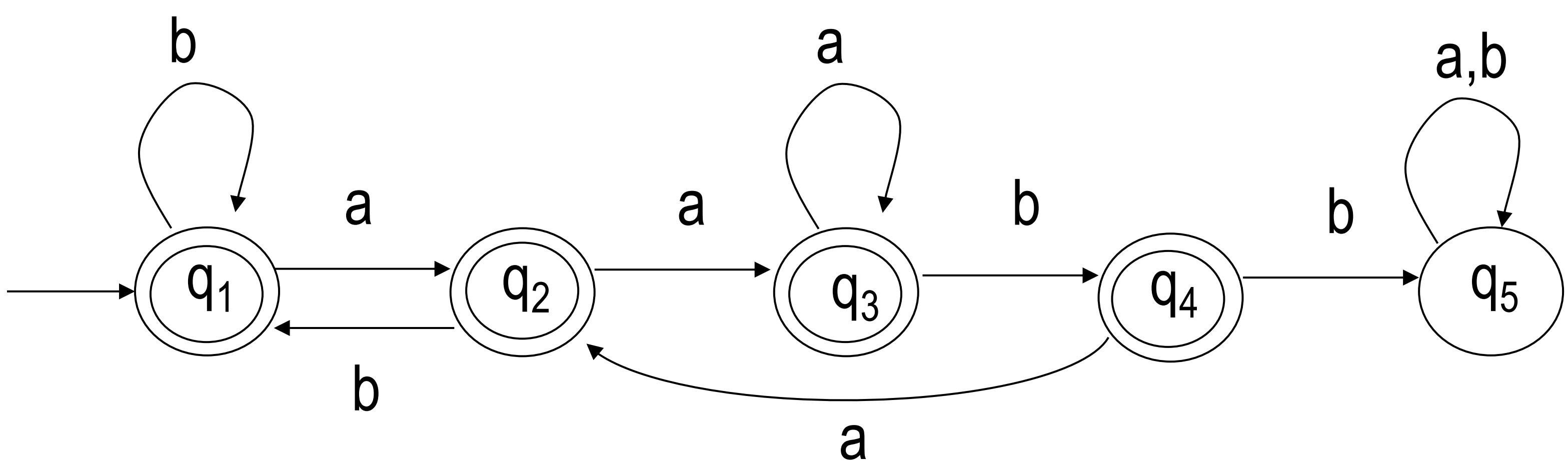


Design a DFA for a language

- Step 1: list all possible states

- Step 2: draw all the transitions between the states

- Step 3: add start and accept states

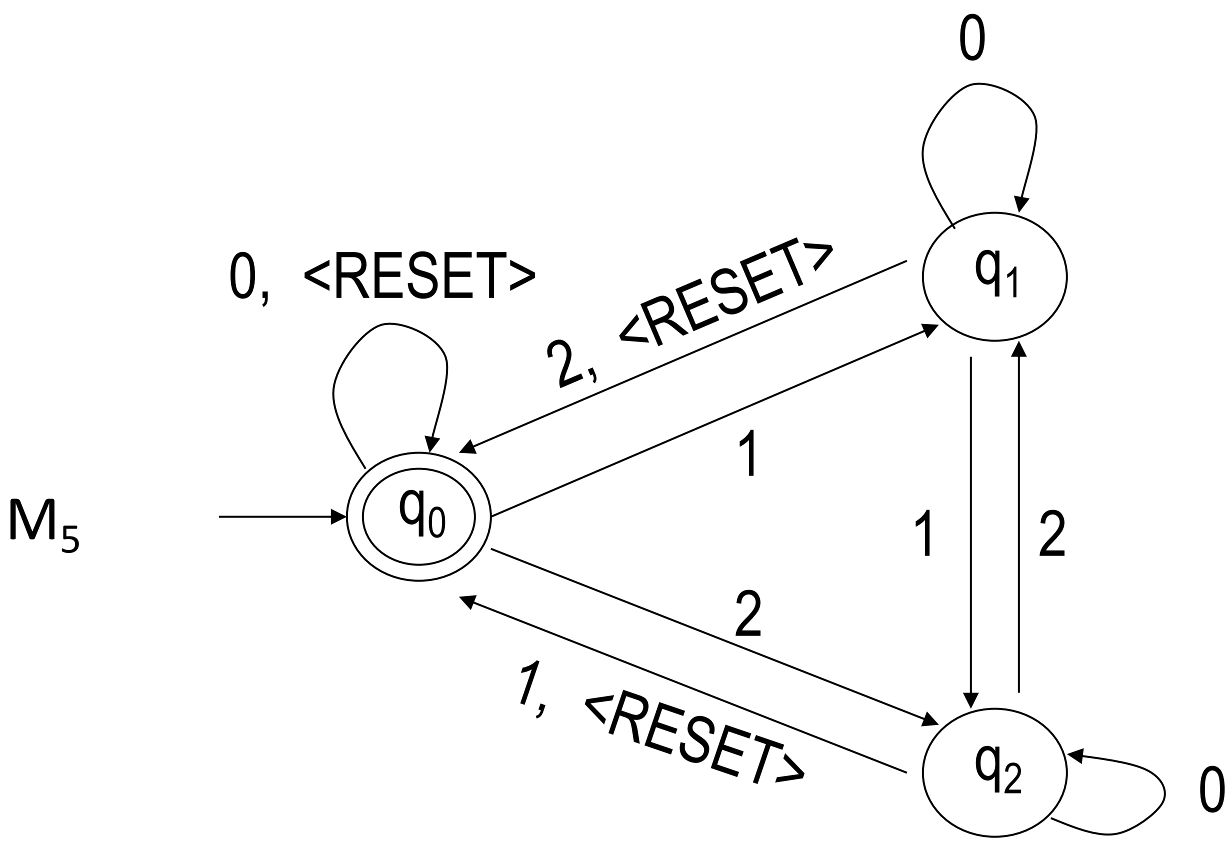
Regular language

- A language is called a regular language if some finite automaton recognizes it

- Regular language:
  - L=L(M)
  - M is finite automaton

Regular language example

- $L(M_5)$ = { w | the sum of the symbols in w is 0 modulo 3, except that ⟨RESET⟩ resets the count to 0 }



Regular language

- What languages are not regular?
  - Not recognized by any DFAs
  - Require memory
    - Memory for DFA is limited, it only stores its current state
    - It cannot store or count strings

- E.g., $a^n b^n$ is not regular language

Regular operations

- A and B be languages
  - Union: $A \cup B = \{x | x \in A \text{ or } x \in B\}$
  - Concatenation: $A^\circ B = \{xy | x \in A \text{ and } y \in B\}$
  - Star: $A^* = \{x_1 x_2 \dots x_k | k \geq 0 \text{ and each } x_i \in A\}$
- Examples:

$$A = \{\text{good}, \text{bad}\} \qquad B = \{\text{boy}, \text{girl}\}$$

$$A \cup B = \{\text{good}, \text{bad}, \text{boy}, \text{girl}\},$$

$$A \circ B = \{\text{goodboy}, \text{goodgirl}, \text{badboy}, \text{badgirl}\}, \text{ and}$$

$$A^* = \{\varepsilon, \text{good}, \text{bad}, \text{goodgood}, \text{goodbad}, \text{badgood}, \text{badbad}, \\ \text{goodgoodgood}, \text{goodgoodbad}, \text{goodbadgood}, \text{goodbadbad}, \dots\}.$$

Regular operations

- Theorem: the regular languages are closed under regular operations
  - Union, $A \cup B$
  - Concatenation, $A \cap B$
  - Star, A^*
  - Complement, $\bar{A}$
  - Boolean operation, $\mathrm{AND}{:}\wedge,\ OR{:}\vee,\ XOR{:}\oplus$

Regular operations

	DFA	PDA	TM
Union	close	?	?
Concatenation	close	?	?
Star	close	?	?
Complement	close	?	?
Boolean operation	close	?	?

Close under the union operation

- Theorem: regular language is closed under the union operation

- Proof:

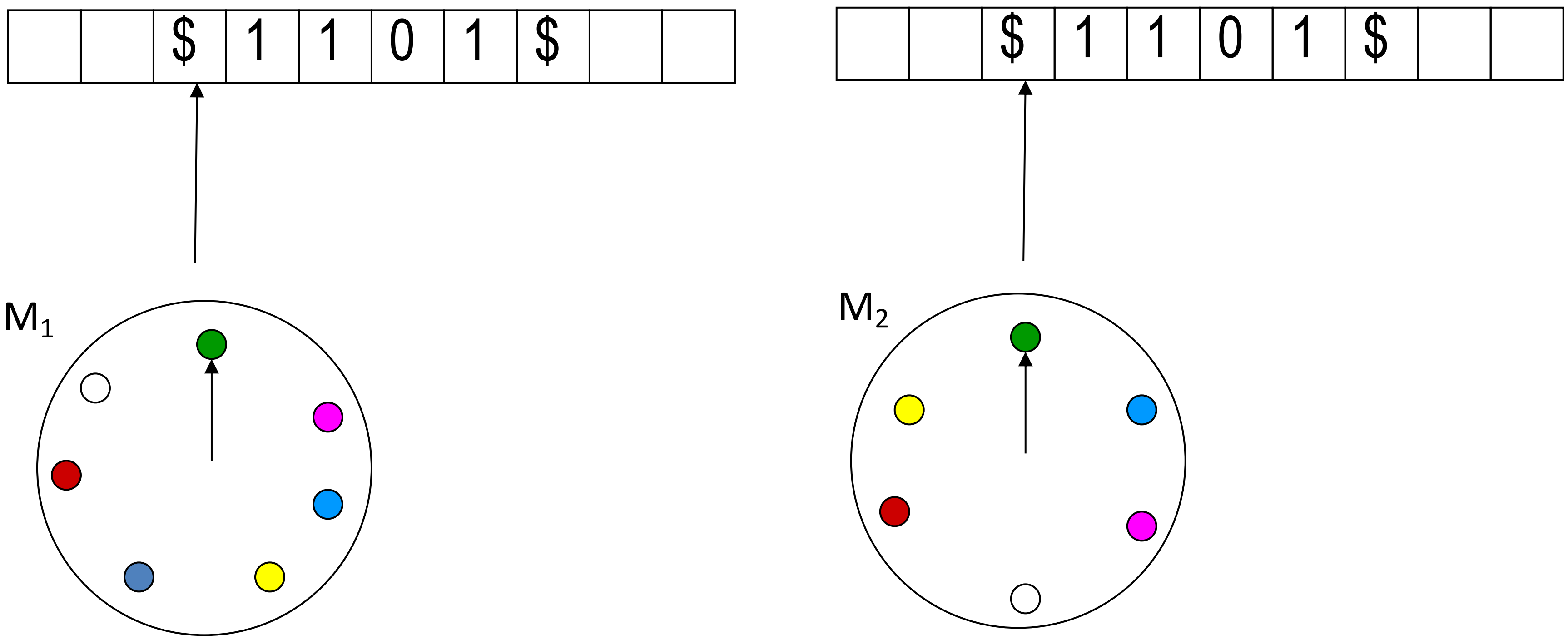
  Let $L_i=L(M_i)$ is a regular language, $M_i=(Q_i,\Sigma,\delta_i,q_i,F_i)$, i=1,2. We need to build a finite automata to recognize $L_1 \cup L_2$

  Build $M_3=(Q_3,\Sigma,\delta_3,q_3,F_3)$.
  $Q_3=Q_1 \times Q_2$;
  $\delta_3((r_1,r_2),a)=(\delta_1(r_1,a),\delta_2(r_2,a))$;
  $q_3=(q_1,q_2)$;
  $F_3=(F_1 \times Q_2) \cup (Q_1 \times F_2)$.

  $L(M_3) = L_1 \cup L_2$, so $L_1 \cup L_2$ is still regular language

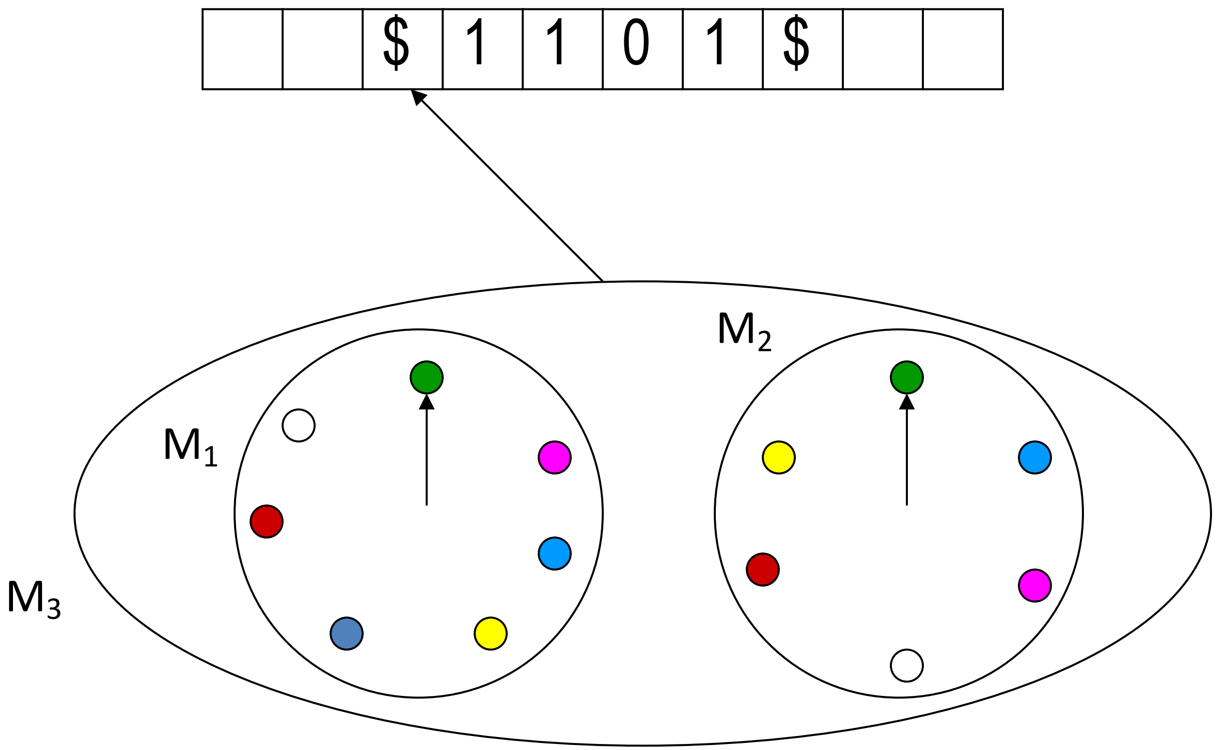
Close under the union operation

- Theorem: regular language is closed under the union operation



Close under the union operation

- Theorem: regular language is closed under the union operation



Close under concatenation operation

- Theorem: regular language is closed under the concatenation operation

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  Let $L_i=L(M_i)$ is a regular language, $M_i=(Q_i,\Sigma,\delta_i,q_i,F_i)$, i=1,2. We need to build a finite automata to recognize $L_1 \cap L_2$

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Close under concatenation operation

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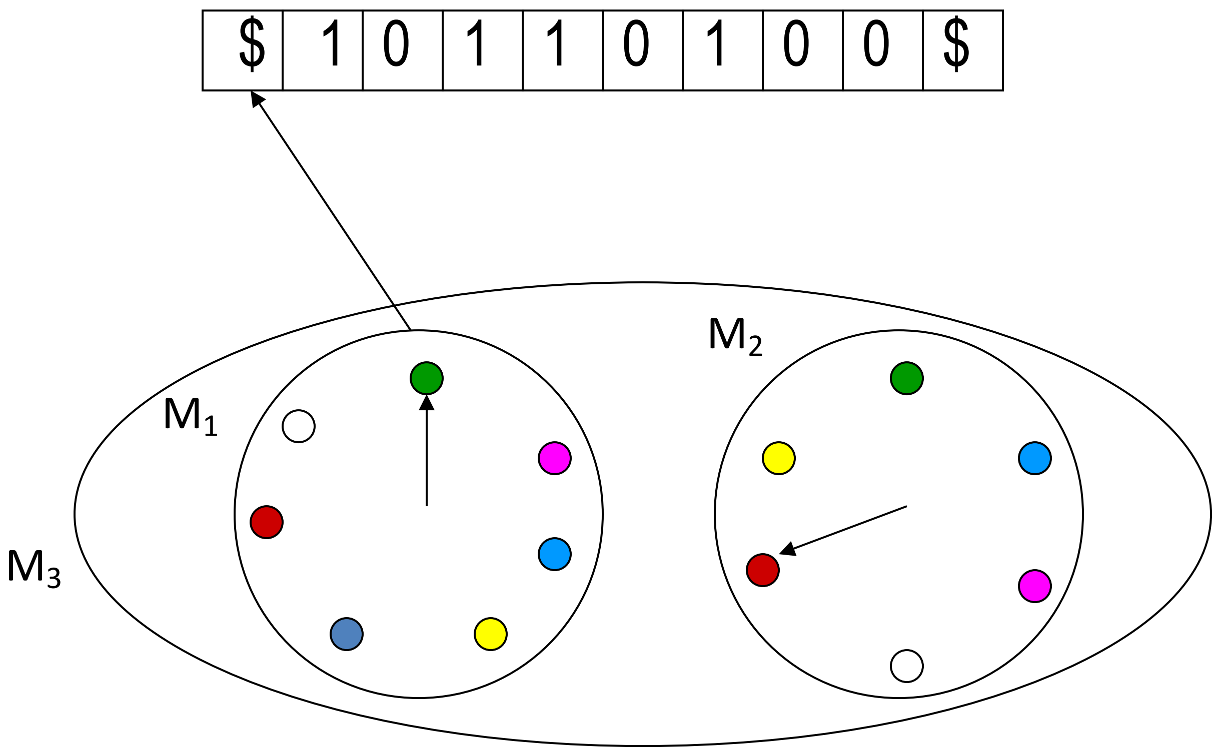

Draw DFA online

- http://madebyevan.com/fsm/

  - Add a state: double-click on the canvas
  - Add an arrow: shift-drag on the canvas
  - Move something: drag it around
  - Delete something: click it and press the delete key (not the backspace key); On Laptop/Macbook, please press “Fn” + “Delete/backspace”.
  - Make accept state: double-click on an existing state
  - Add start state: shift-drag on the canvas to one state
  - Type numeric subscript: put an underscore before the number (like "S_0")
  - Type greek letter: put a backslash before it (like "\beta")