CS 6041
Theory of Computation

Non-regular languages

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Non-regular languages

- If a language is regular, we can create a deterministic finite automaton (DFA), or nondeterministic finite automaton (NFA), or regular expression for it

- How to determine a language is nonregular?
  - $B = \{0^n1^n \mid n \geq 0\}$?
  - C = {w| w has an equal number of 0s and 1s} ?
  - D = {w| w has an equal number of occurrences of 01 and 10 as substrings} ?

Non-regular languages

- $B = \{0^n 1^n \mid n \geq 0\}$
  - --> non-regular

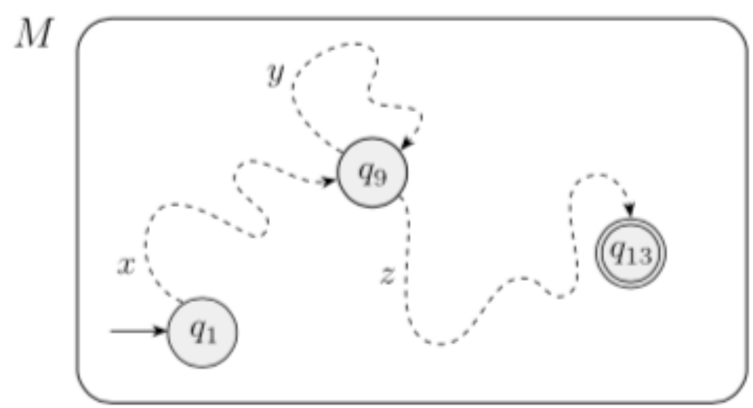
- C = {w| w has an equal number of 0s and 1s}
  - --> non-regular

- D = {w| w has an equal number of occurrences of 01 and 10 as substrings}
  - --> regular

Pumping lemma

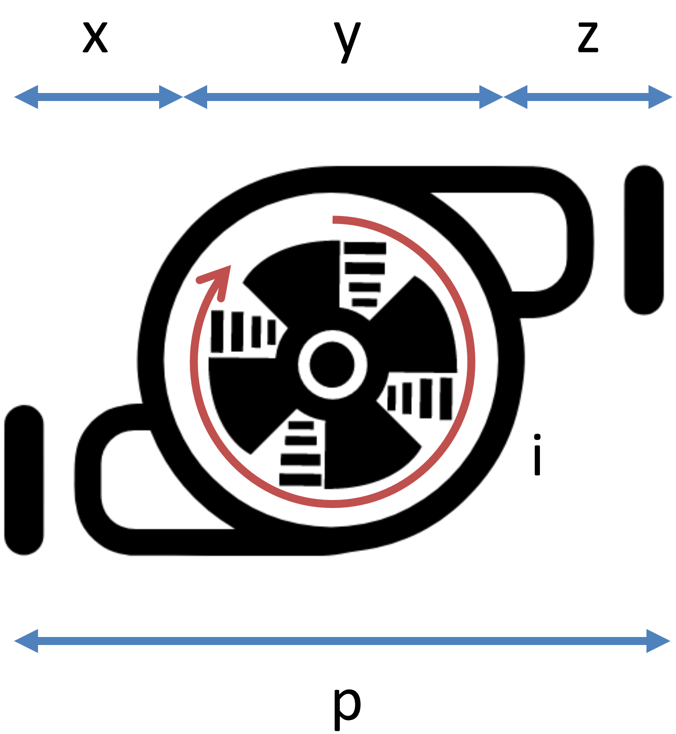
- All regular languages have a special property:

- A is RL, then there is a number p (pumping length), where if $s \in A$ and $|s| \geq p$, then s=xyz, satisfying the following:
  1) $\forall i \geq 0, xy^iz \in A$;
  2) $|y|>0$;
  3) $|xy| \leq p$.



Pumping lemma

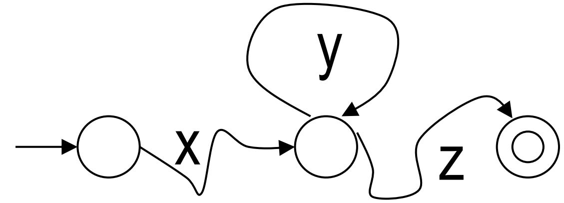
- If we can show that a language does not have this property, we are guaranteed that it is ***not regular***



$xyz \in A$

$xy^iz \in A$

Pumping lemma



- Proof:

p is the number of states

Suppose $A=L(M)$, $M=(Q,\Sigma,\delta,q_1,F)$, $|Q|=p$, $s=s_1s_2\ldots s_n\in A$, $n\geq p$.

$s\in A$, thus the last state r_{n+1} is accept

Computation on M with input s is $r_1,r_2,\ldots,r_{n+1}$, $\delta(r_i,s_i)=r_{i+1}$

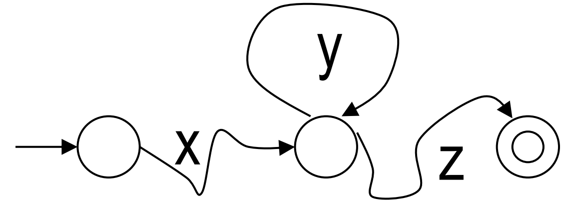
Based on pigeonhole principle, there exist $j<l$ ($l\leq p+1$) to let $r_j=r_l$

We have p states, so as long as we have at most p+1 states, two must be the same

$$r_1,r_2,\ldots,r_j,\ldots,\ r_l,\ldots,r_{n+1}$$

x y z

Pumping lemma



- Proof:

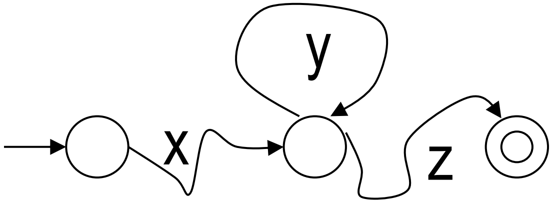
  Let $x=s_1 \ldots s_{j-1}$, $y=s_j \ldots s_{l-1}$, $z=s_l \ldots s_{n+1}$. Because

  r_{n+1} is accept state, thus $\forall i \geq 0$, $xy^iz \in A$.

$$r_1, r_2, \ldots, r_j, \ldots, r_l, \ldots, r_{n+1}$$

x y z

Pumping lemma



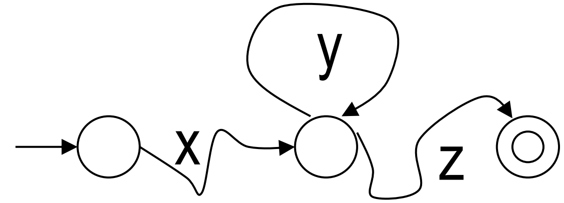
- Proof:

  Because $j \neq l$, thus $|y|>0$.

$$r_1, r_2, \ldots, r_j, \ldots, r_l, \ldots, r_{n+1}$$

x y z

Pumping lemma



- Proof:

  Based on pigeonhole principle, there exist $j<l$ ($l \leq p+1$) to let $r_j = r_l$

  Because $l \leq p+1$, thus $|xy| \leq l-1 \leq p$.

to have two same states (totally p states),
l is at most p+1

$r_1, r_2, \ldots, r_j, \ldots, r_l, \ldots, r_{n+1}$

x y z

Pumping lemma

- All regular languages have a special property:

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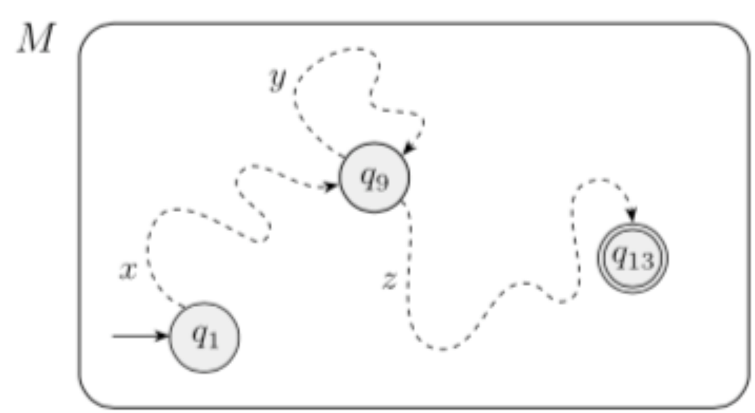

Pumping lemma example

1) $\forall i \geq 0, xy^iz \in A$;

2) $|y| > 0$;

3) $|xy| \leq p$.

- $B = \{0^n1^n \mid n \geq 0\}$ is not regular
- Prove:

Suppose B is regular and p is the pumping length, let $s = 0^p1^p$,

Because $s \in B$ and $|s| > p$,

So $s = xyz = 0^p1^p$ and for each $i \geq 0$, that $xy^iz \in B$

(1) If y only has 0s, then xyyz has more 0s than 1s, so $xyyz \notin B$

(2) If y only has 1s, something happens

(3) If y has 0s and 1s, for xyyz, we will has "1…0" in the substring yy, so $xyyz \notin B$

Contradiction happens. So B is not regular.

Pumping lemma example

1) $\forall i \geq 0, xy^i z \in A$;

2) $|y| > 0$;

3) $|xy| \leq p$.

- C = {w| w has an equal number of 0s and 1s} is not regular
- Prove:

Suppose C is regular and p is the pumping length,

let $s = 0^p 1^p = xyz$,

Because $s \in C$ and $|s| > p$,

so that each $i \geq 0$, that $xy^i z \in C$ and $|xy| \leq p$

If $|xy| \leq p$, then y only has 0s.

Based on the previous prove in language B, we can get $xyyz \notin C$

Contradiction happens. So C is not regular.

Pumping lemma example

1) $\forall i \geq 0, xy^i z \in A$;

2) $|y|>0$;

3) $|xy| \leq p$.

- Let F = {ww| w ∈ $\{0,1\}^*$}. We show that F is nonregular
- Prove:

Suppose F is regular and p is the pumping length,

let s = $0^p 1 0^p 1$ = xyz,

Because s∈F and |s|> p, so that each i≥0,

that xy^iz ∈ F and |xy|≤p

If |xy|≤p, then y only has 0s. Then we can get xyyz ∉ F

Contradiction happens. So F is not regular.

Pumping lemma example

1) $\forall i \geq 0, xy^iz \in A$;

2) $|y|>0$;

3) $|xy| \leq p$.

- $E = \{0^i1^j \mid i > j\}$ is not regular
- Prove:

Suppose E is regular and p is the pumping length,

let $s = 0^{p+1}1^p = xyz$,

Because $s \in F$ and $|s| > p$, so that each $i \geq 0$,

that $xy^iz \in F$ and $|xy| \leq p$

If $|xy| \leq p$, then y only has 0s. We let i=0, then we have xz

Because in s, the number of 0s is only one more than the number of 1s, then in xz, the number of 0s cannot be more than 1s, therefore $xz \notin E$

Contradiction happens. So E is not regular.

Non-regular languages