CS 6041
Theory of Computation

Nondeterministic finite automata

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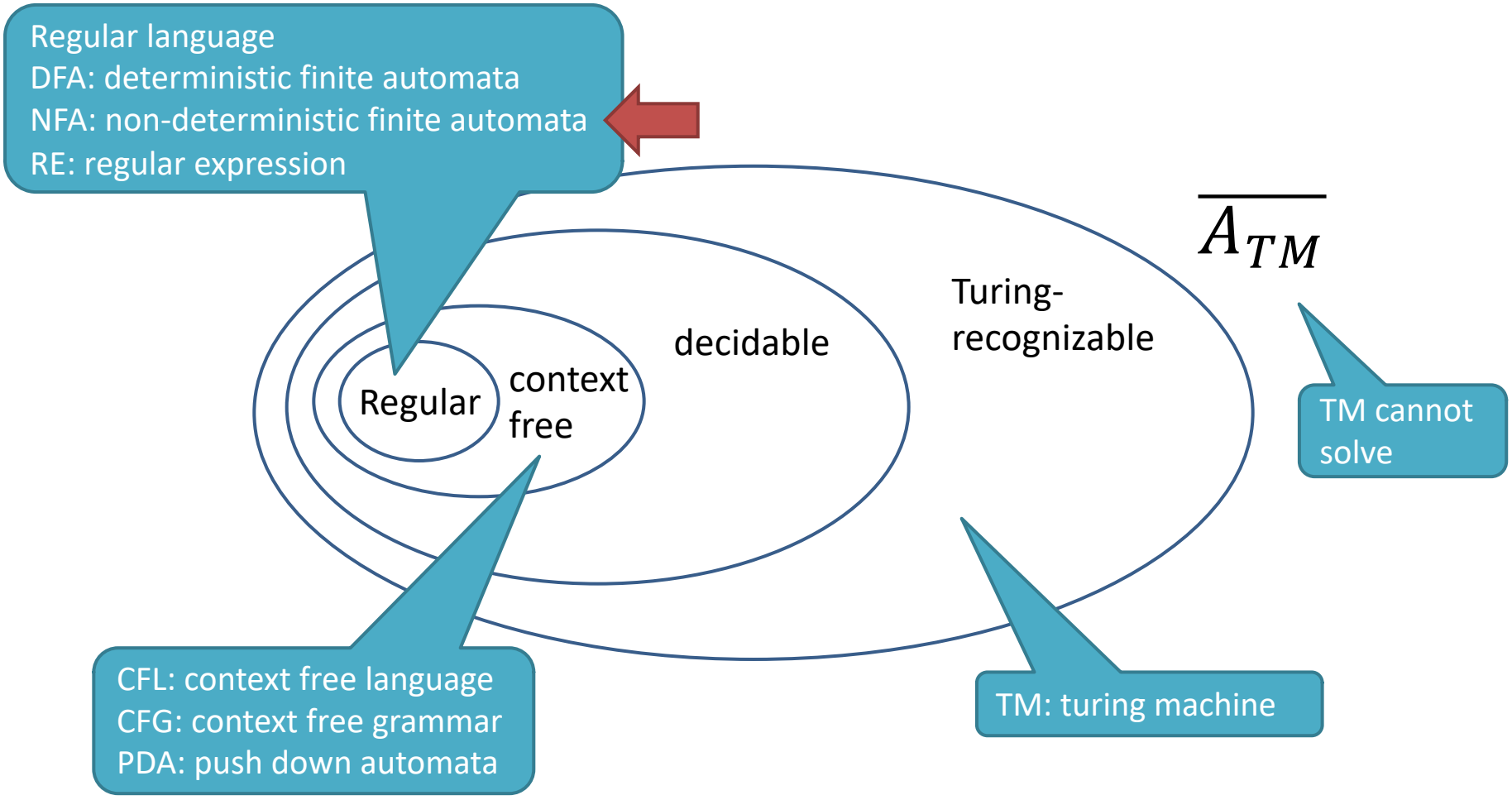
https://kevinsuo.github.io/

Where are we now?



Design a NFA for a language

- Step 1: list all possible states
- Step 2: draw all the transitions between the states
- Step 3: add start and accept states

Some practices

- L_1 = {Set of all strings that end with 0}, Σ = {0,1};

Can anyone draw the NFA?

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q1: all the strings
q2: last letter is 0

NFA of L_1

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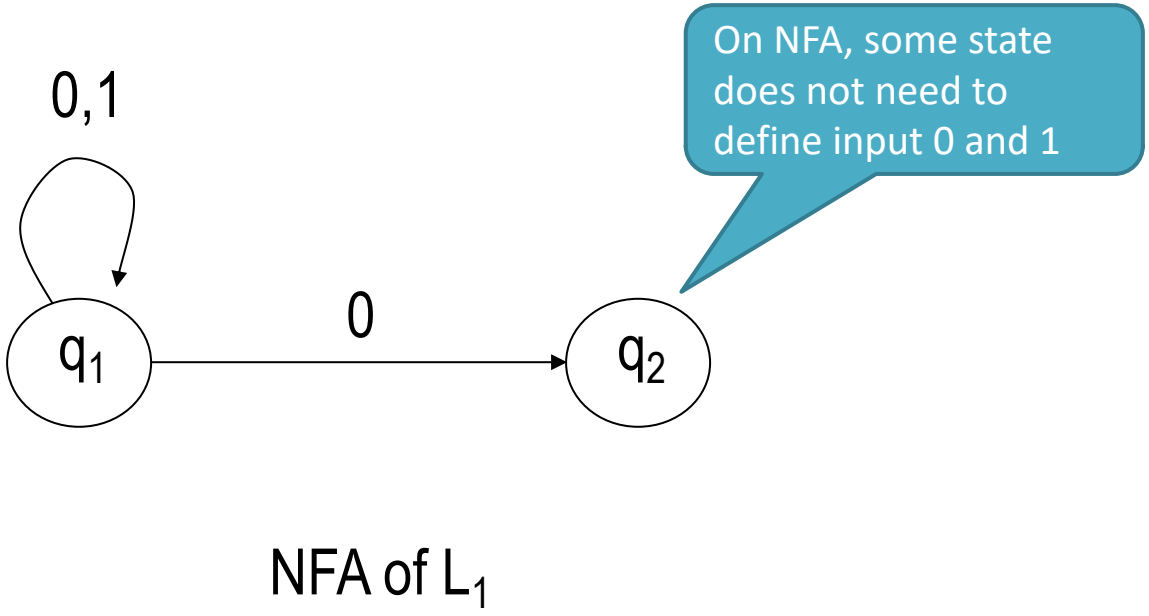

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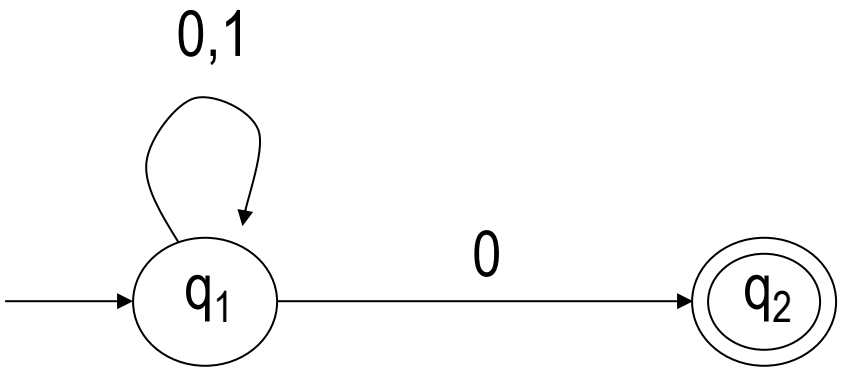

NFA of L_1

Some practices

- L_2 = {Set of all strings that start with 0}, Σ = {0,1};

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NFA of L_2

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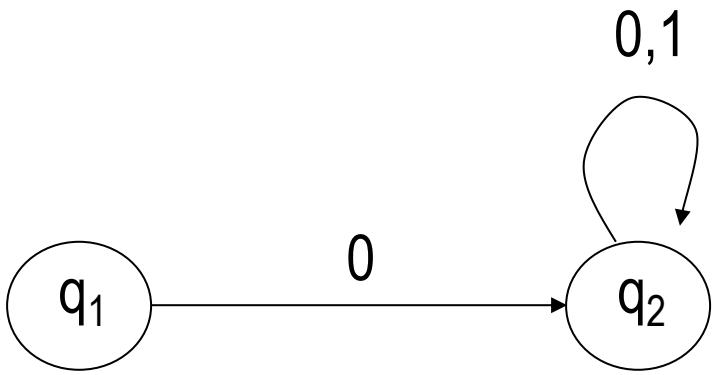

NFA of L_2

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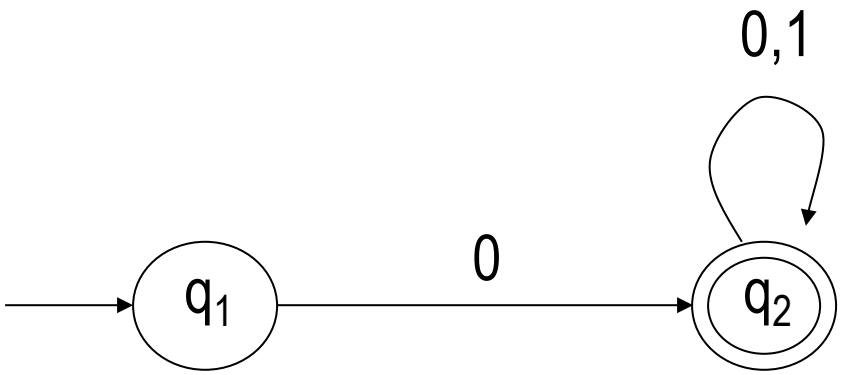

NFA of L_2

Some practices

- L_3 = {Set of all strings that length is 2}, Σ = {0,1};

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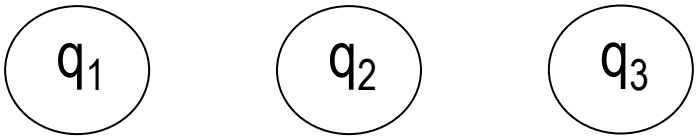


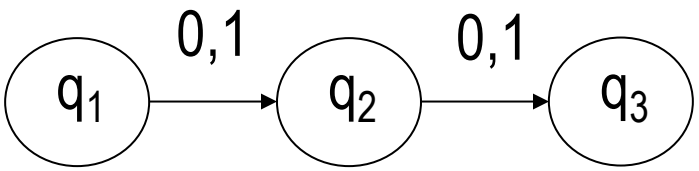
q1: strings that length is 0
q2: strings that length is 1
q3: strings that length is 2

NFA of L_3

Some practices

- L_3 = {Set of all strings that length is 2}, Σ = {0,1};

Can anyone draw the NFA?



NFA of L_3

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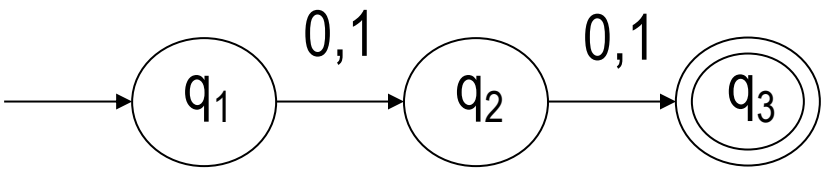

NFA of L_3

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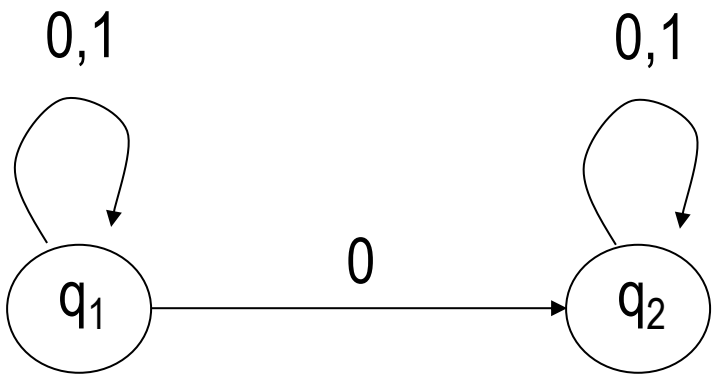

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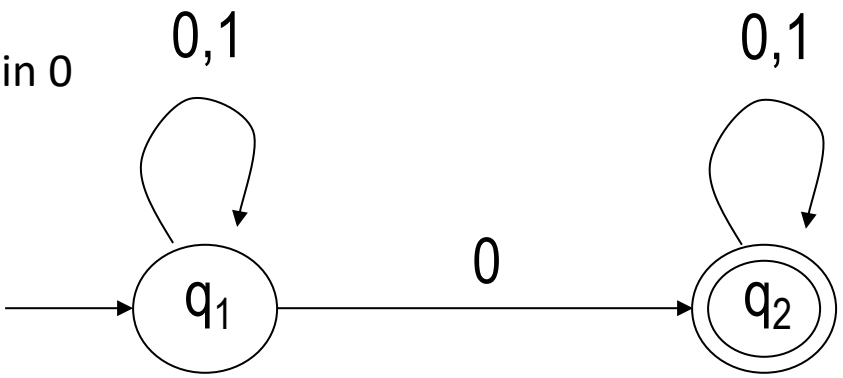

NFA of L_4

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- L_5 = {Set of all strings that starts with '10'}, Σ = {0,1};

Can anyone draw the NFA?

NFA of L_5

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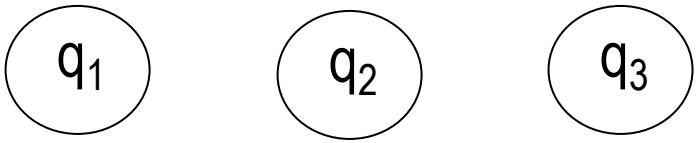
NFA of L_5



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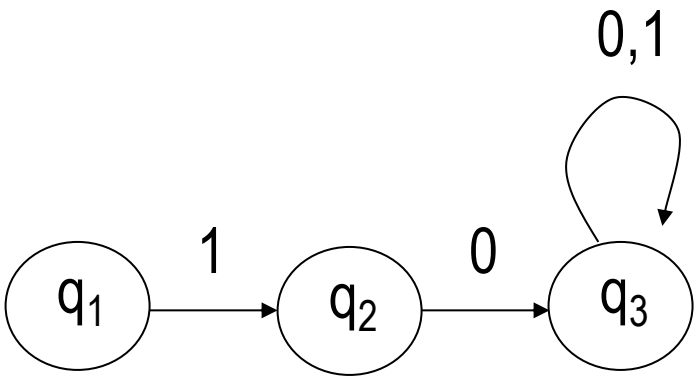

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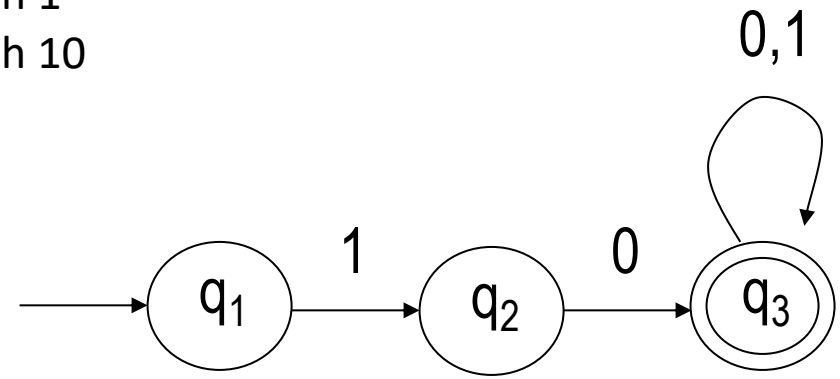

Some practices

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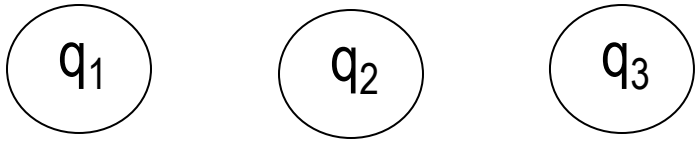
NFA of L_6



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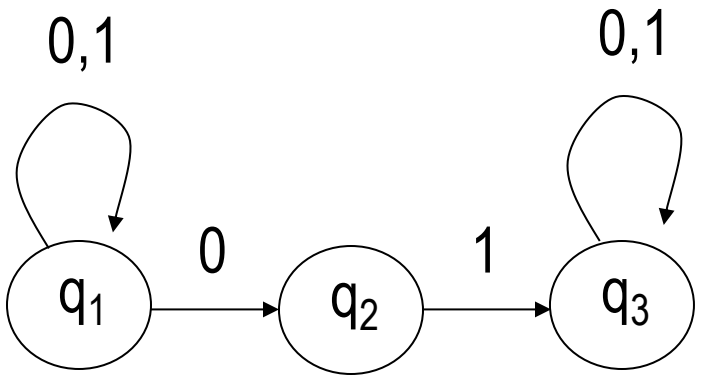

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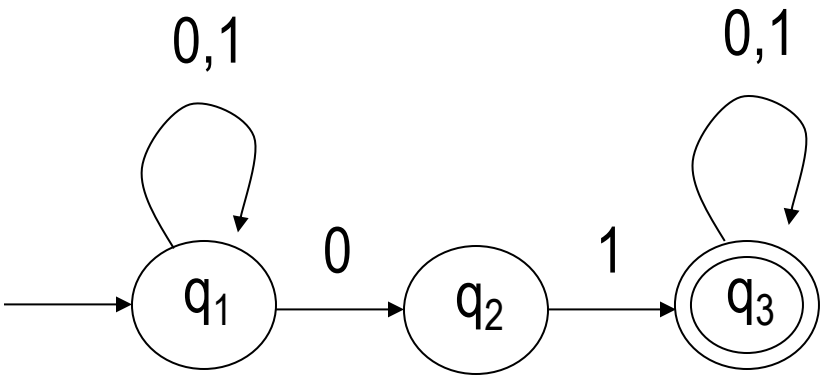

Some practices

- L_7 = {Set of all strings that end with '11'}, Σ = {0,1};

Can anyone draw the NFA?

NFA of L_7

Some practices

- L_7 = {Set of all strings that end with '11'}, Σ = {0,1};

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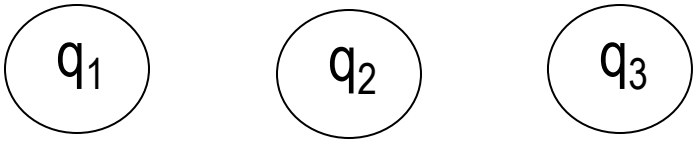
q2: strings that end with 1
q3: strings that end with 11

NFA of L_7



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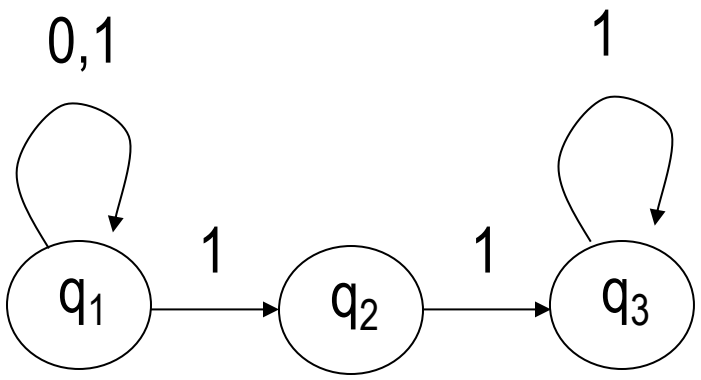
q3: strings that end with 11

NFA of L_7



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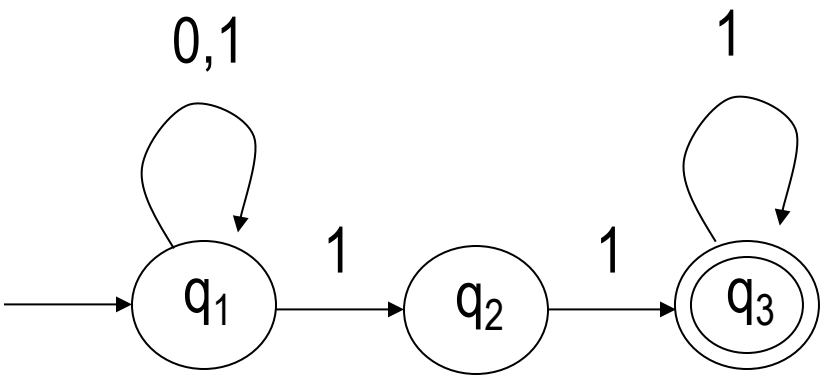


Equivalence of NFAs and DFAs

- Definition of equivalence
  - two machines are ***equivalent*** if they recognize the same language

Equivalence of NFAs and DFAs

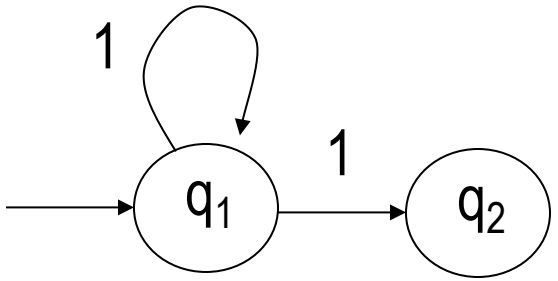
- Theorem: Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

- Proof idea:

  For given NFA, build an equivalent DFA

  Use DFA to simulate NFA DFA records all branches of NFA

  - If NFA has k states, then DFA have totally 2^k states (subset of k states in NFA)

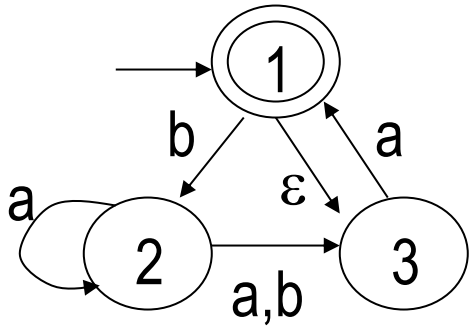


$\{\emptyset, q_1, q_2, \{q_1, q_2\}\}$

Equivalence of NFAs and DFAs

- Closure E(R) on ε

  Collection of states that can be reached from members of R by going only along ε arrows, including the members of R themselves.



What is E({1})?

E({1}) =
{1,3}

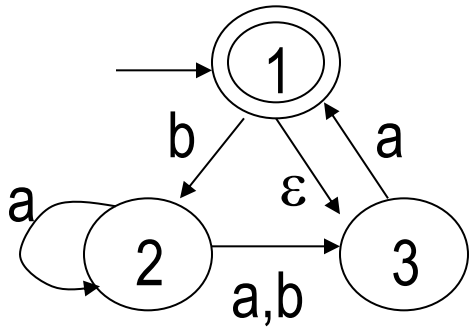
Equivalence of NFAs and DFAs

- Step 1: Draw all the states in DFAs
- Step 2: Define the transitions in DFAs based on the NFAs
- Step 3: Define the start state and accept state in DFAs
- Step 4: Remove all inaccessible states

Example

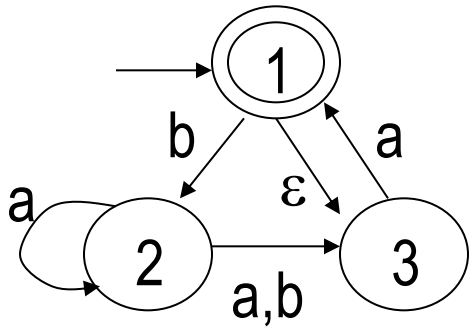
- NFA $N_4=(\{1,2,3\},\{a,b\},\delta,1,\{1\})$

  What is its equivalent DFA?



Example

- List all subset states



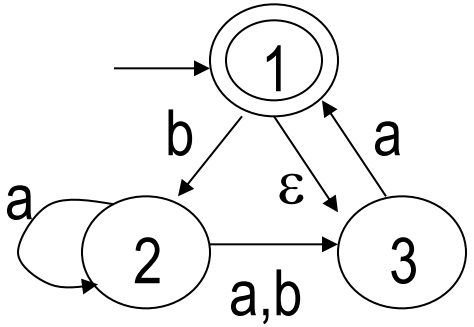
∅ {1} {2} {1,2}

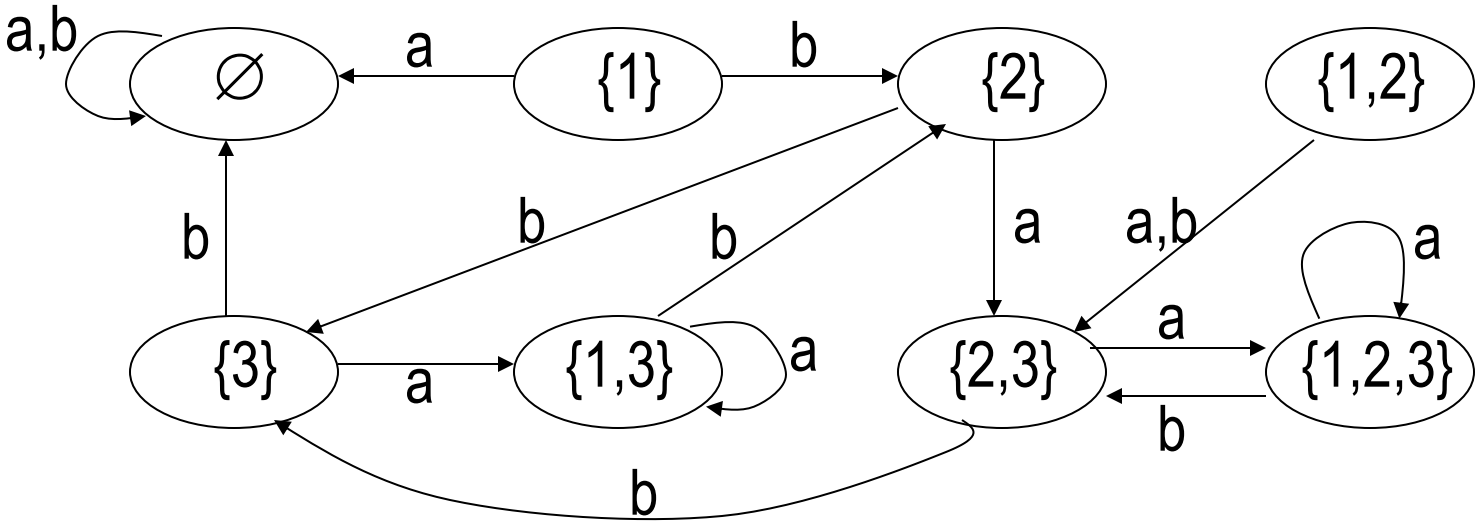
{3} {1,3} {2,3} {1,2,3}

Example

State {1} goes to ∅ on a because no a arrows exit it. It goes to {2} on b. Note that the procedure in Theorem 1.39 specifies that we follow the ε arrows *after* each input symbol is read. An alternative procedure based on following the ε arrows before reading each input symbol works equally well, but that method is not illustrated in this example.

- Add transitions



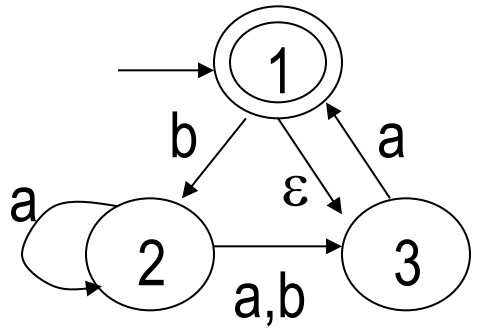


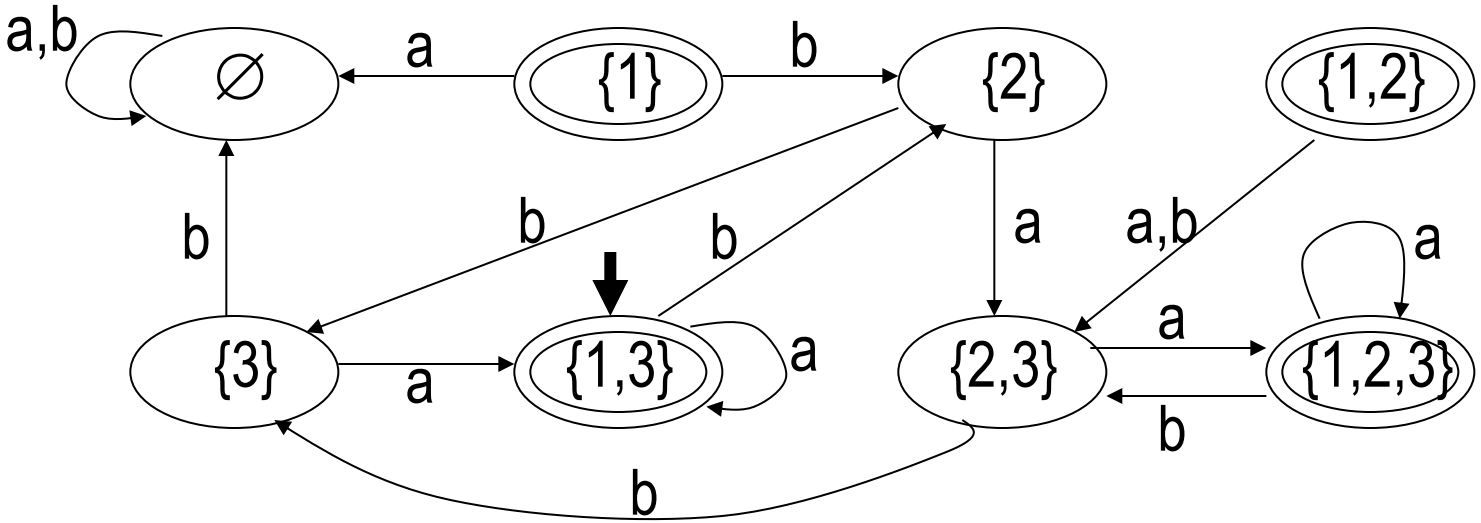
Example

Closure E(R) on ε

Collection of states that can be reached from members of R by going only along ε arrows, including the members of R themselves.

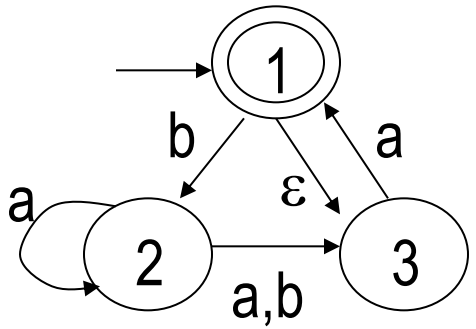
- Start state: E({1}) = {1,3}
- Accept state: all states with 1

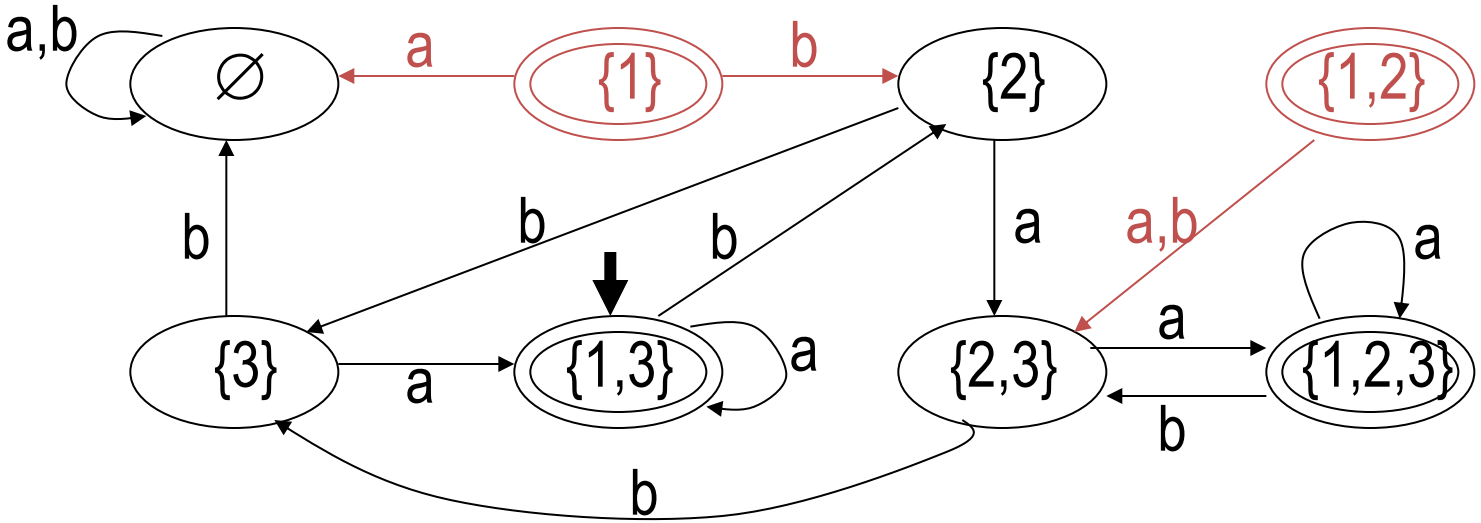




Example

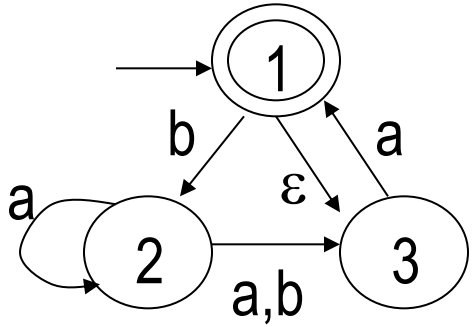
- Remove inaccessible state

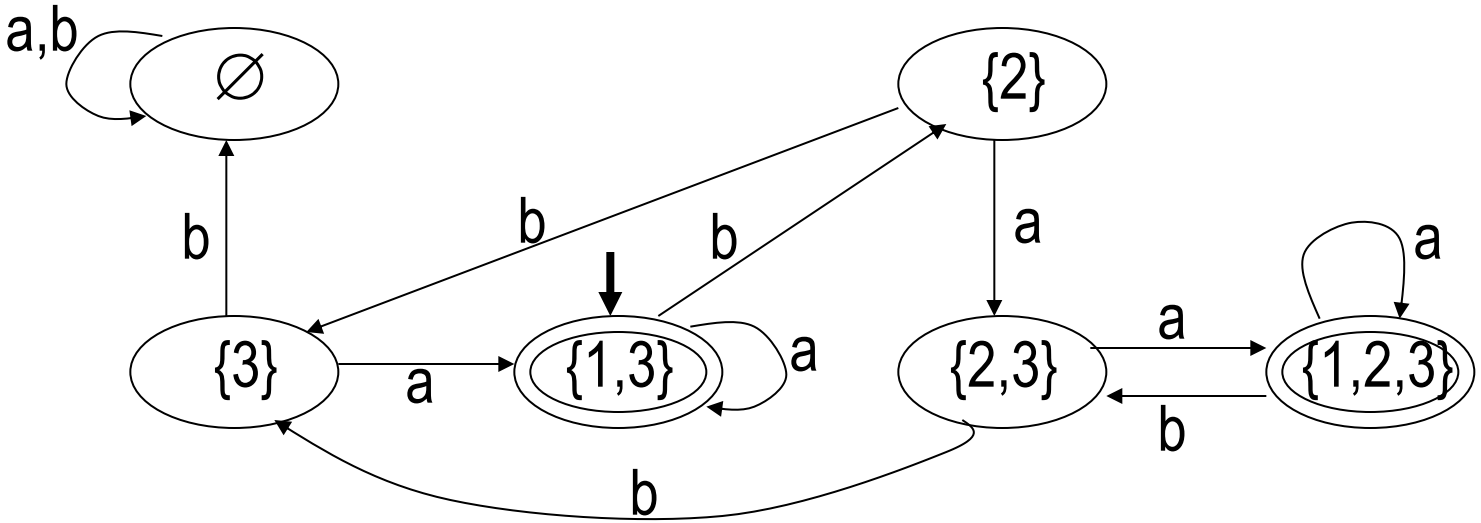




Example

- Remove inaccessible state
  - {1}, {1,2}





Equivalence of NFAs and DFAs

- Theorem: Every NFA has an equivalent DFA.
- Proof:

  let NFA N = $(Q,\Sigma,\delta, q_0, F)$,

  build DFA $M=(Q',\Sigma,\delta',q_0',F')$, $L(M)=L(N)$.

  let $Q'=P(Q)$. // 2^k subsets of Q, k=size of Q.

  $q_0'=E(\{q_0\})$. // Closure q_0 on transition ε

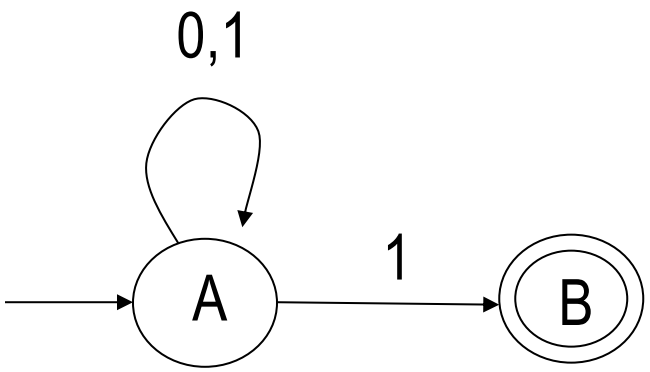
  $F'=\{R \in Q' \mid R \cap F \neq \emptyset\}$. // F' contains accept states in F

  $\delta'(R,a)=U_{r \in R} E(\delta(r,a))$, for $R \in Q'$ and $a \in \Sigma$. // δ': all states that can be reached by going along ε arrows after every step

E(R)={q | reach q from R by going along ε}

Practices: Conversion of NFA to DFA

- L = {Set of all string that ends with ‘1’}, Σ = {0,1};



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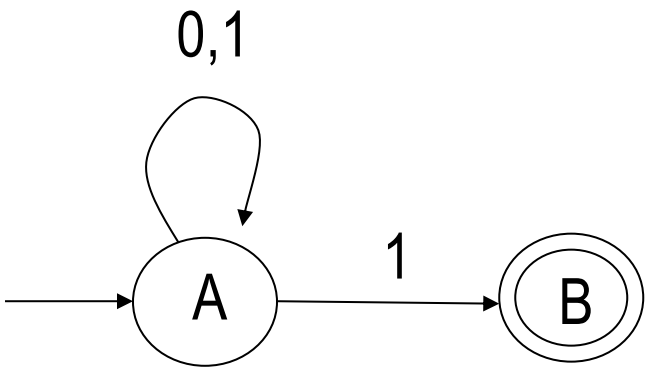

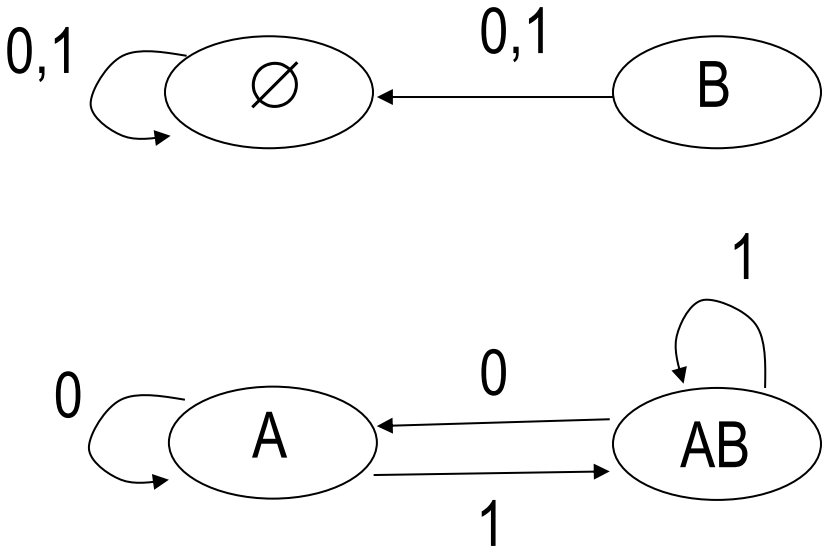

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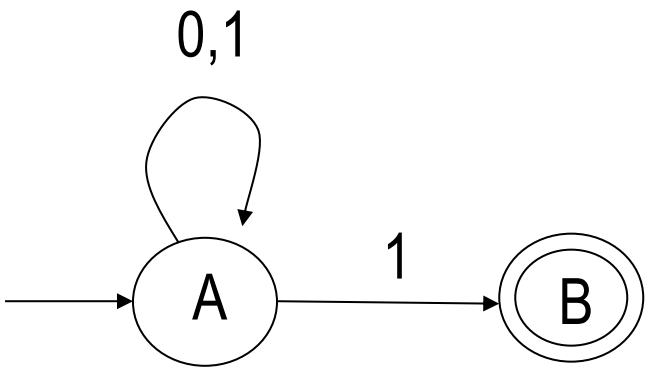

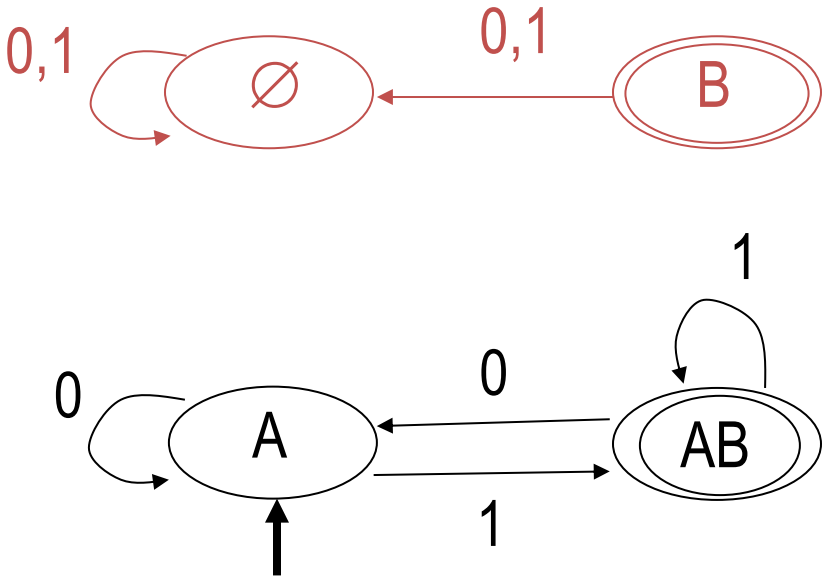

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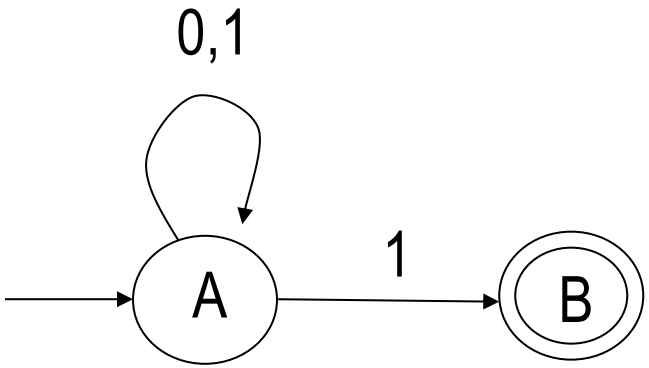

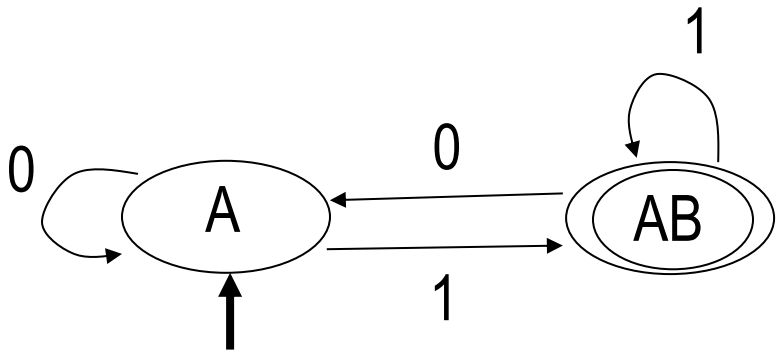

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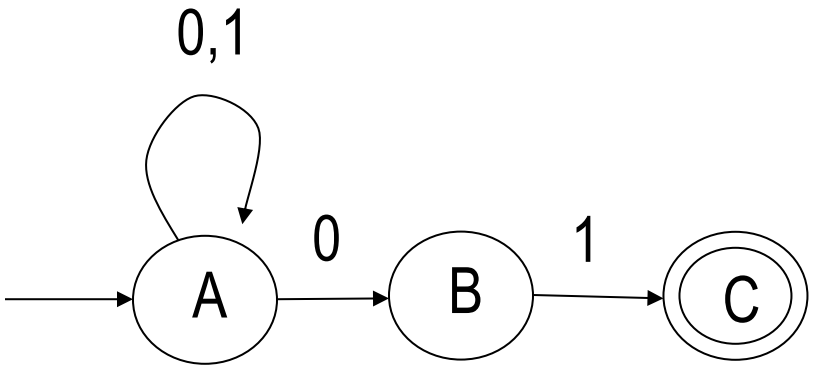

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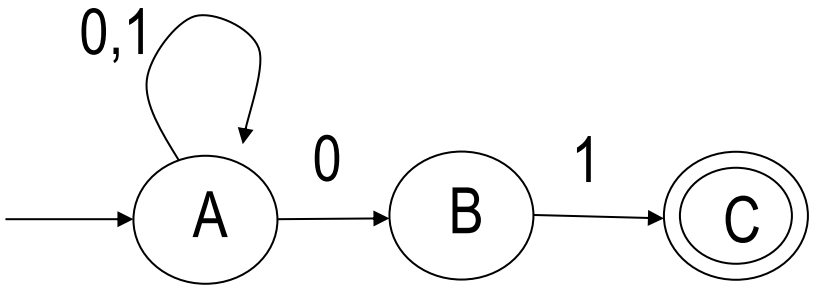
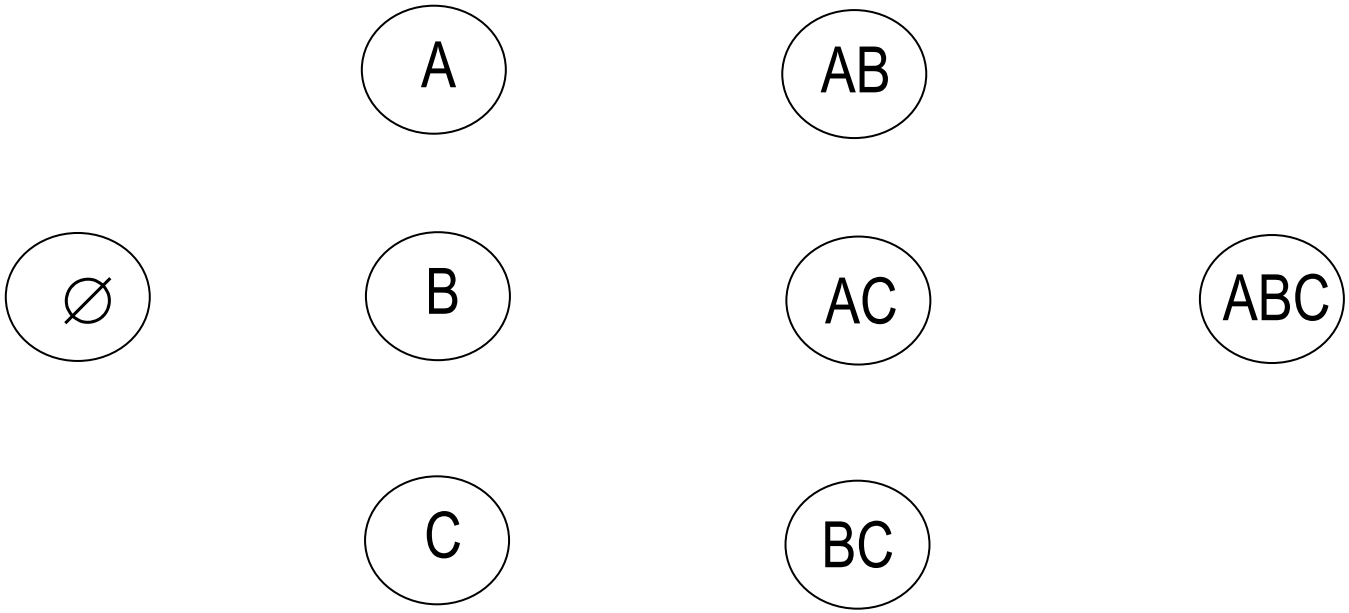
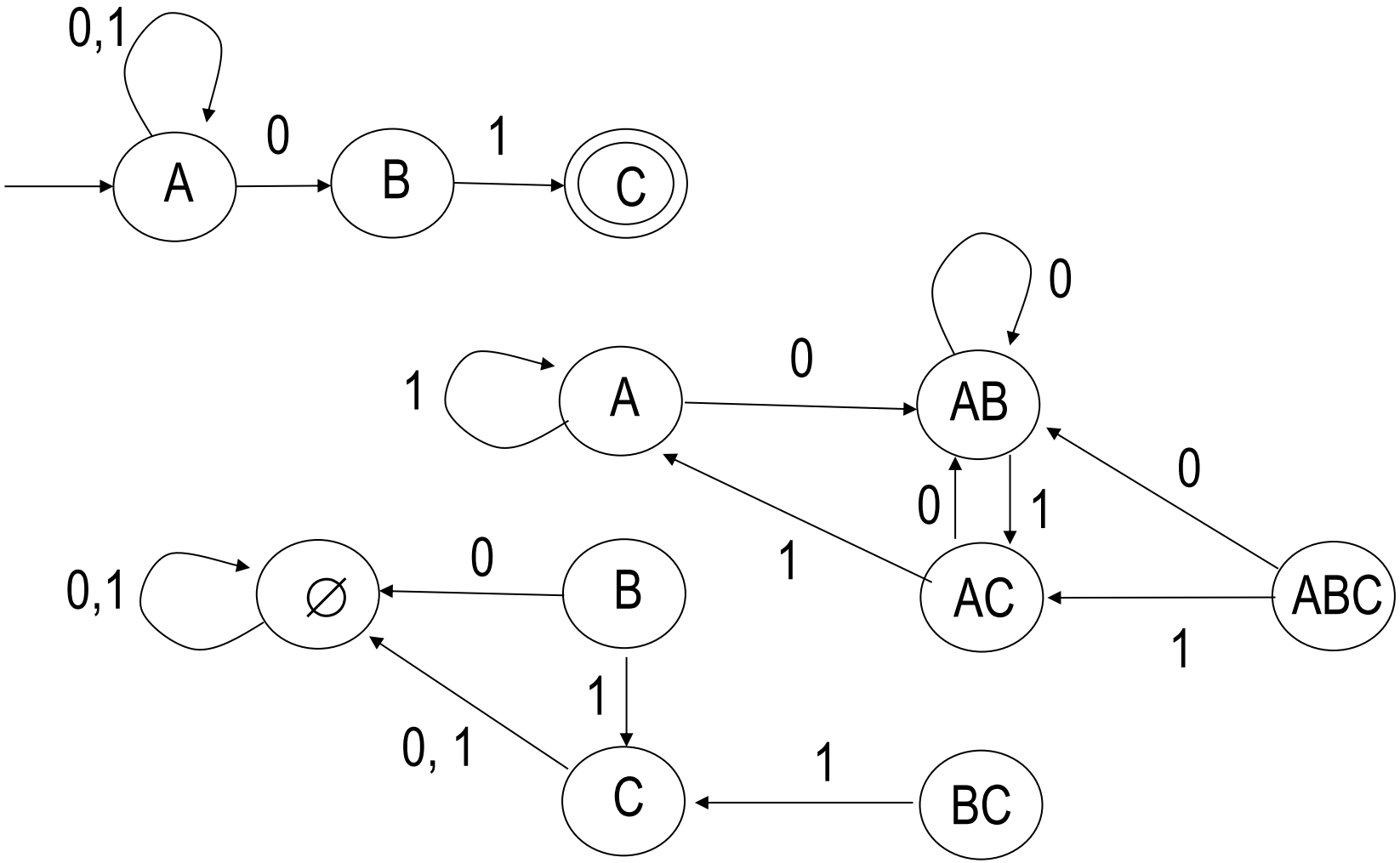
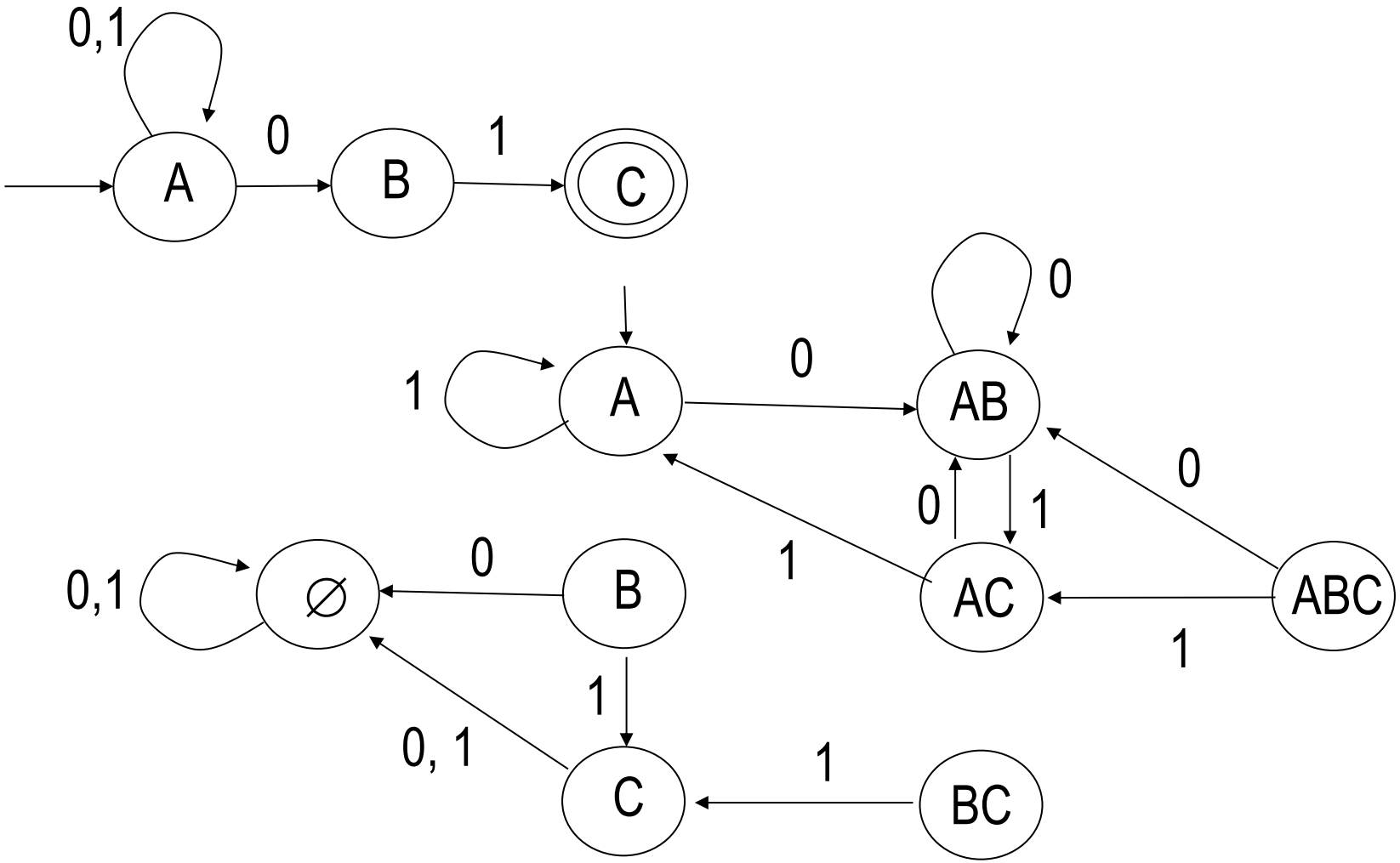




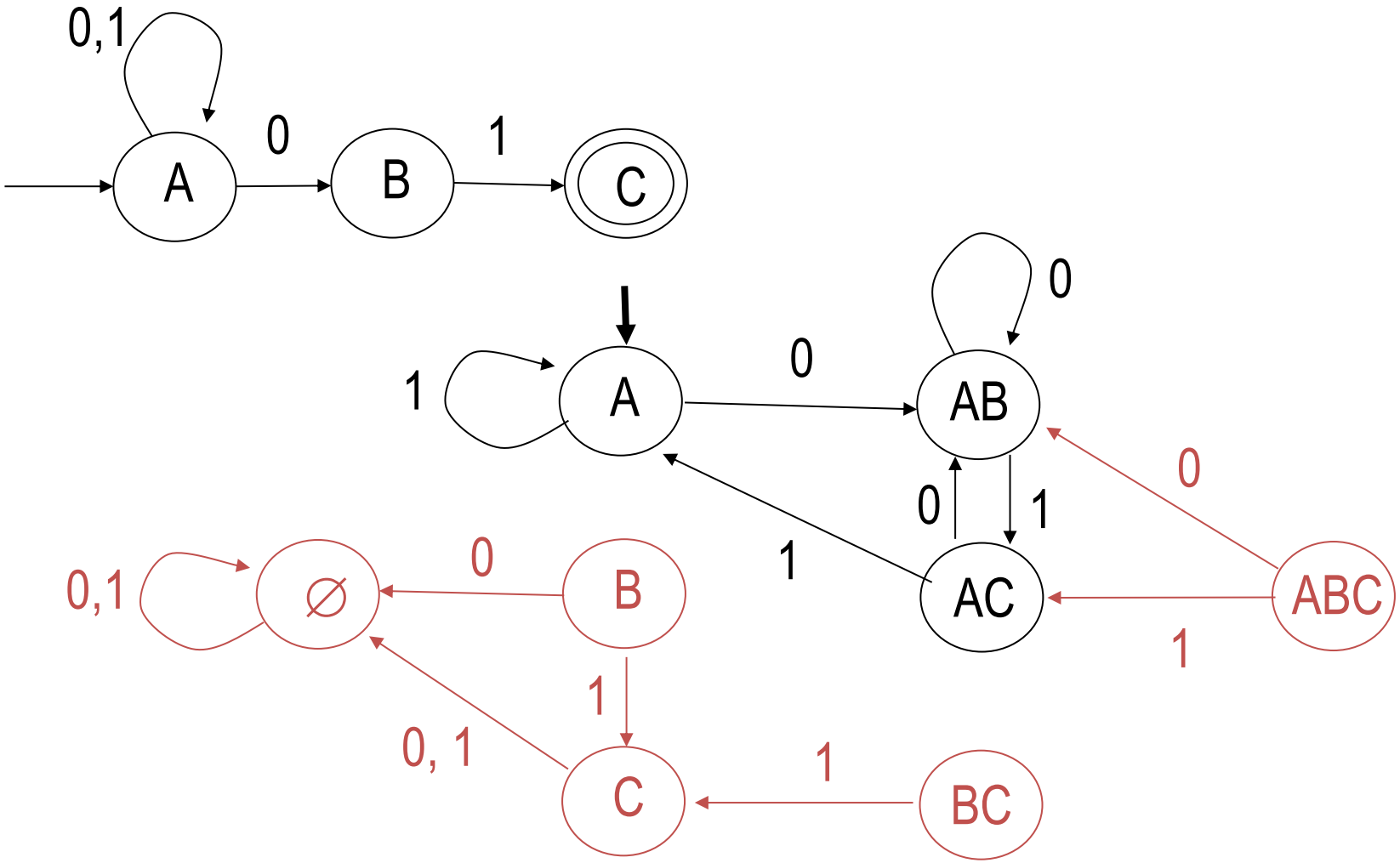
Practices: Conversion of NFA to DFA



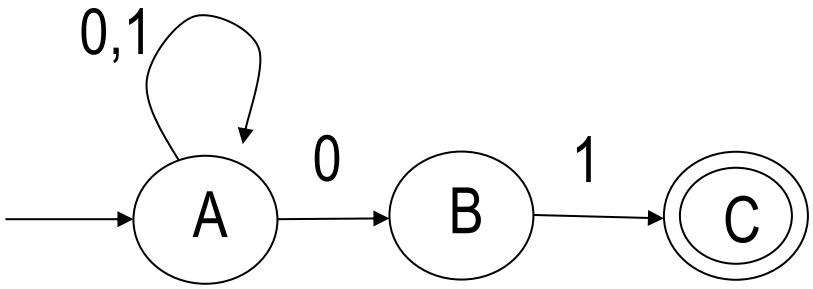
Practices: Conversion of NFA to DFA



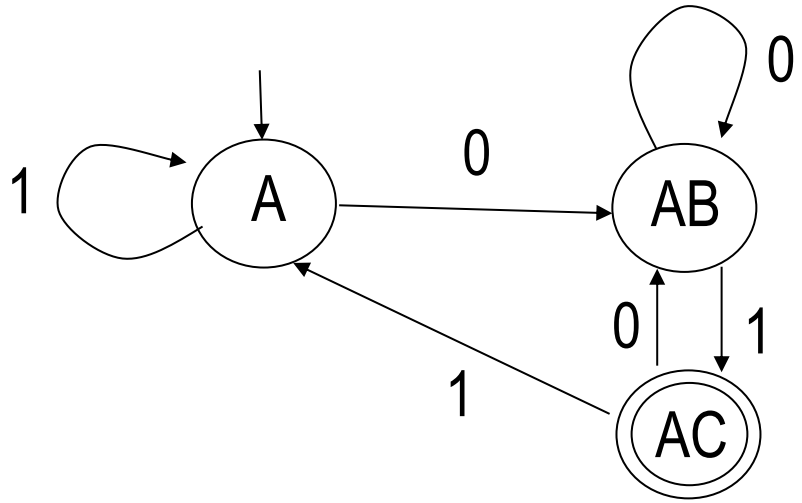
Practices: Conversion of NFA to DFA



Practices: Conversion of NFA to DFA



DFA



Equivalence of NFAs and DFAs

- Step 1: Draw all the states in DFAs
- Step 2: Define the transitions in DFAs based on the NFAs
- Step 3: Define the start state and accept state in DFAs
- Step 4: Remove all inaccessible states

Draw NFA online

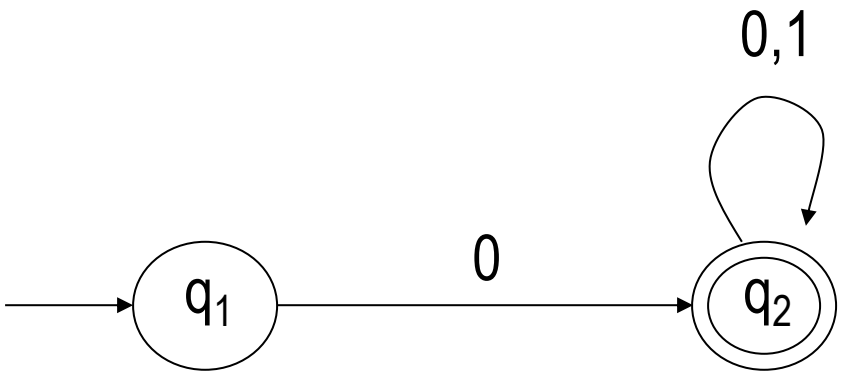
- http://madebyevan.com/fsm/
  - Add a state: double-click on the canvas
  - Add an arrow: shift-drag on the canvas
  - Move something: drag it around
  - Delete something: click it and press the delete key (not the backspace key); On Laptop/Macbook, please press “Fn” + “Delete/backspace”.
  - Make accept state: double-click on an existing state
  - Add start state: shift-drag on the canvas to one state
  - Type numeric subscript: put an underscore before the number (like "S_0")
  - Type greek letter: put a backslash before it (like "\beta")

One language → multiple NFAs?

- L = {Set of all strings that start with 0}, Σ = {0,1};

Can anyone draw the NFA?

q1: empty string
q2: first letter is 0



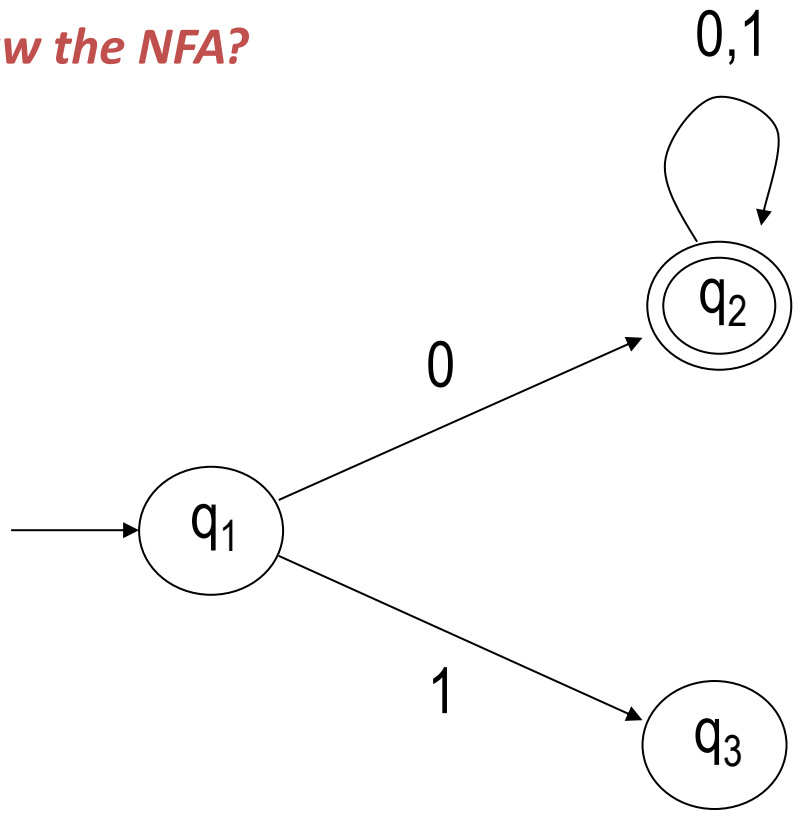
NFA of L

One language → multiple NFAs?

- L = {Set of all strings that start with 0}, Σ = {0,1};

Can anyone draw the NFA?

q1: empty string
q2: first letter is 0
q3: first letter is 1



NFA of L