CS 6041
Theory of Computation

Regular language

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https://kevinsuo.github.io/

Regular language

- A language is called a regular language if some finite automaton (DFA) recognizes it

- A language is called a regular language if some nondeterministic finite automaton (NFA) recognizes it

- Regular language:
  - $L=L(M)$
  - M is DFA or NFA

Closure under the union

- Theorem: regular language is closed under the union operation.

- Proof idea:

  In the past, we prove this by creating a DFA. This time we prove it by creating a NFA.

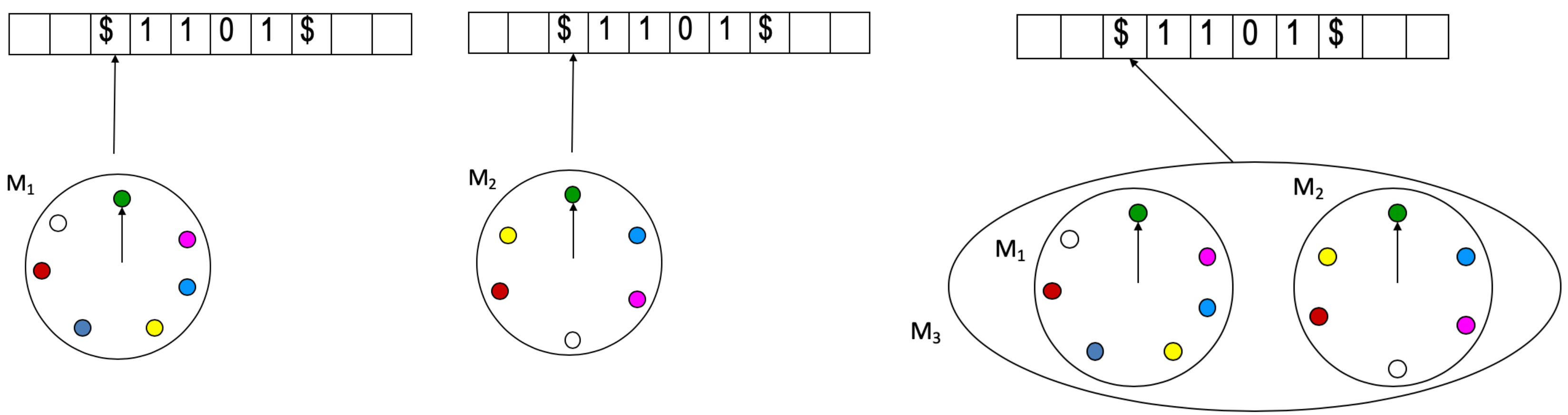
Closure under the union proved by DFA

- Proof:

Let $L_i=L(M_i)$ is a regular language, $M_i=(Q_i,\Sigma,\delta_i,q_i,F_i)$, i=1,2. We need to build a finite automata to recognize $L_1 \cup L_2$

Build $M_3=(Q_3,\Sigma,\delta_3,q_3,F_3)$.

$Q_3=Q_1 \times Q_2$;



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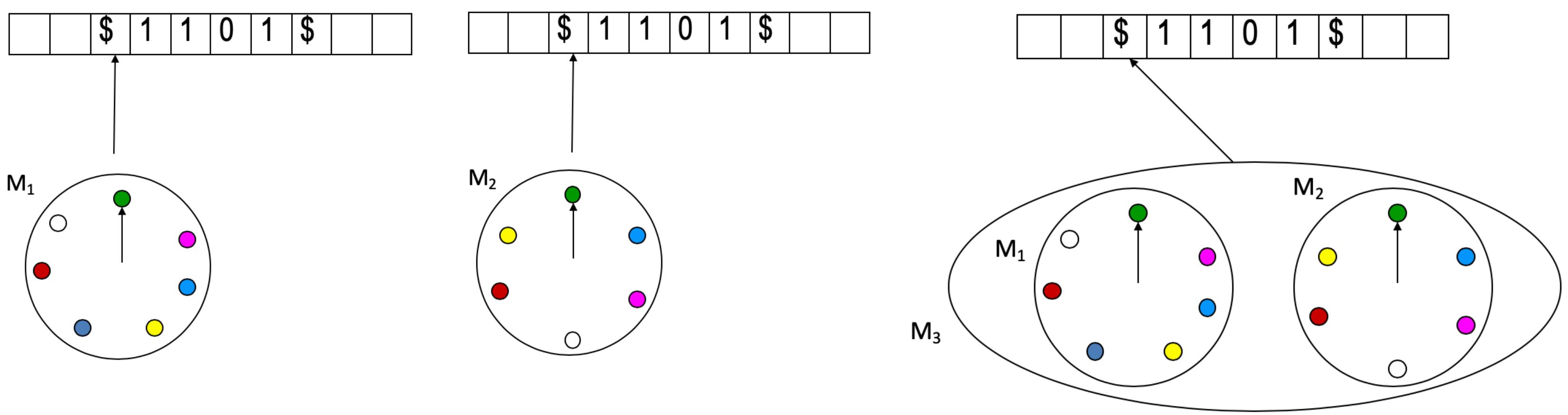
$\delta_3((r_1,r_2),a)=(\delta_1(r_1,a),\delta_2(r_2,a))$;



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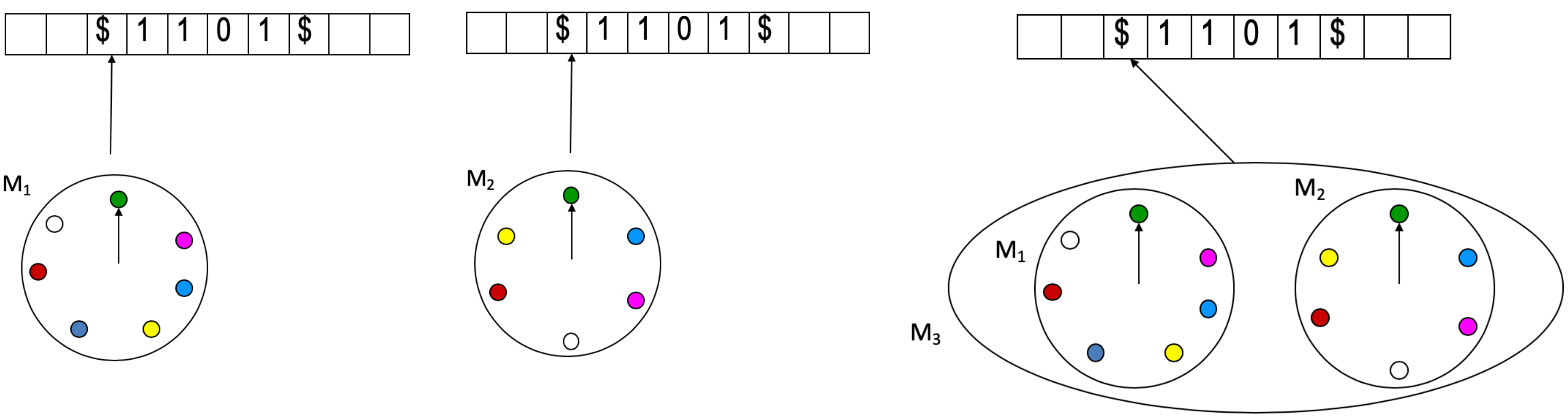

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$F_3=(F_1 \times Q_2) \cup (Q_1 \times F_2)$.

One accept state in M1 with all states in M2
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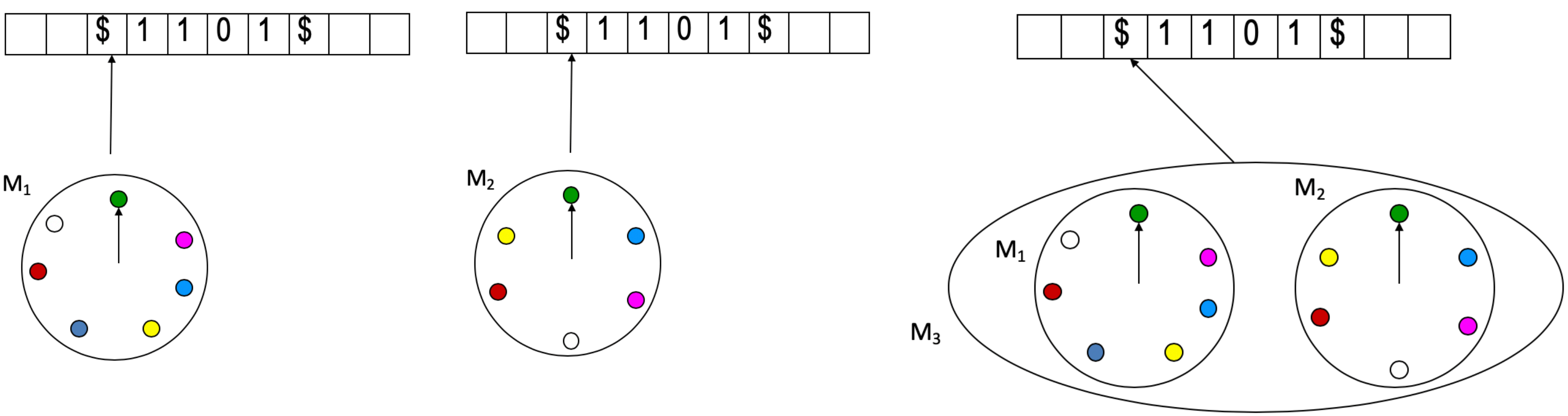

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$q_3=(q_1,q_2)$;
$F_3=(F_1\times Q_2)\cup(Q_1\times F_2)$.

$L(M_3) = L_1\cup L_2$, so $L_1\cup L_2$ is still regular language

Closure under the union

- Theorem: regular language is closed under the union operation.

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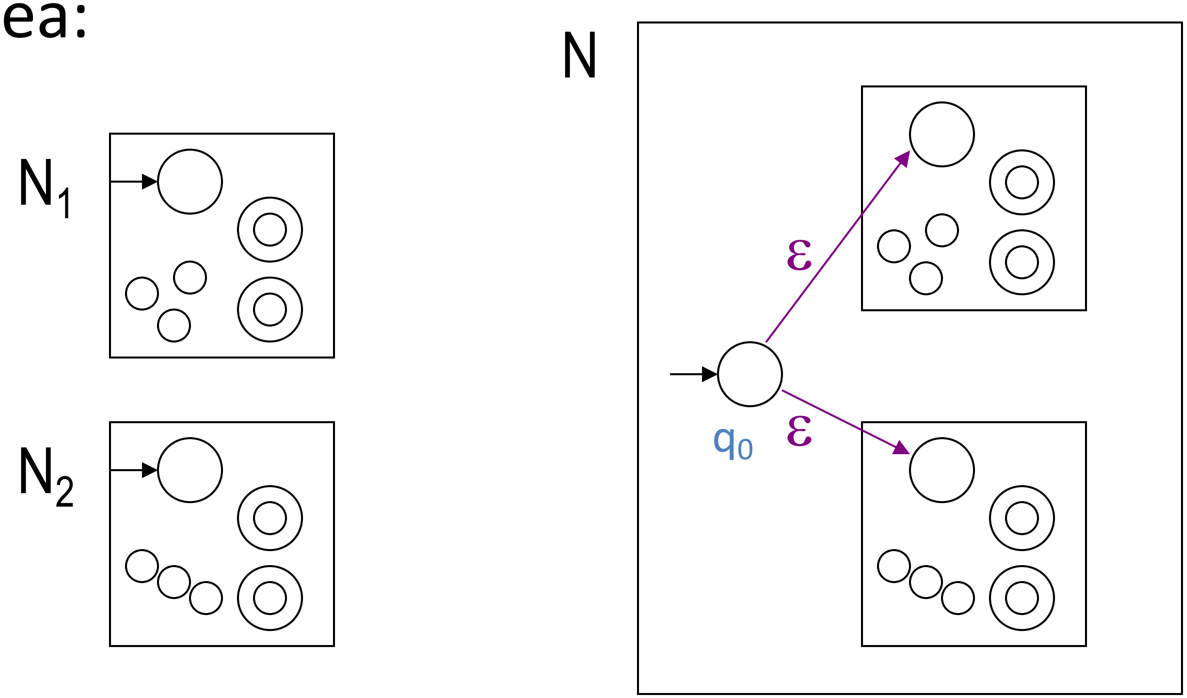


Closure under the union

- Theorem: regular language is closed under the union operation.

- Proof:

  Let NFA $N_i=(Q_i,\Sigma,\delta_i,q_i,F_i)$ recognize A_i, i=1,2.

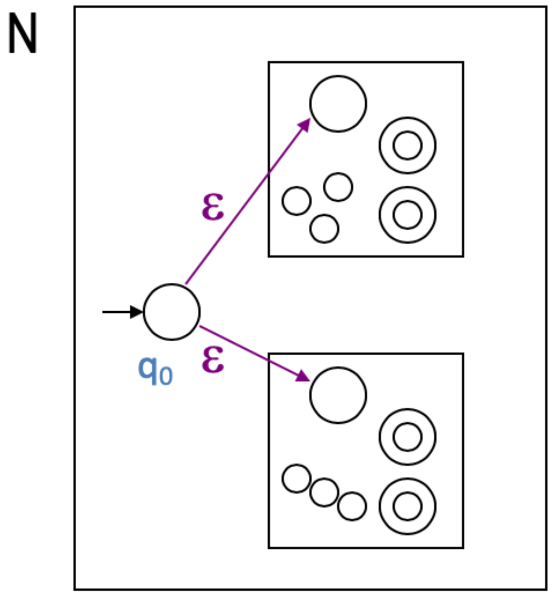
  Create NFA

  $N=(Q,\Sigma,\delta,q_0,F)$ to recognize $A_1 \cup A_2$

  Let $Q=Q_1 \cup Q_2 \cup \{q_0\}$

  $F=F_1 \cup F_2$

$$\delta(q,a) = \begin{cases} \delta_1(q,a), & if\ q \in Q_1 \\ \delta_2(q,a), & if\ q \in Q_2 \\ \{q_1,q_2\}, & if\, q = q_0\ and\ a = \varepsilon \\ \emptyset, & if\, q = q_0\ and\ a \neq \varepsilon \end{cases}$$



Closure under the concatenation

- Theorem: regular language is closed under the concatenation operation.

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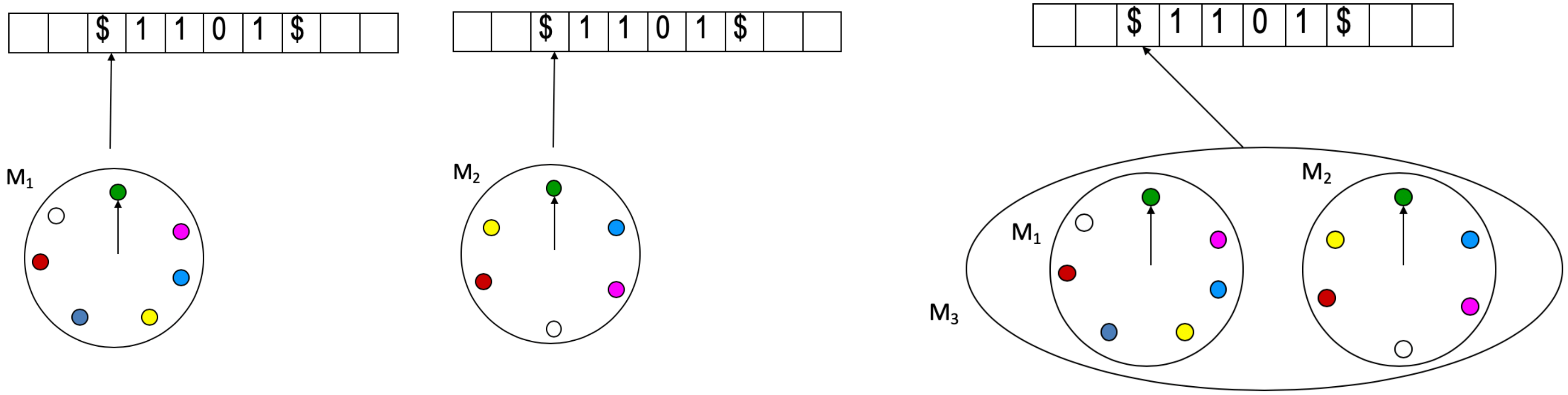

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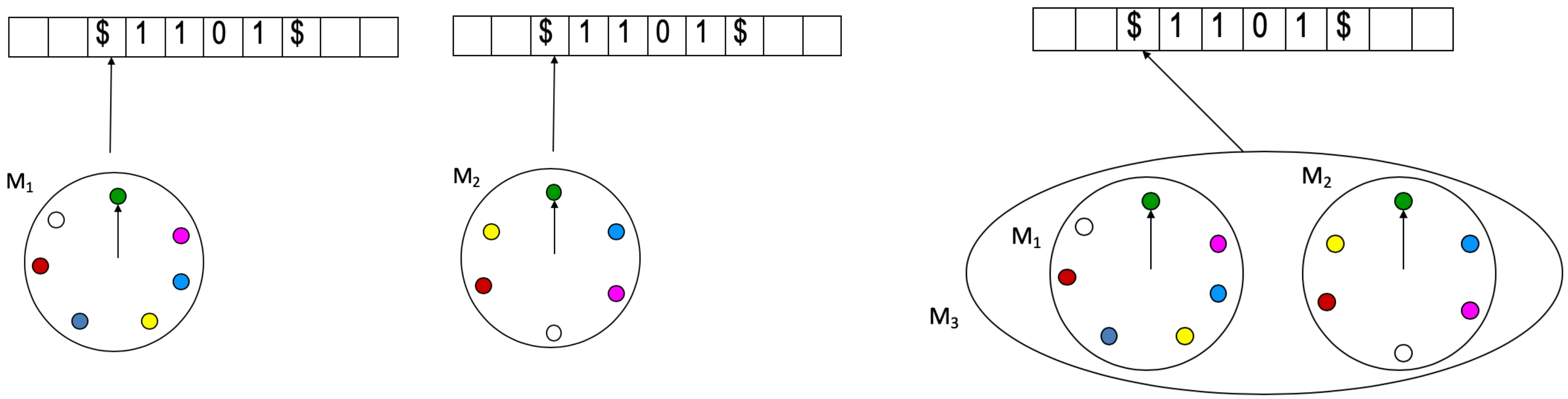

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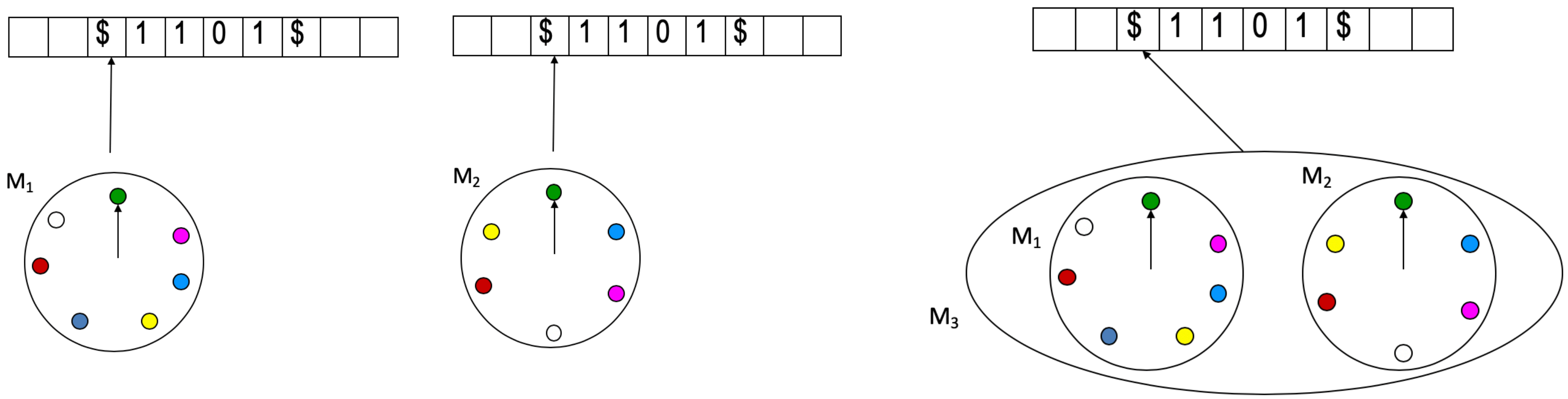

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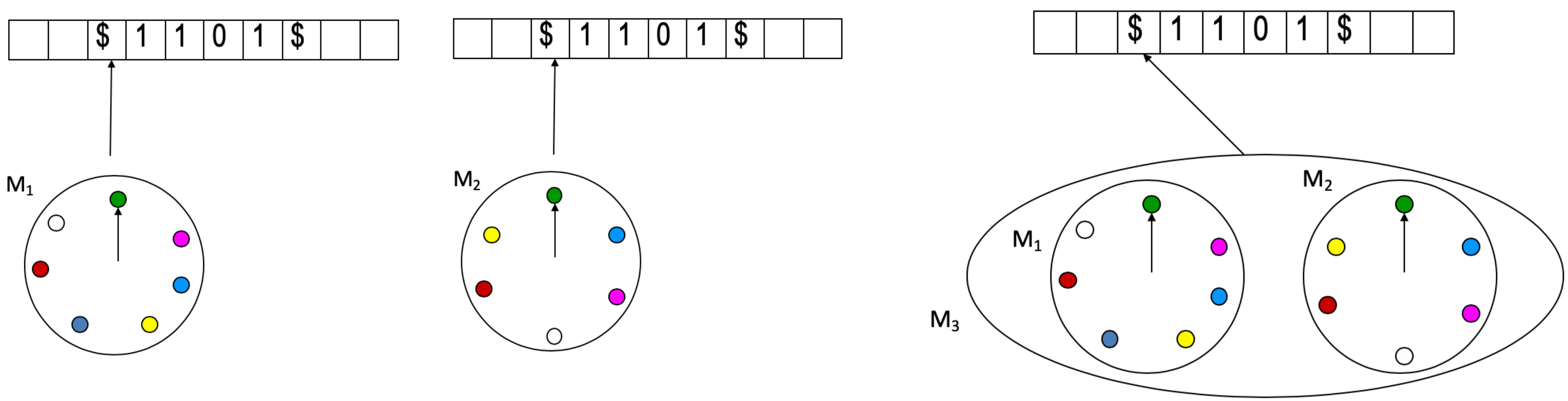

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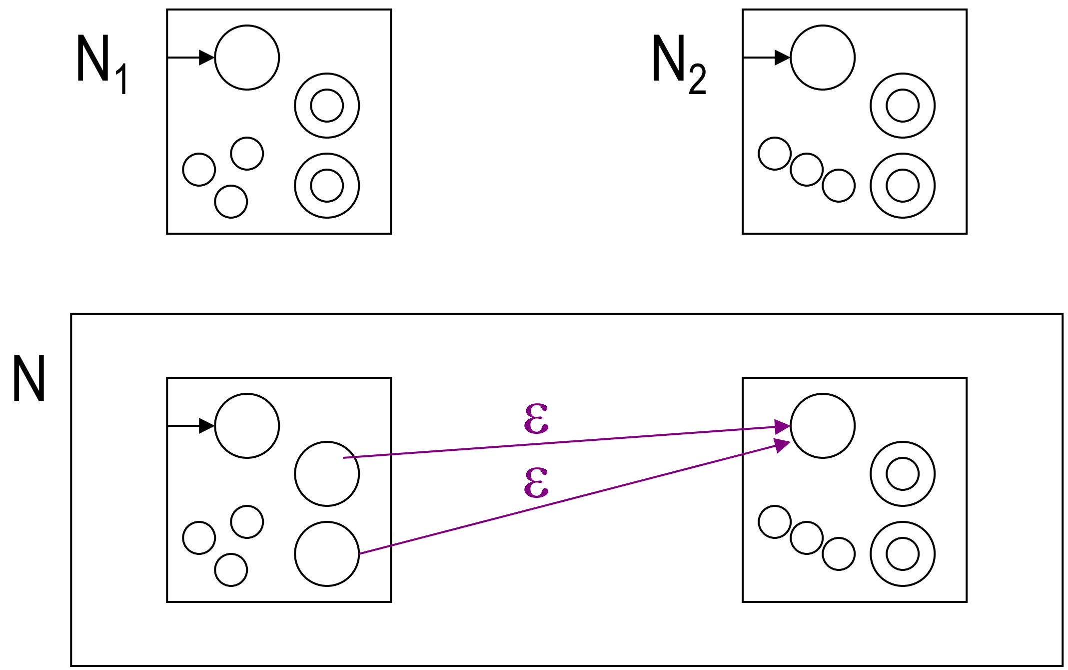

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  Create NFA

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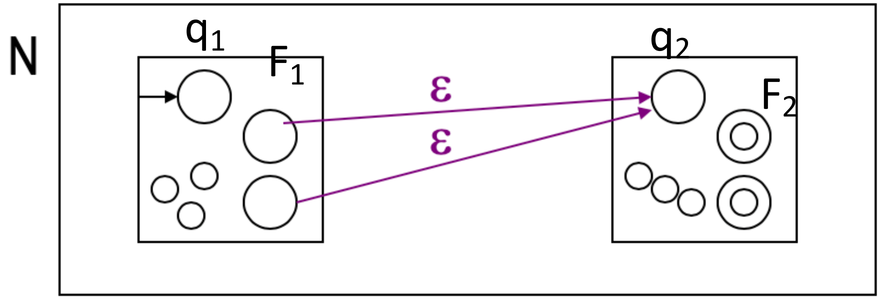
  Let $Q=Q_1 \cup Q_2$

  $F=F_2$

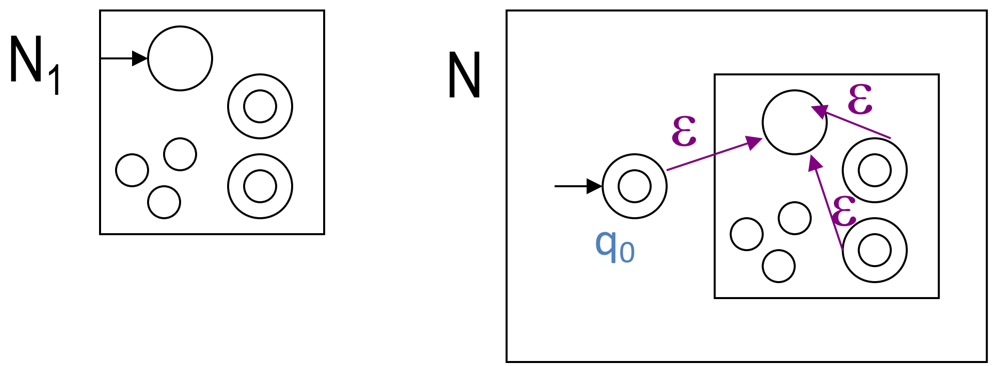
$$\delta(q,a) = \begin{cases} \delta_1(q,a), & if\ q \in Q_1\ and\ q \notin F_1 \\ \delta_1(q,a), & if\ q \in F_1\ and\ a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_2\} & if q \in F_1\ and\ a = \varepsilon \\ \delta_2(q,a), & if q \in Q_2 \end{cases}$$



Closure under the star

- Theorem: regular language is closed under the star operation.

$$A^* = A \times A \times \ldots \times A$$



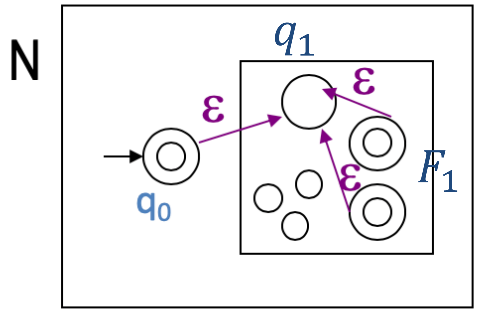
Closure under the star

- Theorem: regular language is closed under the star operation.

- Proof:

let A_1=L(N_1), A_1*=L(N), NFA N_1=(Q_1,Σ,δ_1,q_1,F_1).

create NFA N=(Q,Σ,δ,q_0,F).



$Q=Q_1 \cup \{q_0\}$;

$F=F_1 \cup \{q_0\}$;

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Regular operations

	DFA/NFA	PDA	TM
Union	close	?	?
Concatenation	close	?	?
Star	close	?	?
Complement	close	?	?
Boolean operation	close	?	?