# CS 6041 Theory of Computation

# Homework 1

1. (30 points) Find the error in the following proof that all horses are the same color.

CLAIM: In any set of h horses, all horses are the same color.

PROOF: By induction on h.

***Basis:*** For h = 1. In any set containing just one horse, all horses clearly are the same color.

***Induction step:*** For k ≥ 1, assume that the claim is true for h = k and prove that it is true for h = k + 1. Take any set H of k + 1 horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H1 with just k horses. By the induction hypothesis, all the horses in H1 are the same color. Now replace the removed horse and remove a different one to obtain the set H2. By the same argument, all the horses in H2 are the same color. Therefore, all the horses in H must be the same color, and the proof is complete.

1. a, b ∈ N (positive integers), prove that: a mod b = b mod a iff a = b (30 points)
2. Prove or disapprove that: If a, b ∈ Z (integers), then a2 – 4b ≠ 2 (40 points)