

# 1 Basic

## 1.1 Basic codeblock setting

```
1 Settings -> Editor -> Keyboard shortcuts ->
  Plugins -> Source code formatter (AStyle
  )
2 Settings -> Source Formatter -> Padding
3 Delete empty lines within a function or
  method
4 Insert space padding around operators
5 Insert space padding around parentheses on
  outside
6 Remove extra space padding around
  parentheses
```

## 1.2 Basic vim setting

```
1 /*at home directory*/
2 /* vi ~/.vimrc */
3 syntax enable
4 set smartindent
5 set tabstop=4
6 set shiftwidth=4
7 set expandtab
8 set relativenumber
```

## 1.3 Code Template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4 typedef unsigned long long ull;
5 typedef pair<int, int> pii;
6 #define x first
7 #define y second
8 #define pb push_back
9 #define len length()
10 #define all(p) p.begin(), p.end()
11 #define endl '\n'
12 #define bug(x) cout << "value of " << #x <<
  " is " << x << endl;
13 #define bugarr(x) \
14   for (auto i : x) \
15     cout << i << ' '; \
16   cout << endl;
17
18 int main()
19 {
20   ios::sync_with_stdio(0);
21   cin.tie(0);
22   return 0;
23 }
```

## 1.4 Python

```
1 //輸入
2 import sys
3 line = sys.stdin.readline() // 會讀到換行
4 input().strip()
5
6 array = [0] * (N) //N個0
7 range(0, N) // 0 ~ N-1
8 D, R, N = map(int, line[:-1].split()) // 分
  三個 int 變數
9
10 pow(a, b, c) // a ^ b % c
11
12 print(*objects, sep = ' ', end = '\n')
13 // objects -- 可以一次輸出多個對象
14 // sep -- 分開多個objects
15 // end -- 默認值是\n
16
17 // EOF break
18 try:
19     while True:
20         //input something
21 except EOFError:
22     pass
```

## 1.5 Range data

```
1 int (-2147483648 to 2147483647)
2 unsigned int(0 to 4294967295)
3 long(-2147483648 to 2147483647)
4 unsigned long(0 to 4294967295)
5 long long(-9223372036854775808 to
  9223372036854775807)
6 unsigned long long (0 to
  18446744073709551615)
```

## 1.6 Some Function

```
1 round(double f); // 四捨五入
2 ceil(double f); // 進入
3 floor(double f); // 捨去
4 __builtin_popcount(int n); // 32bit有多少 1
5 to_string(int s); // int to string
6
7 /** 全排列要先 sort !!! **/
8 next_permutation(num.begin(), num.end());
9 prev_permutation(num.begin(), num.end());
10 //用binary search找大於或等於val的最小值的位
  置
11 vector<int>::iterator it = lower_bound(v.
  begin(), v.end(), val);
12 //用binary search找大於val的最小值的位置
13 vector<int>::iterator it = upper_bound(v.
  begin(), v.end(), val);
14 /*queue*/
15
```

```
16 queue<datatype> q;
17 front(); /*取出最前面的值(沒有移除掉)*/
18 back(); /*取出最後面的值(沒有移除掉)*/
19 pop(); /*移掉最前面的值*/
20 push(); /*新增值到最後面*/
21 empty(); /*回傳bool,檢查是不是空的queue*/
22 size(); /*queue 的大小*/
23
24 /*stack*/
25 stack<datatype> s;
26 top(); /*取出最上面的值(沒有移除掉)*/
27 pop(); /*移掉最上面的值*/
28 push(); /*新增值到最上面*/
29 empty(); /*bool 檢查是不是空*/
30 size(); /*stack 的大小*/
31
32 /*unordered_set*/
33 unordered_set<datatype> s;
34 unordered_set<datatype> s(arr, arr + n);
35 /*initial with array*/
36 insert(); /*插入值*/
37 erase(); /*刪除值*/
38 empty(); /*bool 檢查是不是空*/
39 count(); /*判斷元素存在回傳1 無則回傳0*/
```

## 1.7 Time

```
1 cout << 1.0 * clock() / CLOCKS_PER_SEC <<
  endl;
```

# 2 DP

## 2.1 3 維 DP 思路

```
1 解題思路: dp[i][j][k]
2 i 跟 j 代表 range i ~ j 的 value
3 k 在我的理解裡是視題目的要求而定的
4 像是 Remove Boxes 當中 k 代表的是在 i 之前還
  有多少個連續的箱子
5 所以每次區間消去的值就是(k+1) * (k+1)
6 換言之,我認為可以理解成 k 的意義就是題目今
  天所關注的重點,就是老師說的題目所規定的
  運算
```

## 2.2 Knapsack Bounded

```
1 const int N = 100, W = 100000;
2 int cost[N], weight[N], number[N];
3 int c[W + 1];
4 void knapsack(int n, int w)
5 {
```

```
6   for (int i = 0; i < n; ++i)
7   {
8       int num = min(number[i], w / weight[
9           i]);
10      for (int k = 1; num > 0; k -= 2)
11      {
12          if (k > num)
13              k = num;
14          num -= k;
15          for (int j = w; j >= weight[i] *
16              k; --j)
17              c[j] = max(c[j], c[j -
18                  weight[i] * k] + cost[i]
19                  * k);
20      }
21      cout << "Max Prince" << c[w];
22  }
```

## 2.3 Knapsack sample

```
1 int Knapsack(vector<int> weight, vector<int>
  value, int bag_Weight)
2 {
3     // vector<int> weight = {1, 3, 4};
4     // vector<int> value = {15, 20, 30};
5     // int bagWeight = 4;
6     vector<vector<int>> dp(weight.size(),
7         vector<int>(bagWeight + 1, 0));
8     for (int j = weight[0]; j <= bagWeight;
9         j++)
10         dp[0][j] = value[0];
11     // weight數組的大小就是物品個數
12     for (int i = 1; i < weight.size(); i++)
13     { // 遍歷物品
14         for (int j = 0; j <= bagWeight; j++)
15         { // 遍歷背包容量
16             if (j < weight[i]) dp[i][j] = dp
17                 [i - 1][j];
18             else dp[i][j] = max(dp[i - 1][j],
19                 dp[i - 1][j - weight[i]]
20                 + value[i]);
21         }
22     }
23     cout << dp[weight.size() - 1][bagWeight]
24         << endl;
25 }
```

## 2.4 Knapsack Unbounded

```
1 const int N = 100, W = 100000;
2 int cost[N], weight[N];
3 int c[W + 1];
4 void knapsack(int n, int w)
5 {
6     memset(c, 0, sizeof(c));
7     for (int i = 0; i < n; ++i)
8         for (int j = weight[i]; j <= w; ++j)
```

```

9         c[j] = max(c[j], c[j - weight[i
10             ]] + cost[i]);
11     }
    cout << "最高的價值為" << c[w];
}

```

## 2.5 LCIS

```

1 int LCIS_len(vector<int> arr1, vector<int>
    arr2)
2 {
3     int n = arr1.size(), m = arr2.size();
4     vector<int> table(m, 0);
5     for (int j = 0; j < m; j++)
6         table[j] = 0;
7     for (int i = 0; i < n; i++)
8     {
9         int current = 0;
10        for (int j = 0; j < m; j++)
11        {
12            if (arr1[i] == arr2[j])
13                if (current + 1 > table[j])
14                    table[j] = current + 1;
15
16            if (arr1[i] > arr2[j])
17                if (table[j] > current)
18                    current = table[j];
19        }
20    }
21    int result = 0;
22    for (int i = 0; i < m; i++)
23        if (table[i] > result)
24            result = table[i];
25    return result;
26 }

```

## 2.6 LCS

```

1 int LCS(vector<string> Ans, vector<string>
    num)
2 {
3     int N = Ans.size(), M = num.size();
4     vector<vector<int>> LCS(N + 1, vector<
        int>(M + 1, 0));
5     for (int i = 1; i <= N; ++i)
6     {
7         for (int j = 1; j <= M; ++j)
8         {
9             if (Ans[i - 1] == num[j - 1])
10                LCS[i][j] = LCS[i - 1][j -
                    1] + 1;
11            else
12                LCS[i][j] = max(LCS[i - 1][j
                    ], LCS[i][j - 1]);
13        }
14    }
15    cout << LCS[N][M] << '\n';
16    //列印 LCS
17    int n = N, m = M;

```

```

18     vector<string> k;
19     while (n && m)
20     {
21         if (LCS[n][m] != max(LCS[n - 1][m],
                LCS[n][m - 1]))
22         {
23             k.push_back(Ans[n - 1]);
24             n--;
25             m--;
26         }
27         else if (LCS[n][m] == LCS[n - 1][m])
28             n--;
29         else if (LCS[n][m] == LCS[n][m - 1])
30             m--;
31     }
32     reverse(k.begin(), k.end());
33     for (auto i : k)
34         cout << i << " ";
35     cout << endl;
36     return LCS[N][M];
37 }

```

## 2.7 LIS

```

1 vector<int> ans;
2 void printLIS(vector<int> &arr, vector<int>
    &pos, int index)
3 {
4     if (pos[index] != -1)
5         printLIS(arr, pos, pos[index]);
6     // printf("%d", arr[index]);
7     ans.push_back(arr[index]);
8 }
9 void LIS(vector<int> &arr)
10 {
11     vector<int> dp(arr.size(), 1);
12     vector<int> pos(arr.size(), -1);
13     int res = INT_MIN, index = 0;
14     for (int i = 0; i < arr.size(); ++i)
15     {
16         for (int j = i + 1; j < arr.size();
                ++j)
17         {
18             if (arr[j] > arr[i])
19             {
20                 if (dp[i] + 1 > dp[j])
21                     dp[j] = dp[i] + 1;
22                 pos[j] = i;
23             }
24         }
25         if (dp[i] > res)
26         {
27             res = dp[i];
28             index = i;
29         }
30     }
31     cout << res << endl; // length
32     printLIS(arr, pos, index);
33     for (int i = 0; i < ans.size(); i++)
34     {
35         cout << ans[i];
36     }
37 }

```

```

38     if (i != ans.size() - 1)
39         cout << ' ';
40 }
41 cout << '\n';
42 }

```

## 2.8 LPS

```

1 void LPS(string s)
2 {
3     int maxlen = 0, l, r;
4     int n = s.size();
5     for (int i = 0; i < n; i++)
6     {
7         int x = 0;
8         while ((s[i - x] == s[i + x]) && (i
                - x >= 0) && (i + x < n)) //odd
9             length
10            x++;
11        x--;
12        if (2 * x + 1 > maxlen)
13        {
14            maxlen = 2 * x + 1;
15            l = i - x;
16            r = i + x;
17        }
18        x = 0;
19        while ((s[i - x] == s[i + 1 + x]) &&
                (i - x >= 0) && (i + 1 + x < n)) //even length
20            x++;
21        if (2 * x > maxlen)
22        {
23            maxlen = 2 * x;
24            l = i - x + 1;
25            r = i + x;
26        }
27    }
28    cout << maxlen << '\n'; // 最後長度
29    cout << l + 1 << ' ' << r + 1 << '\n';
    //頭到尾
}

```

## 2.9 Max\_subarray

```

1 /*Kadane's algorithm*/
2 int maxSubArray(vector<int>& nums) {
3     int local_max = nums[0], global_max =
        nums[0];
4     for (int i = 1; i < nums.size(); i++) {
5         local_max = max(nums[i], nums[i] +
                local_max);
6         global_max = max(local_max,
                global_max);
7     }
8     return global_max;
9 }

```

## 2.10 Money problem

```

1 //能否湊得某個價位
2 void change(vector<int> price, int limit)
3 {
4     vector<bool> c(limit + 1, 0);
5     c[0] = true;
6     for (int i = 0; i < price.size(); ++i)
7         // 依序加入各種面額
8         for (int j = price[i]; j <= limit;
                ++j) // 由低價位逐步到高價位
9             c[j] = c[j] | c[j - price[i]];
10        // 湊、湊、湊
11        if (c[limit]) cout << "YES\n";
12        else cout << "NO\n";
13    }
14    // 湊得某個價位的湊法總共幾種
15    void change(vector<int> price, int limit)
16    {
17        vector<int> c(limit + 1, 0);
18        c[0] = true;
19        for (int i = 0; i < price.size(); ++i)
20            for (int j = price[i]; j <= limit;
                    ++j)
21                c[j] += c[j - price[i]];
22        cout << c[limit] << '\n';
23    }
24    // 湊得某個價位的最少錢幣用量
25    void change(vector<int> price, int limit)
26    {
27        vector<int> c(limit + 1, 0);
28        c[0] = true;
29        for (int i = 0; i < price.size(); ++i)
30            for (int j = price[i]; j <= limit;
                    ++j)
31                c[j] = min(c[j], c[j - price[i]]
                        + 1);
32        cout << c[limit] << '\n';
33    }
34    //湊得某個價位的錢幣用量，有哪幾種可能性
35    void change(vector<int> price, int limit)
36    {
37        vector<int> c(limit + 1, 0);
38        c[0] = true;
39        for (int i = 0; i < price.size(); ++i)
40            for (int j = price[i]; j <= limit;
                    ++j)
41                c[j] |= c[j - price[i]] << 1; //
                    錢幣數量加一，每一種可能性都
                    加一。
42        for (int i = 1; i <= 63; ++i)
43            if (c[i] & (1 << i))
44                cout << "用" << i << "個錢幣可湊
                    得價位" << m;
45    }

```

## 3 Flow & matching

### 3.1 Dinic

```

1 const long long INF = 1LL<<60;
2 struct Dinic { //O(VVE), with minimum cut
3     static const int MAXN = 5003;
4     struct Edge{
5         int u, v;
6         long long cap, rest;
7     };
8     int n, m, s, t, d[MAXN], cur[MAXN];
9     vector<Edge> edges;
10    vector<int> G[MAXN];
11    void init(){
12        edges.clear();
13        for ( int i = 0 ; i < n ; i++ ) G[i]
14            .clear();
15        n = 0;
16        // min cut start
17        bool side[MAXN];
18        void cut(int u) {
19            side[u] = 1;
20            for ( int i : G[u] ) {
21                if ( !side[ edges[i].v ] &&
22                    edges[i].rest )
23                    cut(edges[i].v);
24            }
25        }
26        // min cut end
27        int add_node(){
28            return n++;
29        }
30        void add_edge(int u, int v, long long
31            cap){
32            edges.push_back( {u, v, cap, cap} );
33            edges.push_back( {v, u, 0, 0LL} );
34            m = edges.size();
35            G[u].push_back(m-2);
36            G[v].push_back(m-1);
37        }
38        bool bfs(){
39            fill(d,d+n,-1);
40            queue<int> que;
41            que.push(s); d[s]=0;
42            while (!que.empty()){
43                int u = que.front(); que.pop();
44                for (int ei : G[u]){
45                    Edge &e = edges[ei];
46                    if (d[e.v] < 0 && e.rest >
47                        0){
48                        d[e.v] = d[u] + 1;
49                        que.push(e.v);
50                    }
51                }
52            }
53            return d[t] >= 0;
54        }
55        long long dfs(int u, long long a){
56            if ( u == t || a == 0 ) return a;
57            long long flow = 0, f;
58            for ( int &i=cur[u]; i < (int)G[u].
59                size(); i++) {
60                Edge &e = edges[ G[u][i] ];
61                if ( d[u] + 1 != d[e.v] )
62                    continue;
63                f = dfs(e.v, min(a, e.rest) );
64                if ( f > 0 ) {
65                    e.rest -= f;
66                    edges[ G[u][i]^1 ].rest += f;
67                    flow += f;
68                    a -= f;
69                    if ( a == 0 ) break;
70                }
71            }
72            return flow;
73        }
74    } dinic;

```

### 3.2 Edmonds\_karp

```

1 /*Flow - Edmonds-karp*/
2 /*Based on UVa820*/
3 #define inf 1000000
4 int getMaxFlow(vector<vector<int>> &capacity
5     , int s, int t, int n){
6     int ans = 0;
7     vector<vector<int>> residual(n+1, vector<
8         int>(n+1, 0)); //residual network
9     while(true){
10        vector<int> bottleneck(n+1, 0);
11        bottleneck[s] = inf;
12        queue<int> q;
13        q.push(s);
14        vector<int> pre(n+1, 0);
15        while(!q.empty() && bottleneck[t] == 0){
16            int cur = q.front();
17            q.pop();
18            for(int i = 1; i <= n; i++){
19                if(bottleneck[i] == 0 && capacity[
20                    cur][i] > residual[cur][i]){
21                    q.push(i);
22                    pre[i] = cur;
23                    bottleneck[i] = min(bottleneck[cur]
24                        , capacity[cur][i] - residual
25                        [cur][i]);
26                }
27            }
28        }
29        if(bottleneck[t] == 0) break;
30        for(int cur = t; cur != s; cur = pre[cur]
31            ){
32                residual[pre[cur]][cur] +=
33                bottleneck[t];

```

```

27         residual[cur][pre[cur]] -=
28         bottleneck[t];
29     }
30     ans += bottleneck[t];
31 }
32 return ans;
33 }
34 int main(){
35     int testcase = 1;
36     int n;
37     while(cin>>n){
38         if(n == 0)
39             break;
40         vector<vector<int>> capacity(n+1, vector
41             <int>(n+1, 0));
42         int s, t, c;
43         cin >> s >> t >> c;
44         int a, b, bandwidth;
45         for(int i = 0 ; i < c ; ++i){
46             cin >> a >> b >> bandwidth;
47             capacity[a][b] += bandwidth;
48             capacity[b][a] += bandwidth;
49         }
50         cout << "Network " << testcase++ << endl
51             << endl;
52         cout << "The bandwidth is " <<
53             getMaxFlow(capacity, s, t, n) << "."
54             << endl;
55     }
56     return 0;
57 }

```

### 3.3 hungarian

```

1 /*bipartite - hungarian*/
2 struct Graph{
3     static const int MAXN = 5003;
4     vector<int> G[MAXN];
5     int n, match[MAXN], vis[MAXN];
6     void init(int _n){
7         n = _n;
8         for (int i=0; i<n; i++) G[i].clear()
9             ;
10    }
11    bool dfs(int u){
12        for (int v:G[u]){
13            if (vis[v]) continue;
14            vis[v]=true;
15            if (match[v]==-1 || dfs(match[v]
16                )){
17                match[v] = u;
18                match[u] = v;
19                return true;
20            }
21        }
22        return false;
23    }
24    int solve(){
25        int res = 0;
26        memset(match,-1,sizeof(match));
27        for (int i=0; i<n; i++){
28            if (match[i]==-1){

```

```

27         memset(vis,0,sizeof(vis));
28         if ( dfs(i) ) res++;
29     }
30 }
31 return res;
32 }
33 } graph;

```

### 3.4 Maximum\_matching

```

1 /*bipartite - maximum matching*/
2 bool dfs(vector<vector<bool>> res,int node,
3     vector<int>& x, vector<int>& y, vector<
4     bool> pass){
5     for (int i = 0; i < res[0].size(); i++){
6         if(res[node][i] && !pass[i]){
7             pass[i] = true;
8             if(y[i] == -1 || dfs(res,y[i],x,
9                 y,pass)){
10                 x[node] = i;
11                 y[i] = node;
12                 return true;
13             }
14         }
15     }
16     return false;
17 }
18 int main(){
19     int n,m,l;
20     while(cin>>n>>m>>l){
21         vector<vector<bool>> res(n, vector<
22             bool>(m, false));
23         for (int i = 0; i < l; i++){
24             int a, b;
25             cin >> a >> b;
26             res[a][b] = true;
27         }
28         int ans = 0;
29         vector<int> x(n, -1);
30         vector<int> y(m, -1);
31         for (int i = 0; i < n; i++){
32             vector<bool> pass(m, false);
33             if(dfs(res,i,x,y,pass))
34                 ans += 1;
35         }
36         cout << ans << endl;
37     }
38     return 0;
39 }
40 /*
41 input:
42 4 3 5 //n matching m, l links
43 0 0
44 0 2
45 1 0
46 2 1
47 3 1
48 answer is 3
49 */

```

### 3.5 MFlow Model

```

1 typedef long long ll;
2 struct MF
3 {
4     static const int N = 5000 + 5;
5     static const int M = 60000 + 5;
6     static const ll oo = 1000000000000LL;
7
8     int n, m, s, t, tot, tim;
9     int first[N], next[M];
10    int u[M], v[M], cur[N], vi[N];
11    ll cap[M], flow[M], dis[N];
12    int que[N + N];
13
14    void Clear()
15    {
16        tot = 0;
17        tim = 0;
18        for (int i = 1; i <= n; ++i)
19            first[i] = -1;
20    }
21    void Add(int from, int to, ll cp, ll flw)
22    {
23        u[tot] = from;
24        v[tot] = to;
25        cap[tot] = cp;
26        flow[tot] = flw;
27        next[tot] = first[u[tot]];
28        first[u[tot]] = tot;
29        ++tot;
30    }
31    bool bfs()
32    {
33        ++tim;
34        dis[s] = 0;
35        vi[s] = tim;
36
37        int head, tail;
38        head = tail = 1;
39        que[head] = s;
40        while (head <= tail)
41        {
42            for (int i = first[que[head]]; i
43                != -1; i = next[i])
44            {
45                if (vi[v[i]] != tim && cap[i]
46                    > flow[i])
47                {
48                    vi[v[i]] = tim;
49                    dis[v[i]] = dis[que[head]]
50                        + 1;
51                    que[++tail] = v[i];
52                }
53            }
54            ++head;
55        }
56        return vi[t] == tim;
57    }
58    ll dfs(int x, ll a)
59    {
60        if (x == t || a == 0)
61            return a;
62        ll flw = 0, f;
63        for (int i = first[x]; i != -1; i = next[i])
64        {
65            if (dis[x] + 1 == dis[v[i]] && (
66                f = dfs(v[i], min(a, cap[i]
67                    - flow[i])) > 0)
68            )
69            {
70                flow[i] += f;
71                flow[i ^ 1] -= f;
72                a -= f;
73                flw += f;
74                if (a == 0)
75                    break;
76            }
77        }
78        return flw;
79    }
80    ll MaxFlow(int s, int t)
81    {
82        this->s = s;
83        this->t = t;
84        ll flw = 0;
85        while (bfs())
86        {
87            for (int i = 1; i <= n; ++i)
88                cur[i] = 0;
89            flw += dfs(s, oo);
90        }
91        return flw;
92    }
93    // MF Net;
94    // Net.n = n;
95    // Net.Clear();
96    // a 到 b (注意從1開始!!!!)
97    // Net.Add(a, b, w, 0);
98    // Net.MaxFlow(s, d)
99    // s 到 d 的 MF

```

## 4 Geometry

### 4.1 Closest Pair

```

1 //最近點對 (距離) //台大
2 vector<pair<double, double>> p;
3 double closest_pair(int l, int r)
4 {
5     // p 要對 x 軸做 sort
6     if (l == r)
7         return 1e9;
8     if (r - l == 1)
9         return dist(p[l], p[r]); // 兩點距離
10    int m = (l + r) >> 1;
11    double d = min(closest_pair(l, m),
12        closest_pair(m + 1, r));
13    vector<int> vec;
14    for (int i = m; i >= l && fabs(p[m].x -
15        p[i].x) < d; --i)
16        vec.push_back(i);

```

```

15    for (int i = m + 1; i <= r && fabs(p[m].
16        x - p[i].x) < d; ++i)
17        vec.push_back(i);
18    sort(vec.begin(), vec.end(), [&](int a,
19        int b)
20    { return p[a].y < p[b].y; });
21    for (int i = 0; i < vec.size(); ++i)
22        for (int j = i + 1; j < vec.size()
23            && fabs(p[vec[j]].y - p[vec[i]].
24                y) < d; ++j)
25            d = min(d, dist(p[vec[i]], p[vec
26                [j]]));
27    return d;
28 }

```

### 4.2 Line

```

1 template <typename T>
2 struct line
3 {
4     line() {}
5     point<T> p1, p2;
6     T a, b, c; //ax+by+c=0
7     line(const point<T> &x, const point<T> &
8         y) : p1(x), p2(y) {}
9     void pton()
10    { //轉成一般式
11        a = p1.y - p2.y;
12        b = p2.x - p1.x;
13        c = -a * p1.x - b * p1.y;
14    }
15    T ori(const point<T> &p) const
16    { //點和有向直線的關係 · >0左邊、=0在線上
17        <0右邊
18        return (p2 - p1).cross(p - p1);
19    }
20    T btw(const point<T> &p) const
21    { //點投影落在線段上<=0
22        return (p1 - p).dot(p2 - p);
23    }
24    bool point_on_segment(const point<T> &p)
25        const
26    { //點是否在線段上
27        return ori(p) == 0 && btw(p) <= 0;
28    }
29    T dis2(const point<T> &p, bool
30        is_segment = 0) const
31    { //點跟直線/線段的距離平方
32        point<T> v = p2 - p1, v1 = p - p1;
33        if (is_segment)
34        {
35            point<T> v2 = p - p2;
36            if (v.dot(v1) <= 0)
37                return v1.abs2();
38            if (v.dot(v2) >= 0)
39                return v2.abs2();
40        }
41        T tmp = v.cross(v1);
42        return tmp * tmp / v.abs2();
43    }
44    T seg_dis2(const line<T> &l) const
45    { //兩線段距離平方

```

```

1     return min({dis2(l.p1, 1), dis2(l.p2
2         , 1), l.dis2(p1, 1), l.dis2(p2,
3             1)});
4     point<T> projection(const point<T> &p)
5         const
6     { //點對直線的投影
7         point<T> n = (p2 - p1).normal();
8         return p - n * (p - p1).dot(n) / n.
9             abs2();
10    }
11    point<T> mirror(const point<T> &p) const
12    {
13        //點對直線的鏡射 · 要先呼叫pton轉成一般式
14        point<T> R;
15        T d = a * a + b * b;
16        R.x = (b * b * p.x - a * a * p.x - 2
17            * a * b * p.y - 2 * a * c) / d;
18        R.y = (a * a * p.y - b * b * p.y - 2
19            * a * b * p.x - 2 * b * c) / d;
20        return R;
21    }
22    bool equal(const line &l) const
23    { //直線相等
24        return ori(l.p1) == 0 && ori(l.p2)
25            == 0;
26    }
27    bool parallel(const line &l) const
28    {
29        return (p1 - p2).cross(l.p1 - l.p2)
30            == 0;
31    }
32    bool cross_seg(const line &l) const
33    {
34        return (p2 - p1).cross(l.p1 - p1) *
35            (p2 - p1).cross(l.p2 - p1) <= 0;
36        //直線是否交線段
37    }
38    int line_intersect(const line &l) const
39    { //直線相交情況 · -1無限多點、1交於一
40        點、0不相交
41        return parallel(l) ? (ori(l.p1) == 0
42            ? -1 : 0) : 1;
43    }
44    int seg_intersect(const line &l) const
45    {
46        T c1 = ori(l.p1), c2 = ori(l.p2);
47        T c3 = l.ori(p1), c4 = l.ori(p2);
48        if (c1 == 0 && c2 == 0)
49        { //共線
50            bool b1 = btw(l.p1) >= 0, b2 =
51                btw(l.p2) >= 0;
52            T a3 = l.btw(p1), a4 = l.btw(p2);
53            ;
54            if (b1 && b2 && a3 == 0 && a4 >=
55                0)
56                return 2;
57            if (b1 && b2 && a3 >= 0 && a4 ==
58                0)
59                return 3;
60            if (b1 && b2 && a3 >= 0 && a4 >=
61                0)
62                return 0;
63        }
64    }

```

```

88     return -1; //無限交點
89 }
90 else if (c1 * c2 <= 0 && c3 * c4 <=
91     0)
92     return 1;
93 return 0; //不相交
94 }
95 point<T> line_intersection(const line &l
96     ) const
97 { /*直線交點*/
98     point<T> a = p2 - p1, b = l.p2 - l.
99     p1, s = l.p1 - p1;
100     //if(a.cross(b)==0)return INF;
101     return p1 + a * (s.cross(b) / a.
102         cross(b));
103 }
104 point<T> seg_intersection(const line &l
105     ) const
106 { //線段交點
107     int res = seg_intersect(l);
108     if (res <= 0)
109         assert(0);
110     if (res == 2)
111         return p1;
112     if (res == 3)
113         return p2;
114     return line_intersection(l);
115 }
116 };

```

```

30     return x * b.x + y * b.y;
31 }
32 T cross(const point &b) const
33 {
34     return x * b.y - y * b.x;
35 }
36 point normal() const
37 { //求法向量
38     return point(-y, x);
39 }
40 T abs2() const
41 { //向量長度的平方
42     return dot(*this);
43 }
44 T rad(const point &b) const
45 { //兩向量的弧度
46     return fabs(atan2(fabs(cross(b)),
47         dot(b)));
48 }
49 T getA() const
50 { //對x軸的弧度
51     T A = atan2(y, x); //超過180度會變負
52     if (A <= -PI / 2)
53         A += PI * 2;
54     return A;
55 }
56 };

```

### 4.3 Point

```

1 const double PI = atan2(0.0, -1.0);
2 template <typename T>
3 struct point
4 {
5     T x, y;
6     point() {}
7     point(const T &x, const T &y) : x(x), y(
8         y) {}
9     point operator+(const point &b) const
10     {
11         return point(x + b.x, y + b.y);
12     }
13     point operator-(const point &b) const
14     {
15         return point(x - b.x, y - b.y);
16     }
17     point operator*(const T &b) const
18     {
19         return point(x * b, y * b);
20     }
21     point operator/(const T &b) const
22     {
23         return point(x / b, y / b);
24     }
25     bool operator==(const point &b) const
26     {
27         return x == b.x && y == b.y;
28     }
29     T dot(const point &b) const
30     {

```

### 4.4 Polygon

```

1 template <typename T>
2 struct polygon
3 {
4     polygon() {}
5     vector<point<T>> p; //逆時針順序
6     T area() const
7     { //面積
8         T ans = 0;
9         for (int i = p.size() - 1, j = 0; j
10             < (int)p.size(); i = j++)
11             ans += p[i].cross(p[j]);
12         return ans / 2;
13     }
14     point<T> center_of_mass() const
15     { //重心
16         T cx = 0, cy = 0, w = 0;
17         for (int i = p.size() - 1, j = 0; j
18             < (int)p.size(); i = j++)
19         {
20             T a = p[i].cross(p[j]);
21             cx += (p[i].x + p[j].x) * a;
22             cy += (p[i].y + p[j].y) * a;
23             w += a;
24         }
25         return point<T>(cx / 3 / w, cy / 3 /
26             w);
27     }
28     char ahas(const point<T> &t) const
29     { //點是否在簡單多邊形內，是的話回傳1、
30         在邊上回傳-1、否則回傳0

```

```

27 bool c = 0;
28 for (int i = 0, j = p.size() - 1; i
29     < p.size(); j = i++)
30     if (line<T>(p[i], p[j]).
31         point_on_segment(t))
32         return -1;
33     else if ((p[i].y > t.y) != (p[j
34         ].y > t.y) &&
35         t.x < (p[j].x - p[i].x)
36         * (t.y - p[i].y) /
37         (p[j].y - p[i].y)
38         + p[i].x)
39         c = !c;
40 return c;
41 }
42 char point_in_convex(const point<T> &x)
43     const
44 {
45     int l = 1, r = (int)p.size() - 2;
46     while (l <= r)
47     { //點是否在凸多邊形內，是的話回傳1
48         、在邊上回傳-1、否則回傳0
49         int mid = (l + r) / 2;
50         T a1 = (p[mid] - p[0]).cross(x -
51             p[0]);
52         T a2 = (p[mid + 1] - p[0]).cross
53             (x - p[0]);
54         if (a1 >= 0 && a2 <= 0)
55         {
56             T res = (p[mid + 1] - p[mid
57                 ]).cross(x - p[mid]);
58             return res > 0 ? 1 : (res >=
59                 0 ? -1 : 0);
60         }
61         else if (a1 < 0)
62             r = mid - 1;
63         else
64             l = mid + 1;
65     }
66     return 0;
67 }
68 vector<T> getA() const
69 { //凸包邊對x軸的夾角
70     vector<T> res; //一定是遞增的
71     for (size_t i = 0; i < p.size(); ++i
72         )
73         res.push_back((p[(i + 1) % p.
74             size()] - p[i]).getA());
75     return res;
76 }
77 bool line_intersect(const vector<T> &A,
78     const line<T> &l) const
79 { //O(logN)
80     int f1 = upper_bound(A.begin(), A.
81         end(), (l.p1 - l.p2).getA()) - A.
82         begin();
83     int f2 = upper_bound(A.begin(), A.
84         end(), (l.p2 - l.p1).getA()) - A.
85         begin();
86     return l.cross_seg(line<T>(p[f1], p[
87         f2]));
88 }
89 polygon cut(const line<T> &l) const
90 { //凸包對直線切割，得到直線l左側的凸包
91     polygon ans;

```

```

for (int n = p.size(), i = n - 1, j
    = 0; j < n; i = j++)
{
    if (l.ori(p[i]) >= 0)
    {
        ans.p.push_back(p[i]);
        if (l.ori(p[j]) < 0)
            ans.p.push_back(l.
                line_intersection(
                    line<T>(p[i], p[j])));
    }
    else if (l.ori(p[j]) > 0)
        ans.p.push_back(l.
            line_intersection(line<T>
                >(p[i], p[j])));
}
return ans;
}
static bool graham_cmp(const point<T> &a
    , const point<T> &b)
{ //凸包排序函數 // 起點點不同
// return (a.x < b.x) || (a.x == b.x
    && a.y < b.y); //最左下角開始
return (a.y < b.y) || (a.y == b.y &&
    a.x < b.x); //Y最小開始
}
void graham(vector<point<T>> &s)
{ //凸包 Convexhull 2D
    sort(s.begin(), s.end(), graham_cmp);
    ;
    p.resize(s.size() + 1);
    int m = 0;
    // cross >= 0 順時針 * cross <= 0 逆
    時針旋轉
    for (size_t i = 0; i < s.size(); ++i
        )
    {
        while (m >= 2 && (p[m - 1] - p[m
            - 2]).cross(s[i] - p[m -
            2]) <= 0)
            --m;
        p[m++] = s[i];
    }
    for (int i = s.size() - 2, t = m +
        1; i >= 0; --i)
    {
        while (m >= t && (p[m - 1] - p[m
            - 2]).cross(s[i] - p[m -
            2]) <= 0)
            --m;
        p[m++] = s[i];
    }
    if (s.size() > 1) // 重複頭一次需扣
        掉
        --m;
    p.resize(m);
}
T diam()
{ //直徑
    int n = p.size(), t = 1;
    T ans = 0;
    p.push_back(p[0]);
    for (int i = 0; i < n; i++)

```



```

118 {
119     point<T> now = p[i + 1] - p[i];
120     while (now.cross(p[t + 1] - p[i]) > now.cross(p[t] - p[i]))
121         t = (t + 1) % n;
122     ans = max(ans, (p[i] - p[t]).abs2());
123 }
124 return p.pop_back(), ans;
125 }
126 T min_cover_rectangle()
127 { //最小覆蓋矩形
128     int n = p.size(), t = 1, r = 1, l;
129     if (n < 3)
130         return 0; //也可以做最小周長矩形
131     T ans = 1e99;
132     p.push_back(p[0]);
133     for (int i = 0; i < n; i++)
134     {
135         point<T> now = p[i + 1] - p[i];
136         while (now.cross(p[t + 1] - p[i]) > now.cross(p[t] - p[i]))
137             t = (t + 1) % n;
138         while (now.dot(p[r + 1] - p[i]) > now.dot(p[r] - p[i]))
139             r = (r + 1) % n;
140         if (!i)
141             l = r;
142         while (now.dot(p[l + 1] - p[i]) <= now.dot(p[l] - p[i]))
143             l = (l + 1) % n;
144         T d = now.abs2();
145         T tmp = now.cross(p[t] - p[i]) * (now.dot(p[r] - p[i]) - now.dot(p[l] - p[i])) / d;
146         ans = min(ans, tmp);
147     }
148     return p.pop_back(), ans;
149 }
150 T dis2(polygon &p1)
151 { //凸包最近距離平方
152     vector<point<T>> &P = p, &Q = p1.p;
153     int n = P.size(), m = Q.size(), l = 0, r = 0;
154     for (int i = 0; i < n; ++i)
155         if (P[i].y < P[l].y)
156             l = i;
157     for (int i = 0; i < m; ++i)
158         if (Q[i].y < Q[r].y)
159             r = i;
160     P.push_back(P[0]), Q.push_back(Q[0]);
161     T ans = 1e99;
162     for (int i = 0; i < n; ++i)
163     {
164         while ((P[l] - P[l + 1]).cross(Q[r + 1] - Q[r]) < 0)
165             r = (r + 1) % m;
166         ans = min(ans, line<T>(P[l], P[l + 1]).seg_dis2(line<T>(Q[r], Q[r + 1])));
167         l = (l + 1) % n;
168     }
169     return P.pop_back(), Q.pop_back(), ans;
170 }
171 static char sign(const point<T> &t)
172 {
173     return (t.y == 0 ? t.x : t.y) < 0;
174 }
175 static bool angle_cmp(const line<T> &A, const line<T> &B)
176 {
177     point<T> a = A.p2 - A.p1, b = B.p2 - B.p1;
178     return sign(a) < sign(b) || (sign(a) == sign(b) && a.cross(b) > 0);
179 }
180 int halfplane_intersection(vector<line<T>> &s)
181 { //半平面交
182     sort(s.begin(), s.end(), angle_cmp);
183     //線段左側為該線段半平面
184     int L, R, n = s.size();
185     vector<point<T>> px(n);
186     vector<line<T>> q(n);
187     q[L = R = 0] = s[0];
188     for (int i = 1; i < n; ++i)
189     {
190         while (L < R && s[i].ori(px[R - 1]) <= 0)
191             --R;
192         while (L < R && s[i].ori(px[L]) <= 0)
193             ++L;
194         q[++R] = s[i];
195         if (q[R].parallel(q[R - 1]))
196         {
197             --R;
198             if (q[R].ori(s[i].p1) > 0)
199                 q[R] = s[i];
200         }
201         if (L < R)
202             px[R - 1] = q[R - 1].line_intersection(q[R]);
203     }
204     while (L < R && q[L].ori(px[R - 1]) <= 0)
205         --L;
206     p.clear();
207     if (R - L <= 1)
208         return 0;
209     px[R] = q[R].line_intersection(q[L]);
210     for (int i = L; i <= R; ++i)
211         p.push_back(px[i]);
212     return R - L + 1;
213 }
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## 4.5 Triangle

```

1 template <typename T>
2 struct triangle
3 {
4     point<T> a, b, c;
5     triangle() {}

```

```

15 triangle(const point<T> &a, const point<
16 T> &b, const point<T> &c) : a(a), b(b), c(c) {}
17 T area() const
18 {
19     T t = (b - a).cross(c - a) / 2;
20     return t > 0 ? t : -t;
21 }
22 point<T> barycenter() const
23 { //重心
24     return (a + b + c) / 3;
25 }
26 point<T> circumcenter() const
27 { //外心
28     static line<T> u, v;
29     u.p1 = (a + b) / 2;
30     u.p2 = point<T>(u.p1.x - a.y + b.y, u.p1.y + a.x - b.x);
31     v.p1 = (a + c) / 2;
32     v.p2 = point<T>(v.p1.x - a.y + c.y, v.p1.y + a.x - c.x);
33     return u.line_intersection(v);
34 }
35 point<T> incenter() const
36 { //內心
37     T A = sqrt((b - c).abs2()), B = sqrt((a - c).abs2()), C = sqrt((a - b).abs2());
38     return point<T>(A * a.x + B * b.x + C * c.x, A * a.y + B * b.y + C * c.y) / (A + B + C);
39 }
40 point<T> perpencenter() const
41 { //垂心
42     return barycenter() * 3 - circumcenter() * 2;
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```

## 5 Graph

### 5.1 Bellman-Ford

```

1 /*SPA - Bellman-Ford*/
2 #define inf 99999 //define by you maximum
3 edges weight
4 vector<vector<int>> > edges;
5 vector<int> dist;
6 vector<int> ancestor;
7 void BellmanFord(int start, int node){
8     dist[start] = 0;
9     for(int i = 0; i < node-1; i++){
10         for(int j = 0; j < node; j++){
11             for(int k = 0; k < node; k++){
12                 if(dist[j] < dist[k] + edges[j][k]){
13                     dist[j] = dist[k] + edges[j][k];
14                     ancestor[j] = i;
15                 }
16             }
17         }
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```

20     else if (edges[i].second == q.
21         front() && pass[edges[i].
22         first] == 0)
23     {
24         p.push(edges[i].first);
25         result[edges[i].first] =
26             count;
27     }
28     else
29         newedges.push_back(edges[i])
30         ;
31     }
32     edges = newedges;
33     newedges.clear();
34     q.pop();
35     if (q.empty() == true)
36     {
37         q = p;
38         queue<int> tmp;
39         p = tmp;
40         count++;
41     }
42 }
43 int main()
44 {
45     int node;
46     cin >> node;
47     vector<pair<int, int>> edges;
48     int a, b;
49     while (cin >> a >> b)
50     {
51         /*a = b = -1 means input edges ended
52         */
53         if (a == -1 && b == -1)
54             break;
55         edges.push_back(pair<int, int>(a, b)
56         );
57     }
58     vector<int> result(node, -1);
59     BFS(result, edges, node, 0);
60     return 0;
61 }

```

### 5.3 DFS-rec

```

1  /*DFS - Recursive version*/
2  map<pair<int,int>,int> edges;
3  vector<int> pass;
4  vector<int> route;
5  void DFS(int start){
6      pass[start] = 1;
7      map<pair<int,int>,int>::iterator iter;
8      for(iter = edges.begin(); iter != edges.
9          end(); iter++){
10         if((*iter).first.first == start &&
11             (*iter).second == 0 && pass[(*)
12             iter).first.second] == 0){
13             route.push_back((*iter).first.
14                 second);
15             DFS((*iter).first.second);
16         }
17     }
18 }

```

```

13     else if((*iter).first.second ==
14         start && (*iter).second == 0 &&
15         pass[(*)iter).first.first] == 0){
16         route.push_back((*iter).first.
17             first);
18         DFS((*iter).first.first);
19     }
20 }
21 int main(){
22     int node;
23     cin>>node;
24     pass.resize(node,0);
25     int a,b;
26     while(cin>>a>>b){
27         if(a == -1 && b == -1)
28             break;
29         edges.insert(pair<pair<int,int>,int>
30             >(pair<int,int>(a,b),0));
31     }
32     int start;
33     cin>>start;
34     route.push_back(start);
35     DFS(start);
36     return 0;
37 }

```

### 5.4 Dijkstra

```

1  /*SPA - Dijkstra*/
2  const int MAXN = 1e5 + 3;
3  const int inf = INT_MAX;
4  typedef pair<int, int> pii;
5  vector<vector<pii>> weight;
6  vector<int> isDone(MAXN, false), dist,
7  ancestor;
8  void dijkstra(int s)
9  {
10     priority_queue<pii, vector<pii>, greater
11         <pii>> pq;
12     pq.push(pii(0, s));
13     ancestor[s] = -1;
14     while (!pq.empty())
15     {
16         int u = pq.top().second;
17         pq.pop();
18         isDone[u] = true;
19         for (auto &pr : weight[u])
20         {
21             int v = pr.first, w = pr.second;
22             if (!isDone[v] && dist[u] + w <
23                 dist[v])
24             {
25                 dist[v] = dist[u] + w;
26                 pq.push(pii(dist[v], v));
27                 ancestor[v] = u;
28             }
29         }
30     }
31 }

```

### 5.5 Euler circuit

```

1  /*Euler circuit*/
2  /*From NTU kiseki*/
3  /*G is graph, vis is visited, la is path*/
4  bool vis[ N ]; size_t la[ K ];
5  void dfs( int u, vector< int >& vec ) {
6      while ( la[ u ] < G[ u ].size() ) {
7          if( vis[ G[ u ][ la[ u ] ].second ]
8              ) {
9              ++ la[ u ];
10             continue;
11         }
12         int v = G[ u ][ la[ u ] ].first;
13         vis[ G[ u ][ la[ u ] ].second ] = true;
14         ++ la[ u ]; dfs( v, vec );
15         vec.push_back( v );
16     }
17 }

```

### 5.6 Floyd-warshall

```

1  /*SPA - Floyd-Warshall*/
2  #define inf 99999
3  void floyd_warshall(vector<vector<int>>&
4      distance, vector<vector<int>>& ancestor,
5      int n){
6      for (int k = 0; k < n; k++){
7          for (int i = 0; i < n; i++){
8              for (int j = 0; j < n; j++){
9                  if(distance[i][k] + distance
10                     [k][j] < distance[i][j])
11                  {
12                      distance[i][j] =
13                          distance[i][k] +
14                          distance[k][j];
15                      ancestor[i][j] =
16                          ancestor[k][j];
17                  }
18              }
19          }
20      }
21      int main(){
22         int n;
23         cin >> n;
24         int a, b, d;
25         vector<vector<int>> distance(n, vector<
26             int>(n,99999));
27         vector<vector<int>> ancestor(n, vector<
28             int>(n,-1));
29         while(cin>>a>>b>>d){
30             if(a == -1 && b == -1 && d == -1)
31                 break;
32             distance[a][b] = d;
33             ancestor[a][b] = a;
34         }
35         floyd_warshall(distance, ancestor, n);
36         /*Negative cycle detection*/
37         for (int i = 0; i < n; i++){
38             if(distance[i][i] < 0){
39                 cout << "Negative cycle!" <<
40                     endl;
41                 break;
42             }
43         }
44         return 0;
45     }

```

```

23         break;
24         distance[a][b] = d;
25         ancestor[a][b] = a;
26     }
27     for (int i = 0; i < n; i++)
28         distance[i][i] = 0;
29     floyd_warshall(distance, ancestor, n);
30     /*Negative cycle detection*/
31     for (int i = 0; i < n; i++){
32         if(distance[i][i] < 0){
33             cout << "Negative cycle!" <<
34                 endl;
35             break;
36         }
37     }
38     return 0;
39 }

```

### 5.7 Hamilton\_cycle

```

1  /*find hamilton cycle*/
2  void hamilton(vector<vector<int>> gp, int k,
3      int cur, vector<int>& solution, vector<
4      bool> pass,bool& flag){
5      if(k == gp.size()-1){
6          if(gp[cur][1] == 1){
7              cout << 1 << " ";
8              while(cur != 1){
9                  cout << cur << " ";
10                 cur = solution[cur];
11             }
12             cout << cur << endl;
13             flag = true;
14             return;
15         }
16     }
17     for (int i = 0; i < gp[cur].size() && !
18         flag; i++){
19         if(gp[cur][i] == 1 && !pass[i]){
20             pass[i] = true;
21             solution[i] = cur;
22             hamilton(gp, k + 1, i, solution,
23                 pass,flag);
24             pass[i] = false;
25         }
26     }
27 }
28 int main(){
29     int n;
30     while(cin>>n){
31         int a,b;
32         bool end = false;
33         vector<vector<int>> gp(n+1,vector<
34             int>(n+1,0));
35         while(cin>>a>>b){
36             if(a == 0 && b == 0)
37                 break;
38             gp[a][b] = 1;
39             gp[b][a] = 1;
40         }
41         vector<int> solution(n + 1, -1);
42         vector<bool> pass(n + 1, false);
43         solution[1] = 0;
44     }

```

```

39     pass[1] = true;
40     bool flag = false;
41     hamilton(gp, 1,1 ,solution,pass,flag
42         );
43     if(!flag)
44         cout << "N" << endl;
45 }
46 return 0;
47 */
48 4
49 1 2
50 2 3
51 2 4
52 3 4
53 3 1
54 0 0
55 output: 1 3 4 2 1
56 */

```

## 5.8 Kruskal

```

1  /*mst - Kruskal*/
2  struct edges{
3      int from;
4      int to;
5      int weight;
6      friend bool operator < (edges a, edges b
7          ){
8          return a.weight > b.weight;
9      }
10 };
11 int find(int x,vector<int>& union_set){
12     if(x != union_set[x])
13         union_set[x] = find(union_set[x],
14             union_set);
15     return union_set[x];
16 }
17 void merge(int a,int b,vector<int>&
18     union_set){
19     int pa = find(a, union_set);
20     int pb = find(b, union_set);
21     if(pa != pb)
22         union_set[pa] = pb;
23 }
24 void kruskal(priority_queue<edges> pq,int n)
25 {
26     vector<int> union_set(n, 0);
27     for (int i = 0; i < n; i++)
28         union_set[i] = i;
29     int edge = 0;
30     int cost = 0; //evaluate cost of mst
31     while(!pq.empty() && edge < n - 1){
32         edges cur = pq.top();
33         int from = find(cur.from, union_set)
34             ;
35         int to = find(cur.to, union_set);
36         if(from != to){
37             merge(from, to, union_set);
38             edge += 1;
39             cost += cur.weight;
40         }
41     }
42     pq.pop();

```

```

37     }
38     if(edge < n-1)
39         cout << "No mst" << endl;
40     else
41         cout << cost << endl;
42 }
43 int main(){
44     int n;
45     cin >> n;
46     int a, b, d;
47     priority_queue<edges> pq;
48     while(cin>>a>>b>>d){
49         if(a == -1 && b == -1 && d == -1)
50             break;
51         edges tmp;
52         tmp.from = a;
53         tmp.to = b;
54         tmp.weight = d;
55         pq.push(tmp);
56     }
57     kruskal(pq, n);
58     return 0;
59 }

```

## 5.9 Prim

```

1  /*mst - Prim*/
2  #define inf 99999
3  struct edges{
4      int from;
5      int to;
6      int weight;
7      friend bool operator < (edges a, edges b
8          ){
9          return a.weight > b.weight;
10     }
11 };
12 void Prim(vector<vector<int>> gp,int n,int
13     start){
14     vector<bool> pass(n,false);
15     int edge = 0;
16     int cost = 0; //evaluate cost of mst
17     priority_queue<edges> pq;
18     for (int i = 0; i < n; i++){
19         if(gp[start][i] != inf){
20             edges tmp;
21             tmp.from = start;
22             tmp.to = i;
23             tmp.weight = gp[start][i];
24             pq.push(tmp);
25         }
26     }
27     pass[start] = true;
28     while(!pq.empty() && edge < n-1){
29         edges cur = pq.top();
30         pq.pop();
31         if(!pass[cur.to]){
32             for (int i = 0; i < n; i++){
33                 if(gp[cur.to][i] != inf){
34                     edges tmp;
35                     tmp.from = cur.to;
36                     tmp.to = i;
37                     tmp.weight = gp[cur.to][i];
38                     pq.push(tmp);
39                 }
40             }
41         }
42     }
43 }

```

```

35     tmp.weight = gp[cur.to][
36         i];
37     pq.push(tmp);
38     }
39     pass[cur.to] = true;
40     edge += 1;
41     cost += cur.weight;
42 }
43 }
44 if(edge < n-1)
45     cout << "No mst" << endl;
46 else
47     cout << cost << endl;
48 }
49 int main(){
50     int n;
51     cin >> n;
52     int a, b, d;
53     vector<vector<int>> gp(n,vector<int>(n,
54         inf));
55     while(cin>>a>>b>>d){
56         if(a == -1 && b == -1 && d == -1)
57             break;
58         if(gp[a][b] > d)
59             gp[a][b] = d;
60     }
61     Prim(gp,n,0);
62     return 0;

```

## 5.10 Union\_find

```

1  // union_find from 台大
2  vector<int> father;
3  vector<int> people;
4  void init(int n)
5  {
6      for (int i = 0; i < n; i++)
7      {
8          father[i] = i;
9          people[i] = 1;
10     }
11 }
12 int Find(int x)
13 {
14     if (x != father[x])
15         father[x] = Find(father[x]);
16     return father[x];
17 }
18
19 void Union(int x, int y)
20 {
21     int m = Find(x);
22     int n = Find(y);
23     if (m != n)
24     {
25         father[n] = m;
26         people[m] += people[n];
27     }
28 }

```

# 6 Mathematics

## 6.1 Catalan

### Catalan number

- 0~19項的catalan number
  - 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190
- 公式:  $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$

## 6.2 Combination

```

1  /*input type string or vector*/
2  for (int i = 0; i < (1 << input.size()); ++i
3      )
4  {
5      string testCase = "";
6      for (int j = 0; j < input.size(); ++j)
7          if (i & (1 << j))
8              testCase += input[j];

```

## 6.3 Extended Euclidean

```

1  // ax + by = gcd(a,b)
2  pair<long long, long long> extgcd(long long
3      a, long long b)
4  {
5      if (b == 0)
6          return {1, 0};
7      long long k = a / b;
8      pair<long long, long long> p = extgcd(b,
9          a - k * b);
10     //cout << p.first << " " << p.second <<
11         endl;
12     //cout << "商數(k)= " << k << endl <<
13         endl;
14     return {p.second, p.first - k * p.second
15         };
16 }
17
18 int main()
19 {
20     int a, b;
21     cin >> a >> b;
22     pair<long long, long long> xy = extgcd(a,
23         b); //(x0,y0)
24     cout << xy.first << " " << xy.second <<
25         endl;
26     cout << xy.first << " * " << a << " + "
27         << xy.second << " * " << b << endl;

```



```

20     return 0;
21 }
22 // ax + by = gcd(a,b) * r
23 /*find |x|+|y| -> min*/
24 int main()
25 {
26     long long r, p, q; /*px+qy = r*/
27     int cases;
28     cin >> cases;
29     while (cases--)
30     {
31         cin >> r >> p >> q;
32         pair<long long, long long> xy =
33             extgcd(q, p); //(x0,y0)
34         long long ans = 0, tmp = 0;
35         double k, k1;
36         long long s, s1;
37         k = 1 - (double)(r * xy.first) / p;
38         s = round(k);
39         ans = llabs(r * xy.first + s * p) +
40             llabs(r * xy.second - s * q);
41         k1 = -(double)(r * xy.first) / p;
42         s1 = round(k1);
43         /*cout << k << endl << k1 << endl;
44             cout << s << endl << s1 << endl;
45             */
46         tmp = llabs(r * xy.first + s1 * p) +
47             llabs(r * xy.second - s1 * q);
48         ans = min(ans, tmp);
49     }
50     cout << ans << endl;
51 }
52 return 0;
53 }

```

## 6.4 Fermat

- $a^{(p-1)} \equiv 1 \pmod{p} \Leftrightarrow a * a^{(p-2)} \equiv 1$ 
  - $a^{(p-2)} \equiv 1/a$
- 同餘因數定理
  - $a \equiv b \pmod{p} \Leftrightarrow k|a - b$
- 同餘加法性質
  - $a \equiv b \pmod{p}$  and  $c \equiv d \pmod{p}$ 
 $\Leftrightarrow a + c \equiv b + d \pmod{p}$
- 同餘相乘性質
  - $a \equiv b \pmod{p}$  and  $c \equiv d \pmod{p}$ 
 $\Leftrightarrow ac \equiv bd \pmod{p}$
- 同餘次方性質
  - $a \equiv b \pmod{p} \Leftrightarrow a^n \equiv b^n \pmod{p}$
- 同餘倍方性質
  - $a \equiv b \pmod{p} \Leftrightarrow am \equiv bm \pmod{p}$

## 6.5 Hex to Dec

```

1 int HextoDec(string num) //16 to 10
2 {
3     int base = 1;
4     int temp = 0;
5     for (int i = num.length() - 1; i >= 0; i--)
6     {
7         if (num[i] >= '0' && num[i] <= '9')
8         {
9             temp += (num[i] - 48) * base;
10            base = base * 16;
11        }
12        else if (num[i] >= 'A' && num[i] <= 'F')
13        {
14            temp += (num[i] - 55) * base;
15            base = base * 16;
16        }
17    }
18    return temp;
19 }
20 void DecToHex(int p) //10 to 16
21 {
22     char *l = new (char);
23     sprintf(l, "%X", p);
24     //int l_intResult = stoi(l);
25     cout << l << "\n";
26     //return l_intResult;
27 }

```

## 6.6 Log

```

1 double mylog(double a, double base)
2 {
3     //a 的對數底數 b = 自然對數 (a) / 自然對數 (b)
4     return log(a) / log(base);
5 }

```

## 6.7 Mod

```

1 int pow_mod(int a, int n, int m) // a ^ n mod m;
2 {
3     // a, n, m < 10 ^ 9
4     if (n == 0)
5         return 1;
6     int x = pow_mid(a, n / 2, m);
7     long long ans = (long long)x * x % m;
8     if (n % 2 == 1)
9         ans = ans * a % m;
10    return (int)ans;
11 }
12 int inv(int a, int n, int p) // n = p-2
13 {
14     long long res = 1;
15     for (; n >= 1, (a *= a) %= p)
16         if (n & 1)
17             (res *= a) %= p;
18     return res;
19 }

```

## 6.8 Mod 性質

加法： $(a + b) \bmod p = (a \bmod p + b \bmod p) \bmod p$   
 減法： $(a - b) \bmod p = (a \bmod p - b \bmod p + p) \bmod p$   
 乘法： $(a * b) \bmod p = (a \bmod p * b \bmod p) \bmod p$   
 次方： $(a^b) \bmod p = ((a \bmod p)^b) \bmod p$   
 加法結合律： $((a + b) \bmod p + c) \bmod p = (a + (b + c)) \bmod p$   
 乘法結合律： $((a * b) \bmod p * c) \bmod p = (a * (b * c)) \bmod p$   
 加法交換律： $(a + b) \bmod p = (b + a) \bmod p$   
 乘法交換律： $(a * b) \bmod p = (b * a) \bmod p$   
 結合律： $((a + b) \bmod p * c) = ((a * c) \bmod p + (b * c) \bmod p) \bmod p$

如果  $a \equiv b \pmod{m}$ ，我們會說  $a, b$  在模  $m$  下同餘。

以下為性質：

- 整除性： $a \equiv b \pmod{m} \Rightarrow c * m = a - b, c \in \mathbb{Z}$   
 $\Rightarrow a \equiv b \pmod{m} \Rightarrow m | a - b$
- 遞移性：若  $a \equiv b \pmod{c}, b \equiv d \pmod{c}$   
 則  $a \equiv d \pmod{c}$
- 保持基本運算：

$$\begin{cases} a \equiv b \pmod{m} \\ c \equiv d \pmod{m} \end{cases} \Rightarrow \begin{cases} a \pm c \equiv b \pm d \pmod{m} \\ a * c \equiv b * d \pmod{m} \end{cases}$$

- 放大縮小模數：

$$k \in \mathbb{Z}^+, a \equiv b \pmod{m} \Leftrightarrow k * a \equiv k * b \pmod{k * m}$$

模逆元是取模下的反元素，即為找到  $a^{-1}$  使得  $aa^{-1} \equiv 1 \pmod{c}$ 。

整數  $a$  在  $\bmod c$  下要有模反元素的充分必要條件為  $a, c$  互質。

模逆元如果存在會有無限個，任意兩相鄰模逆元相差  $c$ 。

費馬小定理

給定一個質數  $p$  及一個整數  $a$ ，那麼： $a^p \equiv a \pmod{p}$  如果  $\gcd(a, p) = 1$ ，則： $a^{p-1} \equiv 1 \pmod{p}$

歐拉定理

歐拉定理是比較 general 版本的費馬小定理，給定兩個整數  $n$  和  $a$ ，如果  $\gcd(a, n) = 1$ ，則： $a^{\phi(n)} \equiv 1 \pmod{n}$  如果  $n$  是質數， $\phi(n) = n - 1$ ，也就是費馬小定理。

Wilson's theorem

給定一個質數  $p$ ，則： $(p - 1)! \equiv -1 \pmod{p}$

## 6.9 PI

```

1 #define PI acos(-1)
2 #define PI M_PI

```

## 6.10 Prime table

```

1 const int maxn = sqrt(INT_MAX);
2 vector<int> p;

```

```

3 bitset<maxn> is_notp;
4 void PrimeTable()
5 {
6     is_notp.reset();
7     is_notp[0] = is_notp[1] = 1;
8     for (int i = 2; i <= maxn; ++i)
9     {
10         if (!is_notp[i])
11             p.push_back(i);
12         for (int j = 0; j < (int)p.size(); ++j)
13         {
14             if (i * p[j] > maxn)
15                 break;
16             is_notp[i * p[j]] = 1;
17             if (i % p[j] == 0)
18                 break;
19         }
20     }
21 }

```

## 6.11 Prime 判斷

```

1 typedef long long ll;
2 ll modmul(ll a, ll b, ll mod)
3 {
4     ll ret = 0;
5     for (; b >= 1, a = (a + a) % mod; b >>= 1)
6         if (b & 1)
7             ret = (ret + a) % mod;
8     return ret;
9 }
10 ll qpow(ll x, ll u, ll mod)
11 {
12     ll ret = 1ll;
13     for (; u >= 1, x = modmul(x, x, mod); u >>= 1)
14         if (u & 1)
15             ret = modmul(ret, x, mod);
16     return ret;
17 }
18 ll gcd(ll a, ll b)
19 {
20     return b ? gcd(b, a % b) : a;
21 }
22 ll Pollard_Rho(ll n, ll c)
23 {
24     ll i = 1, j = 2, x = rand() % (n - 1) + 1, y = x;
25     while (1)
26     {
27         i++;
28         x = (modmul(x, x, n) + c) % n;
29         ll p = gcd((y - x + n) % n, n);
30         if (p != 1 && p != n)
31             return p;
32         if (y == x)
33             return n;
34         if (i == j)
35         {
36             y = x;
37             j <<= 1;
38         }
39     }

```

```

39     }
40 }
41 bool Miller_Rabin(ll n)
42 {
43     ll x, pre, u = n - 1;
44     int i, j, k = 0;
45     if (n == 2 || n == 3 || n == 5 || n == 7 || n == 11)
46         return 1;
47     if (n == 1 || !(n % 2) || !(n % 3) || !(n % 5) || !(n % 7) || !(n % 11))
48         return 0;
49     while (!(u & 1))
50     {
51         k++;
52         u >>= 1;
53     }
54     srand((long long)12234336);
55     for (i = 1; i <= 50; i++)
56     {
57         x = rand() % (n - 2) + 2;
58         if (!(n % x))
59             return 0;
60         x = qpow(x, u, n);
61         pre = x;
62         for (j = 1; j <= k; j++)
63         {
64             x = modmul(x, x, n);
65             if (x == 1 && pre != 1 && pre != n - 1)
66                 return 0;
67             pre = x;
68         }
69         if (x != 1)
70             return 0;
71     }
72     return 1;
73 }
74 // if (Miller_Rabin(n)) puts("Prime");

```

## 6.12 Round(小數)

```

1 double myround(double number, unsigned int bits)
2 {
3     LL integerPart = number;
4     number -= integerPart;
5     for (unsigned int i = 0; i < bits; ++i)
6         number *= 10;
7     number = (LL)(number + 0.5);
8     for (unsigned int i = 0; i < bits; ++i)
9         number /= 10;
10    return integerPart + number;
11 }
12 //printf("%.1f\n", round(3.4515239, 1));

```

## 6.13 二分逼近法

```

1 #define eps 1e-14
2 void half_interval()

```

```

3 {
4     double L = 0, R = /*區間*/, M;
5     while (R - L >= eps)
6     {
7         M = (R + L) / 2;
8         if (/*函數*/ > /*方程式目標*/)
9             L = M;
10        else
11            R = M;
12    }
13    printf("%.3lf\n", R);
14 }

```

## 6.14 公式

$$S_n = \frac{a(1-r^n)}{1-r} \quad a_n = \frac{a_1 + a_n}{2} \quad \sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{k=1}^n k^3 = \left[ \frac{n(n+1)}{2} \right]^2$$

## 6.15 四則運算

```

1 string s = ""; //開頭是負號要補0
2 long long int DFS(int le, int ri) // (0, string final index)
3 {
4     int c = 0;
5     for (int i = ri; i >= le; i--)
6     {
7         if (s[i] == ')')
8             c++;
9         if (s[i] == '(')
10            c--;
11        if (s[i] == '+' && c == 0)
12            return DFS(le, i - 1) + DFS(i + 1, ri);
13        if (s[i] == '-' && c == 0)
14            return DFS(le, i - 1) - DFS(i + 1, ri);
15    }
16    for (int i = ri; i >= le; i--)
17    {
18        if (s[i] == ')')
19            c++;
20        if (s[i] == '(')
21            c--;
22        if (s[i] == '*' && c == 0)
23            return DFS(le, i - 1) * DFS(i + 1, ri);
24        if (s[i] == '/' && c == 0)
25            return DFS(le, i - 1) / DFS(i + 1, ri);
26        if (s[i] == '%' && c == 0)
27            return DFS(le, i - 1) % DFS(i + 1, ri);
28    }
29    if ((s[le] == '(' && (s[ri] == ')'))
30        return DFS(le + 1, ri - 1); //去除刮號

```

```

31 if (s[le] == ' ' && s[ri] == ' ')
32     return DFS(le + 1, ri - 1); //去除左右兩邊空格
33 if (s[le] == ' ')
34     return DFS(le + 1, ri); //去除左邊空格
35 if (s[ri] == ' ')
36     return DFS(le, ri - 1); //去除右邊空格
37 long long int num = 0;
38 for (int i = le; i <= ri; i++)
39     num = num * 10 + s[i] - '0';
40 return num;
41 }

```

## 6.16 因數表

```

1 vector<vector<int>> arr(1000000);
2 const int limit = 10e7;
3 for (int i = 1; i <= limit; i++)
4 {
5     for (int j = i; j <= limit; j += i)
6         arr[j].pb(i); // i 為因數
7 }

```

## 6.17 數字乘法組合

```

1 void dfs(int j, int old, int num, vector<int> > com, vector<vector<int>> &ans)
2 {
3     for (int i = j; i <= sqrt(num); i++)
4     {
5         if (old == num)
6             com.clear();
7         if (num % i == 0)
8         {
9             vector<int> a;
10            a = com;
11            a.push_back(i);
12            finds(i, old, num / i, a, ans);
13            a.push_back(num / i);
14            ans.push_back(a);
15        }
16    }
17 }
18 vector<vector<int>> ans;
19 vector<int> zero;
20 dfs(2, num, num, zero, ans);
21 /*num 為 input 數字*/
22 for (int i = 0; i < ans.size(); i++)
23 {
24     for (int j = 0; j < ans[i].size() - 1; j++)
25         cout << ans[i][j] << " ";
26     cout << ans[i][ans[i].size() - 1] << endl;
27 }

```

## 6.18 數字加法組合

```

1 void recur(int i, int n, int m, vector<int>
  &out, vector<vector<int>> &ans)
2 {
3     if (n == 0)
4     {
5         for (int i : out)
6             if (i > m)
7                 return;
8         ans.push_back(out);
9     }
10    for (int j = i; j <= n; j++)
11    {
12        out.push_back(j);
13        recur(j, n - j, m, out, ans);
14        out.pop_back();
15    }
16 }
17 vector<vector<int>> ans;
18 vector<int> zero;
19 recur(1, num, num, zero, ans);
20 // num 為 input 數字
21 for (int i = 0; i < ans.size(); i++)
22 {
23     for (int j = 0; j < ans[i].size() - 1; j++)
24         cout << ans[i][j] << " ";
25     cout << ans[i][ans[i].size() - 1] <<
26     endl;
27 }

```

## 6.19 羅馬數字

```

1 int romanToInt(string s)
2 {
3     unordered_map<char, int> T;
4     T['I'] = 1;
5     T['V'] = 5;
6     T['X'] = 10;
7     T['L'] = 50;
8     T['C'] = 100;
9     T['D'] = 500;
10    T['M'] = 1000;
11
12    int sum = T[s.back()];
13    for (int i = s.length() - 2; i >= 0; --i)
14    {
15        if (T[s[i]] < T[s[i + 1]])
16            sum -= T[s[i]];
17        else
18            sum += T[s[i]];
19    }
20    return sum;
21 }

```

## 6.20 質因數分解

```

1 LL ans;
2 void find(LL n, LL c) // 配合質數判斷
3 {
4     if (n == 1)
5         return;
6     if (Miller_Rabin(n))
7     {
8         ans = min(ans, n);
9         // bug(ans); // 質因數
10        return;
11    }
12    LL x = n, k = c;
13    while (x == n)
14        x = Pollard_Rho(x, c--);
15    find(n / x, k);
16    find(x, k);
17 }

```

## 6.21 質數數量

```

1 // 10 ^ 11 左右
2 #define LL long long
3 const int N = 5e6 + 2;
4 bool np[N];
5 int prime[N], pi[N];
6 int getprime()
7 {
8     int cnt = 0;
9     np[0] = np[1] = true;
10    pi[0] = pi[1] = 0;
11    for (int i = 2; i < N; ++i)
12    {
13        if (!np[i])
14            prime[++cnt] = i;
15        pi[i] = cnt;
16        for (int j = 1; j <= cnt && i *
17            prime[j] < N; ++j)
18        {
19            np[i * prime[j]] = true;
20            if (i % prime[j] == 0)
21                break;
22        }
23    }
24    return cnt;
25 }
26 const int M = 7;
27 const int PM = 2 * 3 * 5 * 7 * 11 * 13 * 17;
28 int phi[PM + 1][M + 1], sz[M + 1];
29 void init()
30 {
31     getprime();
32     sz[0] = 1;
33     for (int i = 0; i <= PM; ++i)
34         phi[i][0] = i;
35     for (int i = 1; i <= M; ++i)
36     {
37         sz[i] = prime[i] * sz[i - 1];
38         for (int j = 1; j <= PM; ++j)
39             phi[j][i] = phi[j][i - 1] - phi[
40                 j / prime[i]][i - 1];
41     }
42 }

```

```

41 int sqrt2(LL x)
42 {
43     LL r = (LL)sqrt(x - 0.1);
44     while (r * r <= x)
45         ++r;
46     return int(r - 1);
47 }
48 int sqrt3(LL x)
49 {
50     LL r = (LL)cbrt(x - 0.1);
51     while (r * r * r <= x)
52         ++r;
53     return int(r - 1);
54 }
55 LL getphi(LL x, int s)
56 {
57     if (s == 0)
58         return x;
59     if (s <= M)
60         return phi[x % sz[s]][s] + (x / sz[s]
61             ) * phi[sz[s]][s];
62     if (x <= prime[s] * prime[s])
63         return pi[x] - s + 1;
64     if (x <= prime[s] * prime[s] * prime[s]
65         && x < N)
66     {
67         int s2x = pi[sqrt2(x)];
68         LL ans = pi[x] - (s2x * s - 2) * (
69             s2x - s + 1) / 2;
70         for (int i = s + 1; i <= s2x; ++i)
71             ans += pi[x / prime[i]];
72         return ans;
73     }
74     return getphi(x, s - 1) - getphi(x /
75         prime[s], s - 1);
76 }
77 LL getpi(LL x)
78 {
79     if (x < N)
80         return pi[x];
81     LL ans = getphi(x, pi[sqrt3(x)]) + pi[
82         sqrt3(x)] - 1;
83     for (int i = pi[sqrt3(x)] + 1, ed = pi[
84         sqrt2(x)]; i <= ed; ++i)
85         ans -= getpi(x / prime[i]) - i + 1;
86     return ans;
87 }
88 LL lehmer_pi(LL x)
89 {
90     if (x < N)
91         return pi[x];
92     int a = (int)lehmer_pi(sqrt2(sqrt2(x)));
93     int b = (int)lehmer_pi(sqrt2(x));
94     int c = (int)lehmer_pi(sqrt3(x));
95     LL sum = getphi(x, a) + (LL)(b + a - 2)
96         * (b - a + 1) / 2;
97     for (int i = a + 1; i <= b; ++i)
98     {
99         LL w = x / prime[i];
100        sum -= lehmer_pi(w);
101        if (i > c)
102            continue;
103        LL lim = lehmer_pi(sqrt2(w));
104        for (int j = i; j <= lim; ++j)
105            sum -= lehmer_pi(w / prime[j]) -
106                (j - 1);
107    }
108 }

```

```

99 }
100 return sum;
101 }
102 // lehmer_pi(n)

```

## 7 Other

### 7.1 binary search 三類變化

```

1 // 查找和目標值完全相等的數
2 int find(vector<int> &nums, int target)
3 {
4     int left = 0, right = nums.size();
5     while (left < right)
6     {
7         int mid = left + (right - left) / 2;
8         if (nums[mid] == target)
9             return mid;
10        else if (nums[mid] < target)
11            left = mid + 1;
12        else
13            right = mid;
14    }
15    return -1;
16 }
17 // 找第一個不小於目標值的數 == 找最後一個小
18 // 於目標值的數
19 // *(lower_bound)*/
20 int find(vector<int> &nums, int target)
21 {
22     int left = 0, right = nums.size();
23     while (left < right)
24     {
25         int mid = left + (right - left) / 2;
26         if (nums[mid] < target)
27             left = mid + 1;
28        else
29            right = mid;
30    }
31    return right;
32 }
33 // 找第一個大於目標值的數 == 找最後一個不大
34 // 於目標值的數
35 // *(upper_bound)*/
36 int find(vector<int> &nums, int target)
37 {
38     int left = 0, right = nums.size();
39     while (left < right)
40     {
41         int mid = left + (right - left) / 2;
42         if (nums[mid] <= target)
43             left = mid + 1;
44        else
45            right = mid;
46    }
47    return right;
48 }

```

## 7.2 heap sort

```

1 void MaxHeapify(vector<int> &array, int root
2 , int length)
3 {
4     int left = 2 * root,
5         right = 2 * root + 1,
6         largest;
7     if (left <= length && array[left] >
8         array[root])
9         largest = left;
10    else
11        largest = right;
12    if (right <= length && array[right] >
13        array[largest])
14        largest = right;
15    if (largest != root)
16    {
17        swap(array[largest], array[root]);
18        MaxHeapify(array, largest, length);
19    }
20 }
21 void HeapSort(vector<int> &array)
22 {
23     array.insert(array.begin(), 0);
24     for (int i = (int)array.size() / 2; i >=
25         1; i--)
26         MaxHeapify(array, i, (int)array.size()
27             - 1);
28     int size = (int)array.size() - 1;
29     for (int i = (int)array.size() - 1; i >=
30         2; i--)
31     {
32         swap(array[1], array[i]);
33         size--;
34         MaxHeapify(array, 1, size);
35     }
36     array.erase(array.begin());
37 }

```

## 7.3 Merge sort

```

1 void Merge(vector<int> &arr, int front, int
2 mid, int end)
3 {
4     vector<int> LeftSub(arr.begin() + front,
5         arr.begin() + mid + 1);
6     vector<int> RightSub(arr.begin() + mid +
7         1, arr.begin() + end + 1);
8     LeftSub.insert(LeftSub.end(), INT_MAX);
9     RightSub.insert(RightSub.end(), INT_MAX);
10    ;
11    int idxLeft = 0, idxRight = 0;
12
13    for (int i = front; i <= end; i++)
14    {
15        if (LeftSub[idxLeft] <= RightSub[
16            idxRight])
17        {
18            arr[i] = LeftSub[idxLeft];
19            idxLeft++;
20        }
21        else
22        {
23            arr[i] = RightSub[idxRight];
24            idxRight++;
25        }
26    }
27 }

```

```

16 }
17 else
18 {
19     arr[i] = RightSub[idxRight];
20     idxRight++;
21 }
22 }
23 }
24 void MergeSort(vector<int> &arr, int front,
25 int end)
26 {
27     // front = 0 , end = arr.size() - 1
28     if (front < end)
29     {
30         int mid = (front + end) / 2;
31         MergeSort(arr, front, mid);
32         MergeSort(arr, mid + 1, end);
33         Merge(arr, front, mid, end);
34     }
35 }

```

## 7.4 Quick

```

1 int Partition(vector<int> &arr, int front,
2 int end)
3 {
4     int pivot = arr[end];
5     int i = front - 1;
6     for (int j = front; j < end; j++)
7     {
8         if (arr[j] < pivot)
9         {
10            i++;
11            swap(arr[i], arr[j]);
12        }
13    }
14    i++;
15    swap(arr[i], arr[end]);
16    return i;
17 }
18 void QuickSort(vector<int> &arr, int front,
19 int end)
20 {
21     // front = 0 , end = arr.size() - 1
22     if (front < end)
23     {
24         int pivot = Partition(arr, front,
25             end);
26         QuickSort(arr, front, pivot - 1);
27         QuickSort(arr, pivot + 1, end);
28     }
29 }

```

## 7.5 Weighted Job Scheduling

```

1 struct Job
2 {
3     int start, finish, profit;
4 };
5 bool jobComparataor(Job s1, Job s2)

```

```

6 {
7     return (s1.finish < s2.finish);
8 }
9 int latestNonConflict(Job arr[], int i)
10 {
11     for (int j = i - 1; j >= 0; j--)
12     {
13         if (arr[j].finish <= arr[i].start)
14             return j;
15     }
16     return -1;
17 }
18 int findMaxProfit(Job arr[], int n)
19 {
20     sort(arr, arr + n, jobComparataor);
21     int *table = new int[n];
22     table[0] = arr[0].profit;
23     for (int i = 1; i < n; i++)
24     {
25         int inclProf = arr[i].profit;
26         int l = latestNonConflict(arr, i);
27         if (l != -1)
28             inclProf += table[l];
29         table[i] = max(inclProf, table[i -
30             1]);
31     }
32     int result = table[n - 1];
33     delete[] table;
34     return result;
35 }

```

## 7.6 數獨解法

```

1 int getSquareIndex(int row, int column, int
2 n)
3 {
4     return row / n * n + column / n;
5 }
6 bool backtracking(vector<vector<int>> &board
7 , vector<vector<bool>> &rows, vector<
8 vector<bool>> &cols,
9 vector<vector<bool>> &boxes
10 , int index, int n)
11 {
12     int n2 = n * n;
13     int rowNum = index / n2, colNum = index
14         % n2;
15     if (index >= n2 * n2)
16         return true;
17
18     if (board[rowNum][colNum] != 0)
19         return backtracking(board, rows,
20             cols, boxes, index + 1, n);
21
22     for (int i = 1; i <= n2; i++)
23     {
24         if (!rows[rowNum][i] && !cols[colNum
25             ][i] && !boxes[getSquareIndex(
26                 rowNum, colNum, n)][i])
27         {
28             rows[rowNum][i] = true;
29         }
30     }
31 }

```

```

22     cols[colNum][i] = true;
23     boxes[getSquareIndex(rowNum,
24         colNum, n)][i] = true;
25     board[rowNum][colNum] = i;
26     if (backtracking(board, rows,
27         cols, boxes, index + 1, n))
28         return true;
29     board[rowNum][colNum] = 0;
30     rows[rowNum][i] = false;
31     cols[colNum][i] = false;
32     boxes[getSquareIndex(rowNum,
33         colNum, n)][i] = false;
34 }
35 }
36 return false;
37 }
38 /*用法 main*/
39 int n = sqrt(數獨邊長大小) /*e.g. 9*9 n=3*/
40 vector<vector<int>> board(n * n + 1, vector<
41     int>(n * n + 1, 0));
42 vector<vector<bool>> isRow(n * n + 1, vector<
43     bool>(n * n + 1, false));
44 vector<vector<bool>> isColumn(n * n + 1,
45     vector<bool>(n * n + 1, false));
46 vector<vector<bool>> isSquare(n * n + 1,
47     vector<bool>(n * n + 1, false));
48 for (int i = 0; i < n * n; ++i)
49 {
50     for (int j = 0; j < n * n; ++j)
51     {
52         int number;
53         cin >> number;
54         board[i][j] = number;
55         if (number == 0)
56             continue;
57         isRow[i][number] = true;
58         isColumn[j][number] = true;
59         isSquare[getSquareIndex(i, j, n)][
60             number] = true;
61     }
62 }
63 if (backtracking(board, isRow, isColumn,
64     isSquare, 0, n))
65     /*有解答*/
66 else
67     /*解答*/
68 }

```

## 8 String

### 8.1 KMP

```

1 // 用在一個 s 內查找一個詞 w 的出現位置
2 void ComputePrefix(string s, int next[])
3 {
4     int n = s.length();
5     int q, k;
6     next[0] = 0;
7     for (k = 0, q = 1; q < n; q++)
8     {

```

```

9   while (k > 0 && s[k] != s[q])
10       k = next[k];
11   if (s[k] == s[q])
12       k++;
13   next[q] = k;
14   }
15   }
16   void KMPMatcher(string text, string pattern)
17   {
18       int n = text.length();
19       int m = pattern.length();
20       int next[pattern.length()];
21       ComputePrefix(pattern, next);
22
23       for (int i = 0, q = 0; i < n; i++)
24       {
25           while (q > 0 && pattern[q] != text[i])
26               q = next[q];
27           if (pattern[q] == text[i])
28               q++;
29           if (q == m)
30           {
31               cout << "Pattern occurs with shift " << i - m + 1 << endl;
32               q = 0;
33           }
34       }
35   }
36   // string s = "abcdabcdeabcd";
37   // string p = "bcd";
38   // KMPMatcher(s, p);
39   // cout << endl;

```

## 8.3 Sliding window

```

1 string minWindow(string s, string t)
2 {
3     unordered_map<char, int> letterCnt;
4     for (int i = 0; i < t.length(); i++)
5         letterCnt[t[i]]++;
6     int minLength = INT_MAX, minStart = -1;
7     int left = 0, matchCnt = 0;
8     for (int i = 0; i < s.length(); i++)
9     {
10         if (--letterCnt[s[i]] >= 0)
11             matchCnt++;
12         while (matchCnt == t.length())
13         {
14             if (i - left + 1 < minLength)
15             {
16                 minLength = i - left + 1;
17                 minStart = left;
18             }
19             if (++letterCnt[s[left]] > 0)
20                 matchCnt--;
21             left++;
22         }
23     }
24     return minLength == INT_MAX ? "" : s.substr(minStart, minLength);
25 }

```

## 8.4 Split

```

1 vector<string> mysplit(string s, string d)
2 {
3     int ps = 0, pe, dl = d.length();
4     string token;
5     vector<string> res;
6     while ((pe = s.find(d, ps)) != string::npos)
7     {
8         token = s.substr(ps, pe - ps);
9         ps = pe + dl;
10        res.push_back(token);
11    }
12    res.push_back(s.substr(ps));
13    return res;
14 }

```

## 9 data structure

### 9.1 Bigint

```

1 //台大 //非必要請用python
2 struct Bigint
3 {
4     static const int LEN = 60; //
5     maxLEN

```

```

5 static const int BIGMOD = 10000; //10為
6 正常位數
7 int s;
8 int v1, v[LEN];
9 // vector<int> v;
10 Bigint() : s(1) { v1 = 0; }
11 Bigint(long long a)
12 {
13     s = 1;
14     v1 = 0;
15     if (a < 0)
16     {
17         s = -1;
18         a = -a;
19     }
20     while (a)
21     {
22         push_back(a % BIGMOD);
23         a /= BIGMOD;
24     }
25 }
26 Bigint(string str)
27 {
28     s = 1;
29     v1 = 0;
30     int stPos = 0, num = 0;
31     if (!str.empty() && str[0] == '-')
32     {
33         stPos = 1;
34         s = -1;
35     }
36     for (int i = str.length() - 1, q = 1; i >= stPos; i--)
37     {
38         num += (str[i] - '0') * q;
39         if ((q *= 10) >= BIGMOD)
40         {
41             push_back(num);
42             num = 0;
43             q = 1;
44         }
45     }
46     if (num)
47         push_back(num);
48     n();
49 }
50 int len() const
51 {
52     return v1; //return SZ(v);
53 }
54 bool empty() const { return len() == 0; }
55 void push_back(int x)
56 {
57     v[v1++] = x; //v.PB(x);
58 }
59 void pop_back()
60 {
61     v1--; //v.pop_back();
62 }
63 int back() const
64 {
65     return v[v1 - 1]; //return v.back();
66 }
67 void n()

```

```

67 {
68     while (!empty() && !back())
69         pop_back();
70 }
71 void resize(int nl)
72 {
73     v1 = nl; //v.resize(nl);
74     fill(v, v + v1, 0); //fill(ALL(v), 0);
75 }
76 void print() const
77 {
78     if (empty())
79     {
80         putchar('0');
81         return;
82     }
83     if (s == -1)
84         putchar('-');
85     printf("%d", back());
86     for (int i = len() - 2; i >= 0; i--)
87         printf("%.4d", v[i]);
88 }
89 friend std::ostream &operator<<(std::ostream &out, const Bigint &a)
90 {
91     if (a.empty())
92     {
93         out << "0";
94         return out;
95     }
96     if (a.s == -1)
97         out << "-";
98     out << a.back();
99     for (int i = a.len() - 2; i >= 0; i--)
100     {
101         char str[10];
102         snprintf(str, 5, "%.4d", a.v[i]);
103         out << str;
104     }
105     return out;
106 }
107 int cp3(const Bigint &b) const
108 {
109     if (s != b.s)
110         return s - b.s;
111     if (s == -1)
112         return -(*this).cp3(-b);
113     if (len() != b.len())
114         return len() - b.len(); //int
115     for (int i = len() - 1; i >= 0; i--)
116         if (v[i] != b.v[i])
117             return v[i] - b.v[i];
118     return 0;
119 }
120 bool operator<(const Bigint &b) const
121 {
122     return cp3(b) < 0;
123 }
124 bool operator<=(const Bigint &b) const
125 {
126     return cp3(b) <= 0;
127 }
128 bool operator==(const Bigint &b) const

```



```

129 {
130     return cp3(b) == 0;
131 }
132 bool operator!=(const Bigint &b) const
133 {
134     return cp3(b) != 0;
135 }
136 bool operator>(const Bigint &b) const
137 {
138     return cp3(b) > 0;
139 }
140 bool operator>=(const Bigint &b) const
141 {
142     return cp3(b) >= 0;
143 }
144 Bigint operator-() const
145 {
146     Bigint r = (*this);
147     r.s = -r.s;
148     return r;
149 }
150 Bigint operator+(const Bigint &b) const
151 {
152     if (s == -1)
153         return -(*this) + (-b);
154     if (b.s == -1)
155         return (*this) - (-b);
156     Bigint r;
157     int nl = max(len(), b.len());
158     r.resize(nl + 1);
159     for (int i = 0; i < nl; i++)
160     {
161         if (i < len())
162             r.v[i] += v[i];
163         if (i < b.len())
164             r.v[i] += b.v[i];
165         if (r.v[i] >= BIGMOD)
166         {
167             r.v[i + 1] += r.v[i] /
168                 BIGMOD;
169             r.v[i] %= BIGMOD;
170         }
171     }
172     r.n();
173     return r;
174 }
175 Bigint operator-(const Bigint &b) const
176 {
177     if (s == -1)
178         return -(*this) - (-b);
179     if (b.s == -1)
180         return (*this) + (-b);
181     if ((*this) < b)
182         return -(b - (*this));
183     Bigint r;
184     r.resize(len());
185     for (int i = 0; i < len(); i++)
186     {
187         r.v[i] += v[i];
188         if (i < b.len())
189             r.v[i] -= b.v[i];
190         if (r.v[i] < 0)
191         {
192             r.v[i] += BIGMOD;
193             r.v[i + 1]--;

```

## 9.2 DisjointSet

```
1 struct DisjointSet {
```

```

194     }
195     r.n();
196     return r;
197 }
198 Bigint operator*(const Bigint &b)
199 {
200     Bigint r;
201     r.resize(len() + b.len() + 1);
202     r.s = s * b.s;
203     for (int i = 0; i < len(); i++)
204     {
205         for (int j = 0; j < b.len(); j
206             ++){
207             r.v[i + j] += v[i] * b.v[j];
208             if (r.v[i + j] >= BIGMOD)
209             {
210                 r.v[i + j + 1] += r.v[i
211                     + j] / BIGMOD;
212                 r.v[i + j] %= BIGMOD;
213             }
214         }
215     }
216     r.n();
217     return r;
218 }
219 Bigint operator/(const Bigint &b)
220 {
221     Bigint r;
222     r.resize(max(1, len() - b.len() + 1)
223         );
224     int oriS = s;
225     Bigint b2 = b; // b2 = abs(b)
226     s = b2.s = r.s = 1;
227     for (int i = r.len() - 1; i >= 0; i
228         --){
229         int d = 0, u = BIGMOD - 1;
230         while (d < u)
231         {
232             int m = (d + u + 1) >> 1;
233             r.v[i] = m;
234             if ((r * b2) > (*this))
235                 u = m - 1;
236             else
237                 d = m;
238         }
239     }
240     r.v[i] = d;
241     s = oriS;
242     r.s = s * b.s;
243     r.n();
244     return r;
245 }
246 Bigint operator%(const Bigint &b)
247 {
248     return (*this) - (*this) / b * b;
249 }

```

```

2 int p[maxn], sz[maxn], n, cc;
3 vector<pair<int*, int>> his;
4 vector<int> sh;
5 void init(int _n) {
6     n = _n; cc = n;
7     for (int i = 0; i < n; ++i) sz[i] =
8         1, p[i] = i;
9     sh.clear(); his.clear();
10 }
11 void assign(int *k, int v) {
12     his.emplace_back(k, *k);
13     *k = v;
14 }
15 void save() {
16     sh.push_back((int)his.size());
17 }
18 void undo() {
19     int last = sh.back(); sh.pop_back();
20     while (his.size() != last) {
21         int *k, v;
22         tie(k, v) = his.back(); his.
23             pop_back();
24         *k = v;
25     }
26 }
27 int find(int x) {
28     if (x == p[x]) return x;
29     return find(p[x]);
30 }
31 void merge(int x, int y) {
32     x = find(x); y = find(y);
33     if (x == y) return;
34     if (sz[x] > sz[y]) swap(x, y);
35     assign(&sz[y], sz[x] + sz[y]);
36     assign(&p[x], y);
37     assign(&cc, cc - 1);
38 }
39 }

```

## 9.3 Matirx

```

1 template <typename T>
2 struct Matrix
3 {
4     using rt = std::vector<T>;
5     using mt = std::vector<rt>;
6     using matrix = Matrix<T>;
7     int r, c; // [r][c]
8     mt m;
9     Matrix(int r, int c) : r(r), c(c), m(r,
10         rt(c)) {}
11     Matrix(mt a) { m = a, r = a.size(), c =
12         a[0].size(); }
13     rt &operator[](int i) { return m[i]; }
14     matrix operator+(const matrix &a)
15     {
16         matrix rev(r, c);
17         for (int i = 0; i < r; ++i)
18             for (int j = 0; j < c; ++j)
19                 rev[i][j] = m[i][j] + a.m[i]

```

```

20         matrix operator-(const matrix &a)
21         {
22             matrix rev(r, c);
23             for (int i = 0; i < r; ++i)
24                 for (int j = 0; j < c; ++j)
25                     rev[i][j] = m[i][j] - a.m[i]
26                         ][j];
27             return rev;
28         }
29         matrix operator*(const matrix &a)
30         {
31             matrix rev(r, a.c);
32             matrix tmp(a.c, a.r);
33             for (int i = 0; i < a.r; ++i)
34                 for (int j = 0; j < a.c; ++j)
35                     tmp[j][i] = a.m[i][j];
36             for (int i = 0; i < r; ++i)
37                 for (int j = 0; j < a.c; ++j)
38                     for (int k = 0; k < c; ++k)
39                         rev.m[i][j] += m[i][k] *
40                             tmp[j][k];
41             return rev;
42         }
43         bool inverse() //逆矩阵判断
44         {
45             Matrix t(r, r + c);
46             for (int y = 0; y < r; y++)
47             {
48                 t.m[y][c + y] = 1;
49                 for (int x = 0; x < c; ++x)
50                     t.m[y][x] = m[y][x];
51             }
52             if (!t.gas())
53                 return false;
54             for (int y = 0; y < r; y++)
55                 for (int x = 0; x < c; ++x)
56                     m[y][x] = t.m[y][c + x] / t.
57                         m[y][y];
58             return true;
59         }
60         T gas() //行列式
61         {
62             vector<T> lazy(r, 1);
63             bool sign = false;
64             for (int i = 0; i < r; ++i)
65             {
66                 if (m[i][i] == 0)
67                 {
68                     int j = i + 1;
69                     while (j < r && !m[j][i])
70                         j++;
71                     if (j == r)
72                         continue;
73                     m[i].swap(m[j]);
74                     sign = !sign;
75                 }
76                 for (int j = 0; j < r; ++j)
77                 {
78                     if (i == j)
79                         continue;
80                     lazy[j] = lazy[j] * m[i][i];
81                     T mx = m[j][i];
82                     for (int k = 0; k < c; ++k)
83                         m[j][k] = m[j][k] * m[i]
84                             ][i] - m[i][k] * mx;

```

```

81     }
82 }
83 T det = sign ? -1 : 1;
84 for (int i = 0; i < r; ++i)
85 {
86     det = det * m[i][i];
87     det = det / lazy[i];
88     for (auto &j : m[i])
89         j /= lazy[i];
90 }
91 return det;
92 }
93 };

```

## 9.4 Trie

```

1 // biginter字典數
2 struct BigInteger{
3     static const int BASE = 100000000;
4     static const int WIDTH = 8;
5     vector<int> s;
6     BigInteger(long long num = 0){
7         *this = num;
8     }
9     BigInteger operator = (long long num){
10         s.clear();
11         do{
12             s.push_back(num % BASE);
13             num /= BASE;
14         }while(num > 0);
15         return *this;
16 }
17 BigInteger operator = (const string& str)
18 {
19     s.clear();
20     int x, len = (str.length() - 1) /
21         WIDTH + 1;
22     for(int i = 0; i < len; i++){
23         int end = str.length() - i*WIDTH;
24         int start = max(0, end-WIDTH);
25         sscanf(str.substr(start, end-
26             start).c_str(), "%d", &x);
27         s.push_back(x);
28     }
29     return *this;
30 }
31 BigInteger operator + (const BigInteger&
32     b) const{
33     BigInteger c;
34     c.s.clear();
35     for(int i = 0, g = 0; i < s.size() && i
36         <= b.s.size(); i++){
37         if(g == 0 && i >= s.size() && i
38             >= b.s.size()) break;
39         int x = g;
40         if(i < s.size()) x+=s[i];
41         if(i < b.s.size()) x+=b.s[i];
42         c.s.push_back(x % BASE);
43         g = x / BASE;
44     }
45     return c;
46 }

```

```

42 };
43
44 ostream& operator << (ostream &out, const
45     BigInteger& x){
46     out << x.s.back();
47     for(int i = x.s.size()-2; i >= 0; i--){
48         char buf[20];
49         sprintf(buf, "%08d", x.s[i]);
50         for(int j = 0; j < strlen(buf); j++){
51             out << buf[j];
52         }
53     }
54     return out;
55 }
56 istream& operator >> (istream &in,
57     BigInteger& x){
58     string s;
59     if(!(in >> s))
60         return in;
61     x = s;
62     return in;
63 }
64 struct Trie{
65     int c[500005][10];
66     int val[500005];
67     int sz;
68     int getIndex(char c){
69         return c - '0';
70     }
71     void init(){
72         memset(c[0], 0, sizeof(c[0]));
73         memset(val, -1, sizeof(val));
74         sz = 1;
75     }
76     void insert(BigInteger x, int v){
77         int u = 0;
78         int max_len_count = 0;
79         int firstNum = x.s.back();
80         char firstBuf[20];
81         sprintf(firstBuf, "%d", firstNum);
82         for(int j = 0; j < strlen(firstBuf);
83             j++){
84             int index = getIndex(firstBuf[j]);
85             if(!c[u][index]){
86                 memset(c[sz], 0, sizeof(c[
87                     sz]));
88                 val[sz] = v;
89                 c[u][index] = sz++;
90             }
91             u = c[u][index];
92             max_len_count++;
93         }
94         for(int i = x.s.size()-2; i >= 0; i
95             --){
96             char buf[20];
97             sprintf(buf, "%08d", x.s[i]);
98             for(int j = 0; j < strlen(buf)
99                 && max_len_count < 50; j++){
100                 int index = getIndex(buf[j]);
101                 if(!c[u][index]){
102                     memset(c[sz], 0, sizeof
103                         (c[sz]));

```

```

99         val[sz] = v;
100         c[u][index] = sz++;
101     }
102     u = c[u][index];
103     max_len_count++;
104 }
105 if(max_len_count >= 50){
106     break;
107 }
108 }
109 }
110 int find(const char* s){
111     int u = 0;
112     int n = strlen(s);
113     for(int i = 0; i < n; ++i)
114     {
115         int index = getIndex(s[i]);
116         if(!c[u][index]){
117             return -1;
118         }
119         u = c[u][index];
120     }
121     return val[u];
122 }
123 }

```

```

29 {
30     return fraction(n * b.d, d * b.n);
31 }
32 void print()
33 {
34     cout << n;
35     if (d != 1)
36         cout << "/" << d;
37 }
38 };

```

## 9.5 分數

```

1 typedef long long ll;
2 struct fraction
3 {
4     ll n, d;
5     fraction(const ll &n = 0, const ll &d =
6         1) : n(_n), d(_d)
7     {
8         ll t = __gcd(n, d);
9         n /= t, d /= t;
10        if (d < 0)
11            n = -n, d = -d;
12    }
13    fraction operator-() const
14    {
15        return fraction(-n, d);
16    }
17    fraction operator+(const fraction &b)
18        const
19    {
20        return fraction(n * b.d + b.n * d, d * b
21            .d);
22    }
23    fraction operator-(const fraction &b)
24        const
25    {
26        return fraction(n * b.d - b.n * d, d * b
27            .d);
28    }
29    fraction operator*(const fraction &b)
30        const
31    {
32        return fraction(n * b.n, d * b.d);
33    }
34    fraction operator/(const fraction &b)
35        const
36    {
37        return fraction(n * b.d, d * b.n);
38    }
39 }

```

# TO DO WRITING NOT THINKING

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