1 Basic

1.1 Basic codeblock setting

```
Settings -> Editor -> Keyboard shortcuts -> Plugins -> Source code formatter (AStyle)
Settings -> Source Formatter -> Padding
Delete empty lines within a function or method
Insert space padding around operators
Insert space padding around parentheses on outside
Remove extra space padding around parentheses
```

1.2 Basic vim setting

```
/*at home directory*/
/* vi ~/.vimrc */
syntax enable
set smartindent
set tabstop=4
set shiftwidth=4
set expandtab
set relativenumber
```

1.3 Code Template

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef unsigned long long ull;
#define pb push_back
#define len length()
#define all(p) p.begin(), p.end()
#define endl '\n'
#define x first
#define y second
#define bug(k) cout << "value of " << #k << " is " << k << endl;
#define bugarr(k)         \
    for (auto i : k)      \
        cout << i << ' '; \
    cout << endl;
int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    return 0;
}
```

1.4 Python

```python
//輸入
import sys
line = sys.stdin.readline() // 會讀到換行
input().strip()

array = [0] * (N) //N個0
range(0, N) // 0 ~ N-1
D, R, N = map(int, line[:-1].split()) // 分三個 int 變數

pow(a, b, c) // a ^ b % c

print(*objects, sep = ' ', end = '\n')
// objects -- 可以一次輸出多個對象
// sep -- 分開多個objects
// end -- 默認值是\n

// EOF break
try:
    while True:
        //input someithing
except EOFError:
    pass
```

1.5 Range data

```
int (-2147483648 to 2147483647)
unsigned int(0 to 4294967295)
long(-2147483648 to 2147483647)
unsigned long(0 to 4294967295)
long long(-9223372036854775808 to 9223372036854775807)
unsigned long long (0 to 18446744073709551615)
```

1.6 Some Function

```cpp
round(double f);              // 四捨五入
ceil(double f);               // 進入
floor(double f);              //捨去
__builtin_popcount(int n); // 32bit有多少 1
to_string(int s);             // int to string

set_union(all(a), all(b), back_inserter(d)); // 聯集
set_intersection(all(a), all(b), back_inserter(c)); //交集

/** 全排列要先 sort !!! **/
next_permutation(num.begin(), num.end());
prev_permutation(num.begin(), num.end());
//用binary search找大於或等於val的最小值的位置
vector<int>::iterator it = lower_bound(v.begin(), v.end(), val);
//用binary search找大於val的最小值的位置
vector<int>::iterator it = upper_bound(v.begin(), v.end(), val);

/*找到範圍裏面的最大元素*/
max_element(n, n + len);          // n到n+len範圍內最大值
max_element(v.begin(), v.end()); // vector中最大值
/*找到範圍裏面的最大元素*/
min_element(n, n + len);          // n到n+len範圍內最小值
min_element(v.begin(), v.end()); // vector中最小值

/*queue*/
queue<datatype> q;
front(); /*取出最前面的值(沒有移除掉)*/
back();  /*取出最後面的值(沒有移除掉)*/
pop();   /*移掉最前面的值*/
push();  /*新增值到最後面*/
empty(); /*回傳bool,檢查是不是空的queue*/
size();  /*queue 的大小*/

/*stack*/
stack<datatype> s;
top();   /*取出最上面的值(沒有移除掉)*/
pop();   /*移掉最上面的值*/
push();  /*新增值到最上面*/
empty(); /*bool 檢查是不是空*/
size();  /*stack 的大小*/

/*unordered_set*/
unordered_set<datatype> s;
unordered_set<datatype> s(arr, arr + n);
/*initial with array*/
insert(); /*插入值*/
erase();  /*刪除值*/
empty();  /*bool 檢查是不是空*/
count();  /*判斷元素存在回傳1 無則回傳0*/
```

1.7 Time

```cpp
cout << 1.0 * clock() / CLOCKS_PER_SEC << endl;
```

2 DP

2.1 3 維 DP 思路

```
解題思路: dp[i][j][k]
i 跟 j 代表 range i ~ j 的 value
k在我的理解裡是視題目的要求而定的
像是 Remove Boxes 當中 k 代表的是在 i 之前還有多少個連續的箱子
所以每次區間消去的值就是(k+1) * (k+1)
換言之，我認為可以理解成 k 的意義就是題目今天所關注的重點，就是老師說的題目所規定的運算
```

2.2 Knapsack Bounded

```cpp
const int N = 100, W = 100000;
int cost[N], weight[N], number[N];
int c[W + 1];
void knapsack(int n, int w)
{
    for (int i = 0; i < n; ++i)
    {
        int num = min(number[i], w / weight[i]);
        for (int k = 1; num > 0; k *= 2)
        {
            if (k > num)
                k = num;
            num -= k;
            for (int j = w; j >= weight[i] * k; --j)
                c[j] = max(c[j], c[j - weight[i] * k] + cost[i] * k);
        }
    }
    cout << "Max Prince" << c[w];
}
```

2.3 Knapsack sample

```cpp
int Knapsack(vector<int> weight, vector<int> value, int bag_Weight)
{
    // vector<int> weight = {1, 3, 4};
    // vector<int> value = {15, 20, 30};
    // int bagWeight = 4;
    vector<vector<int>> dp(weight.size(), vector<int>(bagWeight + 1, 0));
    for (int j = weight[0]; j <= bagWeight; j++)
        dp[0][j] = value[0];
    // weight數組的大小就是物品個數
    for (int i = 1; i < weight.size(); i++)
    { // 遍歷物品
        for (int j = 0; j <= bagWeight; j++)
        { // 遍歷背包容量
            if (j < weight[i]) dp[i][j] = dp[i - 1][j];
            else dp[i][j] = max(dp[i - 1][j], dp[i - 1][j - weight[i]] + value[i]);
        }
    }
    cout << dp[weight.size() - 1][bagWeight] << endl;
}
```

2.4 Knapsack Unbounded

```
const int N = 100, W = 100000;
int cost[N], weight[N];
int c[W + 1];
void knapsack(int n, int w)
{
    memset(c, 0, sizeof(c));
    for (int i = 0; i < n; ++i)
        for (int j = weight[i]; j <= w; ++j)
            c[j] = max(c[j], c[j - weight[i]] + cost[i]);
    cout << "最高的價值為" << c[w];
}
```

2.5 LCIS

```
int LCIS_len(vector<int> arr1, vetor<int> arr2)
{
    int n = arr1.size(), m = arr2.size();
    vector<int> table(m, 0);
    for (int j = 0; j < m; j++)
        table[j] = 0;
    for (int i = 0; i < n; i++)
    {
        int current = 0;
        for (int j = 0; j < m; j++)
        {

            if (arr1[i] == arr2[j])
                if (current + 1 > table[j])
                    table[j] = current + 1;

            if (arr1[i] > arr2[j])
                if (table[j] > current)
                    current = table[j];
        }
    }
    int result = 0;
    for (int i = 0; i < m; i++)
        if (table[i] > result)
            result = table[i];
    return result;
}
```

2.6 LCS

```
int LCS(vector<string> Ans, vector<string> num)
{
    int N = Ans.size(), M = num.size();
    vector<vector<int>> LCS(N + 1, vector<int>(M + 1, 0));
    for (int i = 1; i <= N; ++i)
    {
        for (int j = 1; j <= M; ++j)
        {
            if (Ans[i - 1] == num[j - 1])
                LCS[i][j] = LCS[i - 1][j - 1] + 1;
            else
                LCS[i][j] = max(LCS[i - 1][j], LCS[i][j - 1]);
        }
    }
    cout << LCS[N][M] << '\n';
    //列印 LCS
    int n = N, m = M;
    vector<string> k;
    while (n && m)
    {
        if (LCS[n][m] != max(LCS[n - 1][m], LCS[n][m - 1]))
        {
            k.push_back(Ans[n - 1]);
            n--;
            m--;
        }
        else if (LCS[n][m] == LCS[n - 1][m])
            n--;
        else if (LCS[n][m] == LCS[n][m - 1])
            m--;
    }
    reverse(k.begin(), k.end());
    for (auto i : k)
        cout << i << " ";
    cout << endl;
    return LCS[N][M];
}
```

2.7 LIS

```
vector<int> ans;
void printLIS(vector<int> &arr, vector<int> &pos, int index)
{
    if (pos[index] != -1)
        printLIS(arr, pos, pos[index]);
    // printf("%d", arr[index]);
    ans.push_back(arr[index]);
}
void LIS(vector<int> &arr)
{
    vector<int> dp(arr.size(), 1);
    vector<int> pos(arr.size(), -1);
    int res = INT_MIN, index = 0;
    for (int i = 0; i < arr.size(); ++i)
    {
        for (int j = i + 1; j < arr.size(); ++j)
        {
            if (arr[j] > arr[i])
            {
                if (dp[i] + 1 > dp[j])
                {
                    dp[j] = dp[i] + 1;
                    pos[j] = i;
                }
            }
        }
        if (dp[i] > res)
        {
            res = dp[i];
            index = i;
        }
    }
    cout << res << endl; // length
    printLIS(arr, pos, index);
    for (int i = 0; i < ans.size(); i++)
    {
        cout << ans[i];
        if (i != ans.size() - 1)
            cout << ' ';
    }
    cout << '\n';
}
```

2.8 LPS

```
void LPS(string s)
{
    int maxlen = 0, l, r;
    int n = n;
    for (int i = 0; i < n; i++)
    {
        int x = 0;
        while ((s[i - x] == s[i + x]) && (i - x >= 0) && (i + x < n)) //odd length
            x++;
        x--;
        if (2 * x + 1 > maxlen)
        {
            maxlen = 2 * x + 1;
            l = i - x;
            r = i + x;
        }
        x = 0;
        while ((s[i - x] == s[i + 1 + x]) && (i - x >= 0) && (i + 1 + x < n)) //even length
            x++;
        if (2 * x > maxlen)
        {
            maxlen = 2 * x;
            l = i - x + 1;
            r = i + x;
        }
    }
    cout << maxlen << '\n';  // 最後長度
    cout << l + 1 << ' ' << r + 1 << '\n'; //頭到尾
}
```

2.9 Max_subarray

```
/*Kadane's algorithm*/
int maxSubArray(vector<int>& nums) {
    int local_max = nums[0], global_max = nums[0];
    for(int i = 1; i < nums.size(); i++){
        local_max = max(nums[i],nums[i]+local_max);
        global_max = max(local_max, global_max);
    }
    return global_max;
}
```

2.10 Money problem

```
//能否湊得某個價位
void change(vector<int> price, int limit)
{
    vector<bool> c(limit + 1, 0);
    c[0] = true;
    for (int i = 0; i < price.size(); ++i) // 依序加入各種面額
        for (int j = price[i]; j <= limit; ++j) // 由低價位逐步到高價位
            c[j] = c[j] | c[j - price[i]]; // 湊、湊、湊
    if (c[limit]) cout << "YES\n";
    else cout << "NO\n";
}
// 湊得某個價位的湊法總共幾種
void change(vector<int> price, int limit)
{
    vector<int> c(limit + 1, 0);
    c[0] = true;
    for (int i = 0; i < price.size(); ++i)
        for (int j = price[i]; j <= limit; ++j)
            c[j] += c[j - price[i]];
    cout << c[limit] << '\n';
}
// 湊得某個價位的最少錢幣用量
void change(vector<int> price, int limit)
{
    vector<int> c(limit + 1, 0);
    c[0] = true;
    for (int i = 0; i < price.size(); ++i)
        for (int j = price[i]; j <= limit; ++j)
            c[j] = min(c[j], c[j - price[i]] + 1);
    cout << c[limit] << '\n';
}
//湊得某個價位的錢幣用量，有哪幾種可能性
void change(vector<int> price, int limit)
{
    vector<int> c(limit + 1, 0);
    c[0] = true;
    for (int i = 0; i < price.size(); ++i)
        for (int j = price[i]; j <= limit; ++j)
            c[j] |= c[j-price[i]] << 1; // 錢幣數量加一，每一種可能性都加一。

    for (int i = 1; i <= 63; ++i)
```

```cpp
        if (c[m] & (1 << i))
            cout << "用" << i << "個錢幣可湊得價位" << m;
}
```

3 Flow & matching

3.1 Dinic

```cpp
const long long INF = 1LL<<60;
struct Dinic { //O(VVE), with minimum cut
    static const int MAXN = 5003;
    struct Edge{
        int u, v;
        long long cap, rest;
    };
    int n, m, s, t, d[MAXN], cur[MAXN];
    vector<Edge> edges;
    vector<int> G[MAXN];
    void init(){
        edges.clear();
        for ( int i = 0 ; i < n ; i++ ) G[i].clear();
        n = 0;
    }
    // min cut start
    bool side[MAXN];
    void cut(int u) {
        side[u] = 1;
        for ( int i : G[u] ) {
            if ( !side[ edges[i].v ] && edges[i].rest )
            cut(edges[i].v);
        }
    }
    // min cut end
    int add_node(){
        return n++;
    }
    void add_edge(int u, int v, long long cap){
        edges.push_back( {u, v, cap, cap} );
        edges.push_back( {v, u, 0, 0LL} );
        m = edges.size();
        G[u].push_back(m-2);
        G[v].push_back(m-1);
    }
    bool bfs(){
        fill(d,d+n,-1);
        queue<int> que;
        que.push(s); d[s]=0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (int ei : G[u]){
                Edge &e = edges[ei];
                if (d[e.v] < 0 && e.rest > 0){
                    d[e.v] = d[u] + 1;
                    que.push(e.v);
                }
            }
        }
        return d[t] >= 0;
    }
    long long dfs(int u, long long a){
        if ( u == t || a == 0 ) return a;
        long long flow = 0, f;
        for ( int &i=cur[u]; i < (int)G[u].size() ; i++) {
            Edge &e = edges[ G[u][i] ];
            if ( d[u] + 1 != d[e.v] ) continue;
            f = dfs(e.v, min(a, e.rest) );
            if ( f > 0 ) {
            e.rest -= f;
            edges[ G[u][i]^1 ].rest += f;
            flow += f;
            a -= f;
            if ( a == 0 ) break;
            }
        }
        return flow;
    }
    long long maxflow(int _s, int _t){
        s = _s, t = _t;
        long long flow = 0, mf;
        while ( bfs() ){
            fill(cur,cur+n,0);
            while ( (mf = dfs(s, INF)) ) flow += mf;
        }
        return flow;
    }
} dinic;
```

3.2 Edmonds_karp

```cpp
/*Flow - Edmonds-karp*/
/*Based on UVa820*/
#define inf 1000000
int getMaxFlow(vector<vector<int>> &capacity, int s, int t, int n){
  int ans = 0;
  vector<vector<int>> residual(n+1, vector<int>(n+1, 0)); //residual network
  while(true){
    vector<int> bottleneck(n+1, 0);
    bottleneck[s] = inf;
    queue<int> q;
    q.push(s);
    vector<int> pre(n+1, 0);
    while(!q.empty() && bottleneck[t] == 0){
      int cur = q.front();
      q.pop();
      for(int i = 1; i <= n ; i++){
        if(bottleneck[i] == 0 && capacity[cur][i] > residual[cur][i]){
          q.push(i);
          pre[i] = cur;
          bottleneck[i] = min(bottleneck[cur], capacity[cur][i] - residual[cur][i]);
        }
      }
    }
    if(bottleneck[t] == 0) break;
    for(int cur = t; cur != s; cur = pre[cur]){
      residual[pre[cur]][cur] += bottleneck[t];
      residual[cur][pre[cur]] -= bottleneck[t];
    }
    ans += bottleneck[t];
  }
  return ans;
}
int main(){
  int testcase = 1;
  int n;
  while(cin>>n){
    if(n == 0)
      break;
    vector<vector<int>> capacity(n+1, vector<int>(n+1, 0));
    int s, t, c;
    cin >> s >> t >> c;
    int a, b, bandwidth;
    for(int i = 0 ; i < c ; ++i){
      cin >> a >> b >> bandwidth;
      capacity[a][b] += bandwidth;
      capacity[b][a] += bandwidth;
    }
    cout << "Network " << testcase++ << endl;
    cout << "The bandwidth is " << getMaxFlow(capacity, s, t, n) << "." << endl;
    cout << endl;
  }
  return 0;
}
```

3.3 hungarian

```cpp
/*bipartite - hungarian*/
struct Graph{
    static const int MAXN = 5003;
    vector<int> G[MAXN];
    int n, match[MAXN], vis[MAXN];
    void init(int _n){
        n = _n;
        for (int i=0; i<n; i++) G[i].clear();
    }
    bool dfs(int u){
            for (int v:G[u]){
            if (vis[v]) continue;
            vis[v]=true;
            if (match[v]==-1 || dfs(match[v])){
                match[v] = u;
                match[u] = v;
                return true;
            }
        }
        return false;
    }
    int solve(){
        int res = 0;
        memset(match,-1,sizeof(match));
        for (int i=0; i<n; i++){
            if (match[i]==-1){
                memset(vis,0,sizeof(vis));
                if ( dfs(i) ) res++;
            }
        }
        return res;
    }
} graph;
```

3.4 Maximum_matching

```cpp
/*bipartite - maximum matching*/
bool dfs(vector<vector<bool>> res,int node, vector<int>& x, vector<int>& y, vector<bool> pass){
    for (int i = 0; i < res[0].size(); i++){
        if(res[node][i] && !pass[i]){
            pass[i] = true;
            if(y[i] == -1 || dfs(res,y[i],x,y,pass)){
                x[node] = i;
                y[i] = node;
                return true;
            }
        }
    }
    return false;
}
int main(){
    int n,m,l;
    while(cin>>n>>m>>l){
        vector<vector<bool>> res(n, vector<bool>(m, false));
        for (int i = 0; i < l; i++){
            int a, b;
            cin >> a >> b;
            res[a][b] = true;
        }
        int ans = 0;
        vector<int> x(n, -1);
        vector<int> y(n, -1);
        for (int i = 0; i < n; i++){
            vector<bool> pass(n, false);
            if(dfs(res,i,x,y,pass))
                ans += 1;
        }
        cout << ans << endl;
    }
    return 0;
}
/*
input:
4 3 5 //n matching m, l links
0 0
0 2
1 0
2 1
```

```
3 1
answer is 3
*/
```

3.5 MFlow Model

```
typedef long long ll;
struct MF
{
    static const int N = 5000 + 5;
    static const int M = 60000 + 5;
    static const ll oo = 10000000000000LL;

    int n, m, s, t, tot, tim;
    int first[N], next[M];
    int u[M], v[M], cur[N], vi[N];
    ll cap[M], flow[M], dis[N];
    int que[N + N];

    void Clear()
    {
        tot = 0;
        tim = 0;
        for (int i = 1; i <= n; ++i)
            first[i] = -1;
    }
    void Add(int from, int to, ll cp, ll flw)
    {
        u[tot] = from;
        v[tot] = to;
        cap[tot] = cp;
        flow[tot] = flw;
        next[tot] = first[u[tot]];
        first[u[tot]] = tot;
        ++tot;
    }
    bool bfs()
    {
        ++tim;
        dis[s] = 0;
        vi[s] = tim;

        int head, tail;
        head = tail = 1;
        que[head] = s;
        while (head <= tail)
        {
            for (int i = first[que[head]]; i != -1; i = next[i])
            {
                if (vi[v[i]] != tim && cap[i] > flow[i])
                {
                    vi[v[i]] = tim;
                    dis[v[i]] = dis[que[head]] + 1;
                    que[++tail] = v[i];
                }
            }
            ++head;
        }
        return vi[t] == tim;
    }
    ll dfs(int x, ll a)
    {
        if (x == t || a == 0)
            return a;
        ll flw = 0, f;
        int &i = cur[x];
        for (i = first[x]; i != -1; i = next[i])
        {
            if (dis[x] + 1 == dis[v[i]] && (f = dfs(v[i], min(a, cap[i] - flow[i]))) > 0)
            {
                flow[i] += f;
                flow[i ^ 1] -= f;
                a -= f;
                flw += f;
                if (a == 0)
                    break;
            }
        }
        return flw;
    }
    ll MaxFlow(int s, int t)
    {
        this->s = s;
        this->t = t;
        ll flw = 0;
        while (bfs())
        {
            for (int i = 1; i <= n; ++i)
                cur[i] = 0;
            flw += dfs(s, oo);
        }
        return flw;
    }
};
// MF Net;
// Net.n = n;
// Net.Clear();
// a 到 b (注意從1開始!!!!)
// Net.Add(a, b, w, 0);
// Net.MaxFlow(s, d)
// s 到 d 的 MF
```

4 Geometry

4.1 Circle

```
bool same(double a, double b)
{
    return abs(a - b) < 0;
}
struct P
{
    double x, y;
    P() : x(0), y(0) {}
    P(double x, double y) : x(x), y(y) {}
    P operator+(P b) { return P(x + b.x, y + b.y); }
    P operator-(P b) { return P(x - b.x, y - b.y); }
    P operator*(double b) { return P(x * b, y * b); }
    P operator/(double b) { return P(x / b, y / b); }
    double operator*(P b) { return x * b.x + y * b.y; }
    // double operator^(P b) { return x * b.y - y * b.x; }
    double abs() { return hypot(x, y); }
    P unit() { return *this / abs(); }
    P rot(double o)
    {
        double c = cos(o), s = sin(o);
        return P(c * x - s * y, s * x + c * y);
    }
    double angle() { return atan2(y, x); }
};
struct C
{
    P c;
    double r;
    C(P c = P(0, 0), double r = 0) : c(c), r(r) {}
};
vector<P> Intersect(C a, C b)
{
    if (a.r > b.r)
        swap(a, b);
    double d = (a.c - b.c).abs();
    vector<P> p;
    if (same(a.r + b.r, d))
        p.pb(a.c + (b.c - a.c).unit() * a.r);
    else if (a.r + b.r > d && d + a.r >= b.r)
    {
        double o = acos((sqrt(a.r) + sqrt(d) - sqrt(b.r)) / (2 * a.r * d));
        P i = (b.c - a.c).unit();
        p.pb(a.c + i.rot(o) * a.r);
        p.pb(a.c + i.rot(-o) * a.r);
    }
    return p;
}
```

4.2 Closest Pair

```
//最近點對 (距離) //台大
vector<pair<double, double>> p;
double closest_pair(int l, int r)
{
    // p 要對 x 軸做 sort
    if (l == r)
        return 1e9;
    if (r - l == 1)
        return dist(p[l], p[r]); // 兩點距離
    int m = (l + r) >> 1;
    double d = min(closest_pair(l, m), closest_pair(m + 1, r));
    vector<int> vec;
    for (int i = m; i >= l && fabs(p[m].x - p[i].x) < d; --i)
        vec.push_back(i);
    for (int i = m + 1; i <= r && fabs(p[m].x - p[i].x) < d; ++i)
        vec.push_back(i);
    sort(vec.begin(), vec.end(), [&](int a, int b)
         { return p[a].y < p[b].y; });
    for (int i = 0; i < vec.size(); ++i)
        for (int j = i + 1; j < vec.size() && fabs(p[vec[j]].y - p[vec[i]].y) < d; ++j)
            d = min(d, dist(p[vec[i]], p[vec[j]]));
    return d;
}
```

4.3 Line

```
template <typename T>
struct line
{
    line() {}
    point<T> p1, p2;
    T a, b, c; //ax+by+c=0
    line(const point<T> &x, const point<T> &y) : p1(x), p2(y) {}
    void pton()
    { //轉成一般式
        a = p1.y - p2.y;
        b = p2.x - p1.x;
        c = -a * p1.x - b * p1.y;
    }
    T ori(const point<T> &p) const
    { //點和有向直線的關係，>0左邊、=0在線上<0右邊
        return (p2 - p1).cross(p - p1);
    }
    T btw(const point<T> &p) const
    { //點投影落在線段上<=0
        return (p1 - p).dot(p2 - p);
    }
    bool point_on_segment(const point<T> &p) const
    { //點是否在線段上
        return ori(p) == 0 && btw(p) <= 0;
    }
    T dis2(const point<T> &p, bool is_segment = 0) const
    { //點跟直線/線段的距離平方
        point<T> v = p2 - p1, v1 = p - p1;
        if (is_segment)
        {
            point<T> v2 = p - p2;
            if (v.dot(v1) <= 0)
                return v1.abs2();
            if (v.dot(v2) >= 0)
                return v2.abs2();
        }
        T tmp = v.cross(v1);
```

```cpp
        return tmp * tmp / v.abs2();
    }
    T seg_dis2(const line<T> &l) const
    { //兩線段距離平方
        return min({dis2(l.p1, 1), dis2(l.p2
            , 1), l.dis2(p1, 1), l.dis2(p2,
            1)});
    }
    point<T> projection(const point<T> &p)
         const
    { //點對直線的投影
        point<T> n = (p2 - p1).normal();
        return p - n * (p - p1).dot(n) / n.
            abs2();
    }
    point<T> mirror(const point<T> &p) const
    {
        //點對直線的鏡射，要先呼叫pton轉成一
            般式
        point<T> R;
        T d = a * a + b * b;
        R.x = (b * b * p.x - a * a * p.x - 2
              * a * b * p.y - 2 * a * c) / d;
        R.y = (a * a * p.y - b * b * p.y - 2
              * a * b * p.x - 2 * b * c) / d;
        return R;
    }
    bool equal(const line &l) const
    { //直線相等
        return ori(l.p1) == 0 && ori(l.p2)
            == 0;
    }
    bool parallel(const line &l) const
    {
        return (p1 - p2).cross(l.p1 - l.p2)
            == 0;
    }
    bool cross_seg(const line &l) const
    {
        return (p2 - p1).cross(l.p1 - p1) *
            (p2 - p1).cross(l.p2 - p1) <= 0;
             //直線是否交線段
    }
    int line_intersect(const line &l) const
    { //直線相交情況，-1無限多點、1交於一
         點、0不相交
        return parallel(l) ? (ori(l.p1) == 0
             ? -1 : 0) : 1;
    }
    int seg_intersect(const line &l) const
    {
        T c1 = ori(l.p1), c2 = ori(l.p2);
        T c3 = l.ori(p1), c4 = l.ori(p2);
        if (c1 == 0 && c2 == 0)
        { //共線
            bool b1 = btw(l.p1) >= 0, b2 =
                btw(l.p2) >= 0;
            T a3 = l.btw(p1), a4 = l.btw(p2)
                ;
            if (b1 && b2 && a3 == 0 && a4 >=
                 0)
                return 2;
            if (b1 && b2 && a3 >= 0 && a4 ==
                 0)
                return 3;
            if (b1 && b2 && a3 >= 0 && a4 >=
                 0)
                return 0;
            return -1; //無限交點
        }
        else if (c1 * c2 <= 0 && c3 * c4 <=
            0)
            return 1;
        return 0; //不相交
    }
    point<T> line_intersection(const line &l
        ) const
    { /*直線交點*/
        point<T> a = p2 - p1, b = l.p2 - l.
            p1, s = l.p1 - p1;
        //if(a.cross(b)==0)return INF;
        return p1 + a * (s.cross(b) / a.
            cross(b));
    }
    point<T> seg_intersection(const line &l)
         const
    { //線段交點
        int res = seg_intersect(l);
        if (res <= 0)
            assert(0);
        if (res == 2)
            return p1;
        if (res == 3)
            return p2;
        return line_intersection(l);
    }
};
```

4.4 Point

```cpp
const double PI = atan2(0.0, -1.0);
template <typename T>
struct point
{
    T x, y;
    point() {}
    point(const T &x, const T &y) : x(x), y(
        y) {}
    point operator+(const point &b) const
    {
        return point(x + b.x, y + b.y);
    }
    point operator-(const point &b) const
    {
        return point(x - b.x, y - b.y);
    }
    point operator*(const T &b) const
    {
        return point(x * b, y * b);
    }
    point operator/(const T &b) const
    {
        return point(x / b, y / b);
    }
    bool operator==(const point &b) const
    {
        return x == b.x && y == b.y;
    }
    T dot(const point &b) const
    {
        return x * b.x + y * b.y;
    }
    T cross(const point &b) const
    {
        return x * b.y - y * b.x;
    }
    point normal() const
    { //求法向量
        return point(-y, x);
    }
    T abs2() const
    { //向量長度的平方
        return dot(*this);
    }
    T rad(const point &b) const
    { //兩向量的弧度
        return fabs(atan2(fabs(cross(b)),
            dot(b)));
    }
    T getA() const
    {                           //對x軸的弧度
        T A = atan2(y, x); //超過180度會變負
            的
        if (A <= -PI / 2)
            A += PI * 2;
        return A;
    }
};
```

4.5 Polygon

```cpp
template <typename T>
struct polygon
{
    polygon() {}
    vector<point<T>> p; //逆時針順序
    T area() const
    { //面積
        T ans = 0;
        for (int i = p.size() - 1, j = 0; j
            < (int)p.size(); i = j++)
            ans += p[i].cross(p[j]);
        return ans / 2;
    }
    point<T> center_of_mass() const
    { //重心
        T cx = 0, cy = 0, w = 0;
        for (int i = p.size() - 1, j = 0; j
            < (int)p.size(); i = j++)
        {
            T a = p[i].cross(p[j]);
            cx += (p[i].x + p[j].x) * a;
            cy += (p[i].y + p[j].y) * a;
            w += a;
        }
        return point<T>(cx / 3 / w, cy / 3 /
            w);
    }
    char ahas(const point<T> &t) const
    { //點是否在簡單多邊形內，是的話回傳1、
         在邊上回傳-1、否則回傳0
        bool c = 0;
        for (int i = 0, j = p.size() - 1; i
             < p.size(); j = i++)
            if (line<T>(p[i], p[j]).
                point_on_segment(t))
                return -1;
            else if ((p[i].y > t.y) != (p[j
                ].y > t.y) &&
                     t.x < (p[j].x - p[i].x)
                         * (t.y - p[i].y) /
                         (p[j].y - p[i].y)
                        + p[i].x)
                c = !c;
        return c;
    }
    char point_in_convex(const point<T> &x)
         const
    {
        int l = 1, r = (int)p.size() - 2;
        while (l <= r)
        { //點是否在凸多邊形內，是的話回傳1
             、在邊上回傳-1、否則回傳0
            int mid = (l + r) / 2;
            T a1 = (p[mid] - p[0]).cross(x -
                 p[0]);
            T a2 = (p[mid + 1] - p[0]).cross
                (x - p[0]);
            if (a1 >= 0 && a2 <= 0)
            {
                T res = (p[mid + 1] - p[mid
                    ]).cross(x - p[mid]);
                return res > 0 ? 1 : (res >=
                     0 ? -1 : 0);
            }
            else if (a1 < 0)
                r = mid - 1;
            else
                l = mid + 1;
        }
        return 0;
    }
    vector<T> getA() const
    {//凸包邊對x軸的夾角
        vector<T> res; //一定是遞增的
        for (size_t i = 0; i < p.size(); ++i
            )
            res.push_back((p[(i + 1) % p.
                size()] - p[i]).getA());
        return res;
    }
    bool line_intersect(const vector<T> &A,
         const line<T> &l) const
    { //O(logN)
        int f1 = upper_bound(A.begin(), A.
            end(), (l.p1 - l.p2).getA()) - A
            .begin();
        int f2 = upper_bound(A.begin(), A.
            end(), (l.p2 - l.p1).getA()) - A
            .begin();
        return l.cross_seg(line<T>(p[f1], p[
            f2]));
    }
    polygon cut(const line<T> &l) const
    { //凸包對直線切割，得到直線l左側的凸包
        polygon ans;
        for (int n = p.size(), i = n - 1, j = 0; j < n; i = j++)
        {
            if (l.ori(p[i]) >= 0)
            {
                ans.p.push_back(p[i]);
                if (l.ori(p[j]) < 0)
                    ans.p.push_back(l.line_intersection(line<T>(p[i], p[j])));
            }
            else if (l.ori(p[j]) > 0)
                ans.p.push_back(l.line_intersection(line<T>(p[i], p[j])));
        }
        return ans;
    }
    static bool graham_cmp(const point<T> &a, const point<T> &b)
    { //凸包排序函數 // 起始點不同
        // return (a.x < b.x) || (a.x == b.x && a.y < b.y);  //最左下角開始
        return (a.y < b.y) || (a.y == b.y && a.x < b.x);  //Y最小開始
    }
    void graham(vector<point<T>> &s)
    { //凸包 Convexhull 2D
        sort(s.begin(), s.end(), graham_cmp);
        p.resize(s.size() + 1);
        int m = 0;
        // cross >= 0 順時針。cross <= 0 逆時針旋轉
        for (size_t i = 0; i < s.size(); ++i)
        {
            while (m >= 2 && (p[m - 1] - p[m - 2]).cross(s[i] - p[m - 2]) <= 0)
                --m;
            p[m++] = s[i];
        }
        for (int i = s.size() - 2, t = m + 1; i >= 0; --i)
        {
            while (m >= t && (p[m - 1] - p[m - 2]).cross(s[i] - p[m - 2]) <= 0)
                --m;
            p[m++] = s[i];
        }
        if (s.size() > 1) // 重複頭一次需扣掉
            --m;
        p.resize(m);
    }
    T diam()
    { //直徑
        int n = p.size(), t = 1;
        T ans = 0;
        p.push_back(p[0]);
        for (int i = 0; i < n; i++)
        {
            point<T> now = p[i + 1] - p[i];
            while (now.cross(p[t + 1] - p[i]) > now.cross(p[t] - p[i]))
                t = (t + 1) % n;
            ans = max(ans, (p[i] - p[t]).abs2());
        }
        return p.pop_back(), ans;
    }
    T min_cover_rectangle()
    { //最小覆蓋矩形
        int n = p.size(), t = 1, r = 1, l;
        if (n < 3)
            return 0; //也可以做最小周長矩形
        T ans = 1e99;
        p.push_back(p[0]);
        for (int i = 0; i < n; i++)
        {
            point<T> now = p[i + 1] - p[i];
            while (now.cross(p[t + 1] - p[i]) > now.cross(p[t] - p[i]))
                t = (t + 1) % n;
            while (now.dot(p[r + 1] - p[i]) > now.dot(p[r] - p[i]))
                r = (r + 1) % n;
            if (!i)
                l = r;
            while (now.dot(p[l + 1] - p[i]) <= now.dot(p[l] - p[i]))
                l = (l + 1) % n;
            T d = now.abs2();
            T tmp = now.cross(p[t] - p[i]) * (now.dot(p[r] - p[i]) - now.dot(p[l] - p[i])) / d;
            ans = min(ans, tmp);
        }
        return p.pop_back(), ans;
    }
    T dis2(polygon &pl)
    { //凸包最近距離平方
        vector<point<T>> &P = p, &Q = pl.p;
        int n = P.size(), m = Q.size(), l = 0, r = 0;
        for (int i = 0; i < n; ++i)
            if (P[i].y < P[l].y)
                l = i;
        for (int i = 0; i < m; ++i)
            if (Q[i].y < Q[r].y)
                r = i;
        P.push_back(P[0]), Q.push_back(Q[0]);
        T ans = 1e99;
        for (int i = 0; i < n; ++i)
        {
            while ((P[l] - P[l + 1]).cross(Q[r + 1] - Q[r]) < 0)
                r = (r + 1) % m;
            ans = min(ans, line<T>(P[l], P[l + 1]).seg_dis2(line<T>(Q[r], Q[r + 1])));
            l = (l + 1) % n;
        }
        return P.pop_back(), Q.pop_back(), ans;
    }
    static char sign(const point<T> &t)
    {
        return (t.y == 0 ? t.x : t.y) < 0;
    }
    static bool angle_cmp(const line<T> &A, const line<T> &B)
    {
        point<T> a = A.p2 - A.p1, b = B.p2 - B.p1;
        return sign(a) < sign(b) || (sign(a) == sign(b) && a.cross(b) > 0);
    }
    int halfplane_intersection(vector<line<T>> &s)
    {  //半平面交
        sort(s.begin(), s.end(), angle_cmp); //線段左側為該線段半平面
        int L, R, n = s.size();
        vector<point<T>> px(n);
        vector<line<T>> q(n);
        q[L = R = 0] = s[0];
        for (int i = 1; i < n; ++i)
        {
            while (L < R && s[i].ori(px[R - 1]) <= 0)
                --R;
            while (L < R && s[i].ori(px[L]) <= 0)
                ++L;
            q[++R] = s[i];
            if (q[R].parallel(q[R - 1]))
            {
                --R;
                if (q[R].ori(s[i].p1) > 0)
                    q[R] = s[i];
            }
            if (L < R)
                px[R - 1] = q[R - 1].line_intersection(q[R]);
        }
        while (L < R && q[L].ori(px[R - 1]) <= 0)
            --R;
        p.clear();
        if (R - L <= 1)
            return 0;
        px[R] = q[R].line_intersection(q[L]);
        for (int i = L; i <= R; ++i)
            p.push_back(px[i]);
        return R - L + 1;
    }
};
```

4.6 Triangle

```cpp
template <typename T>
struct triangle
{
    point<T> a, b, c;
    triangle() {}
    triangle(const point<T> &a, const point<T> &b, const point<T> &c) : a(a), b(b), c(c) {}
    T area() const
    {
        T t = (b - a).cross(c - a) / 2;
        return t > 0 ? t : -t;
    }
    point<T> barycenter() const
    { //重心
        return (a + b + c) / 3;
    }
    point<T> circumcenter() const
    { //外心
        static line<T> u, v;
        u.p1 = (a + b) / 2;
        u.p2 = point<T>(u.p1.x - a.y + b.y, u.p1.y + a.x - b.x);
        v.p1 = (a + c) / 2;
        v.p2 = point<T>(v.p1.x - a.y + c.y, v.p1.y + a.x - c.x);
        return u.line_intersection(v);
    }
    point<T> incenter() const
    { //內心
        T A = sqrt((b - c).abs2()), B = sqrt((a - c).abs2()), C = sqrt((a - b).abs2());
        return point<T>(A * a.x + B * b.x + C * c.x, A * a.y + B * b.y + C * c.y) / (A + B + C);
    }
    point<T> perpencenter() const
    { //垂心
        return barycenter() * 3 - circumcenter() * 2;
    }
};
```

5 Graph

5.1 Bellman-Ford

```cpp
/*SPA - Bellman-Ford*/
#define inf 99999 //define by you maximum edges weight
vector<vector<int> > edges;
vector<int> dist;
vector<int> ancestor;
void BellmanFord(int start,int node){
    dist[start] = 0;
    for(int it = 0; it < node-1; it++){
        for(int i = 0; i < node; i++){
            for(int j = 0; j < node; j++){
                if(edges[i][j] != -1){
                    if(dist[i] + edges[i][j] < dist[j]){
                        dist[j] = dist[i] + edges[i][j];
                        ancestor[j] = i;
                    }
                }
            }
        }
    }

    for(int i = 0; i < node; i++)  // negative cycle detection
        for(int j = 0; j < node; j++)
            if(dist[i] + edges[i][j] < dist[j])
            {
                cout<<"Negative cycle!"<<endl;
                return;
            }
}
int main(){
    int node;
    cin>>node;
    edges.resize(node,vector<int>(node,inf));
    dist.resize(node,inf);
    ancestor.resize(node,-1);
    int a,b,d;
    while(cin>>a>>b>>d){
        /*input: source destination weight*/
        if(a == -1 && b == -1 && d == -1)
            break;
        edges[a][b] = d;
    }
    int start;
    cin>>start;
    BellmanFord(start,node);
    return 0;
}
```

5.2 BFS-queue

```
/*BFS - queue version*/
void BFS(vector<int> &result, vector<pair<int, int>> edges, int node, int start)
{
    vector<int> pass(node, 0);
    queue<int> q;
    queue<int> p;
    q.push(start);
    int count = 1;
    vector<pair<int, int>> newedges;
    while (!q.empty())
    {
        pass[q.front()] = 1;
        for (int i = 0; i < edges.size(); i++)
        {
            if (edges[i].first == q.front() && pass[edges[i].second] == 0)
            {
                p.push(edges[i].second);
                result[edges[i].second] = count;
            }
            else if (edges[i].second == q.front() && pass[edges[i].first] == 0)
            {
                p.push(edges[i].first);
                result[edges[i].first] = count;
            }
            else
                newedges.push_back(edges[i]);
        }
        edges = newedges;
        newedges.clear();
        q.pop();
        if (q.empty())
        {
            q = p;
            queue<int> tmp;
            p = tmp;
            count++;
        }
    }
}
int main()
{
    int node;
    cin >> node;
    vector<pair<int, int>> edges;
    int a, b;
    while (cin >> a >> b)
    {
        /*a = b = -1 means input edges ended*/
        if (a == -1 && b == -1)
            break;
        edges.push_back(pair<int, int>(a, b));
    }
    vector<int> result(node, -1);
    BFS(result, edges, node, 0);

    return 0;
}
```

5.3 DFS-rec

```
/*DFS - Recursive version*/
map<pair<int,int>,int> edges;
vector<int> pass;
vector<int> route;
void DFS(int start){
    pass[start] = 1;
    map<pair<int,int>,int>::iterator iter;
    for(iter = edges.begin(); iter != edges.end(); iter++){
        if((*iter).first.first == start && (*iter).second == 0 && pass[(*iter).first.second] == 0){
            route.push_back((*iter).first.second);
            DFS((*iter).first.second);
        }
        else if((*iter).first.second == start && (*iter).second == 0 && pass[(*iter).first.first] == 0){
            route.push_back((*iter).first.first);
            DFS((*iter).first.first);
        }
    }
}
int main(){
    int node;
    cin>>node;
    pass.resize(node,0);
    int a,b;
    while(cin>>a>>b){
        if(a == -1 && b == -1)
            break;
        edges.insert(pair<pair<int,int>,int>(pair<int,int>(a,b),0));
    }
    int start;
    cin>>start;
    route.push_back(start);
    DFS(start);
    return 0;
}
```

5.4 Dijkstra

```
/*SPA - Dijkstra*/
const int MAXN = 1e5 + 3;
const int inf = INT_MAX;
typedef pair<int, int> pii;
vector<vector<pii>> weight;
vector<int> isDone(MAXN, false), dist, ancestor;
void dijkstra(int s)
{
    priority_queue<pii, vector<pii>, greater<pii>> pq;
    pq.push(pii(0, s));
    ancestor[s] = -1;
    while (!pq.empty())
    {
        int u = pq.top().second;
        pq.pop();

        isDone[u] = true;

        for (auto &pr : weight[u])
        {
            int v = pr.first, w = pr.second;

            if (!isDone[v] && dist[u] + w < dist[v])
            {
                dist[v] = dist[u] + w;
                pq.push(pii(dist[v], v));
                ancestor[v] = u;
            }
        }
    }
}
// weight[a - 1].push_back(pii(b - 1, w));
// weight[b - 1].push_back(pii(a - 1, w));
// dist.resize(n, inf);
// ancestor.resize(n, -1);
// dist[0] = 0;
// dijkstra(0);
```

5.5 Euler circuit

```
/*Euler circuit*/
/*From NTU kiseki*/
/*G is graph, vis is visited, la is path*/
bool vis[N];
size_t la[K];
void dfs(int u, vector<int> &vec)
{
    while (la[u] < G[u].size())
    {
        if (vis[G[u][la[u]].second])
        {
            ++la[u];
            continue;
        }
        int v = G[u][la[u]].first;
        vis[G[u][la[u]].second] = true;
        ++la[u];
        dfs(v, vec);
        vec.push_back(v);
    }
}
```

5.6 Floyd-warshall

```
/*SPA - Floyd-Warshall*/
// 有向圖，正邊    O(V³)
// 有向圖，無負環  O(V³)
// 有向圖，有負環  不適用

// 無向圖，正邊    O(V³)
// 無向圖，無負環  不適用
// 無向圖，有負環  不適用
/*Find min weight cycle*/
#define inf 99999
void floyd_warshall(vector<vector<int>> &distance, vector<vector<int>> &ancestor, int n)
{
    for (int k = 0; k < n; k++)
    {
        for (int i = 0; i < n; i++)
        {
            for (int j = 0; j < n; j++)
            {
                if (distance[i][k] + distance[k][j] < distance[i][j])
                {
                    distance[i][j] = distance[i][k] + distance[k][j];
                    ancestor[i][j] = ancestor[k][j];
                }
            }
        }
    }
}

vector<vector<int>> distance(n, vector<int>(n, inf));
vector<vector<int>> ancestor(n, vector<int>(n, -1));
distance[a][b] = w;
ancestor[a][b] = w;
floyd_warshall(distance, ancestor, n);
/*Negative cycle detection*/
for (int i = 0; i < n; i++)
{
    if (distance[i][i] < 0)
    {
        cout << "Negative cycle!" << endl;
        break;
    }
}
```

5.7 Hamilton_cycle

```
/*find hamilton cycle*/
void hamilton(vector<vector<int>> gp, int k, int cur, vector<int>& solution,vector<bool> pass,bool& flag){
    if(k == gp.size()-1){
        if(gp[cur][1] == 1){
            cout << 1 << " ";
            while(cur != 1){
                cout << cur << " ";
                cur = solution[cur];
            }
            cout << cur << endl;
            flag = true;
            return;
        }
    }
    for (int i = 0; i < gp[cur].size() && !flag; i++){
        if(gp[cur][i] == 1 && !pass[i]){
            pass[i] = true;
            solution[i] = cur;
            hamilton(gp, k + 1, i, solution, pass,flag);
            pass[i] = false;
        }
    }
}
int main(){
    int n;
    while(cin>>n){
        int a,b;
        bool end = false;
        vector<vector<int>> gp(n+1,vector<int>(n+1,0));
        while(cin>>a>>b){
            if(a == 0 && b == 0)
                break;
            gp[a][b] = 1;
            gp[b][a] = 1;
        }
        vector<int> solution(n + 1, -1);
        vector<bool> pass(n + 1, false);
        solution[1] = 0;
        pass[1] = true;
        bool flag = false;
        hamilton(gp, 1,1 ,solution,pass,flag);
        if(!flag)
            cout << "N" << endl;
    }
    return 0;
}
/*
4
1 2
2 3
2 4
3 4
3 1
0 0
output: 1 3 4 2 1
*/
```

5.8 Kruskal

```
/*mst - Kruskal*/
struct edges{
    int from;
    int to;
    int weight;
    friend bool operator < (edges a, edges b){
        return a.weight > b.weight;
    }
};
int find(int x,vector<int>& union_set){
    if(x != union_set[x])
        union_set[x] = find(union_set[x], union_set);
    return union_set[x];
}
void merge(int a,int b,vector<int>& union_set){
    int pa = find(a, union_set);
    int pb = find(b, union_set);
    if(pa != pb)
        union_set[pa] = pb;
}
void kruskal(priority_queue<edges> pq,int n){
    vector<int> union_set(n, 0);
    for (int i = 0; i < n; i++)
        union_set[i] = i;
    int edge = 0;
    int cost = 0; //evaluate cost of mst
    while(!pq.empty() && edge < n - 1){
        edges cur = pq.top();
        int from = find(cur.from, union_set);
        int to = find(cur.to, union_set);
        if(from != to){
            merge(from, to, union_set);
            edge += 1;
            cost += cur.weight;
        }
        pq.pop();
    }
    if(edge < n-1)
        cout << "No mst" << endl;
    else
        cout << cost << endl;
}
int main(){
    int n;
    cin >> n;
    int a, b, d;
    priority_queue<edges> pq;
    while(cin>>a>>b>>d){
        if(a == -1 && b == -1 && d == -1)
            break;
        edges tmp;
        tmp.from = a;
        tmp.to = b;
        tmp.weight = d;
        pq.push(tmp);
    }
    kruskal(pq, n);
    return 0;
}
```

5.9 Minimum Weight Cycle

```
// 最小環
// 圖上無負環 !!!!
#define INF 99999
vector<vector<int>> w, d, p;
vector<int> cycle;
int c = 0;
void trace(int i, int j)
{
    cycle[c++] = i;
    if (i != j)
        trace(p[i][j], j);
}
void init(int n)
{
    for (int i = 0; i < n; ++i)
        d[i][i] = 0;
}
void minimum_cycle(int n)
{
    int weight = 1e9;
    for (int k = 0; k < n; ++k)
    {
        for (int i = 0; i < k; ++i)
            for (int j = 0; j < k; ++j)
                if (i != j)
                    if (w[k][i] + d[i][j] + w[j][k] < weight)
                    {
                        weight = w[k][i] + d[i][j] + w[j][k];
                        c = 0;
                        trace(i, j);
                        cycle[c++] = k;
                    }

        for (int i = 0; i < n; ++i)
        {
            for (int j = 0; j < n; ++j)
            {
                if (d[i][k] + d[k][j] < d[i][j])
                {
                    d[i][j] = d[i][k] + d[k][j];
                    p[i][j] = p[i][k];
                }
            }
        }
    }
    if (weight == 1e9)
        cout << "No exist";
    else
    {
        bug(weight);
        bug(c);
        bugarr(cycle);
    }
}
void simple_minimum_cycle(int n) // No use vector p
{
    int weight = INF;
    for (int k = 0; k < n; ++k)
    {
        for (int i = 0; i < k; ++i)
            for (int j = 0; j < k; ++j)
                if (i != j)
                    weight = min(mp[k][i] + d[i][j] + mp[j][k], weight);

        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                d[i][j] = min(d[i][k] + d[k][j], d[i][j]);
    }
    if (weight == INF)
        cout << "Back to jail\n";
    else
        cout << weight << endl;
}
w.resize(n, vector<int>(n, INF));
d.resize(n, vector<int>(n, INF));
p.resize(n, vector<int>(n));
cycle.resize(n);
//Edge input
w[a][b] = w;
```

```cpp
d[a][b] = w;
p[a][b] = b;
init(n);
minimum_cycle(n);
```

5.10 Prim

```cpp
/*mst - Prim*/
#define inf 99999
struct edges
{
    int from;
    int to;
    int weight;
    friend bool operator<(edges a, edges b)
    {
        return a.weight > b.weight;
    }
};
void Prim(vector<vector<int>> gp, int n, int start)
{
    vector<bool> pass(n, false);
    int edge = 0;
    int cost = 0; //evaluate cost of mst
    priority_queue<edges> pq;
    for (int i = 0; i < n; i++)
    {
        if (gp[start][i] != inf)
        {
            edges tmp;
            tmp.from = start;
            tmp.to = i;
            tmp.weight = gp[start][i];
            pq.push(tmp);
        }
    }
    pass[start] = true;
    while (!pq.empty() && edge < n - 1)
    {
        edges cur = pq.top();
        pq.pop();
        if (!pass[cur.to])
        {
            for (int i = 0; i < n; i++)
            {
                if (gp[cur.to][i] != inf)
                {
                    edges tmp;
                    tmp.from = cur.to;
                    tmp.to = i;
                    tmp.weight = gp[cur.to][i];
                    pq.push(tmp);
                }
            }
            pass[cur.to] = true;
            edge += 1;
            cost += cur.weight;
        }
    }
    if (edge < n - 1)
        cout << "No mst" << endl;
    else
        cout << cost << endl;
}
int main()
{
    int n;
    cin >> n;
    int a, b, d;
    vector<vector<int>> gp(n, vector<int>(n, inf));
    while (cin >> a >> b >> d)
    {
        if (a == -1 && b == -1 && d == -1)
            break;
        if (gp[a][b] > d)
            gp[a][b] = d;
    }
    Prim(gp, n, 0);
    return 0;
}
```

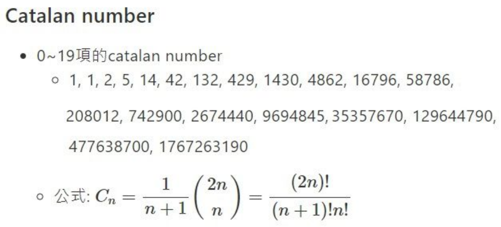
5.11 Union_find

```cpp
// union_find from 台大
vector<int> father;
vector<int> people;
void init(int n)
{
    for (int i = 0; i < n; i++)
    {
        father[i] = i;
        people[i] = 1;
    }
}
int Find(int x)
{
    if (x != father[x])
        father[x] = Find(father[x]);
    return father[x];
}

void Union(int x, int y)
{
    int m = Find(x);
    int n = Find(y);
    if (m != n)
    {
        father[n] = m;
        people[m] += people[n];
    }
}
```

6 Mathematics

6.1 Catalan



Catalan number

- 0~19項的catalan number
    - 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190
    - 公式: $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$

6.2 Combination

```cpp
/*input type string or vector*/
for (int i = 0; i < (1 << input.size()); ++i)
{
    string testCase = "";
    for (int j = 0; j < input.size(); ++j)
        if (i & (1 << j))
            testCase += input[j];
}
```

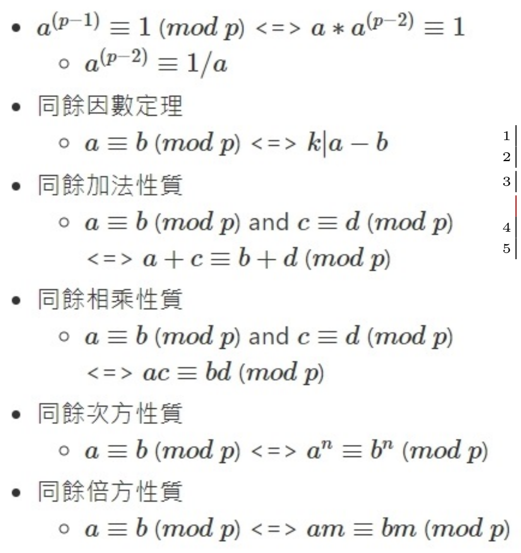
6.3 Extended Euclidean

```cpp
// ax + by = gcd(a,b)
pair<long long, long long> extgcd(long long a, long long b)
{
    if (b == 0)
        return {1, 0};
    long long k = a / b;
    pair<long long, long long> p = extgcd(b, a - k * b);
    //cout << p.first << " " << p.second << endl;
    //cout << "商數(k)=  " << k << endl << endl;
    return {p.second, p.first - k * p.second};
}

int main()
{
    int a, b;
    cin >> a >> b;
    pair<long long, long long> xy = extgcd(a, b); //(x0,y0)
    cout << xy.first << " " << xy.second << endl;
    cout << xy.first << " * " << a << " + " << xy.second << " * " << b << endl;
    return 0;
}
// ax + by = gcd(a,b) * r
/*find |x|+|y| -> min*/
int main()
{
    long long r, p, q; /*px+qy = r*/
    int cases;
    cin >> cases;
    while (cases--)
    {
        cin >> r >> p >> q;
        pair<long long, long long> xy = extgcd(q, p); //(x0,y0)
        long long ans = 0, tmp = 0;
        double k, k1;
        long long s, s1;
        k = 1 - (double)(r * xy.first) / p;
        s = round(k);
        ans = llabs(r * xy.first + s * p) + llabs(r * xy.second - s * q);
        k1 = -(double)(r * xy.first) / p;
        s1 = round(k1);
        /*cout << k << endl << k1 << endl;
            cout << s << endl << s1 << endl; */
        tmp = llabs(r * xy.first + s1 * p) + llabs(r * xy.second - s1 * q);
        ans = min(ans, tmp);

        cout << ans << endl;
    }
    return 0;
}
```

6.4 Fermat



- $a^{(p-1)} \equiv 1\,(mod\;p)$ <=> $a * a^{(p-2)} \equiv 1$
  - $a^{(p-2)} \equiv 1/a$
- 同餘因數定理
  - $a \equiv b\,(mod\;p)$ <=> $k|a-b$
- 同餘加法性質
  - $a \equiv b\,(mod\;p)$ and $c \equiv d\,(mod\;p)$ <=> $a+c \equiv b+d\,(mod\;p)$
- 同餘相乘性質
  - $a \equiv b\,(mod\;p)$ and $c \equiv d\,(mod\;p)$ <=> $ac \equiv bd\,(mod\;p)$
- 同餘次方性質
  - $a \equiv b\,(mod\;p)$ <=> $a^n \equiv b^n\,(mod\;p)$
- 同餘倍方性質
  - $a \equiv b\,(mod\;p)$ <=> $am \equiv bm\,(mod\;p)$

6.5 Hex to Dec

```cpp
int HextoDec(string num) //16 to 10
{
    int base = 1;
    int temp = 0;
    for (int i = num.length() - 1; i >= 0; i--)
    {
        if (num[i] >= '0' && num[i] <= '9')
        {
            temp += (num[i] - 48) * base;
            base = base * 16;
        }
        else if (num[i] >= 'A' && num[i] <= 'F')
        {
            temp += (num[i] - 55) * base;
            base = base * 16;
        }
    }
    return temp;
}
void DecToHex(int p) //10 to 16
{
    char *l = new (char);
    sprintf(l, "%X", p);
    //int l_intResult = stoi(l);
    cout << l << "\n";
    //return l_intResult;
}
```

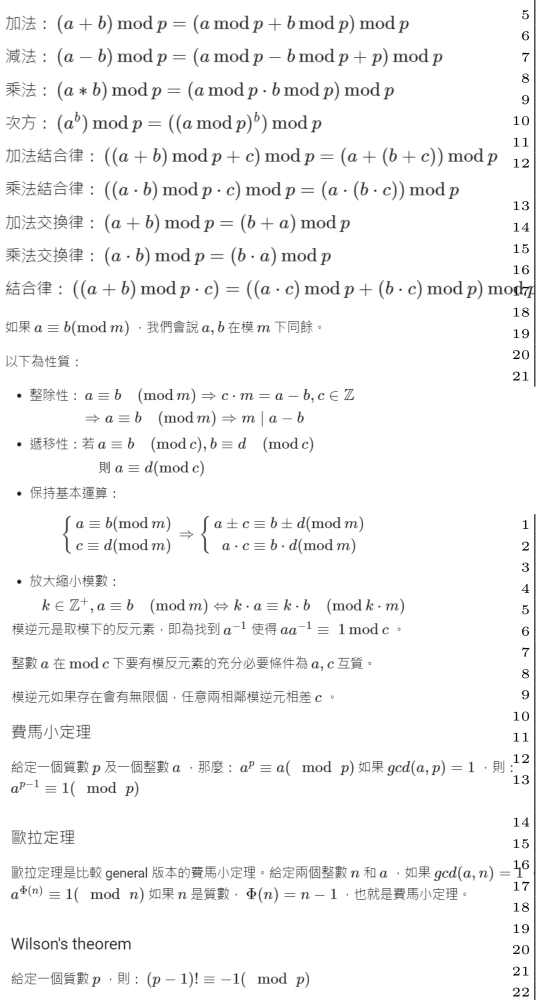
6.6 Log

```cpp
double mylog(double a, double base)
{
    //a 的對數底數 b = 自然對數 (a) / 自然對數 (b)。
    return log(a) / log(base);
}
```

6.7 Mod

```cpp
int pow_mod(int a, int n, int m) // a ^ n mod m;
{                                // a, n, m < 10 ^ 9
    if (n == 0)
        return 1;
    int x = pow_mid(a, n / 2, m);
    long long ans = (long long)x * x % m;
    if (n % 2 == 1)
        ans = ans * a % m;
    return (int)ans;
}
int inv(int a, int n, int p) // n = p-2
{
    long long res = 1;
    for (; n; n >>= 1, (a *= a) %= p)
        if (n & 1)
            (res *= a) %= p;
    return res;
}
```

6.8 Mod 性質



加法： $(a+b) \bmod p = (a \bmod p + b \bmod p) \bmod p$

減法： $(a-b) \bmod p = (a \bmod p - b \bmod p + p) \bmod p$

乘法： $(a*b) \bmod p = (a \bmod p \cdot b \bmod p) \bmod p$

次方： $(a^b) \bmod p = ((a \bmod p)^b) \bmod p$

加法結合律： $((a+b) \bmod p + c) \bmod p = (a + (b+c)) \bmod p$

乘法結合律： $((a \cdot b) \bmod p \cdot c) \bmod p = (a \cdot (b \cdot c)) \bmod p$

加法交換律： $(a+b) \bmod p = (b+a) \bmod p$

乘法交換律： $(a \cdot b) \bmod p = (b \cdot a) \bmod p$

結合律： $((a+b) \bmod p \cdot c) = ((a \cdot c) \bmod p + (b \cdot c) \bmod p) \bmod p$

如果 $a \equiv b (\bmod m)$，我們會說 a, b 在模 m 下同餘。

以下為性質：

- 整除性： $a \equiv b \pmod m \Rightarrow c \cdot m = a - b, c \in \mathbb{Z}$
  $\Rightarrow a \equiv b \pmod m \Rightarrow m \mid a - b$
- 遞移性：若 $a \equiv b \pmod c, b \equiv d \pmod c$
  則 $a \equiv d (\bmod c)$
- 保持基本運算：

$$\begin{cases} a \equiv b(\bmod m) \\ c \equiv d(\bmod m) \end{cases} \Rightarrow \begin{cases} a \pm c \equiv b \pm d(\bmod m) \\ a \cdot c \equiv b \cdot d(\bmod m) \end{cases}$$

- 放大縮小模數：
  $k \in \mathbb{Z}^+, a \equiv b \pmod m \Leftrightarrow k \cdot a \equiv k \cdot b \pmod{k \cdot m}$

模逆元是取模下的反元素，即為找到 a^{-1} 使得 $aa^{-1} \equiv\ 1 \bmod c$。

整數 a 在 $\bmod c$ 下要有模反元素的充分必要條件為 a, c 互質。

模逆元如果存在會有無限個，任意兩相鄰模逆元相差 c。

費馬小定理

給定一個質數 p 及一個整數 a，那麼： $a^p \equiv a(\mod p)$ 如果 $gcd(a,p) = 1$，則： $a^{p-1} \equiv 1(\mod p)$

歐拉定理

歐拉定理是比較 general 版本的費馬小定理。給定兩個整數 n 和 a，如果 $gcd(a,n) = 1$ $a^{\Phi(n)} \equiv 1(\mod n)$ 如果 n 是質數， $\Phi(n) = n - 1$，也就是費馬小定理。

Wilson's theorem

給定一個質數 p，則： $(p-1)! \equiv -1(\mod p)$

6.9 PI

```cpp
#define PI acos(-1)
#define PI M_PI
```

6.10 Prime table

```cpp
const int maxn = sqrt(INT_MAX);
vector<int> p;
bitset<maxn> is_notp;
void PrimeTable()
{
    is_notp.reset();
    is_notp[0] = is_notp[1] = 1;
    for (int i = 2; i <= maxn; ++i)
    {
        if (!is_notp[i])
            p.push_back(i);
        for (int j = 0; j < (int)p.size(); ++j)
        {
            if (i * p[j] > maxn)
                break;
            is_notp[i * p[j]] = 1;
            if (i % p[j] == 0)
                break;
        }
    }
}
```

6.11 Prime 判斷

```cpp
typedef long long ll;
ll modmul(ll a, ll b, ll mod)
{
    ll ret = 0;
    for (; b; b >>= 1, a = (a + a) % mod)
        if (b & 1)
            ret = (ret + a) % mod;
    return ret;
}
ll qpow(ll x, ll u, ll mod)
{
    ll ret = 1ll;
    for (; u; u >>= 1, x = modmul(x, x, mod))
        if (u & 1)
            ret = modmul(ret, x, mod);
    return ret;
}
ll gcd(ll a, ll b)
{
    return b ? gcd(b, a % b) : a;
}
ll Pollard_Rho(ll n, ll c)
{
    ll i = 1, j = 2, x = rand() % (n - 1) + 1, y = x;
    while (1)
    {
        i++;
        x = (modmul(x, x, n) + c) % n;
        ll p = gcd((y - x + n) % n, n);
        if (p != 1 && p != n)
            return p;
        if (y == x)
            return n;
        if (i == j)
        {
            y = x;
            j <<= 1;
        }
    }
}
bool Miller_Rabin(ll n)
{
    ll x, pre, u = n - 1;
    int i, j, k = 0;
    if (n == 2 || n == 3 || n == 5 || n == 7 || n == 11)
        return 1;
    if (n == 1 || !(n % 2) || !(n % 3) || !(n % 5) || !(n % 7) || !(n % 11))
        return 0;
    while (!(u & 1))
    {
        k++;
        u >>= 1;
    }
    srand((long long)12234336);
    for (i = 1; i <= 50; i++)
    {
        x = rand() % (n - 2) + 2;
        if (!(n % x))
            return 0;
        x = qpow(x, u, n);
        pre = x;
        for (j = 1; j <= k; j++)
        {
            x = modmul(x, x, n);
            if (x == 1 && pre != 1 && pre != n - 1)
                return 0;
            pre = x;
        }
        if (x != 1)
            return 0;
    }
    return 1;
}
// if (Miller_Rabin(n)) puts("Prime");
```

6.12 Round(小數)

```cpp
double myround(double number, unsigned int bits)
{
    LL integerPart = number;
    number -= integerPart;
    for (unsigned int i = 0; i < bits; ++i)
        number *= 10;
    number = (LL)(number + 0.5);
    for (unsigned int i = 0; i < bits; ++i)
        number /= 10;
    return integerPart + number;
}
//printf("%.1f\n", round(3.4515239, 1));
```

6.13 二分逼近法

```cpp
#define eps 1e-14
void half_interval()
{
    double L = 0, R = /*區間*/, M;
    while (R - L >= eps)
    {
        M = (R + L) / 2;
        if (/*函數*/ > /*方程式目標*/)
            L = M;
        else
            R = M;
    }
    printf("%.3lf\n", R);
}
```

6.14 公式

$$S_n = \frac{a(1-r^n)}{1-r} \qquad a_n = \frac{a_1 + a_n}{2} \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

6.15 四則運算

```cpp
string s = ""; //開頭是負號要補0
long long int DFS(int le, int ri) // (0, string final index)
{
    int c = 0;
    for (int i = ri; i >= le; i--)
    {
        if (s[i] == ')')
            c++;
        if (s[i] == '(')
            c--;
        if (s[i] == '+' && c == 0)
            return DFS(le, i - 1) + DFS(i + 1, ri);
        if (s[i] == '-' && c == 0)
            return DFS(le, i - 1) - DFS(i + 1, ri);
    }
    for (int i = ri; i >= le; i--)
    {
        if (s[i] == ')')
            c++;
        if (s[i] == '(')
            c--;
        if (s[i] == '*' && c == 0)
            return DFS(le, i - 1) * DFS(i + 1, ri);
        if (s[i] == '/' && c == 0)
            return DFS(le, i - 1) / DFS(i + 1, ri);
        if (s[i] == '%' && c == 0)
            return DFS(le, i - 1) % DFS(i + 1, ri);
    }
    if ((s[le] == '(') && (s[ri] == ')'))
        return DFS(le + 1, ri - 1); //去除刮號
    if (s[le] == ' ' && s[ri] == ' ')
        return DFS(le + 1, ri - 1); //去除左右兩邊空格
    if (s[le] == ' ')
        return DFS(le + 1, ri); //去除左邊空格
    if (s[ri] == ' ')
        return DFS(le, ri - 1); //去除右邊空格
    long long int num = 0;
    for (int i = le; i <= ri; i++)
        num = num * 10 + s[i] - '0';
    return num;
}
```

6.16 因數表

```cpp
vector<vector<int>> arr(10000000);
const int limit = 10e7;
for (int i = 1; i <= limit; i++)
{
    for (int j = i; j <= limit; j += i)
        arr[j].pb(i); // i 為因數
}
```

6.17 數字乘法組合

```cpp
void dfs(int j, int old, int num, vector<int> com, vector<vector<int>> &ans)
{
    for (int i = j; i <= sqrt(num); i++)
    {
        if (old == num)
            com.clear();
        if (num % i == 0)
        {
            vector<int> a;
            a = com;
            a.push_back(i);
            finds(i, old, num / i, a, ans);
            a.push_back(num / i);
            ans.push_back(a);
        }
    }
}
vector<vector<int>> ans;
vector<int> zero;
dfs(2, num, num, zero, ans);
/*/num 為 input 數字*/
for (int i = 0; i < ans.size(); i++)
{
    for (int j = 0; j < ans[i].size() - 1; j++)
        cout << ans[i][j] << " ";
    cout << ans[i][ans[i].size() - 1] << endl;
}
```

6.18 數字加法組合

```cpp
void recur(int i, int n, int m, vector<int> &out, vector<vector<int>> &ans)
{
    if (n == 0)
    {
        for (int i : out)
            if (i > m)
                return;
        ans.push_back(out);
    }
    for (int j = i; j <= n; j++)
    {
        out.push_back(j);
        recur(j, n - j, m, out, ans);
        out.pop_back();
    }
}
vector<vector<int>> ans;
vector<int> zero;
recur(1, num, num, zero, ans);
// num 為 input 數字
for (int i = 0; i < ans.size(); i++)
{
    for (int j = 0; j < ans[i].size() - 1; j++)
        cout << ans[i][j] << " ";
    cout << ans[i][ans[i].size() - 1] << endl;
}
```

6.19 羅馬數字

```cpp
int romanToInt(string s)
{
    unordered_map<char, int> T;
    T['I'] = 1;
    T['V'] = 5;
    T['X'] = 10;
    T['L'] = 50;
    T['C'] = 100;
    T['D'] = 500;
    T['M'] = 1000;

    int sum = T[s.back()];
    for (int i = s.length() - 2; i >= 0; --i)
    {
        if (T[s[i]] < T[s[i + 1]])
            sum -= T[s[i]];
        else
            sum += T[s[i]];
    }
    return sum;
}
```

6.20 質因數分解

```cpp
LL ans;
void find(LL n, LL c) // 配合質數判斷
{
    if (n == 1)
        return;
    if (Miller_Rabin(n))
    {
        ans = min(ans, n);
        // bug(ans); //質因數
        return;
    }
    LL x = n, k = c;
    while (x == n)
        x = Pollard_Rho(x, c--);
    find(n / x, k);
    find(x, k);
}
```

6.21 質數數量

```cpp
// 10 ^ 11 左右
#define LL long long
const int N = 5e6 + 2;
bool np[N];
int prime[N], pi[N];
int getprime()
{
    int cnt = 0;
    np[0] = np[1] = true;
    pi[0] = pi[1] = 0;
    for (int i = 2; i < N; ++i)
    {
        if (!np[i])
            prime[++cnt] = i;
        pi[i] = cnt;
        for (int j = 1; j <= cnt && i * prime[j] < N; ++j)
        {
            np[i * prime[j]] = true;
            if (i % prime[j] == 0)
                break;
        }
    }
    return cnt;
}
const int M = 7;
const int PM = 2 * 3 * 5 * 7 * 11 * 13 * 17;
int phi[PM + 1][M + 1], sz[M + 1];
void init()
{
    getprime();
    sz[0] = 1;
    for (int i = 0; i <= PM; ++i)
        phi[i][0] = i;
    for (int i = 1; i <= M; ++i)
    {
        sz[i] = prime[i] * sz[i - 1];
        for (int j = 1; j <= PM; ++j)
            phi[j][i] = phi[j][i - 1] - phi[j / prime[i]][i - 1];
    }
}
int sqrt2(LL x)
{
    LL r = (LL)sqrt(x - 0.1);
    while (r * r <= x)
        ++r;
    return int(r - 1);
}
int sqrt3(LL x)
{
    LL r = (LL)cbrt(x - 0.1);
    while (r * r * r <= x)
        ++r;
    return int(r - 1);
}
LL getphi(LL x, int s)
{
    if (s == 0)
        return x;
    if (s <= M)
        return phi[x % sz[s]][s] + (x / sz[s]) * phi[sz[s]][s];
    if (x <= prime[s] * prime[s])
        return pi[x] - s + 1;
    if (x <= prime[s] * prime[s] * prime[s] && x < N)
    {
        int s2x = pi[sqrt2(x)];
        LL ans = pi[x] - (s2x + s - 2) * (s2x - s + 1) / 2;
        for (int i = s + 1; i <= s2x; ++i)
            ans += pi[x / prime[i]];
        return ans;
    }
    return getphi(x, s - 1) - getphi(x / prime[s], s - 1);
}
LL getpi(LL x)
{
    if (x < N)
        return pi[x];
    LL ans = getphi(x, pi[sqrt3(x)]) + pi[sqrt3(x)] - 1;
    for (int i = pi[sqrt3(x)] + 1, ed = pi[sqrt2(x)]; i <= ed; ++i)
        ans -= getpi(x / prime[i]) - i + 1;
    return ans;
}
LL lehmer_pi(LL x)
{
    if (x < N)
        return pi[x];
    int a = (int)lehmer_pi(sqrt2(sqrt2(x)));
    int b = (int)lehmer_pi(sqrt2(x));
    int c = (int)lehmer_pi(sqrt3(x));
    LL sum = getphi(x, a) + (LL)(b + a - 2) * (b - a + 1) / 2;
    for (int i = a + 1; i <= b; i++)
    {
        LL w = x / prime[i];
        sum -= lehmer_pi(w);
        if (i > c)
            continue;
        LL lim = lehmer_pi(sqrt2(w));
        for (int j = i; j <= lim; j++)
            sum -= lehmer_pi(w / prime[j]) - (j - 1);
    }
    return sum;
}
// lehmer_pi(n)
```

7 Other

7.1 binary search 三類變化

```cpp
// 查找和目標值完全相等的數
int find(vector<int> &nums, int target)
{
    int left = 0, right = nums.size();
    while (left < right)
    {
        int mid = left + (right - left) / 2;
        if (nums[mid] == target)
            return mid;
        else if (nums[mid] < target)
            left = mid + 1;
        else
            right = mid;
    }
    return -1;
}
// 找第一個不小於目標值的數 == 找最後一個小於目標值的數
/*(lower_bound)*/
int find(vector<int> &nums, int target)
{
    int left = 0, right = nums.size();
    while (left < right)
    {
        int mid = left + (right - left) / 2;
        if (nums[mid] < target)
            left = mid + 1;
        else
            right = mid;
    }
    return right;
}
// 找第一個大於目標值的數 == 找最後一個不大於目標值的數
/*(upper_bound)*/
int find(vector<int> &nums, int target)
{
    int left = 0, right = nums.size();
    while (left < right)
    {
        int mid = left + (right - left) / 2;
        if (nums[mid] <= target)
            left = mid + 1;
        else
            right = mid;
    }
    return right;
}
```

7.2 heap sort

```cpp
void MaxHeapify(vector<int> &array, int root, int length)
{
    int left = 2 * root,
        right = 2 * root + 1,
        largest;
    if (left <= length && array[left] > array[root])
        largest = left;
    else
        largest = root;
    if (right <= length && array[right] > array[largest])
        largest = right;
    if (largest != root)
    {
        swap(array[largest], array[root]);
        MaxHeapify(array, largest, length);
    }
}
void HeapSort(vector<int> &array)
{
    array.insert(array.begin(), 0);
    for (int i = (int)array.size() / 2; i >= 1; i--)
        MaxHeapify(array, i, (int)array.size() - 1);
    int size = (int)array.size() - 1;
    for (int i = (int)array.size() - 1; i >= 2; i--)
    {
        swap(array[1], array[i]);
        size--;
        MaxHeapify(array, 1, size);
    }
    array.erase(array.begin());
}
```

7.3 Josephus

```cpp
/*n people kill k for each turn*/
int josephus(int n, int k)
{
    int s = 0;
    for (int i = 2; i <= n; i++)
    {
        s = (s + k) % i;
    }
    /*index start from 1 -> s+1*/
    return s + 1;
}
/*died at kth*/
int kth(int n, int m, int k)
{
    if (m == 1)
        return n - 1;
    for (k = k * m + m - 1; k >= n; k = k - n + (k - n) / (m - 1))
        ;
    return k;
}
```

7.4 Merge sort

```
void Merge(vector<int> &arr, int front, int
     mid, int end)
{
    vector<int> LeftSub(arr.begin() + front,
          arr.begin() + mid + 1);
    vector<int> RightSub(arr.begin() + mid +
          1, arr.begin() + end + 1);
    LeftSub.insert(LeftSub.end(), INT_MAX);
    RightSub.insert(RightSub.end(), INT_MAX)
        ;
    int idxLeft = 0, idxRight = 0;

    for (int i = front; i <= end; i++)
    {

        if (LeftSub[idxLeft] <= RightSub[
             idxRight])
        {
            arr[i] = LeftSub[idxLeft];
            idxLeft++;
        }
        else
        {
            arr[i] = RightSub[idxRight];
            idxRight++;
        }
    }
}
void MergeSort(vector<int> &arr, int front,
     int end)
{
    // front = 0 , end = arr.size() - 1
    if (front < end)
    {
        int mid = (front + end) / 2;
        MergeSort(arr, front, mid);
        MergeSort(arr, mid + 1, end);
        Merge(arr, front, mid, end);
    }
}
```

7.5 Quick

```
int Partition(vector<int> &arr, int front,
     int end)
{
    int pivot = arr[end];
    int i = front - 1;
    for (int j = front; j < end; j++)
    {
        if (arr[j] < pivot)
        {
            i++;
            swap(arr[i], arr[j]);
        }
    }
    i++;
    swap(arr[i], arr[end]);
    return i;
}
void QuickSort(vector<int> &arr, int front,
     int end)
{
    // front = 0 , end = arr.size() - 1
    if (front < end)
    {
        int pivot = Partition(arr, front,
             end);
        QuickSort(arr, front, pivot - 1);
        QuickSort(arr, pivot + 1, end);
    }
}
```

7.6 Weighted Job Scheduling

```
struct Job
{
    int start, finish, profit;
};
bool jobComparataor(Job s1, Job s2)
{
    return (s1.finish < s2.finish);
}
int latestNonConflict(Job arr[], int i)
{
    for (int j = i - 1; j >= 0; j--)
    {
        if (arr[j].finish <= arr[i].start)
            return j;
    }
    return -1;
}
int findMaxProfit(Job arr[], int n)
{
    sort(arr, arr + n, jobComparataor);
    int *table = new int[n];
    table[0] = arr[0].profit;
    for (int i = 1; i < n; i++)
    {
        int inclProf = arr[i].profit;
        int l = latestNonConflict(arr, i);
        if (l != -1)
            inclProf += table[l];
        table[i] = max(inclProf, table[i -
             1]);
    }
    int result = table[n - 1];
    delete[] table;

    return result;
}
```

7.7 數獨解法

```
int getSquareIndex(int row, int column, int
     n)
{
    return row / n * n + column / n;
}

bool backtracking(vector<vector<int>> &board
    , vector<vector<bool>> &rows, vector<
    vector<bool>> &cols,
                    vector<vector<bool>> &boxs
                        , int index, int n)
{
    int n2 = n * n;
    int rowNum = index / n2, colNum = index
        % n2;
    if (index >= n2 * n2)
        return true;

    if (board[rowNum][colNum] != 0)
        return backtracking(board, rows,
             cols, boxs, index + 1, n);

    for (int i = 1; i <= n2; i++)
    {
        if (!rows[rowNum][i] && !cols[colNum
             ][i] && !boxs[getSquareIndex(
             rowNum, colNum, n)][i])
        {
            rows[rowNum][i] = true;
            cols[colNum][i] = true;
            boxs[getSquareIndex(rowNum,
                 colNum, n)][i] = true;
            board[rowNum][colNum] = i;
            if (backtracking(board, rows,
                 cols, boxs, index + 1, n))
                return true;
            board[rowNum][colNum] = 0;
            rows[rowNum][i] = false;
            cols[colNum][i] = false;
            boxs[getSquareIndex(rowNum,
                 colNum, n)][i] = false;
        }
    }
    return false;
}
/*用法 main*/
int n = sqrt(數獨邊長大小) /*e.g. 9*9 n=3*/
vector<vector<int>> board(n * n + 1, vector<
     int>(n * n + 1, 0));
vector<vector<bool>> isRow(n * n + 1, vector
     <bool>(n * n + 1, false));
vector<vector<bool>> isColumn(n * n + 1,
     vector<bool>(n * n + 1, false));
vector<vector<bool>> isSquare(n * n + 1,
     vector<bool>(n * n + 1, false));

for (int i = 0; i < n * n; ++i)
{
    for (int j = 0; j < n * n; ++j)
    {
        int number;
        cin >> number;
        board[i][j] = number;
        if (number == 0)
            continue;
        isRow[i][number] = true;
        isColumn[j][number] = true;
        isSquare[getSquareIndex(i, j, n)][
             number] = true;
    }
}
if (backtracking(board, isRow, isColumn,
     isSquare, 0, n))
    /*有解答*/
else
    /*解答*/
```

8 String

8.1 KMP

```
// 用在在一個 s 內查找一個詞 w 的出現位置
void ComputePrefix(string s, int next[])
{
    int n = s.length();
    int q, k;
    next[0] = 0;
    for (k = 0, q = 1; q < n; q++)
    {
        while (k > 0 && s[k] != s[q])
            k = next[k];
        if (s[k] == s[q])
            k++;
        next[q] = k;
    }
}
void KMPMatcher(string text, string pattern)
{
    int n = text.length();
    int m = pattern.length();
    int next[pattern.length()];
    ComputePrefix(pattern, next);

    for (int i = 0, q = 0; i < n; i++)
    {
        while (q > 0 && pattern[q] != text[i
             ])
            q = next[q];
        if (pattern[q] == text[i])
            q++;
        if (q == m)
        {
            cout << "Pattern occurs with
                 shift " << i - m + 1 << endl
                 ;
            q = 0;
        }
    }
}
// string s = "abcdabcdebcd";
// string p = "bcd";
// KMPMatcher(s, p);
// cout << endl;
```

8.2 Min Edit Distance

```
int EditDistance(string a, string b)
{
    vector<vector<int>> dp(a.size() + 1, vector<int>(b.size() + 1, 0));
    int m = a.length(), n = b.length();
    for (int i = 0; i < m + 1; i++)
    {
        for (int j = 0; j < n + 1; j++)
        {
            if (i == 0)
                dp[i][j] = j;
            else if (j == 0)
                dp[i][j] = i;
            else if (a[i - 1] == b[j - 1])
                dp[i][j] = dp[i - 1][j - 1];
            else
                dp[i][j] = 1 + min(min(dp[i - 1][j], dp[i][j - 1]), dp[i - 1][j - 1]);
        }
    }
    return dp[m][n];
}
```

8.3 Sliding window

```
string minWindow(string s, string t)
{
    unordered_map<char, int> letterCnt;
    for (int i = 0; i < t.length(); i++)
        letterCnt[t[i]]++;
    int minLength = INT_MAX, minStart = -1;
    int left = 0, matchCnt = 0;
    for (int i = 0; i < s.length(); i++)
    {
        if (--letterCnt[s[i]] >= 0)
            matchCnt++;
        while (matchCnt == t.length())
        {
            if (i - left + 1 < minLength)
            {
                minLength = i - left + 1;
                minStart = left;
            }
            if (++letterCnt[s[left]] > 0)
                matchCnt--;
            left++;
        }
    }
    return minLength == INT_MAX ? "" : s.substr(minStart, minLength);
}
```

8.4 Split

```
vector<string> mysplit(string s, string d)
{
    int ps = 0, pe, dl = d.length();
    string token;
    vector<string> res;
    while ((pe = s.find(d, ps)) != string::npos)
    {
        token = s.substr(ps, pe - ps);
        ps = pe + dl;
        res.push_back(token);
    }
    res.push_back(s.substr(ps));
    return res;
}
```

9 data structure

9.1 Bigint

```
//台大 //非必要請用python
struct Bigint
{
    static const int LEN = 60;       //maxLEN
    static const int BIGMOD = 10000; //10為正常位數
    int s;
    int vl, v[LEN];
    //  vector<int> v;
    Bigint() : s(1) { vl = 0; }
    Bigint(long long a)
    {
        s = 1;
        vl = 0;
        if (a < 0)
        {
            s = -1;
            a = -a;
        }
        while (a)
        {
            push_back(a % BIGMOD);
            a /= BIGMOD;
        }
    }
    Bigint(string str)
    {
        s = 1;
        vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-')
        {
            stPos = 1;
            s = -1;
        }
        for (int i = str.length() - 1, q = 1; i >= stPos; i--)
        {
            num += (str[i] - '0') * q;
            if ((q *= 10) >= BIGMOD)
            {
                push_back(num);
                num = 0;
                q = 1;
            }
        }
        if (num)
            push_back(num);
        n();
    }
    int len() const
    {
        return vl; //return SZ(v);
    }
    bool empty() const { return len() == 0; }
    void push_back(int x)
    {
        v[vl++] = x; //v.PB(x);
    }
    void pop_back()
    {
        vl--; //v.pop_back();
    }
    int back() const
    {
        return v[vl - 1]; //return v.back();
    }
    void n()
    {
        while (!empty() && !back())
            pop_back();
    }
    void resize(int nl)
    {
        vl = nl;            //v.resize(nl);
        fill(v, v + vl, 0); //fill(ALL(v), 0);
    }
    void print() const
    {
        if (empty())
        {
            putchar('0');
            return;
        }
        if (s == -1)
            putchar('-');
        printf("%d", back());
        for (int i = len() - 2; i >= 0; i--)
            printf("%.4d", v[i]);
    }
    friend std::ostream &operator<<(std::ostream &out, const Bigint &a)
    {
        if (a.empty())
        {
            out << "0";
            return out;
        }
        if (a.s == -1)
            out << "-";
        out << a.back();
        for (int i = a.len() - 2; i >= 0; i--)
        {
            char str[10];
            snprintf(str, 5, "%.4d", a.v[i]);
            out << str;
        }
        return out;
    }
    int cp3(const Bigint &b) const
    {
        if (s != b.s)
            return s - b.s;
        if (s == -1)
            return -(-*this).cp3(-b);
        if (len() != b.len())
            return len() - b.len(); //int
        for (int i = len() - 1; i >= 0; i--)
            if (v[i] != b.v[i])
                return v[i] - b.v[i];
        return 0;
    }
    bool operator<(const Bigint &b) const
    {
        return cp3(b) < 0;
    }
    bool operator<=(const Bigint &b) const
    {
        return cp3(b) <= 0;
    }
    bool operator==(const Bigint &b) const
    {
        return cp3(b) == 0;
    }
    bool operator!=(const Bigint &b) const
    {
        return cp3(b) != 0;
    }
    bool operator>(const Bigint &b) const
    {
        return cp3(b) > 0;
    }
    bool operator>=(const Bigint &b) const
    {
        return cp3(b) >= 0;
    }
    Bigint operator-() const
    {
        Bigint r = (*this);
        r.s = -r.s;
        return r;
    }
    Bigint operator+(const Bigint &b) const
    {
        if (s == -1)
            return -(-(*this) + (-b));
        if (b.s == -1)
            return (*this) - (-b);
        Bigint r;
        int nl = max(len(), b.len());
        r.resize(nl + 1);
        for (int i = 0; i < nl; i++)
        {
            if (i < len())
                r.v[i] += v[i];
            if (i < b.len())
                r.v[i] += b.v[i];
            if (r.v[i] >= BIGMOD)
            {
                r.v[i + 1] += r.v[i] / BIGMOD;
                r.v[i] %= BIGMOD;
            }
        }
        r.n();
        return r;
    }
    Bigint operator-(const Bigint &b) const
    {
        if (s == -1)
            return -(-(*this) - (-b));
        if (b.s == -1)
            return (*this) + (-b);
        if ((*this) < b)
            return -(b - (*this));
        Bigint r;
        r.resize(len());
        for (int i = 0; i < len(); i++)
        {
            r.v[i] += v[i];
            if (i < b.len())
                r.v[i] -= b.v[i];
            if (r.v[i] < 0)
            {
                r.v[i] += BIGMOD;
                r.v[i + 1]--;
            }
        }
        r.n();
        return r;
    }
    Bigint operator*(const Bigint &b)
    {
        Bigint r;
        r.resize(len() + b.len() + 1);
        r.s = s * b.s;
        for (int i = 0; i < len(); i++)
        {
            for (int j = 0; j < b.len(); j++)
            {
                r.v[i + j] += v[i] * b.v[j];
                if (r.v[i + j] >= BIGMOD)
                {
                    r.v[i + j + 1] += r.v[i + j] / BIGMOD;
                    r.v[i + j] %= BIGMOD;
                }
            }
        }
        r.n();
        return r;
    }
    Bigint operator/(const Bigint &b)
    {
        Bigint r;
        r.resize(max(1, len() - b.len() + 1));
        int oriS = s;
        Bigint b2 = b; // b2 = abs(b)
        s = b2.s = r.s = 1;
        for (int i = r.len() - 1; i >= 0; i--)
        {
            int d = 0, u = BIGMOD - 1;
            while (d < u)
            {
                int m = (d + u + 1) >> 1;
                r.v[i] = m;
                if ((r * b2) > (*this))
                    u = m - 1;
                else
                    d = m;
            }
            r.v[i] = d;
        }
        s = oriS;
        r.s = s * b.s;
        r.n();
        return r;
    }
    Bigint operator%(const Bigint &b)
    {
        return (*this) - (*this) / b * b;
    }
};
```

9.2 DisjointSet

```
struct DisjointSet {
    int p[maxn], sz[maxn], n, cc;
    vector<pair<int*, int>> his;
    vector<int> sh;
    void init(int _n) {
        n = _n; cc = n;
        for (int i = 0; i < n; ++i) sz[i] = 1, p[i] = i;
        sh.clear(); his.clear();
    }
    void assign(int *k, int v) {
        his.emplace_back(k, *k);
        *k = v;
    }
    void save() {
        sh.push_back((int)his.size());
    }
    void undo() {
        int last = sh.back(); sh.pop_back();
        while (his.size() != last) {
            int *k, v;
            tie(k, v) = his.back(); his.pop_back();
            *k = v;
        }
    }
    int find(int x) {
        if (x == p[x]) return x;
        return find(p[x]);
    }
    void merge(int x, int y) {
        x = find(x); y = find(y);
        if (x == y) return;
        if (sz[x] > sz[y]) swap(x, y);
        assign(&sz[y], sz[x] + sz[y]);
        assign(&p[x], y);
        assign(&cc, cc - 1);
    }
} ;
```

9.3 Matirx

```
template <typename T>
struct Matrix
{
    using rt = std::vector<T>;
    using mt = std::vector<rt>;
    using matrix = Matrix<T>;
    int r, c; // [r][c]
    mt m;
    Matrix(int r, int c) : r(r), c(c), m(r, rt(c)) {}
    Matrix(mt a) { m = a, r = a.size(), c = a[0].size(); }
    rt &operator[](int i) { return m[i]; }
    matrix operator+(const matrix &a)
    {
        matrix rev(r, c);
        for (int i = 0; i < r; ++i)
            for (int j = 0; j < c; ++j)
                rev[i][j] = m[i][j] + a.m[i][j];
        return rev;
    }
    matrix operator-(const matrix &a)
    {
        matrix rev(r, c);
        for (int i = 0; i < r; ++i)
            for (int j = 0; j < c; ++j)
                rev[i][j] = m[i][j] - a.m[i][j];
        return rev;
    }
    matrix operator*(const matrix &a)
    {
        matrix rev(r, a.c);
        matrix tmp(a.c, a.r);
        for (int i = 0; i < a.r; ++i)
            for (int j = 0; j < a.c; ++j)
                tmp[j][i] = a.m[i][j];
        for (int i = 0; i < r; ++i)
            for (int j = 0; j < a.c; ++j)
                for (int k = 0; k < c; ++k)
                    rev.m[i][j] += m[i][k] * tmp[j][k];
        return rev;
    }
    bool inverse() //逆矩陣判斷
    {
        Matrix t(r, r + c);
        for (int y = 0; y < r; y++)
        {
            t.m[y][c + y] = 1;
            for (int x = 0; x < c; ++x)
                t.m[y][x] = m[y][x];
        }
        if (!t.gas())
            return false;
        for (int y = 0; y < r; y++)
            for (int x = 0; x < c; ++x)
                m[y][x] = t.m[y][c + x] / t.m[y][y];
        return true;
    }
    T gas() //行列式
    {
        vector<T> lazy(r, 1);
        bool sign = false;
        for (int i = 0; i < r; ++i)
        {
            if (m[i][i] == 0)
            {
                int j = i + 1;
                while (j < r && !m[j][i])
                    j++;
                if (j == r)
                    continue;
                m[i].swap(m[j]);
                sign = !sign;
            }
            for (int j = 0; j < r; ++j)
            {
                if (i == j)
                    continue;
                lazy[j] = lazy[j] * m[i][i];
                T mx = m[j][i];
                for (int k = 0; k < c; ++k)
                    m[j][k] = m[j][k] * m[i][i] - m[i][k] * mx;
            }
        }
        T det = sign ? -1 : 1;
        for (int i = 0; i < r; ++i)
        {
            det = det * m[i][i];
            det = det / lazy[i];
            for (auto &j : m[i])
                j /= lazy[i];
        }
        return det;
    }
};
```

9.4 Trie

```
// biginter字典數
struct BigInteger{
    static const int BASE = 100000000;
    static const int WIDTH = 8;
    vector<int> s;
    BigInteger(long long num = 0){
        *this = num;
    }
    BigInteger operator = (long long num){
        s.clear();
        do{
            s.push_back(num % BASE);
            num /= BASE;
        }while(num > 0);
        return *this;
    }
    BigInteger operator = (const string& str){
        s.clear();
        int x, len = (str.length() - 1) / WIDTH + 1;
        for(int i = 0; i < len;i++){
            int end = str.length() - i*WIDTH;
            int start = max(0, end-WIDTH);
            sscanf(str.substr(start, end-start).c_str(), "%d", &x);
            s.push_back(x);
        }
        return *this;
    }

    BigInteger operator + (const BigInteger& b) const{
        BigInteger c;
        c.s.clear();
        for(int i = 0, g = 0;;i++){
            if(g == 0 && i >= s.size() && i >= b.s.size()) break;
            int x = g;
            if(i < s.size()) x+=s[i];
            if(i < b.s.size()) x+=b.s[i];
            c.s.push_back(x % BASE);
            g = x / BASE;
        }
        return c;
    }
};

ostream& operator << (ostream &out, const BigInteger& x){
    out << x.s.back();
    for(int i = x.s.size()-2; i >= 0;i--){
        char buf[20];
        sprintf(buf, "%08d", x.s[i]);
        for(int j = 0; j< strlen(buf);j++){
            out << buf[j];
        }
    }
    return out;
}

istream& operator >> (istream &in, BigInteger& x){
    string s;
    if(!(in >> s))
        return in;
    x = s;
    return in;
}

struct Trie{
    int c[5000005][10];
    int val[5000005];
    int sz;
    int getIndex(char c){
        return c - '0';
    }
    void init(){
        memset(c[0], 0, sizeof(c[0]));
        memset(val, -1, sizeof(val));
        sz = 1;
    }
    void insert(BigInteger x, int v){
        int u = 0;
        int max_len_count = 0;
        int firstNum = x.s.back();
        char firstBuf[20];
        sprintf(firstBuf, "%d", firstNum);
        for(int j = 0; j < strlen(firstBuf); j++){
            int index = getIndex(firstBuf[j]);
            if(!c[u][index]){
                memset(c[sz], 0 , sizeof(c[sz]));
                val[sz] = v;
                c[u][index] = sz++;
            }
            u = c[u][index];
            max_len_count++;
        }
        for(int i = x.s.size()-2; i >= 0;i--){
            char buf[20];
            sprintf(buf, "%08d", x.s[i]);
            for(int j = 0; j < strlen(buf) && max_len_count < 50;j++){
                int index = getIndex(buf[j]);
                if(!c[u][index]){
                    memset(c[sz], 0 , sizeof(c[sz]));
                    val[sz] = v;
                    c[u][index] = sz++;
                }
                u = c[u][index];
                max_len_count++;
            }
            if(max_len_count >= 50){
                break;
            }
        }
    }
    int find(const char* s){
        int u = 0;
        int n = strlen(s);
        for(int i = 0 ; i < n;++i)
        {
            int index = getIndex(s[i]);
            if(!c[u][index]){
                return -1;
            }
            u = c[u][index];
        }
        return val[u];
    }
}
```

9.5 分數

```
typedef long long ll;
struct fraction
{
  ll n, d;
  fraction(const ll &_n = 0, const ll &_d = 1) : n(_n), d(_d)
  {
    ll t = __gcd(n, d);
    n /= t, d /= t;
    if (d < 0)
      n = -n, d = -d;
  }
  fraction operator-() const
  {
    return fraction(-n, d);
  }
  fraction operator+(const fraction &b) const
  {
    return fraction(n * b.d + b.n * d, d * b.d);
  }
  fraction operator-(const fraction &b) const
  {
    return fraction(n * b.d - b.n * d, d * b.d);
  }
  fraction operator*(const fraction &b) const
  {
    return fraction(n * b.n, d * b.d);
  }
  fraction operator/(const fraction &b) const
  {
    return fraction(n * b.d, d * b.n);
  }
  void print()
  {
    cout << n;
    if (d != 1)
      cout << "/" << d;
  }
};
```

To do writing not thinking

Contents