1 Basic

1.1 A function

```cpp
round(double f);          // 四捨五入
ceil(double f);           // 無條件捨去
floor(double f);          //無條件進入
__builtin_popcount(int n); // 32bit有多少 1
to_string(int s);         // int to string
/** 全排列要先 sort !!! **/
next_permutation(num.begin(), num.end());
prev_permutation(num.begin(), num.end());
//用binary search找大於或等於val的最小值的位
    置
vector<int>::iterator it = lower_bound(v.
    begin(), v.end(), val);
//用binary search找大於val的最小值的位置
vector<int>::iterator it = upper_bound(v.
    begin(), v.end(), val);
/*queue*/
queue<datatype> q;
front(); /*取出最前面的值(沒有移除掉喔!!)*/
back();  /*取出最後面的值(沒有移除掉!!)*/
pop();   /*移掉最前面的值*/
push();  /*新增值到最後面*/
empty(); /*回傳bool,檢查是不是空的queue*/
size();  /*queue 的大小*/

/*stack*/
stack<datatype> s;
top();   /*取出最上面的值(沒有移除掉喔!!)*/
pop();   /*移掉最上面的值*/
push();  /*新增值到最上面*/
empty(); /*回傳bool,檢查是不是空的stack*/
size();  /*stack 的大小*/

/*unordered_set*/
unordered_set<datatype> s;
unordered_set<datatype> s(arr, arr + n);
/*initial with array*/
insert(); /*插入值*/
erase();  /*刪除值*/
empty();  /*bool 檢查是不是空*/
count();  /*判斷元素存在回傳1 無則回傳0*/
```

1.2 Codeblock setting

```
Settings -> Editor -> Keyboard shortcuts ->
    Plugins -> Source code formatter (AStyle
    )
Settings -> Source Formatter -> Padding
Delete empty lines within a function or
    method
Insert space padding around operators
Insert space padding around parentheses on
    outside
Remove extra space padding around
    parentheses
```

1.3 data range

```
int (-2147483648 to 2147483647)
unsigned int(0 to 4294967295)
long(-2147483648 to 2147483647)
unsigned long(0 to 4294967295)
long long(-9223372036854775808 to
    9223372036854775807)
unsigned long long (0 to
    18446744073709551615)
```

1.4 IO_fast

```cpp
void io()
{
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
    cout.tie(nullptr);
}
```

1.5 Python

```python
/*輸入1*/
import sys
line = sys.stdin.readline()
/*輸入2*/
line = input().strip()
D, R, N = map(int, line[:-1].split()) // 分
    三個 int 變數
pow(a, b, c) // a ^ b % c
print(*objects, sep = ' ', end = '\n')
objects -- /*可以一次輸出多個對象*/
sep -- /*分開多個objects*/
end -- /*默認值是\n*/
```

2 DP

2.1 3 維 DP 思路

```
解題思路: dp[i][j][k]
i 跟 j 代表 range i ~ j 的 value
k在我的理解裡是視題目的要求而定的
像是 Remove Boxes 當中 k 代表的是在 i 之前還
    有多少個連續的箱子
所以每次區間消去的值就是(k+1) * (k+1)
換言之，我認為可以理解成 k 的意義就是題目今
    天所關注的重點，就是老師說的題目所規定的
    運算
```

2.2 Knapsack Bounded

```cpp
const int N = 100, W = 100000;
int cost[N], weight[N], number[N];
int c[W + 1];
void knapsack(int n, int w)
{
    for (int i = 0; i < n; ++i)
    {
        int num = min(number[i], w / weight[
            i]);
        for (int k = 1; num > 0; k *= 2)
        {
            if (k > num)
                k = num;
            num -= k;
            for (int j = w; j >= weight[i] *
                k; --j)
                c[j] = max(c[j], c[j -
                    weight[i] * k] + cost[i]
                    * k);
        }
    }
    cout << "Max Prince" << c[w];
}
```

2.3 Knapsack sample

```cpp
int Knapsack(vector<int> weight, vector<int>
    value, int bag_Weight)
{
    // vector<int> weight = {1, 3, 4};
    // vector<int> value = {15, 20, 30};
    // int bagWeight = 4;
    vector<vector<int>> dp(weight.size(),
        vector<int>(bagWeight + 1, 0));
    for (int j = weight[0]; j <= bagWeight;
        j++)
        dp[0][j] = value[0];
    // weight數組的大小就是物品個數
    for (int i = 1; i < weight.size(); i++)
    { // 遍歷物品
        for (int j = 0; j <= bagWeight; j++)
        { // 遍歷背包容量
            if (j < weight[i]) dp[i][j] = dp
                [i - 1][j];
            else dp[i][j] = max(dp[i - 1][j
                ], dp[i - 1][j - weight[i]]
                + value[i]);
        }
    }
    cout << dp[weight.size() - 1][bagWeight]
        << endl;
}
```

2.4 Knapsack Unbounded

```cpp
const int N = 100, W = 100000;
int cost[N], weight[N];
int c[W + 1];
void knapsack(int n, int w)
{
    memset(c, 0, sizeof(c));
    for (int i = 0; i < n; ++i)
        for (int j = weight[i]; j <= w; ++j)
            c[j] = max(c[j], c[j - weight[i
                ]] + cost[i]);
    cout << "最高的價值為" << c[w];
}
```

2.5 LCIS

```cpp
int LCIS_len(vector<int> arr1, vetor<int>
    arr2)
{
    int n = arr1.size(), m = arr2.size();
    vector<int> table(m, 0);
    for (int j = 0; j < m; j++)
        table[j] = 0;
    for (int i = 0; i < n; i++)
    {
        int current = 0;
        for (int j = 0; j < m; j++)
        {
            if (arr1[i] == arr2[j])
                if (current + 1 > table[j])
                    table[j] = current + 1;

            if (arr1[i] > arr2[j])
                if (table[j] > current)
                    current = table[j];
        }
    }
    int result = 0;
    for (int i = 0; i < m; i++)
        if (table[i] > result)
            result = table[i];
    return result;
}
```

2.6 LCS

```cpp
int LCS(vector<string> Ans, vector<string>
    num)
{
    int N = Ans.size(), M = num.size();
    vector<vector<int>> LCS(N + 1, vector<
        int>(M + 1, 0));
    for (int i = 1; i <= N; ++i)
    {
        for (int j = 1; j <= M; ++j)
        {
            if (Ans[i - 1] == num[j - 1])
                LCS[i][j] = LCS[i - 1][j -
                    1] + 1;
```

```
11              else
12                  LCS[i][j] = max(LCS[i - 1][j
                        ], LCS[i][j - 1]);
13          }
14      }
15      cout << LCS[N][M] << '\n';
16      //列印 LCS
17      int n = N, m = M;
18      vector<string> k;
19      while (n && m)
20      {
21          if (LCS[n][m] != max(LCS[n - 1][m],
                LCS[n][m - 1]))
22          {
23              k.push_back(Ans[n - 1]);
24              n--;
25              m--;
26          }
27          else if (LCS[n][m] == LCS[n - 1][m])
28              n--;
29          else if (LCS[n][m] == LCS[n][m - 1])
30              m--;
31      }
32      reverse(k.begin(), k.end());
33      for (auto i : k)
34          cout << i << " ";
35      cout << endl;
36      return LCS[N][M];
37 }
```

2.7 LIS

```
1  void getMaxElementAndPos(vector<int> &LISTbl
     , vector<int> &LISLen, int tNum, int
     tlen, int tStart, int &num, int &pos)
2  {
3      int max = numeric_limits<int>::min();
4      int maxPos;
5      for (int i = tStart; i >= 0; i--)
6      {
7          if (LISLen[i] == tlen && LISTbl[i] <
                tNum)
8          {
9              if (LISTbl[i] > max)
10             {
11                 max = LISTbl[i];
12                 maxPos = i;
13             }
14         }
15     }
16     num = max;
17     pos = maxPos;
18 }
19 int LIS(vector<int> &LISTbl)
20 {
21     if (LISTbl.size() == 0)
22         return 0;
23     vector<int> LISLen(LISTbl.size(), 1);
24     for (int i = 1; i < LISTbl.size(); i++)
25         for (int j = 0; j < i; j++)
26             if (LISTbl[j] < LISTbl[i])
27                 LISLen[i] = max(LISLen[i],
                       LISLen[j] + 1);
```

```
28     int maxlen = *max_element(LISLen.begin()
         , LISLen.end());
29     int num, pos;
30     vector<int> buf;
31     getMaxElementAndPos(LISTbl, LISLen,
         numeric_limits<int>::max(), maxlen,
         LISTbl.size() - 1, num, pos);
32     buf.push_back(num);
33     for (int len = maxlen - 1; len >= 1; len
         --)
34     {
35         int tnum = num;
36         int tpos = pos;
37         getMaxElementAndPos(LISTbl, LISLen,
             tnum, len, tpos - 1, num, pos);
38         buf.push_back(num);
39     }
40     reverse(buf.begin(), buf.end());
41     for (int k = 0; k < buf.size(); k++) //
         列印
42     {
43         if (k == buf.size() - 1)
44             cout << buf[k] << endl;
45         else
46             cout << buf[k] << ",";
47     }
48     return maxlen;
49 }
```

2.8 LPS

```
1  void LPS(string s)
2  {
3      int maxlen = 0, l, r;
4      int n = n;
5      for (int i = 0; i < n; i++)
6      {
7          int x = 0;
8          while ((s[i - x] == s[i + x]) && (i
             - x >= 0) && (i + x < n)) //odd
             length
9              x++;
10         x--;
11         if (2 * x + 1 > maxlen)
12         {
13             maxlen = 2 * x + 1;
14             l = i - x;
15             r = i + x;
16         }
17         x = 0;
18         while ((s[i - x] == s[i + 1 + x]) &&
             (i - x >= 0) && (i + 1 + x < n)
             ) //even length
19             x++;
20         if (2 * x > maxlen)
21         {
22             maxlen = 2 * x;
23             l = i - x + 1;
24             r = i + x;
25         }
26     }
27     cout << maxlen << '\n';  // 最後長度
```

```
28     cout << l + 1 << ' ' << r + 1 << '\n';
         //頭到尾
29 }
```

2.9 Max_subarray

```
1  /*Kadane's algorithm*/
2  int maxSubArray(vector<int>& nums) {
3      int local_max = nums[0], global_max =
         nums[0];
4      for(int i = 1; i < nums.size(); i++){
5          local_max = max(nums[i],nums[i]+
             local_max);
6          global_max = max(local_max,
             global_max);
7      }
8      return global_max;
9  }
```

2.10 Money problem

```
1  //能否湊得某個價位
2  void change(vector<int> price, int limit)
3  {
4      vector<bool> c(limit + 1, 0);
5      c[0] = true;
6      for (int i = 0; i < price.size(); ++i)
                // 依序加入各種面額
7          for (int j = price[i]; j <= limit;
             ++j) // 由低價位逐步到高價位
8              c[j] = c[j] | c[j - price[i]];
                    // 湊、湊、湊
9      if (c[limit]) cout << "YES\n";
10     else cout << "NO\n";
11 }
12 // 湊得某個價位的湊法總共幾種
13 void change(vector<int> price, int limit)
14 {
15     vector<int> c(limit + 1, 0);
16     c[0] = true;
17     for (int i = 0; i < price.size(); ++i)
18         for (int j = price[i]; j <= limit;
             ++j)
19             c[j] += c[j - price[i]];
20     cout << c[limit] << '\n';
21 }
22 // 湊得某個價位的最少錢幣用量
23 void change(vector<int> price, int limit)
24 {
25     vector<int> c(limit + 1, 0);
26     c[0] = true;
27     for (int i = 0; i < price.size(); ++i)
28         for (int j = price[i]; j <= limit;
             ++j)
29             c[j] = min(c[j], c[j - price[i]]
                 + 1);
30     cout << c[limit] << '\n';
31 }
```

```
32 //湊得某個價位的錢幣用量，有哪幾種可能性
33 void change(vector<int> price, int limit)
34 {
35     vector<int> c(limit + 1, 0);
36     c[0] = true;
37     for (int i = 0; i < price.size(); ++i)
38         for (int j = price[i]; j <= limit;
             ++j)
39             c[j] |= c[j-price[i]] << 1; //
                 錢幣數量加一，每一種可能性都
                 加一。
40
41     for (int i = 1; i <= 63; ++i)
42         if (c[m] & (1 << i))
43             cout << "用" << i << "個錢幣可湊
                 得價位" << m;
44 }
```

3 Flow & matching

3.1 Dinic

```
1  const long long INF = 1LL<<60;
2  struct Dinic { //O(VVE), with minimum cut
3      static const int MAXN = 5003;
4      struct Edge{
5          int u, v;
6          long long cap, rest;
7      };
8      int n, m, s, t, d[MAXN], cur[MAXN];
9      vector<Edge> edges;
10     vector<int> G[MAXN];
11     void init(){
12         edges.clear();
13         for ( int i = 0 ; i < n ; i++ ) G[i
             ].clear();
14         n = 0;
15     }
16     // min cut start
17     bool side[MAXN];
18     void cut(int u) {
19         side[u] = 1;
20         for ( int i : G[u] ) {
21             if ( !side[ edges[i].v ] &&
                 edges[i].rest )
22                 cut(edges[i].v);
23         }
24     }
25     // min cut end
26     int add_node(){
27         return n++;
28     }
29     void add_edge(int u, int v, long long
         cap){
30         edges.push_back( {u, v, cap, cap} );
31         edges.push_back( {v, u, 0, 0LL} );
32         m = edges.size();
33         G[u].push_back(m-2);
34         G[v].push_back(m-1);
35     }
```

```
36    bool bfs(){
37        fill(d,d+n,-1);
38        queue<int> que;
39        que.push(s); d[s]=0;
40        while (!que.empty()){
41            int u = que.front(); que.pop();
42            for (int ei : G[u]){
43                Edge &e = edges[ei];
44                if (d[e.v] < 0 && e.rest >
                       0){
45                    d[e.v] = d[u] + 1;
46                    que.push(e.v);
47                }
48            }
49        }
50        return d[t] >= 0;
51    }
52    long long dfs(int u, long long a){
53        if ( u == t || a == 0 ) return a;
54        long long flow = 0, f;
55        for ( int &i=cur[u]; i < (int)G[u].
               size() ; i++) {
56            Edge &e = edges[ G[u][i] ];
57            if ( d[u] + 1 != d[e.v] )
                   continue;
58            f = dfs(e.v, min(a, e.rest) );
59            if ( f > 0 ) {
60                e.rest -= f;
61                edges[ G[u][i]^1 ].rest += f;
62                flow += f;
63                a -= f;
64                if ( a == 0 ) break;
65            }
66        }
67        return flow;
68    }
69    long long maxflow(int _s, int _t){
70        s = _s, t = _t;
71        long long flow = 0, mf;
72        while ( bfs() ){
73            fill(cur,cur+n,0);
74            while ( (mf = dfs(s, INF)) )
                   flow += mf;
75        }
76        return flow;
77    }
78 } dinic;
```

3.2   Edmonds_karp

```
1  /*Flow - Edmonds-karp*/
2  /*Based on UVa820*/
3  #define inf 1000000
4  int getMaxFlow(vector<vector<int>> &capacity
       , int s, int t, int n){
5      int ans = 0;
6      vector<vector<int>> residual(n+1, vector<
           int>(n+1, 0)); //residual network
7      while(true){
8          vector<int> bottleneck(n+1, 0);
9          bottleneck[s] = inf;
10         queue<int> q;
11         q.push(s);
```

```
12         vector<int> pre(n+1, 0);
13         while(!q.empty() && bottleneck[t] == 0){
14             int cur = q.front();
15             q.pop();
16             for(int i = 1; i <= n ; i++){
17                 if(bottleneck[i] == 0 && capacity[
                       cur][i] > residual[cur][i]){
18                     q.push(i);
19                     pre[i] = cur;
20                     bottleneck[i] = min(bottleneck[cur
                           ], capacity[cur][i] - residual
                           [cur][i]);
21                 }
22             }
23         }
24         if(bottleneck[t] == 0) break;
25         for(int cur = t; cur != s; cur = pre[cur
               ]){
26             residual[pre[cur]][cur] +=
                   bottleneck[t];
27             residual[cur][pre[cur]] -=
                   bottleneck[t];
28         }
29         ans += bottleneck[t];
30     }
31     return ans;
32 }
33 int main(){
34     int testcase = 1;
35     int n;
36     while(cin>>n){
37         if(n == 0)
38             break;
39         vector<vector<int>> capacity(n+1, vector
               <int>(n+1, 0));
40         int s, t, c;
41         cin >> s >> t >> c;
42         int a, b, bandwidth;
43         for(int i = 0 ; i < c ; ++i){
44             cin >> a >> b >> bandwidth;
45             capacity[a][b] += bandwidth;
46             capacity[b][a] += bandwidth;
47         }
48         cout << "Network " << testcase++ << endl
               ;
49         cout << "The bandwidth is " <<
               getMaxFlow(capacity, s, t, n) << "."
               << endl;
50         cout << endl;
51     }
52     return 0;
53 }
```

3.3   hungarian

```
1  /*bipartite - hungarian*/
2  struct Graph{
3      static const int MAXN = 5003;
4      vector<int> G[MAXN];
5      int n, match[MAXN], vis[MAXN];
6      void init(int _n){
7          n = _n;
```

```
8      vector<int> pre(n+1, 0);
9      while(!q.empty() && bottleneck[t] == 0){
...
28     for (int i=0; i<n; i++) G[i].clear()
           ;
31     }
32     bool dfs(int u){
33         for (int v:G[u]){
34             if (vis[v]) continue;
35             vis[v]=true;
36             if (match[v]==-1 || dfs(match[v
                   ])){
37                 match[v] = u;
38                 match[u] = v;
39                 return true;
40             }
41         }
42         return false;
43     }
44     int solve(){
45         int res = 0;
46         memset(match,-1,sizeof(match));
47         for (int i=0; i<n; i++){
48             if (match[i]==-1){
49                 memset(vis,0,sizeof(vis));
50                 if ( dfs(i) ) res++;
51             }
52         }
53         return res;
54     }
55 } graph;
```

3.4   Maximum_matching

```
1  /*bipartite - maximum matching*/
2  bool dfs(vector<vector<bool>> res,int node,
       vector<int>& x, vector<int>& y, vector<
       bool> pass){
3      for (int i = 0; i < res[0].size(); i++){
4          if(res[node][i] && !pass[i]){
5              pass[i] = true;
6              if(y[i] == -1 || dfs(res,y[i],x,
                   y,pass)){
7                  x[node] = i;
8                  y[i] = node;
9                  return true;
10             }
11         }
12     }
13     return false;
14 }
15 int main(){
16     int n,m,l;
17     while(cin>>n>>m>>l){
18         vector<vector<bool>> res(n, vector<
               bool>(m, false));
19         for (int i = 0; i < l; i++){
20             int a, b;
21             cin >> a >> b;
22             res[a][b] = true;
23         }
24         int ans = 0;
25         vector<int> x(n, -1);
26         vector<int> y(n, -1);
27         for (int i = 0; i < n; i++){
28             vector<bool> pass(n, false);
```

```
29             if(dfs(res,i,x,y,pass))
30                 ans += 1;
31         }
32         cout << ans << endl;
33     }
34     return 0;
35 }
/*
input:
4 3 5 //n matching m, l links
0 0
0 2
1 0
2 1
3 1
answer is 3
*/
```

3.5   MFlow Model

```
1  typedef long long ll;
2  struct MF
3  {
4      static const int N = 5000 + 5;
5      static const int M = 60000 + 5;
6      static const ll oo = 1000000000000000LL;
7
8      int n, m, s, t, tot, tim;
9      int first[N], next[M];
10     int u[M], v[M], cur[N], vi[N];
11     ll cap[M], flow[M], dis[N];
12     int que[N + N];
13
14     void Clear()
15     {
16         tot = 0;
17         tim = 0;
18         for (int i = 1; i <= n; ++i)
19             first[i] = -1;
20     }
21     void Add(int from, int to, ll cp, ll flw
           )
22     {
23         u[tot] = from;
24         v[tot] = to;
25         cap[tot] = cp;
26         flow[tot] = flw;
27         next[tot] = first[u[tot]];
28         first[u[tot]] = tot;
29         ++tot;
30     }
31     bool bfs()
32     {
33         ++tim;
34         dis[s] = 0;
35         vi[s] = tim;
36
37         int head, tail;
38         head = tail = 1;
39         que[head] = s;
40         while (head <= tail)
41         {
```

```
42      for (int i = first[que[head]]; i
             != -1; i = next[i])
43      {
44          if (vi[v[i]] != tim && cap[i
               ] > flow[i])
45          {
46              vi[v[i]] = tim;
47              dis[v[i]] = dis[que[head
                   ]] + 1;
48              que[++tail] = v[i];
49          }
50      }
51      ++head;
52  }
53  return vi[t] == tim;
54  }
55  ll dfs(int x, ll a)
56  {
57      if (x == t || a == 0)
58          return a;
59      ll flw = 0, f;
60      int &i = cur[x];
61      for (i = first[x]; i != -1; i = next
           [i])
62      {
63          if (dis[x] + 1 == dis[v[i]] && (
               f = dfs(v[i], min(a, cap[i]
               - flow[i]))) > 0)
64          {
65              flow[i] += f;
66              flow[i ^ 1] -= f;
67              a -= f;
68              flw += f;
69              if (a == 0)
70                  break;
71          }
72      }
73      return flw;
74  }
75  ll MaxFlow(int s, int t)
76  {
77      this->s = s;
78      this->t = t;
79      ll flw = 0;
80      while (bfs())
81      {
82          for (int i = 1; i <= n; ++i)
83              cur[i] = 0;
84          flw += dfs(s, oo);
85      }
86      return flw;
87  }
88  };
89  // MF Net;
90  // Net.n = n;
91  // Net.Clear();
92  // a 到 b (注意從1開始!!!!)
93  // Net.Add(a, b, w, 0);
94  // Net.MaxFlow(s, d);
95  // s 到 d 的 MF
```

4 Geometry

4.1 Closest Pair

```
1  //最近點對 (距離) //台大
2  vector<pair<double, double>> p;
3  double closest_pair(int l, int r)
4  {
5      // p 要對 x 軸做 sort
6      if (l == r)
7          return 1e9;
8      if (r - l == 1)
9          return dist(p[l], p[r]); // 兩點距離
10     int m = (l + r) >> 1;
11     double d = min(closest_pair(l, m),
           closest_pair(m + 1, r));
12     vector<int> vec;
13     for (int i = m; i >= l && fabs(p[m].x -
           p[i].x) < d; --i)
14         vec.push_back(i);
15     for (int i = m + 1; i <= r && fabs(p[m].
           x - p[i].x) < d; ++i)
16         vec.push_back(i);
17     sort(vec.begin(), vec.end(), [&](int a,
           int b)
18         { return p[a].y < p[b].y; });
19     for (int i = 0; i < vec.size(); ++i)
20         for (int j = i + 1; j < vec.size()
               && fabs(p[vec[j]].y - p[vec[i]].
               y) < d; ++j)
21             d = min(d, dist(p[vec[i]], p[vec
                   [j]]));
22     return d;
23 }
```

4.2 Line

```
1  template <typename T>
2  struct line
3  {
4      line() {}
5      point<T> p1, p2;
6      T a, b, c; //ax+by+c=0
7      line(const point<T> &x, const point<T> &
           y) : p1(x), p2(y) {}
8      void pton()
9      { //轉成一般式
10         a = p1.y - p2.y;
11         b = p2.x - p1.x;
12         c = -a * p1.x - b * p1.y;
13     }
14     T ori(const point<T> &p) const
15     { //點和有向直線的關係‧ >0左邊、=0在線上
           <0右邊
16         return (p2 - p1).cross(p - p1);
17     }
18     T btw(const point<T> &p) const
19     { //點投影落在線段上<=0
20         return (p1 - p).dot(p2 - p);
```

```
21     }
22     bool point_on_segment(const point<T> &p)
           const
23     { //點是否在線段上
24         return ori(p) == 0 && btw(p) <= 0;
25     }
26     T dis2(const point<T> &p, bool
           is_segment = 0) const
27     { //點跟直線/線段的距離平方
28         point<T> v = p2 - p1, v1 = p - p1;
29         if (is_segment)
30         {
31             point<T> v2 = p - p2;
32             if (v.dot(v1) <= 0)
33                 return v1.abs2();
34             if (v.dot(v2) >= 0)
35                 return v2.abs2();
36         }
37         T tmp = v.cross(v1);
38         return tmp * tmp / v.abs2();
39     }
40     T seg_dis2(const line<T> &l) const
41     { //兩線段距離平方
42         return min({dis2(l.p1, 1), dis2(l.p2
               , 1), l.dis2(p1, 1), l.dis2(p2,
               1)});
43     }
44     point<T> projection(const point<T> &p)
           const
45     { //點對直線的投影
46         point<T> n = (p2 - p1).normal();
47         return p - n * (p - p1).dot(n) / n.
               abs2();
48     }
49     point<T> mirror(const point<T> &p) const
50     {
51         //點對直線的鏡射‧要先呼叫pton轉成一
               般式
52         point<T> R;
53         T d = a * a + b * b;
54         R.x = (b * b * p.x - a * a * p.x - 2
               * a * b * p.y - 2 * a * c) / d;
55         R.y = (a * a * p.y - b * b * p.y - 2
               * a * b * p.x - 2 * b * c) / d;
56         return R;
57     }
58     bool equal(const line &l) const
59     { //直線相等
60         return ori(l.p1) == 0 && ori(l.p2)
               == 0;
61     }
62     bool parallel(const line &l) const
63     {
64         return (p1 - p2).cross(l.p1 - l.p2)
               == 0;
65     }
66     bool cross_seg(const line &l) const
67     {
68         return (p2 - p1).cross(l.p1 - p1) *
               (p2 - p1).cross(l.p2 - p1) <= 0;
               //直線是否交線段
69     }
70     int line_intersect(const line &l) const
```

```
71     { //直線相交情況‧-1無限多點、1交於一
           點、0不相交
72         return parallel(l) ? (ori(l.p1) == 0
               ? -1 : 0) : 1;
73     }
74     int seg_intersect(const line &l) const
75     {
76         T c1 = ori(l.p1), c2 = ori(l.p2);
77         T c3 = l.ori(p1), c4 = l.ori(p2);
78         if (c1 == 0 && c2 == 0)
79         { //共線
80             bool b1 = btw(l.p1) >= 0, b2 =
                   btw(l.p2) >= 0;
81             T a3 = l.btw(p1), a4 = l.btw(p2)
                   ;
82             if (b1 && b2 && a3 == 0 && a4 >=
                   0)
83                 return 2;
84             if (b1 && b2 && a3 >= 0 && a4 ==
                   0)
85                 return 3;
86             if (b1 && b2 && a3 >= 0 && a4 >=
                   0)
87                 return 0;
88             return -1; //無限交點
89         }
90         else if (c1 * c2 <= 0 && c3 * c4 <=
               0)
91             return 1;
92         return 0; //不相交
93     }
94     point<T> line_intersection(const line &l
           ) const
95     { /*直線交點*/
96         point<T> a = p2 - p1, b = l.p2 - l.
               p1, s = l.p1 - p1;
97         //if(a.cross(b)==0)return INF;
98         return p1 + a * (s.cross(b) / a.
               cross(b));
99     }
100    point<T> seg_intersection(const line &l)
           const
101    { //線段交點
102        int res = seg_intersect(l);
103        if (res <= 0)
104            assert(0);
105        if (res == 2)
106            return p1;
107        if (res == 3)
108            return p2;
109        return line_intersection(l);
110    }
111 };
```

4.3 Point

```
1  template <typename T>
2  struct point
3  {
4      T x, y;
5      point() {}
```

```cpp
    point(const T &x, const T &y) : x(x), y(
        y) {}
    point operator+(const point &b) const
    {
        return point(x + b.x, y + b.y);
    }
    point operator-(const point &b) const
    {
        return point(x - b.x, y - b.y);
    }
    point operator*(const T &b) const
    {
        return point(x * b, y * b);
    }
    point operator/(const T &b) const
    {
        return point(x / b, y / b);
    }
    bool operator==(const point &b) const
    {
        return x == b.x && y == b.y;
    }
    T dot(const point &b) const
    {
        return x * b.x + y * b.y;
    }
    T cross(const point &b) const
    {
        return x * b.y - y * b.x;
    }
    point normal() const
    { //求法向量
        return point(-y, x);
    }
    T abs2() const
    { //向量長度的平方
        return dot(*this);
    }
    T rad(const point &b) const
    { //兩向量的弧度
        return fabs(atan2(fabs(cross(b)),
            dot(b)));
    }
    T getA() const
    {                                //對x軸的弧度
        T A = atan2(y, x); //超過180度會變負
            的
        if (A <= -PI / 2)
            A += PI * 2;
        return A;
    }
};
```

4.4 Polygon

```cpp
template <typename T>
struct polygon
{
    polygon() {}
    vector<point<T>> p; //逆時針順序
    T area() const
    { //面積
        T ans = 0;
        for (int i = p.size() - 1, j = 0; j
            < (int)p.size(); i = j++)
            ans += p[i].cross(p[j]);
        return ans / 2;
    }
    point<T> center_of_mass() const
    { //重心
        T cx = 0, cy = 0, w = 0;
        for (int i = p.size() - 1, j = 0; j
            < (int)p.size(); i = j++)
        {
            T a = p[i].cross(p[j]);
            cx += (p[i].x + p[j].x) * a;
            cy += (p[i].y + p[j].y) * a;
            w += a;
        }
        return point<T>(cx / 3 / w, cy / 3 /
            w);
    }
    char ahas(const point<T> &t) const
    { //點是否在簡單多邊形內，是的話回傳1，
        在邊上回傳-1，否則回傳0
        bool c = 0;
        for (int i = 0, j = p.size() - 1; i
            < p.size(); j = i++)
            if (line<T>(p[i], p[j]).
                point_on_segment(t))
                return -1;
            else if ((p[i].y > t.y) != (p[j
                ].y > t.y) &&
                    t.x < (p[j].x - p[i].x)
                        * (t.y - p[i].y) /
                        (p[j].y - p[i].y)
                        + p[i].x)
                c = !c;
        return c;
    }
    char point_in_convex(const point<T> &x)
        const
    {
        int l = 1, r = (int)p.size() - 2;
        while (l <= r)
        { //點是否在凸多邊形內，是的話回傳1
            、在邊上回傳-1，否則回傳0
            int mid = (l + r) / 2;
            T a1 = (p[mid] - p[0]).cross(x -
                p[0]);
            T a2 = (p[mid + 1] - p[0]).cross
                (x - p[0]);
            if (a1 >= 0 && a2 <= 0)
            {
                T res = (p[mid + 1] - p[mid
                    ]).cross(x - p[mid]);
                return res > 0 ? 1 : (res >=
                    0 ? -1 : 0);
            }
            else if (a1 < 0)
                r = mid - 1;
            else
                l = mid + 1;
        }
        return 0;
    }
    vector<T> getA() const
    {//凸包邊對x軸的夾角
        vector<T> res; //一定是遞增的
        for (size_t i = 0; i < p.size(); ++i
            )
            res.push_back((p[(i + 1) % p.
                size()] - p[i]).getA());
        return res;
    }
    bool line_intersect(const vector<T> &A,
        const line<T> &l) const
    { //O(logN)
        int f1 = upper_bound(A.begin(), A.
            end(), (l.p1 - l.p2).getA()) - A
            .begin();
        int f2 = upper_bound(A.begin(), A.
            end(), (l.p2 - l.p1).getA()) - A
            .begin();
        return l.cross_seg(line<T>(p[f1], p[
            f2]));
    }
    polygon cut(const line<T> &l) const
    { //凸包對直線切割，得到直線l左側的凸包
        polygon ans;
        for (int n = p.size(), i = n - 1, j
            = 0; j < n; i = j++)
        {
            if (l.ori(p[i]) >= 0)
            {
                ans.p.push_back(p[i]);
                if (l.ori(p[j]) < 0)
                    ans.p.push_back(l.
                        line_intersection(
                        line<T>(p[i], p[j]))
                        );
            }
            else if (l.ori(p[j]) > 0)
                ans.p.push_back(l.
                    line_intersection(line<T
                    >(p[i], p[j])));
        }
        return ans;
    }
    static bool graham_cmp(const point<T> &a
        , const point<T> &b)
    { //凸包排序函數 // 起始點不同
        // return (a.x < b.x) || (a.x == b.x
            && a.y < b.y);   //最左下角開始
        return (a.y < b.y) || (a.y == b.y &&
            a.x < b.x);   //Y最小開始
    }
    void graham(vector<point<T>> &s)
    { //凸包 Convexhull 2D
        sort(s.begin(), s.end(), graham_cmp)
            ;
        p.resize(s.size() + 1);
        int m = 0;
        // cross >= 0 順時針，cross <= 0 逆
            時針旋轉
        for (size_t i = 0; i < s.size(); ++i
            )
        {
            while (m >= 2 && (p[m - 1] - p[m
                - 2]).cross(s[i] - p[m -
                2]) <= 0)
                --m;
            p[m++] = s[i];
        }
        for (int i = s.size() - 2, t = m +
            1; i >= 0; --i)
        {
            while (m >= t && (p[m - 1] - p[m
                - 2]).cross(s[i] - p[m -
                2]) <= 0)
                --m;
            p[m++] = s[i];
        }
        if (s.size() > 1) // 重複頭一次需扣
            掉
            --m;
        p.resize(m);
    }
    T diam()
    { //直徑
        int n = p.size(), t = 1;
        T ans = 0;
        p.push_back(p[0]);
        for (int i = 0; i < n; i++)
        {
            point<T> now = p[i + 1] - p[i];
            while (now.cross(p[t + 1] - p[i
                ]) > now.cross(p[t] - p[i]))
                t = (t + 1) % n;
            ans = max(ans, (p[i] - p[t]).
                abs2());
        }
        return p.pop_back(), ans;
    }
    T min_cover_rectangle()
    { //最小覆蓋矩形
        int n = p.size(), t = 1, r = 1, l;
        if (n < 3)
            return 0; //也可以做最小周長矩形
        T ans = 1e99;
        p.push_back(p[0]);
        for (int i = 0; i < n; i++)
        {
            point<T> now = p[i + 1] - p[i];
            while (now.cross(p[t + 1] - p[i
                ]) > now.cross(p[t] - p[i]))
                t = (t + 1) % n;
            while (now.dot(p[r + 1] - p[i])
                > now.dot(p[r] - p[i]))
                r = (r + 1) % n;
            if (!i)
                l = r;
            while (now.dot(p[l + 1] - p[i])
                <= now.dot(p[l] - p[i]))
                l = (l + 1) % n;
            T d = now.abs2();
            T tmp = now.cross(p[t] - p[i]) *
                (now.dot(p[r] - p[i]) - now
                .dot(p[l] - p[i])) / d;
            ans = min(ans, tmp);
        }
        return p.pop_back(), ans;
    }
    T dis2(polygon &pl)
    { //凸包最近距離平方
        vector<point<T>> &P = p, &Q = pl.p;
```

```
153     int n = P.size(), m = Q.size(), l =
             0, r = 0;
154     for (int i = 0; i < n; ++i)
155         if (P[i].y < P[l].y)
156             l = i;
157     for (int i = 0; i < m; ++i)
158         if (Q[i].y < Q[r].y)
159             r = i;
160     P.push_back(P[0]), Q.push_back(Q[0])
             ;
161     T ans = 1e99;
162     for (int i = 0; i < n; ++i)
163     {
164         while ((P[l] - P[l + 1]).cross(Q
                 [r + 1] - Q[r]) < 0)
165             r = (r + 1) % m;
166         ans = min(ans, line<T>(P[l], P[l
                 + 1]).seg_dis2(line<T>(Q[r
                 ], Q[r + 1])));
167         l = (l + 1) % n;
168     }
169     return P.pop_back(), Q.pop_back(),
             ans;
170 }
171 static char sign(const point<T> &t)
172 {
173     return (t.y == 0 ? t.x : t.y) < 0;
174 }
175 static bool angle_cmp(const line<T> &A,
         const line<T> &B)
176 {
177     point<T> a = A.p2 - A.p1, b = B.p2 -
             B.p1;
178     return sign(a) < sign(b) || (sign(a)
             == sign(b) && a.cross(b) > 0);
179 }
180 int halfplane_intersection(vector<line<T
         >> &s)
181 {   //半平面交
182     sort(s.begin(), s.end(), angle_cmp);
             //線段左側為該線段半平面
183     int L, R, n = s.size();
184     vector<point<T>> px(n);
185     vector<line<T>> q(n);
186     q[L = R = 0] = s[0];
187     for (int i = 1; i < n; ++i)
188     {
189         while (L < R && s[i].ori(px[R -
                 1]) <= 0)
190             --R;
191         while (L < R && s[i].ori(px[L])
                 <= 0)
192             ++L;
193         q[++R] = s[i];
194         if (q[R].parallel(q[R - 1]))
195         {
196             --R;
197             if (q[R].ori(s[i].p1) > 0)
198                 q[R] = s[i];
199         }
200         if (L < R)
201             px[R - 1] = q[R - 1].
                 line_intersection(q[R]);
202     }
```

```
203     while (L < R && q[L].ori(px[R - 1])
             <= 0)
204         --R;
205     p.clear();
206     if (R - L <= 1)
207         return 0;
208     px[R] = q[R].line_intersection(q[L])
             ;
209     for (int i = L; i <= R; ++i)
210         p.push_back(px[i]);
211     return R - L + 1;
212 }
213 };
```

4.5 Triangle

```
1 template <typename T>
2 struct triangle
3 {
4     point<T> a, b, c;
5     triangle() {}
6     triangle(const point<T> &a, const point<
         T> &b, const point<T> &c) : a(a), b(
         b), c(c) {}
7     T area() const
8     {
9         T t = (b - a).cross(c - a) / 2;
10        return t > 0 ? t : -t;
11    }
12    point<T> barycenter() const
13    { //重心
14        return (a + b + c) / 3;
15    }
16    point<T> circumcenter() const
17    { //外心
18        static line<T> u, v;
19        u.p1 = (a + b) / 2;
20        u.p2 = point<T>(u.p1.x - a.y + b.y,
             u.p1.y + a.x - b.x);
21        v.p1 = (a + c) / 2;
22        v.p2 = point<T>(v.p1.x - a.y + c.y,
             v.p1.y + a.x - c.x);
23        return u.line_intersection(v);
24    }
25    point<T> incenter() const
26    { //內心
27        T A = sqrt((b - c).abs2()), B = sqrt
             ((a - c).abs2()), C = sqrt((a -
             b).abs2());
28        return point<T>(A * a.x + B * b.x +
             C * c.x, A * a.y + B * b.y + C *
             c.y) / (A + B + C);
29    }
30    point<T> perpencenter() const
31    { //垂心
32        return barycenter() * 3 -
             circumcenter() * 2;
33    }
34 };
```

5   Graph

5.1   Bellman-Ford

```
1 /*SPA - Bellman-Ford*/
2 #define inf 99999 //define by you maximum
     edges weight
3 vector<vector<int> > edges;
4 vector<int> dist;
5 vector<int> ancestor;
6 void BellmanFord(int start,int node){
7     dist[start] = 0;
8     for(int it = 0; it < node-1; it++){
9         for(int i = 0; i < node; i++){
10            for(int j = 0; j < node; j++){
11                if(edges[i][j] != -1){
12                    if(dist[i] + edges[i][j]
                         < dist[j]){
13                        dist[j] = dist[i] +
                             edges[i][j];
14                        ancestor[j] = i;
15                    }
16                }
17            }
18        }
19    }
20
21    for(int i = 0; i < node; i++)  //
             negative cycle detection
22        for(int j = 0; j < node; j++)
23            if(dist[i] + edges[i][j] < dist[
                 j])
24            {
25                cout<<"Negative cycle!"<<
                     endl;
26                return;
27            }
28 }
29 int main(){
30    int node;
31    cin>>node;
32    edges.resize(node,vector<int>(node,inf))
         ;
33    dist.resize(node,inf);
34    ancestor.resize(node,-1);
35    int a,b,d;
36    while(cin>>a>>b>>d){
37        /*input: source destination weight*/
38        if(a == -1 && b == -1 && d == -1)
39            break;
40        edges[a][b] = d;
41    }
42    int start;
43    cin>>start;
44    BellmanFord(start,node);
45    return 0;
46 }
```

5.2   BFS-queue

```
1 /*BFS - queue version*/
2 void BFS(vector<int> &result, vector<pair<
     int, int>> edges, int node, int start)
3 {
4     vector<int> pass(node, 0);
5     queue<int> q;
6     queue<int> p;
7     q.push(start);
8     int count = 1;
9     vector<pair<int, int>> newedges;
10    while (!q.empty())
11    {
12        pass[q.front()] = 1;
13        for (int i = 0; i < edges.size(); i
             ++)
14        {
15            if (edges[i].first == q.front()
                 && pass[edges[i].second] ==
                 0)
16            {
17                p.push(edges[i].second);
18                result[edges[i].second] =
                     count;
19            }
20            else if (edges[i].second == q.
                 front() && pass[edges[i].
                 first] == 0)
21            {
22                p.push(edges[i].first);
23                result[edges[i].first] =
                     count;
24            }
25            else
26                newedges.push_back(edges[i])
                     ;
27        }
28        edges = newedges;
29        newedges.clear();
30        q.pop();
31        if (q.empty() == true)
32        {
33            q = p;
34            queue<int> tmp;
35            p = tmp;
36            count++;
37        }
38    }
39 }
40 int main()
41 {
42    int node;
43    cin >> node;
44    vector<pair<int, int>> edges;
45    int a, b;
46    while (cin >> a >> b)
47    {
48        /*a = b = -1 means input edges ended
             */
49        if (a == -1 && b == -1)
50            break;
51        edges.push_back(pair<int, int>(a, b)
             );
52    }
53    vector<int> result(node, -1);
54    BFS(result, edges, node, 0);
55
```

```
56        return 0;
57 }
```

5.3 DFS-rec

```cpp
1  /*DFS - Recursive version*/
2  map<pair<int,int>,int> edges;
3  vector<int> pass;
4  vector<int> route;
5  void DFS(int start){
6      pass[start] = 1;
7      map<pair<int,int>,int>::iterator iter;
8      for(iter = edges.begin(); iter != edges.
          end(); iter++){
9          if((*iter).first.first == start &&
              (*iter).second == 0 && pass[(*
              iter).first.second] == 0){
10             route.push_back((*iter).first.
                  second);
11             DFS((*iter).first.second);
12         }
13         else if((*iter).first.second ==
              start && (*iter).second == 0 &&
              pass[(*iter).first.first] == 0){
14             route.push_back((*iter).first.
                  first);
15             DFS((*iter).first.first);
16         }
17     }
18 }
19 int main(){
20     int node;
21     cin>>node;
22     pass.resize(node,0);
23     int a,b;
24     while(cin>>a>>b){
25         if(a == -1 && b == -1)
26             break;
27         edges.insert(pair<pair<int,int>,int
              >(pair<int,int>(a,b),0));
28     }
29     int start;
30     cin>>start;
31     route.push_back(start);
32     DFS(start);
33     return 0;
34 }
```

5.4 Dijkstra

```cpp
1  /*SPA - Dijkstra*/
2  const int MAXN = 1e5 + 3;
3  const int inf = INT_MAX;
4  typedef pair<int, int> pii;
5  vector<vector<pii>> weight;
6  vector<int> isDone(MAXN, false), dist,
      ancestor;
7  void dijkstra(int s)
8  {
```

```cpp
9      priority_queue<pii, vector<pii>, greater
          <pii>> pq;
10     pq.push(pii(0, s));
11     ancestor[s] = -1;
12     while (!pq.empty())
13     {
14         int u = pq.top().second;
15         pq.pop();
16
17         isDone[u] = true;
18
19         for (auto &pr : weight[u])
20         {
21             int v = pr.first, w = pr.second;
22
23             if (!isDone[v] && dist[u] + w <
                  dist[v])
24             {
25                 dist[v] = dist[u] + w;
26                 pq.push(pii(dist[v], v));
27                 ancestor[v] = u;
28             }
29         }
30     }
31 }
32 // weight[a - 1].push_back(pii(b - 1, w));
33 // weight[b - 1].push_back(pii(a - 1, w));
34 // dist.resize(n, inf);
35 // ancestor.resize(n, -1);
36 // dist[0] = 0;
37 // dijkstra(0);
```

5.5 Euler circuit

```cpp
1  /*Euler circuit*/
2  /*From NTU kiseki*/
3  /*G is graph, vis is visited, la is path*/
4  bool vis[ N ]; size_t la[ K ];
5  void dfs( int u, vector< int >& vec ) {
6      while ( la[ u ] < G[ u ].size() ) {
7          if( vis[ G[ u ][ la[ u ] ].second ]
              ) {
8              ++ la[ u ];
9              continue;
10         }
11         int v = G[ u ][ la[ u ] ].first;
12         vis[ G[ u ][ la[ u ] ].second ] = true;
13         ++ la[ u ]; dfs( v, vec );
14         vec.push_back( v );
15     }
16 }
```

5.6 Floyd-warshall

```cpp
1  /*SPA - Floyd-Warshall*/
2  #define inf 99999
3  void floyd_warshall(vector<vector<int>>&
      distance, vector<vector<int>>& ancestor,
      int n){
4      for (int k = 0; k < n; k++){
```

```cpp
5          for (int i = 0; i < n; i++){
6              for (int j = 0; j < n; j++){
7                  if(distance[i][k] + distance
                      [k][j] < distance[i][j])
                      {
8                      distance[i][j] =
                          distance[i][k] +
                          distance[k][j];
9                      ancestor[i][j] =
                          ancestor[k][j];
10                 }
11             }
12         }
13     }
14 }
15 int main(){
16     int n;
17     cin >> n;
18     int a, b, d;
19     vector<vector<int>> distance(n, vector<
          int>(n,99999));
20     vector<vector<int>> ancestor(n, vector<
          int>(n,-1));
21     while(cin>>a>>b>>d){
22         if(a == -1 && b == -1 && d == -1)
23             break;
24         distance[a][b] = d;
25         ancestor[a][b] = a;
26     }
27     for (int i = 0; i < n; i++)
28         distance[i][i] = 0;
29     floyd_warshall(distance, ancestor, n);
30     /*Negative cycle detection*/
31     for (int i = 0; i < n; i++){
32         if(distance[i][i] < 0){
33             cout << "Negative cycle!" <<
                  endl;
34             break;
35         }
36     }
37     return 0;
38 }
```

5.7 Hamilton_cycle

```cpp
1  /*find hamilton cycle*/
2  void hamilton(vector<vector<int>> gp, int k,
      int cur, vector<int>& solution,vector<
      bool> pass,bool& flag){
3      if(k == gp.size()-1){
4          if(gp[cur][1] == 1){
5              cout << 1 << " ";
6              while(cur != 1){
7                  cout << cur << " ";
8                  cur = solution[cur];
9              }
10             cout << cur << endl;
11             flag = true;
12             return;
13         }
14     }
15     for (int i = 0; i < gp[cur].size() && !
          flag; i++){
```

```cpp
16         if(gp[cur][i] == 1 && !pass[i]){
17             pass[i] = true;
18             solution[i] = cur;
19             hamilton(gp, k + 1, i, solution,
                  pass,flag);
20             pass[i] = false;
21         }
22     }
23 }
24 int main(){
25     int n;
26     while(cin>>n){
27         int a,b;
28         bool end = false;
29         vector<vector<int>> gp(n+1,vector<
              int>(n+1,0));
30         while(cin>>a>>b){
31             if(a == 0 && b == 0)
32                 break;
33             gp[a][b] = 1;
34             gp[b][a] = 1;
35         }
36         vector<int> solution(n + 1, -1);
37         vector<bool> pass(n + 1, false);
38         solution[1] = 0;
39         pass[1] = true;
40         bool flag = false;
41         hamilton(gp, 1,1 ,solution,pass,flag
              );
42         if(!flag)
43             cout << "N" << endl;
44     }
45     return 0;
46 }
47 /*
48 4
49 1 2
50 2 3
51 2 4
52 3 4
53 3 1
54 0 0
55 output: 1 3 4 2 1
56 */
```

5.8 Kruskal

```cpp
1  /*mst - Kruskal*/
2  struct edges{
3      int from;
4      int to;
5      int weight;
6      friend bool operator < (edges a, edges b
          ){
7          return a.weight > b.weight;
8      }
9  };
10 int find(int x,vector<int>& union_set){
11     if(x != union_set[x])
12         union_set[x] = find(union_set[x],
              union_set);
13     return union_set[x];
14 }
```

```cpp
15 void merge(int a,int b,vector<int>&
       union_set){
16     int pa = find(a, union_set);
17     int pb = find(b, union_set);
18     if(pa != pb)
19         union_set[pa] = pb;
20 }
21 void kruskal(priority_queue<edges> pq,int n)
       {
22     vector<int> union_set(n, 0);
23     for (int i = 0; i < n; i++)
24         union_set[i] = i;
25     int edge = 0;
26     int cost = 0; //evaluate cost of mst
27     while(!pq.empty() && edge < n - 1){
28         edges cur = pq.top();
29         int from = find(cur.from, union_set)
           ;
30         int to = find(cur.to, union_set);
31         if(from != to){
32             merge(from, to, union_set);
33             edge += 1;
34             cost += cur.weight;
35         }
36         pq.pop();
37     }
38     if(edge < n-1)
39         cout << "No mst" << endl;
40     else
41         cout << cost << endl;
42 }
43 int main(){
44     int n;
45     cin >> n;
46     int a, b, d;
47     priority_queue<edges> pq;
48     while(cin>>a>>b>>d){
49         if(a == -1 && b == -1 && d == -1)
50             break;
51         edges tmp;
52         tmp.from = a;
53         tmp.to = b;
54         tmp.weight = d;
55         pq.push(tmp);
56     }
57     kruskal(pq, n);
58     return 0;
59 }
```

5.9   Prim

```cpp
1 /*mst - Prim*/
2 #define inf 99999
3 struct edges{
4     int from;
5     int to;
6     int weight;
7     friend bool operator < (edges a, edges b
       ){
8         return a.weight > b.weight;
9     }
10 };
```

```cpp
11 void Prim(vector<vector<int>> gp,int n,int
       start){
12     vector<bool> pass(n,false);
13     int edge = 0;
14     int cost = 0; //evaluate cost of mst
15     priority_queue<edges> pq;
16     for (int i = 0; i < n; i++){
17         if(gp[start][i] != inf){
18             edges tmp;
19             tmp.from = start;
20             tmp.to = i;
21             tmp.weight = gp[start][i];
22             pq.push(tmp);
23         }
24     }
25     pass[start] = true;
26     while(!pq.empty() && edge < n-1){
27         edges cur = pq.top();
28         pq.pop();
29         if(!pass[cur.to]){
30             for (int i = 0; i < n; i++){
31                 if(gp[cur.to][i] != inf){
32                     edges tmp;
33                     tmp.from = cur.to;
34                     tmp.to = i;
35                     tmp.weight = gp[cur.to][
                       i];
36                     pq.push(tmp);
37                 }
38             }
39             pass[cur.to] = true;
40             edge += 1;
41             cost += cur.weight;
42         }
43     }
44     if(edge < n-1)
45         cout << "No mst" << endl;
46     else
47         cout << cost << endl;
48 }
49 int main(){
50     int n;
51     cin >> n;
52     int a, b, d;
53     vector<vector<int>> gp(n,vector<int>(n,
       inf));
54     while(cin>>a>>b>>d){
55         if(a == -1 && b == -1 && d == -1)
56             break;
57         if(gp[a][b] > d)
58             gp[a][b] = d;
59     }
60     Prim(gp,n,0);
61     return 0;
62 }
```

5.10   Union_find

```cpp
1 int find(int x, vector<int> &union_set)
2 {
3     if (union_set[x] != x)
4         union_set[x] = find(union_set[x],
           union_set); //compress path
```

```cpp
5     return union_set[x];
6 }
7 void merge(int x, int y, vector<int> &
       union_set, vector<int> &rank)
8 {
9     int rx, ry;
10     rx = find(x, union_set);
11     ry = find(y, union_set);
12     if (rx == ry)
13         return;
14     /*merge by rank -> always merge small
           tree to big tree*/
15     if (rank[rx] > rank[ry])
16         union_set[ry] = rx;
17     else
18     {
19         union_set[rx] = ry;
20         if (rank[rx] == rank[ry])
21             ++rank[ry];
22     }
23 }
24 int main()
25 {
26     int node;
27     cin >> node; //Input Node number
28     vector<int> union_set(node, 0);
29     vector<int> rank(node, 0);
30     for (int i = 0; i < node; i++)
31         union_set[i] = i;
32     int edge;
33     cin >> edge; //Input Edge number
34     for (int i = 0; i < edge; i++)
35     {
36         int a, b;
37         cin >> a >> b;
38         merge(a, b, union_set, rank);
39     }
40     /*build party*/
41     vector<vector<int>> party(node, vector<
       int>(0));
42     for (int i = 0; i < node; i++)
43         party[find(i, union_set)].push_back(
           i);
44 }
```

6   Mathematics

6.1   Catalan

Catalan number

- 0~19項的catalan number
  - 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786,
    208012, 742900, 2674440, 9694845, 35357670, 129644790,
    477638700, 1767263190
  - 公式: $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$

6.2   Combination

```cpp
1 /*input type string or vector*/
2 for (int i = 0; i < (1 << input.size()); ++i
       )
3 {
4     string testCase = "";
5     for (int j = 0; j < input.size(); ++j)
6         if (i & (1 << j))
7             testCase += input[j];
8 }
```

6.3   Extended Euclidean

```cpp
1 // ax + by = gcd(a,b)
2 pair<long long, long long> extgcd(long long
       a, long long b)
3 {
4     if (b == 0)
5         return {1, 0};
6     long long k = a / b;
7     pair<long long, long long> p = extgcd(b,
           a - k * b);
8     //cout << p.first << " " << p.second <<
       endl;
9     //cout << "商數(k)=  " << k << endl <<
       endl;
10    return {p.second, p.first - k * p.second
           };
11 }
12
13 int main()
14 {
15     int a, b;
16     cin >> a >> b;
17     pair<long long, long long> xy = extgcd(a
       , b); //(x0,y0)
18     cout << xy.first << " " << xy.second <<
       endl;
19     cout << xy.first << " * " << a << " + "
           << xy.second << " * " << b << endl;
20     return 0;
21 }
22 // ax + by = gcd(a,b) * r
23 /*find |x|+|y| -> min*/
24 int main()
25 {
26     long long r, p, q; /*px+qy = r*/
27     int cases;
28     cin >> cases;
29     while (cases--)
30     {
31         cin >> r >> p >> q;
32         pair<long long, long long> xy =
               extgcd(q, p); //(x0,y0)
33         long long ans = 0, tmp = 0;
34         double k, k1;
35         long long s, s1;
36         k = 1 - (double)(r * xy.first) / p;
37         s = round(k);
38         ans = llabs(r * xy.first + s * p) +
               llabs(r * xy.second - s * q);
```

```
39        k1 = -(double)(r * xy.first) / p;
40        s1 = round(k1);
41        /*cout << k << endl << k1 << endl;
42            cout << s << endl << s1 << endl;
                */
43        tmp = llabs(r * xy.first + s1 * p) +
              llabs(r * xy.second - s1 * q);
44        ans = min(ans, tmp);
45
46        cout << ans << endl;
47    }
48    return 0;
49 }
```

```
20 void DecToHex(int p) //10 to 16
21 {
22     char *l = new (char);
23     sprintf(l, "%X", p);
24     //int l_intResult = stoi(l);
25     cout << l << "\n";
26     //return l_intResult;
27 }
```

6.4   Fermat

Little Fermat

- $a^{(p-1)} \equiv 1\,(mod\,p)$ <=> $a * a^{(p-2)} \equiv 1$
  - $a^{(p-2)} \equiv 1/a$
- 同餘因數定理
  - $a \equiv b\,(mod\,p)$ <=> $k|a-b$
- 同餘加法性質
  - $a \equiv b\,(mod\,p)$ and $c \equiv d\,(mod\,p)$ <=> $a+c \equiv b+d$
- 同餘相乘性質
  - $a \equiv b\,(mod\,p)$ and $c \equiv d\,(mod\,p)$ <=> $ac \equiv bd\,(mod$
- 同餘次方性質
  - $a \equiv b\,(mod\,p)$ <=> $a^n \equiv b^n\,(mod\,p)$
- 同餘倍方性質
  - $a \equiv b\,(mod\,p)$ <=> $am \equiv bm\,(mod\,p)$

6.5   Hex to Dec

```
1 int HextoDec(string num) //16 to 10
2 {
3     int base = 1;
4     int temp = 0;
5     for (int i = num.length() - 1; i >= 0; i
          --)
6     {
7         if (num[i] >= '0' && num[i] <= '9')
8         {
9             temp += (num[i] - 48) * base;
10            base = base * 16;
11        }
12        else if (num[i] >= 'A' && num[i] <=
              'F')
13        {
14            temp += (num[i] - 55) * base;
15            base = base * 16;
16        }
17    }
18    return temp;
19 }
```

6.6   Log

```
1 double mylog(double a, double base)
2 {
3     //a 的對數底數 b = 自然對數 (a) / 自然對
          數 (b)。
4     return log(a) / log(base);
5 }
```

6.7   Mod

```
1 int pow_mod(int a, int n, int m) // a ^ n
      mod m;
2 { // a, n, m < 10 ^ 9
3     if (n == 0)
4         return 1;
5     int x = pow_mid(a, n / 2, m);
6     long long ans = (long long)x * x % m;
7     if (n % 2 == 1)
8         ans = ans * a % m;
9     return (int)ans;
10 }
```

6.8   Mod 性質

加法 : $(a+b)\bmod p = (a\bmod p + b\bmod p)\bmod p$

減法 : $(a-b)\bmod p = (a\bmod p - b\bmod p + p)\bmod p$

乘法 : $(a*b)\bmod p = (a\bmod p \cdot b\bmod p)\bmod p$

次方 : $(a^b)\bmod p = ((a\bmod p)^b)\bmod p$

加法結合律 : $((a+b)\bmod p + c)\bmod p = (a+(b+c))\bmod p$

乘法結合律 : $((a\cdot b)\bmod p \cdot c)\bmod p = (a\cdot(b\cdot c))\bmod p$

加法交換律 : $(a+b)\bmod p = (b+a)\bmod p$

乘法交換律 : $(a\cdot b)\bmod p = (b\cdot a)\bmod p$

結合律 : $((a+b)\bmod p \cdot c) = ((a\cdot c)\bmod p + (b\cdot c)\bmod p)\bmod p$

如果 $a \equiv b(\bmod m)$，我們會說 a, b 在模 m 下同餘。

以下為性質 :

- 整除性 : $a \equiv b \pmod m \Rightarrow c\cdot m = a - b, c \in \mathbb{Z}$
  $\Rightarrow a \equiv b \pmod m \Rightarrow m\,|\,a-b$
- 遞移性 : 若 $a \equiv b \pmod c$, $b \equiv d \pmod c$
  則 $a \equiv d(\bmod c)$
- 保持基本運算 :
  $$\begin{cases} a \equiv b(\bmod m) \\ c \equiv d(\bmod m) \end{cases} \Rightarrow \begin{cases} a \pm c \equiv b \pm d(\bmod m) \\ a\cdot c \equiv b\cdot d(\bmod m) \end{cases}$$
- 放大縮小模數 :
  $k \in \mathbb{Z}^+, a \equiv b \pmod m \Leftrightarrow k\cdot a \equiv k\cdot b \pmod {k\cdot m}$
  模逆元是取模下的反元素，即要找到 a^{-1} 使得 $aa^{-1} \equiv 1\bmod c$。

  整數 a 在 $\bmod c$ 下要有模反元素的充分必要條件為 a, c 互質。

  模逆元如果存在會有無限個，任意兩相鄰模逆元相差 c。

費馬小定理

給定一個質數 p 及一個整數 a，那麼 : $a^p \equiv a(\bmod\,p)$ 如果 $gcd(a,p) = 1$，則 : $a^{p-1} \equiv 1(\bmod\,p)$

歐拉定理

歐拉定理是比較 general 版本的費馬小定理，給定兩個整數 n 和 a，如果 $gcd(a,n) = 1$ $a^{\Phi(n)} \equiv 1(\bmod\,n)$ 如果 n 是質數，$\Phi(n) = n - 1$，也就是費馬小定理。

Wilson's theorem

給定一個質數 p，則 : $(p-1)! \equiv -1(\bmod\,p)$

6.9   PI

```
1 #define PI acos(-1)
2 #define PI M_PI
3 const double PI = atan2(0.0, -1.0);
```

6.10   Prime table

```
1 const int maxn = sqrt(INT_MAX);
```

```
2 vector<int> p;
3 bitset<maxn> is_notp;
4 void PrimeTable()
5 {
6     is_notp.reset();
7     is_notp[0] = is_notp[1] = 1;
8     for (int i = 2; i <= maxn; ++i)
9     {
10        if (!is_notp[i])
11            p.push_back(i);
12        for (int j = 0; j < (int)p.size();
              ++j)
13        {
14            if (i * p[j] > maxn)
15                break;
16            is_notp[i * p[j]] = 1;
17            if (i % p[j] == 0)
18                break;
19        }
20    }
21 }
```

6.11   Prime 判斷

```
1 // n < 4759123141      chk = [2, 7, 61]
2 // n < 1122004669633   chk = [2, 13, 23,
      1662803]
3 // n < 2^64            chk = [2, 325, 9375,
      28178, 450775, 9780504, 1795265022]
4 vector<long long> chk = {};
5 long long fmul(long long a, long long n,
      long long mod)
6 {
7     long long ret = 0;
8     for (; n; n >>= 1)
9     {
10        if (n & 1)
11            (ret += a) %= mod;
12        (a += a) %= mod;
13    }
14    return ret;
15 }
16
17 long long fpow(long long a, long long n,
      long long mod)
18 {
19    long long ret = 1LL;
20    for (; n; n >>= 1)
21    {
22        if (n & 1)
23            ret = fmul(ret, a, mod);
24        a = fmul(a, a, mod);
25    }
26    return ret;
27 }
28 bool check(long long a, long long u, long
      long n, int t)
29 {
30    a = fpow(a, u, n);
31    if (a == 0)
32        return true;
33    if (a == 1 || a == n - 1)
34        return true;
```

```
35    for (int i = 0; i < t; ++i)
36    {
37        a = fmul(a, a, n);
38        if (a == 1)
39            return false;
40        if (a == n - 1)
41            return true;
42    }
43    return false;
44 }
45 bool is_prime(long long n)
46 {
47    if (n < 2)
48        return false;
49    if (n % 2 == 0)
50        return n == 2;
51    long long u = n - 1;
52    int t = 0;
53    for (; u & 1; u >>= 1, ++t)
54        ;
55    for (long long i : chk)
56    {
57        if (!check(i, u, n, t))
58            return false;
59    }
60    return true;
61 }
62
63 // if (is_prime(int num)) // true == prime
   //    反之亦然
```

6.12 Round(小數)

```
1 double myround(double number, unsigned int
     bits)
2 {
3    LL integerPart = number;
4    number -= integerPart;
5    for (unsigned int i = 0; i < bits; ++i)
6        number *= 10;
7    number = (LL)(number + 0.5);
8    for (unsigned int i = 0; i < bits; ++i)
9        number /= 10;
10   return integerPart + number;
11 }
12 //printf("%.1f\n", round(3.4515239, 1));
```

6.13 二分逼近法

```
1 #define eps 1e-14
2 void half_interval()
3 {
4    double L = 0, R = /*區間*/, M;
5    while (R - L >= eps)
6    {
7        M = (R + L) / 2;
8        if (/*函數*/ > /*方程式目標*/)
9            L = M;
10       else
```

```
11           R = M;
12       }
13   printf("%.3lf\n", R);
14 }
```

6.14 公式

$$S_n = \frac{a(1-r^n)}{1-r} \quad a_n = \frac{a_1 + a_n}{2} \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

6.15 四則運算

```
1 string s = ""; //開頭是負號要補0
2 long long int DFS(int le, int ri) // (0,
     string final index)
3 {
4    int c = 0;
5    for (int i = ri; i >= le; i--)
6    {
7        if (s[i] == ')')
8            c++;
9        if (s[i] == '(')
10           c--;
11       if (s[i] == '+' && c == 0)
12           return DFS(le, i - 1) + DFS(i +
                1, ri);
13       if (s[i] == '-' && c == 0)
14           return DFS(le, i - 1) - DFS(i +
                1, ri);
15   }
16   for (int i = ri; i >= le; i--)
17   {
18       if (s[i] == ')')
19           c++;
20       if (s[i] == '(')
21           c--;
22       if (s[i] == '*' && c == 0)
23           return DFS(le, i - 1) * DFS(i +
                1, ri);
24       if (s[i] == '/' && c == 0)
25           return DFS(le, i - 1) / DFS(i +
                1, ri);
26       if (s[i] == '%' && c == 0)
27           return DFS(le, i - 1) % DFS(i +
                1, ri);
28   }
29   if ((s[le] == '(') && (s[ri] == ')'))
30       return DFS(le + 1, ri - 1); //去除刮
              號
31   if (s[le] == ' ' && s[ri] == ' ')
32       return DFS(le + 1, ri - 1); //去除左
              右兩邊空格
33   if (s[le] == ' ')
34       return DFS(le + 1, ri); //去除左邊空
              格
35   if (s[ri] == ' ')
```

```
36       return DFS(le, ri - 1); //去除右邊空
              格
37   long long int num = 0;
38   for (int i = le; i <= ri; i++)
39       num = num * 10 + s[i] - '0';
40   return num;
41 }
```

6.16 數字乘法組合

```
1 void dfs(int j, int old, int num, vector<int
     > com, vector<vector<int>> &ans)
2 {
3    for (int i = j; i <= sqrt(num); i++)
4    {
5        if (old == num)
6            com.clear();
7        if (num % i == 0)
8        {
9            vector<int> a;
10           a = com;
11           a.push_back(i);
12           finds(i, old, num / i, a, ans);
13           a.push_back(num / i);
14           ans.push_back(a);
15       }
16   }
17 }
18 vector<vector<int>> ans;
19 vector<int> zero;
20 dfs(2, num, num, zero, ans);
21 /*num 為 input 數字*/
22 for (int i = 0; i < ans.size(); i++)
23 {
24     for (int j = 0; j < ans[i].size() - 1; j
           ++)
25         cout << ans[i][j] << " ";
26     cout << ans[i][ans[i].size() - 1] <<
           endl;
27 }
```

6.17 數字加法組合

```
1 void recur(int i, int n, int m, vector<int>
     &out, vector<vector<int>> &ans)
2 {
3    if (n == 0)
4    {
5        for (int i : out)
6            if (i > m)
7                return;
8        ans.push_back(out);
9    }
10   for (int j = i; j <= n; j++)
11   {
12       out.push_back(j);
13       recur(j, n - j, m, out, ans);
14       out.pop_back();
15   }
```

```
16 }
17 vector<vector<int>> ans;
18 vector<int> zero;
19 recur(1, num, num, zero, ans);
20 // num 為 input 數字
21 for (int i = 0; i < ans.size(); i++)
22 {
23     for (int j = 0; j < ans[i].size() - 1; j
           ++)
24         cout << ans[i][j] << " ";
25     cout << ans[i][ans[i].size() - 1] <<
           endl;
26 }
```

6.18 羅馬數字

```
1 int romanToInt(string s)
2 {
3    unordered_map<char, int> T;
4    T['I'] = 1;
5    T['V'] = 5;
6    T['X'] = 10;
7    T['L'] = 50;
8    T['C'] = 100;
9    T['D'] = 500;
10   T['M'] = 1000;
11
12   int sum = T[s.back()];
13   for (int i = s.length() - 2; i >= 0; --i
         )
14   {
15       if (T[s[i]] < T[s[i + 1]])
16           sum -= T[s[i]];
17       else
18           sum += T[s[i]];
19   }
20   return sum;
21 }
```

6.19 質因數分解

```
1 void primeFactorization(int n) // 配合質數表
2 {
3    for (int i = 0; i < (int)p.size(); ++i)
4    {
5        if (p[i] * p[i] > n)
6            break;
7        if (n % p[i])
8            continue;
9        cout << p[i] << ' ';
10       while (n % p[i] == 0)
11           n /= p[i];
12   }
13   if (n != 1)
14       cout << n << ' ';
15   cout << '\n';
16 }
```

7 Other

7.1 binary search 三類變化

```
1  // 查找和目標值完全相等的數
2  int find(vector<int> &nums, int target)
3  {
4      int left = 0, right = nums.size();
5      while (left < right)
6      {
7          int mid = left + (right - left) / 2;
8          if (nums[mid] == target)
9              return mid;
10         else if (nums[mid] < target)
11             left = mid + 1;
12         else
13             right = mid;
14     }
15     return -1;
16 }
17 // 找第一個不小於目標值的數 == 找最後一個小
       於目標值的數
18 /*(lower_bound)*/
19 int find(vector<int> &nums, int target)
20 {
21     int left = 0, right = nums.size();
22     while (left < right)
23     {
24         int mid = left + (right - left) / 2;
25         if (nums[mid] < target)
26             left = mid + 1;
27         else
28             right = mid;
29     }
30     return right;
31 }
32 // 找第一個大於目標值的數 == 找最後一個不大
       於目標值的數
33 /*(upper_bound)*/
34 int find(vector<int> &nums, int target)
35 {
36     int left = 0, right = nums.size();
37     while (left < right)
38     {
39         int mid = left + (right - left) / 2;
40         if (nums[mid] <= target)
41             left = mid + 1;
42         else
43             right = mid;
44     }
45     return right;
46 }
```

7.2 heap sort

```
1  void MaxHeapify(vector<int> &array, int root
       , int length)
2  {
3      int left = 2 * root,
4          right = 2 * root + 1,
5          largest;
6      if (left <= length && array[left] >
          array[root])
7          largest = left;
8      else
9          largest = root;
10     if (right <= length && array[right] >
          array[largest])
11         largest = right;
12     if (largest != root)
13     {
14         swap(array[largest], array[root]);
15         MaxHeapify(array, largest, length);
16     }
17 }
18 void HeapSort(vector<int> &array)
19 {
20     array.insert(array.begin(), 0);
21     for (int i = (int)array.size() / 2; i >=
          1; i--)
22         MaxHeapify(array, i, (int)array.size
             () - 1);
23     int size = (int)array.size() - 1;
24     for (int i = (int)array.size() - 1; i >=
          2; i--)
25     {
26         swap(array[1], array[i]);
27         size--;
28         MaxHeapify(array, 1, size);
29     }
30     array.erase(array.begin());
31 }
```

7.3 Merge sort

```
1  void Merge(vector<int> &arr, int front, int
       mid, int end)
2  {
3      vector<int> LeftSub(arr.begin() + front,
          arr.begin() + mid + 1);
4      vector<int> RightSub(arr.begin() + mid +
          1, arr.begin() + end + 1);
5      LeftSub.insert(LeftSub.end(), INT_MAX);
6      RightSub.insert(RightSub.end(), INT_MAX)
          ;
7      int idxLeft = 0, idxRight = 0;
8
9      for (int i = front; i <= end; i++)
10     {
11
12         if (LeftSub[idxLeft] <= RightSub[
             idxRight])
13         {
14             arr[i] = LeftSub[idxLeft];
15             idxLeft++;
16         }
17         else
18         {
19             arr[i] = RightSub[idxRight];
20             idxRight++;
21         }
22     }
23 }
```

```
24 void MergeSort(vector<int> &arr, int front,
       int end)
25 {
26     // front = 0 , end = arr.size() - 1
27     if (front < end)
28     {
29         int mid = (front + end) / 2;
30         MergeSort(arr, front, mid);
31         MergeSort(arr, mid + 1, end);
32         Merge(arr, front, mid, end);
33     }
34 }
```

7.4 Quick

```
1  int Partition(vector<int> &arr, int front,
       int end)
2  {
3      int pivot = arr[end];
4      int i = front - 1;
5      for (int j = front; j < end; j++)
6      {
7          if (arr[j] < pivot)
8          {
9              i++;
10             swap(arr[i], arr[j]);
11         }
12     }
13     i++;
14     swap(arr[i], arr[end]);
15     return i;
16 }
17 void QuickSort(vector<int> &arr, int front,
       int end)
18 {
19     // front = 0 , end = arr.size() - 1
20     if (front < end)
21     {
22         int pivot = Partition(arr, front,
             end);
23         QuickSort(arr, front, pivot - 1);
24         QuickSort(arr, pivot + 1, end);
25     }
26 }
```

7.5 Weighted Job Scheduling

```
1  struct Job
2  {
3      int start, finish, profit;
4  };
5  bool jobComparataor(Job s1, Job s2)
6  {
7      return (s1.finish < s2.finish);
8  }
9  int latestNonConflict(Job arr[], int i)
10 {
11     for (int j = i - 1; j >= 0; j--)
12     {
13         if (arr[j].finish <= arr[i].start)
```

```
14             return j;
15     }
16     return -1;
17 }
18 int findMaxProfit(Job arr[], int n)
19 {
20     sort(arr, arr + n, jobComparataor);
21     int *table = new int[n];
22     table[0] = arr[0].profit;
23     for (int i = 1; i < n; i++)
24     {
25         int inclProf = arr[i].profit;
26         int l = latestNonConflict(arr, i);
27         if (l != -1)
28             inclProf += table[l];
29         table[i] = max(inclProf, table[i -
             1]);
30     }
31     int result = table[n - 1];
32     delete[] table;
33
34     return result;
35 }
```

7.6 數獨解法

```
1  int getSquareIndex(int row, int column, int
       n)
2  {
3      return row / n * n + column / n;
4  }
5
6  bool backtracking(vector<vector<int>> &board
       , vector<vector<bool>> &rows, vector<
       vector<bool>> &cols,
7              vector<vector<bool>> &boxs
                 , int index, int n)
8  {
9      int n2 = n * n;
10     int rowNum = index / n2, colNum = index
          % n2;
11     if (index >= n2 * n2)
12         return true;
13
14     if (board[rowNum][colNum] != 0)
15         return backtracking(board, rows,
             cols, boxs, index + 1, n);
16
17     for (int i = 1; i <= n2; i++)
18     {
19         if (!rows[rowNum][i] && !cols[colNum
             ][i] && !boxs[getSquareIndex(
             rowNum, colNum, n)][i])
20         {
21             rows[rowNum][i] = true;
22             cols[colNum][i] = true;
23             boxs[getSquareIndex(rowNum,
                 colNum, n)][i] = true;
24             board[rowNum][colNum] = i;
25             if (backtracking(board, rows,
                 cols, boxs, index + 1, n))
26                 return true;
27             board[rowNum][colNum] = 0;
```

```
28            rows[rowNum][i] = false;
29            cols[colNum][i] = false;
30            boxs[getSquareIndex(rowNum,
                  colNum, n)][i] = false;
31        }
32    }
33    return false;
34 }
35 /*用法 main*/
36 int n = sqrt(數獨邊長大小) /*e.g. 9*9 n=3*/
37 vector<vector<int>> board(n * n + 1, vector<
      int>(n * n + 1, 0));
38 vector<vector<bool>> isRow(n * n + 1, vector
      <bool>(n * n + 1, false));
39 vector<vector<bool>> isColumn(n * n + 1,
      vector<bool>(n * n + 1, false));
40 vector<vector<bool>> isSquare(n * n + 1,
      vector<bool>(n * n + 1, false));
41
42 for (int i = 0; i < n * n; ++i)
43 {
44    for (int j = 0; j < n * n; ++j)
45    {
46        int number;
47        cin >> number;
48        board[i][j] = number;
49        if (number == 0)
50            continue;
51        isRow[i][number] = true;
52        isColumn[j][number] = true;
53        isSquare[getSquareIndex(i, j, n)][
              number] = true;
54    }
55 }
56 if (backtracking(board, isRow, isColumn,
      isSquare, 0, n))
57    /*有解答*/
58 else
59    /*解答*/
```

8   String

8.1   KMP

```
1 // 用在在一個 S 內查找一個詞 W 的出現位置
2 void ComputePrefix(string s, int next[])
3 {
4    int n = s.length();
5    int q, k;
6    next[0] = 0;
7    for (k = 0, q = 1; q < n; q++)
8    {
9        while (k > 0 && s[k] != s[q])
10            k = next[k];
11        if (s[k] == s[q])
12            k++;
13        next[q] = k;
14    }
15 }
16 void KMPMatcher(string text, string pattern)
```

```
17 {
18    int n = text.length();
19    int m = pattern.length();
20    int next[pattern.length()];
21    ComputePrefix(pattern, next);
22
23    for (int i = 0, q = 0; i < n; i++)
24    {
25        while (q > 0 && pattern[q] != text[i
              ])
26            q = next[q];
27        if (pattern[q] == text[i])
28            q++;
29        if (q == m)
30        {
31            cout << "Pattern occurs with
                  shift " << i - m + 1 << endl
                  ;
32            q = 0;
33        }
34    }
35 }
36 // string s = "abcdabcdebcd";
37 // string p = "bcd";
38 // KMPMatcher(s, p);
39 // cout << endl;
```

8.2   Min Edit Distance

```
1 int EditDistance(string a, string b)
2 {
3    vector<vector<int>> dp(a.size() + 1,
          vector<int>(b.size() + 1, 0));
4    int m = a.length(), n = b.length();
5    for (int i = 0; i < m + 1; i++)
6    {
7        for (int j = 0; j < n + 1; j++)
8        {
9            if (i == 0)
10                dp[i][j] = j;
11            else if (j == 0)
12                dp[i][j] = i;
13            else if (a[i - 1] == b[j - 1])
14                dp[i][j] = dp[i - 1][j - 1];
15            else
16                dp[i][j] = 1 + min(min(dp[i
                      - 1][j], dp[i][j - 1]),
                      dp[i - 1][j - 1]);
17        }
18    }
19    return dp[m][n];
20 }
```

8.3   Sliding window

```
1 string minWindow(string s, string t)
2 {
3    unordered_map<char, int> letterCnt;
4    for (int i = 0; i < t.length(); i++)
5        letterCnt[t[i]]++;
```

```
6    int minLength = INT_MAX, minStart = -1;
7    int left = 0, matchCnt = 0;
8    for (int i = 0; i < s.length(); i++)
9    {
10        if (--letterCnt[s[i]] >= 0)
11            matchCnt++;
12        while (matchCnt == t.length())
13        {
14            if (i - left + 1 < minLength)
15            {
16                minLength = i - left + 1;
17                minStart = left;
18            }
19            if (++letterCnt[s[left]] > 0)
20                matchCnt--;
21            left++;
22        }
23    }
24    return minLength == INT_MAX ? "" : s.
          substr(minStart, minLength);
25 }
```

8.4   Split

```
1 vector<string> mysplit(const string &str,
      const string &delim)
2 {
3    vector<string> res;
4    if ("" == str)
5        return res;
6
7    char *strs = new char[str.length() + 1];
8    char *d = new char[delim.length() + 1];
9    strcpy(strs, str.c_str());
10    strcpy(d, delim.c_str());
11    char *p = strtok(strs, d);
12    while (p)
13    {
14        string s = p;
15        res.push_back(s);
16        p = strtok(NULL, d);
17    }
18    return res;
19 }
```

9   data structure

9.1   Bigint

```
1 //台大 //非必要請用python
2 struct Bigint
3 {
4    static const int LEN = 60;        //
          maxLEN
5    static const int BIGMOD = 10000; //10為
          正常位數
6    int s;
```

```
7    int vl, v[LEN];
8    //  vector<int> v;
9    Bigint() : s(1) { vl = 0; }
10    Bigint(long long a)
11    {
12        s = 1;
13        vl = 0;
14        if (a < 0)
15        {
16            s = -1;
17            a = -a;
18        }
19        while (a)
20        {
21            push_back(a % BIGMOD);
22            a /= BIGMOD;
23        }
24    }
25    Bigint(string str)
26    {
27        s = 1;
28        vl = 0;
29        int stPos = 0, num = 0;
30        if (!str.empty() && str[0] == '-')
31        {
32            stPos = 1;
33            s = -1;
34        }
35        for (int i = str.length() - 1, q =
              1; i >= stPos; i--)
36        {
37            num += (str[i] - '0') * q;
38            if ((q *= 10) >= BIGMOD)
39            {
40                push_back(num);
41                num = 0;
42                q = 1;
43            }
44        }
45        if (num)
46            push_back(num);
47        n();
48    }
49    int len() const
50    {
51        return vl; //return SZ(v);
52    }
53    bool empty() const { return len() == 0;
          }
54    void push_back(int x)
55    {
56        v[vl++] = x; //v.PB(x);
57    }
58    void pop_back()
59    {
60        vl--; //v.pop_back();
61    }
62    int back() const
63    {
64        return v[vl - 1]; //return v.back();
65    }
66    void n()
67    {
68        while (!empty() && !back())
69            pop_back();
70    }
```

```
 71      void resize(int nl)
 72      {
 73          vl = nl;              //v.resize(nl);
 74          fill(v, v + vl, 0); //fill(ALL(v),
                  0);
 75      }
 76      void print() const
 77      {
 78          if (empty())
 79          {
 80              putchar('0');
 81              return;
 82          }
 83          if (s == -1)
 84              putchar('-');
 85          printf("%d", back());
 86          for (int i = len() - 2; i >= 0; i--)
 87              printf("%.4d", v[i]);
 88      }
 89      friend std::ostream &operator<<(std::
              ostream &out, const Bigint &a)
 90      {
 91          if (a.empty())
 92          {
 93              out << "0";
 94              return out;
 95          }
 96          if (a.s == -1)
 97              out << "-";
 98          out << a.back();
 99          for (int i = a.len() - 2; i >= 0; i
                  --)
100          {
101              char str[10];
102              snprintf(str, 5, "%.4d", a.v[i])
                      ;
103              out << str;
104          }
105          return out;
106      }
107      int cp3(const Bigint &b) const
108      {
109          if (s != b.s)
110              return s - b.s;
111          if (s == -1)
112              return -(-*this).cp3(-b);
113          if (len() != b.len())
114              return len() - b.len(); //int
115          for (int i = len() - 1; i >= 0; i--)
116              if (v[i] != b.v[i])
117                  return v[i] - b.v[i];
118          return 0;
119      }
120      bool operator<(const Bigint &b) const
121      {
122          return cp3(b) < 0;
123      }
124      bool operator<=(const Bigint &b) const
125      {
126          return cp3(b) <= 0;
127      }
128      bool operator==(const Bigint &b) const
129      {
130          return cp3(b) == 0;
131      }
132      bool operator!=(const Bigint &b) const
133      {
134          return cp3(b) != 0;
135      }
136      bool operator>(const Bigint &b) const
137      {
138          return cp3(b) > 0;
139      }
140      bool operator>=(const Bigint &b) const
141      {
142          return cp3(b) >= 0;
143      }
144      Bigint operator-() const
145      {
146          Bigint r = (*this);
147          r.s = -r.s;
148          return r;
149      }
150      Bigint operator+(const Bigint &b) const
151      {
152          if (s == -1)
153              return -(-(*this) + (-b));
154          if (b.s == -1)
155              return (*this) - (-b);
156          Bigint r;
157          int nl = max(len(), b.len());
158          r.resize(nl + 1);
159          for (int i = 0; i < nl; i++)
160          {
161              if (i < len())
162                  r.v[i] += v[i];
163              if (i < b.len())
164                  r.v[i] += b.v[i];
165              if (r.v[i] >= BIGMOD)
166              {
167                  r.v[i + 1] += r.v[i] /
                          BIGMOD;
168                  r.v[i] %= BIGMOD;
169              }
170          }
171          r.n();
172          return r;
173      }
174      Bigint operator-(const Bigint &b) const
175      {
176          if (s == -1)
177              return -(-(*this) - (-b));
178          if (b.s == -1)
179              return (*this) + (-b);
180          if ((*this) < b)
181              return -(b - (*this));
182          Bigint r;
183          r.resize(len());
184          for (int i = 0; i < len(); i++)
185          {
186              r.v[i] += v[i];
187              if (i < b.len())
188                  r.v[i] -= b.v[i];
189              if (r.v[i] < 0)
190              {
191                  r.v[i] += BIGMOD;
192                  r.v[i + 1]--;
193              }
194          }
195          r.n();
196          return r;
197      }
198      Bigint operator*(const Bigint &b)
199      {
200          Bigint r;
201          r.resize(len() + b.len() + 1);
202          r.s = s * b.s;
203          for (int i = 0; i < len(); i++)
204          {
205              for (int j = 0; j < b.len(); j
                      ++)
206              {
207                  r.v[i + j] += v[i] * b.v[j];
208                  if (r.v[i + j] >= BIGMOD)
209                  {
210                      r.v[i + j + 1] += r.v[i
                              + j] / BIGMOD;
211                      r.v[i + j] %= BIGMOD;
212                  }
213              }
214          }
215          r.n();
216          return r;
217      }
218      Bigint operator/(const Bigint &b)
219      {
220          Bigint r;
221          r.resize(max(1, len() - b.len() + 1)
                  );
222          int oriS = s;
223          Bigint b2 = b; // b2 = abs(b)
224          s = b2.s = r.s = 1;
225          for (int i = r.len() - 1; i >= 0; i
                  --)
226          {
227              int d = 0, u = BIGMOD - 1;
228              while (d < u)
229              {
230                  int m = (d + u + 1) >> 1;
231                  r.v[i] = m;
232                  if ((r * b2) > (*this))
233                      u = m - 1;
234                  else
235                      d = m;
236              }
237              r.v[i] = d;
238          }
239          s = oriS;
240          r.s = s * b.s;
241          r.n();
242          return r;
243      }
244      Bigint operator%(const Bigint &b)
245      {
246          return (*this) - (*this) / b * b;
247      }
248  };
```

9.2 Matirx

```
  1  template <typename T>
  2  struct Matrix
  3  {
  4      using rt = std::vector<T>;
  5      using mt = std::vector<rt>;
  6      using matrix = Matrix<T>;
  7      int r, c; // [r][c]
  8      mt m;
  9      Matrix(int r, int c) : r(r), c(c), m(r,
              rt(c)) {}
 10      Matrix(mt a) { m = a, r = a.size(), c =
              a[0].size(); }
 11      rt &operator[](int i) { return m[i]; }
 12      matrix operator+(const matrix &a)
 13      {
 14          matrix rev(r, c);
 15          for (int i = 0; i < r; ++i)
 16              for (int j = 0; j < c; ++j)
 17                  rev[i][j] = m[i][j] + a.m[i
                          ][j];
 18          return rev;
 19      }
 20      matrix operator-(const matrix &a)
 21      {
 22          matrix rev(r, c);
 23          for (int i = 0; i < r; ++i)
 24              for (int j = 0; j < c; ++j)
 25                  rev[i][j] = m[i][j] - a.m[i
                          ][j];
 26          return rev;
 27      }
 28      matrix operator*(const matrix &a)
 29      {
 30          matrix rev(r, a.c);
 31          matrix tmp(a.c, a.r);
 32          for (int i = 0; i < a.r; ++i)
 33              for (int j = 0; j < a.c; ++j)
 34                  tmp[j][i] = a.m[i][j];
 35          for (int i = 0; i < r; ++i)
 36              for (int j = 0; j < a.c; ++j)
 37                  for (int k = 0; k < c; ++k)
 38                      rev.m[i][j] += m[i][k] *
                              tmp[j][k];
 39          return rev;
 40      }
 41      bool inverse() //逆矩陣判斷
 42      {
 43          Matrix t(r, r + c);
 44          for (int y = 0; y < r; y++)
 45          {
 46              t.m[y][c + y] = 1;
 47              for (int x = 0; x < c; ++x)
 48                  t.m[y][x] = m[y][x];
 49          }
 50          if (!t.gas())
 51              return false;
 52          for (int y = 0; y < r; y++)
 53              for (int x = 0; x < c; ++x)
 54                  m[y][x] = t.m[y][c + x] / t.
                          m[y][y];
 55          return true;
 56      }
 57      T gas() //行列式
 58      {
 59          vector<T> lazy(r, 1);
 60          bool sign = false;
 61          for (int i = 0; i < r; ++i)
 62          {
 63              if (m[i][i] == 0)
 64              {
```

```cpp
                int j = i + 1;
                while (j < r && !m[j][i])
                    j++;
                if (j == r)
                    continue;
                m[i].swap(m[j]);
                sign = !sign;
            }
            for (int j = 0; j < r; ++j)
            {
                if (i == j)
                    continue;
                lazy[j] = lazy[j] * m[i][i];
                T mx = m[j][i];
                for (int k = 0; k < c; ++k)
                    m[j][k] = m[j][k] * m[i][i] - m[i][k] * mx;
            }
        }
        T det = sign ? -1 : 1;
        for (int i = 0; i < r; ++i)
        {
            det = det * m[i][i];
            det = det / lazy[i];
            for (auto &j : m[i])
                j /= lazy[i];
        }
        return det;
    }
};
```

9.3 Trie

```cpp
// biginter字典數
struct BigInteger{
    static const int BASE = 100000000;
    static const int WIDTH = 8;
    vector<int> s;
    BigInteger(long long num = 0){
        *this = num;
    }
    BigInteger operator = (long long num){
        s.clear();
        do{
            s.push_back(num % BASE);
            num /= BASE;
        }while(num > 0);
        return *this;
    }
    BigInteger operator = (const string& str){
        s.clear();
        int x, len = (str.length() - 1) / WIDTH + 1;
        for(int i = 0; i < len;i++){
            int end = str.length() - i*WIDTH;
            int start = max(0, end-WIDTH);
            sscanf(str.substr(start, end-start).c_str(), "%d", &x);
            s.push_back(x);
        }
        return *this;
    }

    BigInteger operator + (const BigInteger& b) const{
        BigInteger c;
        c.s.clear();
        for(int i = 0, g = 0;;i++){
            if(g == 0 && i >= s.size() && i >= b.s.size()) break;
            int x = g;
            if(i < s.size()) x+=s[i];
            if(i < b.s.size()) x+=b.s[i];
            c.s.push_back(x % BASE);
            g = x / BASE;
        }
        return c;
    }
};

ostream& operator << (ostream &out, const BigInteger& x){
    out << x.s.back();
    for(int i = x.s.size()-2; i >= 0;i--){
        char buf[20];
        sprintf(buf, "%08d", x.s[i]);
        for(int j = 0; j< strlen(buf);j++){
            out << buf[j];
        }
    }
    return out;
}

istream& operator >> (istream &in, BigInteger& x){
    string s;
    if(!(in >> s))
        return in;
    x = s;
    return in;
}
struct Trie{
    int c[5000005][10];
    int val[5000005];
    int sz;
    int getIndex(char c){
        return c - '0';
    }
    void init(){
        memset(c[0], 0, sizeof(c[0]));
        memset(val, -1, sizeof(val));
        sz = 1;
    }
    void insert(BigInteger x, int v){
        int u = 0;
        int max_len_count = 0;
        int firstNum = x.s.back();
        char firstBuf[20];
        sprintf(firstBuf, "%d", firstNum);
        for(int j = 0; j < strlen(firstBuf);
            j++){
            int index = getIndex(firstBuf[j]);
            if(!c[u][index]){
                memset(c[sz], 0 , sizeof(c[
                    sz]));
                val[sz] = v;
                c[u][index] = sz++;
            }
            u = c[u][index];
            max_len_count++;
        }
        for(int i = x.s.size()-2; i >= 0;i
            --){
            char buf[20];
            sprintf(buf, "%08d", x.s[i]);
            for(int j = 0; j < strlen(buf)
                && max_len_count < 50;j++){
                int index = getIndex(buf[j])
                    ;
                if(!c[u][index]){
                    memset(c[sz], 0 , sizeof
                        (c[sz]));
                    val[sz] = v;
                    c[u][index] = sz++;
                }
                u = c[u][index];
                max_len_count++;
            }
            if(max_len_count >= 50){
                break;
            }
        }
    }
    int find(const char* s){
        int u = 0;
        int n = strlen(s);
        for(int i = 0 ; i < n;++i)
        {
            int index = getIndex(s[i]);
            if(!c[u][index]){
                return -1;
            }
            u = c[u][index];
        }
        return val[u];
    }
}
```

9.4 分數

```cpp
typedef long long ll;
struct fraction
{
  ll n, d;
  fraction(const ll &_n = 0, const ll &_d =
      1) : n(_n), d(_d)
  {
    ll t = __gcd(n, d);
    n /= t, d /= t;
    if (d < 0)
      n = -n, d = -d;
  }
  fraction operator-() const
  {
    return fraction(-n, d);
  }
  fraction operator+(const fraction &b)
      const
  {
    return fraction(n * b.d + b.n * d, d * b
        .d);
  }
  fraction operator-(const fraction &b)
      const
  {
    return fraction(n * b.d - b.n * d, d * b
        .d);
  }
  fraction operator*(const fraction &b)
      const
  {
    return fraction(n * b.n, d * b.d);
  }
  fraction operator/(const fraction &b)
      const
  {
    return fraction(n * b.d, d * b.n);
  }
  void print()
  {
    cout << n;
    if (d != 1)
      cout << "/" << d;
  }
};
```

TO DO WRITING NOT THINKING

Contents