序號:姓名

1. 以遞迴方式定義級數$1^3+3^3+5^3+......+(2n-1)^3, n\geq 1$
2. 拔 29882160 的因數中為 280 的倍數有幾個？
3. 以遞迴方式定義數列 1,7,13,19,25,… …,(6n-5)
4. 以遞迴方式定義數列 1,3,6,10,15, … …。
5. 設 A={9,18,27,36,45,54,63,72,81,90}，則 A 的子集合中包含 18 或 81 的元素有幾個?
6. A={2,3},B={4,8},列出 A 到 B 的所有關係
7. A={2,22,32,42}, B={2,3,5,7}, B 到 A 的關係中包含(2,2)或(7,22)者有幾個?
8. A={1,2,3,4,5,6,7}，以 0/1 矩陣來表示$R=\{(x,y)\mid x,y\in A 且 x\mid y\}$

1.
$$S(n)=1^3+3^3+......+(2n-1)^3$$
$$\begin{cases} S(1)=1, n=1 \\ S(n)=S(n-1)+(2n-1)^3, n>1 \end{cases}$$

2.
$$29882160=2^4 3^2 5^1 7^3 11^2$$
因數個數 $(4+1)(2+1)(1+1)(3+1)(2+1)=360$
$$280=2^3 5^1 7^1$$
29882160為280倍數有幾個
$$(4-3+1)(2+1)(1-1+1)(3-1+1)(2+1)=54$$

3.
$$a_n=6n-5$$
$$a_{n-1}=6(n-1)-5=6n-11$$
$$a_n-a_{n-1}=6\Rightarrow$$
$$\begin{cases} a_1=1, n=1 \\ a_n=a_{n-1}+6, n>1 \end{cases}$$

3. 1,3,6,10,15,… …

$$a_1=1$$
$$a_2=3$$
$$a_3=6$$
$$a_n=a_{n-1}+n, n>1$$
$$a_1=1, n=1$$

5. $2^9+2^9-2^8$

6. A={2,3},B={4,8} 2^{2x2} 個

個數	
0	Ø
1	{(2,4)},{2,8},{(3,4)},{(3,8)}
2	{(2,4),(2,8)},{(2,4),(3,4)},{(2,4),(3,8)},{(2,8),(3,4)},{(2,8)},(3,8)}
3	{},{},{},{}

7. $2^{15}+2^{15}-2^{14}$

	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	0	1	0	1	0	1	0
3			1			1	
4				1			
5					1		
6						1	
7							1

解遞迴式$a_n = 3a_{n-1}, a_0 = 2$

特徵 方程式 $x-3=0 \Rightarrow x=3$

$a_n = A\cdot 3^n \Rightarrow a_0 = A\cdot 3^0 = A = 2$

$a_n = 2\cdot 3^n$

解遞迴式$a_n = -2a_{n-1} + 15a_{n-2}, n \geq 2, a_0 = 3, a_1 = 1$

特徵 方程式 $x^2+2x-15=0 \Rightarrow (x-3)(x+5)=0 \Rightarrow x=3,-5$

$x \quad -3$

$x \quad 5$

$-3x \quad 5x$

$a_n = A\cdot 3^n + B\cdot(-5)^n \Rightarrow a_0 = A\cdot 3^0 + B(-5)^0 = A+B=3$

$n=1, \Rightarrow A\cdot 3^1 + B\cdot(-5)^1 = 1 \Rightarrow 3A-5B=1$

$A+B=3 \Rightarrow 3A+3B=9 \Rightarrow B=1 \Rightarrow A=2$

$a_n = 2\cdot 3^n + 1\cdot(-5)^n$

解遞迴式$a_n = a_{n-1} + 12a_{n-2}, n \ge 2, a_0 = 0, a_1 = 2$

$a_n - a_{n-1} - 12a_{n-2} = 0$

特徵 方程式 $x^2 - x - 12 = 0 \Rightarrow (x-4)(x+3) = 0 \Rightarrow x = 4, -3$

$a_n = A \cdot 4^n + B \cdot (-3)^n \Rightarrow a_0 = A \cdot 4^0 + B(-3)^0 = A + B = 0 \Rightarrow 3A + 3B = 0$

$n = 1, \Rightarrow A \cdot 4^1 + B \cdot (-3)^1 = 2 \Rightarrow 4A - 3B = 2 \Rightarrow 7A = 2 \Rightarrow A = \frac{2}{7}, B = \frac{-2}{7}$

$a_n = \frac{2}{7} \cdot 4^n + \frac{-2}{7} \cdot (-3)^n$

解遞迴式$a_n = 10a_{n-1} - 25a_{n-2}, n \ge 2, a_0 = -3, a_1 = -1$

已註解 [AU1]:

$a_n - 10a_{n-1} + 25a_{n-2} = 0$

特徵 方程式 $x^2 - 10x + 25 = 0 \Rightarrow (x-5)(x-5) = 0 \Rightarrow x = 5, 5$

$a_n = A \cdot 5^n + B \cdot n \cdot 5^n \Rightarrow a_0 = A \cdot 5^0 + B \cdot 0 \cdot (5)^0 = A = -3$

$n = 1, \Rightarrow A \cdot 5^1 + B \cdot 1 \cdot (5)^1 = -1 \Rightarrow 5A + 5B = -1 \Rightarrow 5B = -1 + 15 = 14, B = \frac{14}{5}$

$a_n = -3 \cdot 5^n + \frac{14}{5} \cdot n(-3)^n$

解遞迴式$a_n = 4a_{n-1} - 4a_{n-2}, n \ge 2, a_0 = 2, a_1 = 3$

解遞迴式$a_n = a_{n-1} + 9a_{n-2} - 9a_{n-3}, n \ge 3, a_0 = -1, a_1 = 0, a_2 = 1$

ex6.13

解遞迴式 $a_n = 6a_{n-1} - 3a_{n-2} - 10a_{n-3}, n \geq 3, a_0 = 0, a_1 = 1, a_2 = 2$

特徵方程式 $x^3\text{-}6x^2 + 3x + 10 = 0$

$(x^3\text{-}5x^2)\text{-}(x^2\text{-}3x\text{-}10) = 0$

$x^2(x\text{-}5)\text{-}(x+2)(x\text{-}5) = 0$

$(x-5)(x^2-x-2) = 0$

$(x-5)(x-2)(x+1) = 0 \Rightarrow x = 5,2,-1$

$a_n = A \cdot 5^n + B \cdot 2^n + C \cdot (-1)^n$

$a_0 = A + B + C = 0 ......(1)$

$a_1 = 5A + 2B - C = 1 ......(2)$

$a_2 = 25A + 4B + C = 2 ......(3)$

$(1)+(2) \Rightarrow 6A + 3B = 1 \Rightarrow 12A + 6B = 2$

$(2)+(3) \Rightarrow 30A + 6B = 3 \Rightarrow 18A = 1 \Rightarrow A = \dfrac{1}{18} \Rightarrow B = \dfrac{2}{9}, C = \dfrac{-5}{18}$

$a_n = \dfrac{1}{18} \cdot 5^n + \dfrac{2}{9} \cdot 2^n + \dfrac{-5}{18}(-1)^n$

6.16 解遞迴式 $a_n = 10a_{n-1} - 25a_{n-2}, n \geq 2, a_0 = 0, a_1 = -1$

解遞迴式 $a_n = 10a_{n-1} - 25a_{n-2}, n \geq 2, a_0 = 0, a_1 = -1$

特徵方程式 $x^2 - 10x + 25 = 0 \Rightarrow (x-5)^2 = 0 \Rightarrow x = 5,5$

$a_n = A \cdot 5^n + B \cdot n \cdot 5^n$

$a_0 = A = 0$

$a_1 = 5A + 5B = -1 \Rightarrow B = \dfrac{-1}{5}$

$a_n = \dfrac{-1}{5} \cdot n \cdot 5^n$

解 $a_n = a_{n-1} + 6, a_0 = 1$

$a_n = a_{n-1} + 6$

$a_n = (a_{n-2} + 6) + 6$

$a_n = (a_{n-3} + 6) + 6 + 6$

……

$a_n = (a_{n-n} + 6) + 6 + \ldots\ldots + 6$

$a_n = a_0 + 6n = 1 + 6n$

$$a_n = a_{n-1} + k \Rightarrow a_0 + \sum_{j=1}^{n} k = a_0 + nk$$

$$a_n = a_{n-1} + (n+3) = a_0 + \sum_{j=1}^{n} (n+3)$$

$$= a_0 + \sum_{j=1}^{n} n + \sum_{j=1}^{n} 3 = a_0 + \frac{n(n+1)}{2} + 3n$$

$$\sum_{j=1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{j=1}^{n} n = \frac{n(n+1)}{2}$$

解 $a_n = a_{n-1} + 3n^2, a_0 = 3$

$$a_n = a_0 + \sum_{j=1}^{n} 3n^2 = 3 + 3\sum_{j=1}^{n} n^2 = 3 + 3(\frac{n(N+1)(2n+1)}{6})$$

$$= 3 + \frac{n(N+1)(2n+1)}{2}$$