Computer Architecture–Homework IV

107 Fall semester, Chapter 5

5.11 Suppose an 8-bit data word stored in memory is 11000010. Using the Hamming algorithm, determine what check bits would be stored in memory with the data word. Show how you got your answer.

Sol:

i) Firstly, M data bits require K check bits such that $2^K \geq M + K + 1$.
8 data bits require 4 check bits since $2^3 \ngeq 8 + 3 + 1$ but $2^4 \geq 8 + 4 + 1$.

ii) Secondly, arrange the 8 data bits D8∼D1 and the 4 check bits C8, C4, C2, and C1 as follows.

Position$^{(10)}$	12	11	10	9	8	7	6	5	4	3	2	1
Position$^{(2)}$	1100	1011	1010	1001	1000	0111	0110	0101	0100	0011	0010	0001
Data bit	D8	D7	D6	D5		D4	D3	D2		D1		
Check bit					C8				C4		C2	C1

Those positions whose numbers are powers of 2 are designated as check bits. Those check bits are calculated as follows.
C8 = D8 ⊕ D7 ⊕ D6 ⊕ D5
C4 = D8 ⊕ D4 ⊕ D3 ⊕ D2
C2 = D7 ⊕ D6 ⊕ D4 ⊕ D3 ⊕ D1
C1 = D7 ⊕ D5 ⊕ D4 ⊕ D2 ⊕ D1

iii) Given data word 11000010, we can compute check bits as follows.

Position$^{(10)}$	12	11	10	9	8	7	6	5	4	3	2	1
Position$^{(2)}$	1100	1011	1010	1001	1000	0111	0110	0101	0100	0011	0010	0001
Data bit	D8 1	D7 1	D6 0	D5 0		D4 0	D3 0	D2 1		D1 0		
Check bit					C8 0				C4 0		C2 1	C1 0
Data stored	1	1	0	0	0	0	0	1	0	0	1	0

C8 = D8 ⊕ D7 ⊕ D6 ⊕ D5 = 1 ⊕ 1 ⊕ 0 ⊕ 0 = 0
C4 = D8 ⊕ D4 ⊕ D3 ⊕ D2 = 1 ⊕ 0 ⊕ 0 ⊕ 1 = 0
C2 = D7 ⊕ D6 ⊕ D4 ⊕ D3 ⊕ D1 = 1 ⊕ 0 ⊕ 0 ⊕ 0 ⊕ 0 = 1
C1 = D7 ⊕ D5 ⊕ D4 ⊕ D2 ⊕ D1 = 1 ⊕ 0 ⊕ 0 ⊕ 1 ⊕ 0 = 0

iv) Finally, the data word stored is 1100$\underline{0}$001$\underline{0}$0$\underline{1}\underline{0}$

5.12 For the 8-bit word 00111001, the check bits stored with it would be 0111. Suppose when the word is read from memory, the check bits are calculated to be 1101. What is the data word that was read from memory?

Sol:

Position$^{(10)}$	12	11	10	9	8	7	6	5	4	3	2	1
Position$^{(2)}$	1100	1011	1010	1001	1000	0111	0110	0101	0100	0011	0010	0001
Data bit	D8	D7	D6	D5		D4	D3	D2		D1		
Check bit					C8				C4		C2	C1

Doing an exclusive-OR of 0111 and 1101 yields 1010 indicating an error in position 10, i.e., data bit D6. Thus, the data word read from memory was 00$\underline{0}$11001.

5.13 How many check bits are needed if the Hamming error correction code is used to detect single bit errors in a 1024-bit data word?

Sol:
M data bits require K check bits such that $2^K \geq M + K + 1$.
1024 data bits require 11 check bits since $2^{10} \ngeq 1024 + 10 + 1$ but $2^{11} \geq 1024 + 11 + 1$.