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filename: Math115 homework14
date: 05/05/2022
desc: https://courses.csail.mit.edu/6.042/spring18/mcs.pdf (Links to an external site.) please do these problems: 15.2 15.4, 15.5, 15.12, additional problem

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15.2

In how many different ways is it possible to answer the next chapter's practice problems if:

- the first problem has four *true/false* questions,

  First problem: $2 * 2 * 2 * 2 = 16$ different ways.

- the second problem requires choosing one of four alternatives, and

  Second problem: 4 different ways.

- the answer to the third problem is an *integer* ≥ 15 and ≤ 20?

  Third problem: $15, 16, 17, 18, 19, 20$ results in 6 different ways.

- This results in $16 * 4 * 6 = 384$ different ways.

15.4

Let X be the six element set $\{x_1, x_2, x_3, x_4, x_5, x_6\}$.

- a) How many subsets of X contain x_1?

  Subsets of X containing x_1: $1 * 2 * 2 * 2 * 2 * 2 = 2^5 = 32$ ways.

- b) How many subsets of X contain x_2 and x_3 but do not contain x_6?

  Subsets of X containing x_2 and x_3 but do not containing x_6: $(1 * 1 * 2 * 2 * 2 * 2) - (1 * 1 * 2 * 2 * 2) = 2^4 - 2^3 = 8$ ways.

15.5

A license plate consists of either:

- 3 letters followed by 3 digits(standard plate)

  combinations: $26 * 26 * 26 * 10 * 10 * 10 = 17576000$.

- 5 letter (vanity plate)

  combinations: $26 * 26 * 26 * 26 * 26 = 11881376$.

- 2 characters-letters or numbers (big shot plate) case 1: 2 characters-letters: $26 * 26 = 676$ combinations.

case 2: 2 numbers: $10 * 10 = 100$ combinations.

case 3: 1 character-letter and 1 number: $26 * 10 = 260$ combinations.

case 4: 1 number and 1 character-letter: $10 * 26 = 260$ combinations.

Total combinations for 2 characters-letters or numbers: $676 + 100 + 260 + 260 = 1296$.

Let L be the set of all possible license plates.

- a) Express L in terms of

  $\mathcal{A} = \{A, B, C, \ldots, Z\}$

  $\mathcal{D} = \{0, 1, 2, \ldots, 9\}$

  using unions ($\cup$) and set products ($\times$).

  - $L = (\mathcal{A}^3 \times \mathcal{D}^3) \cup (\mathcal{A}^5) \cup ((\mathcal{A}^2) \cup (\mathcal{D}^2) \cup (\mathcal{A} \times \mathcal{D}) \cup (\mathcal{D} \times \mathcal{A}))$
- b) Compute $|L|$, the number of different license plates, using the sum and product rules.
  - The total number of combinations for different license plates is $17576000 + 11881376 + 1296 = 29458672$.
  - or $|L| = 29458672$

15.12

Eight students—Anna, Brian, Caine,. . . —are to be seated around a circular table in a circular room. Two seatings are regarded as defining the same *arrangement* if each student has the same student on his or her right in both seatings: it does not matter which way they face. We'll be interested in counting how many arrangements there are of these 8 students, given some restrictions.

- a) As a start, how many different arrangements of these 8 students around the table are there without any restrictions?

  Each arrangement has 8 equivalent arrangements because of the circular table. No restrictions combinations: $8!/8 = 7! = 5040$ ways.
- b) How many arrangements of these 8 students are there with Anna sitting next to Brian?

  There are $(1 * 2 * 6!)/8 = 180$ ways.
- c) How many arrangements are there with if Brian sitting next to both Anna AND Caine?

  There are $(1 * 2 * 1 * 5!)/8 = 30$ ways.
- d) How many arrangements are there with Brian sitting next to Anna OR Caine?

  combinations with Brian sitting next to Anna OR Caine: $180 + 180 - 30 = 330$ ways.

5

Students A, B, C, D, E, F, G are running a footrace.

One possible outcome to the race is (G, D, F, A, E, D, C).

Compute the probability that...

- a. A comes in first.

  Let E be the event that A comes in first.

  $P(E) = \frac{6!}{7!} = 0.14$

- b. A or B come in first.

  Let E_1 be the event that A comes in first and E_2 be the event that B comes in first.

  $P(E_1 \cup E_2) = \frac{(1*6!+1*6!-0)}{7!} = 0.28$

- c. A comes in first or last.

  Let E_1 be the event that A comes in first and E_2 be the event that A comes in last.

  $P(E_1 \cup E_2) = \frac{(1*6!+6!*1-0)}{7!} = 0.28$

- d. A comes in first or B comes in last.

  Let E_1 be the event that A comes in first and E_2 be the event that B comes in last.

  $P(E_1 \cup E_2) = \frac{((1*6!)+(1*6!)-(1*5!*1))}{7!} = 0.26$

- e. A doesn't finish in the top four.

  Let E be the event that A doesn't finish in the top four.

  $P(E) = \frac{(6!*1)*3}{7!} = 0.42$

- f. A finishes before B.

  Let E be the event that A finishes before B.

  Case: A is 1st: $6!$ outcomes have B after.

  Case: A is 2nd: $5 * 5!$ outcomes have B after.

  Case: A is 3rd: $4 * 5!$ outcomes have B after.

  Case: A is 4th: $3 * 5!$ outcomes have B after.

  Case: A is 5th: $2 * 5!$ outcomes have B after.

  Case: A is 6th: $1 * 5!$ outcomes have B after.

  Case: A is 7th: $0 * 5!$ outcomes have B after.

  $P(E) = \frac{5!(6+5+4+3+2+1)}{7!} = 0.50$

- g. A finishes before B, given that B is not last.

  Let E_1 be the event that A finishes before B and E_2 be the event that B is not last.

  $P(E_1 \cap E_2) = 0.50 - 0.14 = 0.36$

END