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filename: Math115 homework3
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desc: https://courses.csail.mit.edu/6.042/spring18/mcs.pdf (Links to an external site.) please do these problems: 1.12, 1.13, 1.15, 1.17

1.12

Prove that for any $n > 0$, if a^n is even, then a is even.

Prove by contradiction

There exist a positive a where a^n is even and a is odd.

We know that any odd number to the power of any positive integer will be odd.

If we substitute (2m + 1) to represent an odd number, where "m" is in the same domain as "n". Upon expanding the any $(2m+1)^n$ we are always left with a odd number.

This is an invalid contradiction meaning the preposition is true.

1.13

Prove that if $a * b = n$, then either a or b must be $\leq \sqrt{n}$, where a, b, and n are non-negative real numbers.

Prove by contradiction

There exist an $a * b = n$ where $a > \sqrt{n}$ and $b > \sqrt{n}$

so there exist a $a > \sqrt{n}$ and $b > \sqrt{n}$

or so there exist a $a^2 > n$ and $b^2 > n$

then set $a^2 > n = a * b$ and $b^2 > n = a * b$

The product of two non-negative real numbers real numbers is always larger than its factors. So if a is set to a larger number, b has to become smaller to satisfy the equation. There is no combination where a and b can be large enough to make both equations true.

So, I'm concluding the contradiction is invalid meaning the initial preposition is valid.

1.15

Give an example of two distinct positive integers m, n such that n^2 is a multiple of m, but n is not a multiple of m. How about have m be less than n?

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example 1

$n = 6$ and $m = 9$, $n^2 = 36$

n^2 is multiple of m, but n is not a multiple of m.

example 2

$n = 6$ and $m = 4$, $n^2 = 36$

n^2 is multiple of m, n is not a multiple of m, and $m < n$

1.17

Prove that $log_4 6$ is irrational.

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prove by contradiction

There exist $log_4 6$ is rational

let m/n be represent a rational number where m and n are integers

we'll set $log_4 6 = \frac{m}{n}$

raise both sides by 4, $4^{log_4 6} = 4^{\frac{m}{n}}$

we end up with $6 = 4^{\frac{m}{n}}$

square root both sides by n, $6^n = 4^m$

divide both sides by 2^n, $3^n = 2^m$

We know that an odd number to the power of any integer is an odd and an even number to the power of any integer is an even so the contradiction is not valid meaning the preposition is valid.

Boolean Function problem

X	Y	Z	G
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

a) Find the sum-of-products and product-of-sums expansion for this Boolean Function G(x,y,z).

sum of products expansion

$$G(x,y,z) = xy\overline{z} + x\overline{y}\overline{z} + \overline{x}yz + \overline{x}\overline{y}z$$

product of sums expansion

$$G(x,y,z) = (\overline{x} + \overline{y} + \overline{z}) * (\overline{x} + y + \overline{z}) * (x + \overline{y} + z) * (x + y + z)$$

x/c problem

Prove or disprove that $\sqrt{3} - 3\sqrt{2}$ is irrational

To the above equation is irrational I believe only $\sqrt{3}$ or $3\sqrt{2}$ needs to be proven irrational, since a the subtraction of a rational and irrational number would result in an irrational number. Also the subtraction of an irrational number with an irrational number would be irrational assuming the two numbers are not equal.

Seeing as the $\sqrt{3}$ is a prime number there doesn't exist a rational number taken to the power of 3 would result in 3 I would say the $\sqrt{3}$ is irrational. I believe this same argument can be made for the $\sqrt{2}$. So I believe this proves $\sqrt{3}$ or $3\sqrt{2}$ is irrational.