M115 Homework1

3.5 a)

P	Q	R	1. P—>Q	2. Q—>R	3. R—>P	1 and 2 and 3	P and Q and R
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F
T	F	T	F	T	T	F	F
T	F	F	F	T	T	F	F
F	T	T	T	T	F	F	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	T	T	F

3.5 b) You could conclude that all three of P,Q and R are true if you knew that one of them were true. However, it's possible that they're all false. In this case, all the implications are going to be true, but (P and Q and R) won't be.

3.14

	P	Q	R	P or (Q and R)	P or Q	P or R	(P or Q) and (P or R)
1	T	T	T	T	T	T	T
2	T	T	F	T	T	T	T
3	T	F	T	T	T	T	T
4	T	F	F	T	T	T	T
5	F	T	T	T	T	T	T
6	F	T	F	F	T	F	F
7	F	F	T	F	F	T	F
8	F	F	F	F	F	F	F

In all 8 cases you have P or (Q and R) has the same value as (P or Q) and (P or R), so they're equivalent.

3.26

domains in this question:
a) nonnegative integers = $\mathbf{N} = \{0,1,2,3,4,...\}$
b) integers = $\mathbf{Z} = \{-3, -2, -1,0,1,2,3,...\}$
c) real numbers = $\mathbf{R}$
(real numbers are hard to list out, but they can be represented by a point on a number line.)

i)$\forall x \exists y \ \ 2x - y = 0$. This is true in all cases, when the domain is $\mathbf{N}$, $\mathbf{Z}$ or $\mathbf{R}$.

ii) $\forall x \exists y \ \ x - 2y = 0$. This isn't true when the domain is $\mathbf{N}$ or $\mathbf{Z}$. In either case, $x = 1$ is a counterexample that demonstrates falsehood of the proposition, because if the proposition is true then when $x = 1$ there's supposed to exists a $y \in \mathbf{N}$ or $\mathbf{Z}$ so that $x = 2y$, but the only y which would fit here is $y = 1/2$, which isn't in the domains $\mathbf{N}$ or $\mathbf{Z}$.

iii) $\forall x \ x < 10 \to [\forall y \ (y < x) \to y < 9]$
In English, you might say it this way: For all x in the domain, if x is less than 10, and if y is any numbers less than x, then y is less than 9. This is true when the domain is $\mathbf{N}$ or $\mathbf{Z}$, but when the domain is $\mathbf{R}$ you could have x and y like $(x, y) = (9.9,9.8)$. The conditions $x < 10$ and $y < x$ are satisfied, but $y < 9$ is not, so the implication isn't true.

iv)$\forall x \exists y \ [y > x \wedge \exists z \ y + z = 100]$.
This isn't true when the domain is $\mathbf{N}$ because you could have $x = 100$ (or anything larger), but then you couldn't find a y and z in $\mathbf{N}$ so that $y > x$ and $y + z = 100$. With $y > x = 100$, z would have to be negative, which doesn't happen with a domain of $\mathbf{N}$.

3.27 Q(x,y) := "x has been a contestant on TV quiz show y."

the domain for x is students at our school, the domain for y is the set of all quiz shows on TV.

"No student at our school has been a contestant on a TV show."

There are two equivalent expression to this, given in (a) and (d):
$\forall x \forall y \neg Q(x, y) \equiv \neg[\exists x \exists y \ Q(x, y)]$.

The statement on the left says something like 'considering all students x at your school, and all TV quiz shows y, Q(x,y) hasn't happened.' The statement on the right says something like 'It's not the case that there is a student x at your school, and a quiz show on TV, y, such that Q(x,y).' These two statements are equivalent because of deMorgan's Law.

3.28

You can use implications ‘if…, then …’ in all parts a,b,c,d !

a) ‘No monkeys like to eat pizza.’ You could also think of it like this: If x is a monkey, then x doesn’t like eating pizza. In math talk: $\forall x\ P(x) \to \neg S(x)$
b) ‘Nobody from the 23rd century dislikes eating pizza.’ You could say it this way ‘everyone from the 23rd century likes eating pizza’ or ‘if x is from the 23rd century, then x likes eating pizza,’ so $\forall x\ R(x) \to S(x)$
c) ‘All 6.042 TAs are monkeys’ i.e. ‘if x is a 6.042 TA, then x is a monkey.’ $\forall x\ Q(x) \to P(x)$
d) ‘No 6.042 TA comes from the 23rd century,’ i.e. ‘if x is a 6.042 TA, then x does not come from the 23rd century’. $\forall x\ Q(x) \to \neg R(x)$
e) Yes (d) does follow from parts (a) (b) and (c). Because if you have a living thing x, and Q(x) is satisfied then we can eventually conclude ¬S(x): If Q(x) is satisfied, i.e. x is a 6.042 TA, then x is a monkey, according to (c). Since x is a monkey, x does not like to eat pizza according to (a), but every one from the 23rd century likes eating pizza by (b), so our x can’t come from the 23rd century, i.e. ¬S(x) is true. This shows that for all x, Q(x) implies ¬S(x).

f) There are lots of ways to translate $\forall x(R(x) \vee S(x) \to Q(x))$ depending on where you put some parentheses. $\forall x((R(x) \vee S(x)) \to Q(x))$, you could translate to ‘for all x, if x comes from the 23rd century or likes eating pizza, then x is a 6.042 TA.’ Said a little more concisely, ‘everyone who comes from the 23rd century or who likes eating pizza is a 6.042 TA’

If you have parentheses around the implication $\forall x(R(x) \vee (S(x) \to Q(x)))$ this might translate to ‘for all x, x comes from the 23rd century, or if they like pizza, then they’re a 6.042 TA.’ You could also say ‘either every creature comes from the 23rd century, or everyone who likes pizza is a 6.042 TA (or both).’

g). One possible translation is ‘If there’s a living creature that comes from the 23rd century or is not a 6.042 TA, then all monkeys like eating pizza.’

3.29 (this is optional extra credit, which you didn’t have to do).
To show that $\forall x \exists y\ P(x, y) \to \forall z P(z, z)$ is not valid (i.e. not true all the time), I need to find a specific predicate $P(x, y)$ and a specific domain so that when I put them into this implication I get something false. Here’s one possibility. Say the domain for x and y is the set of real numbers $\mathbf{R}$, and the predicate is $P(x, y) := x > y$. With this predicate and domain $\forall x \exists y\ P(x, y)$ is true because no matter what x you pick, there’s always a y where $x > y$ is true ($y = x - 1$ would work for all x values.). But $\forall z\ P(z, z)$ is not true, because $z < z$ actually never happens for any z in the domain. All together the implication $\forall x \exists y\ P(x, y) \to \forall z P(z, z)$ becomes $T \to F$, which is a false implication. This is why it’s not valid.