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filename: Math115 homework1
date: 1/24/2022
desc: https://courses.csail.mit.edu/6.042/spring18/mcs.pdf (Links to an external site.) Please do these problems: 3.5, 3.14, 3.26, 3.27, 3.28. x/c 3.29, 3.31, 3.33

3.5

P	Q	R	P -> Q	Q -> R	R -> P	P -> Q ^ Q -> R ^ R -> P	P ^ Q ^ R
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F
T	F	F	F	T	T	F	F
T	F	T	F	T	T	F	F
F	F	F	T	T	T	T	F
F	F	T	T	T	F	F	F
F	T	T	T	T	F	F	F
F	T	F	T	F	T	F	F

```
Sloppy Joe is incorrect when he concluded "all three of P, Q and R are true,"
because "P -> Q ^ Q -> R ^ R -> P" is also true when P, Q and R are all false.
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3.14

Problem 3.14. Prove by truth table that OR distributes over AND, namely, P OR (Q AND R) is equivalent to (P OR Q) AND (P OR R)

P	Q	R	Q ^ R	P v Q	P v R	P v (Q ^ R)	(P v Q) ^ (P v R)
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	F	F	F	F	F	F	F
F	F	T	F	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F

```
Base on the truth table we can conclude that "P v (Q ^ R)" ≡ "(P v Q) ^ (P v R)"
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3.26

For each of the following propositions:
(i) ∀x∃y.2x-y=0
(ii) ∀x∃y.x-2y=0
(iii) ∀x.x < 10 -> (∀y.y < x -> y < 9)
(iv) ∀x∃y.[y > x ^ ∃z.y + z = 100]

indicate which propositions are False when the variables range over:

(a) the nonnegative integers(ℤ+)
(b) the integers(ℤ)
(c) the real numbers(ℝ)

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- (i)
   (a) True, ∀x∃y ∈ ℤ+.2x-y=0
   (b) True, ∀x∃y ∈ ℤ.2x-y=0
   (c) True, ∀x∃y ∈ ℝ.2x-y=0
- (ii)
   (a) False, ∀x∃y ∈ ℤ+.x-2y=0
   (b) False, ∀x∃y ∈ ℤ.x-2y=0
   (c) True, ∀x∃y ∈ ℝ.x-2y=0
- (iii)
   (a) True, ∀x∀y ∈ ℤ+.x < 10 -> (y < x -> y < 9)
   (b) True, ∀x∀y ∈ ℤ.x < 10 -> (y < x -> y < 9)
   (c) True, ∀x∃y ∈ ℝ.x < 10 -> (y < x -> y < 9)
- (iv)
   (a) False, ∀x∃y∃z ∈ ℤ+.[y > x ^ y + z = 100]
   (b) True, ∀x∃y∃z ∈ ℤ.[y > x ^ y + z = 100]
   (c) True, ∀x∃y∃z ∈ ℝ.[y > x ^ y + z = 100]
```

3.27

Let Q(x, y) be the statement "x has been a contestant on television show y."

The domain of discourse for x is the set of all students at your school and for y is the set of all quiz shows that have ever been on television.

Indicate which of the following expressions are logically equivalent to the sentence: “No student at your school has ever been a contestant on a television quiz show.”

```
(a) ∀x∀y.¬(Q(x,y)) Yes
(b) ∃x∃y.¬(Q(x,y)) No
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(c) ¬(∀x∀y.Q(x,y)) No
(d) ¬(∃x∃y.Q(x,y)) No
```

3.28

Express each of the following statements using quantifiers, logical connectives, and/or the following predicates

P(x): x is a monkey,
Q(x): x is a 6.042 TA,
R(x): x comes from the 23rd century,
S(x): x likes to eat pizza

where x ranges over all living things.

```
(a) No monkeys like to eat pizza
∀x(P(x)^¬S(x))

(b) Nobody from the 23rd century dislikes eating pizza.
∀x(R(x)^S(x))

(c) All 6.042 TAs are monkeys.
∀x(Q(x) -> P(x))

(d) No 6.042 TA comes from the 23rd century.
∀x(Q(x)^¬R(x))

(e) Does part (d) follow logically from parts (a), (b), (c)? If so, give a proof.
If not, give a counterexample.
"d" doesn't conflict with a, b, or c.

A counter-example would be if there P(x) ^ Q(x) ^ R(x) ^ S(x) are all true. Recall
Nobody from the 23rd century dislikes eating pizza (b), which conflicts with no
monkeys like to eat pizza (a), and we know 6.042 TAs are all monkeys from (c).

(f) Translate into English: (∀x)(R(x) OR S(x) IMPLIES Q(x)).
If all living things from the 23rd century or that like to eat pizza then they are
a 6.042 TA

(g) Translate into English: [∃x.R(x) AND NOT(Q(x))] IMPLIES ∀x.(P(x) IMPLIES
S(s)).
If some living things from the 23rd century are not a 6.042 TA, then if they are
all monkeys then they like eating pizza.
```

3.29

Show that
(∀x∃y.P(x,y)) -> ∀z.P(z,z)

is not valid by describing a counter-model.

```
Let P(x,y) = x > y
Let x,y,z ∈ ℤ
Therefore
[(∀x∃y.P(x,y)) -> (∀z.P(x,y))] = [(∀x∃y.x > y) -> (∀z.z > z)] = [T -> F] = F
```

3.31

Find a counter-model showing the following is not valid. [∃x.P(x) ^ ∃x.Q(x)] -> ∃x.[P(x) ^ Q(x)] (Just define your counter-model. You do not need to verify that it is correct.)

```
Let x ∈ ℤ
Let P(x) = x > 10
and
Let Q(x) = x < 10
Therefore
{[∃x.P(x) ^ ∃x.Q(x)] -> ∃x.[x > 10 ^ x < 10]} = {[∃x.x > 10 ^ ∃x.x < 10] -> ∃x.[x
> 10 ^ x < 10]} = {(T ^ T) -> F} = F
```

3.33

No attempt