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filename: Math115 homework2
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desc: https://courses.csail.mit.edu/6.042/spring18/mcs.pdf (Links to an external site.) please do these problems: 1.3, 1.7, 1.8, 1.10. x/c: 1.1, 1.4, 1.5

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first problem

Please do this problem first: 1. Prove $\neg P \vee (R \to \neg Q) \equiv \neg(P \wedge Q \wedge R)$ without using a truth table.

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Proof by equivalence chains (Patterns of Proof)
¬P ∨ (R → ¬Q)
¬P ∨ (¬R ∨ ¬Q))    Logical equivalency R → Q ≡ ¬R ∨ Q
¬P ∨ ¬(R ∧ Q))     De Morgan's 2nd law
¬(P ∧ (R ∧ Q))     De Morgan's 2nd law
¬(P ∧ R ∧ Q)       Associative law

Conclusion: ¬P ∨ (R → ¬Q) ≡ ¬(P ∧ Q ∧ R) is plausible
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1.3

Identify exactly where the bugs are in each of the following bogus proofs. (a) Bogus Claim: 1/8 > 1/4

Bogus proof:
$3 > 2$
$3\log_{10}(1/2) > 2\log_{10}(1/2)$

$\log_{10}(1/2)^3 > \log_{10}(1/2)^2$

$(1/2)^3 > (1/2)^2$

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It is a mathematical role to flip the inequality sign when you multiply or divide
both sides by a negative number. log(1/2) is a negative number, so the inequality
symbol ">" should have flipped to "<" in step 2.

Also the log property in the last step I don't believe is a property of log.
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(b) Bogus proof : 1¢ = \$0.01 = (\$0.1)^2 = (10¢)^2 = 100¢ = \$1:

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(10¢)^2 = (10¢)(10¢), this expression doesn't mean anything in terms of U.S
currency. So saying (10¢)^2 = 100¢ doesn't hold.
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(c) Bogus Claim: If a and b are two equal real numbers, then a = 0.

Bogus proof:

a = b

$a^2 = ab$

$a^2 - b^2 = ab - b^2$

(a-b)(a+b) = (a-b)b

a + b = b

a = 0

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if a = b then dividing both sides by (a-b) is dividing by zero. Which is done in
step 4 and isn't defined.
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1.7

Prove by cases that
max(r, s) + min(r, s) = r + s
for all real numbers r, s.

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Prove by cases
r, s ∈ ℝ
case 1: r = s:
    max(r, s) + min(r, s)
    r + r = s + s = r + s
case 2: r > s:
    max(r, s) + min(r, s)
    r + s
case 3: r < s:
    max(r, s) + min(r, s)
    s + r
    r + s     commutative law

Based on the cases considered we can conclude that ∀r, ∀s ∈ ℝ.max(r, s) + min(r,
s) = r + s
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1.8

If we raise an irrational number to an irrational power, can the result be rational? Show that it can by considering $\sqrt{2}$$^{\$\backslash sqrt\{2\}\$}$ and arguing by cases.

Prove by cases
case 1:

irrational_number $^{\text{irrational\_number}}$ = irrational_number

$\sqrt{2}^{\sqrt{2}} = 1.6325$

The above concludes an irrational number to the power of an irrational power can result in an irrational number.

case 2:

irrational_number$^{\text{irrational_number}}$ = rational_number

$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = 2^{1/2 * \sqrt{2} * \sqrt{2}} = 2^{1/2 * 2^{1/2} * 2^{1/2}} = 2^{1/2 * 2^{1/2 + 1/2}} = 2^{1/2 * 2^{2/2}} = 2^{1/2 * 2} = 2^{2/2} = 2$

The above concludes an irrational number to the power of an irrational power can result in a rational number.

1.10

Hint: An odd number equals 2m + 1 for some integer m, so its square equals $4(m^2 + m) + 1$.

(a) Suppose that

a + b + c = d

where a, b, c, d $\in \mathbb{Z}^+$

Let P be the assertion that d is even. Let W be the assertion that exactly one among a, b, c are even, and let T be the assertion that all three are even.

Prove by cases that P IFF [W OR T]

$P \leftrightarrow W \vee T$

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proof by cases

- case 1: $P \rightarrow W$ or $P \rightarrow T$
  Let P be True
  This implies d is an even positive integer and some multiple of 2.

  Let d = 2x, where x $\in \mathbb{Z}^+$
  2x = a + b + c
  Conclusion: Based on the above equation we can conclude that either a + b + c are all even satisfying assertion T or either only one of a, b, or c is even satisfying W to solve to an even number.

- case 2: $W \rightarrow P$
  Let W be True
  This implies only one of a, b, or c is even.
  Let a, b, c = 2x, 2y + 1, 2z + 1

  where x, y, z $\in \mathbb{Z}^+$
  2x + (2y + 1) + (2z + 1) = d

$2(x + y + z) + 2 = d$
Conclusion: Based on the above equation we know d is some multiple of 2 plus 2 which implies d is a positive integer satisfying assertion P.

- case 3: $T \rightarrow P$
  T is True T is True implies a, b, and c are all even.
  $a + b + c = d$
  let $a, b, c = 2x, 2y, 2z$
  where $x, y, z \in \mathbb{Z}^+$
  $2x + 2y + 2z = 2m$
  $2(x + y + z) = 2m$
  Conclusion: Based on the above equation we know d is some multiple of 2 which implies d is a positive integer satisfying assertion P.

(b) Now suppose that

$w^2 + x^2 + y^2 = z^2$

where $w, x, y, z \in \mathbb{Z}^+$

Let P be the assertion that z is even, and let R be the assertion that all three of w, x, y are even.

Prove by cases that P IFF R
$P \leftrightarrow R$

- case 1: $P \rightarrow R$
  Proof by contradiction
  Let P be true, so z is even
  Let $z = 2x$
  where $x \in \mathbb{Z}^+$
  so, we get
  $2x = \sqrt{w^2 + x^2 + y^2}$
  - case 1 subcase 1: w, x, y are all odd.
    By substituting the hint given in the problem we get
    $2x = \sqrt{4(w^2 + w) + 1 + 4(x^2 + x) + 1 + 4(y^2 + y) + 1}$
    $2x = \sqrt{4(w^2 + w) + 4(x^2 + x) + 4(y^2 + y) + 3}$
    Using the above equation we know $\sqrt{4(w^2 + w) + 4(x^2 + x) + 4(y^2 + y) + 3}$ will solve to an odd number.
  - case 1 subcase 2: w, x are odd and y is even.
    By substituting the hint given in the problem we get
    $2x = \sqrt{4(w^2 + w) + 1 + 4(x^2 + x) + 1 + (2y)^2}$
    $2x = \sqrt{(4w^2 + 4w) + (4x^2 + 4x) + (2y)^2 + 2}$
    $2x = \sqrt{(4w^2 + 4w) + (4x^2 + 4x) + 4y^2 + 2}$
    $2x = \sqrt{2(2w^2 + 2w + 2x^2 + 2x + 2y^2 + 1)}$

    2x = $\sqrt{2}$ $\sqrt{2w^2 + 2w + 2x^2 + 2x + 2y^2 + 1}$
    Using the above equation we know $\sqrt{2}$ is an irrational number so anything multiplied by it will not be a perfect number.

  - case 1 subcase 3: w is odd and x, y are even.
    By substituting the hint given in the problem we get
    2x = $\sqrt{4(w^2 + w) + 1 + (2x)^2 + (2y)^2}$
    2x = $\sqrt{4(w^2 + w) + (2x)^2 + (2y)^2 + 1}$
    Using the above equation we know $\sqrt{4(w^2 + w) + (2x)^2 + (2y)^2 + 1}$ will solve to an odd number.

  - Conclusion: Using case 1 and it's subcases P would imply R would have to be true in other words, if z is even then w, x, and y would all have to be even.

- case 2: R → P
  Let R be true, so w, x, y are even.
  let w, x, y = 2x, 2y, 2z
  where w, x, y, z ∈ $\mathbb{Z}^+$
  $2x^2 + 2y^2 + 2z^2 = z^2$
  2x = $\sqrt{(2w)^2 + 1 + (2x)^2 + (2y)^2}$
  2x = $\sqrt{4w^2 + 4x^2 + 4y^2}$
  2x = $\sqrt{4(w^2 + x^2 + y^2)}$
  2x = $\sqrt{4}$ * $\sqrt{w^2 + x^2 + y^2}$
  2x = 2 * $\sqrt{w^2 + x^2 + y^2}$
  Based on the equation above we can conclude that there does exist an w, x, y where R → P would be true. For example 2 * $\sqrt{4^2 + 4^2 + 2^2}$ would equal to 2 * 6 = 12.
