

## Problem 1 (Decision tree)

R05631027 楊皓文 ML HW #8

No.

Date

$$1. \quad E_n(K) = -\left(\frac{9}{14} \log_2 \left(\frac{9}{14}\right) + \frac{5}{14} \log_2 \left(\frac{5}{14}\right)\right) = 0.9403$$

$$\text{Gain}(K, \text{Temp}) = E_n(K) - \frac{6}{14} E_n(K_1) - \frac{4}{14} E_n(K_2) - \frac{4}{14} E_n(K_3)$$

$$\left( \text{其中 } E_n(K_1) = -\left(\frac{2}{6} \log_2 \left(\frac{2}{6}\right) + \frac{4}{6} \log_2 \left(\frac{4}{6}\right)\right) = 0.9183 \right)$$

$$E_n(K_2) = -\left(\frac{4}{4} \log_2 \left(\frac{4}{4}\right)\right) = 0$$

$$E_n(K_3) = -\left(\frac{3}{4} \log_2 \left(\frac{3}{4}\right) + \frac{1}{4} \log_2 \left(\frac{1}{4}\right)\right) = 0.8113$$

$$= 0.9403 - \frac{6}{14} \cdot 0.9183 - 0 - \frac{4}{14} \cdot 0.8113 = 0.3149 \quad \checkmark$$

$$\text{Gain}(K, \text{Midterm}) = E_n(K) - \frac{4}{14} E_n(K_1) - \frac{6}{14} E_n(K_2) - \frac{4}{14} E_n(K_3)$$

$$\left( \text{其中 } E_n(K_1) = -\left(\frac{3}{4} \log_2 \left(\frac{3}{4}\right) + \frac{1}{4} \log_2 \left(\frac{1}{4}\right)\right) = 0.8113 \right)$$

$$E_n(K_2) = -\left(\frac{4}{6} \log_2 \left(\frac{4}{6}\right) + \frac{2}{6} \log_2 \left(\frac{2}{6}\right)\right) = 0.9183$$

$$E_n(K_3) = -\left(\frac{3}{4} \log_2 \left(\frac{3}{4}\right) + \frac{1}{4} \log_2 \left(\frac{1}{4}\right)\right) = 1$$

$$= 0.9403 - \frac{4}{14} \cdot 0.8113 - \frac{6}{14} \cdot 0.9183 - \frac{4}{14} \cdot 1 = 0.029$$

$$\text{Gain}(K, \text{HW done}) = E_n(K) - \frac{6}{14} E_n(K_1) - \frac{2}{14} E_n(K_2)$$

$$\left( \text{其中 } E_n(K_1) = -\left(\frac{6}{7} \log_2 \left(\frac{6}{7}\right) + \frac{1}{7} \log_2 \left(\frac{1}{7}\right)\right) = 0.5917 \right)$$

$$E_n(K_2) = -\left(\frac{3}{7} \log_2 \left(\frac{3}{7}\right) + \frac{4}{7} \log_2 \left(\frac{4}{7}\right)\right) = 0.9152$$

$$= 0.9403 - \frac{6}{14} \cdot 0.5917 - \frac{2}{14} \cdot 0.9152 = 0.1519$$

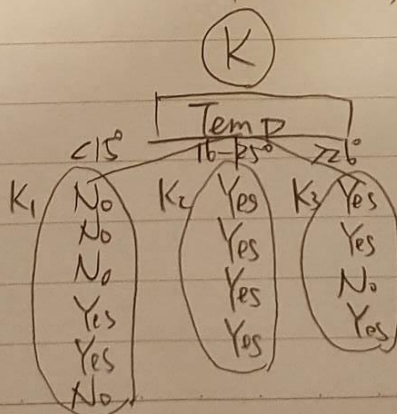
$$\text{Gain}(K, \text{Quiz}) = E_n(K) - \frac{6}{14} E_n(K_1) - \frac{2}{14} E_n(K_2)$$

$$\left( \text{其中 } E_n(K_1) = -\left(\frac{3}{6} \log_2 \left(\frac{3}{6}\right) + \frac{3}{6} \log_2 \left(\frac{3}{6}\right)\right) = 1 \right)$$

$$E_n(K_2) = -\left(\frac{6}{8} \log_2 \left(\frac{6}{8}\right) + \frac{2}{8} \log_2 \left(\frac{2}{8}\right)\right) = 0.8113$$

$$= 0.9403 - \frac{6}{14} \cdot 1 - \frac{2}{14} \cdot 0.8113 = 0.0481$$

$\therefore \text{Gain}(K, \text{Temp})$  最大, 因此選 "Temp" 做為 root node



接下來搭配觀察法，可看出  $<15^\circ$  再選擇 HW-done attribute,  $>26^\circ$  再選擇 Quiz attribute, 即可完全分類

$$\begin{aligned}
 <15^\circ: E_n(K) &= -\left(\frac{2}{6}\log_2\left(\frac{2}{6}\right) + \frac{4}{6}\log_2\left(\frac{4}{6}\right)\right) = 0.9183 \\
 \text{Gain}(K, \text{HW-done}) &= E_n(K) - \frac{2}{6}E_n(K_1) - \frac{4}{6}E_n(K_2) \\
 &\quad \left( \text{其中 } E_n(K_1) = -\left(\frac{2}{2}\log_2\left(\frac{2}{2}\right)\right) = 0 \right. \\
 &\quad \quad \quad E_n(K_2) = -\left(\frac{4}{4}\log_2\left(\frac{4}{4}\right)\right) = 0 \\
 &= 0.9183 - 0 - 0 = 0.9183 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 >26^\circ: E_n(K) &= -\left(\frac{3}{4}\log_2\left(\frac{3}{4}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right)\right) = 0.8113 \\
 \text{Gain}(K, \text{Quiz}) &= E_n(K) - \frac{3}{4}E_n(K_1) - \frac{1}{4}E_n(K_2) \\
 &\quad \left( \text{其中 } E_n(K_1) = -\left(\frac{1}{1}\log_2(1)\right) = 0 \right. \\
 &\quad \quad \quad E_n(K_2) = -\left(\frac{3}{3}\log_2\left(\frac{3}{3}\right)\right) = 0 \\
 &= 0.8113 - 0 - 0 = 0.8113 \quad \checkmark
 \end{aligned}$$

所以 model 為:

