Metamodeling of M/M/1 call center

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This example is adapted from Cheng, R and Kleijnen, J. (1999). *Improved Design of Queueing Simulation Experiments with Highly Heteroscedastic Responses.* Operations Research, v. 47, n. 5, pp. 762-777.

Let $\bar{f}(x_i)$ be the average waiting time in a M/M/1 queue as determined through the simulation of t_i arrivals when the utilization rate is x_i ($x_i = \frac{\lambda}{\mu_i}$, where λ and μ are the arrival and service rates, respectively). Furthermore, let x be the vector $(x_1, x_2, \ldots, x_n)$ of utilization rates at which the average waiting time is estimated. Lastly, let

$$\hat{f}(x) = \frac{\beta_0 + \beta_1 x + \beta_2 x^2}{1 - x}.$$

The goal is to find β_0, β_1 and β_2 in order to approximate $\bar{f}(x)$ through $\hat{f}(x)$ as accurately as possible, i.e.

$$\min\ (\bar{f}(x) - \hat{f}(x))\Gamma^{-1}(\bar{f}(x) - \hat{f}(x))$$

where Γ is the covariance matrix for $\bar{f}(x)$. It accounts for any correlation, such as the use of common random numbers, in the estimation of the average waiting times through simulation. If the simulation at each x_i is done independently, then Γ would be a diagonal matrix.

Recommended Parameter Settings: Use $n = 5$ and $x = (0.5, 0.564, 0.706, 0.859, 0.950)$. Lastly, take $T = 50,000(0.007, .024, .064, .258, .647)$, and estimate $\bar{f}(x_i)$ independently for each x_i.

Starting Solutions: Take $\beta_1 = 1$, and $\beta_1 = \beta_2 = 0$. If multiple solutions are needed, take $\beta_0, \beta_1, \beta_2$ uniformly distributed on [0,2].

Measurement of Time: Number of estimations of $\bar{f}(x_i)$ made.

Optimal Solution: $\beta_0 = \beta_2 = 0$ and $\beta_1 = 1$.