GI/G/1 Queue

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(Integer-ordered variables, constrained.)

This example is adapted from the article Ecuyer, P. L., & Glynn, P. W. (1994). *Stochastic Optimization by Simulation: Convergence Proofs for the GI/G/*1 *Queue in Steady-state*. Management, 40(11), 1562-1578. [1]

Problem Statement:

Consider a GI/G/1 queue. Let A and B_θ denote the distribution of its interarrival and service time respectively, and suppose that they both have finite expectation and variance. B_θ depends on θ which is restricted to an interval $\bar{\Theta} = [\theta_{\min}, \theta_{\max}] \subset \mathbb{R}$. Assume that the system is stable for each $\theta \in \bar{\Theta}$, and denote the average sojourn time per customer in steady-state by $w(\theta)$. Let C be a continuously differentiable function, and our objective is to minimize:

$$\alpha(\theta) \quad = \quad w(\theta) + C(\theta)$$

over $[\theta_{\min}, \theta_{\max}]$.

Recommended Parameter Settings:

Let A be an exponential distribution with $\lambda = 2$, B_θ an exponential distribution with $\lambda = \theta$, $\theta \in \bar{\Theta} = [1, 2]$. Define $C(\theta) = \theta^2$.

Set a warm-up period of 20 periods and then compute the average profit during the next 50 periods.

Starting Solutions: Generate θ_0 uniformly from $[\theta_{\min}, \theta_{\max}]$.

Measurement of Time: Total length of period simulated.

Optimal Solutions: Unknown.

Known Structure: None.

References

[1] Ecuyer, P. L., & Glynn, P. W. (1994). *Stochastic Optimization by Simulation: Convergence Proofs for the GI/G/*1 *Queue in Steady-state*. Management, 40(11), 1562-1578.