Economic Order Quantity

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(Continuous variables, unconstrained, smooth, gradients available)

This example is adapted from Dellino, G., & Kleijnen, J. P. (2009). Robust Simulation-Optimization Using Metamodels. Proceedings of the 2009 Winter Simulation Conference (WSC), 540-550 and the book Zipkin, P. H. (2000). Foundations of Inventory Management. McGraw-Hill.

Problem Statement: The Economic-Order-Quantity model is a classical inventory model concerning the management of a single product at a single location. Demand occurs continuously and deterministically at a constant rate $a > 0$. When the inventory reaches 0 it is instantaneously replenished to the level $q > 0$ at a fixed ordering cost of $k > 0$ and a per-unit cost of $c > 0$, so that each replenishment costs $k + cq$. There is a holding cost $h > 0$ that is charged per unit of inventory per unit of time. We are interested in minimizing the long-run cost rate per unit time.

Assuming that all quantities are deterministic, the inventory process is deterministic and cyclic. On each cycle it starts at level q, decreases at rate a until it hits 0 and then returns instantaneously to level q. The average cost per unit time is therefore

$$ca + \frac{ka}{q} + \frac{hq}{2},$$

the optimal choice of q is $q^*(a) = \sqrt{2ka/h}$, and the resulting optimal cost per unit time is $\sqrt{2kah} + ca$. Now suppose that the demand rate a is random and given by the random variable A, and we must choose q before learning the realized value of A. (In practice one would probably learn the realized value of A over time and adapt q accordingly, but we instead assume, for the purposes of this problem, that q is chosen and fixed before the realized value of A is learned.) Now we want to minimize the expected long-run average cost

$$cE(A) + \frac{kE(A)}{q} + \frac{hq}{2}. \tag{1}$$

Take A to be gamma distributed with mean μ_a and standard deviation δ_a so that

$$A \sim \text{Gamma}\left(\frac{\mu_a^2}{\delta_a^2}, \frac{\delta_a^2}{\mu_a}\right),$$

where the two parameters are the shape and scale respectively.

We approximate (1) by a sample average,

$$c\bar{A}_m + \frac{k\bar{A}_m}{q} + \frac{hq}{2},$$

where m is the simulation runlength and $\bar{A}_m$ is the usual sample average $m^{-1}(A_1 + A_2 + \cdots + A_m)$ of iid observations.

This sample function is differentiable with IPA derivative

$$\frac{-k\bar{A}_m}{q^2} + \frac{h}{2}.$$

Recommended Parameter Settings: We use the parameters from Chase, Jacobs and Aquilano (2006) *Operations Management for Competitive Advantage*, 11th ed. McGraw Hill: $\mu_a = 1040, \delta_a = 104, k = 10, c = 15, h = 2.5$.

Starting Solutions: Take $q = 300$. If multiple starting solutions are required, then take them to be iid uniform on $(0, 500)$.

Measurement of Time: The number of gamma random variables generated.

Optimal Solutions: From (1) we see that the true optimal solution is $q^* = \sqrt{2k\mu_a/h} = 91$ (to the nearest integer).

Known Structure: Both the sample functions and the true objective are strictly convex and differentiable.