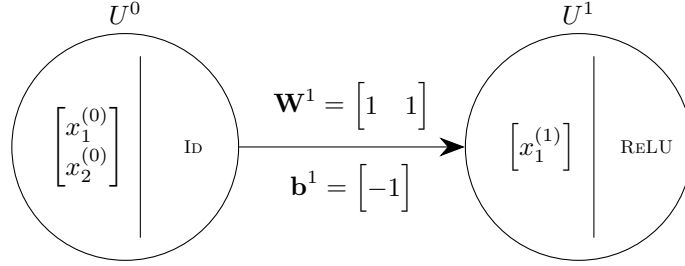


1 Logical AND

$$\{0, 1\}^2 \rightarrow \{0, 1\}$$

$$U_0 = \text{Id} \left(\begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \end{bmatrix} \right)$$

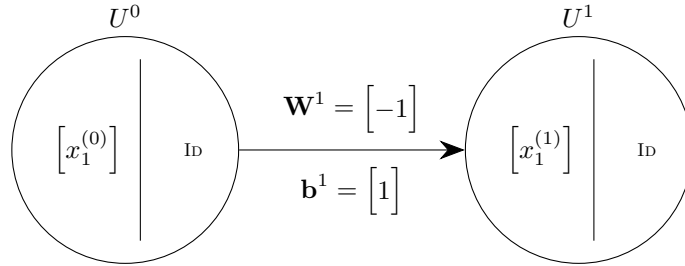
$$U_1 = \text{ReLU} \left(\left[\text{Id} \left(x_1^{(0)} \right) + \text{Id} \left(x_2^{(0)} \right) - 1 \right] \right)$$



2 Logical NOT

$$U_0 = \text{Id} \left(\begin{bmatrix} x_1^{(0)} \end{bmatrix} \right)$$

$$U_1 = \text{Id} \left(\left[-\text{Id} \left(x_1^{(0)} \right) + 1 \right] \right)$$



3 Logical OR

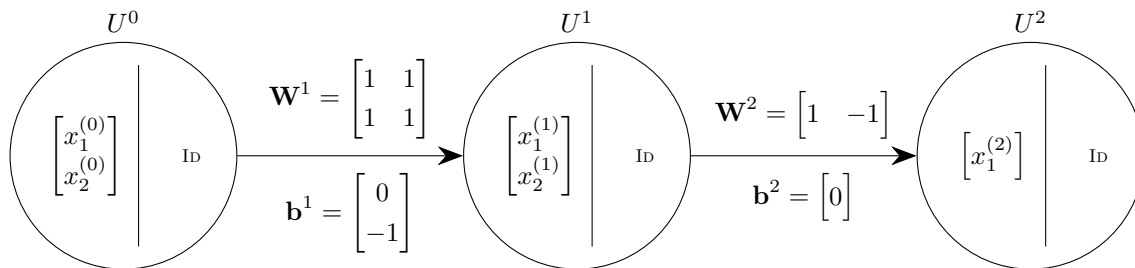
$$\{0, 1\}^2 \rightarrow \{0, 1\}$$

Let S be the sum of the inputs, which is $x_1^{(0)} + x_2^{(0)}$. We output whether $S > 0$ by the taking the ReLU of S and $S - 1$. If $S \leq 0$, then $0 - 0 = 0$. If $S > 0$, then $S - (S - 1) = 1$.

$$U_0 = \text{Id} \left(\begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \end{bmatrix} \right)$$

$$U_1 = \text{ReLU} \left(\begin{bmatrix} \text{Id} \left(x_1^{(0)} + x_2^{(0)} \right) \\ \text{Id} \left(x_1^{(0)} + x_2^{(0)} \right) - 1 \end{bmatrix} \right)$$

$$U_2 = \text{Id} \left(\left[\text{ReLU} \left(\text{Id} \left(x_1^{(0)} + x_2^{(0)} \right) \right) - \text{ReLU} \left(\text{Id} \left(x_1^{(0)} + x_2^{(0)} \right) - 1 \right) \right] \right)$$



4 Logical XOR

5 Logistic Regression

6 Two-Variable Function