Exercises for Introduction to Mathematical Arguments

Kevin Ma

May 23, 2025

Based on notes by M. Hutchings*

2 How to prove things

1. Prove the following statements; what is the negation of each of these statements?

   (a) For every integer x, if x is even, then for every integer y, xy is even.

   Proof. Since x is even, choose an integer w such that $x = 2w$.
   Then, $xy = 2wy$.
   Let $z = wy$; then $xy = 2z$, so xy is even. □

   Negation: x is even, and there is an integer y where xy is odd.

   (b) For every integer x and for every integer y, if x is odd and y is odd then $x + y$ is even.

   Proof. Since x is odd, choose an integer v such that $x = 2v + 1$.
   Since y is odd, choose an integer w such that $y = 2w + 1$.
   Then, $x + y = 2v + 2w + 2$.
   Let $z = v + w + 1$; then $x + y = 2z$, so $x + y$ is even. □

   Negation: There is an integer x and an integer y such that x is odd, y is odd, and $x + y$ is odd.

   (c) For every integer x, if x is odd then x^3 is odd.

   Proof. Since x is odd, choose an integer w such that $x = 2w + 1$.
   Then, $x^3 = 8w^3 + 12w^2 + 6w + 1$.
   Let $z = 4w^3 + 6w^2 + 3w$; then $x^3 = 2z + 1$, so x^3 is odd. □

   Negation: There is an integer x such that x is odd and x^3 is even.

*https://math.berkeley.edu/∼hutching/teach/proofs.pdf

2. Prove that for every integer x, $x+4$ is odd if and only if $x+7$ is even.

   Proof. ($\Rightarrow$) Suppose $x+4$ is odd.
   Choose an integer v such that $x+4 = 2v+1$.
   Then, $x+7 = 2v+4$.
   Let $z = v+2$; then $x+7 = 2z$, so $x+7$ is even.
   ($\Leftarrow$) Suppose $x+7$ is even.
   Choose an integer w such that $x+7 = 2w$.
   Then, $x+4 = 2w-3 = 2(w-2)+1$.
   Let $z = w-2$; then $x+4 = 2z+1$, so $x+4$ is odd. $\square$

3. Figure out whether the statement we negated in §1.3 is true or false, and prove it (or its negation).

4. Prove that for every integer x, if x is odd then there exists an integer y such that $x^2 = 8y+1$.