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Sustituyendo (IV) y (VI) en (I)
en cada caso por separado

$$y'' = Ae^{-x} - 9B \cos(3x) - 9C \sin(3x)$$

$$-3y' = 3Ae^{-x} + 9B \sin(3x) - 9C \cos(3x)$$

$$2y = 2Ae^{-x} + 2B \cos(3x) + 2C \sin(3x)$$

$$y'' - 3y' + 2y = 6Ae^{-x} + (-9B - 9C + 2B) \cos(3x) + (9C + 9B + 2C) \sin(3x)$$

$$y'' - 3y' + 2y = 6Ae^{-x} + (-7B - 9C) \cos(3x) + (9B - 7C) \sin(3x) \quad (VII)$$

Considerando $y'' - 3y' + 2y = 3e^{-x} - 10 \cos(3x) + 0 \sin(3x) \quad (VIII)$

Iguando (VII) y (VIII)

$$-7B - 9C = 10$$

$$9B - 7C = 0$$

$$9(-7B - 9C) = (-10)(9)$$

$$7(9B - 7C) = 0$$

$$63B - 81C = -90$$

$$63B - 49C = 0$$

$$-130C = -90$$

$$C = \frac{9}{13} \quad (IX) \rightarrow 9B - 7C = 0$$

$$B = \left(\frac{7}{9}\right) \left(\frac{9}{13}\right)$$

$$B = \frac{7}{13} \quad (X)$$

Sustituyendo (IX) y (X) en (IV)

$$y_p = \frac{1}{2} e^{-x} + \frac{7}{13} \cos(3x) + \frac{9}{13} \sin(3x) \quad (XI)$$

$$(1) \quad y'' - 3y' + 2y = 0 \quad (I)$$

Solucion Característica

$$\boxed{y = e^{mx} \quad y' = m e^{mx} \quad y'' = m^2 e^{mx}} \quad (II)$$

Substituyendo II en I

$$m^2 e^{mx} - 3m e^{mx} + 2e^{mx} = 0$$

$$e^{mx}(m^2 - 3m + 2) = 0 \quad e^{mx} \neq 0$$

$$m^2 - 3m + 2 = 0$$

$$m = \frac{3 \pm \sqrt{9-8}}{2 \cdot 1} \quad \begin{cases} m = \frac{3+1}{2} = 2 \\ m = \frac{3-1}{2} = 1 \end{cases}$$

$$y_c = C_1 e^{2x} + C_2 e^x \quad (III)$$

Solucion Particular

$$y_p = A e^x + B \cos(3x) + C \sin(3x) \quad (IV)$$

$$y_p' = -A e^{-x} - 3B \sin(3x) + 3C \cos(3x) \quad (V)$$

$$y_p'' = A e^{-x} - 9B \cos(3x) - 9C \sin(3x) \quad (VI)$$

De (II) y (III) la Solución General es:

$$Y(x) = C_1 e^x + C_2 e^{-x} + \frac{1}{2} e^x + \frac{2}{13} \cos(3x) + \frac{9}{13} \sin(3x) \quad \text{(XIII)}$$

ajo las condiciones

$$Y(0) = C_1 e^0 + C_2 e^0 + \frac{1}{2} e^0 + \frac{2}{13} \cos(0) + \frac{9}{13} \sin(0)$$

$$1 = C_1 + C_2 + \frac{1}{2} + \frac{2}{13} + \frac{9}{13}(0)$$

$$C_1 + C_2 = 1 - \frac{1}{2} - \frac{2}{13}$$

$$C_1 + C_2 = \frac{1}{26} \quad \text{(XIV)}$$

la derivada de (XIII)

$$Y'(x) = 2C_1 e^{2x} + C_2 e^x - \frac{1}{2} e^{-x} - \frac{21}{13} \sin(3x) + \frac{27}{13} \cos(3x)$$

$$Y'(0) = 2C_1 e^0 + C_2 e^0 - \frac{1}{2} e^0 - \frac{21}{13} \sin(0) + \frac{27}{13} \cos(0)$$

$$Y'(0) = 2C_1 + C_2 - \frac{1}{2} + \frac{27}{13}$$

$$2 = 2C_1 + C_2 + \frac{41}{26}$$

$$2C_1 + C_2 = 2 - \frac{41}{26}$$

$$2C_1 + C_2 = \frac{11}{26} \quad \text{(XV)}$$

De (XIV) y (XV)

$$-C_1 - C_2 = \frac{1}{26}$$

$$2C_1 + C_2 = \frac{11}{26}$$

$$C_1 = \frac{6}{13} \quad \text{(XVI)}$$

substituyendo (XVI) y (XIV)

$$C_2 = -\frac{1}{26} - \frac{6}{13}$$

$$C_2 = -\frac{1}{2} \quad \text{(XVII)}$$

Sustituyendo $\textcircled{\text{XVI}}$ $\textcircled{\text{XVII}}$ en

$$y(x) = \frac{6}{13} e^{2x} - \frac{1}{2} e^x + \frac{1}{2} e^{-x} + \frac{7}{13} \cos(3x) + \frac{9}{13} \sin(3x)$$