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$$y'' + 9y = 2\,Sec(3X) \quad (I)$$

Caso 1

$$y'' + 9y = 0 \quad (II)$$

Sea $y = e^{mx}$ $\quad y' = me^{mx} \quad y'' = m^2 e^{mx}$ (III)

Sustituyendo (III) en (I)

$$m^2 e^{mx} + 9e^{mx} = 0$$

$$e^{mx}(m^2 + 9) = 0 \qquad e^{mx} \neq 0$$

$$m^2 = -9$$

$$m = \pm 3i \Rightarrow m = 0 \pm 3i$$

$$Y_c = e^{0x}(C_1 Cos(3X) + C_2 Sen(3X))$$

$$Y_c = C_1 Cos(3X) + C_2 Sen(3X) \quad (IV)$$

Veamos el Wosrkiano

$Y_1 = Cos(3X) \qquad Y_2 = 3Cos(3X)$

$$W_{(x)} = \begin{vmatrix} Cos(3X & Sen(3X) \\ -3Sen(3X) & 3Cos(3X) \end{vmatrix} \Rightarrow W = 3Cos^2(3X) + 3Sen^2(x)$$

$$\boxed{W_{(x)} = 3} \quad (VI)$$

Escaneado con CamScanner

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$$Y_p = -Y_1(x) \int \frac{Y_2(x) f(x)}{W(x)} + Y_2(x) \int \frac{Y_1(x) f(x)}{W(x)} dx \quad \text{VII}$$

Considerando $f(x) = 2 Sec(3x)$

$$Y_{p(x)} = -Cos(3x) \int \frac{Sen(3x) 2 Sec(3x)}{3} dx + Sen(3x) \int \frac{Cos(3x)\, 2 Sec(3x)}{3} dx$$

$$Y_{p(x)} = -\frac{2}{3} Cos(3x) \int \frac{Sen(3x)}{Cos(3x)} dx + \frac{2}{3} Sen(3x) \int \frac{Cos(3x)}{Cos(3x)} dx$$

$$Y_{p(x)} = -\frac{2}{3} Cos(3x) \int tg(3x)\, dx + \frac{2}{3} Sen(3x) \int dx \quad \text{VIII}$$

$$\int tg(u) = -ln(Cos(u)) + C \quad \text{IX}$$

$\int tg(3x) dx$ $\quad m = 3x$ $\quad \int dx = x + c$

$\frac{1}{3} dm = dx$

$$\int tg(3x) dx = \frac{1}{3} \int tg(m)\, dm = -\frac{1}{3} ln(Cos(m)) + C \quad \text{Por IX} \rightarrow = \frac{-1}{3} ln(Cos(3x)) + c$$

Sustituyendo

$$Y_{p(x)} = \frac{2}{3} Cos(3x) \left(-ln(Cos(3x))\right) + \frac{2}{3} sen(3x)(x)$$

$$Y_{p(x)} = \frac{2}{3} Cos(3x)\, ln(Cos(3x)) + \frac{2}{3} x\, Sen(3x)$$

Solucion

$$Y_{(x)} = Y_c(x) + Y_{p(x)}$$

$$Y_{(x)} = C_1 Cos(3x) + C_2 Sen(3x) + \frac{2}{3} Cos(3x) ln(Cos(3x)) + \frac{2}{3} x Sen(3x)$$