

# Analytical Hierarchy Process

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CIV E 601

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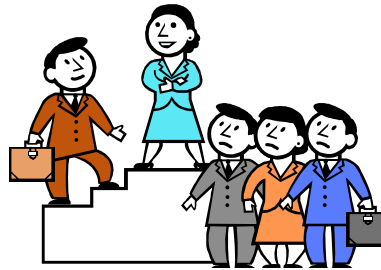
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Fall 2004

# Analytical Hierarchy Process

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- Analytical Hierarchy Process (AHP) can be defined as:
  - A well established mathematically proven formal method used for scaling or evaluating the weights of a criterion with respect to other criteria



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# Analytical Hierarchy Process

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- AHP is a consistent procedure not like other pair wise comparison techniques. The method is accounted for the consistency of thought, and consistency index has been introduced to account of this important issue



# Applying AHP

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- As mentioned earlier that AHP is a formal structural process, and in order to apply the AHP we need to do the following:
  1. Determine the requirements of the system:
    - Answering (What do we need to do?)
  2. Generating alternatives to satisfy those requirements
    - Answering (What are the possible courses of action?)
  3. Set priorities
  4. Chose the best policy alternative or mix of the best policy alternatives

# AHP Procedure

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1. Define the set of alternatives (2)
2. Define a set of criteria
3. Perform the pair wise comparison between those criteria
4. Calculate the priorities (weight)
5. Assign a score for each criteria
6. Calculate the total score, and select the maximum

# How to Calculate the Weights

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- Given Example:
  - Assume that the success of a person in a university position is characterized by:
    - Hard work (HD)
    - Productivity (PR)
    - Intelligence (IN)
    - Perseverance (PE)

# How to Calculate the Weights

- A committee come up with the following pair wise comparison matrix.

	<i>HD</i>	<i>PR</i>	<i>I</i>	<i>PE</i>
<i>HD</i>	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{5}$
<i>PR</i>	6	1	4	3
<i>I</i>	3	$\frac{1}{4}$	1	1
<i>PE</i>	5	$\frac{1}{3}$	1	1

## Intensity of Importance Scale

- 1 Equal Importance
- 3 Weak Importance
- 5 Strong Importance
- 7 Demonstrated Importance
- 9 Absolute Importance
- 2,4 Intermediate Values
- 6,8 Between the two Adjacent Judgments

# How to Calculate the Weights

1- Sum the elements in each row and normalize by dividing each sum by the total of all sums.

The first entry of the resulting vector the priority of the first criteria and so on

	<i>HD</i>	<i>PR</i>	<i>I</i>	<i>PE</i>	Sum	Priority
<i>HD</i>	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{5}$	1.70	0.060
<i>PR</i>	6	1	4	3	14.0	0.495
<i>I</i>	3	$\frac{1}{4}$	1	1	5.25	0.186
<i>PE</i>	5	$\frac{1}{3}$	1	1	7.33	0.259
					Sum = 28.28	1.000



# How to Calculate the Weights

2- Sum the elements in each column and take the reciprocals of those sums, then normalize by dividing each sum by the total of all sums.

The first entry of the resulting vector the priority of the first criteria and so on

	<i>HD</i>	<i>PR</i>	<i>I</i>	<i>PE</i>	
<i>HD</i>	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{5}$	
<i>PR</i>	6	1	4	3	
<i>I</i>	3	$\frac{1}{4}$	1	1	
<i>PE</i>	5	$\frac{1}{3}$	1	1	
Sum	15	1.75	6.333	5.2	<i>sum</i>
Reciprocals	0.0676	0.5714	0.1579	0.1923	0.9883
Priority	0.0683	0.5782	0.1598	0.1946	1.000

# How to Calculate the Weights

- 3- Divide the elements of each column by the sum of that column and then add the elements in each resulting row and divide this sum by the number of elements in that row.

	<i>HD</i>	<i>PR</i>	<i>I</i>	<i>PE</i>
<i>HD</i>	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{5}$
<i>PR</i>	6	1	4	3
<i>I</i>	3	$\frac{1}{4}$	1	1
<i>PE</i>	5	$\frac{1}{3}$	1	1

Sum      15   1.75   6.333   5.2

0.0667	0.0952	0.0526	0.0385
0.4000	0.5714	0.6316	0.5769
0.2000	0.1429	0.1579	0.1923
0.3333	0.1905	0.1579	0.1923

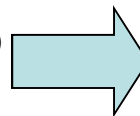
Sum

0.2530

0.5449

0.1732

0.2185



Priority

0.0632

0.5450

0.1732

0.2185

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# How to Calculate the Weights

4- Multiply the n element in each row and take the n<sup>th</sup> root, and then normalize the resulting number.

	Multiplication	4 <sup>th</sup> root	Weight
$\begin{bmatrix} 1 & 1/6 & 1/3 & 1/5 \\ 6 & 1 & 4 & 3 \\ 3 & 1/4 & 1 & 1 \\ 5 & 1/3 & 1 & 1 \end{bmatrix}$	$\Rightarrow \begin{Bmatrix} 0.01111 \\ 72 \\ 0.75 \\ 1.6667 \end{Bmatrix}$	$\Rightarrow \begin{Bmatrix} 0.32467 \\ 2.91295 \\ 0.93061 \\ 1.13622 \end{Bmatrix}$	$\Rightarrow \begin{Bmatrix} 0.0612 \\ 0.5492 \\ 0.1733 \\ 0.2185 \end{Bmatrix}$
	sum	5.3044	1.0000

# AHP Example

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- Calculate Consistency Ratio
  - 1- Multiply the pair wise comparison matrix by the weight vector, the resultant is a vector {B}
  - 2- Divide vector {B} by the weight vector and calculate the average  $\lambda$
  - 3- Calculate Consistency Index (C.I.)
  - 4- Calculate Consistency Ratio (C.R.)

# AHP Example

$$\begin{matrix} & & \text{Weight} & & \text{Vector B} \\ \begin{bmatrix} 1 & 1/6 & 1/3 & 1/5 \\ 6 & 1 & 4 & 3 \\ 3 & 1/4 & 1 & 1 \\ 5 & 1/3 & 1 & 1 \end{bmatrix} & * & \begin{Bmatrix} 0.0612 \\ 0.5492 \\ 0.1733 \\ 0.2185 \end{Bmatrix} & = & \begin{Bmatrix} 0.2541 \\ 2.2608 \\ 0.71060 \\ 0.87870 \end{Bmatrix}
 \end{matrix}$$

$$\begin{matrix} \text{Vector B} & & \text{Weight} \\ \begin{Bmatrix} 0.2541 \\ 2.2608 \\ 0.71060 \\ 0.87870 \end{Bmatrix} & / & \begin{Bmatrix} 0.0612 \\ 0.5492 \\ 0.1733 \\ 0.2185 \end{Bmatrix} & = & \begin{Bmatrix} 4.1507 \\ 4.1168 \\ 4.0501 \\ 4.1023 \end{Bmatrix} \Rightarrow \lambda = 4.1050
 \end{matrix}$$

# AHP Example

$$C.I. = \frac{\lambda - n}{n - 1} = \frac{4.105 - 4}{3} = 0.035$$

$$C.R. = \frac{C.I.}{RC} = \frac{0.035}{0.90} = 0.0389 < 0.1 \quad Ok.$$

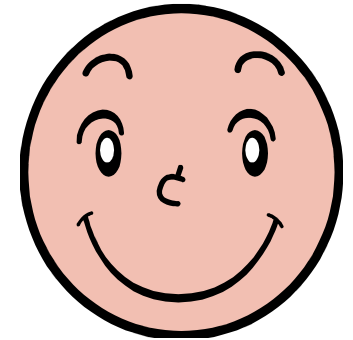
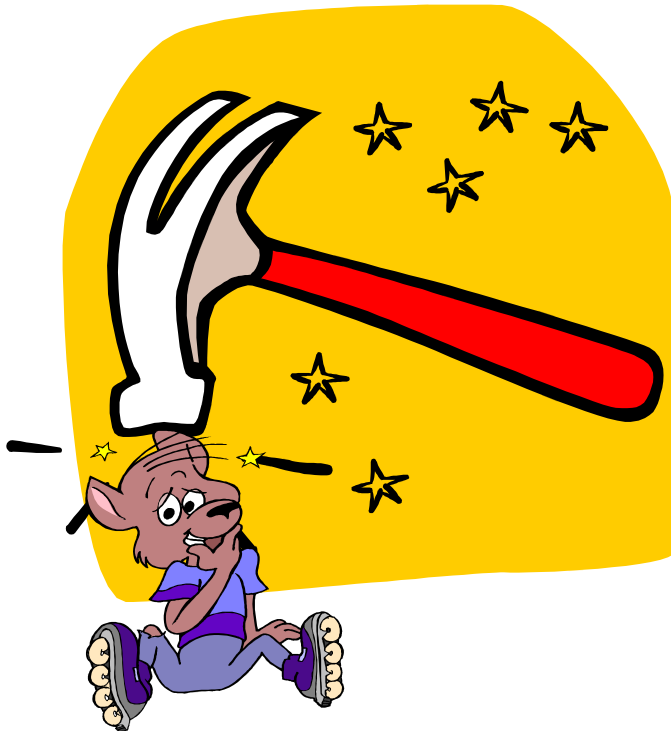
C.R. Should be less than 0.10  
To accept the calculated weight

RC from Table

matrix size	RC
1	0.00
2	0.00
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.49

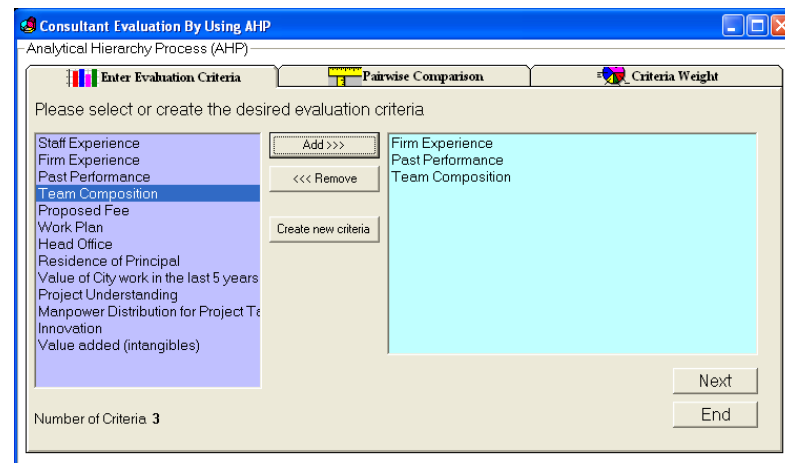
# What is in you tool box?

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# AHP Program

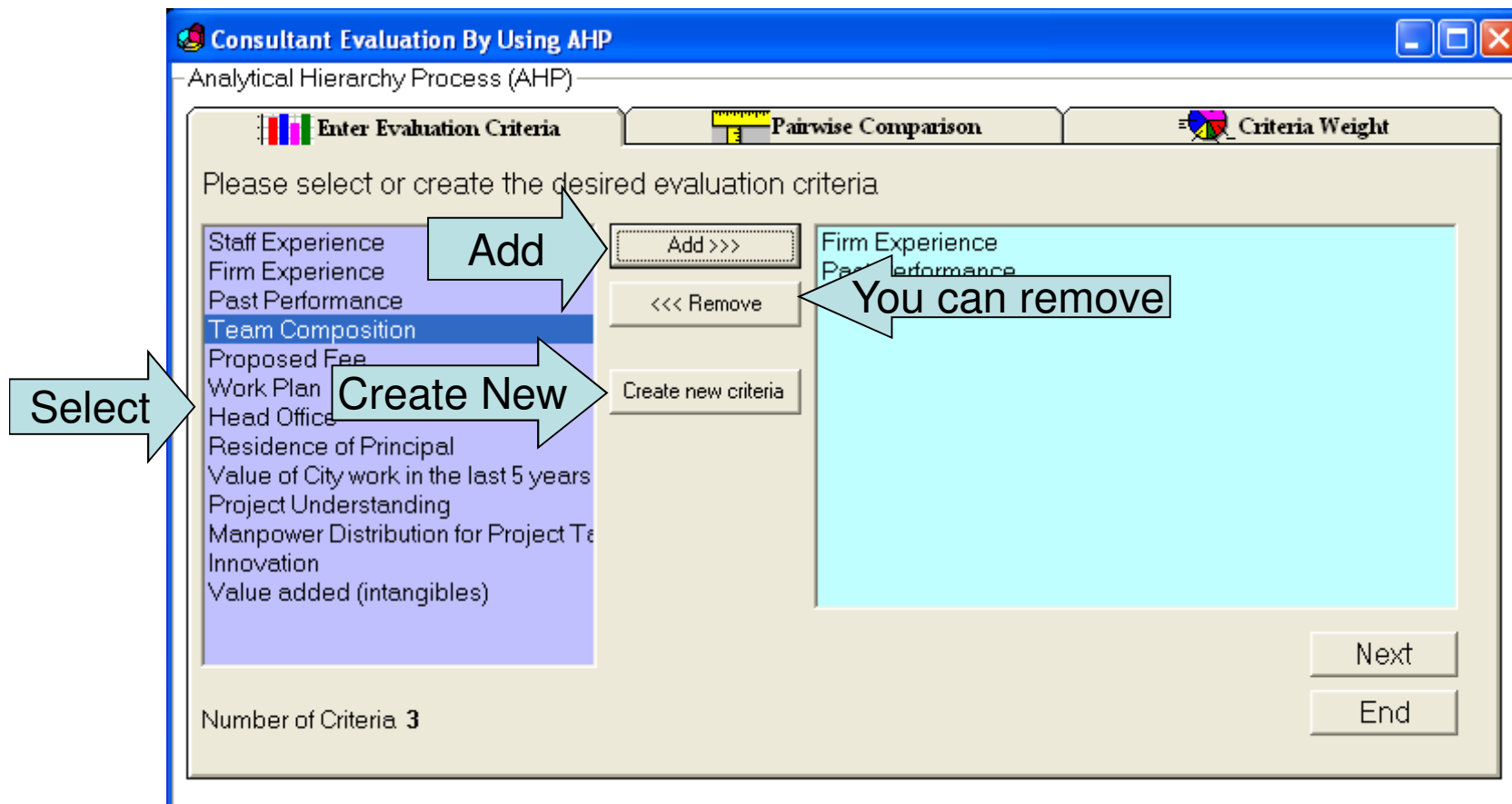
- A computer program has been developed for the purpose of facilitating AHP comparison
- The tool generates a spreadsheet file with all information required to complete the evaluation process





# AHP Procedure

- Defining a set of evaluation criteria



# AHP Procedure

- Perform pair wise comparison

**Consultant Evaluation By Using AHP**

Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** | **Pairwise Comparison** | **Criteria Weight**

Please enter the pairwise comparisons, the number you select represent the weight of the associated row criteria over the column criteria ( $W_r/W_c$ )

Comparison Criteria				
	Firm Experience	Past Performance	Team Composition	
Firm Experience	1	3	6	
Past Performance	1/3	1		
Team Composition	1/6			

Intensity of Importance Scale

- 1 Equal Importance
- 3 Weak Importance
- 5 Strong Importance
- 7 Demonstrated Importance
- 9 Absolute Importance
- 2,4 Intermediate Values
- 6,8 Between the two Adjacent Judgments

Calculate

End

# AHP Procedure

- Calculate criteria relative weights

**Consultant Evaluation By Using AHP**

Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** | **Pairwise Comparison** | **Criteria Weight**

Selection Criteria	Weight
Firm Experience	0.65481
Past Performance	0.24986
Team Composition	0.09534

Number of Firms:

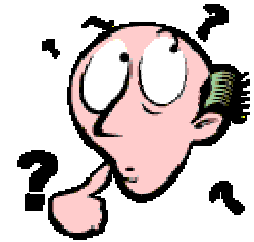
Project No.:

Project Name:

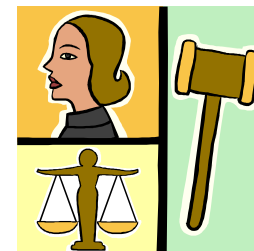
Consistency Ratio (C.R.) = 1.58 %

Note: The value of (C.R.) should be around 10% or less to be acceptable

# Consistency Ratio

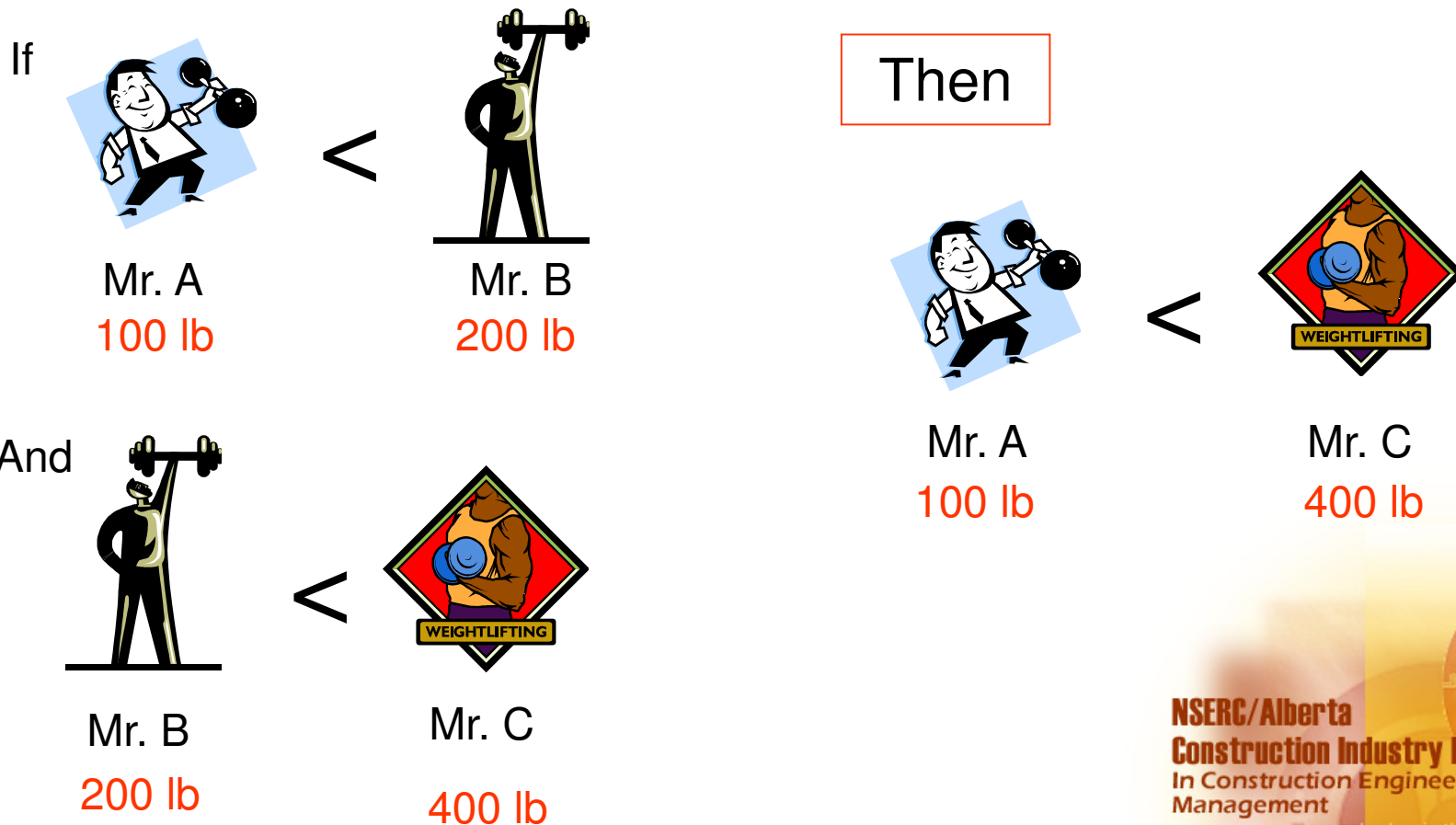


- A number which indicate if there are *significant contradictions in the pair wise comparison values* or not.
- acceptable consistency value is less than or equal to 10%.



# Consistency Ratio ?

- Compare the weight of three people



# Consistency Ratio

- Compare the weight of three people

**Consultant Evaluation By Using AHP**

Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** | **Pairwise Comparison** | **Criteria Weight**

Please enter the pairwise comparisons, the number you select represent the weight of the associated row criteria over the column criteria ( $W_r/W_c$ )

Comparison Criteria				
		Mr. A	Mr. B	Mr. C
	Mr. A	1	1/2	1/4
	Mr. B	2	1	1/2
▶	Mr. C	4	2	1

Intensity of Importance Scale

- 1 Equal Importance
- 3 Weak Importance
- 5 Strong Importance
- 7 Demonstrated Importance
- 9 Absolute Importance
- 2,4 Intermediate Values
- 6,8 Between the two Adjacent Judgments

Calculate

End

# Consistency Ratio

- Compare the weight of three people

**Consultant Evaluation By Using AHP**  
Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** | **Pairwise Comparison** | **Criteria Weight**

Selection Criteria	Weight
Mr. A	0.14286
Mr. B	0.28571
Mr. C	0.57143

Number of Firms:

Project No.:


Project Name:

Consistency Ratio (C.R.) = 0.00 %



Note: The value of (C.R.) should be around 10% or less to be acceptable

# Consistency Ratio

- If we said that Mr.A and Mr.C are equal:

 **Consultant Evaluation By Using AHP**

Analytical Hierarchy Process (AHP)

 **Enter Evaluation Criteria**       **Pairwise Comparison**

Please enter the pairwise comparisons, the number you select represent the weight of the associated row criteria over the column criteria ( $W_r/W_c$ )

Comparison Criteria				
		Mr. A	Mr. B	Mr. C
	Mr. A	1	1/2	1
	Mr. B	2	1	1/2
▶	Mr. C	1	2	1



# Consistency Ratio

- Consistency ratio is  $>10\%$

**Consultant Evaluation By Using AHP**  
Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** | **Pairwise Comparison** | **Criteria Weight**

Selection Criteria	Weight
Mr. A	0.25992
Mr. B	0.32748
Mr. C	0.41260

Number of Firms:

Project No.:

Project Name:

**Consistency Ratio (C.R.) = 18.74 %**

Note: The value of (C.R.) should be around 10% or less to be acceptable

# AHP Procedure

- If all have the same weight (100 kg), then

**Consultant Evaluation By Using AHP**  
Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** **Pairwise Comparison**

Please enter the pairwise comparisons, the number you select represent the weight of the associated row criteria over the column criteria (Wr/Wc)

Comparison Criteria				
	Mr. A	Mr. B	Mr. C	
Mr. A	1	1	1	
Mr. B	1	1	1	
Mr. C	1	1	1	

**Consultant Evaluation By Using AHP**  
Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** **Pairwise Comparison**

Selection Criteria	Weight
Mr. A	0.33333
Mr. B	0.33333
Mr. C	0.33333

Consistency Ratio (C.R.) = 0.00 %

The criteria are equal then the weight is Simply

$$W = \frac{1}{\text{\# of Criteria}}$$

# AHP Procedure

- Fill Project Information and Number of Consultants

**Consultant Evaluation By Using AHP**

Analytical Hierarchy Process (AHP)

**Enter Evaluation Criteria** | **Pairwise Comparison** | **Criteria Weight**

Selection Criteria	Weight
Firm Experience	0.65481
Past Performance	0.24986
Team Composition	0.09534

Number of Firms:

Project No.:

Project Name:

Consistency Ratio (C.R.) = 1.58 %

Note: The value of (C.R.) should be around 10% or less to be acceptable

**Click**

# AHP Procedure

- Evaluate score for each consultant firm

Microsoft Excel - Book1

File Edit View Insert Format Tools Data Window Help

A1 = Consultant Evaluation - Scoring Summary

	A	B	C	D	E	F	G	H	I	J
1	Consultant Evaluation - Scoring Summary									
2										
3	Project No. :12030-03-444888									
4	Project Name : Presentation Test Project									
5										
6	Item	Weight	Firm 1 Rating (100)	Firm 1 Score	Firm 2 Rating (100)	Firm 2 Score	Firm 3 Rating (100)	Firm 3 Score	Firm 4 Rating (100)	Firm 4 Score
7	1. Firm Experience	0.654806738	0	0	0	0	0	0	0	0
8	2. Past Performance	0.249855533	0	0	0	0	0	0	0	0
9	3. Team Composition	0.095337729	0	0	0	0	0	0	0	0
10	Firm Total Score		0	0	0	0	0	0	0	0
11										
12										

Fill Score

# AHP Procedure

- Select the highest scoring firm:

	A	B	C	D	E	F	G	H	I	J
1	Consultant Evaluation -Scoring Summary									
2										
3	Project No. :12030-03-444888									
4	Project Name : Presentation Test Project									
5			Firm 1		My Firm		Firm 3		Firm 4	
6	Item	Weight	Rating (100)	Score	Rating (100)	Score	Rating (100)	Score	Rating (100)	Score
7	1. Firm Experience	0.654806738	85	55.659	99	64.826	85	55.659	26	17.025
8	2. Past Performance	0.249855533	55	13.742	87	21.737	88	21.987	75	18.739
9	3. Team Composition	0.095337729	62	5.9109	45	4.2902	24	2.2881	65	6.197
10	Firm Total Score			75.312		90.853		79.934		41.961