

# The Book of Math (Notes)

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# Forward and Disclaimer

These are math notes made by a student (with a physics major and math minor) based off text books. It may contain misconceptions and misinterpretations, thus should not be viewed in the same light of a text book. Use at your own risk and mental sanity.

## Symbols

### Logic

Name	Symbol	Comment
Exists	$\exists$	There exists at least one
For all	$\forall$	
Not exists	$\nexists$	There does not exist
Exists one	$\exists!$	There only exists one and only one
And	$\wedge$	
Or	$\vee$	Inclusive or
Not	$\neg$	
Logically implies	$\implies$	If
Logically implied by	$\impliedby$	Only if
Logically equivalent	$\iff$	If and only if
Implies	$\longrightarrow$	
Implied by	$\longleftarrow$	
Double Implication	$\longleftrightarrow$	

### Set Notation

Name	Symbol	Comment
Empty Set	$\emptyset$	The set that is empty
Natural Numbers	$\mathbb{N}$	Set of natural numbers not containing 0, equivalent to the set of positive integers
Integers	$\mathbb{Z}$	Set of integers
Rational Numbers	$\mathbb{Q}$	
Algebraic Numbers	$\mathbb{A}$	
Real Numbers	$\mathbb{R}$	
Complex Numbers	$\mathbb{C}$	
In	$\in$	
Not in	$\notin$	
Owns	$\ni$	Has an element
Proper Subset	$\subset$	Subset that is not itself
Subset	$\subseteq$	
Superset	$\supset$	Superset that is not itself
Proper Superset	$\supsetneq$	

Power set	$\wp$
Union	$\cup$
Intersection	$\cap$
Difference	$\setminus$

## Relationships

Name	Symbol	Comment
Defined	$\doteq$	
Approximate	$\approx$	
Equivalent	$\equiv$	Isomorphic (Group Theory)
Congruent	$\cong$	Homomorphic (Group Theory)
Proportional	$\propto$	

## Operators

Name	Symbol	Comment
	$\oplus$	
	$\otimes$	
	$\odot$	
	$\circ$	Convolution
Dagger	$\dagger$	Complex conjugate transpose of a matrix

## Arrows

Name	Symbol	Comment
Maps to	$\mapsto$	

## Hebrew

Name	Symbol	Comment
Aleph	$\aleph$	Carnality of infinite sets that can be well ordered

## Other

Name	Symbol	Comment
Real part	$\Re$	Real part of a number
Imaginary part	$\Im$	Imaginary part of a number



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# Part I

## Logic



# Chapter 1

## Proofs



# **Part II**

## **Numbers**



# Chapter 2

## Natural $\mathbb{N}$





# Chapter 3

## Integers $\mathbb{Z}$



# Chapter 4

## Rationals $\mathbb{Q}$



# Chapter 5

## Constructible



# Chapter 6

## Algebraic $\mathbb{A}$





# Chapter 7

## Reals $\mathbb{R}$



# Chapter 8

## Complex $\mathbb{C}$



# Part III

## Real Analysis



Books Used:

1. Kenneth A. Ross - Elementary Analysis (2nd Ed.) [1]





# Part IV

## Complex Analysis



Books Used:

1. Brown and Churchill - Complex Variables and Applications [2]



## Chapter 9

# Conformal Mapping



## Part V

# Ordinary Differential Equations





# Part VI

## Nonlinear Dynamics



## Part VII

# Partial Differential Equations



## Calculus of Variations



# Part VIII

## Integral Equations





# Part IX

## Linear Algebra



# Chapter 10

## Markov Chains



# Part X

## Tensors



# Part XI

## Riemann Geometry





# Part XII

## Abstract Algebra



# Chapter 11

## Groups



# Chapter 12

## Rings

### 12.1 Ideals



# Chapter 13

## Integral Domains





# Chapter 14

## GCD Domains



## Chapter 15

# Unique Factorization Domains



## Chapter 16

# Principal Ideal Domains



# Chapter 17

## Fields





# **Part XIII**

## **Galois Theory**



## Lie Algebra



# Part XIV

## C-Star Algebra



# **Part XV**

## **Set Theory**





**Part XVI**

**Model Theory**



# Part XVII

## Statistics



# **Part XVIII**

## **Tips and Tricks**



# Chapter 18

## Integration Techniques

### 18.1 DI Method (Integration Table)

### 18.2 Feynman Integration





# Part XIX

## Index



# Part XX

## Bibliography



# Bibliography

- [1] Kenneth A. Ross. *Elementary Analysis*. Springer, 2 edition, 2013.
- [2] James Ward Brown and Ruel V. Churchill. *Complex Variables and Applications*. McGraw-Hill Education, 9 edition, 2014.