

# The Book of Math (Notes)

Kevin Kuo

November 14, 2020

# Forward and Disclaimer

These are math notes made by a student (with a physics major and math minor) based off text books. It may contain misconceptions and misinterpretations, thus should not be viewed in the same light of a text book. Use at your own risk and mental sanity.

## Symbols

### Logic

Name	Symbol	Comment
Exists	$\exists$	There exists at least one
For all	$\forall$	
Not exists	$\nexists$	There does not exist
Exists one	$\exists!$	There only exists one and only one
And	$\wedge$	
Or	$\vee$	Inclusive or
Not	$\neg$	
Logically implies	$\implies$	If
Logically implied by	$\impliedby$	Only if
Logically equivalent	$\iff$	If and only if
Implies	$\longrightarrow$	
Implied by	$\longleftarrow$	
Double Implication	$\longleftrightarrow$	

### Set Notation

Name	Symbol	Comment
Empty Set	$\emptyset$	The set that is empty
Natural Numbers	$\mathbb{N}$	Set of natural numbers not containing 0, equivalent to the set of positive integers
Integers	$\mathbb{Z}$	Set of integers
Rational Numbers	$\mathbb{Q}$	
Algebraic Numbers	$\mathbb{A}$	
Real Numbers	$\mathbb{R}$	
Complex Numbers	$\mathbb{C}$	
In	$\in$	
Not in	$\notin$	
Owns	$\ni$	Has an element
Proper Subset	$\subset$	Subset that is not itself
Subset	$\subseteq$	
Superset	$\supset$	Superset that is not itself
Proper Superset	$\supsetneq$	

Power set	$\wp$
Union	$\cup$
Intersection	$\cap$
Difference	$\setminus$

## Relationships

Name	Symbol	Comment
Defined	$\doteq$	
Approximate	$\approx$	
Equivalent	$\equiv$	Isomorphic (Group Theory)
Congruent	$\cong$	Homomorphic (Group Theory)
Proportional	$\propto$	

## Operators

Name	Symbol	Comment
	$\oplus$	
	$\otimes$	
	$\odot$	
	$\circ$	Convolution
Dagger	$\dagger$	Complex conjugate transpose of a matrix

## Arrows

Name	Symbol	Comment
Maps to	$\mapsto$	

## Hebrew

Name	Symbol	Comment
Aleph	$\aleph$	Carnality of infinite sets that can be well ordered

## Other

Name	Symbol	Comment
Real part	$\Re$	Real part of a number
Imaginary part	$\Im$	Imaginary part of a number

# Book Constitution

## Intents and Purpose

The goal of this book is to organize mathematical knowledge of topics related to the study of physics or the author's interest. It is meant to be used as a source of for future reference, not as a textbook for students new to the topics. It is a notebook of a student, thus should be treated as one and not as a textbook. At most, it could be used as a study guide along side a textbook. Definitely not as the main source for acquiring knowledge.

## Layout and Organization

The book is split into parts each containing a field of study mathematics, or a topic large enough to justify giving it its own part. Each part contains chapters that focuses on a particular topic required to understand the field, with sections dedicated to describing a particular knowledge required for the topic.

As axioms, definitions, theorems, corollary, and proofs are integral and abundant to the study of mathematics, each will have a unique style. Each environment and its styles are displayed as follows:

### **Axiom 0.1: Axiom name**

*Example Axiom Axioms are the “ground rules” of the set.*

### **Theorem 0.1.1: Theorem name or citation**

*Example Theorem An important logical result from the axioms, with proof.*

### **Conjecture 0.1.1: Name of conjecture or citation**

*Example Conjecture A hypothesis, without proof.*

### **Corollary 0.1.1.1:**

*Example Corollary An implication as a result of a theorem.*

### **Lemma 0.1.1.1:**

*Example Lemma Small theorems that build up to a larger theorem.*

### **Proposition 0.1.1.1:**

*Example Proposition Example proposition.*

*Proof:* Logical deductions that results in a theorem. Proofs I've written will be in grey, which may or may not be correct. □

### **Definition 0.1: Word**

*Example Definition The definition of a word.*

**Example 0.1.1.** *An example.*

**Remark.** *Remark A comment by the author in the textbooks used.*

**Observation.** *Example Observation A remark by me.*

**Question.** *Example Question A question from me for a mystery to be answered later.*



# Contents

<b>I</b>	<b>Logic</b>	<b>1</b>
1	Proofs	3
<b>II</b>	<b>Numbers</b>	<b>5</b>
2	Natural $\mathbb{N}$	7
3	Integers $\mathbb{Z}$	9
4	Rationals $\mathbb{Q}$	11
5	Constructible	13
6	Algebraic $\mathbb{A}$	15
7	Reals $\mathbb{R}$	17
8	Complex $\mathbb{C}$	19
<b>III</b>	<b>Real Analysis</b>	<b>21</b>
<b>IV</b>	<b>Complex Analysis</b>	<b>25</b>
9	Metric Spaces	29
10	Conformal Mapping	31

V	Ordinary Differential Equations	33
VI	Nonlinear Dynamics	35
VII	Partial Differential Equations	37
VIII	Integral Equations	41
IX	Linear Algebra	43
11	Markov Chains	45
X	Tensors	47
XI	Riemann Geometry	49
XII	Abstract Algebra	51
12	Groups	53
13	Rings	55
13.1	Ideals . . . . .	55
14	Integral Domains	57
15	GCD Domains	59
16	Unique Factorization Domains	61
17	Principal Ideal Domains	63
18	Fields	65



<b>XIII</b>	<b>Galois Theory</b>	<b>67</b>
<b>XIV</b>	<b>C-Star Algebra</b>	<b>71</b>
<b>XV</b>	<b>Set Theory</b>	<b>73</b>
<b>XVI</b>	<b>Model Theory</b>	<b>75</b>
<b>XVII</b>	<b>Statistics</b>	<b>77</b>
<b>XVIII</b>	<b>Tips and Tricks</b>	<b>79</b>
<b>19</b>	<b>Integration Techniques</b>	<b>81</b>
19.1	DI Method (Integration Table) . . . . .	81
19.2	Feynman Integration . . . . .	81
<b>XIX</b>	<b>Index</b>	<b>83</b>
<b>XX</b>	<b>Bibliography</b>	<b>85</b>



# Part I

## Logic



# Chapter 1

## Proofs



# **Part II**

## **Numbers**





# Chapter 2

## Natural $\mathbb{N}$



# Chapter 3

## Integers $\mathbb{Z}$



# Chapter 4

## Rationals $\mathbb{Q}$



# Chapter 5

## Constructible





# Chapter 6

## Algebraic $\mathbb{A}$



# Chapter 7

## Reals $\mathbb{R}$



# Chapter 8

## Complex $\mathbb{C}$



# Part III

## Real Analysis





Books Used:

1. Kenneth A. Ross - Elementary Analysis (2nd Ed.) [1]



# Part IV

## Complex Analysis



Books Used:

1. Brown and Churchill - Complex Variables and Applications [2]



# Chapter 9

## Metric Spaces





# Chapter 10

## Conformal Mapping



## Part V

# Ordinary Differential Equations



# **Part VI**

## **Nonlinear Dynamics**



## Part VII

# Partial Differential Equations





## Calculus of Variations



# Part VIII

## Integral Equations



# Part IX

## Linear Algebra



# Chapter 11

## Markov Chains





# Part X

## Tensors



# Part XI

## Riemann Geometry



# Part XII

## Abstract Algebra



# Chapter 12

## Groups





# Chapter 13

## Rings

### 13.1 Ideals



## Chapter 14

# Integral Domains



# Chapter 15

## GCD Domains



## Chapter 16

# Unique Factorization Domains





## Chapter 17

# Principal Ideal Domains



# Chapter 18

## Fields



**Part XIII**

**Galois Theory**



## Lie Algebra





**Part XIV**

**C-Star Algebra**



# **Part XV**

## **Set Theory**



**Part XVI**

**Model Theory**



# Part XVII

## Statistics





# **Part XVIII**

## **Tips and Tricks**



# Chapter 19

## Integration Techniques

### 19.1 DI Method (Integration Table)

### 19.2 Feynman Integration



# Part XIX

## Index



# Part XX

## Bibliography





# Bibliography

- [1] Kenneth A. Ross. *Elementary Analysis*. Springer, 2 edition, 2013.
- [2] James Ward Brown and Ruel V. Churchill. *Complex Variables and Applications*. McGraw-Hill Education, 9 edition, 2014.