

The Book of Math (Notes)

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Forward and Disclaimer

These are math notes made by a student (with a physics major and math minor) based off text books. It may contain misconceptions and misinterpretations, thus should not be viewed in the same light of a text book. Use at your own risk and mental sanity.

Symbols

Logic

Name	Symbol	Comment
Exists	\exists	There exists at least one
For all	\forall	
Not exists	\nexists	There does not exist
Exists one	$\exists!$	There only exists one and only one
And	\wedge	
Or	\vee	Inclusive or
Not	\neg	
Logically implies	\implies	If
Logically implied by	\impliedby	Only if
Logically equivalent	\iff	If and only if
Implies	\longrightarrow	
Implied by	\longleftarrow	
Double Implication	\longleftrightarrow	

Set Notation

Name	Symbol	Comment
Empty Set	\emptyset	The set that is empty
Natural Numbers	\mathbb{N}	Set of natural numbers not containing 0, equivalent to the set of positive integers
Integers	\mathbb{Z}	Set of integers
Rational Numbers	\mathbb{Q}	
Algebraic Numbers	\mathbb{A}	
Real Numbers	\mathbb{R}	
Complex Numbers	\mathbb{C}	
In	\in	
Not in	\notin	
Owns	\ni	Has an element
Proper Subset	\subset	Subset that is not itself
Subset	\subseteq	
Superset	\supset	Superset that is not itself
Proper Superset	\supsetneq	

Power set	\wp
Union	\cup
Intersection	\cap
Difference	\setminus

Relationships

Name	Symbol	Comment
Defined	\doteq	
Approximate	\approx	
Equivalent	\equiv	Isomorphic (Group Theory)
Congruent	\cong	Homomorphic (Group Theory)
Proportional	\propto	

Operators

Name	Symbol	Comment
	\oplus	
	\otimes	
	\odot	
	\circ	Convolution
Dagger	\dagger	Complex conjugate transpose of a matrix

Arrows

Name	Symbol	Comment
Maps to	\mapsto	

Hebrew

Name	Symbol	Comment
Aleph	\aleph	Carnality of infinite sets that can be well ordered

Other

Name	Symbol	Comment
Real part	\Re	Real part of a number
Imaginary part	\Im	Imaginary part of a number

Book Constitution

Intents and Purpose

The goal of this book is to organize mathematical knowledge of topics related to the study of physics or the author's interest. It is meant to be used as a source of for future reference, not as a textbook for students new to the topics. It is a notebook of a student, thus should be treated as one and not as a textbook. At most, it could be used as a study guide along side a textbook. Definitely not as the main source for acquiring knowledge.

Layout and Organization

The book is split into parts each containing a field of study mathematics, or a topic large enough to justify giving it its own part. Each part contains chapters that focuses on a particular topic required to understand the field, with sections dedicated to describing a particular knowledge required for the topic.

As axioms, definitions, theorems, corollary, and proofs are integral and abundant to the study of mathematics, each will have a unique style. Each environment and its styles are displayed as follows:

Axiom 0.1: Axiom name

Example Axiom Axioms are the “ground rules” of the set.

Theorem 0.1.1: Theorem name or citation

Example Theorem An important logical result from the axioms, with proof.

Conjecture 0.1.1: Name of conjecture or citation

Example Conjecture A hypothesis, without proof.

Corollary 0.1.1.1:

Example Corollary An implication as a result of a theorem.

Lemma 0.1.1.1:

Example Lemma Small theorems that build up to a larger theorem.

Proposition 0.1.1.1:

Example Proposition Example proposition.

Proof: Logical deductions that results in a theorem. Proofs I've written will be in grey, which may or may not be correct. □

Definition 0.1: Word

Example Definition The definition of a word.

Example 0.1.1. *An example.*

Remark. *Remark A comment by the author in the textbooks used.*

Observation. *Example Observation A remark by me.*

Question. *Example Question A question from me for a mystery to be answered later.*

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Part I

Logic

Chapter 1

Proofs

Part II

Numbers

Chapter 2

Natural \mathbb{N}

Chapter 3

Integers \mathbb{Z}

Chapter 4

Rationals \mathbb{Q}

Chapter 5

Constructible

Chapter 6

Algebraic \mathbb{A}

Chapter 7

Reals \mathbb{R}

Chapter 8

Complex \mathbb{C}

Part III

Real Analysis

Books Used:

1. Kenneth A. Ross - Elementary Analysis (2nd Ed.) [1]

Part IV

Complex Analysis

Books Used:

1. Brown and Churchill - Complex Variables and Applications [2]

Chapter 9

Metric Spaces

Chapter 10

Conformal Mapping

Part V

Ordinary Differential Equations

Part VI

Nonlinear Dynamics

Part VII

Partial Differential Equations

Calculus of Variations

Part VIII

Integral Equations

Part IX

Linear Algebra

Chapter 11

Markov Chains

Part X

Tensors

Part XI

Riemann Geometry

Part XII

Abstract Algebra

Chapter 12

Groups

Chapter 13

Rings

13.1 Ideals

Chapter 14

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Chapter 15

GCD Domains

Chapter 16

Unique Factorization Domains

Chapter 17

Principal Ideal Domains

Chapter 18

Fields

Part XIII

Galois Theory

Lie Algebra

Part XIV

C-Star Algebra

Part XV

Set Theory

Part XVI

Model Theory

Part XVII

Statistics

Part XVIII

Tips and Tricks

Chapter 19

Integration Techniques

19.1 DI Method (Integration Table)

19.2 Feynman Integration

Part XIX

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Part XX

Bibliography

Bibliography

- [1] Kenneth A. Ross. *Elementary Analysis*. Springer, 2 edition, 2013.
- [2] James Ward Brown and Ruel V. Churchill. *Complex Variables and Applications*. McGraw-Hill Education, 9 edition, 2014.