The Book of Math (Notes)

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Forward and Disclaimer

These are math notes made by a student (with a physics major and math minor) based off text books. It may contain misconceptions and misinterpretations, thus should not be viewed in the same light of a text book. Use at your own risk and mental sanity.

Symbols

Logic

Name	Symbol	Comment
Exists	$\exists$	There exists at least one
For all	$\forall$	
Not exists	$\nexists$	There does not exist
Exists one	$\exists!$	There only exists one and only one
And	$\wedge$	
Or	$\vee$	Inclusive or
Not	$\neg$	
Logically implies	$\Longrightarrow$	If
Logically implied by	$\Longleftarrow$	Only if
Logically equivalent	$\Longleftrightarrow$	If and only if
Implies	$\longrightarrow$	
Implied by	$\longleftarrow$	
Double Implication	$\longleftrightarrow$	

Set Notation

Name	Symbol	Comment
Empty Set	$\varnothing$	The set that is empty
Natural Numbers	$\mathbb{N}$	Set of natural numbers not containing 0, equivalent to the set of positive integers
Integers	$\mathbb{Z}$	Set of integers
Rational Numbers	$\mathbb{Q}$	
Algebraic Numbers	$\mathbb{A}$	
Real Numbers	$\mathbb{R}$	
Complex Numbers	$\mathbb{C}$	
In	$\in$	
Not in	$\notin$	
Owns	$\ni$	Has an element
Proper Subset	$\subset$	Subset that is not itself
Subset	$\subseteq$	
Superset	$\supset$	Superset that is not itself
Proper Superset	$\supseteq$	

Power set	$\wp$
Union	$\cup$
Intersection	$\cap$
Difference	$\setminus$

Relationships

Name	Symbol	Comment
Defined	$\doteq$	
Approximate	$\approx$	
Equivalent	$\equiv$	Isomorphic (Group Theory)
Congruent	$\cong$	Homomorphic (Group Theory)
Proportional	$\propto$	

Operators

Name	Symbol	Comment
	$\oplus$	
	$\otimes$	
	$\odot$	
	$\circ$	Convolution
Dagger	$\dagger$	Complex conjugate transpose of a matrix

Arrows

Name	Symbol	Comment
Maps to	$\mapsto$	

Hebrew

Name	Symbol	Comment
Aleph	$\aleph$	Carnality of infinite sets that can be well ordered

Other

Name	Symbol	Comment
Real part	$\Re$	Real part of a number
Imaginary part	$\Im$	Imaginary part of a number

Book Constitution

Intents and Purpose

The goal of this book is to organize mathematical knowledge of topics related to the study of physics or the author's interest. It is meant to be used as a source of for future reference, not as a textbook for students new to the topics. It is a notebook of a student, thus should be treated as one and not as a textbook. At most, it could be used as a study guide along side a textbook. Definitely not as the main source for acquiring knowledge.

Layout and Organization

The book is split into parts each containing a field of study mathematics, or a topic large enough to justify giving it its own part. Each part contains chapters that focuses on a particular topic required to understand the field, with sections dedicated to describing a particular knowledge required for the topic.

As axioms, definitions, theorems, corollary, and proofs are integral and abundant to the study of mathematics, each will have a unique style. Each environment and its styles are displayed as follows:

Axiom 0.1: Axiom name

Example Axiom Axioms are the "ground rules" of the set.

Theorem 0.1.1: Theorem name or citation

Example Theorem An important logical result from the axioms, with proof.

Conjecture 0.1.1: Name of conjecture or citation

Example Conjecture A hypothesis, without proof.

Corollary 0.1.1.1:

Example Corollary An implication as a result of a theorem.

Lemma 0.1.1.1:

Example Lemma Small theorems that build up to a larger theorem.

Proposition 0.1.1.1:

Example Proposition Example proposition.

Proof: Logical deductions that results in a theorem. Proofs I've written will be in grey, which may or may not be correct. □

Definition 0.1: Word

Example Definition The definition of a word.

Example 0.1.1. *An example.*

Remark. *Remark A comment by the author in the textbooks used.*

Observation. *Example Observation A remark by me.*

Question. *Example Question A question from me for a mystery to be answered later.*

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Reals $\mathbb{R}$

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Part III

Real Analysis

Books Used:

1. Kenneth A. Ross - Elementary Analysis (2nd Ed.) [1]

Part IV

Complex Analysis

Books Used:

1. Brown and Churchill - Complex Variables and Applications [2]

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Bibliography

[1] Kenneth A. Ross. *Elementary Analysis*. Springer, 2 edition, 2013.

[2] James Ward Brown and Ruel V. Churchill. *Complex Variables and Applications*. McGraw-Hill Education, 9 edition, 2014.