Lab 1: Hydrogen-Deuterium Mass Ratio

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Note: Answers to prompts 1 and 2 are split into sections I to IV for clarity in answers

I. INTRODUCTION

The experiment performed aimed to derive a value for the mass ratio of Hydrogen to Deuterium through finding a Rydberg constant for various emission lines of Hydrogen and Deuterium isotopes. The Balmer formula can be generalized to $1/\lambda_n = R(1/2^2 - 1/n^2)$, where R is the Rydberg constant, which, for Hydrogen is $R_H = 109677.5810cm^{-1}$. This Rydberg constant can be reformulated to be $R_H = \frac{me^4}{8\epsilon_0^2 h^3 c}$. However, it was then found that the electron mass m should be replaced by a reduced mass term μ of form $\mu = mM/(M+m)$, where M is the mass of the nucleus. This leads to the Rydberg formula being corrected for the inclusion of the reduced mass term, giving $E_i - E_f = \frac{hc}{\lambda} = \frac{\mu_X Z^2 e^4}{(4\pi\epsilon_0)^2 2\hbar^2}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$, where μ_X is the reduced mass of some atom X.

Thus, using this corrected Rydberg formula, the light emitted from some transition is inversely proportional to the reduced mass, $1/\lambda_X \propto \mu_X$. Thus, the nuclear masses of Hydrogen and Deuterium are related to the wavelengths of light produced by each isotope, allowing for the experimental verification of the ratio of masses of Hydrogen and Deuterium, which has the established value of

$$\frac{M_H}{M_D} = \frac{1.007276}{2.013553} = 0.500248$$

II. EXPERIMENTAL METHODS

The measuring of the mass ratio was done through analysis of emission peaks of Hydrogen and Deuterium. Five different transitions in the Balmer series were utilized, with one run of data for alpha, beta, gamma, and delta transitions, and two runs of data for the epsilon transition.

Initially, emission spectra of Sodium D lines was used to calibrate the data output. An automated stepper motor was used to linearly sweep through wavelengths that include both Sodium D emission spectra peaks, giving data of intensity vs. time. Comparing the time taken for the motor to sweep between peaks and the known wavelength separation between peaks, one can convert raw time data to a wavelength separation in Angstroms.

Peaks were determined through a least squares fitting in MATLAB of a two-term Gaussian curve, as can be seen in figure 1. Experimental uncertainty was estimated through manually observing the raw data output from the oscilloscope. An example of this manual determination is explored later, with an example in figure 3.

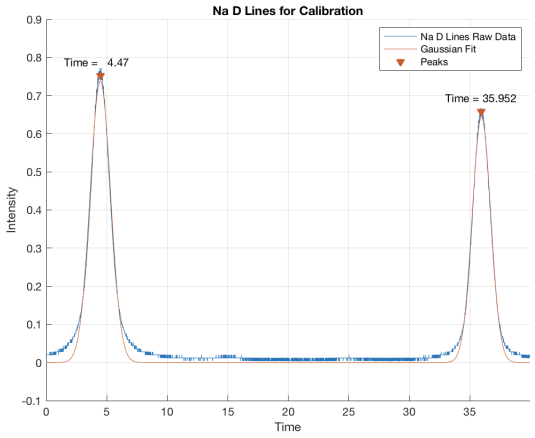

FIG. 1. Raw Calibration Data

Once this calibration was completed, raw data from the oscilloscope giving the separation of peaks in units of time were converted to Angstroms. As the mass ratio depends solely on $\Delta\lambda_{HD}/\lambda_H$ and other constants, the wavelength separation for each emission spectrum was plotted vs. the wavelength. This slope was then used to find the mass ratio.

III. RESULTS

The mass ratio M_H/M_D was experimentally found to be 0.499 ± 0.015. This, with uncertainty, includes the accepted literature value of 0.500248. The final plotting of experimental data, with propagated experimental uncertainties, can be seen in figure 2.

Individual experimentally observed values for wavelength separation of peaks are detailed in table I, with their respective uncertainties.

TABLE I. Experimental Values for Wavelength Separation

	Wavelength Separation (Å)	Uncertainty (Å)
Alpha	1.76	0.08
Beta	1.27	0.07
Gamma	1.15	0.12
Delta	1.10	0.08
Epsilon	1.0	0.3

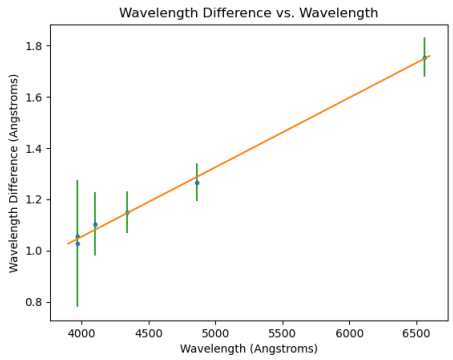

FIG. 2. Final Results for Mass Ratio Derivation

IV. DETERMINATION OF UNCERTAINTIES

When manually considering the raw data, experimental uncertainty was observed to be much more significant than any fitting error output by MATLAB in the least squares Gaussian fits. Thus, experimental uncertainty further propagated in this report were manually estimated. Observation of the noise in the peak was considered, and two points to the outside of the "flat" peak were used to estimate a position where the peak were confidently assumed to be. An example of two points chosen for a reasonable boundary in the epsilon data can be seen in figure 3. In retrospect, uncertainties of peak location may have been overestimated, however this overestimation was deemed to be preferable to underestimation.

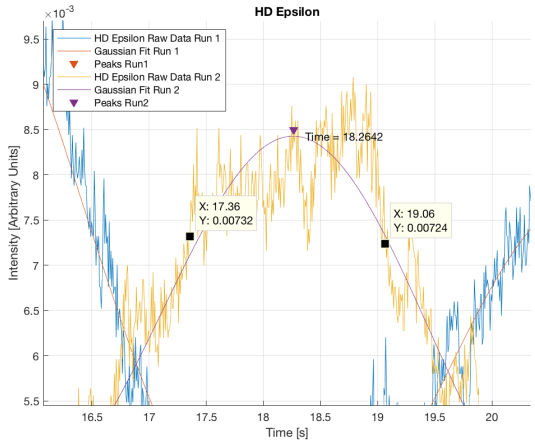

FIG. 3. Example of Uncertainty Determination

There were no significant sources of systematic error evident that affected final results. Final results for the mass ratio include accepted literature values when taking into account propagated uncertainties. Individual data points also generally adhere to the trends expected, with decreasing wavelength separation as one moves from alpha to epsilon emission spectra.

However, the individual points do suggest some systematic error causing calculated mass ratios to be higher than expected if calculated through individual peak separation data for each emission line, rather than plotted and a gradient taken. i.e., All points on figure 2 appear to be slightly higher on the y-axis than expected. However, this does not affect final results as individual values were not utilized to form the final mass ratio, but instead the gradient of a linear best fit line on figure 2 was used, which was not affected by the vertical shifting of data points possibly caused by some systematic error.

One possible source of systematic error may be from the stepper motor used to rotate the diffraction grating, allowing for the progressive scanning through various wavelength values to produce the raw output of two emission spectra peaks. Due to the angle and wavelength not being exactly linearly proportional, the rate at which wavelengths are swept through by the motor may change at various wavelengths, causing a slightly non-linear plot once plotted as in figure 2.

Another possible source of systematic error would be any mistake made in taking the calibration data, and thus utilizing an incorrect calibration constant. This would affect all further conversions from raw time data to peak wavelength separations in Angstroms, and affect the final mass ratio derived from experimental data.

V. DISCUSSION OF CALIBRATION

If one assumes a mistake where the stepper motor was paused for one second when between the two Na D lines used for calibration, this would affect further calculations, as calibration data affects the wavelength separation in Angstroms which is utilized to determine the mass ratio. This is due to the method used for determining the mass ratio being dependent upon the slope of figure 2, which would change if the multiplier (calibration constant) is incorrect, and the time separation is multiplied by this incorrect value.

Should there be a pause, the Na D lines would appear further apart than they actually are in the raw data output from the oscilloscope. This would affect the conversion factor used to convert from the wavelength separation being in units of time (raw output of oscilloscope) compared to the final units of Angstroms used for wavelength separation. The conversion factor would become smaller in magnitude, and thus the slope in the final plot of wavelength separation vs. wavelength (similar to figure 2) would become lower than the correct amount should this mistake have been corrected.

The result of this added second between peaks in the calibration data would be an erroneously large experimentally observed mass ratio M_H/M_D. Suppose there was an extra second, the calibration constant would then become roughly 0.1837941 Å s^{-1} as opposed to the more correct 0.1896 Å s^{-1}.

The originally calculated mass ratio is 0.500248, but

with the new calibration constant, this would end up being roughly 0.514.

VI. PHOTOMULTIPLIER TUBES

An essential component to this experiment and similar are photomultiplier tubes (PMTs). These vacuum tubes allow for the detection and amplification of very small amounts of light. This occurs through multiplying the amount of current that any incident light may produce. Once a photocathode is struck by incident photons, electrons are ejected due to the photoelectric effect. These would towards some electron multiplier segment of the apparatus, where multiply dynodes increas the number of electrons produced, and eventually detected at the end of the chain. The end of the dynode chain includes an anode, which sends out a sharp pulse of current as the large number of electrons produced by the chain of dynodes hit it.

For the PMT used in this experimental setup, the sensitivity peaks at a wavelength of 380 nm, and rapidly falls off as one goes away from this sensitivity peak.

Understanding the mechanics of such PMTs are highly relevant to the data analysis conducted within this experiment. Most notably, the noise produced in certain data sets, such as both runs of the epsilon data.