Wireless Communication

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1 Introduction to Wireless Communication

- **Current Wireless System**
- **Current:** 4G Cellular Systems (LTE-Advanced), 6G Wireless LANs, WiFi (802.11ax), mmWave massive MIMO systems, Satellite Systems, Bluetooth, Zigbee, WiGig.

1.1 Cellular Networks

- **4G/LTE Cellular**
- Much higher data rates than 3G (50-100Mbps), while 3G systems has 384 Kbps peak rates.
- Greater spectral efficiency (bit/s/Hz). More bandwidth, adaptive OFDM-MIMO, reduced interference.
- Flexible use of up to 100MHZ of spectrum. 10-20 MHz spectrum allocation common.
- Low packet latency (<5ms).
- Reduced cost-per-bit/ All IP networks.

1.2 Wireless LAN Standards

- 802.11b (Old-1990s)
  - Standard for 2.4GHz ISM Band (80MHz)
  - Direct sequence spread spectrum (DSSS)
  - Speeds of 11 Mbps, approx. $500ft$ range
- 802.11a/g (Middle Age- mid-late 1990s)
  - Standard for $5GHz$ band $(300MHz)/$ also $2.4GHz$
  - OFDM in 20 MHz with adaptive rate/codes
  - Speeds of 54 Mbps, approx. $100-200ft$ range
- 802.11n / ac / ax or Wi-Fi 6 (current gen)
  - Standard in $2.4GHz$ and $5GHz$ band
  - Adaptive OFDM /MIMO in 20/40/80/160 MHz
  - Antennas:2-4, up to 8.
  - Speeds up to 1 Gbps (10 Gbps for ax), approx. $200ft$ range
  - Other advances in packetization, antenna use, multiuser MIMO

1.3 Emerging Systems

- New cellular system architectures
- mmWave/massive MIMO communications
- Software-defined network architectures
- Ad-hoc/mesh wireless networks
- Cognitive radio networks

- Enery-constrained radios
- Distributed control networks
- Chemical Communications

2 Signal Propagation and Path Loss Models

- **Signal Propagation Characteristics:**
  - Path loss: power falloff relative to distance
  - Shadowing: random fluctuations due to obstructions
  - Flat and frequency selective fading: caused by multipath
- **Transmitted and received signals:**
  - **Transmitted Signal:**

$$s(t) = \mathrm{Re}\left\{u(t)e^{j(2\pi f_c t)}\right\} = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \tag{1}$$

    where f_c is the carrier frequency and $u(t) = s_I(t) + js_Q(t)$ is the equivalent lowpass signal of $s(t)$ with bandwidth B_u, power P_u, in-phase component $s_I(t) = \mathrm{Re}\{u(t)\}$ and quadrature component $s_Q(t) = \mathrm{Im}\{u(t)\}$. The phase of $u(t)$ includes any carrier phase offset.

  - **Received signal:**

$$r(t) = \mathrm{Re}\left\{v(t)e^{j(2\pi f_c t)}\right\} \tag{2}$$

    where, $v(t) = u(t) * c(t)$ for $c(t)$ the equivalent lowpass channel impulse reponse for $h(t)$.

  - **Doppler frequency shift:**

$$f_D = (v/\lambda)\cos(\theta) \tag{3}$$

    It will be introduce in the received signal. It can be ignored on path loss and will have big impact on fading.

- **Free space path loss model:**
  - Typically used for unobstructed LOS signal path.
  - Received signal is,

$$r(t) = \Re\left\{\frac{u(t)\sqrt{G_t G_r}\lambda e^{j2\pi d/\lambda}}{4\pi d} e^{j(2\pi f_c t)}\right\} \tag{4}$$

  - Receiver power is

$$P_r = P_t\left[\frac{\sqrt{G_t G_r}\lambda}{4\pi d}\right]^2 \Rightarrow \frac{P_r}{P_t} = G_t G_r\left[\frac{\lambda}{4\pi d}\right]^2 \tag{5}$$

  - Power falls off proportional to $(1/d)^2$ and to $\lambda^2 = (1/f_c)^2$. This dependence on the inverse of the square of the carrier frequency is due to the effective aperture of the receiver.
  - Power falls off proportional to net antenna gain $G_t G_r$ which combines the transmit and receive antenna gains G_t and G_r, respectively.
  - Model is not accurate for general environments.

- **Standards-based Models for WiFi (802.11) and Cellular (3GPP):**
  - The standards bodies for cellular (3GPP) and WiFi (802.11) have developed classes of propagation models (e.g. indoor, outdoor high speed, outdoor low speed) which are used to evaluate different technology proposals to the standard.
  - Simulation packages often integrate these models into their software for ease of simulations.
  - We usually describes cellular 3GPP and $5G$ channel models as well as WiFi $802.11n$ and 802.11ac models. New models for future cellular and WiFi systems are under development.

3 Shadowing, Combined Path Loss/Shadowing, Data Model Parameters, Statistical Multipath Model

3.1 Log-normal Shadowing

- Statistical model for variations in the received signal amplitude due to blockage.
- The received signal power with the combined effect of path loss (power falloff model) and shadowing is, in dB, given by

$$P_r(\,dB) = P_t(\,dB) + 10\log_{10} K - 10\gamma \log_{10}(d/d_r) - \psi(dB) \tag{6}$$

- Empirical measurements support the log-normal distribution for ψ :

$$p(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi_{dB}}\psi} exp\left[-\frac{\left(10\log_{10}\psi - \mu_{\psi_{dB}}\right)^2}{2\sigma^2_{\psi_{dB}}}\right], \psi > 0 \tag{7}$$

  where $\xi = 10/\ln 10$, $\mu_{\psi_{dB}}$ is the mean of $\psi_{dB} = 10\log_{10}\psi$ in dB and $\sigma_{\psi_{dB}}$ is the standard deviation of ψ_{dB}, also in dB.
- With a change of variables, setting $\psi_{dB} = 10\log_{10}\psi$, we get

$$p\left(\psi_{dB}\right) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}} exp\left[-\frac{\left(\psi_{dB} - \mu_{\psi_{dB}}\right)^2}{2\sigma^2_{\psi_{dB}}}\right], -\infty < \psi_{dB} < \infty \tag{8}$$

- This empirical distribution can be justified by a CLT argument.

3.2 Combined Path Loss and Shadowing

- Linear Model:

$$\frac{P_r}{P_t} = K\left(\frac{d}{d_r}\right)^{\gamma}\psi \tag{9}$$

- dB Model:

$$\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma\log_{10}(d/d_r) - \psi_{dB} \tag{10}$$

- Average shadowing attenuation: when $K_{dB} = 10\log_{10} K$ captures average dB shadowing, $\mu_{\psi_{dB}} = 0$, otherwise $\mu_{\psi_{dB}} > 0$ since shadowing causes positive attenuation.

3.3 Outage Probability under Path Loss and Shadowing

- With path loss and shadowing, the received power at any given distance between transmitter and receiver is random.
- Leads to a non-circular coverage area around the transmitter, i.e. non-circular contours of constant power above which performance (e.g. in WiFi or cellular) is acceptable.

- Outage probability $P_{\text{out}}\left(P_{\min}, d\right)$ is defined as the probability that the received power at a given distance $d, P_r(d)$, is below a target $P_{\min} : P_{\text{out}}\left(P_{\min}, d\right)=p\left(P_r(d)<P_{\min}\right)$.
- For the simplified path loss model and log normal shadowing this becomes

$$p\left(P_r(d) \leq P_{\min}\right)=1-Q\left(\frac{P_{\min}-\left(P_t+K_{dB}-10 \gamma \log_{10}\left(d / d_r\right)\right)}{\sigma_{\psi_{dB}}}\right) \tag{11}$$

3.4 Model Parameters from Empirical Data:

- Constant K_{dB} typically obtained from measurement at distance d_0.
- Power falloff exponent γ obtained by minimizing the MSE of the predicted model versus the data (assume N samples):

$$F(\gamma)=\sum_{i=1}^N\left[M_{\text{measured}}\left(d_i\right)-M_{\text{model}}\left(d_i\right)\right]^2 \tag{12}$$

where $M_{\text{measured}}\left(d_i\right)$ is the i th path loss measurement at distance d_i and $M_{\text{model}}\left(d_i\right)=K_{dB}-10 \gamma \log_{10}\left(d_i\right)$. The minimizing γ is obtained by differentiating with respect to γ, setting this derivative to zero, and solving for γ

- The resulting path loss model will include average attenuation, so $\mu_{\psi_{dB}}=0$.
- Can also solve simultaneously for $\left(K_{dB}, \gamma\right)$ via a least squares fit of both parameters to the data. Using the line equation for each data point y_i that $y_i=m x_i+K_{dB}$ for $m=-10 \gamma$ and $x_i=\log_{10}\left(d_i\right)$, the error of the straight line fit is

$$F(K, \gamma)=\sum_{i=1}^N\left[M_{\text{measured}}\left(d_i\right)-\left(m x_i+K_{dB}\right)\right]^2 \tag{13}$$

- The shadowing variance $\sigma_{\psi_{dB}}^2$ is obtained by determining the MSE of the data versus the empirical path loss model with the minimizing $\gamma=\gamma_0$:

$$\sigma_{\psi_{dB}}^2=\frac{1}{N} \sum_{i=1}^N\left[M_{\text{measured}}\left(d_i\right)-M_{\text{model}}\left(d_i\right)\right]^2 ; M_{\text{model}}\left(d_i\right)=K_{dB}-10 \gamma_0 \log_{10}\left(d_i\right) \tag{14}$$

3.5 Statistical Multipath Model:

- At each time instant there are a random number $N(t)$ of multipath signal components.
- At time t the i th component has a random amplitude $\alpha_i(t)$, angle of arrival $\theta_i(t)$ Doppler shift $f_{D_i}=\frac{v}{\lambda} \cos \theta_i(t)$ and associated phase shift $\phi_{D_i}=\int_t f_{D_i}(t) d t$, and path delay relative to the LOS component $\tau_i(t)=\left(x_i(t)\right) / c$.
- Thus, the received signal is given by the following expression, which implies the channel has a time-varying impulse response.

$$r(t)=\operatorname{Re}\left\{\left[\sum_{n=1}^N \alpha_n u\left(t-\tau_n\right) e^{-j \phi_n}\right] e^{j 2 \pi f_c t}\right\}=\operatorname{Re}\left\{\left[\left(\sum_{n=1}^N \alpha_n \delta\left(t-\tau_n\right) e^{-j \phi_n}\right) * u(t)\right] e^{j 2 \pi f_c t}\right\} \tag{15}$$

- The complex baseband channel model can be written as,

$$h(t)=\sum_{n=1}^N \alpha_n \delta\left(t-\tau_n\right) e^{-j \phi_n} \tag{16}$$

$$h = \sum_{n=1}^{N} \alpha_n e^{-j\phi_n} \Leftrightarrow \mathrm{Re}(h) = \sum_{n=1}^{N} \alpha_n \cos(\phi_n), \mathrm{Im}(h) = -\sum_{n=1}^{N} \alpha_n \sin(\phi_n) \tag{17}$$

- Using central limit theorem, we have,

$$h \sim C\mathcal{N}\left(0, \sum_n \mathbb{E}\left[\alpha_n^2\right]\right) \tag{18}$$

- The envelope $z = |h|$ is Rayleigh distributed with p.d.f.

$$p(z) = \frac{z}{\sigma_h^2} exp\left[-\frac{z^2}{2\sigma_h^2}\right], \quad \sigma_h^2 \triangleq \frac{1}{2}\sum_n \mathbb{E}\left[\alpha_n^2\right] \tag{19}$$

4 Rician and Nakagami Fading. Wideband Fading.

4.1 Rician Fading

- A LOS component leads to a received signal with non-zero mean. The Rician distribution models signal envelope in this case, with K factor dictating the relative power of the LOS component:

$$p_Z(z) = \frac{z}{\sigma^2} exp\left[\frac{-\left(z^2+s^2\right)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad z \geq 0 \tag{20}$$

- The average received power in the Rician fading is $P_r = \int_0^\infty z^2 p_Z(z) dz = s^2 + 2\sigma^2$.
- The Rician distribution is often described in terms of a fading parameter K, defined by $K = \frac{s^2}{2\sigma^2}$. The distribution in terms of K is:

$$p_Z(z) = \frac{2z(K+1)}{P_r} exp\left[-K - \frac{(K+1)z^2}{P_r}\right] I_0\left(2\sqrt{\frac{K(K+1)z}{P_r}}\right) \tag{21}$$

4.2 Nakagami Fading Distribution

- Experimental results support a Nakagami distribution for the signal envelope for some environments. Nakagami is similar to Rician, but can model "worse than Rayleigh."
- Model generally leads to closed-form expressions in BER and diversity analysis.
- Distribution is $p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m) P_r^m} exp\left[\frac{-mz^2}{P_r}\right], \quad m \geq 0.5$. By change of variables, power distribution is $p_{Z^2}(x) = \left(\frac{m}{P}\right)^m \frac{x^{m-1}}{\Gamma(m)}$

4.3 Wideband Channel Models

- In wideband multipath channels the individual multipath components can be resolved by the receiver. True if $T_m > 1/B$.
- If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).

4.4 Channel Scattering Function:

- Typically time-varying channel impulse response $c(\tau, t)$ is unknown, so its wideband model must be characterized statistically.
- Since under our random model with a large number of scatterers, $c(\tau, t)$ is Gaussian. We assume it is WSS, so we only need to characterize its mean and correlation, which is independent of time. Similar to narrowband model, for ϕ_n uniformly distributed, $c(\tau, t)$ has mean zero.
- Autocorrelation of $c(\tau, t)$ is $A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t)\, \delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t)$ since we assume channel response associated with different scatterers is uncorrelated.
- Statistical scattering function defined as $S(\tau, \rho) = \mathcal{F}_{\Delta t}\left[A_c(\tau, \Delta t)\right]$. This function measures the average channel gain as a function of both delay τ and Doppler ρ.
- $S(\tau, \rho)$ easy to measure empirically and is used to get average delay spread T_M, rms delay spread σ_τ, and Doppler spread B_d for empirical channel measurements.

5 Shannon Capacity of Wireless Channels

5.1 Shannon Capacity

- The maximum mutual information of a channel. Its significance comes from Shannon's coding theorem and converse, which show that capacity is the maximum error-free data rate a channel can support.
- Capacity is a channel characteristic - not dependent on transmission or reception techniques or limitation.
- In AWGN, $C = B \log_2(1 + \gamma)$bps, where B is the signal bandwdith and $\gamma = P/\left(N_0 B\right)$ is the received signal-to-noise power ratio.

5.2 Capacity of Flat-Fading Channels:

- Depends on what is known about the channel.
- Three cases: 1) Fading statistics known; 2) Fade value known at receiver; 3) Fade value known at transmitter and receiver.
- When only fading statistics known, capacity difficult to compute. Only known results are for Finite State Markov channels, Rayleigh fading channels, and block fading.
- Fading Known at the Receiver:
  - Capacity given by $C = \int_0^\infty B \log_2(1 + \gamma) p(\gamma) d\gamma$ bps, where $p(\gamma)$ is the distribution of the fading SNR γ.
  - By Jensen's inequality this capacity always less than that of an AWGN channel.
  - "Average" capacity formula, but transmission rate is fixed.
- Capacity with Fading Known at Transmitter and Receiver
  - For fixed transmit power, same capacity as when only receiver knows fading.
  - Transmit power as well as rate can be adapted.
  - Under variable rate and power $C = \max_{P(\gamma): \int P(\gamma) p(\gamma) d\gamma = \bar{P}} \int_0^\infty B \log_2\left(1 + \frac{P(\gamma)\gamma}{\bar{P}}\right) p(\gamma) d\gamma$ where $P(\gamma)$ is power adaptation

5.3 Optimal Power and Rate Adaptation

- Optimal adaptation found via Lagrangian differentiation.
- Optimal power adaptation is a "water-filling" in time: power $P(\gamma) = \gamma_0^{-1} - \gamma^{-1}$ increases with channel quality γ above an optimized cutoff value γ_0.

- Rate adaptation relative to $\gamma \geq \gamma_0$ is $B \log_2(\gamma/\gamma_0)$: also increases with γ above cutoff.
- Resulting capacity is $C = \int_{\gamma_0}^{\infty} B \log_2(\gamma/\gamma_0)\, p(\gamma) d\gamma$
- Capacity with power and rate adaptation not much larger than when just receiver knows channel, but has lower complexity and yields more insight into practical schemes.
- Capacity in flat-fading can exceed the capacity in AWGN, typically at low SNRs.

6 Linear Digital Modulation and its Performance in AWGN and in Fading

6.1 Performance of Linear Modulation in AWGN:

- ML detection corresponds to decision regions.
- For coherent modulation, probability of symbol error P_s depends on the number of nearest neighbors α_M, and the ratio of their distance $d_{\min}$ to the square root $\sqrt{N_0}$ of the noise power spectral density (this ratio is a function of the SNR γ_s).
- P_s approximated by $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$, where α_M and β_M depend on the constellation size and modulation type (MPSK vs. MQAM).
- Alternate Q function representation $Q(z) = \frac{1}{\pi} \int_0^{.5\pi} exp\left[-z^2/\left(2\sin^2\phi\right)\right] d\phi$ leads to closed form expression for error probability of PSK in AWGN and, more importantly, greatly simplifies fading/diversity analysis.

6.2 Performance of Linear Modulation in Fading:

- In fading γ_s and therefore P_s are random variables.
- Three performance metrics to characterize the random P_s.
  - Outage: $p\left(P_s > P_{\text{target}}\right) = p\left(\gamma < \gamma_{\text{target}}\right)$
    - Outage probability used when fade duration long compared to a symbol time.
    - Obtained directly from fading distribution and target γ_s
    - Can obtain simple formulas for outage in log-normal shadowing or in Rayleigh fading.
  - Average P_s $\left(\bar{P}_s = \int P_s(\gamma) p(\gamma) d\gamma\right)$
    - Rarely leads to close form expressions for general $p(\gamma_s)$ distributions.
    - Can be hard to evaluate numerically.
    - Can obtain closed form expressions for general linear modulation in Rayleigh fading (using approximation $P_s \approx \alpha Q\left(\sqrt{\beta\gamma_s}\right)$ in AWGN)..
  - Combined outage and average P_s

6.3 Delay Spread (ISI) Effects on Performance

- Delay spread exceeding a symbol time causes ISI (self-interference).
- ISI leads to an irreducible error floor. Approximated as $\bar{P}_{b,floor} \approx \left(\sigma_{T_m}/T_s\right)^2$
- Without ISI compensation, avoid error floor by reducing data rate: $T_s >> T_m$ or $R \leq \log_2(M) \times \sqrt{\bar{P}_{b,floor}/\sigma_{T_m}^2}$

6.4 Doppler Spread Effects on Performance

- Doppler causes the channel phase to decorrelate.

- Doppler impacts coherent modulation if an accurate coherent phase reference cannot be obtained at the receiver. A noisy phase estimate leads to large errors.
- Phase decorrelation between symbols leads to an irreducible error floor for differential modulation.

7 Diversity

7.1 Diversity Combining Techniques

- Selection Combining: largest fading path chosen.
- Maximal Ratio Combining: all paths cophased and summed with optimal weighting to maximize SNR at combiner output.
- Equal Gain Combining: all paths cophased and summed with equal weighting.
- We use space diversity as a reference for analysis; same analysis applies for any mechanism used to obtain independent fading paths.

7.2 Array and Diversity Gain

- Array gain is the gain in SNR from noise averaging over the multiple antennas. Gain in both AWGN and fading channels.
- Diversity gain is the change in slope of the probability of error due to diversity. Only applies to fading channels.

7.3 Selection Combining (SC) and its Performance

- Combiner SNR γ_Σ is the maximum of the branch SNRs.
- This gives diminishing returns, in terms of power gain, as the number of antennas increases.
- CDF of γ_Σ easy to obtain, then pdf found by differentiating.
- Typically get $10 - 15\ dB$ of gain for $2 - 3$ antennas.

7.4 Maximal Ratio Combining (MRC)

- Branch weights optimized to maximize output SNR of combiner.
- Optimal weights are proportional to branch SNR.
- Resulting combiner SNR γ_Σ is sum of branch SNRs.
- Distribution obtained by characteristic function analysis (can be hard).

8 Adaptive Modulation

- Basic idea: adapt at transmitter relative to channel fade level (borrows from capacity ideas).
- Parameters to adapt (degrees of freedom) include constellation size, transmit power, instantaneous BER, symbol time, coding rate/scheme, and combinations.
- Optimization criterion for adaptation is typically maximizing average rate, minimizing average power, or minimizing average BER.
- Few degrees of freedom need be exploited for near-optimal performance.

8.1 Variable-Rate Variable-Power MQAM

- Constellation size and power adapted to maximize average throughput given an instantaneous BER constraint.

- BER bound $\text{BER}(\gamma) = .2exp[-1.5\gamma P(\gamma)/((M-1)\bar{S})]$ inverted to get adaptive con- stellation size $M[\gamma]$ below with $K = -1.5/\ln(5 \cdot BER)$ that meets the BER constraint for any adaptive power policy $P[\gamma]$:

$$M[\gamma] = 1 + \frac{-1.5\gamma}{-\ln(5 \cdot BER)} \frac{P(\gamma)}{\bar{P}} = 1 + K\gamma P(\gamma)/\bar{P} \tag{22}$$

8.2 Optimal Rate and Power Adaptation for Maximum Throughput

- Optimal power adaptation $P(\gamma)$ found by maximizing average throughput $E\left[\log_2(M[\gamma])\right] = E\left[\log_2(1 + K\gamma P(\gamma)/\bar{P}\right)$ relative to $P(\gamma)$
- Optimal power adaptation is the same waterfilling as the capacity-achieving strategy with an effective power loss K.
- Optimal rate adapation found by substituting optimal power adaptation into $M(\gamma)$ yielding $R(\gamma) = \log_2\left(\gamma/\gamma_K\right), \gamma > \gamma_K$, where γ_K is cutoff value for the water-filling power policy.
- Same optimal power and rate adaptation as the capacity-acheiving strategies with an effective power reduction $K = -1.5/\ln(5 \cdot BER)$. Throughput is within $5 - 6\ dB$ of channel capacity.
- Different modulations and BER bounds result in different adaptive policies.

8.3 Finite Constellations

- Constellation restricted to finite set $\{M_0 = 0, M_1, \ldots, M_{N-1}\}$
- Divide the fading range of γ into N discrete fading regions R_j. Within each region "conservatively" assign constellation $M_j : M_j \leq M(\gamma) \leq M_{j+1}$, where $M(\gamma) = \gamma/\gamma_K^*$ for some optimized γ_K^*.
- Power control based on channel inversion; maintains constant BER within region R_j.
- Using large enough constellation set results in near-optimal performance.
- Additional power penalty of $1.5 - 2\ dB$ if each constellation restricted to a single transmit power

9 MIMO Communications and Capacity

9.1 MIMO Systems

- MIMO systems have multiple antennas at the transmitter and receiver.
- The antennas can be used for capacity gain and/or diversity gain.
- MIMO system design and analysis complex since it requires vector signal processing.
- The performance and complexity of MIMO systems depends on what is known about the channel at both the transmitter and receiver

9.2 MIMO Channel Decomposition

- With perfect channel estimates at the transmitter and receiver, the MIMO channel decomposes into $R_{\mathbf{H}}$ independent parallel channels, where $R_{\mathbf{H}}$ is the rank of the channel matrix $(\min\left(M_t, M_r\right)$ for M_t transmit and M_r receive antennas under rich scattering).
- With this decomposition there is no need for vector signal processing.
- Decomposition is obtained by transmit precoding and receiver shaping.

9.3 MIMO Channel Capacity: Static Channels

- Capacity depends on whether the channel is static or fading, and what is known about the channel at the transmitter and receiver.

- For a static channel known at the transmitter and receiver capacity is given by

$$C = \max_{P_i : \sum_i P_i \leq P} \sum_i B \log_2 \left(1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right) = \max_{P_i : \sum_i P_i \leq P} \sum_i B \log_2 \left(1 + \frac{P_i \gamma_i}{P}\right)$$

  This leads to a water-filling power allocation in space.

- Without transmitter knowledge, outage probability is the right metric for capacity.
- In the limit of a large antenna array (Massive MIMO), even without TX CSI, random matrix theory dictates that the singular values of the channel matrix converge to the same constant. Hence, the capacity of each spatial dimension is the same, and the total system capacity is $C = \min(M_t, M_r)\, B \log(1 + \rho)$. So capacity grows linearly with the size of the antenna arrays in Massive MIMO systems.

9.4 MIMO Channel Capacity: Fading Channels

- In fading, if the channel is unknown at transmitter, uniform power allocation is optimal, but this leads to an outage probability since the transmitter doesn't know what rate to transmit at:

$$P_{out} = p\left(\mathbf{H} : B \log_2 \det \left[\mathbf{I}_{M_r} + \frac{\rho}{M_t} \mathbf{H}\mathbf{H}^H\right] > C\right) \tag{23}$$

- Capacity with both transmitter and receiver knowledge of the fading is the average of the capacity for the static channel, with power allocated either by an instantaneous or average power constraint. Under the instantaneous constraint power is optimally allocated over the spatial dimension only. Under the average constraint it is allocated over both space and time.
- Massive MIMO: When the number of TX (or RX) antennas is large, the channel becomes "static" due to the law of large numbers. Specifically,

$$\lim_{M_t \to \infty} \frac{1}{M_t} \mathbf{H}\mathbf{H}^{\mathbf{H}} = \mathbf{I}_{\mathbf{M_r}} \tag{24}$$

  The MIMO channel capacity then becomes,

$$\lim_{M_t \to \infty} I(\mathbf{x}; \mathbf{y}) = \lim_{\mathbf{M_t} \to \infty} \mathbf{B} \log_{\mathbf{2}} \det \left[\mathbf{I}_{\mathbf{M_r}} + \frac{\rho}{\mathbf{M_t}} \mathbf{H}\mathbf{H}^{\mathbf{H}}\right] = \mathbf{B} \log_{\mathbf{2}} \det \left[\mathbf{I}_{\mathbf{M_r}} + \rho \mathbf{I}_{\mathbf{M_r}}\right] = \mathbf{M_r} \mathbf{B} \log_{\mathbf{2}}(\mathbf{1} + \rho) \tag{25}$$

  Defining $M = \min(M_t, M_r)$, this implies that as M grows large, the MIMO channel capacity in the absence of TX CSI approaches $C = MB \log_2(1 + \rho)$ for ρ the SNR, and hence grows linearly in M.

9.5 MIMO Systems: Beamforming

- Beamforming sends the same symbol over each transmit antenna with a different scale factor.
- At the receiver, all received signals are coherently combined using a different scale factor.
- This produces a transmit/receiver diversity system, whose SNR can be maximized by optimizing the scale factors (MRC).
- Beamforming leads to a much higher SNR than on the individual channels in the parallel channel decomposition.
- Thus, there is a design tradeoff in MIMO systems between capacity and diversity.

9.6 MIMO Receiver Design

- Optimal MIMO receiver is maximum-likelihood (ML) receiver. Finds input vector $\mathbf{x}$ that minimizes $|\mathbf{y} - \mathbf{H}\mathbf{x}|_F^2$ for $|\cdot|_F$ the Frobenius (matrix) norm.
- This receiver is exponentially complex in the constellation size and number of transmitted data streams.

- Can reduce complexity through linear processing of input vector $\mathbf{Ax}$. Zero-forcing receiver forces all interference from other symbols to zero. This can result in significant noise enhancement.
- MMSE receiver: trades off cancellation of interference from other symbols for noise enhancement. Reduces to zero forcing in the absence of noise.
- Sphere decoder: uses upper triangular decomposition of H to reduce complexity. Finds constellation point within a sphere of a given radius. Provides near-ML performance with near-linear complexity.

9.7 Linear MIMO Receivers:

- Multiplies $\mathbf{y}$ with a MIMO equalization matrix $\mathbf{A} \in \mathbf{C}^{\mathbf{M_t} \times \mathbf{M_r}}$ to get $\tilde{\mathbf{x}}(\mathbf{y}) \in \mathbf{C}^{\mathbf{M}_t}$
- Zero-forcing receiver forces all interference from other symbols to zero:

  $$\tilde{\mathbf{x}}(\mathbf{y}) = \mathbf{H}^{\dagger}\mathbf{y}$$

  where $\mathbf{H}^{\dagger}$ is $\mathbf{H}^{-1}$ if the matrix is square and invertible, otherwise we use the pseudo inverse.
- When $M_t \leq M_r$, and there are at least M_t linearly independent columns in $\mathbf{H}$ (typical for many propagation environments), the pseudo inverse (Moore-Penrose pseudoin-

  verse) is $\mathbf{H}^{\dagger} = \left(\mathbf{H^H H}\right)^{\mathbf{-1}} \mathbf{H^H}$.
- Complexity of obtaining $\mathbf{H}^{\dagger}$ is roughly cubic in M_t for square matrices. However obtaining $\hat{\mathbf{x}}(\mathbf{y})$ from $\tilde{\mathbf{x}}(\mathbf{y})$ is linear in M_t. ZF can result in significant noise enhancement.
- L-MMSE receiver: trades off cancellation of interference from other symbols for noise enhancemen with a regularization term that depends on SNR. The solution is

  $$\tilde{\mathbf{x}}(\mathbf{y}) = \left(\mathbf{H^H H} + \lambda\mathbf{I}\right)^{\mathbf{-1}} \mathbf{H^H y} = \mathbf{H^H}\left(\mathbf{H H^H} + \lambda\mathbf{I}\right)^{\mathbf{-1}} \mathbf{y}$$

  which reduces to zero forcing in the absence of noise (infinite SNR).
- Compared to the ML detector, both the ZF and MMSE linear detectors are simpler to implement, but their BER performances are worse.

10 OFDM

10.1 OFDM: IFFT/FFT Implementation of MCM

- Complexity of implementing N separate modulators/demodulators is prohibitive.
- MCM effectively implemented using IFFT at transmitter and FFT at receiver.
- The IFFT shifts modulated symbols to desired subcarriers.
- A cyclic prefix is inserted in the data to remove ISI between blocks and make the linear convolution with the channel circular.
- The received symbol is just a scaled version of the transmitted symbol.

10.2 Fading across Subcarriers

- Different subcarriers experience different fading (H_i for subcarrier i) and hence different received SNRs.
- Can invert fading at the transmitter such that received SNR is constant across all subcarriers. This technique, called precoding, is most common in wireline systems. It is wasteful of power when channel has deep nulls since it is effectively inverting the channel.
- A more common technique to address fading across subcarriers is adaptive loading, which adapts power and rate relative to the fading in each subchannel. This technique is capacity achieving.

- Can also use coding across subcarriers, so that a subchannel whose symbols are affected by deep fading is compensated by coding across subcarriers with high SNRs.

10.3 Implementation Challenges in OFDM

- Timing and frequency offsets cause subchannels to interfere with each other.
- Interference between subchannels mitigated by minimizing the number of subchannels and using pulse shapes robust to timing errors.
- OFDM/DMT consists of multiple sinusoids summed together, can have a large peakto-average power ratio (PAPR), which leads to amplifier inefficiencies.
- PAPR compensated through clipping or coding.

10.4 MIMO-OFDM Systems

- Most next-generation wireless systems combine OFDM and MIMO, e.g. $4G$ cellular and Wifi $(802.11n/ac/ax)$ and future generations.
- These systems use OFDM modulation over each spatial dimension
- OFDM compensates for ISI, while MIMO is used for its diversity/multiplexing benefits.
- MIMO-OFDM systems adapt over space, time, and frequency.
- Receiver complexity depends on both the MIMO parameters and the number of OFDM tones: high data rates lead to more tones to compensate for ISI, hence more complexity.
- Both OFDM and MIMO systems can be represented by a matrix. MIMO-OFDM represented by a combined matrix.