Assignment 2

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Problem 2

Playing with a toy model of SRE

```
# FRT for the Project STAR data in the Imbens-Rubin book
# Note that here we collect the image manually
data <- read_excel("STAR.xlsx")

# Preparations
# Step 1: Write a function that gives you all the statistics you want in SRE
stat_SRE <- function(stratum, treatment, y){
  # Assume in our case that the stratum in arranged and indexed.
  # If not, then re-code it to an index.
  number = length(unique(stratum))
  tau = 0
  wil = 0
  r = 0
  # Calculate the three statistics as defined
  for (i in 1:number){
    tempy = y[stratum == i]
    tempt = treatment[stratum == i]
    n = length(tempy)
    pi = n/length(y)
    tau = tau + pi*(mean(tempy[tempt == 1] - mean(tempy[tempt == 0])))
    wil = wil + wilcox.test(tempy[tempt == 1], tempy[tempt == 0])$statistic / (n+1)
    tempy = tempy - mean(tempy)
  }
  y <- rank(y)
  for (i in 1:length(y)){
    if (treatment[i] == 1){
      r = r + y[i]
    }
  }
  return(c(taus = tau, wilcoxon = wil, alignedRank = r))
}
# Then we can calculate the observed value
obsValue <- stat_SRE(data$Stratum, data$Treatment, data$Y)
obsValue
```

```
##         taus   wilcoxon.W  alignedRank
##    0.2278897    8.3000000 1362.5000000
```

```
# Step 2: Write a function that permutes your data in strata
permute <- function(stratum, treatment){
  ptreat <- vector()
```

```r
  for (i in 1:length(unique(stratum))){
    ptreat <- c(ptreat, sample(treatment[stratum == i]))
  }
  return(ptreat)
}
```

```r
# Step 3: Carry out Stratified Randomization Test
MC = 2000
extreme = rep(0,3)
for (i in 1:MC){
  mcStat = stat_SRE(data$Stratum, permute(data$Stratum, data$Treatment), data$Y)
  for (j in 1:3){
    if (abs(mcStat[j]) > abs(obsValue[j])){
      extreme[j] = extreme[j] + 1
    }
  }
}
# Tidy display of our result
display <- data.frame("Taus" = extreme[1]/MC, "V" = extreme[2]/MC, "Aligned Rank" = extreme[3]/MC)
display
```

```
##     Taus     V Aligned.Rank
## 1 0.0325 0.086       0.0235
```

```r
# At 95% significance level, we rejeect the sharp null hypothesis that there's no significant differenc
```

Problem 3

```r
# Baseline Model with NO Strata
# Compare it with the normal complete randomized experiment
# Part 1: FRT
# Step 4: Compare our results with the CRE
library(Matching)
data("lalonde")
z <- lalonde$treat
y <- lalonde$re78

# Monte-Carlo Simulation of data
MC = 2000
Tauhat   = rep(0, MC)
Student  = rep(0, MC)
Wilcox   = rep(0, MC)
Ks       = rep(0, MC)
tau = t.test(y ~ z, var.equal = TRUE)$statistic
t = t.test(y ~ z, var.equal = FALSE)$statistic
w = wilcox.test(y ~ z)$statistic
ks = ks.test(y[z == 1], y[z == 0])$statistic

extreme_tau = 0
extreme_t = 0
```

```
extreme_w = 0
extreme_ks = 0
for(mc in 1:MC){
   zperm = sample(z)
   temptau = t.test(y ~ zperm, var.equal = TRUE)$statistic
   tempt = t.test(y ~ zperm, var.equal = FALSE)$statistic
   tempw = wilcox.test(y ~ zperm)$statistic
   tempks = ks.test(y[zperm == 1], y[zperm == 0])$statistic
   if (abs(temptau) > abs(tau)){
     extreme_tau <- extreme_tau + 1
   }
   if (abs(tempt) > abs(t)){
     extreme_t <- extreme_t + 1
   }
   if (abs(tempw) < abs(w)){
     extreme_w <- extreme_w + 1
   }
   if (abs(tempks) > abs(ks)){
     extreme_ks <- extreme_ks + 1
   }
}
# Tidy display of our result
display_CRE <- data.frame("Tau" = extreme_tau/MC, "t" = extreme_t/MC, "Wilcoxon" = extreme_w/MC, "KS" =
display_CRE
```

```
##      Tau     t Wilcoxon    KS
## 1 0.0055 0.009   0.0055 0.039
```

```
# Part 2: Neymanian Inference
library(Matching)
data(lalonde)
head(lalonde)
```

```
##   age educ black hisp married nodegr re74 re75     re78 u74 u75 treat
## 1  37   11     1    0       1      1    0    0  9930.05   1   1     1
## 2  22    9     0    1       0      1    0    0  3595.89   1   1     1
## 3  30   12     1    0       0      0    0    0 24909.50   1   1     1
## 4  27   11     1    0       0      1    0    0  7506.15   1   1     1
## 5  33    8     1    0       0      1    0    0   289.79   1   1     1
## 6  22    9     1    0       0      1    0    0  4056.49   1   1     1
```

```
z = lalonde$treat
y = lalonde$re78

## Neymanian inference
n1= sum(z)
n0= length(z) - n1
tauhat = mean(y[z==1]) - mean(y[z==0])
vhat   = var(y[z==1])/n1 + var(y[z==0])/n0
sehat  = sqrt(vhat)
tauhat
```

```
## [1] 1794.343
```

```
sehat
```

```
## [1] 670.9967
```

```
# Step 0: Some data-cleaning presumed here as I'm implementing my own function of SRE
library(Matching)
data(lalonde)
data <- lalonde
data <- data %>%
  mutate(race = ifelse(black==1, 1, 0)) %>%
  mutate(race = ifelse(hisp == 1, 2, race))
data <- data[,c(-3,-4)]
data$race <- data$race + 1
data <- data %>%
  arrange(by = race)
```

```
# Step 1: Pretend that the SRE is done by blocking race
# Part 1: Fisher Randomization test
MC = 2000
extreme = rep(0,3)
obsValue <- stat_SRE(data$race, data$treat, data$re78)
obsValue
```

```
##        taus  wilcoxon.W alignedRank
##  1794.96905    60.50269 44607.50000
```

```
for (i in 1:MC){
  mcStat = stat_SRE(data$race, permute(data$race, data$treat), data$re78)
  for (j in 1:3){
    if (abs(mcStat[j]) > abs(obsValue[j])){
      extreme[j] = extreme[j] + 1
    }
  }
}
# Tidy display of our result
display1 <- data.frame("Taus" = extreme[1]/MC, "V" = extreme[2]/MC, "Aligned Rank" = extreme[3]/MC)
display1
```

```
##    Taus      V Aligned.Rank
## 1 0.005 0.0045        0.004
```

```
# Step 1: Pretend that the SRE is done by blocking race
# Part 2: Neymanian Inference
print(c ("The point estimator is", obsValue[1]))
```

```
##                                               taus
## "The point estimator is"       "1794.96904513932"
```

```r
var_neyman <- function(stratum, treatment, y){
  V = 0
  for(i in 1:length(unique(stratum))){
    tempy = y[stratum == i]
    tempt = treatment[stratum == i]
    n = length(tempy)
    y0 = tempy[tempt == 0]
    y1 = tempy[tempt == 1]
    V = V + (length(y0)/n)^2 * (sd(y0)/length(y0) + sd(y1)/length(y1))
  }
  return(V)
}
SRE_race <- var_neyman(data$race, data$treat, data$re78)
SRE_race
```

```
## [1] 610.8122
```

```r
# Step 2: Pretend that the SRE is done by blocking marital status
# Part 1: FRT
data$married <- data$married + 1
data <- data %>% arrange(by=married)
MC = 2000
extreme = rep(0,3)
obsValue <- stat_SRE(data$married, data$treat, data$re78)
obsValue
```

```
##        taus  wilcoxon.W alignedRank
##  1767.17517    61.02082 44607.50000
```

```r
for (i in 1:MC){
  mcStat = stat_SRE(data$married, permute(data$married, data$treat), data$re78)
  for (j in 1:3){
    if (abs(mcStat[j]) > abs(obsValue[j])){
      extreme[j] = extreme[j] + 1
    }
  }
}
# Tidy display of our result
display2 <- data.frame("Taus" = extreme[1]/MC, "V" = extreme[2]/MC, "Aligned Rank" = extreme[3]/MC)
display2
```

```
##     Taus     V Aligned.Rank
## 1 0.0035 0.004        0.004
```

```r
# Step 2: Pretend that the SRE is done by blocking marital status
# Part 2: Neymanian Inference
SRE_marriage <- var_neyman(data$married, data$treat, data$re78)
SRE_marriage
```

```
## [1] 127.7127
```

```r
# Step 3: Pretend that the SRE is done by blocking nodegr
# Part 1: FRT
data$nodegr = data$nodegr + 1
data <- data %>% arrange(by = nodegr)
MC = 2000
extreme = rep(0,3)
obsValue <- stat_SRE(data$nodegr, data$treat, data$re78)
obsValue
```

```
##        taus  wilcoxon.W alignedRank
##  1598.28122    59.17541 44607.50000
```

```r
for (i in 1:MC){
  mcStat = stat_SRE(data$nodegr, permute(data$nodegr, data$treat), data$re78)
  for (j in 1:3){
    if (abs(mcStat[j]) > abs(obsValue[j])){
      extreme[j] = extreme[j] + 1
    }
  }
}
# Tidy display of our result
display3 <- data.frame("Taus" = extreme[1]/MC, "V" = extreme[2]/MC, "Aligned Rank" = extreme[3]/MC)
display3
```

```
##     Taus     V Aligned.Rank
## 1 0.0135 0.014       0.0125
```

```r
# Step 3: Pretend that the SRE is done by blocking nodegr
# Part 2: Neymanian Inference
SRE_edu <- var_neyman(data$nodegr, data$treat, data$re78)
SRE_edu
```

```
## [1] 88.97333
```

3.2 Regression adjusments for Penn

```r
penndata = read.table("Penn46_ascii.txt")
head(penndata)
```

```
##     duration treatment female black hispanic ndependents recall young old
## 1  18.011343         0      0     0        0           2      0     0   0
## 4   1.003399         0      0     0        0           0      0     0   0
## 5  26.960396         0      0     0        0           0      0     0   0
## 6   7.009044         1      0     0        0           0      0     0   0
## 12  9.022409         1      0     0        0           0      0     1   0
## 13 26.991390         0      0     0        0           1      0     0   1
##    quarter durable lusd
## 1        5       0    0
## 4        5       0    1
## 5        4       0    1
```

```
## 6         2       0     0
## 12        3       0     0
## 13        5       1     1
```

```
z = penndata$treatment
penndata$duration = log(penndata$duration)
y = lm(duration ~ .-treatment, data = penndata)$residuals
penndata <- penndata %>%
  mutate(quarter = quarter + 1) %>%
  arrange(by = quarter)
obsValue = stat_SRE(penndata$quarter, penndata$treatment, y)
# The point estimator
obsValue[1]
```

```
##        taus
## -0.01150982
```

```
SRE_adjusted <- var_neyman(penndata$quarter, penndata$treatment, y)
SRE_adjusted
```

```
## [1] 0.02450576
```

```
# Interval estimation
print(paste("[",obsValue[1] - SRE_adjusted*1.96,",",obsValue[1] + SRE_adjusted*1.96,"]"), sep = "")
```

```
## [1] "[ -0.0595410962445224 , 0.0365214636256135 ]"
```

```
Neyman_SRE = function(z, y, x)
{
       xlevels = unique(x)
       K       = length(xlevels)
       PiK     = rep(0, K)
       TauK    = rep(0, K)
       varK    = rep(0, K)
       for(k in 1:K)
       {
             xk         = xlevels[k]
             zk         = z[x == xk]
             yk         = y[x == xk]
             PiK[k]     = length(zk)/length(z)
             TauK[k]    = mean(yk[zk==1]) - mean(yk[zk==0])
             varK[k]    = var(yk[zk==1])/sum(zk) +
                               var(yk[zk==0])/sum(1 - zk)
       }

       return(c(sum(PiK*TauK), sum(PiK^2*varK)))
}

## pennsylvania re-employment bonus experiment
## description of the DATA:
## Koenker and Xiao 2002 Econometrica
```

```r
## "Inference on the Quantile Regression Process"

penndata = read.table("Penn46_ascii.txt")
head(penndata)
```

```
##     duration treatment female black hispanic ndependents recall young old
## 1  18.011343         0      0     0        0           2      0     0   0
## 4   1.003399         0      0     0        0           0      0     0   0
## 5  26.960396         0      0     0        0           0      0     0   0
## 6   7.009044         1      0     0        0           0      0     0   0
## 12  9.022409         1      0     0        0           0      0     1   0
## 13 26.991390         0      0     0        0           1      0     0   1
##    quarter durable lusd
## 1        5       0    0
## 4        5       0    1
## 5        4       0    1
## 6        2       0    0
## 12       3       0    0
## 13       5       1    1
```

```r
z = penndata$treatment
y = log(penndata$duration)
block = penndata$quarter
est = Neyman_SRE(z, y, block)
est[1]
```

```
## [1] -0.08990646
```

```r
sqrt(est[2])
```

```
## [1] 0.03079775
```

```r
print(paste("[",est[1]-1.96*sqrt(est[2]),",",est[1]+sqrt(est[2]),"]"))
```

```
## [1] "[ -0.150270048386588 , -0.0591087093146378 ]"
```