<u>Econ C103: Introduction to Mathematical Economics</u>

UC Berkeley, Fall 2019

Assignment 1 *(total points: 100)*

Problem 1 (10 points): Consider the preference relationship (also depicted below) over four choices $\{x_1, x_2, x_3, x_4\}$ which includes the following elements:

$$x_i \succcurlyeq x_i, \forall i = 1, \dots, 4, x_2 \succcurlyeq x_4, x_3 \succcurlyeq x_2, x_4 \succcurlyeq x_1, x_4 \succcurlyeq x_3$$

	x_1	x_2	x_3	x_4
x_1	●			
x_2		●		●
x_3		●	●	
x_4	●		●	●

Is the preference relationship complete? Is it transitive? Find the smallest set of additional elements (an example of an "additional element" to the preference relationship is $x_1 \succcurlyeq x_2$) that makes the preference relationship rational.

Problem 2 (10 points): Formulate the following game (i.e. define the game's elements). Do so first as a static game (i.e. define each element in $\Gamma = \langle N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$), then as its mixed extension (i.e. define each element in $\Delta\Gamma = \langle N, \{\Delta(A_i)\}_{i \in N}, \{U_i\}_{i \in N} \rangle$). Find the set of all PNE and MNE of the game. For this, graph together the best response correspondences of each player, and find their intersection. What game discussed in lecture is this game most similar to?

		2	
		L	R
1	T	$2, 1$	$0, 2$
	B	$1, 2$	$3, 0$

Problem 3 (10 points): In the following game, what strategies survive iterated elimination of strictly dominated strategies (IESDS)? Find the set of MNE.

	L	C	R
T	$2, 0$	$1, 1$	$4, 2$
M	$3, 4$	$1, 2$	$2, 3$
B	$1, 3$	$0, 2$	$3, 0$

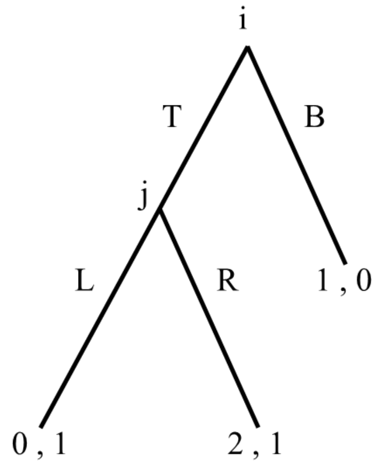
Problem 4 (5 points): With the following game, verify that the order in which you eliminate strictly dominated strategies in IESDS does not carry any implication for the set of actions that survive. Note that this independence of IESDS can be proven for general finite static games.

	L	C	R
T	4 , 4	2 , 3	1 , 2
M	2 , 4	4 , 3	3 , 3
B	3 , 2	1 , 3	0 , 1

Problem 5 (15 points): Consider the following “Hotelling Model” game (named after its inventor): There is a continuum of political platforms that candidates can choose (imagine the voters distributed along the unit interval $[0,1]$). Candidates gather the density of votes for voters that are closest to that candidate along the continuum (than any other candidate). If any number of candidates chose the same platform, these candidates split the voters that are closest to their location along the continuum. Candidates strictly prefer: winning, to a tie (for 1st place) among n candidates ($n > 1$), to a tie among $n + 1$ candidates, to losing. The voters are distributed via cumulative distribution function F along the continuum $[0,1]$. Let m be the median of F. Formulate this game (i.e. define the game’s elements). Find the set of PNE when there are two candidates.

Now assume that candidates also have the option to exit the race, which is preferred to running and loosing, but tying for 1st is strictly preferred to exiting. How does the formulation of the game change? Will the set of Nash change for the two-candidate case? Find the set of PNE when there are three candidates.

Problem 6 (10 points): Consider the following game tree, representing an extensive form game.



Find the set of PNE and MNE of the game. Which of these equilibria are SPNE?

Problem 7 (10 points): Consider the following game. Two people, $i \in \{1,2\}$, work on a joint project by choosing an effort level $x_i \in [0,1]$ (i’s action). The outcome of the project is worth $V(x_1, x_2)$ and each faces a cost $c(x_i)$. The value of the project is split evenly between the two, regardless of their relative effort levels. Find the PNE of the game when:

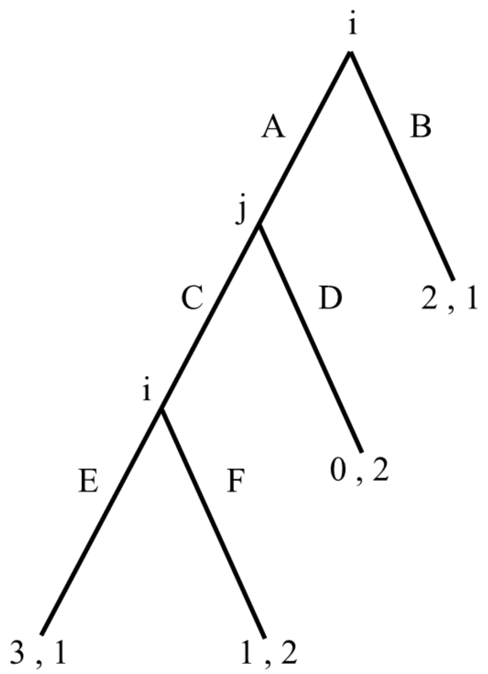
a) $V(x_1, x_2) = 3x_1x_2$ and $c(x_i) = x_i^2$, and
b) $V(x_1, x_2) = 4x_1x_2$ and $c(x_i) = x_i$.

For both cases, find a Pareto efficient outcome such that both players exert the same effort. Compare this value to each PNE value(s).

Problem 8 (10 points): Consider the follows *third-price sealed-bid auction*. Bidder $N = \{1, \dots, n\}$, $n > 2$, simultaneously place non-negative bids for an indivisible good. Each player $i \in N$ has value $v_i \geq 0$ for the good. Assume that $v_1 > v_2 \cdots > v_n$. The winner is the bidder who submits the highest bid, and ties are broken at random. The winner must pay the third highest bid to the auctioneer.

a) Show that for each $i \in N$, the bid $b_i = v_i$ weakly dominates low bids, but does not weakly dominate any higher bid.
b) Show that the action profile in which each bidder bids their value is not a PNE.
c) Find a symmetric PNE.

Problem 9 (10 points): Consider the two-player extensive game in the figure below. Formulate the following game (i.e. define the game's elements: recall the definition of an extensive form game). Write the game in its strategic form and find the set of all PNE. Find the set of all SPNE of the game. Is there a unique SPNE?



Problem 10 (10 points): Consider the following variation of the two-player infinite horizon bargaining game studied in class. At any stage t, after player i makes an offer (x_i, x_j), if player j accepts, then with probability $\lambda \in (0,1]$ they reach an agreement and receive payoffs $(\delta^{t-1}x_i, \delta^{t-1}x_j)$. However, with probability $1 - \lambda$, player i misunderstands j's "accept" response, and they proceed to the next period as if the offer was rejected. A "reject" response is never misunderstood. The rest of the game remains the same. (That is, player i starts making an offer in period 1; in subsequent periods players alternate in making offers; the payoffs, the timeline, and the disagreement payoff of (0,0) remain the same; and the players have the common discount factor $\delta \in (0,1)$.)

Conjecture an SPNE. Formally, write down the strategy profile and verify that it is indeed an SPNE by using the single deviation property.