Econ C103: Game Theory and Networks

Module I (Game Theory): Lecture 11

Instructor: Matt Leister

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Readings:

1. Diamond, Douglas and Philip Dybvig. "Bank Runs, Deposit Insurance, and Liquidity", Journal of Political Economy, 1983.

Bayes' Cournot Duopoly

- Two firms i and j compete by producing quantities $q_i \in \mathbb{R}$ and $q_j \in \mathbb{R}$, resp., under price function $P(q_i, q_j) = (a + \omega) - b(q_i + q_j)$, $a, b > 0$, $\omega \sim \mathcal{N}(0, \sigma^2)$. Marginal cost of production is $c > 0$.
- Each firm $k = i, j$ observes signal $\theta_k = \omega + \epsilon_k$, $\epsilon_k \sim \mathcal{N}(0, 1/\gamma_k)$; ϵ_k is the noise in k's signal θ_k. The "state space" is $\{(\omega, \epsilon_i, \epsilon_j)\}$, $\gamma_k \in \mathbb{R}_+$ is the "precision" of k's signal; $\gamma_k = 0$ is "no information".
- Each firm maximizes profit given their signal, and the other firm's production strategy $s_k : \mathbb{R} \mapsto \mathbb{R}$, a function of t_k.
- Player i's Bayesian updating yields the following expectations (similar for j's expectations; *feel free to take these next expressions as given*):

  E1. $\mathbb{E}[\omega|\theta_i] = \mathbb{E}[\omega|t_i] = e_i t_i$, and

  E2. $\mathbb{E}[\theta_j|\theta_i] = \mathbb{E}[t_j|t_i]/e_j = e_i t_i$, so $\mathbb{E}[t_j|t_i] = e_i e_j t_i$

  where:

$$\begin{aligned} e_i &\equiv \sqrt{\frac{\sigma^2}{\sigma^2 + \gamma_i^{-1}}} \in [0, 1], \\ t_i &\equiv e_i \theta_i. \end{aligned}$$

Bayes' Cournot Duopoly

- For firm i, given s_j and upon observing t_i, her optimal production is:

$$s_i^*(t_i) = \underset{q_i}{\text{argmax}}((a + \mathbb{E}[\omega|t_i]) - b(q_i + \mathbb{E}[s_j(t_j)|t_i]))q_i - cq_i.$$

- First-order condition: $a + \mathbb{E}[\omega|t_i] - 2bq_i - b\mathbb{E}[s_j(t_j)|t_i] - c = 0.$
- Solving for q_i gives firm i's best response:

$$s_i^*(t_i) = \frac{a + e_i t_i - b\mathbb{E}[s_j(t_j)|t_i] - c}{2b}.$$

Bayes' Cournot Duopoly

- Assume j uses linear strategy $s_j(t_j) = \alpha_j + \beta_j t_j$. Then:

$$\mathbb{E}[s_j(t_j)|t_i] = \alpha_j + \beta_j \mathbb{E}[t_j|t_i] = \alpha_j + \beta_j e_i e_j t_i.$$

- This gives firm i's optimal strategy:

$$s_i^*(t_i) = \frac{a + e_i t_i - b(\alpha_j + \beta_j e_i e_j t_i) - c}{2b},$$

  which implies:

$$\begin{aligned} \alpha_i^* &= \frac{a - b\alpha_j - c}{2b}, \\ \beta_i^* &= \frac{e_i - b\beta_j e_i e_j}{2b}. \end{aligned}$$

  β_i^* decreasing in e_j: better information of competition $\Rightarrow$ less responsive to private information (learning ω is high $\Rightarrow$ j also learns ω is high $\Rightarrow$ q_j increases $\Rightarrow$ price and marginal revenue decrease!).

- Similarly for j...

Bayes' Cournot Duopoly

$$\begin{aligned}
\alpha_i^* &= (a - b\alpha_j^* - c)/(2b); & \alpha_j^* &= (a - b\alpha_i^* - c)/(2b), \\
\beta_i^* &= (e_i - b\beta_j^* e_i e_j)/(2b); & \beta_j^* &= (e_j - b\beta_i^* e_j e_i)/(2b).
\end{aligned}$$

- Solving this system yields:

$$\begin{aligned}
\alpha_k^* &= \frac{a-c}{3b}, \ \forall k = i, j \\
\beta_i^* &= \frac{e_i(2 - e_j^2)}{b(4 - e_i^2 e_j^2)}, \\
\beta_j^* &= \frac{e_j(2 - e_i^2)}{b(4 - e_i^2 e_j^2)}.
\end{aligned}$$

Bayes' Cournot Duopoly

$$\alpha_i^* = \frac{a-c}{3b},$$
$$\beta_i^* = \frac{e_i(2-e_j^2)}{b(4-e_i^2 e_j^2)}.$$

- α_k^* corresponds to the PNE in the Cournot Duopoly.
- β_i^* increasing in e_i: better information $\Rightarrow$ optimal strategy more responsive to private information.
- β_i^* decreases in e_j: better information of competition ($e_j \uparrow$) $\Rightarrow$ less responsive to private information ($\beta_i^* \downarrow$) $\Rightarrow \beta_j^* \uparrow \Rightarrow \beta_i^* \downarrow \ldots$! Players' information use become interdependent in equilibrium...

Bayes' Cournot Duopoly

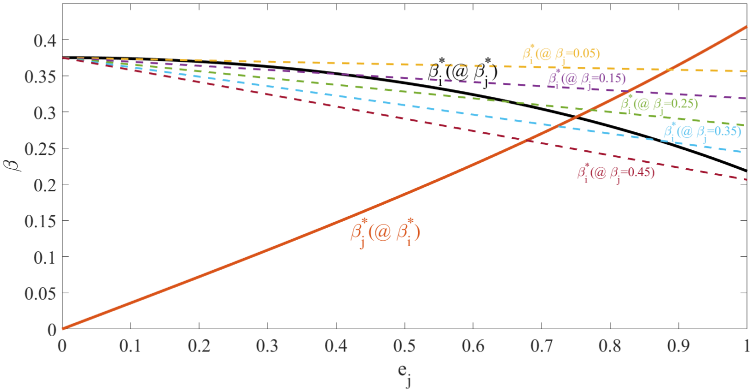

Figure: Information use: $\beta =$ slope of strategy to private signal.

- In the figure, $e_i = 0.75$, and $e_j \in [0, 1]$ (the x-axis). Equilibrium β_i^* (black line) decreases in e_j, at an increasing rate as e_j increases; in contrast, β_i^* when β_j is held fixed (dashed lines) has a fixed slope.

Diamond-Dybvig (1983) bank-run model

- Three periods $t = 1, 2, 3$. A unit measure of consumers deposit wealth in a bank at $t = 1$; consumption occurs in $t = 2, 3$.
- Measure θ values consumption only in $t = 2$ (type A), measure $1 - \theta$ values consumption in both $t = 2, 3$ (type B):

$$U_i(c_1, c_2) = \begin{cases} u(c_2) & \textit{with prob. } \theta \text{ (type A)} \\ \rho u(c_2 + c_3) & \textit{with prob. } 1 - \theta \text{ (type B)} \end{cases} ;$$

  where $u(\cdot)$ is twice continuously differentiable, increasing and concave, and $0 < \rho < 1$. So, types A's have greater need.
- Bank holds 1 unit of liquidity (money); withdrawals at $t = 2$ and $t = 3$ yield consumption value 1 and $R > 1$, respectively.
- Consumption type is private information to each consumer $i \in [0, 1]$; the "state" determines which consumers are of types A/B. BUT, on aggregate measure θ is of type A (so, no aggregate risk!).

Diamond-Dybvig (1983) bank-run model

- Socially efficient consumption allocation has $c_3^A = 0$ and $c_2^B = 0$.
- If the bank could require only type A's withdraw in period $t = 2$, then bank's budget constraint becomes:

$$\theta c_2^A + (1-\theta)(c_3^B / R) = 1 \Leftrightarrow c_3^B = \frac{(1-\theta c_2^A)R}{1-\theta}.$$

- Planner's problem then solves:

$$\max_{c_2^A \geq 0} \theta u(c_2^A) + (1-\theta)\rho u(\frac{(1-\theta c_2^A)R}{1-\theta}).$$

First-order condition for efficient c_2^{A*}:

$$u'(c_2^{A*}) = R\rho u'(\frac{(1-\theta c_2^{A*})R}{1-\theta}) = R\rho u'(c_3^{B*}).$$

If $R\rho > 1$ then $c_3^{B*} > c_2^{A*}$, and type B's prefer to wait (greater c_3^B).

Diamond-Dybvig (1983) bank-run model

- So, if $R\rho > 1$ and bank promises c_2^{A*} to any consumer withdrawing at $t = 2$, then it is a BNE for type A's to withdraw at $t = 2$, type B's at $t = 3$, which yields efficient consumption profile (c_2^{A*}, c_3^{B*}).
- Bank-run BNE: also assume that $R\rho \in (1, \bar{R\rho}]$, where $\bar{R\rho}$ satisfies:

$$u'(1) = \bar{R\rho} u'(R).$$

  Then, $1 < c_2^{A*} < c_3^{B*}$ (while $\theta c_2^A + (1-\theta)(c_3^B/R) = 1$ still holds).
- What if type B's believe all consumers will withdraw at $t = 2$?
- Then, all consumer "run to the bank", but only first $1/c_2^{A*}$ consume!
- Two "sunspot" BNE:
  1. an efficient (liquid) BNE, in which only type A withdraws in $t = 2$, and
  2. a bank-run (illiquid) BNE, in which all consumers withdraw in $t = 2$.