Econ C103: Introduction to Mathematical Economics

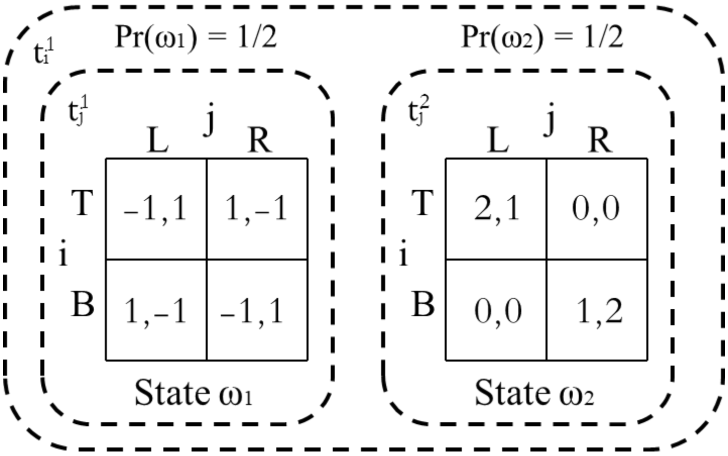
UC Berkeley, Fall 2019

Assignment 2 *(total points: 75)*. Due Date: October 16.

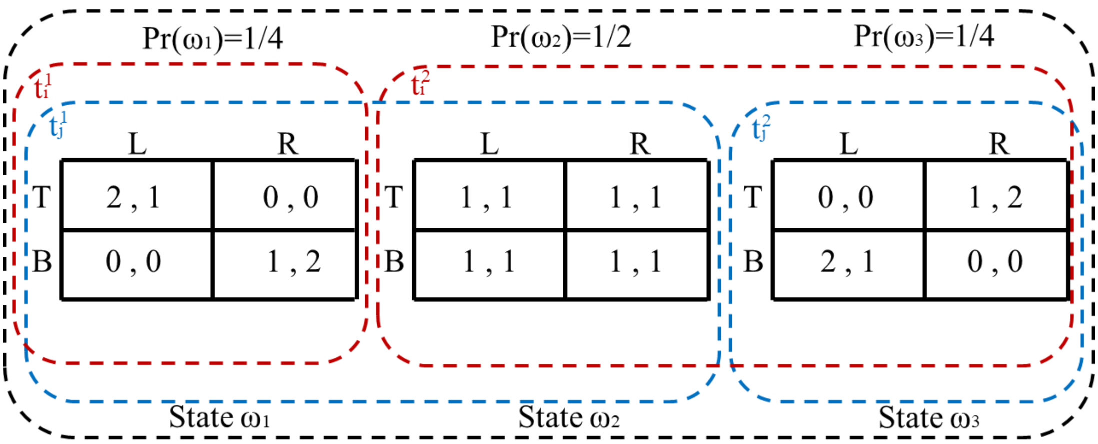
Problem 1 (10 points): Construct the Stackelberg games (i.e. one player moves before the other) of the following static game, and determine which players have a first-mover advantage by comparing SPNE payoffs in each Stackleberg game to the set of MNE expected payoffs in the static game. *Hint: First show that L is never played by j in any MNE of the static game.*

i \ j	L	C	R
T	1,-1	3,-3	-3,3
B	2,4/5	-3,3	3,-1

Problem 2 (10 points): Find the BNE of the following game. To do this, construct the corresponding normal-form game in which players' action sets equal their strategies in the Bayesian game, and certain payoffs equal their ex-ante expected payoffs in the Bayesian game as a function of the strategy profile.



Problem 3 (10 points): Find the BNE of the following game. To do this, complete the normal-form game (below) in which players' action sets equal their strategies in the Bayesian game, and certain payoffs equal the ex-ante expected payoffs in the Bayesian game as a function of the strategy profile.



j

		LL	LR	RL	RR
	TT	¼2+½1+¼0 =1, ¼1+½1+¼0 =3/4	¼2+½1+¼1 =5/4, ¼1+½1+¼2 =5/4	¼0+½1+¼0 =1/2, ¼0+½1+¼0 =1/2	
i	TB				¼0+½1+¼0 =1/2, ¼0+½1+¼0 =1/2
	BT	¼0+½1+¼0 =1/2, ¼0+½1+¼0 =1/2	¼0+½1+¼1 =3/4, ¼0+½1+¼2 =1		
	BB	¼0+½1+¼2 =1, ¼0+½1+¼1 =3/4			

Problem 4 (10 points): Recall the Diamond-Dybvig model of lecture 11. Now consider the following variation of the model where the central bank secures consumers' deposits. If a consumer arrives at the bank at $t = 2$ and the bank is illiquid (i.e. has no funds left to offer the consumer), then the central bank will reimburse the consumer an amount $C' > 0$. What are the set of symmetric BNE of the game for each C' value (by "symmetric" I mean the BNE in which all consumers of each given type use the same strategy)?

Problem 5 (10 points): This problem tests your intuition for BNE in auctions.

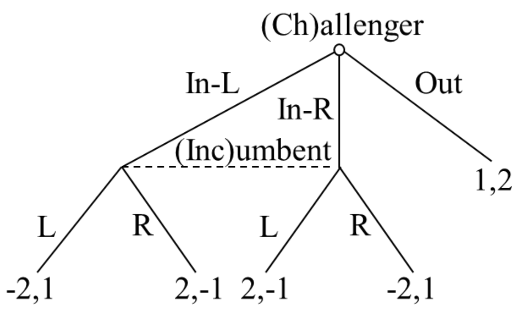
Part 1) Consider a sealed-bid auction with i.i.d. values distributed via cdf $F(\cdot)$. Prove that if the mechanism $P(\boldsymbol{b})$ is given by the k^{th}-highest bid (i.e. a k^{th}-price auction) for $k > 2$ then the symmetric BNE involves equilibrium bidding $b^*(v)$ above ones values (i.e. $b^*(v) > v$; show this must hold for some values of v, though this is actually true for all values). You can use all results/theorems provided in lecture. *Hint: use the fact that* $F^{[k]}(v) < F^{[k+1]}(v)$ *for all* v *and* k, where $F^{[k]}$ gives the cdf of the k^{th}-highest statistic, *and the fact that*:

$$\int_{\underline{v}}^{\overline{v}} f(v)dF^{[k]}(v) > \int_{\underline{v}}^{\overline{v}} f(v)dF^{[k+1]}(v)$$

for any monotonically increasing $f(v)$. *The proof will require limited math, but careful use of results.*

Part 2) Again consider a sealed-bid auction with i.i.d. values distributed via cdf $F(\cdot)$. By considering the equilibrium bidding of the 1^{st}- and 2^{nd}- price auctions, and using your intuition, postulate what equilibrium bidding in the k^{th}-price auction, for $k > 2$, will converge to as the number of bidders grows very large. Do you expect $b^*(v)$ to be increasing or decreasing in n for $k > 2$? Is expected revenue increasing or decreasing in n? Explain.

Problem 6 (10 points): Find the set of beliefs of the (Inc)umbent regarding the probability that (Ch)allenger chose In-L, and the set of strategies of Inc that are supported in PBNE.



Problem 7 (15 points): Find the set of PBNE of the following signaling game.