Econ C103: Game Theory and Networks

Module I (Game Theory): Lecture 8

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Fall 2019, UC Berkeley

Readings:

1. Osborne (2004) Example 233.3
2. Osborne and Rubinstein (1994) Section 6.5.2

The Centipede Game

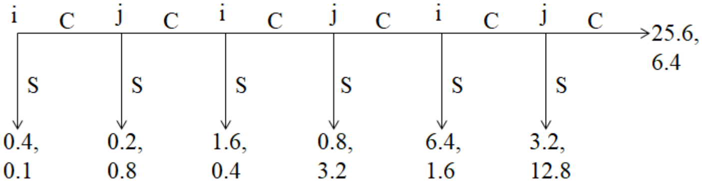

Figure: The Centipede Game

- $N = \{i, j\}$,
- $S_i = S_j = \{(C,C,C), (C,C,S), (C,S,S), (S,S,S), (S,S,C), (S,C,C), (C,S,C), (S,C,S)\}$,

The Centipede Game

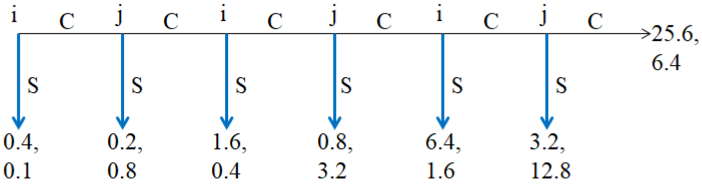

Figure: The Centipede Game

- $N = \{i, j\}$,
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- $SPNE = \{($(S,S,S), (S,S,S)$)\}$,

The Centipede Game

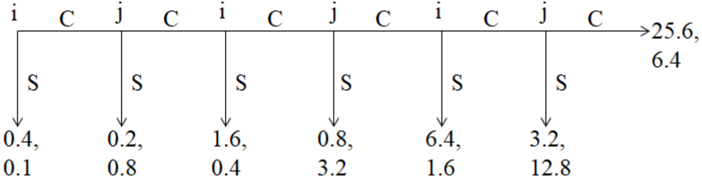

Figure: The Centipede Game

- $N = \{i, j\}$,
- $S_i = S_j =$ $\{$(C,C,C), (C,C,S), (C,S,S), (S,S,S), (S,S,C), (S,C,C), (C,S,C), (S,C,S)$\}$,
- $SPNE = \{$((S,S,S), (S,S,S))$\}$,
- $PNE = \{$((S,X,X'), (S,Y,Y')) : $X, X', Y, Y' \in \{S, C\}\}$.

The Centipede Game in the laboratory

(McKelvey and Palfrey 1992) Subjects played the Centipede Game 10 times, each time with a different opponent.

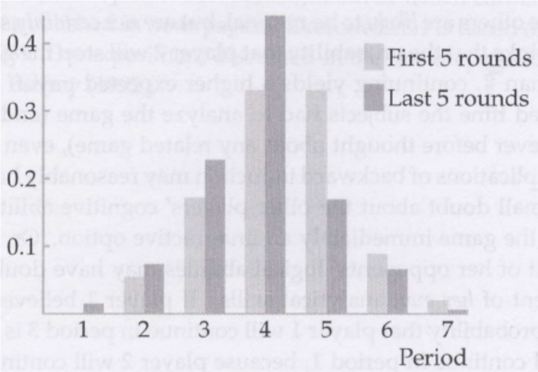

Figure: Distribution of terminal nodes

- Subjects were "learning" to play the game, and moved toward SPNE. But what game?...

The Centipede Game with "other regarding preferences"

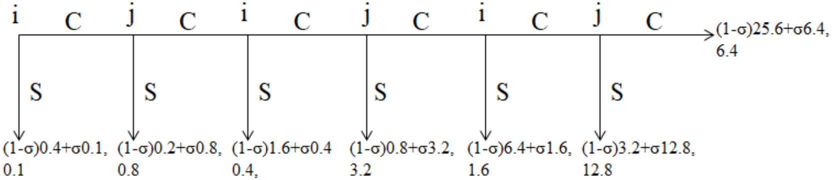

Figure: The Centipede Game

- $\sigma \in [0, 1]$ measures *i*'s regard for *j*'s payoff...

The Centipede Game with “other regarding preferences”

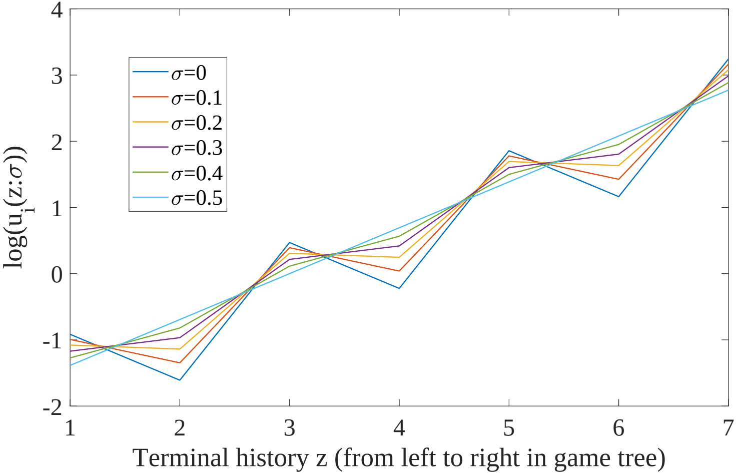

Figure: $u_i(z:\sigma) = (1-\sigma)u_i(z:0) + \sigma u_j(z:0)$.

The Centipede Game with "other regarding preferences"

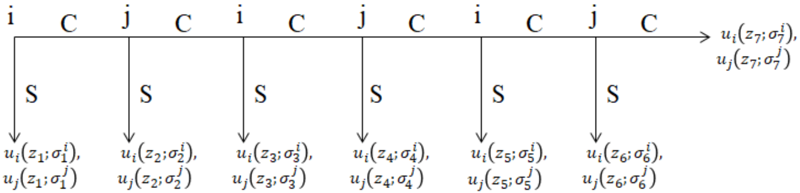

Figure: The Centipede Game

- $\sigma_t^k \in [0,1]$ measures k's regarding for $-k$, $k = i, j$, in outcome z_t.
- $\{\sigma_t^k\}$ can "rationalize" any observed behavior: for any outcome z, there is some $\{\sigma_t^k\}$ such that z is a PNE.

Stackelberg Duopoly

- Two firms i and j compete by producing quantities $q_i \geq 0$ and $q_j \geq 0$, resp., under price function $P(q_i, q_j) = a - b(q_i + q_j)$, $a, b > 0$. Marginal cost of production is $c > 0$.
- Timing: firm i sets q_i (first stage), then firm j sets q_j (second stage).
- Second stage: Take history where i set q_i in the first stage. Firm j maximizes profit given q_i:

$$\max_{q_j}(a - b(q_i + q_j)) - cq_j.$$

- Solving for q_j gives firm j's best response:

$$q_j^*(q_i) = \begin{cases} \frac{a-bq_i-c}{2b} & \textit{if } \frac{a-c}{b} > q_i \\ 0 & \textit{otherwise} \end{cases}.$$

- First stage: firm i chooses q_i^* taking $q_j^*(q_i)$ as given...

Stackelberg Duopoly

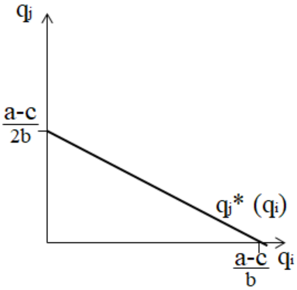

Figure: Stackelberg Duopoly: best response of second mover

Stackelberg Duopoly

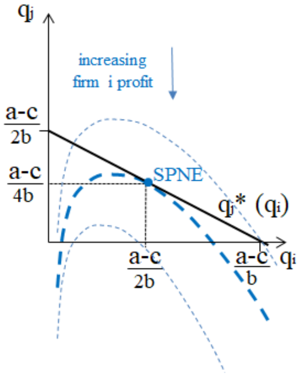

Figure: Stackelberg Duopoly: first mover's problem

Stackelberg Duopoly

- First-mover i chooses q_i^* to maximize profit, given $q_j^*(q_i)$:

$$\max_{q_i}(a - b(q_i + q_j^*(q_i)))q_i - cq_i.$$

- Firm i's first-order condition becomes:

$$\begin{aligned}
\frac{d}{dq_i}\pi_i(q_i, q_j^*(q_i)) &= \frac{\partial}{\partial q_i}\left(\left(a - b\left(q_i + \frac{a - bq_i - c}{2b}\right)\right) - c\right) q_i \\
&= a - 2q_i b - \underbrace{\frac{a - 2q_i b - c}{2}}_{\text{affect of } q_i \text{ on } q_j^*} - c \ = \ 0 \\
\Leftrightarrow q_i^* &= \frac{a-c}{2b} \ \Leftrightarrow \ q_j^*(q_i^*) \ = \ \frac{a-c}{4b}.
\end{aligned}$$

- Recall PNE in Cournot Duopoly: $q_i^* = q_j^* = \frac{a-c}{3b}$.
- Stackelberg Duopoly: first-mover firm i commits to larger production.

Stackelberg Duopoly

- Firm $k = i, j$ profit in Cournot Duopoly:

$$\pi_k(q_i^*, q_j^*) = \left(a - b\left(\frac{a-c}{3b} + \frac{a-c}{3b}\right)\right)\frac{a-c}{3b} - c\frac{a-c}{3b} = \frac{(a-c)^2}{9b}.$$

- First-mover profit in Stackelberg Duopoly:

$$\pi_i(q_i^*, q_j^*) = \left(a - b\left(\frac{a-c}{2b} + \frac{a-c}{4b}\right)\right)\frac{a-c}{2b} - c\frac{a-c}{2b} = \frac{(a-c)^2}{8b}.$$

- Second-mover profit in Stackelberg Duopoly:

$$\pi_j(q_i^*, q_j^*) = \left(a - b\left(\frac{a-c}{2b} + \frac{a-c}{4b}\right)\right)\frac{a-c}{4b} - c\frac{a-c}{4b} = \frac{(a-c)^2}{16b}.$$

- Firm i has a "first-mover" advantage in Stackelberg Duopoly.

Stackelberg and the Centipede Game

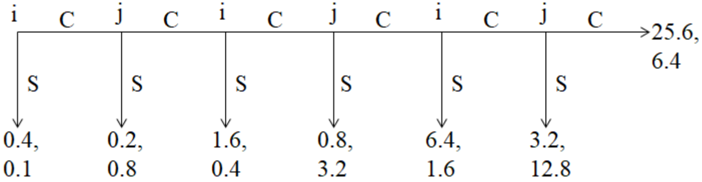

Figure: The Centipede Game

- Let i commit to strategy in S_i, then j chooses strategy in S_j.
- $BR_j(s_i) =$ "stop before i's first stop, otherwise continue".
- $SPNE$ outcome: i commits to (C, C, C), j chooses (C, C, S), players realize payoffs $(3.2, 12.8)$.
  $\Rightarrow$ Both i and j advantaged from i's commitment!

Stackelberg and Matching Pennies

Matching Pennies

	L	R
T	$1, -1$	$-1, 1$
B	$-1, 1$	$1, -1$

- Let i commit to strategy in A_i, then j chooses strategy in A_j.
- $SPNE = \{(T, (R, L)), (B, (R, L))\}$.
  Respective $SPNE$ outcomes: (T, R) and (B, L).
  $\Rightarrow$ Player j has "second-mover" advantage in Stackelberg game.

First/second-mover advantage in Stackelberg games

Exercise: work through the Stackelberg versions of other 2×2 *games.*

- If commitment implies a PNE of the simultaneous game is selected (among multiple PNE), then there's a first-mover advantage. Second-mover may also be advantaged (e.g., Coordination Game) or disadvantaged (e.g., Battle of the Sexes, Hawk-Dove).
- If there is a unique PNE in the simultaneous game, then the first mover is never disadvantaged (e.g., Cournot), but both players can be indifferent (e.g., Prisoner's Dilemma).
- If there are (non-degenerate) MNE, then there may be a first-mover disadvantage (e.g., Matching Pennies), but not always (e.g., Coordination Game).