Econ C103: Game Theory and Networks

Module I (Game Theory): Lecture 9

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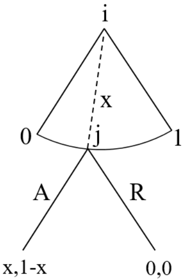
Readings:

1. Osborne (2004) Chapter 16
2. Osborne and Rubinstein (1994) Sections 7.2-7.4

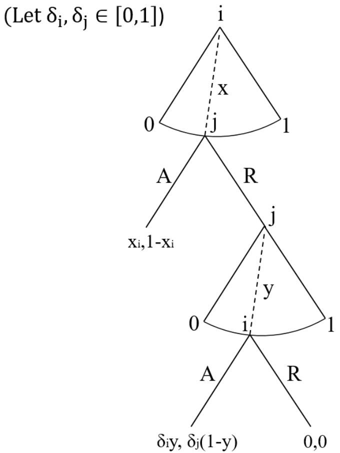
The Ultimatum Game



- $N = \{i, j\}$,
- $H = \{\emptyset, x, (x, A), (x, R) : x \in [0, 1]\}$.
- $S_i = [0, 1]$,
  $S_j = \mathcal{R} \equiv \{\text{Reply}(x) \in \{\text{(A)ccept}, \text{(R)eject}\},\ \textit{for } x \in [0, 1]\}$
- $SPNE = (1, \text{Reply}(x) = \text{A})$ (*assume indifferent respondents accept*).
- *PNE* includes incredible threats; Example:

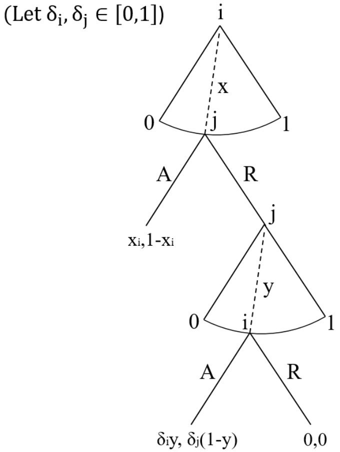
$$(1/2, \text{Reply}(x) = \text{A } \textit{iff } x \leq 1/2).$$

Non-cooperative Bargaining: one counter offer



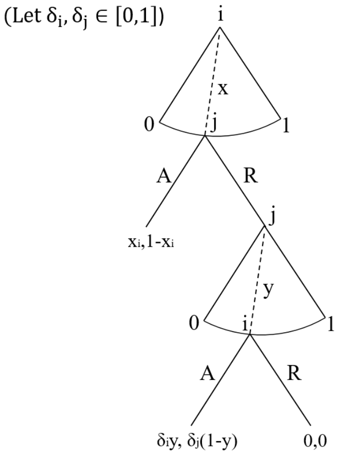
- $H = \left\{ \begin{array}{c} \emptyset, x, (x, A), (x, R, y), \\ (x, R, y, A), (x, R, y, R) \end{array} : x, y \in [0, 1] \right\}.$
- $S_i = [0, 1] \times ([0, 1] \times \mathcal{R}), \;\; S_j = \mathcal{R} \times [0, 1].$

Non-cooperative Bargaining: one counter offer



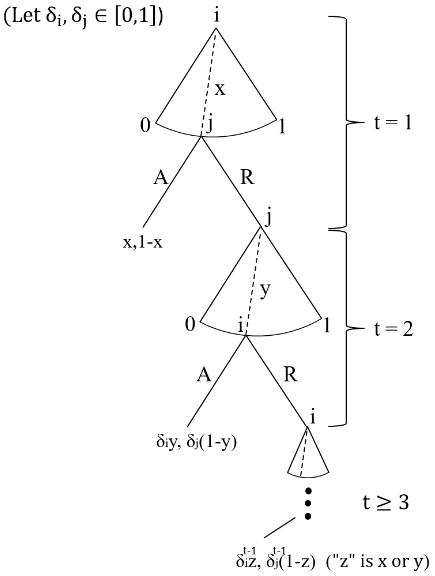
- $H = \left\{ \begin{array}{c} \emptyset, x, (x, A), (x, R), (x, R, y), \\ (x, R, y, A), (x, R, y, R) \end{array} : x, y \in [0, 1] \right\}.$
- i's "stationary" strategies: $S_i = [0, 1] \times \mathcal{R}$.

Non-cooperative Bargaining: one counter offer



- Backward induction: i will accept $y = 0$, so $y^* = 0$. Thus, j will reject x if $1 - x < \delta_j$, and accept otherwise. Thus, $x^* = 1 - \delta_j$.
- $SPNE = ((1 - \delta_j, \text{Reply}(y) = \text{A}), (\text{Reply}(x) = \text{A } \textit{iff } x \leq 1 - \delta_j, 0))$.

Non-cooperative Bargaining: infinite horizon



One-deviation Property

Definition (One-deviation Property)

No player can increase her payoff by changing her action at the start of any subgame in which she is the first-mover, given the other payer's strategies and the rest of her own strategy.

Proposition (SPNE in finite games and One-deviation Property)

A strategy profile in a finite-horizon extensive game with perfect information is a SPNE if and only if it satisfies the One-deviation Property.

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Proposition (SPNE in bargaining and One-deviation Property)

A strategy profile in the infinite-horizon bargaining game of alternating offers is a SPNE if and only if it satisfies the One-deviation Property.

- By the above Proposition, finding an SPNE involves finding stationary strategies (i.e. in which players behave the same in all subgames in which they are the proposer, and the same in all subgames in which they are the responder) which satisfy the One-deviation Property.

Non-cooperative Bargaining: infinite horizon

- Rubinstein 1982: Characterizes SPNE of infinite-horizon alternating-offer bargaining game.
- Player i's stationary strategy (x^*, c_i^*): independent of history, player i makes offer x^*, and accepts any offer y iff $y \geq c_i^*$.
- Player j's stationary strategy (y^*, c_j^*): independent of history, player j makes offer y^*, and accepts any offer x iff $1 - x \geq 1 - c_j^*$.
- Fact: Under perfect/complete information, SPNE offers are immediately accepted, independent of the history.
- For player j to accept i's offer x^*:

$$1 - x^* \geq \delta_j(1 - y^*).$$

- And for x^* to be optimal, i sets x^* so that this equality holds.
- Likewise, for player i to accept j's (optimal) offer y^*:

$$y^* = \delta_i x^*.$$

Non-cooperative Bargaining: infinite horizon

- Rubinstein 1982: Characterizes SPNE of infinite-horizon alternating-offer bargaining game.
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- Player j's stationary strategy (y^*, c_j^*): independent of history, player j makes offer x_j^*, and accepts any offer x iff $x \leq c_j^*$.
- Player j's and i's indifference conditions, $1 - x^* = \delta_j(1 - y^*)$ and $y^* = \delta_i x^*$, respectively, give a system of equations with solution:

$$x^* = \frac{1-\delta_j}{1-\delta_i\delta_j}, \quad y^* = \frac{\delta_i(1-\delta_j)}{1-\delta_i\delta_j}.$$

- Cutoffs that leave each player indifferent: $c_i^* = y^*$ and $c_j^* = x^*$.
- Equilibrium outcome: i offers x_i^* and j accepts and receives $1 - x_i^*$.
- If $1 \approx \delta_i \gg \delta_j$: $x^* \approx 1$; i more patient & receives most of the "pie".
- If $1 \approx \delta_j \gg \delta_i$: $x^* \approx 0$; j more patient & receives most of the "pie".

Non-cooperative Bargaining: infinite horizon

- Rubinstein 1982 shows stationary SPNE is unique!
- No delay: long the equilibrium path, first offer x_i^* accepted.
- Rubinstein 1982 also considers fixed costs of delay (rather than discounting). Unique stationary SPNE, also solved via indifference conditions, also exhibits immediate agreement (i.e. no rejections along the equilibrium path): *no delay is a general feature of complete information bargaining*...
- ...Admati and Perry "Strategic Delay in Bargaining" (REStud 1987) incorporate incomplete information (weeks 4-6). They show private information of one's value to agreement can introduce delay: player who privately knows she has a high value to agreement may reject offers and maintain a (false) reputation that she *doesn't* value agreement very much, which causes the other player to offer more.