MCR3U0

3.8 - Linear–Quadratic Systems

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Learning Goal: Solve problems involving the intersection of a linear and a quadratic function.

Key Ideas

- **A linear function and a quadratic function can intersect at a maximum of two points.**
- **The point(s) of intersection of a line and a parabola can be found**
    - **graphically**
    - **algebraically**

Need to Know

- **To determine the points of intersection algebraically, use substitution to replace in the quadratic function with the expression for from the linear function. This results in a quadratic equation whose solutions correspond to the x-coordinates of the points of intersection.**
- **In many situations, one of the two solutions will be inadmissible.**

1P198] Find the point(s) of intersection by graphing.

$y \quad\quad\quad\quad y$

a) $f(x) = x^2, g(x) = x + 6$

b) $f(x) = -2x^2 + 3, g(x) = 0.5x + 3$

c) $f(x) = (x - 3)^2 + 1, g(x) = -2x - 2$

$y_Q = y_L$

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x_1 = -2$; $y_1 = (-2)^2$; $y_1 = 4$

$PI_1(-2, 4)$

$x_2 = 3$

$y_2 = 3^2 = 9$

$PI_2 = (3, 9)$

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$PI_2 = (3, 9)$

c) $f(x) = (x-3)^2 + 1 \ ; \quad g(x) = -2x - 2$

$$f(x) = g(x)$$

$$(x-3)^2 + 1 = -2x - 1$$

$$x^2 - 6x + 9 + 1 + 2x + 1 = 0$$

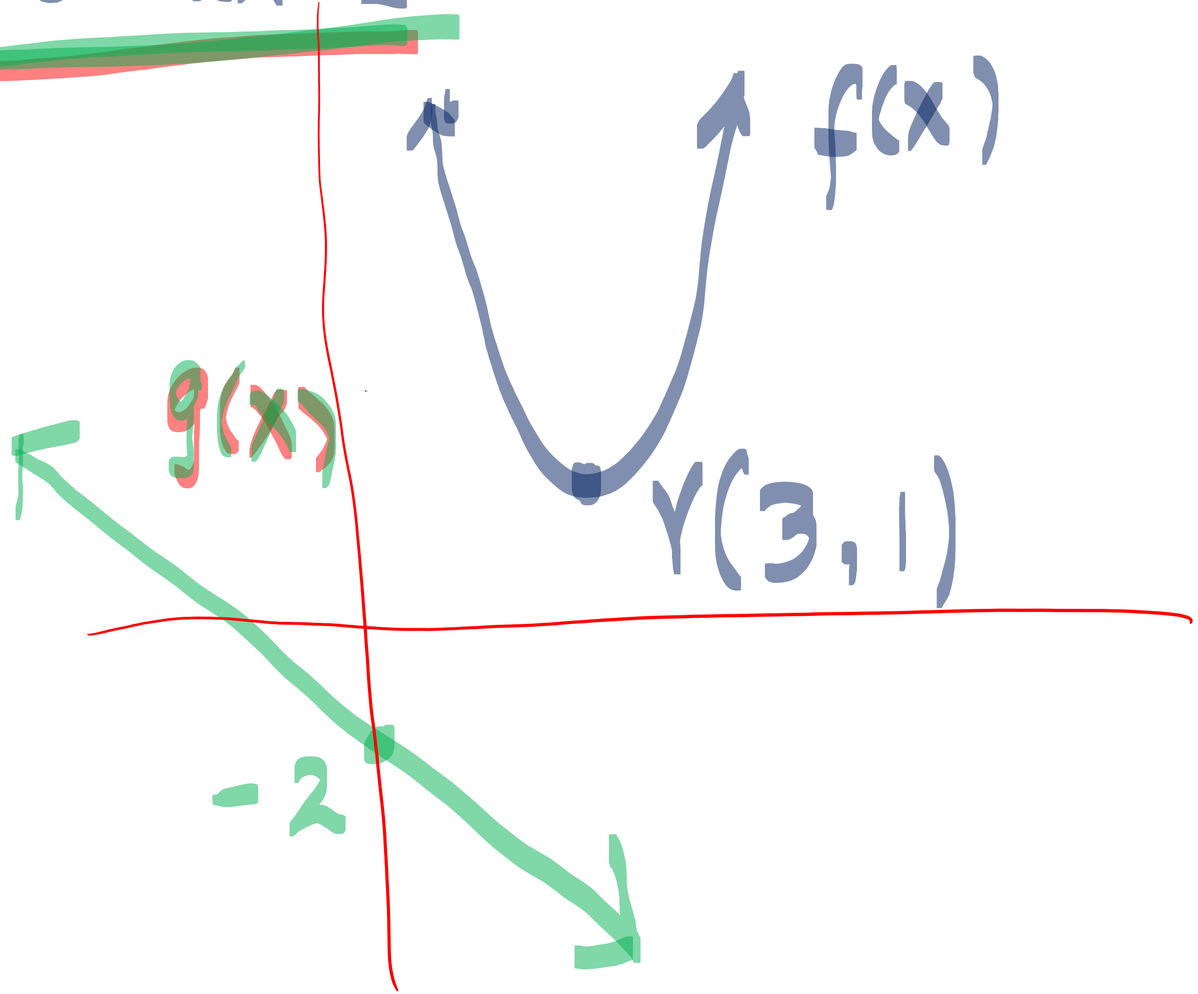
$$x^2 - 4x + 11 = 0$$

$$b^2 - 4ac = (-4)^2 - 4(1)(11)$$

$$= 16 - 44$$

$$b^2 - 4ac = -28$$

$\therefore$ There is no sol'n



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$$f(x) = g(x)$$

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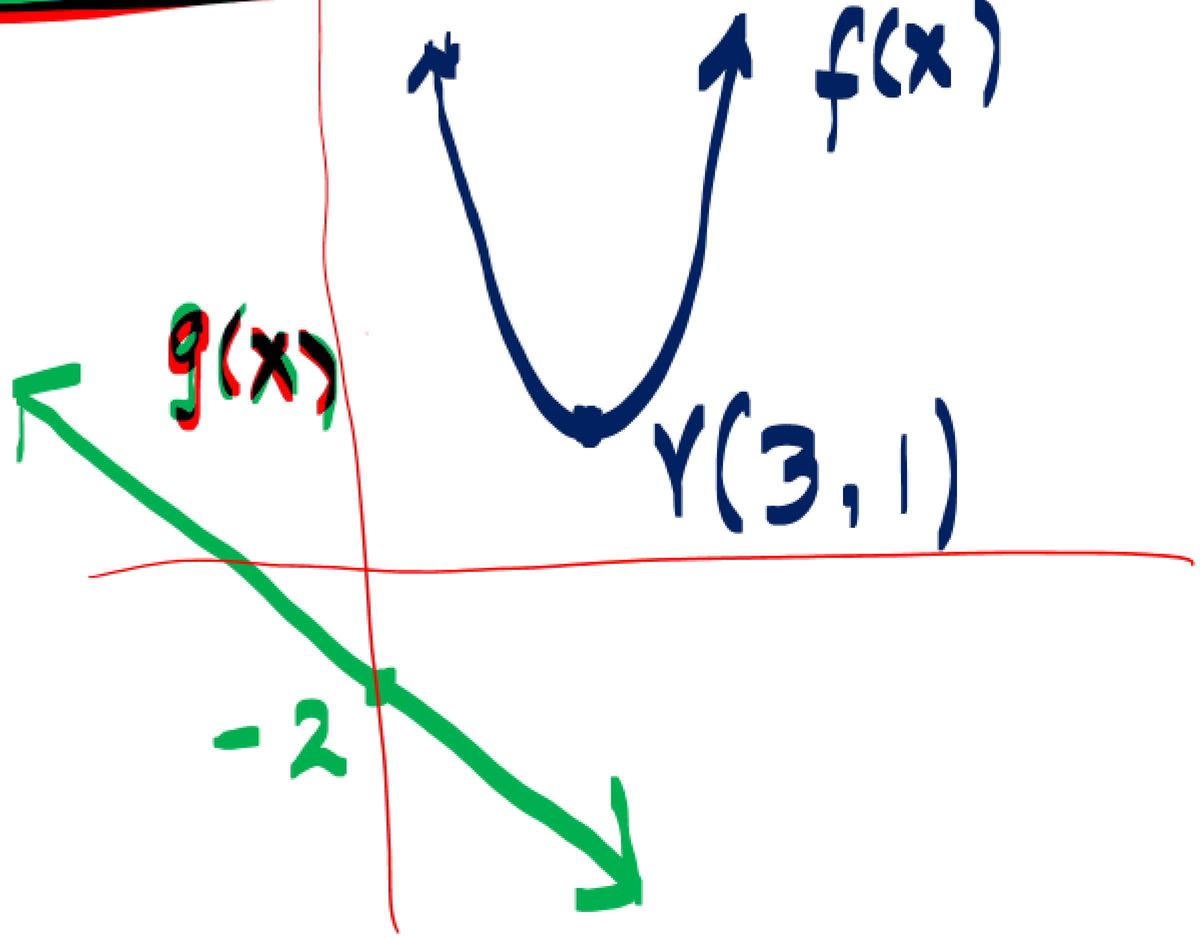

2P198] Determine the point(s) of intersection algebraically.

a) $f(x) = -x^2 + 6x - 5, g(x) = -4x + 19$

b) $f(x) = 2x^2 - 1, g(x) = 3x + 1$

c) $f(x) = 3x^2 - 2x - 1, g(x) = -x - 6$

2a) $f(x) = -x^2 + 6x - 5$

$f(x) = g(x)$

$-x^2 + 6x - 5 = -4x + 19$

$-x^2 + 6x - 5 + 4x - 19 = 0$

$-x^2 + 10x - 24 = 0$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x_1 = 4$; $x_2 = 6$

$g(x) = -4x + 19$

$g(4) = -4(4) + 19$

$= -16 + 19$

$g(4) = 3$ $PI_1 (4,3)$

$g(6) = -4(6) + 19$

$= -24 + 19$

$g(6) = -5$

$PI_2 (6,-5)$

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$g(x) = -4x + 19$

$f(x) = g(x)$

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$-x^2 + 6x - 5 + 4x - 19 = 0$

$-x^2 + 10x - 24 = 0$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x_1 = 4 \; ; \; x_2 = 6$

$g(4) = -4(4) + 19$

$= -16 + 19$

$g(4) = 3 \quad PI_1(4,3)$

$g(6) = -4(6) + 19$

$= -24 + 19$

$g(6) = -5$

$PI_2(6,-5)$

2c) $f(x) = 3x^2 - 2x - 1$; $g(x) = -x - 6$

$$f(x) = g(x)$$

$$3x^2 - 2x - 1 = -x - 6$$

$$3x^2 - x + 5 = 0$$

$$b^2 - 4ac = (-1)^2 - 4(3)(5)$$

$$= 1 - 60$$

$$= -59 < 0$$

NO POINT OF INTERSECTION

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$$f(x) = g(x)$$

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NO POINT OF INTERSECTION

3P198]

Determine the number of points of intersection of $f(x) = 4x^2 + x - 3$ and $g(x) = 5x - 4$ without solving.

$f(x) = g(x)$

$4x^2 + x - 3 = 5x - 4$

$4x^2 - 4x + 1 = 0$

$b^2 - 4ac$

$= (-4)^2 - 4(4)(1)$

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5P198] An integer is two more than another integer. Twice the larger integer is one more than the square of the smaller integer. Find the two integers.

x - FIRST INT.

$x+2$ - 2nd INT.

$2(x+2) = x^2 + 1$

$2x + 4 = x^2 + 1$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x_1 = -1$

$x+2 = 1$

$x_2 = 3$

$x+2 = 5$

SET A: $\{-1, 1\}$

SET B: $\{3, 5\}$

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$$2x + 4 = x^2 + 1$$

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$$(x+1)(x-3) = 0$$

$x_1 = -1$	$x_2 = 3$
$x+2 = 1$	$x+2 = 5$

SET A: $\{-1, 1\}$

SET B: $\{3, 5\}$

6P199] $P(x)$ - PROFIT $f(x)$

The revenue function for a production by a theatre group is $R(t) = -50t^2 + 300t$, where t is the ticket price in dollars. The cost function for the production is $C(t) = 600 - 50t$. Determine the ticket price that will allow the production to break even.

$P(t) = R(t) - C(t)$

$0 = -50t^2 + 300t - (600 - 50t)$

$[0 = -50t^2 + 350t - 600] \div (-50)$

$t^2 - 7t + 12 = 0$

$(t-4)(t-3) = 0$

BREAK EVEN TICKET PRICES = \$3, \$4

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