Stochastic Programming and Applications

Computational Techniques

C. Odhiambo[1]

[1] Institute of Mathematical Sciences
Strathmore University

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Introduction to Stochastic Programming

Stochastic programming is an optimization framework that incorporates uncertainty into decision-making models. Unlike deterministic optimization, which assumes perfect information about all parameters, stochastic programming accounts for randomness in constraints and objectives.

Key areas:

- **Decision Variables:** Represent choices to be optimized.
- **Uncertainty (Random Variables):** Captures variability in parameters.
- **Objective Function:** Typically involves expected value optimization.
- **Constraints:** Incorporate probabilistic constraints or chance constraints.

Role of Uncertainty in Decision-Making Models

Uncertainty plays a critical role in many real-world problems. Decision-makers often face incomplete or noisy data, leading to suboptimal choices if uncertainty is ignored.

Uncertainty

Stochastic programming provides mechanisms to:

- Model uncertainty explicitly.
- Make decisions that are robust to variability.
- Optimize expected performance while mitigating risk.

Applications in Finance, Logistics, and Healthcare

Finance

- Portfolio Optimization: Incorporating uncertainty in stock returns to minimize risk while maximizing expected returns.
- Risk Management: Managing credit and operational risks by optimizing capital allocation under uncertainty.

Applications in Finance, Logistics, and Healthcare

Logistics

- Supply Chain Management: Handling uncertain demand and transportation costs to optimize distribution.
- Inventory Control: Adjusting stock levels dynamically under demand uncertainty.

Applications in Finance, Logistics, and Healthcare

Health care

- Resource Allocation: Assigning limited resources (e.g., ICU beds) under uncertain patient arrivals.
- Disease Screening: Optimizing screening strategies for diseases with uncertain prevalence.

Interpreting Two-Stage Stochastic Optimization

The optimal decision x=7.57 suggests that the best strategy to minimize the expected cost under uncertain demand scenarios (10, 20, and 30 units) is to set $\approx$ 7.57.

- Underproduction Costs: Since $x = 7.57$ is less than all demand scenarios (10, 20, and 30), there will always be a shortfall. It is more cost-effective to underproduce and accept penalties for unmet demand rather than overproduce and incur holding costs.
- Expected Cost Minimization: The model finds the balance where the expected penalty cost (due to demand exceeding supply) is minimized. The decision variable x is positioned optimally given the probabilities of different demand levels.
- Practical Implication: In a real-world setting, this solution implies that producing or stocking around 7.57 units is the best choice, given the stochastic nature of demand and the assumed cost function.

Interpretation of Portfolio Optimization Results

The portfolio weights, Asset 1: 30.61%, Asset 2: 24.02%, Asset 3: 22.51%, Asset 4: 22.86% represent the proportion of total capital that should be invested in each of the four assets to maximize expected returns while controlling risk (as measured by the L2-norm of return fluctuations).

- Diversification: No single asset dominates the portfolio, meaning the optimizer has selected a well-diversified allocation.
- Risk vs. Return Tradeoff: The optimizer considers both expected returns (mean returns of assets) and risk (variance proxy using L2-norm).
- The fact that Asset 1 has the highest allocation (30.61%) implies it likely has a higher expected return compared to the others.
- Assets 2, 3, and 4 have similar allocations, suggesting they have comparable risk-adjusted returns.