

Computational Techniques in Data Science. Assignment 5

Deadline on Wednesday 16th 1159PM

One: Steady-State Probability Calculation

You are given the following 3×3 transition matrix representing the state changes of a system over time:

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$$

Tasks:

1. Represent the matrix in Python using NumPy.
2. Verify if this matrix is a valid stochastic matrix.
3. Compute the steady-state probabilities analytically using Python.

Two: Markov Chain Simulation and Visualization

Using the same transition matrix P in question One, simulate a Markov chain for 50 time steps starting from state 0.

Tasks:

1. Simulate the Markov chain and record the state at each time step.
2. Count how many times each state was visited.
3. Plot the sequence of visited states over time using `matplotlib`.
4. Visualize the frequency distribution of states visited using a bar plot.

Three: Network Visualization of State Transitions

Using the transition matrix matrix P in question One again, create a directed graph of the Markov chain.

Tasks:

1. Use `networkx` to create a directed graph where:
 - Nodes represent the states (0, 1, 2).
 - Edges represent transitions with weights corresponding to transition probabilities.
2. Draw the graph using `matplotlib`.
3. Use edge thickness or labels to indicate the strength of the transition probabilities.