

### 1. Transportation Problem: Optimal Shipping Plan

A logistics company supplies goods from **three warehouses (W1, W2, W3)** to **four retail stores (S1, S2, S3, S4)**. The transportation cost per unit from each warehouse to each store is given in the table below. Each warehouse has a limited supply, and each store has a demand requirement. The goal is to minimize the total transportation cost.

To / From	S1	S2	S3	S4	Supply
W1	4	3	6	5	250
W2	2	5	3	4	300
W3	7	6	4	3	400
Demand	200	200	250	300	

#### Decision Variables:

Let  $x_{ij}$  be the number of units transported from warehouse  $i$  to store  $j$ .

#### Objective Function:

Minimize total transportation cost:

$$\text{Minimize } Z = 4x_{11} + 3x_{12} + 6x_{13} + 5x_{14} + 2x_{21} + 5x_{22} + 3x_{23} + 4x_{24} + 7x_{31} + 6x_{32} + 4x_{33} + 3x_{34}$$

#### Constraints:

##### 1. Supply Constraints:

- $x_{11} + x_{12} + x_{13} + x_{14} \leq 250$  (Warehouse W1)
- $x_{21} + x_{22} + x_{23} + x_{24} \leq 300$  (Warehouse W2)
- $x_{31} + x_{32} + x_{33} + x_{34} \leq 400$  (Warehouse W3)

##### 2. Demand Constraints:

- $x_{11} + x_{21} + x_{31} = 200$  (Store S1)
- $x_{12} + x_{22} + x_{32} = 200$  (Store S2)
- $x_{13} + x_{23} + x_{33} = 250$  (Store S3)
- $x_{14} + x_{24} + x_{34} = 300$  (Store S4)

##### 3. Non-Negativity: $x_{ij} \geq 0$ for all $i, j$

### 2. Manufacturing Problem: Maximizing Profit (Product Mix)

A company produces **two types of products (A and B)** using **two machines (M1 and M2)**. The processing time (in hours per unit) and the profit per unit are given below. The company has a limited number of available hours for each machine. The objective is to maximize profit.

Product	M1 Hours per unit	M2 Hours per unit	Profit per unit (\$)
A	3	2	50
B	5	4	80

- Machine M1 has **600 hours** available.

- **Machine M2** has **500 hours** available.

**Decision Variables:**

Let  $x_1$  be the number of units of product A produced.

Let  $x_2$  be the number of units of product B produced.

**Objective Function:**

Maximize profit:

$$\text{Maximize } Z = 50x_1 + 80x_2$$

**Constraints:**

- 1) **Machine Time Constraints:**
  - a.  $3x_1 + 5x_2 \leq 600$  (M1 capacity)
  - b.  $2x_1 + 4x_2 \leq 500$  (M2 capacity)
- 2) **Non-Negativity:**  $x_1, x_2 \geq 0$ .

**3. Manufacturing Problem: Minimizing Production Cost**

A furniture company manufactures **chairs and tables**. The company has **limited resources of wood and labor** and wants to **minimize the total production cost**.

Product	Wood Required (cubic ft.)	Labor Required (hours)	Cost per unit (\$)
Chair	5	2	30
Table	8	3	50

- **Available wood:** 800 cubic feet.
- **Available labor:** 300 hours.

**Decision Variables:**

Let  $x_1$  be the number of chairs produced.

Let  $x_2$  be the number of tables produced.

**Objective Function:**

Minimize cost:

$$\text{Minimize } Z = 30x_1 + 50x_2$$

**Constraints:**

1. **Resource Constraints:**
  - $5x_1 + 8x_2 \leq 800$  (Wood availability)
  - $2x_1 + 3x_2 \leq 300$  (Labor availability)
2. **Non-Negativity:**  $x_1, x_2 \geq 0$ .