

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be uniformly continuous. For $a \in \mathbb{R}$, define $f_a(x) = f(x - a)$. Prove that $\{f_a : a \in \mathbb{R}\}$ is equicontinuous.
2. Let (S, d) be a metric space. Let $E \subseteq S$ be connected. Prove \overline{E} is connected. Prove or disprove that if \overline{E} is connected, then E is connected.
3. Prove that $[0, 1]$ and $E = \{(x, y) : x^2 + y^2 = 1\}$ are not homeomorphic.
4. Prove or disprove that \mathbb{R} and \mathbb{R}^2 are homeomorphic.
5. Let (S, d) be a metric space and $\{E_k\}_{k \in \mathbb{N}}$ be connected subsets of S such that for every $k \in \mathbb{N}$, $E_k \cap E_{k+1} \neq \emptyset$. Prove that $\bigcup_{k=1}^{\infty} E_k$ is connected.