

1. Let (S, d) be a metric space. Prove that $E \subseteq S$ is nowhere dense if and only every open ball B contains an open ball B' such that $B' \cap E = \emptyset$.
2. Is $\mathbb{R} \setminus \mathbb{Q}$ first category, second category, both, or neither in \mathbb{R} ? Be sure to prove your claims.
3. Let (S, d) be a complete metric space such that $S = \bigcup_{n=1}^{\infty} F_n$ with $F_n \subseteq S$ are closed. Prove that there exists an open ball B such that $B \subseteq F_n$ for some n .
4. Prove that an infinite dimensional Banach space cannot have countably infinite algebraic dimension.
5. Let $f_k : \mathbb{R} \rightarrow \mathbb{R}$ be continuous nonnegative functions such that $\sum_{k=1}^{\infty} f_k(t)$ converges for every $t \in \mathbb{R}$. Prove that there exists an interval $I \subseteq \mathbb{R}$ where the series converges uniformly.