Chapter 21: Electric fields

Section 21-1: Electric Charge

- Electricity is caused by electric charges
- Can be positive or negative
- Like charges repel, opposites attract
- Charge is conserved: the total charge is always the same
- Unit of charge: Coulomb (C)

Section 21-2: Electric Charge in the Atom

- Electric charge is found on subatomic particles
- It is a fundamental, unchangeable property of certain particles
- Protons are positive, electrons negative, neutrons have zero charge
- "Electron charge" is $e = 1.6 \times 10^{-19}$ C

    e is always positive, so technically the proton charge

Section 21-3: Insulators and conductors

- Conductors allow electrons to move through them
- Insulators do not; the electrons are stuck to their atom

Section 21-5: Coulomb's Law

- Force between two charges is:

Magnitude: $\boxed{F = |\vec{F}| = \frac{k|Q_1 Q_2|}{r^2}}$

$k = 8.99 \times 10^9$ Nm^2/C^2, *r* is distance between centers

Direction: opposites attract, like repel, always draw a diagram!

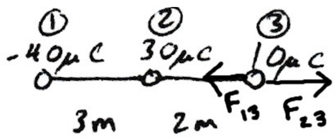
- Permittivity of free space:

$$k = \frac{1}{4\pi\epsilon_0}, \text{ using value for } k \text{ gives } \epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$$

Different way of expressing the same constant of nature

Both are used so be familiar with both!

Example: A 30μC charge has a 10μC charge 2m to its right and a -40μC charge 3m to its left. What is the force on the 10μC charge?



From -40μC: $F_{13} = \frac{kQ_1Q_3}{r_{13}^2} = \frac{8.99\times10^9\,\mathrm{Nm^2/C^2}\cdot 40\times10^{-6}\,\mathrm{C}\cdot 10\times10^{-6}\,\mathrm{C}}{(5\,\mathrm{m})^2}$

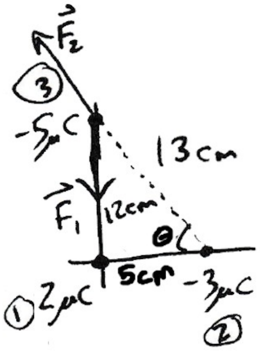
$= 0.144\,\mathrm{N}$ Direction: left (attractive)

From 30μC: $F = \frac{kQ_2Q_3}{r_{23}^2} = \frac{8.99\times10^9\,\mathrm{Nm^2/C^2}\cdot 30\times10^{-6}\,\mathrm{C}\cdot 10\times10^{-6}\,\mathrm{C}}{(2\,\mathrm{m})^2}$

$= 0.674\,\mathrm{N}$ Direction: right (repulsive)

$F_{net} = -0.144\,\mathrm{N} + 0.674\,\mathrm{N} = \boxed{0.530\,\mathrm{N}}$ (right)

Example: A 2μC charge sits at the origin, -3μC sits at (5cm, 0), and -5μC sits at (0, 12cm). Find the magnitude and direction of the force on the -5μC charge?

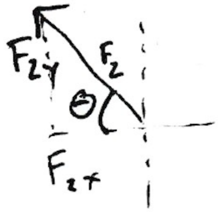


$F_1 = \frac{kQ_1Q_3}{r_{13}^2} = \frac{8.99\times10^9\,\mathrm{Nm^2/C^2}\cdot 2\times10^{-6}\,\mathrm{C}\cdot 5\times10^{-6}\,\mathrm{C}}{(0.12\,\mathrm{m})^2} = 6.24\,\mathrm{N}$

$F_{1x} = 0 \quad F_{1y} = -6.24\,\mathrm{N}$ (straight down)

$F_2 = \frac{kQ_2Q_3}{r_{23}^2} = \frac{8.99\times10^9\,\mathrm{Nm^2/C^2}\cdot 3\times10^{-6}\,\mathrm{C}\cdot 5\times10^{-6}\,\mathrm{C}}{(0.13\,\mathrm{m})^2} = 7.98\,\mathrm{N}$

Same angle θ in each diagram



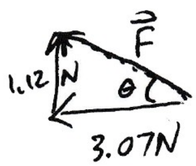
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5\text{ cm}}{13\text{ cm}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{12\text{ cm}}{13\text{ cm}}$$

$$F_{2x} = -7.98\text{ N}\cos\theta = -7.98\text{ N}\cdot\frac{5}{13} = -3.07\text{ N (left)}$$

$$F_{2y} = 7.98\text{ N}\sin\theta = 7.98\text{ N}\cdot\frac{12}{13} = 7.37\text{ N}$$

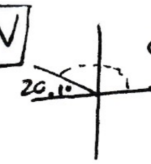
$$F_x = 0 - 3.07\text{ N} = -3.07\text{ N}$$

$$F_y = -6.24\text{ N} + 7.37\text{ N} = 1.12\text{ N}$$



$$F = \sqrt{(1.12\text{N})^2 + (3.07\text{N})^2} = \boxed{3.27\text{ N}}$$

$$\theta = \tan^{-1}\frac{1.12\text{N}}{3.07\text{N}} = 20.1^\circ$$



ccw: $180 - 20.1^\circ = \boxed{160^\circ}$

<u>Section 21-6: Electric Field</u>

- Kinds of forces:

    Contact force: pushing/pulling by touching, friction, tension

    Action at a distance: gravity, magnets…and now electric force

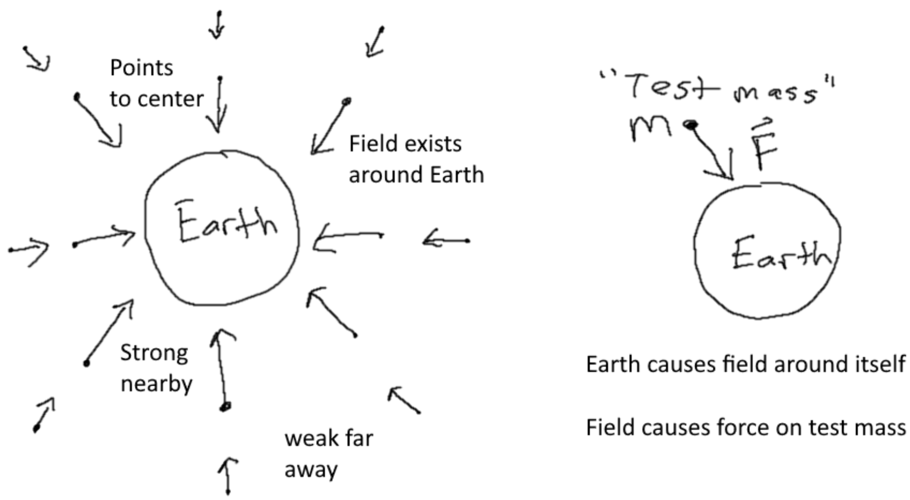
- Action at a distance forces are conveyed by <u>fields</u>

    A field is created by some object

    The field exists in the space around the object

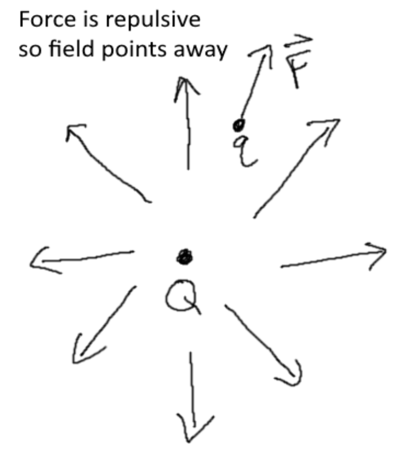
    There is a value of the field at each point in space; if asked to "Find what the field is" you must ask "<u>Where?</u>"

    If another object is placed in the field, it feels a force that depends on the field value at its location

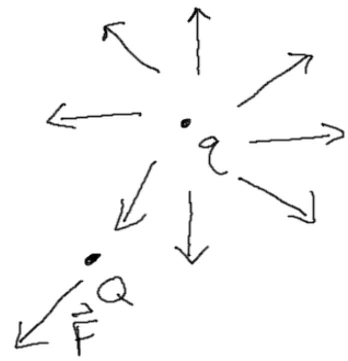


- Gravitational force is proportional to mass
- Field strength is defined as force over mass, $g = \dfrac{F}{m}$.

  Just the gravitational acceleration
- Newtonian gravity: $F = \dfrac{GMm}{r^2}$, so $g = \dfrac{GM}{r^2}$
  - m <u>feels</u> the field, M <u>makes</u> the field
- Direction is towards center, $\vec{g} = -\dfrac{GM}{r^2}\hat{r}$
- Electric force is proportional to charge, so strength is $E = \dfrac{F}{q}$
  - q is the charge which <u>feels</u> the field, not the one who <u>made</u> it
  - Solving for F: $\boxed{\vec{F} = q\vec{E}}$
- Coulomb: $F = \dfrac{kQq}{r^2}$ so $\boxed{E = \dfrac{k|Q|}{r^2}}$ (mag. only, so abs. value)
  - Strength depends on Q because it <u>made</u> the field



Q makes the field and q feels the field



q makes a field and Q feels the field
Newton's 3rd, action/reacion

- $\vec{E}$ points <u>away</u> from positive charges and <u>towards</u> negative ones

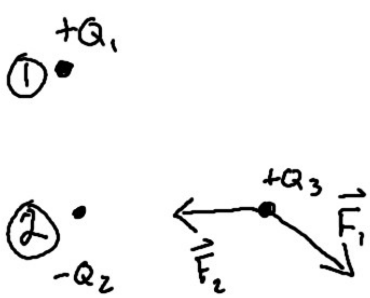
  $E = \dfrac{k|Q|}{r^2}$ is <u>magnitude only</u>!

- Diagrams: Force versus Electric Field

- Force: forces act on particles

  (Vectors start on particles)

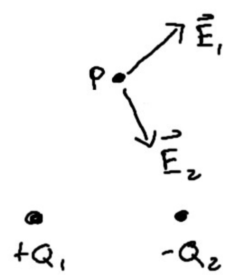
  Find force on particle 3:



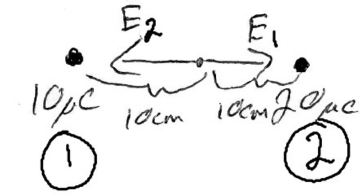
- Electric Field at points in space

  (Vectors start at location of $\vec{E}$)

  Find electric field at P:



Example: A $10\mu C$ charge is 20 cm from a $20\mu C$ charge. What is the magnitude of $\vec{E}$ halfway between?



From $10\mu C$: $E_1 = \dfrac{8.99\times10^9 \frac{Nm^2}{C^2}\cdot 10\mu C}{(0.1\,m)^2}$ ← Not 0.2m!

$= 8.99\times10^6\ N/C$

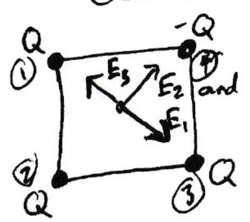
From $20\mu C$: $E_2 = \dfrac{8.99\times10^9 \frac{Nm^2}{C^2}\cdot 20\mu C}{(0.1\,m)^2}$

$= 1.80\times10^7\ N/C$

Opp. dir

$E_{net} = E_1 - E_2 = 1.80\times10^7 N/C - 8.99\times10^6 N/C = \boxed{8.99\times10^6 N/C}$

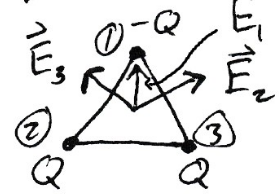
Example: Find the electric field direction at the center of each figure below.



$\vec{E}_1$ cancels $\vec{E}_3$

$\vec{E}_2 = \vec{E}_4$

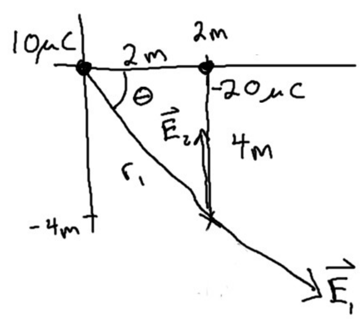
up-and-right (45°)



E_{2x} opposite of E_{3x}

$\vec{E}_2 + \vec{E}_3$ up, $\vec{E}_1$ also up

Example: A charge $Q_1 = 10\mu C$ sits at the origin and a charge $Q_2 = -20\mu C$ sits at $x = 2$ m on the x-axis. What is the electric field at (2m, −4m)?



$E_1 = \dfrac{kQ_1}{r_1^2}$

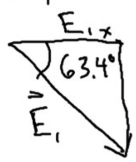
$r_1^2 = (2m)^2 + (4m)^2 = 20\ m^2$ (don't need r_1 itself!)

$E_1 = \dfrac{8.99\times10^9 \frac{Nm^2}{C^2}\cdot 10\times10^{-6}C}{20\ m^2} = 4500 N/C$

$\tan\theta = \frac{opp}{adj} = \frac{4m}{2m} = 2$

$\theta = \tan^{-1} 2 = 63.4°$

$\vec{E}_1$ is at the same angle as the hypoteneuse!



$E_{1x} = E_1 \cos\theta = 4500\frac{N}{C}\cos 63.4° = 2010\frac{N}{C}$

$E_{1y} = -E_1 \sin\theta = -4500\frac{N}{C}\sin 63.4° = -4020\frac{N}{C}$

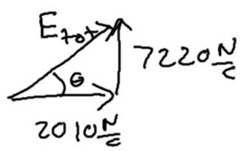
down is negative

$$E_2 = \frac{kQ_2}{r_2^2} = \frac{8.99\times10^9\frac{Nm^2}{C^2}\cdot 20\times10^{-6}C}{(4m)^2} = 11200\frac{N}{C}$$

$\vec{E}_2$ is straight up: $E_{2x} = 0$, $E_{2y} = 11200\frac{N}{C}$

$E_{tot,x} = E_{1x} + E_{2x} = 2010\frac{N}{C} + 0 = 2010\frac{N}{C}$

$E_{tot,y} = E_{1y} + E_{2y} = -4020\frac{N}{C} + 11200\frac{N}{C} = 7220\frac{N}{C}$



$$E_{tot} = \sqrt{\left(2010\frac{N}{C}\right)^2 + \left(7220\frac{N}{C}\right)^2} = \boxed{7490\frac{N}{C}}$$

$$\tan\theta = \frac{7220\frac{N}{C}}{2010\frac{N}{C}} = 3.59$$

$$\theta = \tan^{-1} 3.59 = \boxed{74.4°}$$

Homework: Do Chapter 21, Part 1 in Mastering Physics

<u>Section 21-7: Continuous Distributions</u>

- Charges can be spread out over 1D, 2D, or 3D object

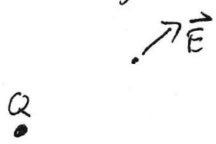
1D: $\lambda = Q/\ell$ or $\boxed{Q = \lambda\ell}$

2D: $\sigma = Q/A$ or $\boxed{Q = \sigma A}$

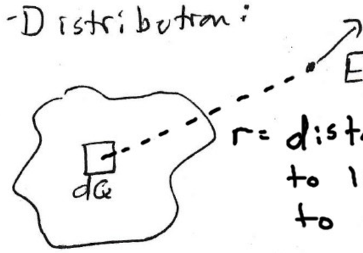
3D: $\rho = Q/V$ or $\boxed{Q = \rho V}$

- Find electric field by dividing object into “points” (infinitesimals)

- Point charge:



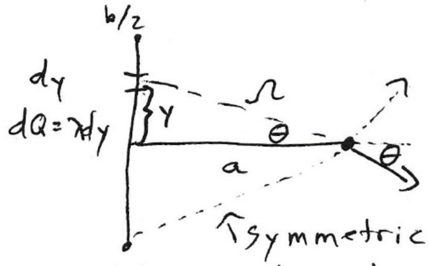
- Distribution:



r = distance from source to location we want to find $\vec{E}$ at.

- Add (integrate) contributions from all of the *dQ*

<u>Example</u>: Find the electric field a distance a from the center of a thin rod of length b and charge per unit length λ. Use ε_0 instead of k.



↑ symmetric point on lower half cancels E_y

$$r = \sqrt{a^2 + y^2}$$

$$dE = \frac{k\lambda dy}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dy}{a^2+y^2}$$

$$dE_x = dE\cos\theta = dE \cdot \frac{a}{r}$$

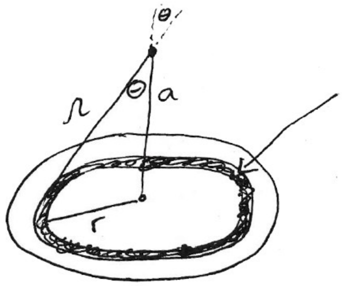
$$= \frac{1}{4\pi\varepsilon_0}\frac{a\lambda dy}{(a^2+y^2)^{3/2}}$$

$$E_x = \int_{y=-\frac{b}{2}}^{y=b/2} dE_x = \frac{a\lambda}{4\pi\varepsilon_0}\int_{-b/2}^{b/2}\frac{dy}{(a^2+y^2)^{3/2}}$$

This integral is very difficult, so here is the answer:

$$\boxed{E_x = \frac{\lambda b}{4\pi\varepsilon_0 a\sqrt{a^2+b^2/4}}}$$

<u>Example</u>: Find the magnitude of the electric field a distance a above the center of a disk of radius R and charge per unit area σ. Use ε_0 instead of k.



Ring of radius r and width dr. Every point is a distance $\mathscr{r} = \sqrt{a^2+r^2}$ from the point we want. By symmetry, horizontal $\vec{E}$ comp's cancel and only vertical remains.

$dQ = \sigma \cdot$ area

stretch ring into a strip: Length $2\pi r$, width dr

area $= 2\pi r\,dr$ so $dQ = 2\pi\sigma r\,dr$

$$dE = \frac{k\,dQ}{\mathscr{r}^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi\sigma\,r\,dr}{a^2+r^2} = \frac{\sigma}{2\varepsilon_0}\,\frac{r\,dr}{a^2+r^2}$$

$$dE_z = dE\cos\theta = dE\cdot\frac{a}{\mathscr{r}} = \frac{\sigma a}{2\varepsilon_0}\,\frac{r\,dr}{(a^2+r^2)^{3/2}}$$

$$E = \int_{r=0}^{r=R} dE = \frac{a\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(a^2+r^2)^{3/2}}$$

Let $u = r^2$ or $du = 2r\,dr$ or $r\,dr = \frac{1}{2}du$

$$= \frac{a\sigma}{2\varepsilon_0}\int_{r=0}^{r=R}\frac{\frac{1}{2}du}{(a^2+u)^{3/2}} = \frac{a\sigma}{4\varepsilon_0}\cdot -2\,(a^2+u)^{-1/2}\Big|_{r=0}^{r=R}$$

Keep track of which variable limits represent!

$$= -\frac{a\sigma}{2\varepsilon_0}\,\frac{1}{\sqrt{a^2+r^2}}\Big|_{r=0}^{R}$$

$$= -\frac{a\sigma}{2\varepsilon_0}\left(\frac{1}{\sqrt{a^2+R^2}} - \frac{1}{a}\right)$$

$$= \boxed{\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{a}{\sqrt{a^2+R^2}}\right)}$$

Infinite plane:

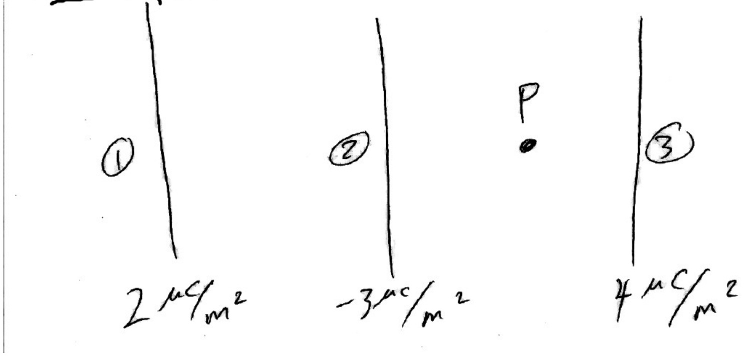
- We can find the electric field of an infinite plane by making R big:

$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{a}{\infty}\right)$, so:

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Direction: away from pos., towards neg.

Example: Find $\vec{E}$ at P



$$E_1 = \frac{\sigma_1}{2\varepsilon_0} = \frac{2\ \mu C/m^2}{2 \cdot 8.85 \times 10^{-12}\ C^2/Nm^2} = 1.13 \times 10^5\ N/C$$

$$E_2 = \frac{\sigma_2}{2\varepsilon_0} = \frac{3\ \mu C/m^2}{2 \cdot 8.85 \times 10^{-12}\ C^2/Nm^2} = 1.69 \times 10^5\ N/C$$

$$E_3 = \frac{4\ \mu C/m^2}{2 \cdot 8.85 \times 10^{-12}\ C^2/Nm^2} = 2.26 \times 10^5\ N/C$$

$$E_{net} = E_1 - E_2 - E_3 = -2.82 \times 10^5\ N/C$$

left

Section 21-8: Electric Field Lines

- Field lines are a way to graphically represent a field
- They are not literal: fields are not composed of lines
- Rules:

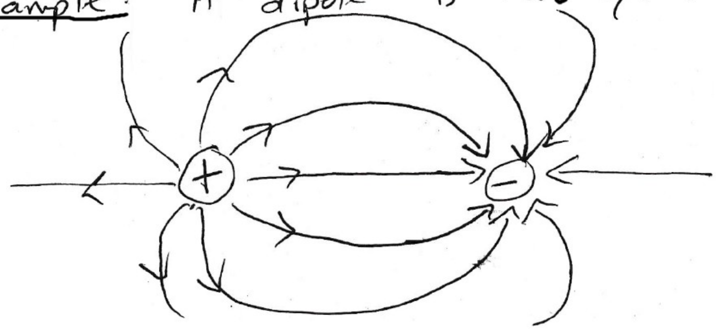
    Start/stop on charges or infinity (if charges not balanced)

    Point in direction of $\vec{E}$ (from pos. to neg.)

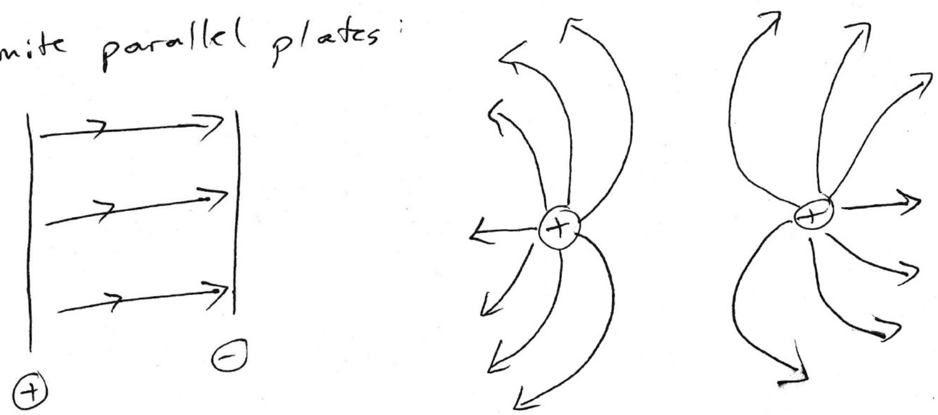
    Density of lines proportional to field strength

    Lines cannot cross (so must avoid each other)

Example: A "dipole" is two equal but opposite charges



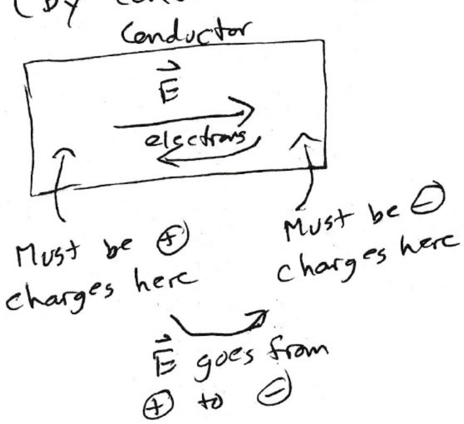
Infinite parallel plates:



Section 21-9: Electric Fields and Conductors

- Fact #1: Electric fields cannot exist inside a conductor (for long)
- Proof by contradiction – suppose there was:

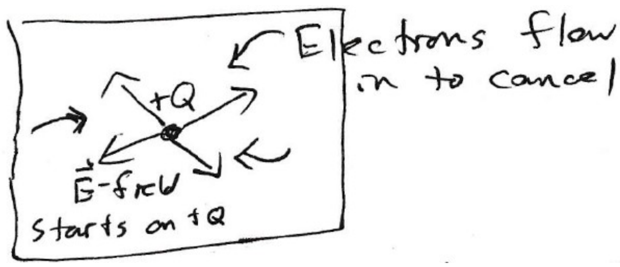
Proof (by contradiction): Suppose there was.



Electrons will flow contrary to $\vec{E}$-field, quickly cancelling it.

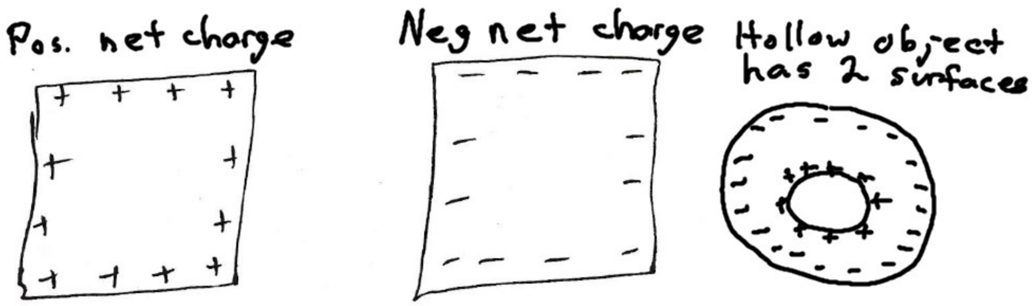
- Fact #2: No charge can exist (for long) in the interior

- Proof by contradiction:



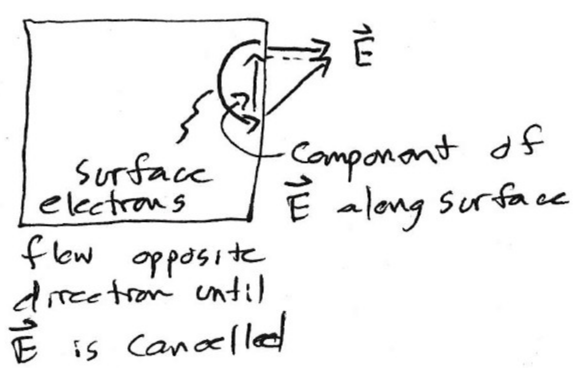
- Note: charges can exist on the surface

  Net charge quickly spread over surface of conductor



- Fact #3: The electric field is perpendicular to surface just outside

- Proof by contradiction:

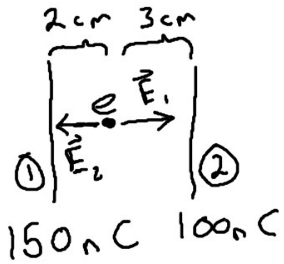


Section 21-10: Motion of Charges in an Electric Field

- $\vec{E} = \dfrac{\vec{F}}{q}$ so $\boxed{\vec{F} = q\vec{E}}$

- Then $\vec{F} = m\vec{a}$, and use equations with $\vec{r}$, $\vec{v}$, and $\vec{a}$

Example: An electron at rest is positioned between two plates, each with area $200 cm^2$, as shown below. How long does it take to hit one of the plates?



$$A = 200 cm^2 \times \left(\frac{1 m}{100 cm}\right)^2 = 0.02 m^2$$

$$\sigma_1 = \frac{Q_1}{A} = \frac{150 \times 10^{-9} C}{0.02 m^2} = 7.5 \times 10^{-6} C/m^2$$

$$\sigma_2 = \frac{Q_2}{A} = \frac{100 \times 10^{-9} C}{0.02 m^2} = 5 \times 10^{-6} C/m^2$$

$$E_1 = \frac{\sigma_1}{2\varepsilon_0} = \frac{7.5 \times 10^{-6} C/m^2}{2 \cdot 8.85 \times 10^{-12} C^2/Nm^2} = 424000 N/C \text{ (right)}$$

$$E_2 = \frac{\sigma_2}{2\varepsilon_0} = \frac{5 \times 10^{-6} C/m^2}{2 \cdot 8.85 \times 10^{-12} C^2/Nm^2} = 282000 N/C \text{ (left)}$$

$$E = E_1 - E_2 = 424000 \tfrac{N}{C} - 282000 \tfrac{N}{C} = 141000 \tfrac{N}{C}$$

$$F = qE = -1.6 \times 10^{-19} C \cdot 141000 N/C = -2.26 \times 10^{-14} N$$

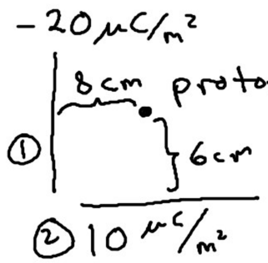
$$a = \frac{F}{m} = \frac{-2.26 \times 10^{-14} N}{9.11 \times 10^{-31} kg} = -2.48 \times 10^{16} m/s^2$$

Negative means left; 2 cm to left plate, so $x = -0.02 m$

$$x = \overset{\text{rest}}{\cancel{v_0 t}} + \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2 \cdot (-0.02 m)}{-2.48 \times 10^{16} m/s^2}} = \boxed{1.27 \times 10^{-9} s}$$

Example: A proton (mass 1.67×10^{-27} kg) with initial velocity $\vec{v}_0 = 8 \times 10^5 m/s\ \hat{i} - 6 \times 10^5 m/s\ \hat{j}$ sits beside two plane charges as shown. What are the x and y components of the velocity 5 ns later?



Note: $\mu C/m^2$ is charge per area σ, not Q!

Also: distance from a plate does not matter!

$$E_1 = \frac{\sigma_1}{2\varepsilon_0} = \frac{20\times10^{-6}\,C/m^2}{2\cdot 8.85\times10^{-12}\,C^2/Nm^2} = 1.13\times10^{6}\,\frac{N}{C}\ (\text{left})$$

$$E_2 = \frac{\sigma_2}{2\varepsilon_0} = \frac{10\times10^{-6}\,C/m^2}{2\cdot 8.85\times10^{-12}\,C^2/Nm^2} = 5.65\times10^{5}\,\frac{N}{C}\ (\text{up})$$

$$F_x = qE_x = \overset{\text{proton } q}{1.6\times10^{-19}}\,C\cdot\left(\overset{\text{left}}{-}1.13\times10^{6}\,\frac{N}{C}\right) = -1.81\times10^{-13}\,N$$

$$a_x = \frac{F_x}{m} = \frac{-1.81\times10^{-13}\,N}{1.67\times10^{-27}\,kg} = -1.08\times10^{14}\,m/s^2$$

$$v_x = v_{0x} + a_x t = 8\times10^{5}\,m/s - 1.08\times10^{14}\,m/s^2\cdot 5\times10^{-9}\,s = \boxed{2.59\times10^{5}\,m/s}$$

$$F_y = qE_y = 1.6\times10^{-19}\,C\cdot 5.65\times10^{5}\,\frac{N}{C} = 9.04\times10^{-14}\,N$$

$$a_y = \frac{F_y}{m} = \frac{9.04\times10^{-14}\,N}{1.67\times10^{-27}\,kg} = 5.41\times10^{13}\,m/s^2$$

$$v_y = v_{0y} + a_y t = -6\times10^{5}\,m/s + 5.41\times10^{13}\,m/s^2\cdot 5\times10^{-9}\,s = \boxed{-3.29\times10^{5}\,m/s}$$

Section 21-11: Electric Dipoles

- Dipole: Two equal but opposite charges a fixed distance apart

  Could be as simple as two charges attached to wood stick

  More useful example: water molecule

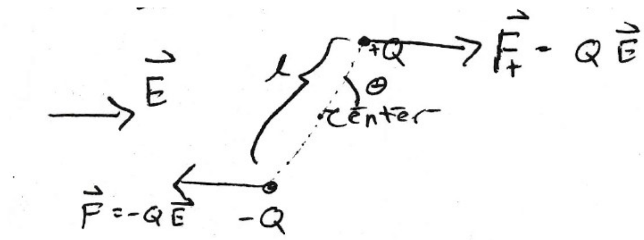
- Dipole moment: $\boxed{\vec{p} = Q\vec{\ell}}$

  Q is charge of just positive charge; don't double it!

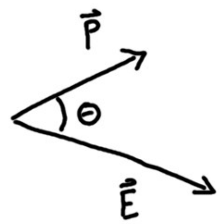
  $\vec{\ell}$ points from negative to positive charge

Dipole in an electric field:

- Net charge is zero, so net force is zero
- But there is a torque:



- After lengthy derivation, torque is $\vec{\tau} = \vec{p} \times \vec{E}$
- Magnitude of torque is $\boxed{\tau = pE\sin\theta}$
- θ is angle between $\vec{p}$ and $\vec{E}$



- Direction of cross products will be discussed in Chapter 27

- Work is $W = \int \tau d\theta = \int pE\sin\theta d\theta = -pE\cos\theta$

  So $W = -\vec{p} \cdot \vec{E}$

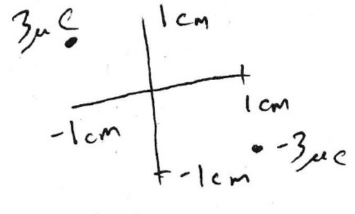
  Work done to dipole equals the potential energy of dipole:

  $\boxed{U = -\vec{p} \cdot \vec{E}}$ or $\boxed{U = -pE\cos\theta}$
- Lowest energy state (equilibrium) has energy $-pE$ <u>not</u> 0
- Equilibrium state is when $\vec{p}$ aligns with $\vec{E}$

  Highest energy is antialigned

Example: A $3\,\mu C$ charge sits at (-1cm, 1cm, 0cm) and a $-3\,\mu C$ charge sits at (1cm, -1cm, 0cm). There is an electric field $-500{,}000\ N/C\ \hat{j}$.

a) What is the dipole moment?



$Q = 3\,\mu C$

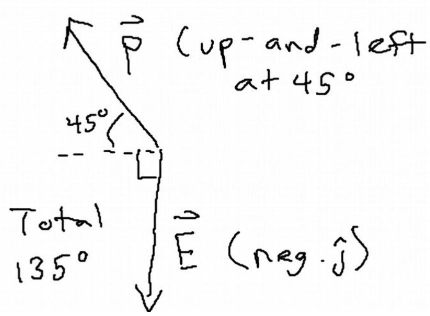
$|\vec{\ell}| = \sqrt{(2cm)^2 + (2cm)^2} = 2\sqrt{2}\ cm$

Direction: 135° counterclockwise from x

$\vec{p} = Q\ell = 6\sqrt{2}\ \mu C \cdot cm$

$= \boxed{8.49 \times 10^{-8}\ Cm \text{ at } 135°}$

(b) What is the magnitude of the torque?



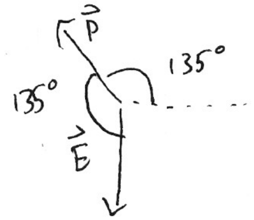
$\tau = pE\sin\theta$

Mag of E (no-sign)

$= 8.49 \times 10^{-8}\ Cm \cdot 500000\frac{N}{C} \sin 135°$

$= \boxed{0.03\ Nm}$

(c) What is the change in potential energy if the dipole is allowed to rotate to equilibrium?



$U_0 = -\vec{p}\cdot\vec{E} = -8.49 \times 10^{-8}\ Cm \cdot 500{,}000\frac{N}{C} \cos 135°$

$= 0.03\ J$

Equilibrium: $\vec{p}$ parallel to $\vec{E}$ minimizes U

$U = -pE\cos 0° = -pE$

$= -8.49 \times 10^{-8}\ Cm \cdot 500000\frac{N}{C} = -0.0424\ J$

$\Delta U = U - U_0 = -0.0424J - 0.03J = \boxed{-0.0724\ J}$

Homework: Do Chapter 21, Part 2 in Mastering Physics