Name:______________________________

MA 4633/6633 Section 01 *Homework Set 6* *Due: 10/18/2024*

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1. Let $f : (a, b) \to \mathbb{R}$ be differentiable on (a, b) and $x_0 \in (a, b)$, compute(with proof) the following limit:

$$\lim_{h\to 0} \frac{f(x_0 + h) - f(x_0 - h)}{2h} .$$

Prove or disprove the converse, that is: If the above limit exists for each $x_0 \in (a, b)$, then f must be differentiable on (a, b).

2. Properly define the inverse sine, inverse cosine and inverse tangent function and compute their respective derivatives. This requires justification, don't just calculate without proof.

3. Let f and g be n-times differentiable. Prove the general product rule: that is:

$$(fg)^{(n)}(x) = \sum_{j=0}^{n} \binom{n}{j} f^{(j)}(x)g^{(n-j)}(x) .$$

4. Let $f : (a, b) \to \mathbb{R}$ be differentiable on (a, b) and suppose there exists an $M > 0$ such that $|f'(x)| \leq M$ for all $x \in (a, b)$. Prove that f is uniformly continuous on (a, b). Moreover prove that $\lim_{x\to a^+} f(x)$ and $\lim_{x\to b^-} f(x)$ both exist.

5. Let $f : (0, 1) \to \mathbb{R}$ be differentiable such that $|f'(x)| \leq 1$ for all $x \in (0, 1)$. Define a sequence $\{a_n\}$ by $a_n = f(1/n)$. Prove that $\{a_n\}$ converges.