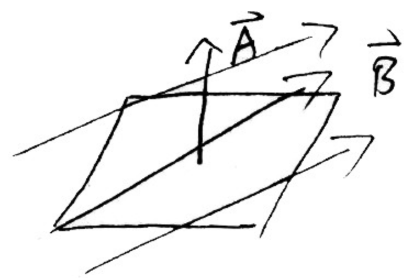
Chapter 29: Induction and Faraday's Law

Section 29-1: Induced EMF

- Moving charges cause magnetic fields
- There's a symmetry: moving magnets cause electric field
- Induced emf: E-field (or potential) caused by a magnetic field

Section 29-2: Faraday's Law

- Magnetic flux is just like electric flux:



$$\boxed{\Phi = \vec{B} \cdot \vec{A}} \text{ or } \Phi = \int \vec{B} \cdot d\vec{A}$$

Unit: Weber

Flux (no adjective) generally means magnetic flux

- Faraday's Law:

Loop of wire: $\mathcal{E} = -\dfrac{d\Phi}{dt}$

Multiple loops: $\boxed{\mathcal{E} = -N\dfrac{d\Phi}{dt}}$

- Lenz's Law

Point thumb in direction of $\vec{B}$ which cause the Φ

Suppose current in loop due to emf

$\mathcal{E} > 0$: current flows in direction of finger curl

$\mathcal{E} < 0$: current flows opposite to fingers

Alternative: current flows in a direction which will create a magnetic field that resists the change that caused it

Example: A magnet falls through a loop North pole first. In what direction does the current flow (a) as it approaches, (b) as it leaves?

(a)

B grows in magnitude as magnet approaches

$\Phi = \vec{B} \cdot \vec{A}$ grows

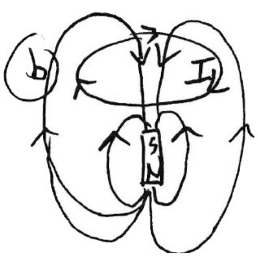
$\frac{d\Phi}{dt} > 0$

$\mathcal{E} = -\frac{d\Phi}{dt} < 0$

$\vec{B}$ is down $\Rightarrow$ c.w. is positive direction by RHR

I is $\boxed{\text{C.C.W.}}$ because $\mathcal{E}$ is negative

(b)



$\vec{B}$ decreases in magnitude

$\Phi = \vec{B} \cdot \vec{A}$ shrinks

$\frac{d\Phi}{dt} < 0$

$\mathcal{E} = -\frac{d\Phi}{dt} > 0$

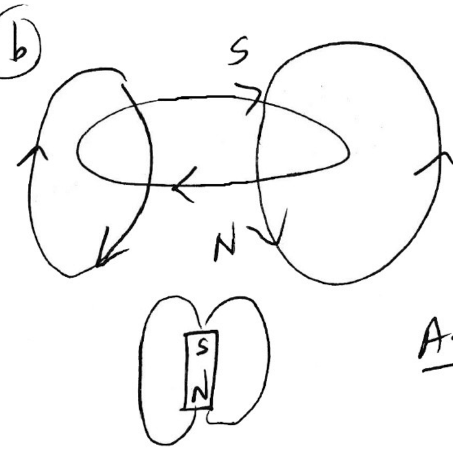
$\vec{B}$ is down $\Rightarrow I$ is c.w. by RHR

Example, continued: In which direction does the $\vec{B}$ field of the induced current point in (a) and (b) above

(a)

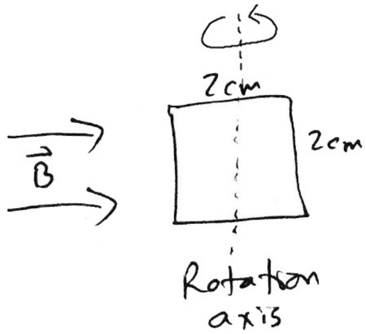
Repel: Slow Fall

(b)



Attract: Also slows fall

Example: A square loop 2 cm on a side is in a magnetic field $B = 0.02T$ in the plane of the loop. The loop rotates $30°$ around an axis perpendicular to $\vec{B}$ in 0.01 s. What is the magnitude of the average emf?



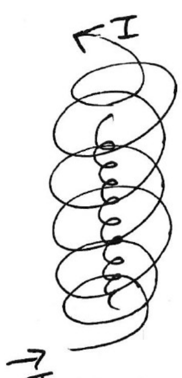
$\Theta_0 = 90°$ $\quad \Phi_0 = AB\cos 90° = 0$

$\Theta = 90° - 30° = 60°$ $\quad \Phi = AB\cos 60° = \frac{1}{2}AB$

Magnitude only (no - sign)

$$\bar{\mathcal{E}} = \frac{\Delta\Phi}{\Delta t} = \frac{\frac{1}{2}AB}{\Delta t} = \frac{1}{2}\cdot\frac{(0.02m)^2\cdot 0.02T}{0.01s} = \boxed{4\times10^{-4}V}$$

Example: A solenoid 1 m long with 1000 turns and $r = 5cm$ has a current which steadily rises from 0 to 5 A over 2s. A solenoid with $r = 4cm$, length 20 cm, $R = 10\Omega$ and 200 turns is inside the larger solenoid. What is (a) the magnitude of current in the small solenoid and (b) the direction of its $\vec{B}$ field relative to the $\vec{B}$ of the larger one?



- Let I be ccw in larger
- $\vec{B}_{large}$ is up
- $B_{large} = \mu_0 n_{large} I$
- $\Phi_{small} = B_{large} A_{small}$ (Let $\vec{A}_{small}$ point up)

$$= \mu_0 \cdot \frac{N_{large}}{l_{large}} \cdot I A_{small}$$

$$\frac{d\Phi}{dt} = \mu_0 \cdot \frac{N_{large}}{\ell_{large}} \cdot A_{small} \frac{\Delta I}{\Delta t}$$

$$= 4\pi\times10^{-7}\,\frac{Tm}{A} \cdot \frac{1000}{1\,m} \cdot \pi\,(0.04m)^2 \cdot \frac{5A}{2s}$$

$$= 1.58\times10^{-5}\ V$$

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -200\cdot1.58\times10^{-5}\,V = -3.16\times10^{-3}\,V$$

$$I = \frac{\mathcal{E}}{R} = -\frac{3.16\times10^{-3}V}{10\,\Omega} = -\boxed{3.16\times10^{-4}\,A}$$

current is c.w. by RHR

(b) Clockwise current means $\vec{B}$ points down

This is $\boxed{\text{opposite}}$ to $\vec{B}$ which created it

Section 29-3: Motional EMF

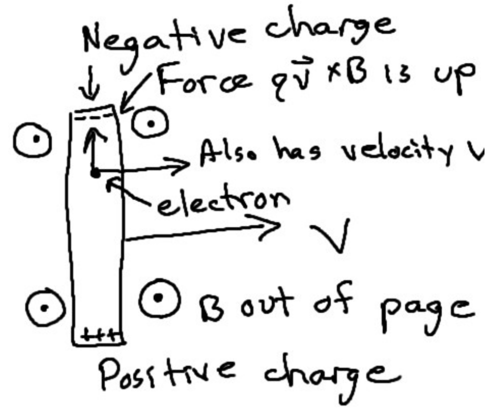
- Lorentz force in a moving conductor:

  Electrons in conductor move with it

  Therefore Lorentz force on electrons

  Electrons move to one end

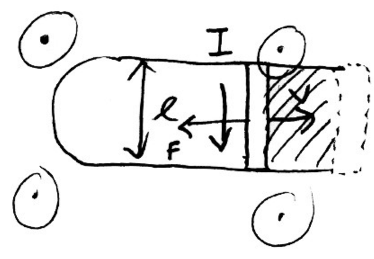
  Voltage difference between two ends



[*May skip above discussion on Lorentz force if time is short.*]

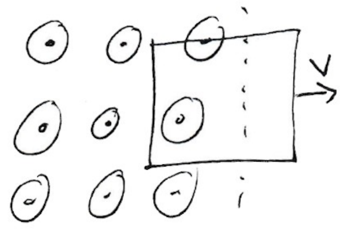
$dA = \ell \cdot vdt$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt} = -B\cdot\frac{\ell vdt}{dt}$$



$\boxed{\mathcal{E} = B\ell v}$ (magnitude only; ignore sign)

Example: A 10 cm square loop with 150 turns is pulled out of a region where B is 0.004 T out of the page at a constant speed of 0.2 m/s. What is (a) the emf, (b) the current if the resistance is $4\ \Omega$, (c) the electric power, and (d) the power of the person pulling it?



(a) Left side feels motional emf

$$\mathcal{E} = B\ell v = 0.004\,T \cdot 0.1\,m \cdot 0.2\,m/s$$
$$= 8 \times 10^{-5}\ V$$

Each turn experiences same emf

$$\mathcal{E}_{total} = 150 \cdot 8 \times 10^{-5}\ V = \boxed{0.012V}$$

(b) $I = \frac{\mathcal{E}}{R} = \frac{0.012\ V}{4\ \Omega} = \boxed{3 \times 10^{-3}\ A}$

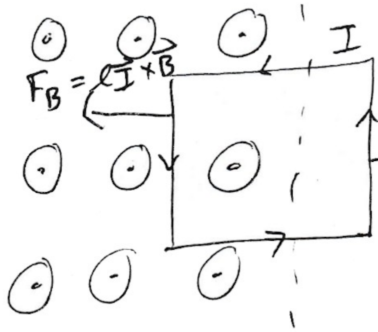
(c) $P = VI = 0.012\ V \cdot 3 \times 10^{-3}\ A = \boxed{3.6 \times 10^{-5}\ W}$

(d) $F_B = \ell I B$ (one wire)

$$= 0.1m \cdot 3 \times 10^{-3} A \cdot 0.004 T$$
$$= 1.2 \times 10^{-6}\ N$$

$$F_{B,\ total} = 150 \cdot F_B$$
$$= 1.8 \times 10^{-4}\ N$$

Direction of force must be left (opp. to velocity) since energy cons. says it must slow down (lose energy)



$F_{pull} = F_{B,total}$ (constant speed)

$= 1.8 \times 10^{-4} N$

$P = Fv = 1.8 \times 10^{-4} N \cdot 0.2\ m/s = \boxed{3.6 \times 10^{-5} W}$

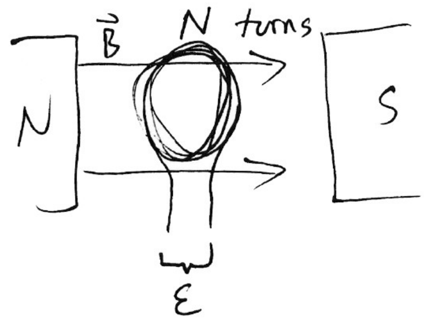
Section 29-4: AC Generators

- Wire wrapped around rod

- Rod is called armature

- Spin armature to make emf

- Mechanical to electrical energy



$$\mathcal{E} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(BA\cos\theta)$$

Let $\theta = \omega t$, ω in radians so $\boxed{\omega = 2\pi f}$

Then $\mathcal{E} = NBA\omega\sin(\omega t)$

Peak voltage $\boxed{\mathcal{E}_0 = NBA\omega}$

Frequency is same as armature

[*May skip example below if time is short.*]

Example: If an armature spins at 60 Hz in a 0.1 T B-field and the wire loop has an area of $10 cm^2$, how many turns are needed to achieve an rms voltage of 120V?

$$A = 10 cm^2 \cdot \frac{1 m}{(100 cm)^2} = 0.001\ m^2$$

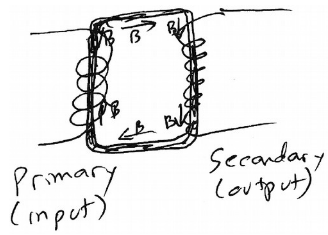
$$\omega = 2\pi f = 2\pi \cdot 60\ Hz = 378\ rad/s$$

$$\mathcal{E}_{rms} = \frac{1}{\sqrt{2}}\mathcal{E}_0 \quad \text{or} \quad \mathcal{E}_0 = \sqrt{2}\,\mathcal{E}_{rms} = \sqrt{2}\ 120\ V = 170\ V$$

$\mathcal{E}_0 = NBA\omega$

$N = \frac{\mathcal{E}_0}{BA\omega} = \frac{170\,V}{0.1\,T \cdot 0.001\,m^2 \cdot 378\,rad/s} = \boxed{4490}$

Section 29-6: Transformers



- Two coils around iron core

  Primary coil connects to *power source*

  Secondary coil connects to *power consumer*

- Iron amplifies magnetic fields; ignore B-fields outside iron

  $V = N\frac{d\Phi}{dt}$ for each, or $\frac{d\Phi}{dt} = \frac{V}{N}$

  Equal fluxes, so equal *V/N* or $\frac{V_P}{N_P} = \frac{V_S}{N_S}$

  $\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}}$ (voltage in same ratio as number of coils)

- Types of transformers

  $N_S > N_P$: step up (voltage increases)

  $N_S < N_P$: step down

- Current:

  $P = VI$, and energy must be conserved

  Step up *V*, must step down *I* by same amount

  $\boxed{\frac{I_S}{I_P} = \frac{N_P}{N_S}}$ (current in inverse ratio as number of coils)

Example: A 20 Ω resistor with 5 W of power is connected to a transformer with primary peak current 0.4 A and 80 secondary coils. How many primary coils does it have?

$$P = I_{rms}^2 R \quad \text{or} \quad I_{rms} = \sqrt{\frac{P}{R}} = \sqrt{\frac{5\,W}{20\,\Omega}} = 0.5\,A$$

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{2} \cdot 0.5\,A = 0.707\,A \text{ (secondary)}$$

$$\frac{I_s}{I_p} = \frac{0.707\,A}{0.4\,A} = 1.77 \text{ (step down transformer, since current increased)}$$

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \quad \text{so} \quad \frac{N_p}{N_s} = 1.77 \quad \text{or} \quad N_p = 1.77\,N_s = 1.77 \cdot 80 = \boxed{141}$$

Example: An AC generator has an armature area of 75 cm², 120 turns of wire, and rotates at 200 Hz in a 0.25 T field. It is connected to a transformer with 30 primary and 90 secondary coils, which is connected to a device that draws 5 W power. What is the resistance of this device?

$$A = 75\,cm^2 \times \left(\frac{1\,m}{100\,cm}\right)^2 = 0.0075\,m^2$$

$$\omega = 2\pi f = 2\pi \cdot 200\,Hz = 1260\,rad/s$$

$$\mathcal{E}_0 = NBA\omega = 120 \cdot 0.25\,T \cdot 0.0075\,m^2 \cdot 1260\,rad/s = 283\,V$$

$$\mathcal{E}_{rms} = \frac{1}{\sqrt{2}}\mathcal{E}_0 = \frac{1}{\sqrt{2}} \cdot 283\text{ V} = 200\text{V}$$

$$\frac{N_s}{N_p} = \frac{90}{30} = 3 \quad \text{(step up)} \quad \text{so} \quad \mathcal{E}_s = 200\text{V} \cdot 3 = 600\text{V (rms)}$$

$$P = \frac{V_{rms}^2}{R} \quad \text{or} \quad R = \frac{V_{rms}^2}{P} = \frac{(600\text{V})^2}{5\text{ W}} = \boxed{71900\,\Omega}$$

Homework: Do Chapter 29 in Mastering Physics

Exam #3 on Chapters 27-29

Exam review materials available:

- These lecture notes (Canvas home page)
- Webex recordings (in Canvas under Cisco Webex link)
- Practice exam (Canvas home page)
- Conceptual review practice assignment (in Mastering Physics)
    - Practice for multiple-choice conceptual questions
    - Not for credit but strongly recommended