Name:________________________________

MA 4633/6633 Section 01 *Homework Set 2* *Due: 9/6/2024*

1. Let n be a square free natural number. Prove that $\sqrt{n}$ is an irrational number.

2. Define the following set:

$$X = \left\{3 - \frac{1}{\sqrt{2+n}} : n \in \mathbb{N}\right\}.$$

Compute, with proof, $\inf X$ and $\sup X$.

3. Let X be a set of real numbers. If $\alpha = \sup X$, prove that $-\alpha = \inf(-X)$, here the set $-X = \{-x : x \in X\}$.

4. Let A and B be nonempty sets in $\mathbb{R}$ and define the following:

$$A + B = \{a + b : a \in A, b \in B\}.$$

Prove that $\sup(A+B) \le \sup A + \sup B$ and that $\inf A + \inf B \le \inf(A+B)$. Can you find a necessary and sufficient condition to make the inequalities become equalities? Be sure to justify.

5. Let I and J be intervals in $\mathbb{R}$ and $f : I \times J \to \mathbb{R}$ be a function. Prove that

$$\sup\{f(x,y) : (x,y) \in I \times J\} = \sup_{y\in J}\sup_{x\in I} f(x,y) = \sup_{x\in I}\sup_{y\in J} f(x,y).$$

Prove or disprove that:

$$\inf_{y\in J}\sup_{x\in I} f(x,y) = \sup_{x\in I}\inf_{y\in J} f(x,y).$$