

Name: \_\_\_\_\_

MA 4643/6643 Section 01

*Homework Set 6*

Due: 3/28/2025

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- 1.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be uniformly continuous. For  $a \in \mathbb{R}$ , define  $f_a(x) = f(x - a)$ . Prove that  $\{f_a : a \in \mathbb{R}\}$  is equicontinuous.
  
- 2.** Let  $(S, d)$  be a metric space. Let  $E \subseteq S$  be connected. Prove  $\overline{E}$  is connected. Prove or disprove that if  $\overline{E}$  is connected, then  $E$  is connected.
  
- 3.** Prove that  $[0, 1]$  and  $E = \{(x, y) : x^2 + y^2 = 1\}$  are not homeomorphic.
  
- 4.** Prove or disprove that  $\mathbb{R}$  and  $\mathbb{R}^2$  are homeomorphic.
  
  
- 5.** Let  $(S, d)$  be a metric space and  $\{E_k\}_{k \in \mathbb{N}}$  be connected subsets of  $S$  such that for every  $k \in \mathbb{N}$ ,  $E_k \cap E_{k+1} \neq \emptyset$ . Prove that  $\bigcup_{k=1}^{\infty} E_k$  is connected.