Chapter 27: Magnetism

Section 27-1: Magnets and Magnetic Fields

- Magnets have North and South poles
- Opposites attract

    N attracts S, N repels N and S repels S

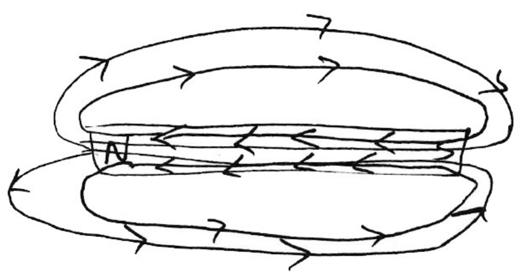
    N points toward's Earth's north pole, S points south

    Earth's geographical north pole is magnetic south pole

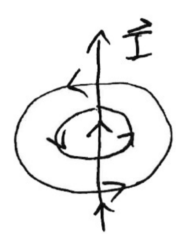
- No "magnetic monopole" exists

    "Monopole" means charge; there's no magnetic charge

- Recall: electric fields begin/end on electric charges
- No magnetic charge means magnetic field lines are loops



Section 27-2: Current Produces Magnet Fields



- Magnetic fields form loops around wires

    Will discuss in detail in the next chapter

Section 27-3: Force on a Current-Carrying Wire

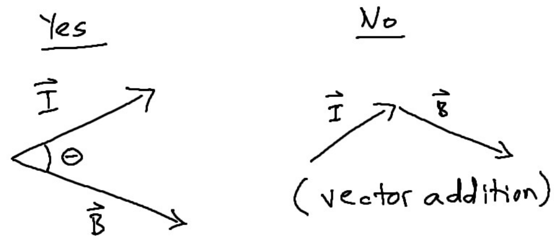
- Force on a wire is $\boxed{\vec{F} = \ell\vec{I} \times \vec{B}}$
- $\vec{B}$ is magnetic field. Units: Tesla (T)

- Magnitude: $\boxed{F = \ell I B \sin\theta}$

    θ is angle between $\vec{I}$ and $\vec{B}$

    Draw diagram where they start

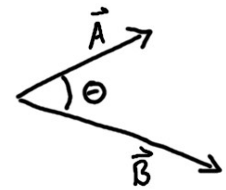
        at same point



- Mathematics of Cross Product: Let $\vec{A} \times \vec{B} = \vec{C}$
- Magnitude: $\boxed{C = AB \sin\theta}$

    No vector sign means magnitude

    Magnitude is always positive



    θ is angle between (diagram where both start at same point)

- Direction: Right-Hand Rule (for Cross Products)

    Fingers straight, point in direction of $\vec{A}$

    Do not ever change this direction – only lower arm twist now

        No shoulder or elbow motion, do not bend wrist

        Two bones in lower arm rotate around each other

    Bend fingers in direction of $\vec{B}$

    Thumb gives direction of $\vec{C}$

- Orthogonality:

    $\vec{C}$ must be perpendicular to $\vec{A}$ and $\vec{B}$

    $\vec{A}$ and $\vec{B}$ can be at any angle to each other

- Cyclic order:

$$\vec{i} \times \vec{j} = \vec{k} \qquad \vec{j} \times \vec{i} = -\vec{k}$$

$$\vec{k} \times \vec{i} = \vec{j} \qquad \vec{i} \times \vec{k} = -\vec{j}$$

$$\vec{j} \times \vec{k} = \vec{i} \qquad \vec{k} \times \vec{j} = -\vec{i}$$

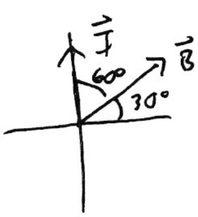
Rule: alphabetic order is positive, opposite is negative

- Parallel cross to zero

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

- Anticommutative, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

<u>Example:</u> A 3A current flows through a wire 10 cm long towards the north. A B-field of 0.01 T points 30° c.c.w. from east. What is the force on the wire?



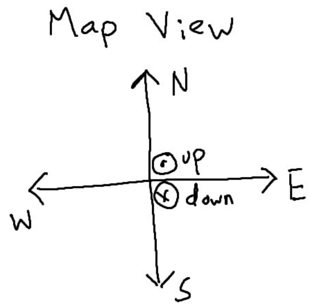
Magnitude:

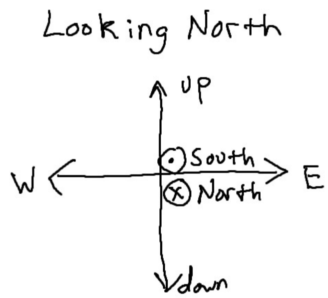
$$F = I \ell B \sin 60°$$
$$= 3A \cdot 0.1m \cdot 0.01T \cdot \sin 60°$$
$$= \boxed{2.60 \times 10^{-3}\ N}$$

Direction: $\boxed{\text{down}}$ (into board)

- Henceforth, ⊙=out of board, ⊗=into board

- NSEW up/down diagrams:





Example: A 2m wire with a 150 mA current has the largest force on it when oriented with the current going upward. The force is 0.06 N south. What is the magnetic field?

$$F = I \ell B$$

$$B = \frac{F}{I\ell} = \frac{0.06\,N}{0.150\,A \cdot 2m} = 0.2\ T$$

Direction: $\hat{\imath}$ = east, $\hat{\jmath}$ = north, $\hat{k}$ = up

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

$$F \cdot -\hat{\jmath} = I\ell \cdot \hat{k} \times B \cdot ?$$

Must be $\pm\hat{\imath}$ since perp. to other two.

$\hat{k} \times \hat{\imath} = \hat{\jmath}$, so $\hat{k} \times (-\hat{\imath}) = -\hat{\jmath}$

$\hookrightarrow -\hat{\imath}$ = west

$$\boxed{\vec{B} = 0.2\ T \text{ west}}$$

- Cross product distributes over addition (<u>watch order!</u>):

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

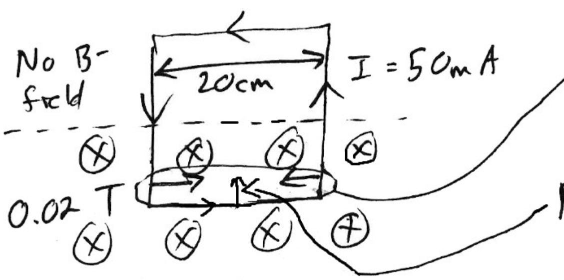
$$(\vec{B} + \vec{C}) \times \vec{A} = \vec{B} \times \vec{A} + \vec{C} \times \vec{A}$$

Example: A 0.5 A current in the y-direction moves through a uniform B-field $\vec{B} = 1.5\times10^{-4}\,T\,\hat{\imath} + 2.6\times10^{-4}\,T\,\hat{\jmath} - 3\times10^{-4}\,\hat{k}$. What is the force if the length is 4m?

$\hat{\jmath}\times\hat{\jmath} = 0$

$$\vec{F} = \ell\vec{I}\times\vec{B}$$
$$= 4m\cdot 0.5A\,\hat{\jmath}\times\left(1.5\times10^{-4}\,T\,\hat{\imath} + 2.6\times10^{-4}\,T\,\hat{\jmath} - 3\times10^{-4}\,T\,\hat{k}\right)$$
$$= 3\times10^{-4}\,N\,\hat{\jmath}\times\hat{\imath} - 6\times10^{-4}\,N\,\hat{\jmath}\times\hat{k}$$
$$= \boxed{-3\times10^{-4}\,N\,\hat{k} - 6\times10^{-4}\,N\,\hat{\imath}}$$

Example: A rectangular loop 20 cm wide with 50 mA counterclockwise current has one end in a 0.02T B-field into the page. What is the force on it?



Sides have inward, thus cancelling, forces

$$F = I\ell B\sin\theta$$
$$= 0.05A\cdot 0.2\,m\cdot 0.02T\sin 90°$$
$$= \boxed{2\times10^{-4}\,N \text{ up}}$$

Section 27-4: Lorentz Force

- Why is there a force on currents?

  Current is moving charges

  B-field must apply force to charge in motion

  q and $\vec{v}$ must be in equation

  Must be a cross product

  Lorentz force: $\boxed{\vec{F} = q\vec{v}\times\vec{B}}$

<u>Example</u>: A 2mC charge has a velocity $\vec{v} = 2.5\,m/s\,\hat{\imath}$ and experiences a force $\vec{F} = 2\times10^{-3}\,N\,\hat{\jmath} - 3\times10^{-3}\,N\,\hat{k}$. What can you know about the B-field, and what can't you know?

$$q\vec{v}\times\vec{B} = \vec{F}$$

$$\vec{v}\times\vec{B} = \frac{\vec{F}}{q}$$

$$2.5\,m/s\,\hat{\imath}\times\left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right) = \frac{2\times10^{-3}N}{2mC}\hat{\jmath} - \frac{3\times10^{-3}N}{2mC}\hat{k}$$

$$2.5\,m/s\cdot B_y\,\hat{k} + 2.5\,m/s\,B_z(-\hat{\jmath}) = 1\,\tfrac{N}{C}\hat{\jmath} - 1.5\,\tfrac{N}{C}\hat{k}$$

$$B_y = \frac{-1.5\frac{N}{C}}{2.5\,m/s} = \boxed{-0.6\,T = B_y}$$

$$B_z = \frac{1\,N/C}{-2.5\,m/s} = \boxed{-0.4\,T = B_z}$$

$$\boxed{B_x \text{ is unknowable}}$$

- Motion under (just) Lorentz force:
- First, break $\vec{v}$ into $v_\perp$ and $v_{||}$

    Perp. or parallel to magnetic field

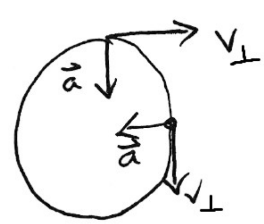
    $v_{||}$ causes no force (parallel cross to zero)

    $v_\perp$ causes a force

- Perpendicular part:

    $\vec{F}$ perp. to $\vec{v}$ (nature of right-hand-rule)

    $\vec{F}$ perp. to $\vec{v}$ is circular motion:



Centripetal force is $F_c = \dfrac{mv^2}{r}$

So $F = qv_\perp B = \dfrac{mv_\perp^2}{r}$

Solve for r: $\boxed{r = \dfrac{mv_\perp}{qB}}$

- Parallel part:

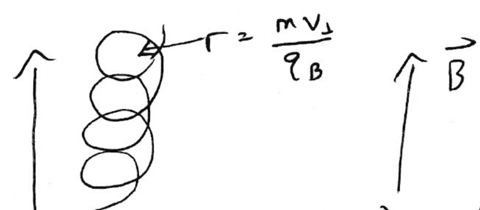
No force parallel to $\vec{B}$

$v_\parallel$ is constant

Pitch (distance between spiral rings): $\boxed{z = v_\parallel T}$

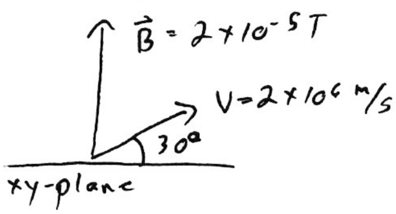
T is period, $\boxed{v_\perp = \dfrac{2\pi r}{T}}$ (note $v_\perp$ is related to period, not $v_\parallel$!)

- Overall motion: helix (spiral)



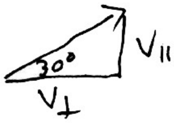
Constant $V_\parallel$ in the $\vec{B}$-field direction

Example: An electron moves in a uniform B-field of 2×10^{-5} T. Its velocity is 2×10^{6} m/s angled upward 30° above the xy plane, and the B-field is pointed up. Find the radius and the pitch of the helical motion.



$$r = \frac{m V_\perp}{q B}$$

$$= \frac{9.11\times10^{-31}\,\text{kg} \cdot 1.73\times10^{6}\,\frac{m}{s}}{1.6\times10^{-19}\,C \cdot 2\times10^{-5}\,T}$$



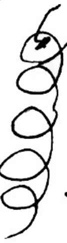
$$= \boxed{0.493\ m\ \ (\text{radius})}$$

$$V_{\perp} = 2\times10^{6}\tfrac{m}{s}\cos 30^{\circ}$$
$$= 1.73\times10^{6}\tfrac{m}{s}$$
$$V_{\parallel} = 2\times10^{6}\tfrac{m}{s}\sin 30^{\circ}$$
$$= 1\times10^{6}\tfrac{m}{s}$$

$$V_{\perp} = \frac{2\pi r}{T} \qquad T = \text{Period}$$

$$T = \frac{2\pi r}{V_{\perp}} = \frac{2\pi \cdot 0.493m}{1.73\times10^{6}\frac{m}{s}} = 1.79\times10^{-6}s$$

$r = 0.493\,m$



Distance from one coil to next:

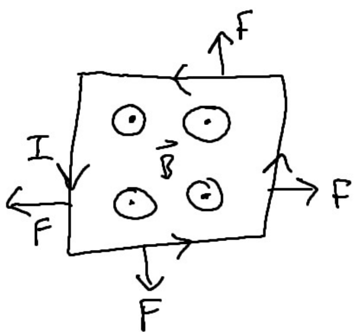
$$Z = V_{\parallel}\cdot T = 1\times10^{6}\tfrac{m}{s}\cdot 1.79\times10^{-6}s = \boxed{1.79m}$$

"pitch"

$Z = 1.79m$

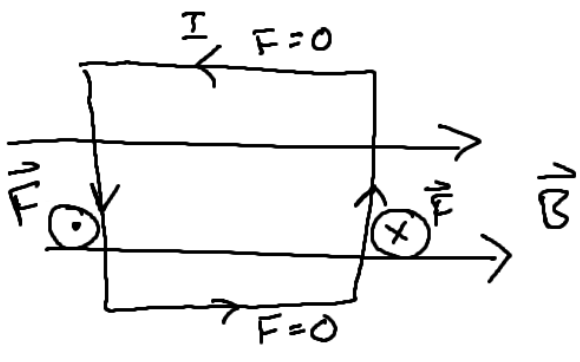
<u>Section 27-5: Magnetic Dipoles</u>

- Consider B pointing through a current loop



(nothing happens)

- Consider B in plane of loop:



- No net force, but there is a torque

- Magnitude of torque is $\tau = IAB$ (won't prove)

  If there are N loops: $\tau = NIAB$

- Recall: Area is a vector $\vec{A}$ which is perpendicular to the surface

- There's no torque when $\vec{B}$ parallel to $\vec{A}$ (first case above)

  $\tau = NIAB\sin\theta$ (sine is zero for parallel, full value for perp.)

- Direction: $\vec{\tau} = NI\vec{A} \times \vec{B}$ (won't prove direction is right)

- <u>Dipole moment</u>

  $\boxed{\vec{\mu} = NI\vec{A}}$

- Direction: Right-Hand Rule (for loops)

  Fingers curl in loop direction (I here)

  Thumb is in vector direction ($\vec{\mu}$ and $\vec{A}$ here)

- Torque in terms of dipole moment: $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$

  Compare to $\vec{\tau} = \vec{p} \times \vec{E}$

- Potential energy: same as for electric dipole

  $\boxed{U = -\vec{\mu} \cdot \vec{B}}$

- Equilibrium (lowest energy) when parallel; antiparallel high energy

  Equilibrium energy is negative <u>not zero</u>

<u>Example</u>: A wire wraps 10 times around a circular loop of radius 3 cm, and a 2.5 A current flows counterclockwise in the x-y plane. If $\vec{B} = 0.01\,T\,\hat{\imath} - 0.03\,T\,\hat{k}$, what is the torque?

$$\mu = NIA = 10 \cdot 2.5\,A \cdot \pi \cdot (0.03\,m)^2$$
$$= 0.0707\ Am^2$$

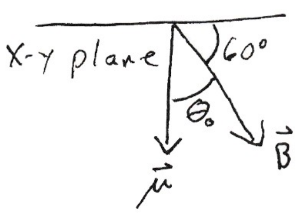
Thumb points up (positive z)

$$\mu = 0.0707\ Am^2\ \hat{k}$$

$$\hat{k} \times \hat{k} = 0$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = 0.0707\ Am^2\ \hat{k} \times (0.01\,T\ \hat{\imath} - \cancel{0.03\,T\ \hat{k}})$$
$$= \boxed{7.07 \times 10^{-4}\ Nm\ \hat{\jmath}} \quad (\hat{k} \times \hat{\imath} = \hat{\jmath})$$

Example: A 10 cm × 5 cm loop with 3 turns has 0.5 A current clockwise in the x-y plane. If $\vec{B}$ is 0.25 T at 60° below the x-y plane, what is the change in potential energy if it rotates to equilibrium?



$$\mu = NIA = 3 \cdot 0.5\,A \cdot 0.1\,m \cdot 0.05\,m = 7.5 \times 10^{-3}\ Am^2$$
$$\theta_0 = 90° - 60° = 30°$$
$$U_0 = -\mu B \cos\theta_0$$
$$= -7.5 \times 10^{-3}\ Am^2 \cdot 0.25\,T \cos 30° = -1.62 \times 10^{-3}\,J$$

$$U = -\mu B \cos 0° = -7.5 \times 10^{-3} Am^2 \cdot 0.25\,T = -1.88 \times 10^{-3}\ J$$

$$\Delta U = U - U_0 = 1.88 \times 10^{-3}\,J - (-1.62 \times 10^{-3}\,J) = \boxed{-2.51 \times 10^{-4}\,J}$$

Homework: Do Chapter 27 in Mastering Physics