Name:______________________________

1. For $1 \leq p < \infty$, give an example of a closed and bounded subset of $\ell^p(\mathbb{N})$ which is not compact. Be sure to prove you claims.

2. For $1 \leq p < \infty$, prove that $\ell^p(\mathbb{N})$ is separable.

3. Let (S, d) be a metric space. Prove or disprove that $K \subseteq S$ is compact if and only if every continuous real-valued function on K is uniformly continuous.

4. Let (S, d) be a metric space and let $K_1, K_2 \subseteq S$ be compact subsets. Prove that $K_1 \cup K_2$ is compact. Let $\{K_n\}$ be compact subsets of S. Prove or disprove $\displaystyle\bigcap_{n=1}^{\infty} K_n$ is compact. Prove or disprove $\displaystyle\bigcup_{n=1}^{\infty} K_n$ is compact.

5. Let (X, d) be a compact metric space and $\{K_n\}_{n=1}^{\infty}$ a decreasing sequence of closed sets in X. If $f : X \to X$ is continuous, prove that

$$f\left(\bigcap_{n=1}^{\infty} K_n\right) = \bigcap_{n=1}^{\infty} f(K_n)\,.$$