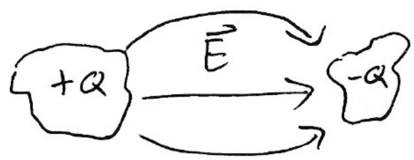
Chapter 24: Capacitors

Section 24-1: Capacitors

- A capacitor is:

  Two conductors with equal and opposite charges

  Not connected by a conductor (vacuum/insulator between)



- Electric field between them

  E is proportional to *Q*

  V is proportional to *E*

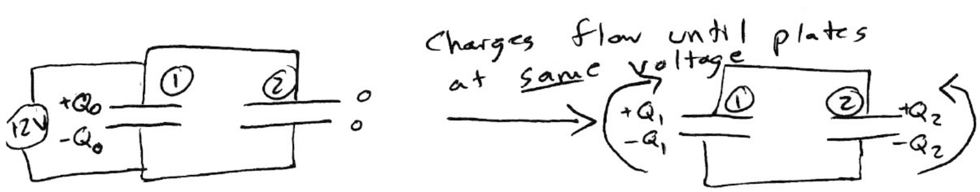
- So *Q* proportional to *V*: $\boxed{Q = CV}$
- *C* is called the capacitance

  Constant of proportionality between *Q* and *V*

  Depends on conductor shape, distance, and material between

- Units are Farads: F = C/V

Example: A 100 nF capacitor is charged to 12 V and disconnected from the battery. It is then connected to an (uncharged) 50 nF capacitor. What is the new voltage on each capacitor?



$$Q_0 = C_1 V = 100\times10^{-9}\,\text{F}\cdot 12\,\text{V} = 1.2\times10^{-6}\,\text{C}$$

Same voltage (but opposite directions around loop)

$$Q_1 = C_1 V$$
$$Q_2 = C_2 V$$
$$Q_1 + Q_2 = Q_0$$
$$C_1 V + C_2 V = Q_0$$
$$V = \frac{Q_0}{C_1 + C_2} = \frac{1.2\times10^{-6}\,\text{C}}{100\times10^{-9}\,\text{F} + 50\times10^{-9}\,\text{F}} = \boxed{8\text{V}}$$

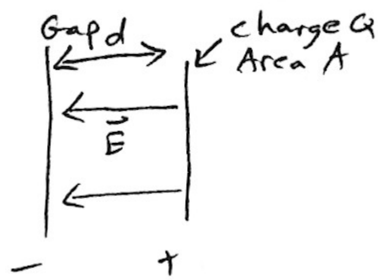
Section 24-2: Determination of Capacitance

- Most capacitor are parallel plates

- If size of plates larger than distance between, use infinite plane

$$E = \frac{\sigma}{2\epsilon_0} \quad \text{(per plate)}$$

$$E = -\frac{\sigma}{\epsilon_0} \quad \text{(two plates, left)}$$



$$V = -E\Delta x = -\left(-\frac{\sigma}{\epsilon_0}\right) d = \frac{Qd}{A\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\epsilon_0}\right)} \quad \text{so} \quad \boxed{C = \frac{A\epsilon_0}{d}}$$

Section 24-3: Series and Parallel Capacitors

- Circuit diagram symbols:


Voltage source (Battery or power supply)

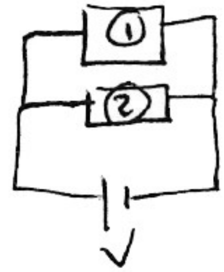

Capacitor

- Parallel:

  Choice: one or other never both

  Voltage V the same

  Charges add $Q = Q_1 + Q_2$

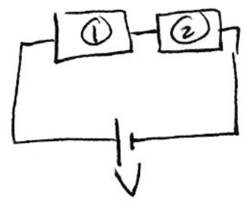


- Series:

  No choices: every charge through both

  Charge Q the same

  $V = V_1 + V_2$ since energy changes add



- Parallel Capacitors:

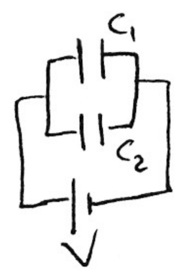
  Same voltage V (no subscript)

  $Q_1 = C_1 V$ and $Q_2 = C_2 V$

  $Q = Q_1 + Q_2 = C_1 V + C_2 V = (C_1 + C_2) V$

  Acts like one capacitor with $\boxed{C_{par} = C_1 + C_2}$

  Three capacitors: $C_{par} = C_1 + C_2 + C_3$, etc.



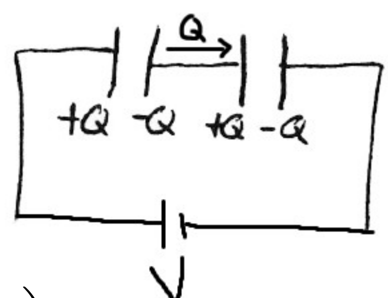
- Series Capacitors:

  Same charge Q (no subscript)

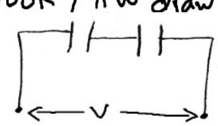
  $V_1 = \dfrac{Q}{C_1}$ and $V_2 = \dfrac{Q}{C_2}$

  $$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_1}\right) Q$$

  $$Q = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} V$$



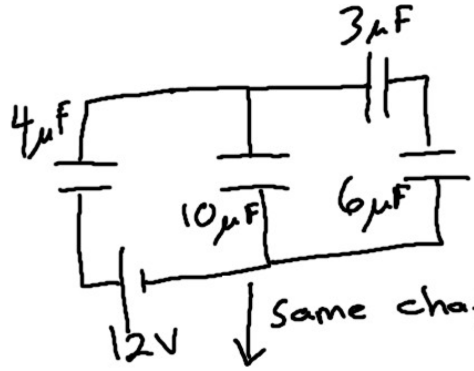
Book / HW draw as:



Acts like single capacitor with $\boxed{\dfrac{1}{C_{ser}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}}$

C_{ser} necessarily smaller than both C_1 and C_2

Example: Find the voltage on the $3\mu F$ capacitor.

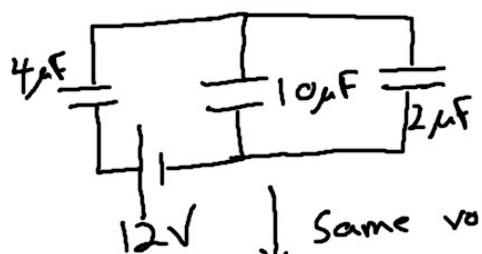


$3\mu F$ in series with $6\mu F$

$$\frac{1}{C_{ser}} = \frac{1}{3\mu F} + \frac{1}{6\mu F} = 0.5\mu F^{-1}$$

$$C_{ser} = \frac{1}{0.5\mu F^{-1}} = 2\mu F$$

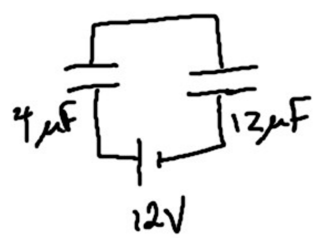
$\downarrow$ same charge ($6\mu C$: see below)



$10\mu F$ and $2\mu F$ in parallel

$$C_{par} = 10\mu F + 2\mu F = 12\mu F$$

$\downarrow$ same volts ($3V$: see below)



Series

$$\frac{1}{C_{ser}} = \frac{1}{4\mu F} + \frac{1}{12\mu F} = 0.333\mu F^{-1}$$

$$C_{ser} = \frac{1}{0.333\mu F^{-1}} = 3\mu F$$

$$Q = CV = 3\mu F \cdot 12V = 36\mu C$$

Voltage on $12\mu F$ in last diagram:

$$V = \frac{Q}{C} = \frac{36\mu C}{12\mu F} = 3V$$ (same volts in middle diagram)

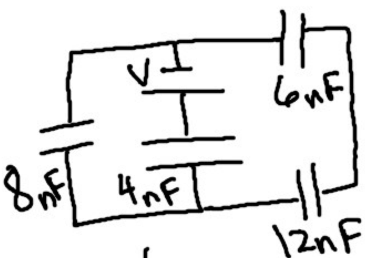
Charge on $2\mu F$ in middle diagram:

$$Q = CV = 2\mu F \cdot 3V = 6\mu C$$ (same charge in 1st diagram)

Voltage on $3\mu F$ in 1st diagram:

$$V = \frac{Q}{C} = \frac{6\mu C}{3\mu F} = \boxed{2V}$$

Example: If the $6nF$ has $4V$, find the voltage of the battery



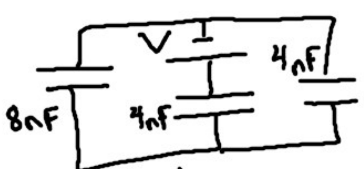
↓ same Q (24nC)

Q on 6nF: $Q = CV = 6nF \cdot 4V = 24nC$

6nF in series with 12nF:

$$\frac{1}{C_{ser}} = \frac{1}{6nF} + \frac{1}{12nF} = 0.25\,nF^{-1}$$

$$C_{ser} = \frac{1}{0.25\,nF^{-1}} = 4\,nF$$

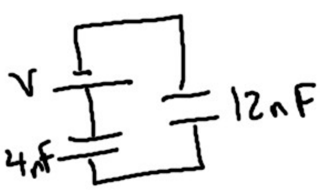


↓ Same V (6V)

V on right-hand 4nF: $V = \frac{Q}{C} = \frac{24nC}{4nF} = 6V$

4nF in parallel with 8nF

$C_{par} = 4nF + 8nF = 12nF$



Q on 12nF: $Q = CV = 12nF \cdot 6V = 72nC$

Series: $\frac{1}{C_{ser}} = \frac{1}{4nF} + \frac{1}{12nF} = 0.333\,nF^{-1}$

$$C_{ser} = \frac{1}{0.333\,nF^{-1}} = 3\,nF$$

$$V = \frac{Q}{C} = \frac{72nC}{3nF} = \boxed{24\,V}$$

Section 24-4: Electric Field Energy

- Charging a capacitor to final charge Q:

    Small dQ goes from pos. to neg.

    Energy change $dU = V\,dQ$

- Final energy is U, initially empty so $U_0 = 0$

- Total energy gain:

$$\Delta U = \int dU$$

$$U - U_0 = \int_0^Q V\,dQ \quad \text{(initial charge 0, final is Q)}$$

$$U = \int_0^Q \frac{Q}{C}\,dQ \quad \text{(using Q = CV and also using } U_0 = 0\text{)}$$

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C}} \quad (\Delta U = U \text{ since } U_0 = 0)$$

- Plugging in $Q = CV$ gives $\boxed{U = \frac{1}{2}CV^2}$

Example: A 1μF and a 2μF capacitor are connected in series to a 15 V supply. What is the energy of the capacitors?

$$\frac{1}{C_{ser}} = \frac{1}{1\mu F} + \frac{1}{2\mu F} = 1.5\,\mu F^{-1}$$

$$C_{ser} = 0.667\,\mu F$$

$$U = \frac{1}{2}C_{ser}V^2 = \frac{1}{2}(0.667\times10^{-6}\,F)(15\,V)^2 = \boxed{7.5\times10^{-5}\,J}$$

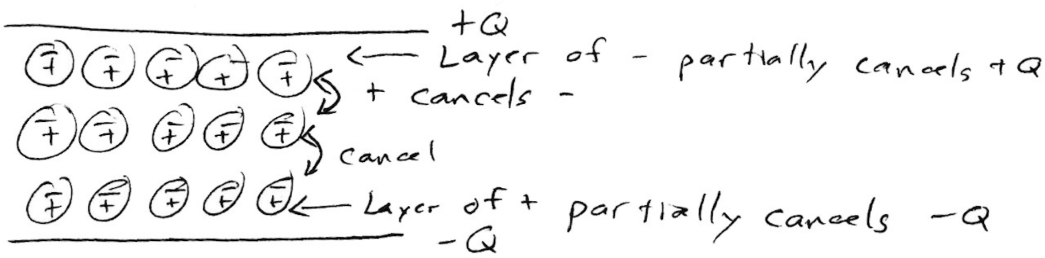
Section 24-5: Dielectrics

- Dielectric materials are the same thing as insulators

When discussing conduction of electricty: insulator

When discussing capacitors: dielectric

- For a given amount of charge, will reduce voltage



- Replace ϵ_0 by ϵ in formulas

Vacuum: $C_{vac} = \dfrac{A\epsilon_0}{d}$

Dielectric: $C = \dfrac{A\epsilon}{d}$

ϵ (permittivity of some material) larger than ϵ_0

- <u>Dielectric constant</u> *K*:

$$K = \frac{\epsilon}{\epsilon_0}$$

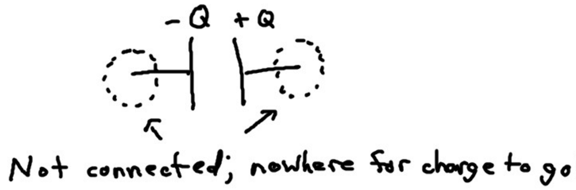
$$\boxed{C = KC_{vac}}$$

- Since $\epsilon > \epsilon_0$, $K > 1$
- <u>Use table in book</u>, not values you find online (given on exams)

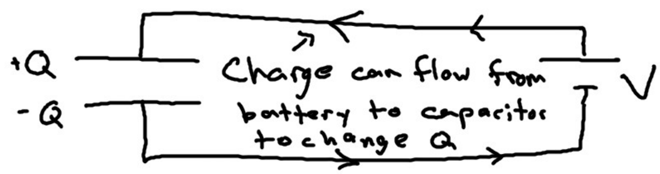
Table on page 703 (print edition), Section 24-5

- Scaling:

Not connected to battery: *Q* must stay constant



Connected to battery: battery maintains constant *V*



Batteries raise voltage of connected device to battery's voltage

Example: A capacitor is charged up to 12V and disconnected from the battery. A K=3 material is then inserted. What is the new voltage?

If disconnected from battery, Q = constant

$Q = C V$ ← V must be multiplied by $\frac{1}{3}$ since Q must stay the same while C triples

same (Q) times 3 (C)

$12V \times \frac{1}{3} = \boxed{4\ V}$

Example: An air filled capacitor is made of square plates 10cm on a side and 2mm apart. It is charged up to 15V and kept connected to the battery. What is the change in energy if it is then filled with a K = 4 material?

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\,Nm^2/c^2 \cdot (0.1m)^2}{0.002m} = 4.425\times10^{-11}\,F$$

$$U_0 = \frac{1}{2} C V^2 = \frac{1}{2}\cdot 4.425\times10^{-11}F\cdot(15V)^2 = 4.98\times10^{-9}\,J$$

$U = \frac{1}{2} C V^2$ ← Constant (V) so $U = 4U_0 = 4\cdot 4.98\times10^{-9}J = 1.99\times10^{-8}J$

Must be 4 times (U) 4 times (C)

$$\Delta U = U - U_0 = 1.99\times10^{-8}J - 4.98\times10^{-9}J = \boxed{1.49\times10^{-8}J}$$

Example: While connected to a battery, the distance between plates of a capacitor is doubled. What happens to (a) the charge, (b) the energy?

$C = \frac{A\varepsilon_0}{d}$ Double $d \Rightarrow C$ cut in half.

Connected to battery: $V = $ const

(a) $Q = CV$

C is $\frac{1}{2}$, $V = $ const, so $Q = \boxed{\frac{1}{2}}$

(b) $U = \frac{1}{2}CV^2$ $\qquad$ $U = \frac{1}{2}\frac{Q^2}{C}$

Easier ($V = $ const) $\qquad$ Harder to analyze (both Q and C change)

C is half, so U cut in $\boxed{\text{half}}$

Homework: Do Chapter 24 in Mastering Physics