Chapter 25: Current and Resistance

Section 25-1: Batteries

- Batteries cause current to flow

Section 25-2: Current

- Current is the rate of charge flow (charge per time)

Average: $\boxed{I = \frac{Q}{t}}$

Instantaneous: $I = \frac{dQ}{dt}$

Units: Amperes, or Amps, A=C/s

Section 25-3: Ohm's Law and Resistance

- For many materials (not all!), *V* is proportional to *I*
- Constant of proportionality is called resistance, *R*:

Ohm's Law: $\boxed{V = IR}$

Units: Ohms, Ω = V/A

- Compare to $Q = CV$: In both cases, *V* proportional to something
- Ohm's Law not universal law; badly wrong for some materials

Example: 1200 C of charge flows down a wire over 1 hour. If there is 0.4 V potential difference across the wire, what is its resistance?

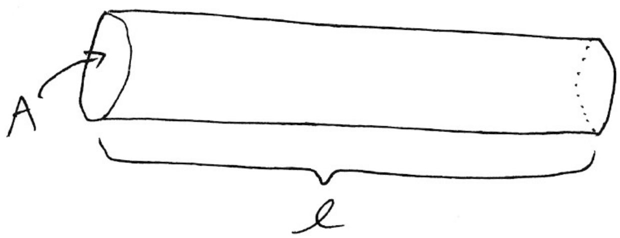
$I = \frac{Q}{t} = \frac{1200C}{3600s} = 0.333\ A$ (Convert time to seconds!)

$V = IR$ or $R = \frac{V}{I} = \frac{0.4V}{0.333A} = \boxed{1.2\ \Omega}$

Section 25-4: Resistivity

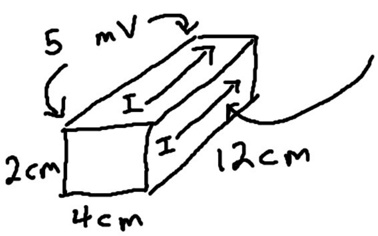
- Resistance does not depend on V or I (constant of proportionality)
- Depends on shape and material

  Just like C is const. of prop. that depends on shape and material
- Resistance is proportional to length and inversely to area



- $\boxed{R = \dfrac{\rho \ell}{A}}$
- ρ is resistivity: depends on material
- Use table in book (p 724, Section 25-4), not numbers you find online (will be given on exams if needed)

Example: The prism below has resistivity $5.6 \times 10^{-8}\ \Omega\cdot m$ and a 5 mV difference between the front and back. How much current flows through it?



Voltage between front/back means current runs from front to back along 12 cm direction

Thus 12 cm is length ℓ

Other dimensions are area A

$$R = \frac{\rho \ell}{A} = \frac{5.6 \times 10^{-8}\ \Omega\cdot m \cdot 0.12 m}{0.02 m \cdot 0.04 m} = 8.4 \times 10^{-6}\ \Omega$$

$$V = IR \text{ or } I = \frac{V}{R} = \frac{0.005 V}{8.4 \times 10^{-6}\ \Omega} = \boxed{595 A}$$

Section 25-5: Power

- Energy is $U = QV$

- Power is $P = \dfrac{\text{energy}}{\text{time}} = \dfrac{U}{t} = \dfrac{QV}{t}$

- Using $I = \dfrac{Q}{t}$ gives or $\boxed{P = VI}$

- Using $V = IR$:

$$P = I(IR) \text{ or } \boxed{P = I^2R}$$

$$P = \frac{V}{R} \cdot V \text{ or } \boxed{P = \frac{V^2}{R}}$$

Example: Two wires have the same voltage, but the second has double the length and radius. If the first has 0.2W of power, what is the power of the second?

$A = \pi r^2$ 4 times the area

$\underset{\times 4}{A} = \pi \underset{\times 2}{r}^2$

$\underset{\times\frac{1}{2}}{R} = \dfrac{\overset{\text{same}}{\rho}\,\overset{\times 2}{l}}{\underset{\times 4}{A}}$ Half the resistance

$\underset{\text{double}}{P} = \dfrac{\overset{\text{same}}{V^2}}{\underset{\text{half}}{R}}$ Double power

$P_2 = 2P_1 = 2 \cdot 0.2\,W = 0.4\,W$

- New unit: kWhr

Power = energy/time or energy = power $\times$ time

Power unit times time unit is an amount of energy

So kWhr (kW times hours) is an energy unit

Example: How much does it cost to run a 50Ω device hooked up to 120 V for 3 days if electricity costs 10¢ per kWhr?

$V = IR$ or $I = \frac{V}{R} = \frac{120V}{50\Omega} = 2.4A$

$P = VI = 120V \cdot 2.4A = 288\ W \times \frac{1\,kW}{1000\,W} = 0.288\ kW$

$E = Pt = 0.288\ kW \cdot 3\ days \times \frac{24\ hrs}{1\ day} = 20.7\ kWhr$

$Cost = 20.7\ kWhr \times \frac{10¢}{1\ kWhr} = 207¢$ or $\boxed{\$2.07}$

Section 25-7: Alternating Current

- Direct current

    So far we have assumed direct current

    Direct current (DC) is constant current

- Alternating current (AC)

    Current can be any function of time, $I = f(t)$

    sin or cos function is of most interest so we focus on those

    $I = I_0 \sin(\omega t)$ or $I = I_0 \cos(\omega t)$

    ω is in radians per second

- Voltage in resistors

    $V = IR$ so $V = V_0 \sin(\omega t)$ with $V_0 = I_0 R$

- Power

    $P = VI$ so if *I* and *V* change with time, *P* does too

    Instantaneous power: $P = I_0 V_0 \sin^2(\omega t)$

    For practical purposes, we just want to know average power

Average of $\sin^2$ function is ½ (will leave for calculus class)

$\bar{P} = \frac{1}{2} I_0 V_0$ (usually will not use bar in the future)

By "power" we will almost always mean average power

- Root-mean-square

$I_{rms} = \sqrt{\bar{I^2}} = \sqrt{\frac{1}{2} I_0^2}$ so $\boxed{I_{rms} = \frac{1}{\sqrt{2}} I_0}$

Likewise $\boxed{V_{rms} = \frac{1}{\sqrt{2}} V_0}$

$P = \frac{1}{2} I_0 V_0 = \frac{1}{\sqrt{2}} I_0 \frac{1}{\sqrt{2}} V_0$ so $\boxed{P = V_{rms} I_{rms}}$

Also $\boxed{P = I_{rms}^2 R}$ and $\boxed{P = \frac{V_{rms}^2}{R}}$

Example: A resistor with $10\,\Omega$ resistance has voltage $V = 5\cos(250t)$ in SI units. (a) What is the frequency in Hz? (b) What is the power? (c) What is rms current?

Note: $V = \underbrace{5}_{\text{peak value}} \cos(\underbrace{250}_{\text{angular frequency in rad/s}} t)$

(a) $\omega = 2\pi f$ or $f = \frac{\omega}{2\pi} = \frac{250\,\text{rad/s}}{2\pi} = \boxed{39.8\,\text{Hz}}$

(b) $V_{rms} = \frac{1}{\sqrt{2}} V_0 = \frac{1}{\sqrt{2}} \cdot 5\,\text{V} = 3.54\,\text{V}$

$P = \frac{V_{rms}^2}{R} = \frac{(3.54\,\text{V})^2}{10\,\Omega} = \boxed{1.25\,\text{W}}$

(c) $V_{rms} = I_{rms} R$ or $I_{rms} = \frac{V_{rms}}{R} = \frac{3.54\,\text{V}}{10\,\Omega} = \boxed{0.354\,\text{A}}$

Example: A 3 m long wire is to have a peak AC current of 3 A. If the resistivity is $3.8 \times 10^{-8}\ \Omega \cdot m$ and the power lost can't be over 0.25 W, what is the minimum diameter the wire can have, in mm?

Note: smaller radius ⇒ More resistance ⇒ More power $P = I^2 R$
Thus, maximum allowed power lost in wire will correspond to a minimum radius/diameter

$$I_{rms} = \frac{1}{\sqrt{2}} I_0 = \frac{1}{\sqrt{2}} \cdot 3\,A = 2.12\,A$$

$$P = I_{rms}^2 R \quad \text{or} \quad R = \frac{P}{I_{rms}^2} = \frac{0.25\,W}{(2.12\,A)^2} = 0.0556\,\Omega$$

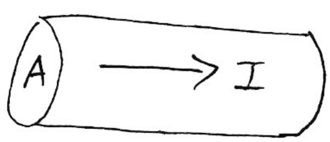
$$R = \frac{\rho l}{A} \quad \text{or} \quad A = \frac{\rho l}{R} = \frac{3.8 \times 10^{-8}\,\Omega \cdot m \cdot 3\,m}{0.0556\,\Omega} = 2.05 \times 10^{-6}\,m^2$$

$$A = \pi r^2 \quad \text{or} \quad r = \sqrt{\frac{A}{\pi}} = 0.000808\,m$$

$$d = 2r = 2 \cdot 0.000808\,m = 0.00162\,m = \boxed{1.62\,mm}$$

Section 25-8: Current Density

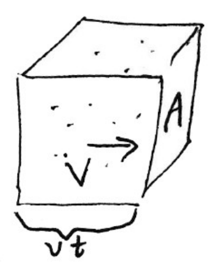
- Current density is current per area:



$$\boxed{j = \frac{I}{A}}$$

- Let *n* be the number of electrons per unit volume (units m^{-3})

In time t, all charges in box move on



Number in box is *nV*

So $Q = enV = envtA$

$$I = \frac{Q}{t} = envA$$

$j = \dfrac{I}{A}$ so $\boxed{j = nev}$ where *e* is 1.6×10^{-19} C

Example. If the current through a 6 mm diameter wire is 2A and there are 5.8×10^{28} electrons/m^3 in the metal what is the drift velocity?

$$A = \pi r^2 = \pi (0.003m)^2 = 2.83 \times 10^{-5} m^2$$

$$j = \frac{I}{A} = \frac{2A}{2.83 \times 10^{-5} m^2} = 70,700 \ A/m^2$$

$$v = \frac{j}{ne} = \frac{70,700}{5.8 \times 10^{28} m^{-3} \cdot 1.6 \times 10^{-19} C} = \boxed{7.6 \times 10^{-6} \ m/s}$$

[*May skip the following example if time is short.*]

Example: A 20 A current passes through a 1 mm^2 wire with a drift velocity of 2×10^{-4} m/s. How many electrons are there per cubic meter?

$$A = 1 mm^2 \times \left(\frac{1 \ m}{1000 mm}\right)^2 = 10^{-6} \ m^2$$

$$j = \frac{I}{A} = \frac{20A}{10^{-6} m^2} = 2 \times 10^7 \ A/m^2$$

$$j = nev \quad \text{or} \quad n = \frac{j}{ev} = \frac{2 \times 10^7 A/m^2}{1.6 \times 10^{-19} C \cdot 2 \times 10^{-4} \ m/s} = \boxed{6.25 \times 10^{29} \ m^{-3}}$$

Example: Suppose a metal has $\rho = 3 \times 10^{-8} \ \Omega m$ and $n = 2 \times 10^{28}$ valence electrons/m^3. If a wire is 5m long and has radius 2mm and 20mV is put across it, what is the drift velocity?

$$A = \pi r^2 = \pi (0.002m)^2 = 1.26 \times 10^{-5} \ m^2$$

$$R = \frac{\rho l}{A} = \frac{3\times10^{-8}\,\Omega m \cdot 5m}{\pi(0.002\,m)^2} = 0.0119\,\Omega$$

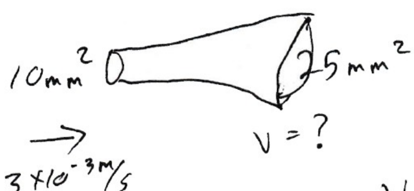
$$V = IR \text{ or } I = \frac{V}{R} = \frac{0.02V}{0.0119\,\Omega} = 1.68\,A$$

$$j = \frac{I}{A} = \frac{1.68A}{\pi\,(0.002m)^2} = 133000\ A/m^2$$

$$j = n e v_d$$

$$v_d = \frac{j}{ne} = \frac{133000\ A/m^2}{2\times10^{28}\,m^{-3} \cdot 1.6\times10^{-19}C} = \boxed{4.17\times10^{-5}\,m/s}$$

Example: A wire has a radius which changes over its length. On the narrow end it has a $10\,mm^2$ area and on the wide end $25\,mm^2$. The drift velocity entering the narrow end is 3×10^{-3} m/s. What is the exit velocity?



$$I_{in} = I_{out}$$

$$j_{in}A_{in} = j_{out}A_{out}$$

$$\cancel{ne}V_{in}A_{in} = \cancel{ne}V_{out}A_{out}$$

$$V_{out} = \frac{V_{in}A_{in}}{A_{out}} = \frac{3\times10^{-3}\,m/s \cdot 10\,mm^2}{25\,mm^2} = \boxed{1.2\times10^{-3}\,m/s}$$

- Notice the velocity is <u>slow</u> at the <u>wide</u> end, <u>fast</u> at the <u>narrow</u> end

Homework: Do Chapter 25 in Mastering Physics