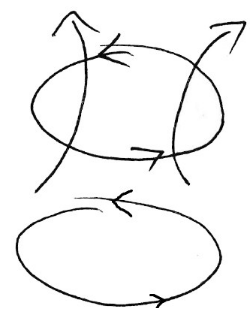
Chapter 30: AC Circuits

Section 30-1: Mutual Induction

- Two wire loops:



Change current in one

Makes changing flux in other

Thus, induced current in other

But this makes changing flux back in the first loop

- Recall currents from flux are induced, thus mutual inductance

Section 30-2: Self-Inductance

- Self-inductance:

Magnetic field from current goes through loop itself

Changing current in loop makes changing flux in itself

Thus, makes an induced current (in addition to original current)

$L = \frac{N\Phi}{I}$ Units: Henri (H)

Solve for Φ: $\Phi = \frac{IL}{N}$

$\mathcal{E} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\frac{IL}{N}$ so $\boxed{\mathcal{E} = -L\frac{dI}{dt}}$

- Inductance of a solenoid:

$$B = \frac{\mu_0 NI}{\ell}$$

$$\Phi = BA = \frac{\mu_0 NI\pi r^2}{\ell}$$

$$L = \frac{N\Phi}{I} \text{ so } \boxed{L = \frac{\mu_0 \pi N^2 r^2}{\ell}}$$

Example: A solenoid with 200 turns, length 10 cm, and radius 1 cm has a current which rises from 2.5A to 3A in 2 ms. What is the mag. of the average emf?

$$L = \frac{\mu_0 \pi N^2 r^2}{\ell} = \frac{4\pi\times10^{-7}\frac{Tm}{A}\cdot\pi\cdot200^2\cdot(0.01m)^2}{0.1\ m} = 1.58\times10^{-4}\ H$$

$$\overline{\mathcal{E}} = -L\frac{\Delta I}{\Delta t} = -1.58\times10^{-4}\ H\cdot\frac{(3A-2.5A)}{0.002s} = \boxed{0.0395\text{ V}}$$

(mag. only; ignore − sign)

Example: A solenoid has 160 turns, a diameter of 4 cm, and length 12 cm. The current is $I = 6\sin(200t)$ in SI units. What is the emf as a function of time?

$$L = \frac{\mu_0 \pi N^2 r^2}{\ell} = \frac{4\pi\times10^{-7}\frac{Tm}{A}\cdot\pi\cdot160^2\cdot(0.04m)^2}{0.12m} = 3.37\times10^{-4}\text{H}$$

$$\mathcal{E} = -L\frac{dI}{dt} = -3.37\times10^{-4}\cdot\frac{d}{dt}(6\sin 200t)$$

chain rule

$$= -3.37\times10^{-4}\cdot6\cdot\cos(200t)\cdot200$$

$$= \boxed{-0.404\cos(200t)}$$

Section 30-3: Energy in magnetic fields

- Finding work to put current through an inductor:

$$P = \mathcal{E}I$$

$$\frac{dW}{dt} = I\cdot\left(-L\frac{dI}{dt}\right)$$

$$dW = -LIdI$$

$$W = -\frac{1}{2}LI^2$$

Inductor did negative work on current

Thus work done to inductor is positive

This is a potential energy stored in the magnetic fields:

$$\boxed{U = \frac{1}{2}LI^2}$$

Section 30-7: AC Circuits and Reactance

- Want to explore series circuit in the remainder of this chapter
- Therefore want same current in all devices: $I = I_0 \cos \omega t$

  $\boxed{\omega = 2\pi f}$ always convert Hertz to rad/s

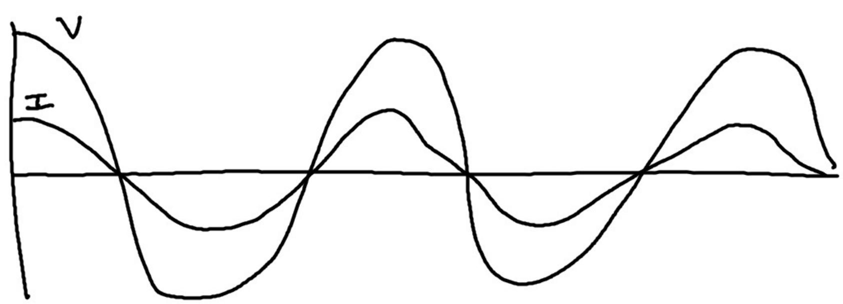
- Resistors

  $V = IR = I_0 R \cos \omega t$

  Peak voltage $V_0 = I_0 R$ and $V_{rms} = I_{rms} R$

  V and *I* have same phase

  Graph:



Power $P = V_{rms} I_{rms}$ or $P = \frac{V_{rms}^2}{R}$ or $\boxed{P = I_{rms}^2 R}$

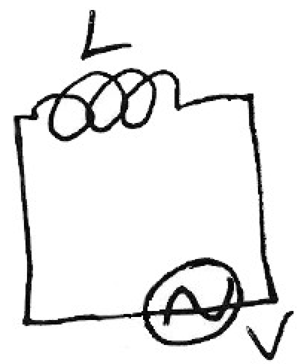
Warning: voltage must be resistor's voltage, not power supply!

Last eqn. is best for series circuit (same current for all in series)

Only resistors have power output

Inductors/capacitors turn energy into field energy and back

- Inductors (watch signs!)



$$V - L\frac{dI}{dt} = 0 \text{ (Kirchhoff)}$$

$$V = L\frac{d}{dt}(I_0 \cos\omega t)$$

$$V = -\omega L I_0 \sin\omega t$$

Peak voltage $V_0 = \omega L I_0$

Define (inductive) reactance as $\boxed{X_L = \omega L}$ Units: Ω (Ohms)

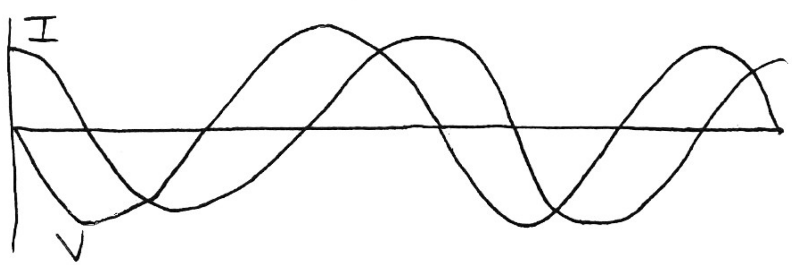
$\boxed{V_0 = I_0 X_L}$ and $\boxed{V_{rms} = I_{rms} X_L}$

Reactance is not resistance!

Resistance: $V = IR$ at all times (in phase)

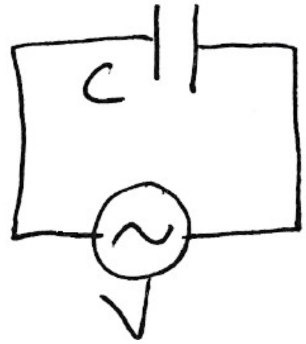
Reactance: $V_0 = I_0 X$ peak only, and peaks at diff. times

Graphs:



Voltage leads current by 90° (or current lags voltage)

- <u>Capacitors</u>



$$V = \frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{d}{dt}\frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{1}{C}\frac{dQ}{dt} = \frac{I}{C} = \frac{I_0}{C}\cos\omega t$$

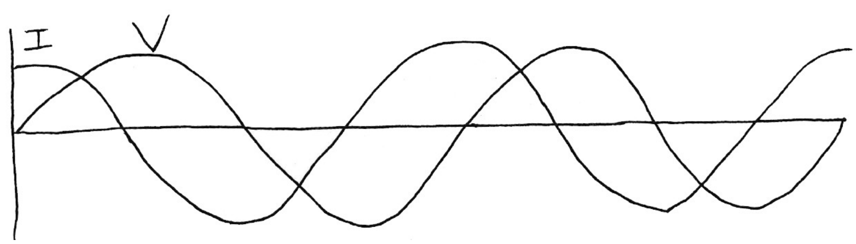
$$V = \int dV = \int \frac{I_0}{C}\cos\omega t\,dt = \frac{I_0}{C\omega}\sin\omega t$$

(Capacitive) <u>reactance</u>: $\boxed{X_C = \frac{1}{C\omega}}$

Peak voltage $\boxed{V_0 = I_0 X_C}$ and $\boxed{V_{rms} = I_{rms} X_C}$

Again, <u>this is not Ohm's Law</u>

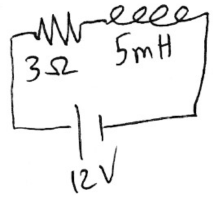
Graphs:



Voltage <u>lags</u> current by 90^{O} (or current <u>leads</u> voltage)

[*May skip first example below if time is short.*]

<u>Example</u>: A 5 mH inductor has 3 Ω resistance. What energy does it store when connected to a 12 V battery?

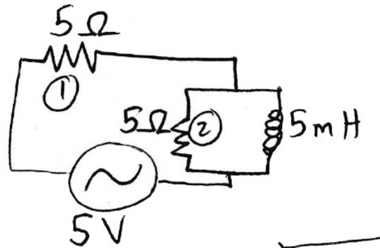


Battery ⇒ DC ⇒ $\frac{dI}{dt} = 0$ ⇒ $\mathcal{E} = 0$ on inductor

$I = \frac{V}{R} = \frac{12V}{3\Omega} = 4A$ (All volts on resistor)

$U = \frac{1}{2}LI^2 = \frac{1}{2} \cdot 0.005H \cdot (4A)^2 = \boxed{0.04J}$

<u>Example:</u> Find the current in each resistor for (a) $\omega = 0$ (DC) and (b) $\omega = \infty$ (high frequency AC)



(a) $X_L = \omega L = 0$

- No reactance: just like a wire
- <u>All</u> current goes through it
  - Electrons take easiest path
- Parallel part counts as 0 resistance/reactance
- other: $I_1 = \frac{5V}{5\Omega} = \boxed{1A}$

$\boxed{I_2 = 0}$

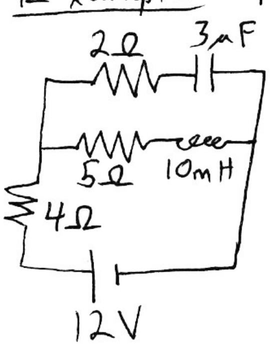
(b) $X_L = \omega L = \infty$

<u>No</u> current in L, so all through R

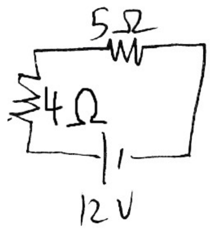
$R_{series} = 5\Omega + 5\Omega = 10\Omega$

$I = \frac{V}{R} = \frac{5V}{10\Omega} = \boxed{0.5A}$ (Same for both resistors)

<u>Example:</u> Find the current in each resistor.



- DC source ⇒ $\omega = 0$
- $X_L = \omega L = 0$ (just a wire)
- $X_C = \frac{1}{\omega C} = \infty$ (blocks current completely)
  ↳ Only once full charged
- Current: through 4 Ω and 5Ω, then back to battery



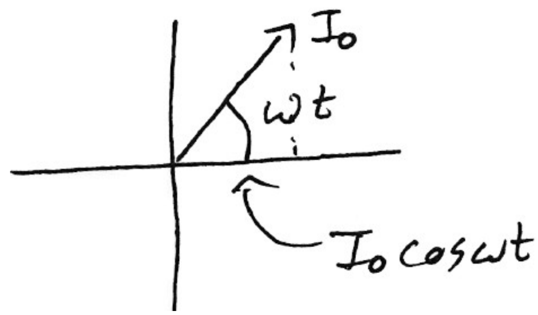
$$R_{series} = 4\Omega + 5\Omega = 9\Omega$$

$$I = \frac{V}{R} = \frac{12V}{9\Omega} = \boxed{1.33A} \text{ for } 4\Omega, 5\Omega$$

$$\boxed{I = 0} \text{ for } 2\Omega$$

<u>Section 30-8: Series LRC Circuit</u>

- <u>Phasor diagram</u>



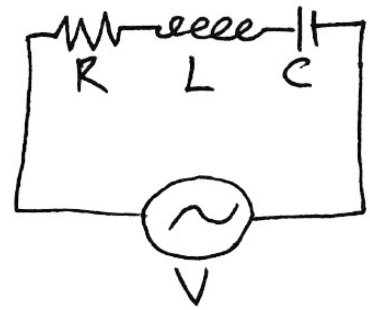
Vector of length I_0 and angle ωt

Current is x-component

y-component is non-physical

Allows us to use geometric, vector reasoning to solve problems

- <u>LRC Series</u>:



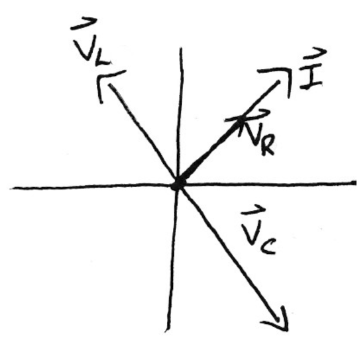
Same current $I = I_0 \cos\omega t$ (already done)

Series, add volts: $V = V_R + V_L + V_C$

Add volts at <u>one moment in time</u>

<u>Never</u> add peaks: $V_0 \neq V_{R0} + V_{L0} + V_{C0}$!

- LRC phasor diagram:

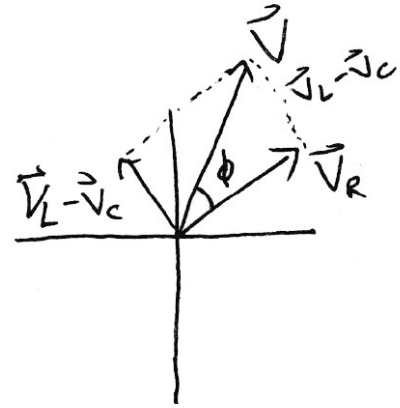


V_R is in phase with I (angle ωt)

V_L is 90$^{\mathrm{O}}$ ahead

V_C is 90$^{\mathrm{O}}$ behind

- Add $\vec{V}_L$ to $\vec{V}_C$ (diagram):



- Magnitude:

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2}$$

$$V_0 = \sqrt{I_0^2 R^2 + (I_0 X_L - I_0 X_C)^2}$$

$$V_0 = \sqrt{R^2 + (X_L - X_C)^2}\, I_0$$

Note again $V_0 \neq V_{R0} + V_{L0} + V_{C0}$: <u>Never add peak values</u>!

- <u>Impedance</u>:

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

$\boxed{V_0 = I_0 Z}$ and $\boxed{V_{rms} = I_{rms} Z}$

- Phase:

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}}$$

$$\phi = \tan^{-1}\frac{I_0 X_L - I_0 X_C}{R}$$

$$\boxed{\phi = \tan^{-1}\frac{X_L - X_C}{R}}$$

- Resistance vs. Reactance vs. Impedance:

Resistance when voltage in phase with current

Reactance is 90° out of phase

Impedance is in-between

Example: A 10 mH inductor is in series with a 20 Ω resistor and an AC source with voltage (in SI) $V = 10\cos(6280t)$. Find the power.

inside cosine is ang. freq.: $\omega = 6280\,\text{rad/s}$. Also $V_0 = 10\text{ V}$

$$X_L = \omega L = 6280\,\text{rad/s} \cdot 10\times10^{-3}\text{ H} = 62.8\,\Omega$$

$X_C = 0$ (no capacitor, no reactance)

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(20\,\Omega)^2 + (62.8\,\Omega)^2} = 65.9\,\Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{10\text{ V}}{65.9\,\Omega} = 0.152\text{ A}$$

$$I_{rms} = \frac{1}{\sqrt{2}} I_0 = 0.107\text{ A}$$

$$\bar{P} = I_{rms}^2 R = (0.107\text{ A})^2 \cdot 20\,\Omega = \boxed{0.230\text{ W}}$$

- Only resistors have power

Use current of resistor in power equation!

Use resistance not impedance

Use rms current not peak current

Example: A 100 Ω resistor is in series with a 100 Hz AC source and a 20 μF capacitor. (a) Does voltage lag or lead current? (b) By how much?

(a) Lag: Capacitors make volts trail current (inductors lead)

(b) $\omega = 2\pi f = 2\pi \cdot 100\text{ Hz} = 628\text{ Hz}$

$$X = \frac{1}{\omega C} = \frac{1}{628\text{ Hz} \cdot 20\times10^{-6}\text{ F}} = 79.6\,\Omega$$

$$\tan\phi = \frac{X_L - X_C}{R} = -\frac{X_C}{R} = -\frac{79.6\,\Omega}{100\,\Omega} = -0.796$$

$$\phi = \tan^{-1}(-0.796) = \boxed{-38.5^\circ} \text{ (Negative for lag)}$$

Example: A $2\,k\Omega$ resistor, a $0.03\,H$ inductor, and a $6\,nF$ capacitor are in series with a $6\,V$ rms, $10\,kHz$ AC source. Find the rms voltage on each component.

$$\omega = 2\pi f = 2\pi \cdot 10{,}000\,Hz = 62{,}800\,Hz$$

$$X_L = \omega L = 62{,}800\,Hz \cdot 0.03\,H = 1885\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{62800\,Hz \cdot 6\times10^{-9}\,F} = 2653\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2000\,\Omega)^2 + (2653\,\Omega - 1885\,\Omega)^2}$$

$$= 2142\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{6\,V}{2142\,\Omega} = 0.0028\,A$$

rms volts

$$\begin{cases} V_R = I_{rms}\,R = 0.0028\,A \cdot 2000\,\Omega = \boxed{5.6\,V} \\ V_C = I_{rms}\,X_C = 0.0028\,A \cdot 2653\,\Omega = \boxed{7.43\,V} \\ V_L = I_{rms}\,X_L = 0.0028\,A \cdot 1885\,\Omega = \boxed{5.28\,V} \end{cases}$$

Larger than source!

Section 30-9: Resonance

- How can volts on one component be bigger than source?
- Resonance:

Child on swing: small force in sync makes big motion

LC circuits oscillate like a swing

Small “force” (source voltage) makes large voltage in LC

- Resonance frequency:

$$I = \frac{V}{Z}$$

Maximize I by minimizing Z, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Only X_L and X_C change with ω

Resonates when $X_L = X_C$

$$\omega L = \frac{1}{\omega C} \quad \text{so} \quad \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

- At resonance frequency

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{so} \quad \boxed{Z = R \quad \text{(on resonance)}}$$

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} \quad \text{so} \quad \boxed{\phi = 0 \quad \text{(on resonance)}}$$

Example: A circuit needs to be tuned to 80 kHz. If a 15 mH inductor is used, what must the capacitance be?

$$\omega = \sqrt{\frac{1}{LC}} \Rightarrow (2\pi f)^2 = \frac{1}{LC}$$

$$C = \frac{1}{L(2\pi f)^2} = \frac{1}{0.015\,\text{H}\,(2\pi \cdot 80000\,\text{Hz})^2} = \boxed{2.6 \times 10^{-10}\,\text{F}}$$

Example: $R = 10\,\Omega$, $L = 10\,\text{mH}$, $C = 10\,\mu\text{F}$, (a) what is I on resonance if $V = 12\,\text{V}$? (rms V and I) (b) what is the phase?

On resonance $X_L = X_C$, so $Z = \sqrt{R^2 + 0^2} = R$

(a) $I_{rms} = \frac{V_{rms}}{R} = \frac{12\,\text{V}}{10\,\Omega} = \boxed{1.2\,\text{A}}$

(b) $\tan\phi = \frac{X_L - X_C}{R} = 0$

$$\boxed{\phi = 0}$$

Homework: Do Chapter 30 in Mastering Physics