Name:______________________________

MA 4633/6633 Section 01 *Homework Set 11* *Due: 12/6/2024*

1. Show that $\lim_{p\to\infty} \|x\|_p = \|x\|_\infty$ for all $x \in \mathbb{R}^n$.

2. Show that $c_{00}(\mathbb{N}) \subsetneqq c_0(\mathbb{N}) \subsetneqq c(\mathbb{N}) \subsetneqq \ell^\infty(\mathbb{N})$.

3. Show that $\ell^1(\mathbb{N})$ is a proper subset of $\ell^2(\mathbb{N})$ and that if $x \in \ell^1(\mathbb{N})$, then $\|x\|_2 \le \|x\|_1$. Can you give a more general statement for arbitrary $1 \le p \le \infty$? Be sure to justify.

4. Let $f_k(t) = \dfrac{\sin(kt)}{\sqrt{k}}$ for $0 \le t \le 2\pi$. Show that $f_k \to 0$ in $C[0, 2\pi]$ with respect to $\|\cdot\|_\infty$. Does $f_k \to 0$ in $C^1[0, 2\pi]$ with respect to $\|\cdot\|_{\infty,1}$? Why or why not?

5. Let $(X, \|\cdot\|)$. A formal series $\sum_k x_k$ in X is **absolutely convergent** if $\sum_k \|x_k\|$ is convergent. Prove that X is complete if and only if every absolutely convergent series is also convergent.