Name:______________________________

1. Using the ε-N definition(ie: Proposition 1.3.3), prove the following limits:

$$(a)\ \lim_{n\to\infty} \frac{3n^2+4}{2n^2+17} = \frac{3}{2} \quad , \ (b)\ \lim_{n\to\infty} \frac{e^{1/n}+3}{n^2} = 0 \quad \text{and} \quad (c)\ \lim_{n\to\infty} \frac{5n-7}{10n+3} = \frac{1}{2}\,.$$

2. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers. Prove or disprove the following:

$$\overline{\lim_n}\,(a_n+b_n) = \overline{\lim_n}\,a_n + \overline{\lim_n}\,b_n\,.$$

3. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers and define the sequence $\{b_n\}_{n=1}^{\infty}$ by:

$$b_n = \frac{1}{n}\sum_{k=1}^{n} a_k\,.$$

Prove that if $\lim_n a_n = a$, then $\lim_n b_n = a$. Is it possible that $\{b_n\}$ converges to a real number, yet $\{a_n\}$ has no limit? (Be sure to justify.)

4. Let $\sum_n a_n$ be an absolutely convergent series. Prove that if $p > 1$ then the series $\sum_n a_n^p$ converge absolutely. Give an example in which if $0 < p < 1$, then $\sum_n a_n^p$ can either converge or diverge. Be sure to justify.

5. Let $\{a_{j,k}\}_{j,k=1}^{\infty}$ be a doubly indexed sequence of non-negative real numbers. Suppose that $a_{j,k} \leq a_{j+1,k}$ for all k and $a_{j,k}$ is bounded in j for every k. Suppose $\displaystyle\sum_{k=1}^{\infty} a_{j,k} < \infty$ for all j. Prove that

$$\lim_{j\to\infty}\sum_{k=1}^{\infty} a_{j,k} = \sum_{k=1}^{\infty}\lim_{j\to\infty} a_{j,k}\,.$$