Name:________________________________

MA 4643/6643 Section 01 *Homework Set 1* *Due: 1/24/2025*

1. Let (X, d) be a metric space and $E, F \subseteq X$. (a) Prove $\overline{E \cup F} = \overline{E} \cup \overline{F}$. (b) Is it true that $\overline{E \cap F} = \overline{E} \cap \overline{F}$? If so, prove it, otherwise give a counterexample. (c) Show $E^o \cap F^o = (E \cap F)^o$. (d) Is it true that $E^o \cup F^o = (E \cup F)^o$? If so, prove it, otherwise give a counterexample.

2. Let (X, d) be a metric space and $E \subseteq X$. x is said to be a **boundary point** of E if every ε-neighborhood of x contains points of both E and $X \setminus E$. The set of all boundary points of E is denoted by ∂E. Show that ∂E is closed.

3. Let (X, d) be a metric space. Let $F \subseteq X$, prove that F is closed if and only if $F = \overline{F}$.

4. Let $c_0(\mathbb{N})$ be space of all sequences that converge to zero. Prove that $c_{00}(\mathbb{N})$ is dense in $c_0(\mathbb{N})$ with respect to $\|\cdot\|_\infty$.

5. Let $(X, \|\cdot\|)$ be a normed space and L a subspace. For $x \in X$, define $[x] = x + L$ be the class in the quotient space X/L induced by x. Show that $\|[x]\|_L := \inf\{\|x + l\| : l \in L\}$ defines a seminorm on X/L and that this seminorm is a norm if and only if L is closed.