Chapter 28: Sources of Magnetic Fields

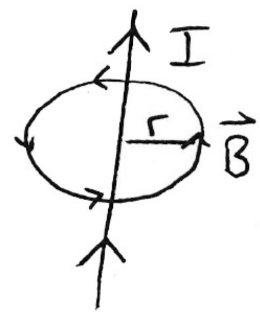
Section 28-1: Magnetic Field of a Straight Wire

- Current creates magnetic field
- Magnetic field is a loop around current

    Right-hand rule

    Thumb in current direction

    Fingers curl in *B* direction around current



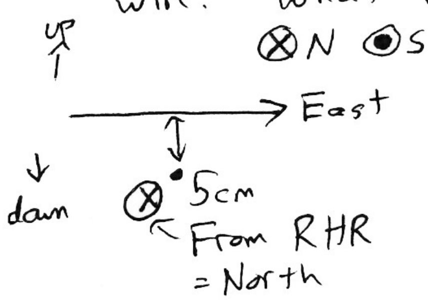
- Field strength is $\boxed{B = \frac{\mu_0}{2\pi}\frac{I}{R}}$

    μ_0 is "permeability of free space"

    Value is $4\pi \times 10^{-7}$ Tm/A

    Similar to ϵ_0, but for magnetic fields

Example: A 3A current runs eastward through a wire. What is $\vec{B}$ 5 cm below the wire?

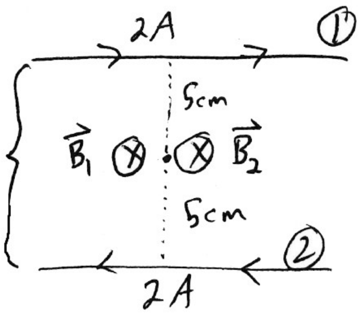


$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r} \text{ north}$$

$$= \frac{4\pi \times 10^{-7}\,\frac{\text{Tm}}{\text{A}}}{2\pi}\frac{3A}{0.05\text{ m}} \text{ north}$$

$$= \boxed{1.2 \times 10^{-5}\text{ T north}}$$

Example: Two wires are parallel and 10 cm apart. Their 2A currents run in opposite directions. What is the magnitude of B half-way between them?



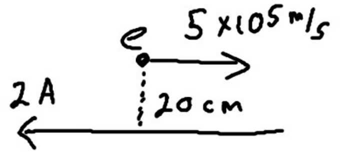
$$B_1 = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{4\pi \times 10^{-7}\ \frac{Tm}{A}}{2\pi}\ \frac{2A}{0.05\ m}$$

$$= 8 \times 10^{-6}\ T$$

$B_2 = 8 \times 10^{-6}\ T$ too

$$B = 2 \times 8 \times 10^{-6}\ T = \boxed{1.6 \times 10^{-5}\ T}$$

Example: An electron going 5×10^5 m/s east is 20 cm north of a wire with a 2A westward current. What is the force (mag. and dir.) on the electron?



$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7}\ \frac{Tm}{A} \cdot 2A}{2\pi \cdot 0.2m} = 2 \times 10^{-6}\ T$$

Direction (by RHR): into page (down)

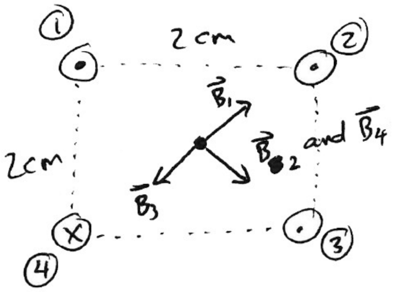
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB \sin 90° = 1.6 \times 10^{-19} C \cdot 5 \times 10^5 m/s \cdot 2 \times 10^{-6} T = \boxed{1.6 \times 10^{-19} N}$$

RHR: $\underset{\text{east}}{\vec{v}} \times \underset{\text{down}}{\vec{B}}$ = north

But $\vec{F} = q\vec{v} \times \vec{B}$ and $q < 0$ so $\boxed{\text{south}}$

Example: 4 1A currents pass through the corners of a 2cm square perpendicular to the plane. 3 go up and one goes down. What is the magnitude of B in the center?



$\vec{B}_1$ cancels $\vec{B}_3$

$\vec{B}_2$ and $\vec{B}_4$ are equal

$$B_2 = \frac{\mu_0}{2\pi}\frac{I}{r}$$

r = half diagonal

$= \frac{1}{2} \cdot 2\sqrt{2}$ cm $= \sqrt{2}$ cm

$= 0.0141$ m

$$B_2 = \frac{4\pi\times10^{-7}\,\frac{\mathrm{Tm}}{\mathrm{A}}}{2\pi}\,\frac{1\,\mathrm{A}}{0.0141\,\mathrm{m}}$$

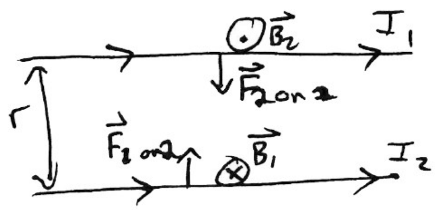
$$= 1.41\times10^{-5}\ \mathrm{T}$$

$$B_4 = 1.41\times10^{-5}\ \mathrm{T}$$

$$B = B_2 + B_4 = \boxed{2.83\times10^{-5}\ \mathrm{T}}$$

<u>Section 28-2: Force Between Parallel Wires</u>

- Two parallel currents of length ℓ:



Right hand rule (twice): Attractive

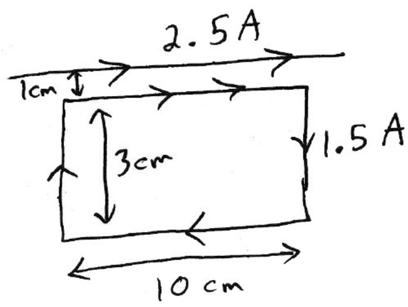
$$F_1 = I_1 \ell B_2 = I_1 \ell \cdot \frac{\mu_0}{2\pi}\frac{I_2}{r}$$

Newton's 3rd: $\boxed{F = \frac{\mu_0}{2\pi}\frac{I_1 I_2 \ell}{r}}$ (forces equal)

- Opposite direction: forces become repulsive

Like currents <u>attract</u>, opposites <u>repel</u>

Example: Find the net force on the wire loop



- Left/right wires cancel
- Top wire:

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}\ell$$

$$= \frac{4\pi\times10^{-7}\,\frac{Tm}{A}}{2\pi}\,\frac{2.5\,A\cdot1.5\,A}{0.01\,m}\cdot0.1\,m$$

$$= 7.5\times10^{-6}\ N\ \underline{up}$$

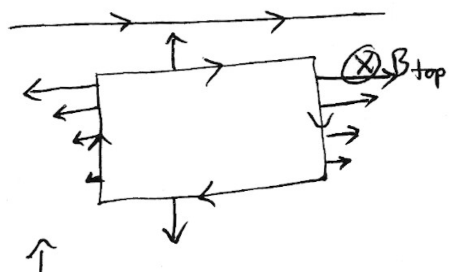
- Bottom wire:

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}\ell$$

$$= \frac{4\pi\times10^{-7}\,\frac{Tm}{A}}{2\pi}\cdot\frac{2.5A\cdot1.5A}{0.04\,m}\cdot0.1\,m$$

$$= 1.88\times10^{-6}\ N\ \underline{down}$$

Forces:



Force weakens with distance as B decreases

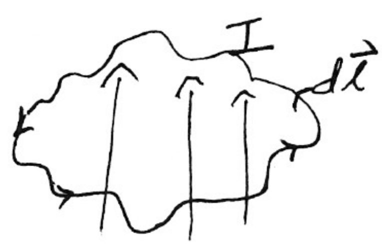
$$\boxed{\vec{F}_{net} = 5.63\times10^{-6}\ N\ up}$$

Section 28-4: Ampere's Law

- Recall Gauss's Law:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

- Equivalent law for B-fields: Ampere's Law



$$\boxed{\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{in}}$$

- I_{in} is current that went through loop

    Ignore current that passes outside the loop

    If given current density, use $\boxed{I = jA}$

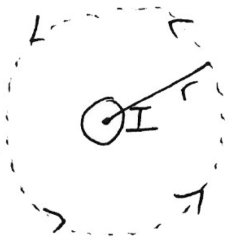
- Directions: Right-hand rule

  Point thumb in I direction

  Fingers give direction of B

  If two currents, opposite direction: call one pos., other neg.

Example: Show $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ for a long wire.



- By symmetry, B = constant around loop
- By RHR, $\vec{B}$ points in direction of $d\vec{\ell}$

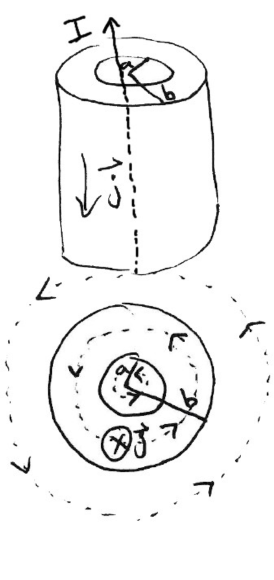
$$\oint \vec{B}\cdot d\vec{\ell} = \int B\,d\ell = B\int d\ell = B\cdot 2\pi r$$

$$\oint B\cdot d\ell = \mu_0 I_{enc}$$

$$B\cdot 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Example: A cylindrical shell of inner radius a and outer radius b has a current density $-j\hat{k}$ going through it and a wire with current $I\hat{k}$ in the center. Find $\vec{B}$ everywhere.



$r < a$: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$

$$B\cdot 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$a < r < b$: $I_{enc} = -j\,A_{enc} + I$

Counter-Clockwise loop means I_{enc} out of page
our I is into page so a negative I_{enc}

$$I_{enc} = -j\underbrace{(\pi r^2 - \pi a^2)}_{\text{Area between a and r}} + I$$

$$\oint B\cdot d\ell = \mu_0 I_{enc}$$

$$B\cdot 2\pi r = -\mu_0 j(\pi r^2 - \pi a^2) + \mu_0 I$$

$$\boxed{B = \frac{\mu_0 j(\pi a^2 - \pi r^2) + \mu_0 I}{2\pi r}}$$ (ccw in xy plane)

$r > b$: $I_{enc} = -j A_{enc} + I$

$$= -j(\pi b^2 - \pi a^2) + I$$

Full area of cross-section is enclosed

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

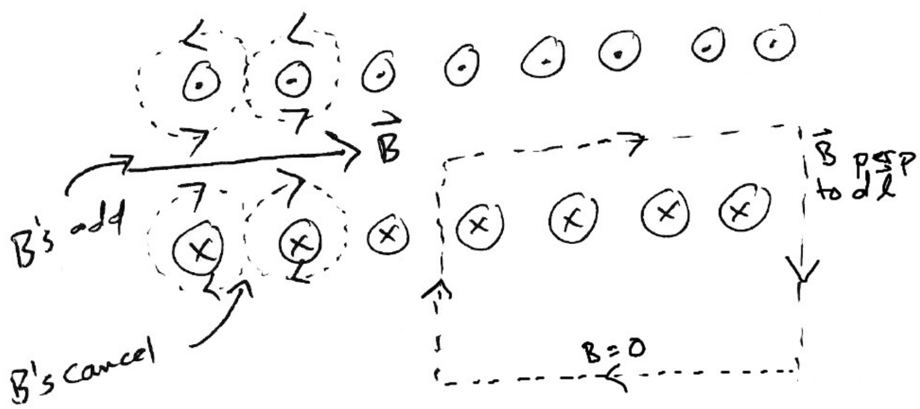
$$B\cdot 2\pi r = -\mu_0 j(\pi b^2 - \pi a^2) + \mu_0 I$$

$$\boxed{B = -\frac{\mu_0 j(\pi b^2 - \pi a^2) + \mu_0 I}{2\pi r} \quad \text{ccw in x-y plane}}$$

Section 28-5: Solenoid and Torus

- Solenoid: just a coil of wire

- Use Ampere's Law:

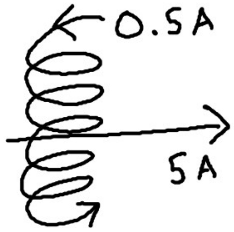


$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{in}$$

$$B\ell = \mu_0 N I \quad \text{or} \quad \boxed{B = \frac{\mu_0 N I}{\ell}}$$

[*May skip first example below if time is short.*]

Example: A solenoid has 200 turns, 30 cm length, 5 cm radius and 0.5 A in the direction shown. A wire with 5 A current goes through the center, perpendicular to the long axis. What is the force (mag. and dir.) on the wire?



$$B = \frac{\mu_0 N I}{l} = \frac{4\pi\times10^{-7}\,\frac{Tm}{A}\cdot 200\cdot 0.5A}{0.3m} = 4.19\times10^{-4}\,T$$

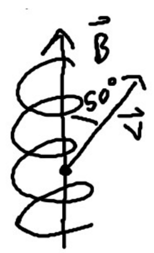
Direction (RHR): down

$\vec{F} = l\vec{I}\times\vec{B}$ $\quad$ l = length inside solenoid = diameter = $2r$

$F = 2\cdot 0.05m \cdot 5A \cdot 4.19\times10^{-4}\,T = \boxed{2.09\times10^{-4}\,N}$

Direction: $\vec{F} = l\underset{\text{right}}{\vec{I}}\times\underset{\text{down}}{\vec{B}}$ $\boxed{\text{into page}}$ by RHR

Example: An electron is moving $6.2\times10^{5}\,m/s$ at a $50°$ angle from the long axis of a solenoid with 350 turns, 1.5 m length and 12 A current. What is the radius and pitch of the helical motion?



$$B = \frac{\mu_0 N I}{l} = \frac{4\pi\times10^{-7}\,\frac{Tm}{A}\cdot 350\cdot 12A}{1.5\,m} = 0.00352\,T$$

$$V_{\parallel} = v\cos\theta = 6.2\times10^{5}\,m/s\,\cos 50° = 3.99\times10^{5}\,m/s$$

$$V_{\perp} = v\sin\theta = 6.2\times10^{5}\,m/s\,\sin 50° = 4.75\times10^{5}\,m/s$$

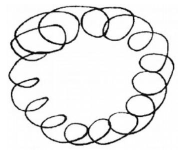
$$r = \frac{mV_{\perp}}{qB} = \frac{9.11\times10^{-31}\,kg\cdot 4.75\times10^{5}\,m/s}{1.6\times10^{-19}\,C\cdot 0.00352\,T} = \boxed{7.69\times10^{-4}\,m}$$

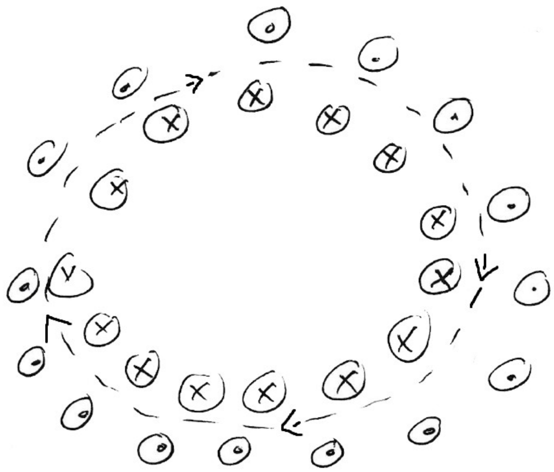
$$V_{\perp} = \frac{2\pi r}{T} \text{ or } T = \frac{2\pi r}{V_{\perp}} = \frac{2\pi\cdot 7.69\times10^{-4}\,m}{4.75\times10^{5}\,m/s} = 1.02\times10^{-8}\,s$$

$$z = V_{\parallel}T = 3.99\times10^{5}\,m/s\cdot 1.02\times10^{-8}\,s = \boxed{0.00405\,m}$$

[*May skip discussion of the torus if time is short.*]

- Torus





Inside: $I_{enc} = 0$

$B = 0$

Between:

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$

$B \cdot 2\pi r = \mu_0 N I$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

<u>Section 28-6: Biot-Savart Law</u>

- Recall:

Electric field is inverse square law: $E = \dfrac{kQ}{r^2}$

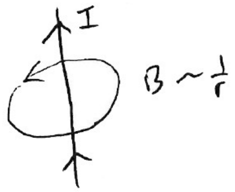
Magnetic field of a line is $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ $\quad\ldots\quad$ inverse law???

- Magnetic fields also inverse square law

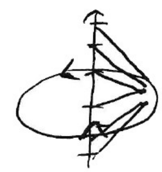
Can't compare points to lines

Each <u>infinitesimal</u> (point) line segment has inverse square law

- B-fields from infinite <u>wire</u> are $\frac{1}{r}$ law



- But this comes from a $\frac{1}{r^2}$ law for each wire <u>segment</u>



sum of $\frac{1}{r^2}$ contributions is a $\frac{1}{r}$ law

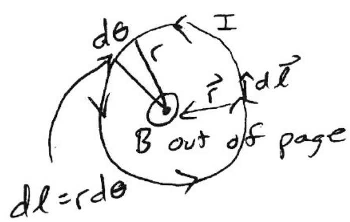
- <u>Biot-Savart Law</u>:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

$\hat{r} = \frac{\vec{r}}{r}$ (unit vector that points in r-direction)

$\hat{r}$ <u>from</u> source <u>to</u> point *P* (where $\vec{B}$ is being calculated)

- *B*-field at the center of a circular loop with current *I* and radius *r*:



$d\vec{\ell} \times \hat{r}$ = out of page

$|d\vec{\ell} \times \hat{r}| = d\ell \cdot 1 \cdot \sin 90° = d\ell$

$dB = \frac{\mu_0 I}{4\pi} \frac{d\ell}{r^2}$ $\quad d\ell = r d\theta$ (definition of radians)

$$B = \int_0^{2\pi} \frac{\mu_0 I}{4\pi} \frac{r d\theta}{r^2} = \frac{\mu_0 I}{4\pi r} \int_0^{2\pi} d\theta$$

$\boxed{B = \frac{\mu_0 I}{2r}}$ Do not confuse with straight wire $B = \frac{\mu_0 I}{2\pi r}$!

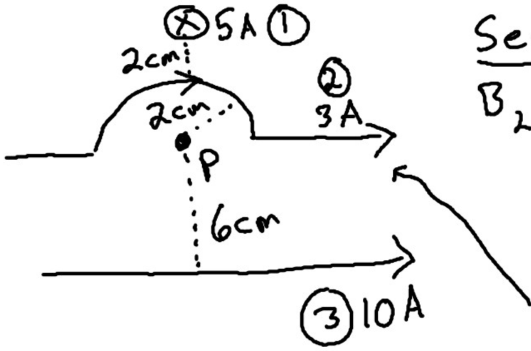
Direction: out of page for counterclockwise current

Follows RHR: Curl fingers in I-dir, thumb is in B-dir

<u>Outside</u> the loop B-field points other direction

Why? B-field lines must curl back to their starting point

Example: Find the magnitude of the magnetic field at P.



Semi-circle: Factor of $\frac{1}{2}$

$$B_2 = \frac{1}{2} \cdot \frac{\mu_0 I_2}{2r_2} = \frac{1}{2} \cdot \frac{4\pi \times 10^{-7} \frac{Tm}{A} \cdot 3A}{2 \cdot 0.02m} = 4.71 \times 10^{-5}\,T$$

RHR: into page

Straight segments: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} = 0$ since $d\vec{l} \times \hat{r} = 0$

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{4\pi \times 10^{-7} \frac{Tm}{A} \cdot 5A}{2\pi (0.02m + 0.02m)} = 2.5 \times 10^{-5}\,T \quad \text{RHR: left}$$

$$B_3 = \frac{\mu_0 I_3}{2\pi r_3} = \frac{4\pi \times 10^{-7} \frac{Tm}{A} \cdot 10A}{2\pi \cdot 0.06\,m} = 3.33 \times 10^{-5}\,T \quad \text{out of page}$$

B_1 parallel to B_3: $B_3 - B_1 = 3.33 \times 10^{-5}\,T - 4.71 \times 10^{-5}\,T = -1.38 \times 10^{-5}\,T$

$B_3 - B_1$ perp to B_2: $B = \sqrt{(-1.38 \times 10^{-5}\,T)^2 + (2.5 \times 10^{-5}\,T)^2} = \boxed{2.86 \times 10^{-5}\,T}$

Homework: Do Chapter 28 in Mastering Physics