Name:______________________________

MA 4633/6633 Section 01 *Homework Set 9* *Due: 11/15/2024*

1. Prove that the function $f(x) = \dfrac{\sin x}{x^{1/3}}$ is absolutely Riemann integrable over $[0, 1]$.

2. Let A be an interval in $\mathbb{R}$. Let f and g be absolutely Riemann integrable over A. Prove or disprove that fg must be Riemann integrable over A.

3. Prove that the function $f(x) = \dfrac{\sin x}{x}$ is not absolutely Riemann integrable over $[1, \infty)$.

4. A real number α is said to be an algebraic number if there exists a polynomial p with integer coefficients such that $p(\alpha) = 0$. Let $E = \{\alpha \in \mathbb{R} : \alpha \text{ is an algebraic number}\}$. Prove or disprove that E has measure zero.

5. Let $f : [a, b] \to \mathbb{R}$ be monotonic. Prove or disprove that f is Riemann integrable over $[a, b]$.