Chapter 23: Electric Potential

Section 23-1: Electric Potential

- Coulomb's law is just like Newton's law of gravity
- Therefore both have a potential energy

$$\Delta E = \Delta K + \Delta U = 0$$

$\Delta U = -\Delta K = -W$ (work-energy theorem)

- For constant electric field (therefore constant force)

$$\vec{F} = q\vec{E}$$

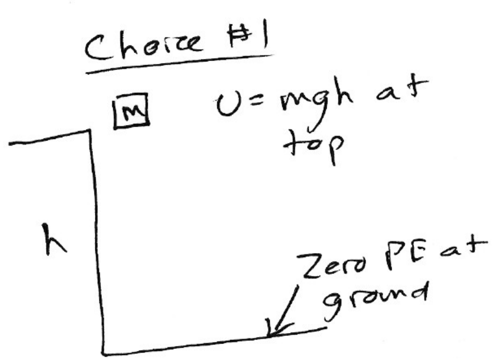
$$W = \vec{F} \cdot \Delta\vec{x} = -q\vec{E} \cdot \Delta\vec{x}$$

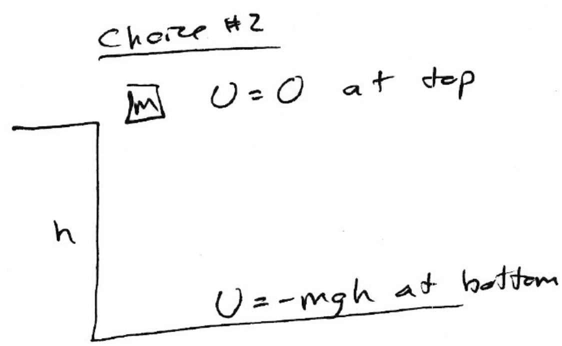
$$\Delta U = -q\vec{E} \cdot \Delta x$$

- Zero energy at arbitrary location

Zero gravity PE can be anywhere (floor, tabletop, ground, etc.)

Any choice is OK physically (but might be inconvenient)





- Electric potential (same as voltage) is defined as:

$\Delta V = \Delta U/q$ or $\boxed{\Delta U = q\Delta V}$

$$\boxed{\Delta V = -\vec{E} \cdot \Delta\vec{x}}$$

Units: Volts, V=J/C

- Particle motion:

Forces <u>always</u> push towards lower PE (rocks roll <u>down</u>hill)

Therefore ΔU is negative

Since $\Delta U = q\Delta V$:

Positive charge pushed towards lower V

Negative charge pushed towards higher V

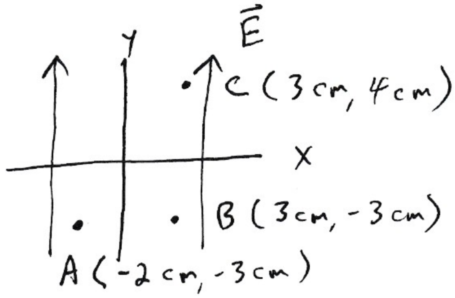
A 3mC charge moves from (1cm, 3cm) to (4cm, -2cm) in a uniform electric field $\vec{E} = 20{,}000 \frac{N}{C} \hat{\imath}$. What is the change in PE, ΔU?

$$\Delta \vec{x} = (4cm - 1cm)\hat{\imath} + (-2cm - 3cm)\hat{\jmath}$$
$$= 0.03m\,\hat{\imath} - 0.05\,m\,\hat{\jmath}$$

$$\Delta V = -\vec{E} \cdot \Delta\vec{x} = -(0.03m\,\hat{\imath} - 0.05m\,\hat{\jmath}) \cdot 20000\tfrac{N}{C}\,\hat{\imath}$$
$$= -600\ V$$

$$\Delta U = q\Delta V = 3 \times 10^{-3}\,C \cdot -600\,V = \boxed{-1.8\,J}$$

<u>Example:</u> The electric field below is $5\frac{V}{m}$. Find ΔV from A to B and from B to C.



From A to B: Δx is to the right, $\vec{E}$ is up (perp), so $\Delta V = -\vec{E} \cdot \Delta \vec{x} = \boxed{0}$

From B to C: $\Delta \vec{x}$ is up, 7 cm distance

$\Delta V = -\vec{E} \cdot \Delta \vec{x} = -5\frac{V}{m} \cdot 0.07m$ (same direction, dot prod. positive)

$= \boxed{-0.35\ V}$

Section 23-2: Electric Potential from Electric Field

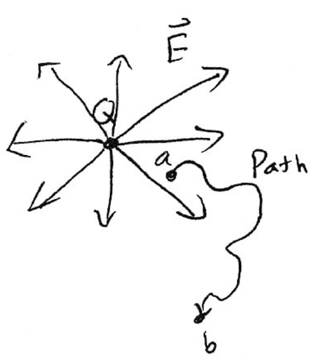
- For non-constant force:

$$W = \int \vec{F} \cdot d\vec{x}$$

$$\Delta U = -\int q\vec{E} \cdot d\vec{x}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{x}$$

Section 23-3: Potential Due to Point Charge



$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{x}$$

Because $\vec{E}$ points out in radial direction, only changes in r matter as we move along a path

$$\Delta V = -\int_a^b E\,dr$$

$$\Delta V = -\int_a^b E dr$$
$$= kQ \cdot \frac{1}{r}\Big|_{r_a}^{r_b}$$
$$= \frac{kQ}{r_b} - \frac{kQ}{r_a}$$

- Can choose V=0 where we want, just like choosing U=0 for gravity
- Convenient choice: V=0 when $r = \infty$
- So if we choose $V_a = 0$ and $r_a = \infty$, we get:

$V = \frac{kQ}{r}$ (drop the subscript *b*)

Use negative values for *Q*; neg. charge makes neg. voltage

A 3μC charge sits at the origin and a
-2μC charge is at y = 12 cm on the y-axis.
A 5nC charge moves from (0, 5 cm) to (5cm, 0).
What is its change in potential energy?

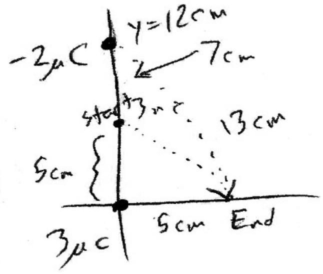
- Note on problems like this:

Moving charge feels different voltage at start and end points

This charge (5 nC) will go into $\Delta U = q\Delta V$

Fixed (unmoving) charges created the voltage at start/end points

These charges (3 μC and -2 μC) go in $V = \frac{kQ}{r}$



$$V_0 = \frac{8.99\times10^9\,\frac{Nm^2}{C^2}\cdot 3\times10^{-6}C}{0.05m} + \frac{8.99\times10^9\,\frac{Nm^2}{C^2}\cdot -2\times10^{-6}C}{0.07m}$$

$$= 283\,000\,V$$

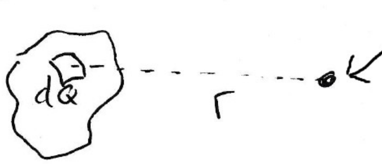
$$V = \frac{8.99\times10^9\,\frac{Nm^2}{C^2}\cdot 3\times10^{-6}C}{0.05m} + \frac{8.99\times10^9\,\frac{Nm^2}{C^2}\cdot -2\times10^{-6}C}{0.13m}$$

$$= 401\,000\,V$$

$$\Delta V = 401\,000\,V - 283\,000\,V$$
$$= 118000\,V$$
$$\Delta U = q\Delta V = 5\times10^{-9}C\cdot 118\,000\,V$$
$$= \boxed{5.92\times10^{-4}\,J}$$

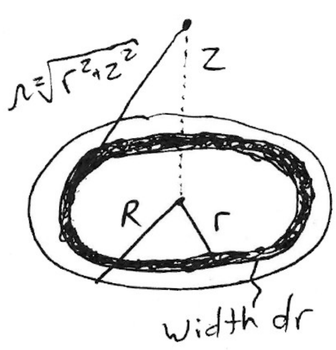
Section 23-4: Potential Due to a Distribution

- Divide a distribution into infinitesimal parts and integrate:



$V = \int \frac{k\,dq}{r}$

Example: Find V a distance z above the center of a disk with σ charge density and radius R



$dQ = \sigma\,dA$

Cut ring, stretch out:

$dA = 2\pi r\,dr$

$dQ = 2\pi\sigma\,r\,dr$

$$dV = \frac{k\,dQ}{r} = \frac{k \cdot 2\pi\sigma r\,dr}{\sqrt{r^2+z^2}}$$

$$V = \int dV$$

$$V = \int_0^R \frac{k \cdot 2\pi\sigma r}{\sqrt{r^2+z^2}}\,dr$$

Let $u = r^2$, $du = 2r\,dr$

$$V = \int_{r=0}^{r=R} \frac{\pi k\sigma}{\sqrt{u+z^2}}\,du$$

$$= \pi k\sigma \cdot 2(u+z^2)^{1/2}\Big|_{r=0}^{r=R}$$

$$= 2\pi k\sigma\sqrt{r^2+z^2}\Big|_{r=0}^{r=R}$$

$$= 2\pi k\sigma\left(\sqrt{R^2+z^2} - \sqrt{z^2}\right)$$

using $k = \frac{1}{4\pi\varepsilon_0}$:

$$\boxed{V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - z\right)}$$

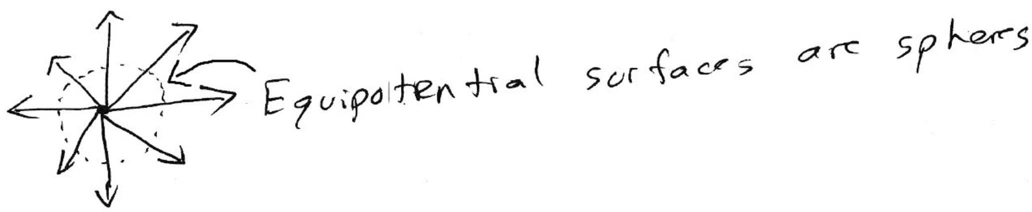
[*May skip Section 23-5 if time is short.*]

Section 23-5: Equipotential Surfaces

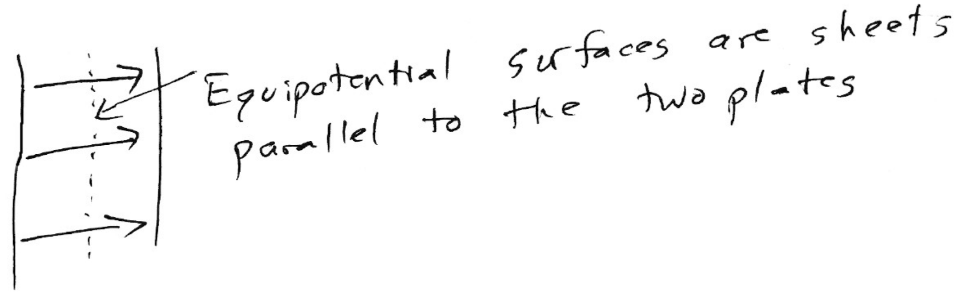
- Since $\Delta V = -\int \vec{E} \cdot d\vec{x}$, *V* is constant if $d\vec{x}$ is perp. to $\vec{E}$

- Equipotential means *V* is consant (“equi”=equal)

- <u>Point charge</u>:



- <u>Parallel plates</u>:



<u>Section 23-7: Electric field from potential</u>

- ΔV is (minus) the integral of $\vec{E}$, $\Delta V = -\int \vec{E} \cdot d\vec{x}$
- Therefore $\vec{E}$ should be the opposite (derivative) of V
- Not just a regular integral, since V is a scalar and $\vec{E}$ is a vector
- It's a (negative) <u>gradient</u>:

$$\boxed{E_x = -\frac{\partial V}{\partial x}} \qquad \boxed{E_y = -\frac{\partial V}{\partial y}} \qquad \boxed{E_z = -\frac{\partial V}{\partial z}}$$

- Partial derivative: treat other variables as if constant

<u>Example:</u> If $V = \frac{Axy^2}{z}$, find $\vec{E}$ (ie its components)

$$E_x = -\frac{\partial V}{\partial x} = \boxed{-\frac{Ay^2}{z}}$$

$$E_y = -\frac{\partial V}{\partial y} = \boxed{-\frac{2Axy}{z}}$$

$$E_z = -\frac{\partial V}{\partial z} = -Axy^2 \cdot -\frac{1}{z^2} = \boxed{\frac{Axy^2}{z^2}}$$

<u>Example:</u> A 200 g particle with 20 mC charge is at (1, 2, −1) in a voltage given by $V = 3x^2 + 2xy - yz^2$. Find the acceleration.

$$E_x = -\frac{\partial V}{\partial x} = -(6x + 2y) \xrightarrow{\text{at } (1,2,-1)} E_x = -(6+4) = -10$$

$$E_y = -\frac{\partial V}{\partial y} = -(2x - z^2) \longrightarrow E_y = -(2-1) = -1$$

$$E_z = -\frac{\partial V}{\partial z} = -(-2yz) \longrightarrow E_z = 2 \cdot 2 \cdot -1 = -4$$

$$\vec{F} = q\vec{E}$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

$$a_x = \frac{q}{m}E_x = \frac{0.02C}{0.2kg} \cdot \left(-10\frac{N}{C}\right) = \boxed{-1\ m/s^2}$$

$$a_y = \frac{q}{m}E_y = \frac{0.02C}{0.2kg} \cdot \left(-1\frac{N}{C}\right) = \boxed{-0.1\ m/s^2}$$

$$a_z = \frac{q}{m}E_z = \frac{0.02C}{0.2kg} \cdot \left(-4\frac{N}{C}\right) = \boxed{-0.4\ m/s^2}$$

<u>Section 23-8: The Electron Volt</u>

- Voltage is energy per unit charge, V=J/C
- So a voltage times a charge is an energy
- If we take electron charge times volts, we have an energy unit:

$$\text{eV} = 1.6 \times 10^{-19}\ \text{C} \times 1\ \text{V} = 1.6 \times 10^{-19}\ \text{J}$$

- Note: if you want energy in eV, <u>do not substitute</u> for *e*!

A proton with 2 MeV of kinetic energy is released 20 cm from a 50 μC charge. How fast is it moving after moving to 50 cm away? The proton mass is 1.67×10^{-27} kg.

$$V_0 = \frac{kQ}{r_0} = \frac{8.99 \times 10^9 \frac{Nm^2}{C^2} \cdot 50 \times 10^{-6} C}{0.2 m} = 2.25 \times 10^6 V$$

$$V = \frac{kQ}{r} = \frac{8.99 \times 10^9 Nm^2/C^2 \cdot 50 \times 10^{-6} C}{0.5 m} = 8.99 \times 10^5 V$$

$$U_0 = q V_0 = e \cdot 2.25 \times 10^6 V = 2.25 MeV$$

$$K_0 = 2 MeV$$

$$U = qV = e \cdot 8.99 \times 10^5 V = 0.899 MeV$$

$$K = ?$$

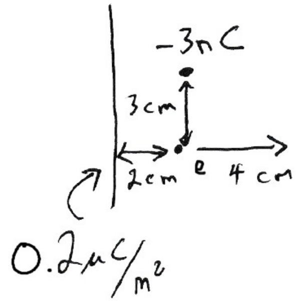
$$K_0 + U_0 = K + U$$

$$2 MeV + 2.25 MeV = K + 0.899 MeV$$

$$K = 3.35 MeV \times \frac{1.6 \times 10^{-19} J}{1 eV} = 5.36 \times 10^{-13} J$$

$$V = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \cdot 5.36 \times 10^{-13} J}{1.67 \times 10^{-27} kg}} = \boxed{2.53 \times 10^7 m/s}$$

<u>Example</u>: An electron moves as shown below. If its initial KE is 1 KeV, what is its final KE?



Plane charge: $E = \frac{\sigma}{2\varepsilon_0} = \frac{0.2 \times 10^{-6}\ C/m^2}{2 \cdot 8.85 \times 10^{-12}\ C^2/Nm^2}$

$= 11300 \frac{V}{m}$

$\vec{E}$ and $\Delta\vec{x}$ in same direction

$\Delta V = -\vec{E} \cdot \Delta\vec{x} = -11300 \frac{V}{m} \cdot 0.04\,m = -452\,V$

Point charge: $V_0 = \frac{kQ}{r_0} = \frac{8.99 \times 10^9\ Nm^2/C^2 \cdot -3 \times 10^{-9}\ C}{0.03\ m}$

$= -899\,V$

$V = \frac{kQ}{r} = \frac{8.99 \times 10^9\ Nm^2/C^2 \cdot -3 \times 10^{-9}\,C}{0.05\,m}$

$= -539\,V$

$\Delta V_{charge} = -539\,V - (-899\,V) = 360\,V$

$\Delta V_{total} = 360\,V - 452\,V = -92\,V$

$\Delta U = q\Delta V = -e \cdot -92\,V = 92\,eV$

$K_0 + U_0 = K + U$

$K = K_0 + U_0 - U = K_0 - \Delta U$ since $\Delta U = U - U_0$

$= 1000\,eV - 92\,eV = \boxed{908\,eV}$

Homework: Do Chapter 23 in Mastering Physics

Exam #1 on Chapters 21-23

Exam review materials available:

- These lecture notes (Canvas home page)
- Webex recordings (in Canvas under Cisco Webex link)
- Practice exam (Canvas home page)
- Conceptual review practice assignment (in Mastering Physics)
    - Practice for multiple-choice conceptual questions