Chapter 26: DC Circuits

Section 26-1: EMF

- Batteries have resistance called internal resistance *r*

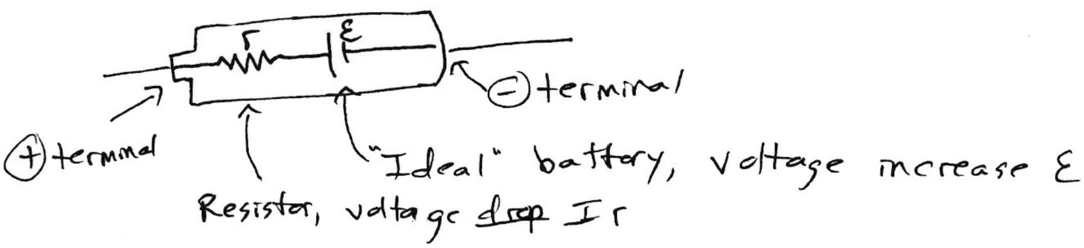
- Resistance decreases voltage by *Ir* (Ohm's Law)

- Terminal voltage is actual voltage of battery, including resistance

- Electromotive force (emf) is voltage when no current flows

$$\boxed{V = \mathcal{E} - Ir}$$

- A real battery can be thought of as a series circuit:

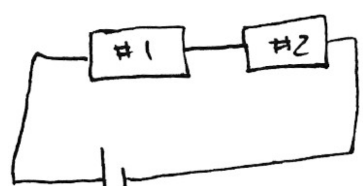


- Overall voltage across both is $\mathcal{E} - Ir$

Section 26-2: Series and Parallel Resistors

- Recall rules of circuits:

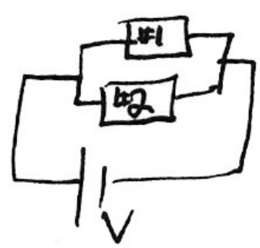
- Series:



Voltages add, $V = V_1 + V_2$

Same charge *Q*, so same current *I*

- Parallel:



Same voltage *V*

Currents add, $I = I_1 + I_2$

- Series resistors

$$V = V_1 + V_2 = IR_1 + IR_2 = I(R_1 + R_2)$$

So series circuit <u>acts</u> like single resistor $\boxed{R_{ser} = R_1 + R_2}$

Note this is the same as <u>parallel</u> capacitors

N identical resistors: $R_{ser} = R + R + R + \ldots$ or $\boxed{R_{ser} = NR}$

More in series gives bigger resistance

- Parallel resistors

$$I = I_1 + I_2$$

$$\frac{V}{R_{par}} = \frac{V}{R_1} + \frac{V}{R_2}$$

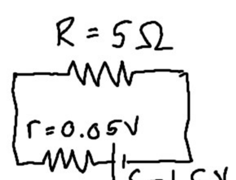
$$\boxed{\frac{1}{R_{par}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Note this is the same as <u>series</u> capacitors

N identical in parallel: $\frac{1}{R_{par}} = \frac{1}{R} + \frac{1}{R} + \ldots = \frac{N}{R}$ or $\boxed{R_{par} = \frac{R}{N}}$

More resistors in parallel makes resistance <u>smaller</u>

<u>Example</u>: A 1.5V battery has 0.05Ω internal resistance. How much power will it deliver to a 5Ω light bulb?



Treat real battery as ideal battery and resistor in series

$R_{ser} = 5\Omega + 0.05\Omega = 5.05\Omega$

$V = IR_{ser}$ so $I = \frac{V}{R_{ser}} = \frac{1.5V}{5.05\Omega} = 0.297A$

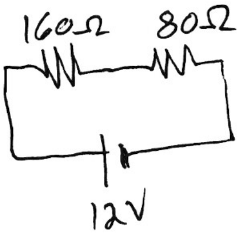
Don't use $P = VI$ or $P = \frac{V^2}{R}$ since we don't know V for light bulb

$P = I^2R = (0.297A)^2 \cdot 5\Omega = \boxed{0.441W}$

Resistance of bulb to find bulb's power

[*May skip this example if time is short.*]

Example: A 12 V battery is connected in series to a 160 Ω and an 80 Ω resistor. What is the voltage drop on the 80 Ω resistor?



Whole circuit: $R_{eq} = 160\,\Omega + 80\,\Omega = 240\,\Omega$

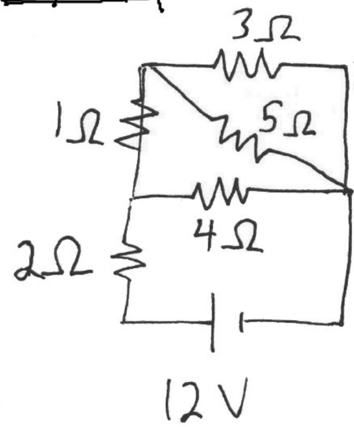
$$V = I R_{eq}$$

$$I = \frac{V}{R_{eq}} = \frac{12\,V}{240\,\Omega} = 0.05\,A$$

80 Ω resistor: $V = IR = 0.05\,A \cdot 80\,\Omega$

$$= \boxed{4\,V}$$

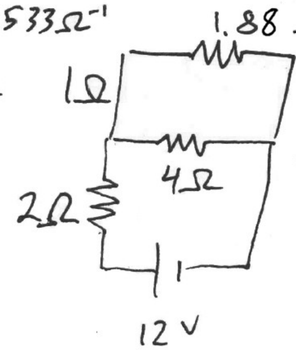
Example: Find the current in the 3 Ω resistor



$$\frac{1}{R_{par}} = \frac{1}{3\,\Omega} + \frac{1}{5\,\Omega} = 0.533\,\Omega^{-1}$$

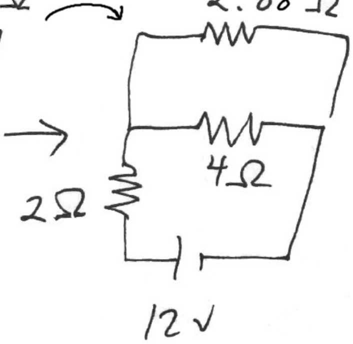
$$R_{par} = 1.88\,\Omega$$

→



$R_{ser} = 1.88\,\Omega + 1\,\Omega = 2.88\,\Omega$

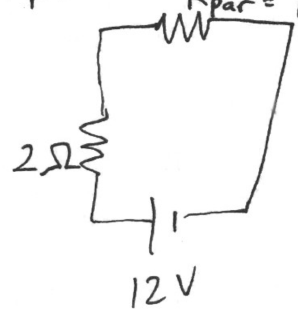
→



$$\frac{1}{R_{par}} = \frac{1}{2.875\,\Omega} + \frac{1}{4\,\Omega} = 0.598\,\Omega^{-1}$$

$$R_{par} = 1.67\,\Omega$$

→



$R_{ser} = 2\,\Omega + 1.67\,\Omega = 3.67\,\Omega$

Total resistance: 3.67 Ω

$$I = \frac{V}{R} = \frac{12\,V}{3.67\,\Omega} = 3.27\,A$$

Voltage on 1.67Ω (last diagram):

$V = IR = 3.27\,A \cdot 1.67\,\Omega = 5.46\,V$ Parallel: Same V

Current in $2.88\,\Omega$ (3^{rd} diagram):

$I = \frac{V}{R} = \frac{5.46V}{2.88\Omega} = 1.90A$ series: Same I

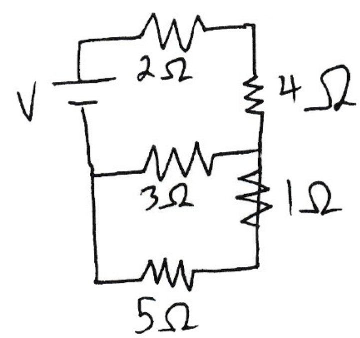
Voltage on 1.88Ω (2^{nd} diagram)

$V = IR = 1.90A \cdot 1.88\Omega = 3.56V$ Parallel: Same V

Current in 3Ω (1^{st} diagram)

$I = \frac{V}{R} = \frac{3.56V}{3\Omega} = \boxed{1.19A}$

Example: If there is 2V on the 1Ω resistor, what is V, the battery voltage?

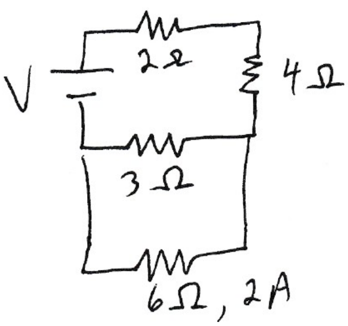


On 1Ω: $V = IR$ or $I = \frac{V}{R} = \frac{2V}{1\Omega} = 2A$

1Ω is in series with 5Ω

$R_{ser} = 5\Omega + 1\Omega = 6\Omega$

Series: Current is the same



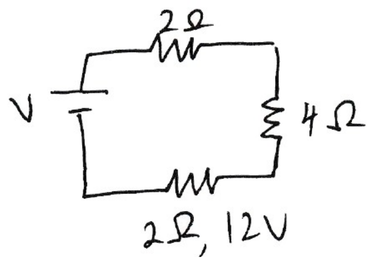
On 6Ω: $V = IR = 2A \cdot 6\Omega = 12V$

6Ω in parallel with 3Ω

$\frac{1}{R_{par}} = \frac{1}{3\Omega} + \frac{1}{6\Omega} = 0.5\,\Omega^{-1}$

$R_{par} = 2\Omega$

Parallel: Voltage the same



On bottom 2Ω: $I = \frac{V}{R} = \frac{12\,V}{2\,\Omega} = 6\,A$

Series circuit: same current for all

$R_{ser} = 2\,\Omega + 2\,\Omega + 4\,\Omega = 8\,\Omega$

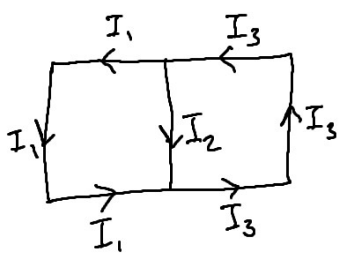
$V = IR = 6\,A \cdot 8\,\Omega = \boxed{48\,V}$

Section 26-3: Kirchhoff's Rules

- Intersection Rule: Current into junction equals current out

  Think about traffic at an intersection

  When analyzing circuits: currents go from one intersection to the next



$I_1 + I_2 = I_3$ in above diagram

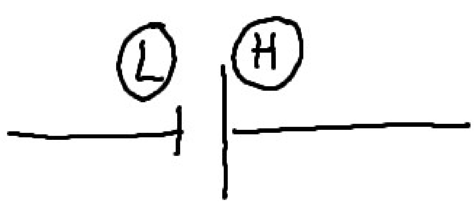
- Loop Rule: $\Delta V = 0$ around a loop

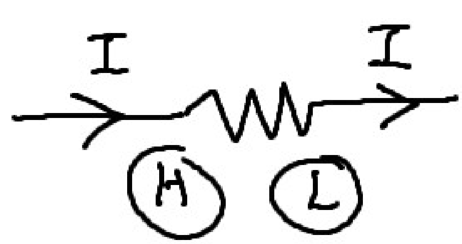
  $\Delta V = -\int \vec{E} \cdot d\vec{\ell} = 0$ (conservative force, like gravity)

- Voltage changes:

  V, for batteries — *IR*, for resistors

  Short end is low voltage — High in, low out

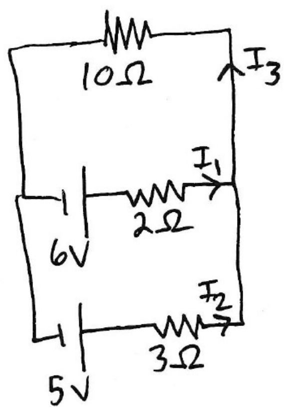




- Steps:
    - Label currents, if not given

        Currents from one intersection all the way to another
    - Do intersection rule
    - Label High/Low for each battery/resistor
    - Do loop rule
        - Starting point / direction do not matter

Example: Find the voltage drop on 10 Ω resistor



Left junction: $I_3 = I_1 + I_2$ ①

Right junction: $\cancel{I_1 + I_2 = I_3}$ (redundant)

Top loop: $-10 I_3 + 6 - 2 I_1 = 0$ ②

Bottom loop: $5 - 3 I_2 + 2 I_1 - 6 = 0$ ③

Outer loop: $5 - 3 I_2 - 10 I_3 = 0$ (redundant)

- <u>Must</u> use both rules for every problem!

    Some intersections and loops <u>always</u> redundant
- System of equations:

    Homework: Use whatever (Wolfram Alpha is free)

    Exam: Don’t solve for currents, just write down equations
- Above solution is $I_1 = 0.5$ A, $I_2 = 0$, $I_3 = 0.5$ A
- If a current is negative: it goes opposite direction to what is drawn

Section 26-5: RC Circuits

- Charging a capacitor

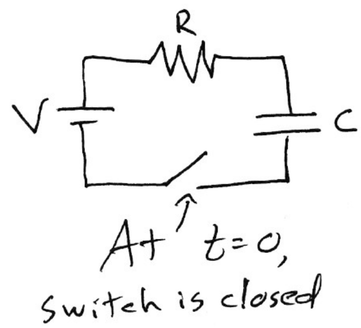
$$V = V_R + V_C$$

$$= IR + \frac{Q}{C}$$

$$= \frac{dQ}{dt} \cdot R + \frac{Q}{C}$$

$$V - \frac{Q}{C} = R\frac{dQ}{dt}$$

$$dt = \frac{R}{V - Q/C} dQ = \frac{RC}{CV - Q} dQ$$



- Variables are separated, so integrate:

$$\int_0^t dt = \int_0^Q \frac{RC}{CV - Q} dQ$$

$$t = -RC(\ln(CV - Q) - \ln(CV)) = -RC \ln \frac{CV - Q}{CV}$$

$$t = -RC \ln \left(1 - \frac{Q}{CV}\right)$$

$$1 - \frac{Q}{CV} = e^{-t/RC}$$

Solve for Q: $\boxed{Q = CV\left(1 - e^{-t/RC}\right)}$

- How to think of this equation:

$Q = CV$ would be charge if there were no resistor

$1 - e^{-t/RC}$ determines how fast it "turns on"

It is fraction/percent of total of total charge at time t

- Other variables:

$$V_C = \frac{Q}{C} \text{ or } \boxed{V_C = V\left(1 - e^{-t/RC}\right)}$$

$$V_R = V - V_C \text{ or } \boxed{V_R = Ve^{-t/RC}}$$

$$I = \frac{V_R}{R} \text{ so } \boxed{I = \frac{V}{R}e^{-t/RC}}$$

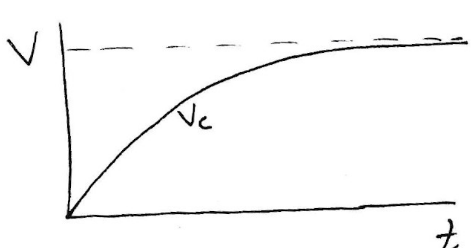
- How to think of this equation:

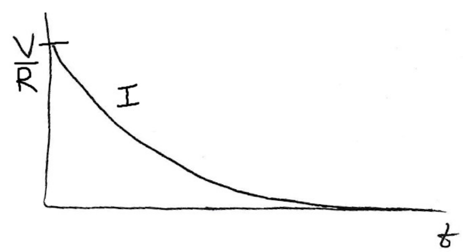
  With no capacitor, current is $I = \dfrac{V}{R}$

  As capacitor fills up, current will turn off

  $e^{-t/RC}$ is fraction/percent of starting current that remains

- Graphs:





- V_C approaches battery voltage
- Also Q approaches full CV

- I approaches zero (cap. is full, so no more current can flow)
- Full current initially $(I = V/R)$

Example: A 20 μF capacitor is charged up by a 6 V battery through a 12 Ω resistor. What is the current when the charge on the capacitor is 30 μC ?

$$Q = CV\left(1 - e^{-t/RC}\right)$$

$$\underbrace{30\mu C}_{\text{current } 30\mu C} = \underbrace{20\mu F \cdot 6\,V}_{\text{Full charge is } 120\mu C}\left(1 - e^{-t/RC}\right)$$

$$\frac{30\mu C}{120\mu C} = 1 - e^{-t/RC}$$

$$\underbrace{0.25}_{25\%\text{ charged}} = \underbrace{1 - e^{-t/RC}}_{\text{current fraction}}$$

$$-0.75 = -e^{-t/RC}$$

$$e^{-t/RC} = 0.75$$

$$I = \frac{V}{R} e^{-t/RC}$$

$$= \underbrace{\frac{6\,V}{12\,\Omega}}_{\substack{\text{starting}\\ \text{current}}} \cdot \underbrace{0.75}_{\substack{75\% \text{ remains when}\\ 25\% \text{ charged}}}$$

$$= 0.5\,A \cdot 0.75 = \boxed{0.375\,A}$$

- Discharging a capacitor

At $t=0$, capacitor already charged to V and switch is closed

$V_R = V_C$

Very similar derivation

$\boxed{Q = CV_0 e^{-t/RC}}$ V_0 is initial voltage

$V_C = \dfrac{Q}{C}$ so $\boxed{V_C = V_0 e^{-t/RC}}$

$V_R = V_C$ so $\boxed{V_R = V_0 e^{-t/RC}}$

$I = \dfrac{V_R}{R}$ so $\boxed{I = \dfrac{V_0}{R} e^{-t/RC}}$

- Time constant

Because RC determines how fast capacitor is discharged it is called the *time constant* τ (Greek tau), $\boxed{\tau = RC}$

Has units of seconds (exponents must have no units)

<u>Example</u>: A 100 nF capacitor is charged to 20V. It then discharges through a 15 Ω resistor. After 1.5 time constants, (a) What is the charge on the capacitor? (b) What is the current?

"1.5 time constants" means $t = 1.5\tau = 1.5RC$

$$e^{-t/RC} = e^{-1.5\cancel{RC}/\cancel{RC}} = e^{-1.5} = 0.223$$

(a) $Q = CVe^{-t/RC} = 100\,nF \cdot 20V \cdot 0.223 = \boxed{44.6\,nC}$

(b) $I = \frac{V}{R}e^{-t/RC} = \frac{20V}{15\Omega} \cdot 0.223 = \boxed{0.298\,A}$

Homework: Do Chapter 26 in Mastering Physics

Exam #2 on Chapters 24-26

Exam review materials available:

- These lecture notes (Canvas home page)
- Webex recordings (in Canvas under Cisco Webex link)
- Practice exam (Canvas home page)
- Conceptual review practice assignment (in Mastering Physics)
  - Practice for multiple-choice conceptual questions
  - Not for credit but strongly recommended