Assignment 3 Variational Autoencoders

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1 Implementing the Model [5 points]

```
# Load all the packages, provided functions, in order for the jmd to compile
using Flux
using MLDatasets
using Statistics
using Logging
using Test
using Random
using StatsFuns: log1pexp, logit
using Distributions
Random.seed!(412414);

# log-pdf of x under Factorized or Diagonal Gaussian N(x|μ,σI)
function factorized_gaussian_log_density(mu, logsig,xs)
  """
  mu and logsig either same size as x in batch or same as whole batch
  returns a 1 x batchsize array of likelihoods
  """
  σ = exp.(logsig)
  return sum((-1/2)*log.(2π*σ.^2) .+ -1/2 * ((xs .- mu).^2)./(σ.^2),dims=1)
end

# log-pdf of x under Bernoulli
function bernoulli_log_density(logit_means,x)
  """Numerically stable log_likelihood under bernoulli by accepting μ/(1-μ)"""
  b = x .* 2 .- 1 # {0,1} -> {-1,1}
  return - log1pexp.(-b .* logit_means)
end

# sample from Diagonal Gaussian x~N(μ,σI) (hint: use reparameterization trick here)
sample_diag_gaussian(μ,logσ) = (ϵ = randn(size(μ)); μ .+ exp.(logσ).*ϵ)
# sample from Bernoulli (this can just be supplied by library)
sample_bernoulli(θ) = rand.(Bernoulli.(θ))

## Amortized Inference
function unpack_gaussian_params(θ)
  μ, logσ = θ[1:2,:], θ[3:end,:]
  return  μ, logσ
end

# Load MNIST data, binarise it, split into train and test sets (10000 each) and
partition train into mini-batches of M=100.
# You may use the utilities from A2, or dataloaders provided by a framework
```

```
function load_binarized_mnist(train_size=1000, test_size=1000)
  train_x, train_label = MNIST.traindata(1:train_size);
  test_x, test_label = MNIST.testdata(1:test_size);
  @info "Loaded MNIST digits with dimensionality $(size(train_x))"
  train_x = reshape(train_x, 28*28,:)
  test_x = reshape(test_x, 28*28,:)
  @info "Reshaped MNIST digits to vectors, dimensionality $(size(train_x))"
  train_x = train_x .> 0.5; #binarize
  test_x = test_x .> 0.5; #binarize
  @info "Binarized the pixels"
  return (train_x, train_label), (test_x, test_label)
end

function batch_data((x,label)::Tuple, batch_size=100)
  """
  Shuffle both data and image and put into batches
  """
  N = size(x)[end] # number of examples in set
  rand_idx = shuffle(1:N) # randomly shuffle batch elements
  batch_idx = Iterators.partition(rand_idx,batch_size) # split into batches
  batch_x = [x[:,i] for i in batch_idx]
  batch_label = [label[i] for i in batch_idx]
  return zip(batch_x, batch_label)
end
# if you only want to batch xs
batch_x(x::AbstractArray, batch_size=100) =
first.(batch_data((x,zeros(size(x)[end])),batch_size))

## Model Dimensionality
# #### Set up model according to Appendix C (using Bernoulli decoder for Binarized
MNIST)
# Set latent dimensionality=2 and number of hidden units=500.
Dz, Dh = 2, 500
Ddata = 28^2
```

```
784
```

(a) [1 point] Implement a function log_prior that computes the log of the prior.

```
function log_prior(z)
  factorized_gaussian_log_density(0,0,z)
end
```

```
log_prior (generic function with 1 method)
```

(b) [2 points] Implement a function decoder.

```
decoderm = Chain(Dense(Dz, Dh, tanh), Dense(Dh, Ddata)) # The MLP model outside the
function to pass to train function

function decoder(z)
    m = decoderm(z)
    return m
end
```

```
decoder (generic function with 1 method)
```

(c) [1 point] Implement a function log_likelihood

```
function log_likelihood(x,z)
  """ Compute log likelihood log_p(x|z)"""
```

```
  return sum(bernoulli_log_density(decoder(z),x), dims=1) # return likelihood for each
element in batch
end
```

```
log_likelihood (generic function with 1 method)
```

(d) [1 point] Implement a function joint*log*density.

```
function joint_log_density(x,z)
  return log_prior(z) .+ log_likelihood(x,z)
end
```

```
joint_log_density (generic function with 1 method)
```

2 Amortized Approximate Inference and training [13 points]

(a) [2 points] Write a function encoder.

```
encoderm=Chain(Dense(Ddata, Dh, tanh),Dense(Dh, Dz*2),)#The MLP model outside the
function to pass to train function

function encoder(x)
    m = unpack_gaussian_params(encoderm(x))
    return m
end
```

```
encoder (generic function with 1 method)
```

(b) [1 points] Write a function log_q.

```
function log_q(q_μ, q_logσ, z)
    return factorized_gaussian_log_density(q_μ, q_logσ, z)
end
```

```
log_q (generic function with 1 method)
```

(c) [5 points] Implement a function elbo.

```
function elbo(x)
  q_μ, q_logσ = encoder(x)#variational parameters from data
  zq = sample_diag_gaussian(q_μ,q_logσ) #sample from variational distribution
  log_q_z =log_q(q_μ, q_logσ, zq)#likelihood of z under variational distribution
  joint_ll = joint_log_density(x,zq)# joint likelihood of z and x under model
  elbo_estimate = sum(joint_ll .- log_q_z) ./ size(q_μ)[2]#Scalar value, mean
variational evidence lower bound over batch
  return elbo_estimate
end
```

```
elbo (generic function with 1 method)
```

(d) [2 points] Write a loss function called loss that returns the negative elbo estimate over a batch of data.

```
function loss(x)
  return -1 .* elbo(x)#scalar value for the variational loss over elements in the
batch
end
```

```
loss (generic function with 1 method)
```

(e) [3 points] Write a function that initializes and optimizes the encoder and decoder parameters jointly on the training set.

```
# Training with gradient optimization:

function train_model_params!(loss, encoder, decoder, train_x, test_x; nepochs=10)
  # model params
  ps = Flux.params(encoder, decoder)#parameters to update with gradient descent
  # ADAM optimizer with default parameters
  opt = ADAM()
  # over batches of the data
  for i in 1:nepochs
    for d in batch_x(train_x)
      gs = Flux.gradient(ps) do
        batch_loss = loss(d)
        return batch_loss# compute gradients with respect to variational loss over
batch
      end
      # update the paramters with gradients
      Flux.Optimise.update!(opt,ps,gs)
    end
    if i%1 == 0 # change 1 to higher number to compute and print less frequently
      @info "Test loss at epoch $i: $(loss(batch_x(test_x)[1]))"
    end
  end
  @info "Parameters of encoder and decoder trained!"
end

## Load the Data
train_data, test_data = load_binarized_mnist(10000, 10000)
train_x, train_label = train_data;
test_x, test_label = test_data;
```

Use the following line to train the model, but comment out during compiling of jmd as it takes long time. The result was saved using provided method.

```
## Train the model
# train_model_params!(loss,encoderm,decoderm,train_x,test_x, nepochs=100)
```

Final ELBO on test set:

```
# Load the saved result from training
using BSON:@load
cd(@__DIR__)
@info "Changed directory to $(@__DIR__)"
load_dir = "trained_models"
@load joinpath(load_dir,"encoder_params.bson") encoderm
@load joinpath(load_dir,"decoder_params.bson") decoderm
@info "Load model params from $load_dir"

elbo(test_x)

-155.66149211739722
```

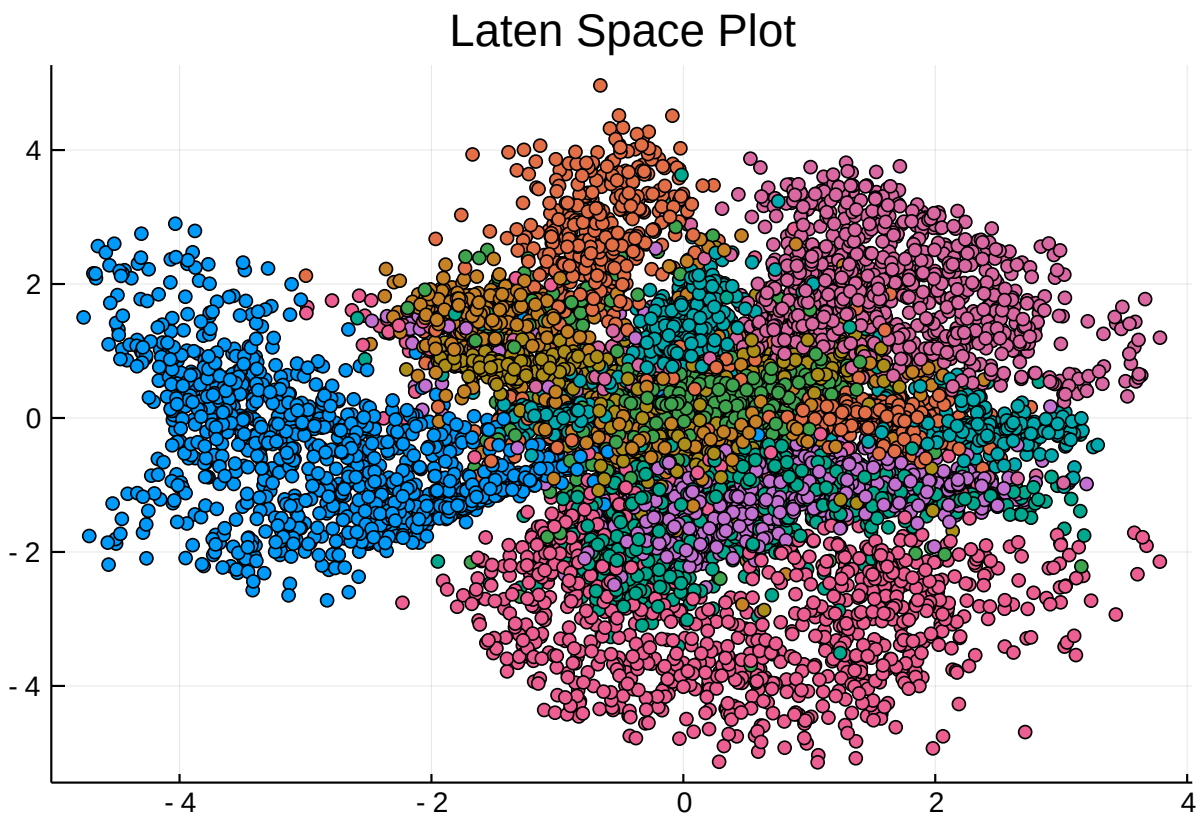
3 Visualizing Posteriors and Exploring the Model [15 points]

(a) [5 points] Plot samples from the trained generative model using ancestral sampling.

```
using Images
using Plots
# make vector of digits into images, works on batches also
mnist_img(x) = ndims(x)==2 ? Gray.(reshape(x,28,28,:)) : Gray.(reshape(x,28,28))

plot_list=Matrix{Union{Nothing, Any}}(nothing, 2, 10)
for i in 1:10
  z = randn(Dz,1)
  logitz = decoder(z)
  means = exp.(logitz)./ (1 .+ exp.(logitz))
  x = sample_bernoulli(means)
  plot_list[1,i] = plot(mnist_img(vec(means)),framestyle=:none)
  plot_list[2,i] = plot(mnist_img(vec(x)),framestyle=:none)
end

display(plot(plot_list[1,:]...,plot_list[2,:]...,layout=(2,10),size = (800,400)))
```

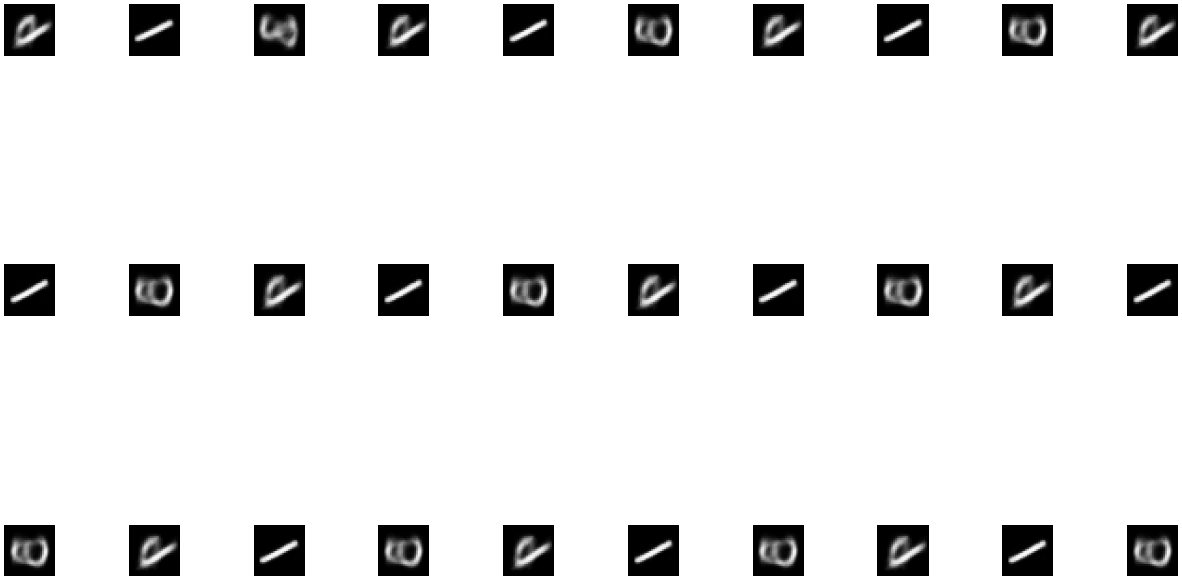
(b) [5 points] Produce a scatter plot in the latent space, where each point in the plot represents a different image in the training set.

```
p1=plot()
q_μ= encoder(train_x[:,1])[1]
p1=plot!(q_μ[1,:], q_μ[2,:], seriestype = :scatter, title = "Laten Space Plot",
    color=train_label[1],legend=false)
for i in 2:10000
  d= train_x[:,i]
  q_μ= encoder(d)[1]
  colour=train_label[i]
  plot!(p1, q_μ[1,:], q_μ[2,:], seriestype = :scatter, color=colour,legend=false)
end
display(p1)
```



(c) Display generative distribution along linear interpolation

```
function linear_interp(Za, Zb, alpha)
  Za = alpha .* Za .+ (1 .- alpha) .* Zb
  return Za
end

plot_list=Matrix{Union{Nothing, Any}}(nothing, 3, 10)
p2=plot()
for i in 1:3
  x = rand(1:size(train_x)[2])
  y = rand(1:size(train_x)[2])
  while train_label[x] == train_label[y]
    y = rand(1:size(train_x)[2])
  end
  Za = encoder(train_x[:,x])[1]
  Zb = encoder(train_x[:,y])[1]
  for j in 1:10
    alpha = j/10
    Za = linear_interp(Za, Zb, alpha)
    logitz = decoder(Za)
    means=exp.( logitz) ./ (1 .+ exp.( logitz))
    plot_list[i,j] = plot(mnist_img(vec(means)),framestyle=:none)
  end
end
display(plot(plot_list...,layout=(3,10),size = (800,500)))
```



4 Predicting the Bottom of Images given the Top [15 points]

(a) [5 points] Write a function that computes p(z, top half of image x) Note the image plotted by given mnist_img(x) function needs rotate to normal direction. So here the reshaped single digit 28×28 data is transposed.

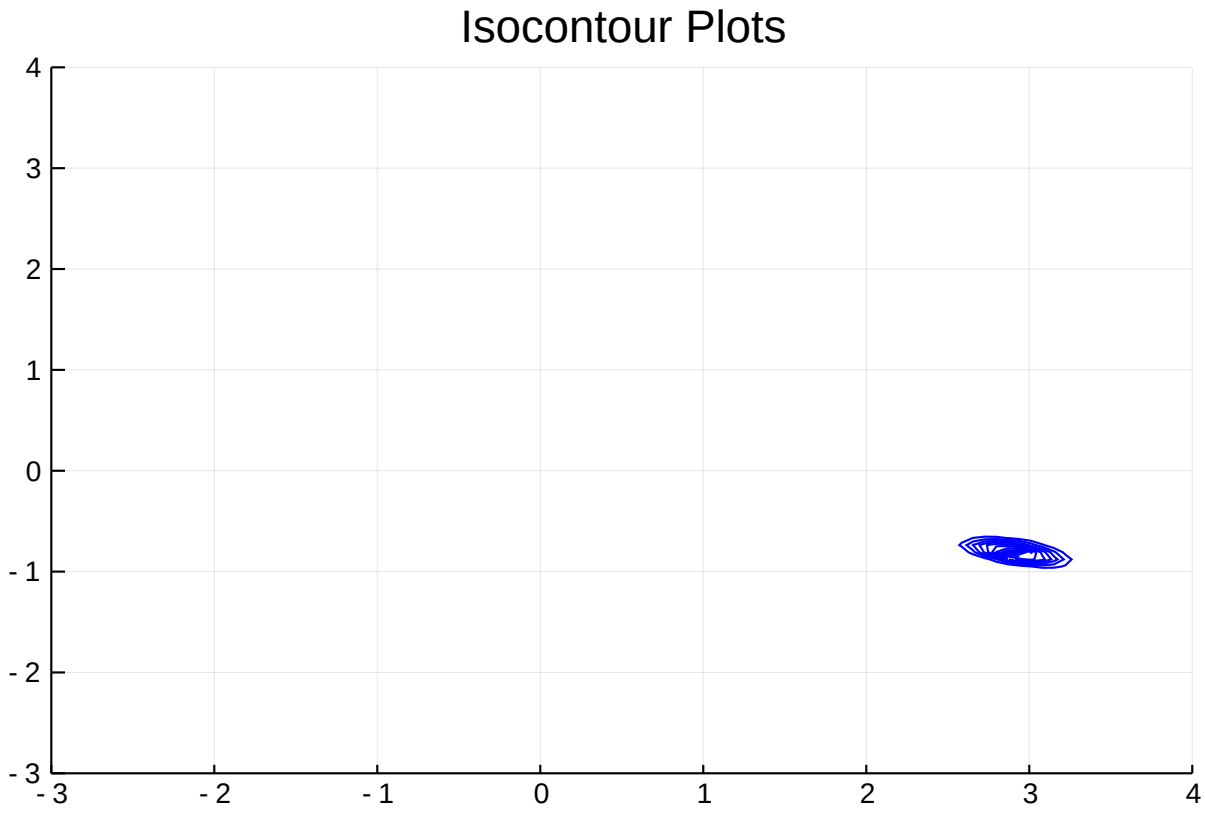
```
# function which returns only the top half of a 28 by 28 array.
function top_half(x)
  x = permutedims(reshape(x,28,28)) # The digits in mnist need rotate
  return x[1:14,:]
end

#function that computes log p(top half of image x|z)
function logp_top(x,z)
  z_top = decoder(z)[1:392,:]
  sum_top = sum(bernoulli_log_density(z_top,x),dims=1)
  z_bottom = decoder(z)[393:784,:]
  kones = ones(1,k)
  kzeros = zeros(1,k)
  # Integrate over the bottom half for every pixels being 1 and 0
  sum_bottom = sum(log.(exp.(bernoulli_log_density(z_bottom,kones))
    .+ exp.(bernoulli_log_density(z_bottom,kzeros))),dims=1)
  return sum_top .+ sum_bottom
end

# Function compute log joint density log p(z, top half of image x)
function log_joint_density(x,z)
  return log_prior(z) .+ logp_top(x, z)
end
```

```
log_joint_density (generic function with 1 method)
```

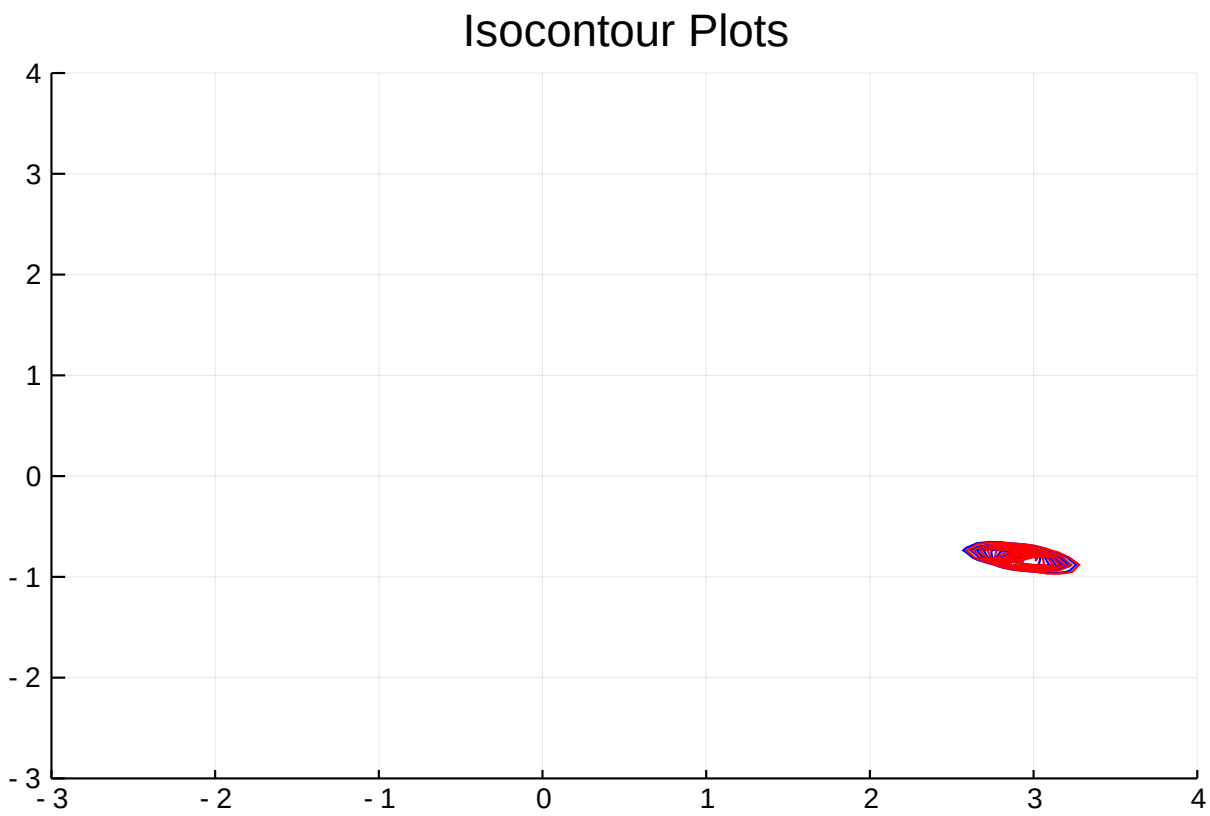
(b) [5 points] Now, to approximate p(zjtop half of image x) in a scalable way, we'll use stochastic varia- tional inference. For a digit of your choosing from the training set (choose one that is modelled well, i.e. the resulting plot looks reasonable).

```
using Zygote

# Function for compute ELBO
function elbosv(params, logp, k)
  z = sample_diag_gaussian(params[1],params[2])
  log_q_z =log_q(params[1], params[2], z)
  logp = logp(z)
  return sum(logp .- log_q_z) ./k
end

# Function for compute loss
function loss(params, x, k)
  logp(z) = log_joint_density(x,z)
  return -1 .* elbosv(params,logp, k)
end

# Function for training to optimize parameters by maximize ELBO
function opti_elbo(init_params, x; num_itrs=200, lr= 1e-2, k = 10)
  params_cur = init_params
  elbo_val = loss(params_cur, x, k)
  for i in 1:num_itrs
    f(param) = loss(param, x, k)
    grad_params = gradient(f, params_cur)[1]
    params_cur =  params_cur .- grad_params .* lr
    elbo_val = loss(params_cur, x, k)
    @info "loss: $(elbo_val)"
  end
  return params_cur, elbo_val
end

k = 100
init_params = (randn(Dz,k), randn(Dz,k)) #Initialize variational parameters
x=top_half(train_x[:,2]) # Choose the second digit of mnist data set, which is the
image of "0"
x = reshape(x, 392,1)
# Traing the data, optimaize parameters by maximize ELBO
trained_params = opti_elbo(init_params, x; num_itrs=200, lr= 1e-2, k = 100)

# Function for plot contour, basically the same from assignment 2
function plotcontour!(f; colour=nothing)
  n = 100
  x = range(-3,stop=4,length=n)
  y = range(-3,stop=4,length=n)
  z_grid = Iterators.product(x,y) # meshgrid for contour
  z_grid = reshape.(collect.(z_grid),:,1) # add single batch dim
  z = f.(z_grid)
  z = getindex.(z,1)'
  max_z = maximum(z)
  levels = [.99, 0.9, 0.8, 0.7,0.6,0.5, 0.4, 0.3, 0.2] .* max_z
  if colour==nothing
  p1 = contour!(x, y, z, fill=false, levels=levels)
  else
  p1 = contour!(x, y, z, fill=false, c=colour,levels=levels,colorbar=false)
```

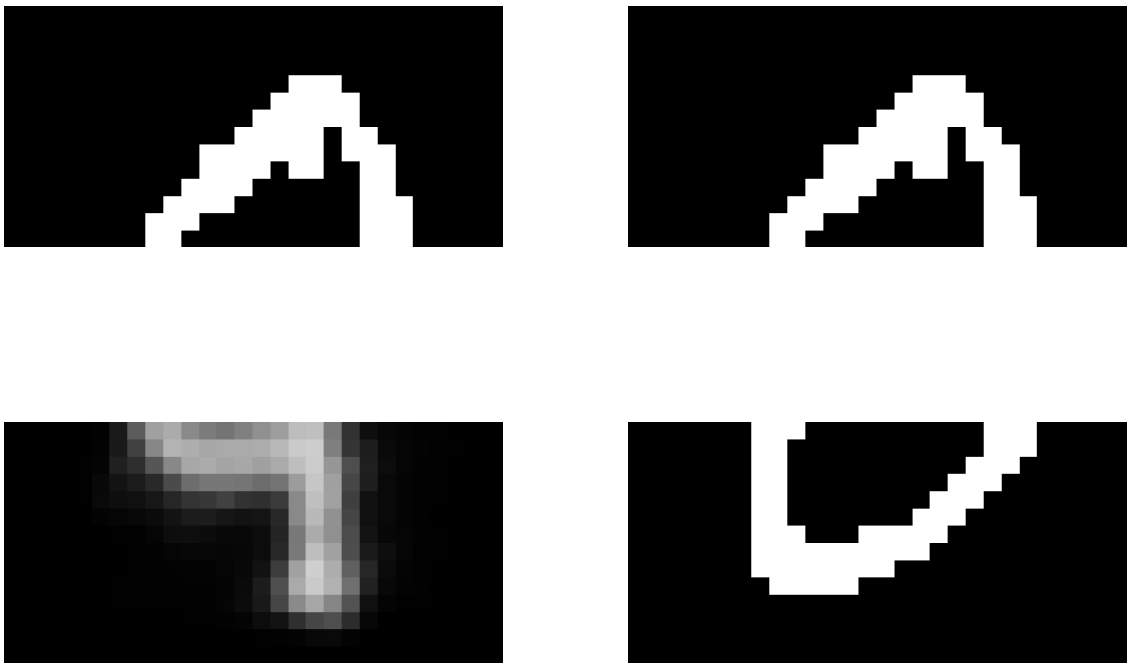
```
  end
  plot!(p1)
end

# Plot the contours
params_res = (trained_params[1][1], trained_params[1][2])# Get result parameters from
training
z = sample_diag_gaussian(params_res[1],params_res[2]) # Sample z by reparameterization
jointq(z)=exp.(log_q(params_res[1],params_res[2],z) .+ logp_top(x,z)) # Posterior
q(z|top half of image x)
jointp(z) = exp.(log_joint_density(x,z)) # Joint p(z, top half of image x),
plot(title="Isocontour Plots")
display(plotcontour!(jointp ,colour="blue"))
```




```
display(plotcontour!(jointq, colour="red"))
```




```
# See the image of predictimg bottom half
mnist_img_half(x) = ndims(x)==2 ? Gray.(reshape(x,14,28,:)) : Gray.(reshape(x,14,28)) #
for plot half image
s = rand(1:k) # Sample one z from k samples
param = (trained_params[1][1][:,s], trained_params[1][2][:,s]) # Get the sample
parameters in trained parameters
z = sample_diag_gaussian(param[1],param[2]) #sample a z with the parameters
logitz = permutedims(reshape(decoder(z),28,28)) #Use decoder get the logits and
transpose
bottom_half = logitz[15:28,:]
bottom_means=exp.( bottom_half) ./ (1 .+ exp.( bottom_half)) # Transfer from logit
p_sim_bottom=plot(mnist_img_half(vec(bottom_means)),framestyle=:none) #The similated
bottom half

true_top_means = top_half(train_x[:,2]) # True top half means of first mnist data
p_top1 = plot(mnist_img_half(vec(true_top_means)),framestyle=:none)
p_top2 = plot(mnist_img_half(vec(true_top_means)),framestyle=:none)

true_bottom_means = permutedims(reshape(train_x[:,2],28,28))[15:28,:]
p_bottom = plot(mnist_img_half(vec(true_bottom_means)),framestyle=:none)
#Display concatenated images
display(plot(p_top1, p_top2, p_sim_bottom,p_bottom,layout=(2,2)))
```



(c) True or false.

(a) Yes

(b) Yes

(c) No

(d) No

(e) No