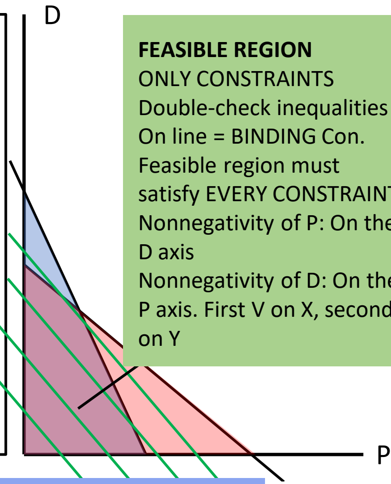
Common Ramen
// Variables
Let P = # of Plain
Let D = # of Deluxe
// Objective Function
max/min 1.5P + 2.5D
// Constraints
Subject to:
0.18P+2.5D <= 496
2P+2.5D <= 4800
P >=0, D >=0
^ read as TWO non-neg constraints



FEASIBLE REGION
ONLY CONSTRAINTS
Double-check inequalities
On line = BINDING Con.
Feasible region must satisfy EVERY CONSTRAINT
Nonnegativity of P: On the D axis
Nonnegativity of D: On the P axis. First V on X, second on Y

Solve graphically: Only use feasible region CORNERS. Given only FR = can't find opt.
EACH CORNER = **BINDING CONSTRAINTS = Intersections**
Solve by enumeration: plug in corner coords, choose max
Solve graphically: construct ISOPROFIT LINE, choose LAST POINT TO TOUCH ISO. LINE
Given max C1X + C2Y, the slope is –C1/C2. Graph with y-int C1, x-int C2. FR = +inf.0 iso
ALL ISO. POINTS HAVE SAME PROFIT VALUE. +inf.0 on cons.
Relax = BIG FR, Tight = SMall

SOLVER:
Found solution: optimal solution found
Could not find a feasible solution: LP INFEASIBLE, no FR, no satisfy cons
Objective cell values do not converge: LP is unbounded, infinite FR, solver gives up
CHECK OBJ. FUNCTION: Unbounded FR can still have opt. solution (min)

BINDING CONSTRAINT:
LHS == RHS (Use up all)
NON-BINDING:
LHS != RHS (don't use)
NONNEGATIVITY:
0 == binding,
REDUNDANT:
DOES NOT MAKE UP THE FEASIBLE REGION
LP needs linear obj. func. + constraints
NON-INTEGER okay (iffy for scheduling)

SENSITIVTY ANALYSIS: OBJECTIVE COEFFICIENTS

Measures how sensitive the OPTIMAL SOLUTION is to changes in the OBJECTIVE COEFFICIENTS
For COMM 290 we only change one input at a time. CHANGING SEVERAL INPUTS: We are not sure what happens (out of scope of course)
Changing the OBJECTIVE COEFFICIENTS CHANGES THE SLOPE of the OBJECTIVE FUNCTION / THE ISOPROFIT LINE
If we keep this slope within the two binding constraints of the corner, the OPTIMAL SOLUTION DOES NOT CHANGE
If isoprofit line lies on constraint line: MULTIPLE (infinite) OPTIMA / OPTIMAL SOLUTIONS
STEPS:
1. Write out SLOPES OF CONSTRAINTS and OBJECTIVE IN Order:
-Infinity <= OBJ -C1/C2 <= +Infinity
2. Pick either C1 or c2 to check. Leave other one alone
3. Isolate each sign
4. Multiply by NEGATIVE and **FLIP THE INEQUALITY SIGN**
5. Isolate the C1/C2 and rewrite both in original format
Recall: Slope = -3/5. C1 = -C1/5, C2 = -3/C2
Report this RANGE as how much we can INCREASE/DECREASE the objective coefficient from that ORIGINAL VALUE
Allowable → Before optimal changes, regarding OBJECTIVE COEFFIEICNTS

Within allowable range: Target cell change (unless 0 made), solution no
Outside range: Solution and target cell change

SENSITIVITY ANALYSIS: RHS OF CONSTRAINTS

Shadow price = New profit – Old profit (Add + 1 to constraint equation that you want to calculate shadow price for)
SHADOW PRICE = Contribution to target cell with one more of that constraint (aka measured in +!. Important! Negative s. price can exist
Remember: It all deals with CHANGES IN THE RHS VALUE OF THE CONSTRAINT
Steps:
1. Take a binding constraint and + 1 to its RHS
2. Find new optimal solution (intersection of + 1 binding constraint and other binding constraint that make up corner)
3. Calculate new objective and 4. new objective – old objective

THE ABOVE ONLY WORKS FOR BINDING CONSTRAINTS!
Shadow price of a NON-BINDING CONSTRAINT IS 0
ALLOWABLE INCREASE + DECREASE:
Allowable increase + decrease of a binding constraint is the Range over which that shadow price remains true
Calculate: Distance before a new binding constraint takes over as a Binding constraint for the optimal solution (in both directions)

NON-BINDING CONSTRAINT: Allowable increase/decrease of the RHS is how much RHS must change to become binding. (RHS -> Value). Other value will be infinity

Within allowable range: Target cell and solution both change
Outside Range: Don't know what happens beyond the range. AT LEAST

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease

Final Values – Optimal solution value for that variable
Objective Coefficient – Multiply this by final values to get the total target cell value

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease

Constraint R.H. Side – The value of the constraint RHS
For blending constraints RHS will most likely be 0
Shadow Price – 0 for non-binding constraint, but some binding may have 0. Check if final = RHS. Measured in +1

Allowable Increase / Decrease
This is the allowable increase/decrease to the OBJECTIVE COEFFICIENT before the optimal solution changes
If any of them are 0, this signifies **MULTIPLE OPTIMA**
If it is a very small value (e.g. 537E-16) this is basically 0, treat it as a 0
Double-check question if it is decrease BY x or decrease TO x

Allowable Increase / Decrease
This is the RANGE for which the SHADOW PRICE HOLDS
If the allowable increase/decrease is 0, it may signal that demand cannot surpass supply, or something along those lines in a transportation LP
Non-binding -> One is how to change RHS to final value, other is infinity (1E+30)
Beyond the RHS allowable range we don't know what happens → Thus we can only talk about what will happen AT LEAST.

Lucky Strike Oil Company

Input Data							
		Cost per litre ($)	Amount Available		Selling Price Regular	Selling Price Ultra	
	Oil A	$ 0.32	500,000		$ 0.42	$ 0.51	
	Oil B	$ 0.38	275,000				
	Oil C	$ 0.34	425,000				
	Regular gas must be at least 45% Oil A				45%		
	Regular gas must be at most 25% Oil B				25%		
	Ultra gas must be at least 35% Oil B				35%		
	Ultra gas must be at most 35% Oil C				35%		
Action Plan				Model Output		Model	
		Regular	Ultra			Requirement	*Units*
	OilA	186428.5714	313571.4286	500000	<=	500,000	*litres*
	OilB	0	275000	275000	<=	275,000	*litres*
	OilC	227857.1429	197142.8571	425000	<=	425,000	*litres*
	Total	414285.7143	785714.2857				
Blending Constraints							
							Units
	Regular gas must be at least 45% Oil A			186428.5714	>=	186428.6	*litres*
	Regular gas must be at most 25% Oil B			0	<=	103571.4	*litres*
	Ultra gas must be at least 35% Oil B			275000	>=	275000	*litres*
	Ultra gas must be at most 35% Oil C			197142.8571	<=	275000	*litres*
Revenue and Cost Info							
		Regular	Ultra	Total			
	Revenue	$ 174,000.00	$ 400,714.29	$ 574,714.29			
	Costs						
	Oil A	$ 59,657.14	$ 100,342.86	$ 160,000.00			
	Oil B	$ -	$ 104,500.00	$ 104,500.00			
	Oil C	$ 77,471.43	$ 67,028.57	$ 144,500.00			
	Profit	$ 36,871.43	$ 128,842.86	$ 165,714.29			

Several inputs BLENDING into different outputs, # variables is usually # inputs * # outputs.
BLENDING CONSTRAINT: E/E+S+X = .8 must be linearized by multiplying bottom over
SEPARATE from normal constraints
ALGEBRAIC OBJECTIVE FUNCTION:
Careful:
e.g. max profit → max revenue – costs.
R = R1(AR + BR + CR) + R2(AU + BU + BU)
C = C1(AR+AU) + C2(BR+BU) + C3(CR+CU)
Collect like terms. Final objective function should have SAME # OF VARIABLES AS LET
ACTION PLAN: =SUM() for model output and total with respective rows/columns

Ladner Police

		Time Periods covered by different shift workers						
		8:00 - Noon	Noon - 4:00pm	4:00 - 8:00pm	8:00 - midnight	Midnight - 4:00 am	4:00am - 8:00 am	
Shift Starting at	8:00am	3	3					3
	12 Noon		3	3				3
	4:00pm			7	7			7
	8:00:00pm				0	0		0
	12 Midnight					4	4	4
	4:00am	2					2	2
	Supply	5	6	10	7	4	6	19
		>=	>=	>=	>=	>=	>=	
	Demand	5	6	10	7	4	6	

Answers HOW MANY RESOURCES/PEOPLE to have in each shift at a time. Coefficient usually 1
MIXED REFERENCE SELECT RED CELLS
Think about how **LONG SHIFTS ARE**
Green Target cell: =SUM() RED CELLS.

THE SUPPLY is the total number of people/resources that are scheduled for that time
The DEMAND is the total number of people/resources that are NEEDED for that time
INTEGER SOLUTION? – With other models it is okay, but for scheduling it is more iffy
Don't forget to add the LAST CONDITION at the bottom of first (since shift carries over)
Alternative: DIAGONAL RED VARIABLE CELLS. Each "first left" stair step will be red cell

Power Company Distribution Problem

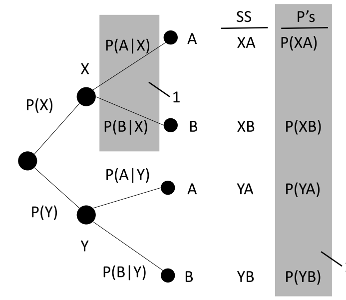
Input data									
		To							
	Costs ($)	City1	City2	City3	City4				
From	Plant1	8	6	10	9				
	Plant2	9	12	13	7				
	Plant3	14	9	16	5				
Action Plan									
		To							
	Shipments	City1	City2	City3	City4	**Shipped**		Supply	*units*
From	Plant1	0	10	25	0	35	<=	35	kWh millions
	Plant2	45	0	5	0	50	<=	50	kWh millions
	Plant3	0	10	0	30	40	<=	40	kWh millions
	Received	45	20	30	30				
		>=	>=	>=	>=				
	Demand	45	20	30	30	*kWh millions*			
Total Cost	$ 1,020.00								

Answers HOW MUCH TO TRANSPORT **FROM** location(s) **TO** location(s). # variables = FROM * TO
From = Locations where we are transporting things from (start). To = End location for things
Shipped = TOTAL AMOUNT that has left each "from" location. Received = for "to" locations
Supply is AMOUNT AVAILABLE at each "from" location. Demand is AMOUNT NEEDED FOR "to"
Total Cost = =SUMPRODUCT() the cost chunk with the large red variables chunk
Supply and demand do not have to be equal, however Supply >= Demand or else INFEASIBLE
If a from CANNOT SHIP TO a TO, then you can either SET CONSTRAINT THAT THAT PATH = 0, OR YOU CAN SET A VERY HIGH COST for that transportation path
BE FLEXIBLE IN SUPPLY VS. DEMAND. SOMETIMES BOTTOM HAS TO BE LESS THAN RIGHT, ETC.

RELATIVE: =A15. ABSOLUTE: =A15. MIXED: $A15 KEEP A COLUMN, A$15 KEEP 15 ROW. =SUMPRODUCT(C10:D10, C15:D15) = C10*D15+D10*D15

PROBABILITY: Number between 0 and 1
SIMULATION WITH EXCEL:
=RAND() generates the number
=IF(Predicate, True Answer, False Answer)
=COUNTIF(Range, Condition) → Counts NUMBER of
Conditional Formatting: Styling cells based on smth
Notation: P(A u B) = A or B, P(A n B) = A and B
OUTCOME: What happens as result of experiment
List of all possible outcomes: The SAMPLE SPACE
EVENT = A SUBSET of the sample space
Marginal = P(X). Joint = P(A + B)

INDEPENDENT VS. DEPENDENT
I: The outcome of FIRST EVENT DOES NOT AFFECT the outcome of the second
D: First event OUTCOMES HAVE AN EFFECT ON the second effect's probabilities
MUTUAL EXCLUSIVE: If A and B CANNOT HAPPEN SIMULTANEOUSLY. If P(A|B) = 0, they are mut. Ex.
CONDITIONAL PROBABILITY: P(A|B). P(A) given B
P(A|B) = P(AB) / P(B)
BAYES THEOREM: P(A|B) = (P(B|A)*P(B))/P(A)
With Replacement = Inde, Without = Depe



OR: Mut. Ex: = P(A) + P(B).
P(A or B) = P(A) + P(B) – P(AB)
AND: Indep. = P(W)*P(R)
P(A and B) = P(A)*P(B|A). A is first, B is second
Cond=Joint/Mar
Use Joint Prob. Table, find all

AND	A	B	Total
Y	P(AY)	P(BY)	P(Y)
X	P(AX)	P(BX)	P(X)
Total	P(A)	P(B)	1

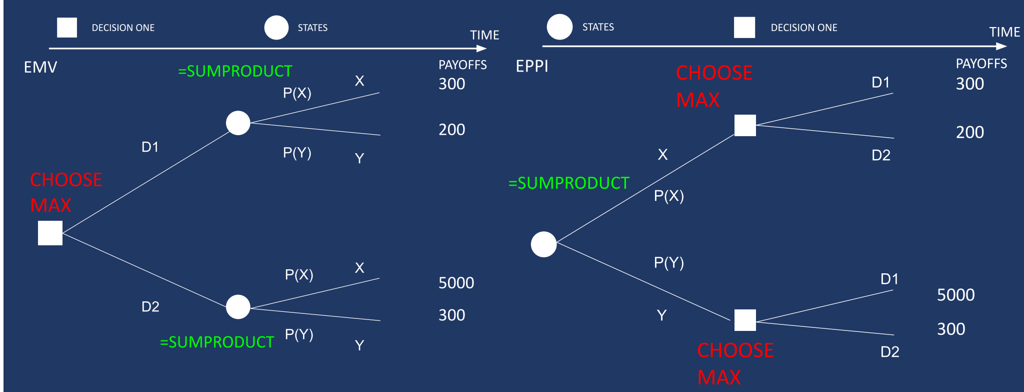
DECISION ANALYSIS: 3 ELEMENTS:
1. Decision (Choices, alternatives, actions)
2. States of Nature (States), no control over
3. Consequences (Outcomes, Payoffs, Results)

PAYOFF MATRIX: Three Decision APPROACHES GIVEN STATES → AND DECISIONS ^ v

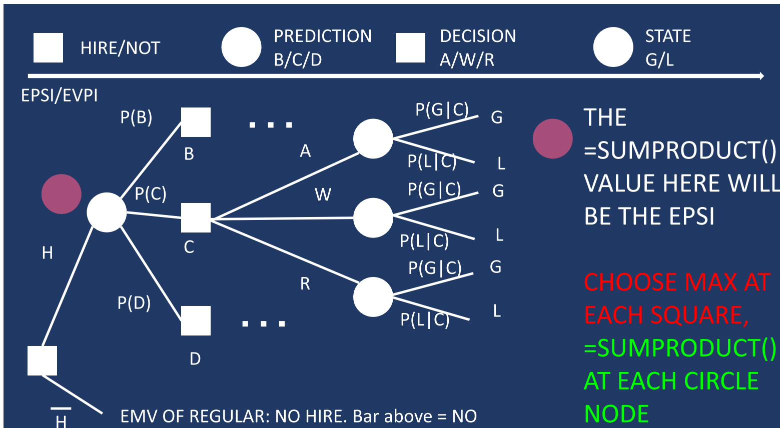
PROFIT	X	Y	Z	CON.	OPT.	REG.
d1	50	80	105	50	105	45 -
d2	70	85	90	70 -	90	60
d3	20	70	150	20	150 -	50

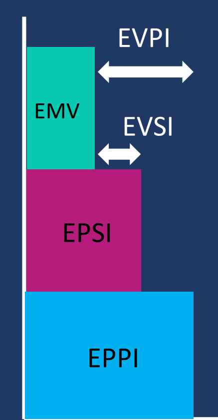
CONSERVATIVE: Looking for the worst possibilities
FOR EACH DECISION CHOOSE WORST. Out of those best choose the **HIGHEST OF THE MINIMUMS**
OPTIMISTC: Looking for the best possibilities
FOR EACH DECISION CHOOSE BEST. Out of those best choose the **HIGHEST OF THE MAXIMUMS**
REGRET: Looking for the DIFFERENCES in payouts
STEPS:
1. Pinpoint the HIGHEST PAYOUT FOR EACH **STATE**
2. Write out the difference from EACH decision's payout to the best payout.
3. CHOOSE THE HIGHEST REGRET FOR EACH DECISION
4. In the original matrix, choose lowest R. value

← This is given no prob's + outcomes = UNCERTAINTY. With prob's and payoffs = RISK, can find EMV
EMV = EXPECTED MONETARY VALUE = Σ (Payoff)*Probability = Weighted average of each decision



EVPI = EPPI – EMV: THE EVPI IS THE MOST YOU ARE WILLING TO PAY FOR PERFECT INFORMATION
EVSI = EPSI – EMV: MOST WILLING TO PAY FOR SAMPLE INFO. EFFICIENCY: EVSI/EVPI





CONTINUOUS VS. DISCRETE VARIABLES:

DISCRETE: X=0,1,2,3.	CONTINUOUS: 0 <= X < 4
P(X=2) = 0.25	P(X=2) = 0
P(X<=2) = 0.75	P(X<=2) = 0.50
P(X<2) = 0.50	P(X<2) = 0.50

RANDOM VARIABLE: Numerical descriptions of the outcome of an experiment. PR. DISTRIBUTION of X:

X	5	6	7	8	9	10
P(X)	.3	.1	.2	.1	.1	.2

EXPECTED VALUE E(X):
Σ X * P(X). IF it's f(x), Σ f(x) * P(X)
With replacement E(X) = without r. E(X)
VARIANCE σ2(X) or VAR(X):
Σ [X – E(X)]^2 * P(X), E(X^2)*[E(X)]^2
DIFFERENCE between each X and E(X) squared, multiplied by probability
STANDARD DEVIATION σ(X):
(sqrt (VAR(X))
UNITS: E(X), ST.DEV = Original, VAR = ^2

EXCEL: =RANDBETWEEN(LO, HI) produces INTEGER VALUE IN THAT RANGE, INCLUSIVE
OR CAN USE =RAND(), AND IF(=G5<=1/5, "T", "F") are examples of ways to do this with excel
CONSTANT TIMES VARIABLE E(C*X)
E(X) of E(C*X) → E(C*X) = C*E(X)
VAR(X) of VAR(C*X) -> VAR(C*X) = C^2*VAR(X)
ST.DEV(X) of ST.DEV(C*X) -> ST.DEV(C*X) = |C|*ST.DEV(X)

ADDITION OF TWO RANDOM VARIABLES (Given T = A + B), Indep + Dep
INDEPENDENT = P(X and Y) = P(X) * P(Y) holds for all X and Y. No = DEP.
E(T) = E(A) + E(B) no matter independent or dependent
VAR(T) = VAR(A) + VAR(B) if INDEPENDENT, does not hold if dependent

JOINT PROBABILITY DISTRIBUTION BETWEEN 2 RANDOM VARIABLES

AND	A	B	Total
Y	P(AY)	P(BY)	P(Y)
X	P(AX)	P(BX)	P(X)
Total	P(A)	P(B)	1

Recall: E(X – Y) = E(X) + -E(Y)
If relationship between X and Y is NOT addition/subtraction, construct p.tree and use definitions of VAR, ST.DEV, E(X) and calculate manually