

Time Series Forecasting report

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```
library(readxl)
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method      from
##   as.zoo.data.frame zoo
```

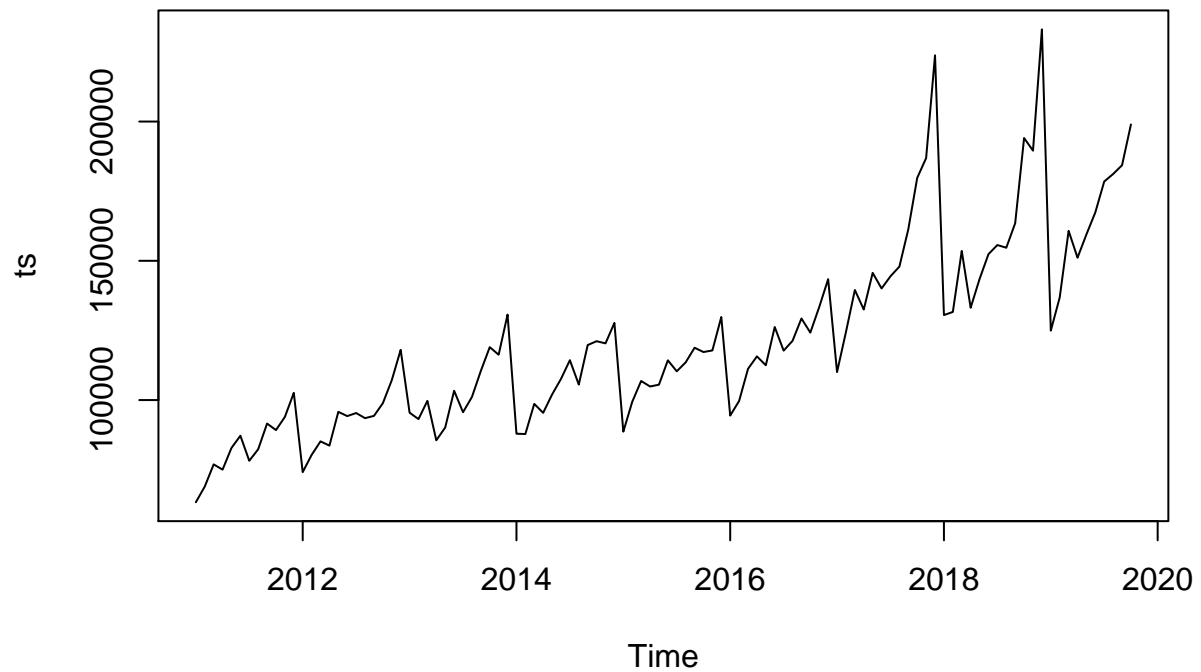
```
df <- read_xls('economy.xls', sheet='2011-2019 NACE 2')
```

```
## New names:
## * ' ' -> ...2
## * ' ' -> ...3
## * ' ' -> ...4
## * ' ' -> ...5
## * ' ' -> ...6
## * ...
```

```
df = df[4,]
df = df[-c(1,3)]
rownames(df) = df[1]
```

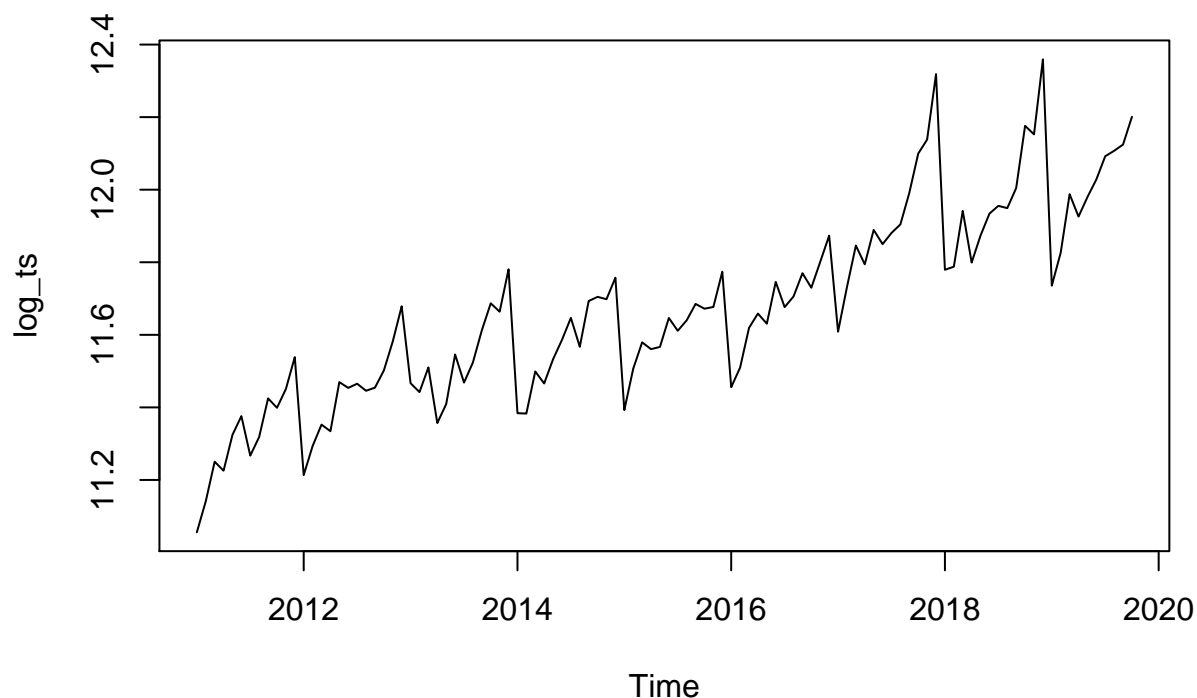
```
## Warning: Setting row names on a tibble is deprecated.
```

```
df = df[-1]
df = t(df)
df[] <- sapply(df[],function(x) as.numeric(as.character(x)))
df = as.numeric(df)
# df= df * 1000000
ts = ts(df, start = c(2011,1), frequency = c(12))
```



In this case, it appears that an additive model is not appropriate for describing this time series, since the size of the seasonal fluctuations and random fluctuations seem to increase with the level of the time series. Thus, we may need to transform the time series in order to get a transformed time series that can be described using an additive model. For example, we can transform the time series by calculating the natural log of the original data:

```
log_ts <- log(ts)
plot.ts(log_ts)
```



##Decomposing Time Series

Decomposing a time series means separating it into its constituent components, which are usually a trend component and an irregular component, and if it is a seasonal time series, a seasonal component.

###Decomposing Seasonal Data A seasonal time series consists of a trend component, a seasonal component and an irregular component. Decomposing the time series means separating the time series into these three components: that is, estimating these three components.

```
ts_components <- decompose(ts)
```

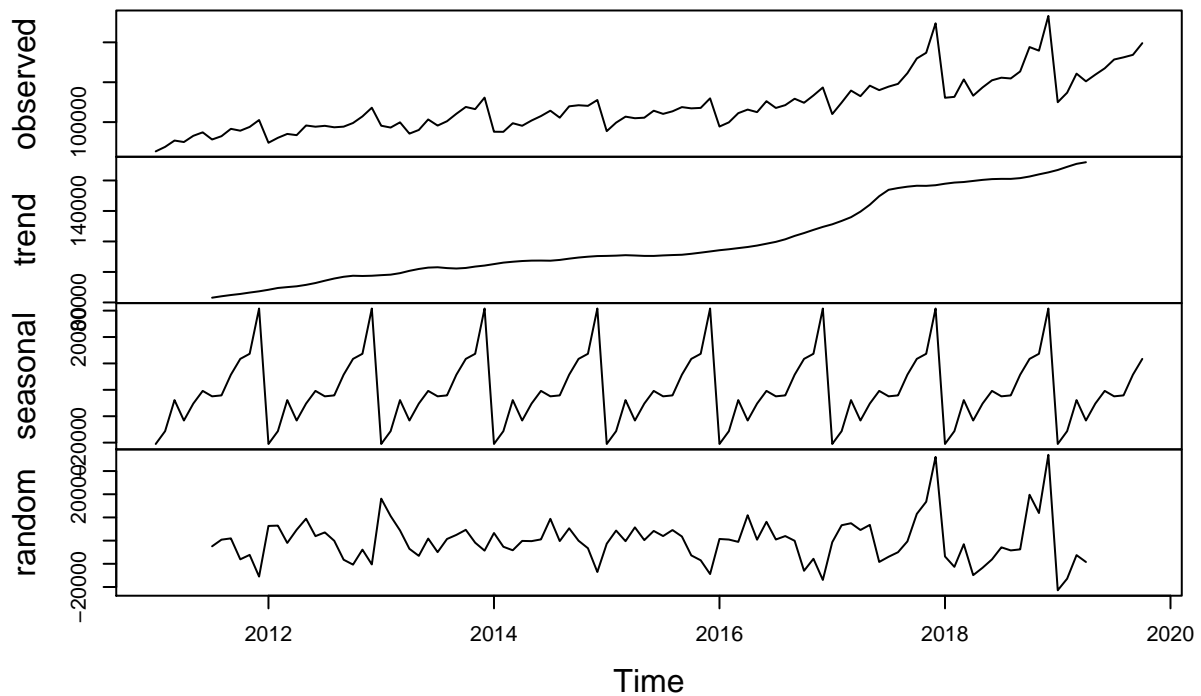
we can print out the estimated values of the seasonal component

```
ts_components$seasonal
```

##		Jan	Feb	Mar	Apr	May	Jun
##	2011	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2012	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2013	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2014	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2015	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2016	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2017	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2018	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##	2019	-20517.7010	-15661.9010	-3889.3766	-11627.5854	-5217.1400	-407.0709
##		Jul	Aug	Sep	Oct	Nov	Dec

```
## 2011 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2012 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2013 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2014 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2015 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2016 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2017 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2018 -2465.4208 -2114.1271 5731.3250 11714.6380 13652.3146 30802.0453
## 2019 -2465.4208 -2114.1271 5731.3250 11714.6380
```

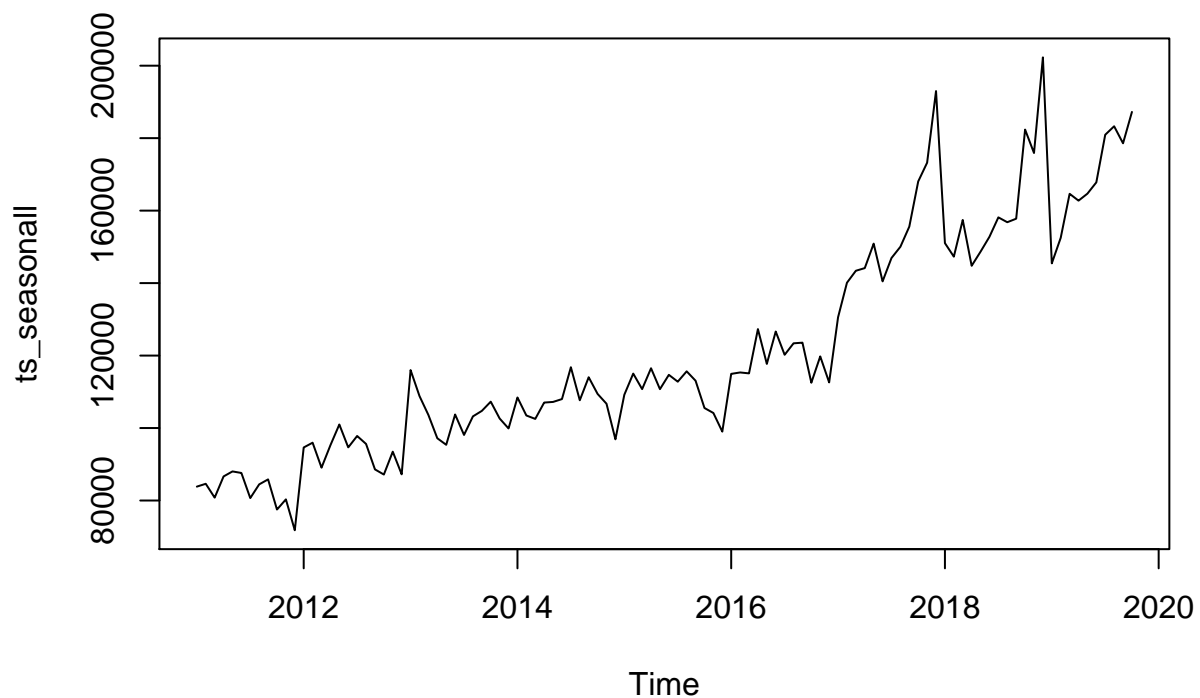
Decomposition of additive time series



The plot above shows the original time series (top), the estimated trend component (second from top), the estimated seasonal component (third from top), and the estimated irregular component (bottom)

Seasonally Adjusting

```
ts_seasonall <- ts - ts_components$seasonal
```



```
## Holt-Winters Exponential Smoothing
```

```
ts_forcaste <- HoltWinters(ts)
```

```
## Warning in HoltWinters(ts): optimization difficulties: ERROR:
## ABNORMAL_TERMINATION_IN_LNSRCH
```

```
ts_forcaste
```

```
## Holt-Winters exponential smoothing with trend and additive seasonal component.
##
## Call:
## HoltWinters(x = ts)
##
## Smoothing parameters:
##  alpha: 0.3836132
##  beta : 0
##  gamma: 1
##
## Coefficients:
##           [,1]
## a  172867.8976
## b    859.0055
## s1  23294.3107
## s2  56302.6708
## s3 -47702.4669
```

```
## s4 -33062.7461
## s5 -7303.0840
## s6 -17399.9736
## s7 -7593.9919
## s8 447.7895
## s9 7608.8518
## s10 6202.5650
## s11 8966.0791
## s12 26086.7024
```

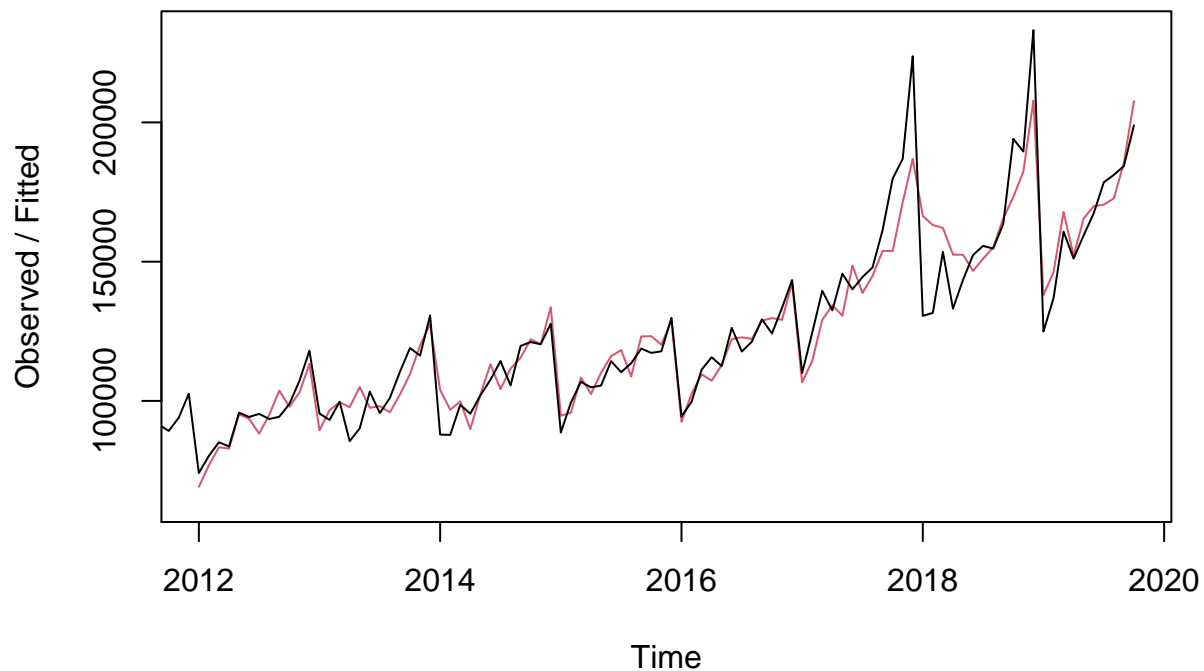
```
#
```

The value of alpha (0.41) is relatively low, indicating that the estimate of the level at the current time point is based upon both recent observations and some observations in the more distant past. The value of beta is 0.00, indicating that the estimate of the slope b of the trend component is not updated over the time series, and instead is set equal to its initial value. This makes good intuitive sense, as the level changes quite a bit over the time series, but the slope b of the trend component remains roughly the same. In contrast, the value of gamma (0.96) is high, indicating that the estimate of the seasonal component at the current time point is just based upon very recent observations

```
ts_forcaste$SSE
```

```
## [1] 9343490075
```

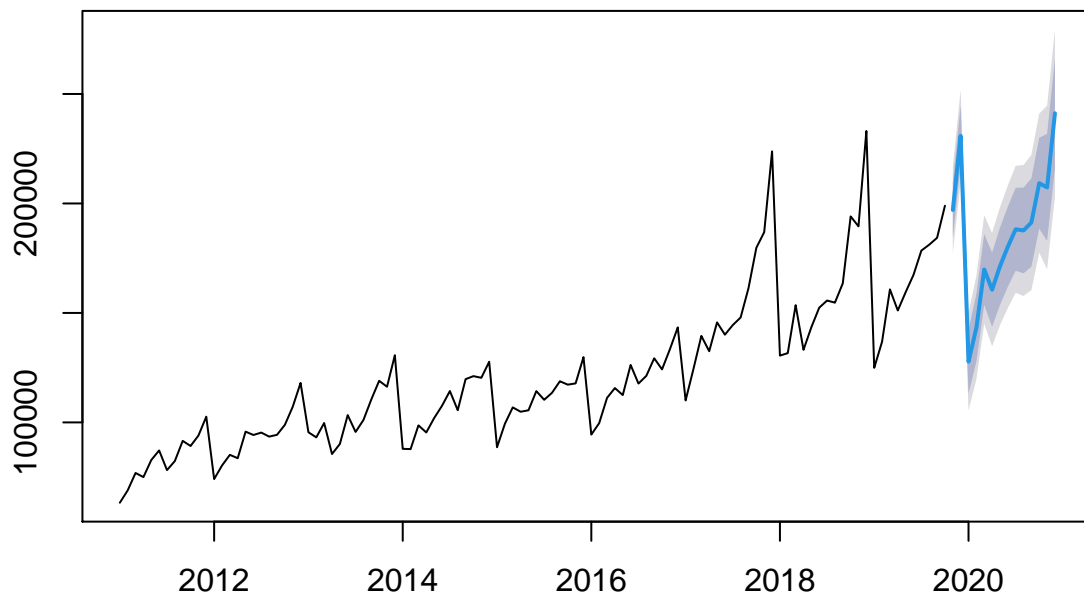
Holt-Winters filtering



```
ts_forcaste2 = forecast::forecast.HoltWinters(ts_forcaste, h= 14)
(as.data.frame(ts_forcaste2))[1]
```

```
##      Point Forecast
## Nov 2019      197021.2
## Dec 2019      230888.6
## Jan 2020      127742.4
## Feb 2020      143241.2
## Mar 2020      169859.8
## Apr 2020      160622.0
## May 2020      171286.9
## Jun 2020      180187.7
## Jul 2020      188207.8
## Aug 2020      187660.5
## Sep 2020      191283.0
## Oct 2020      209262.7
## Nov 2020      207329.3
## Dec 2020      241196.6
```

Forecasts from HoltWinters



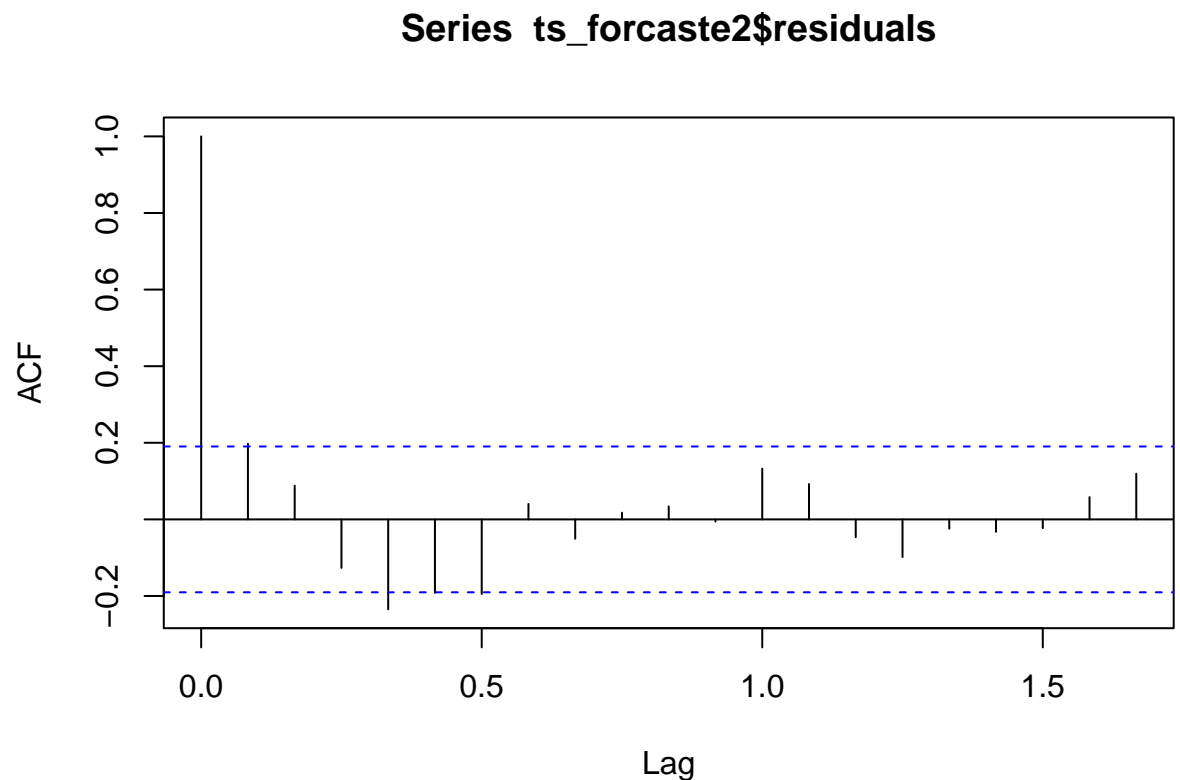
Growth

```
year_2019 <- window(ts, 2019)
year_2019_predict_HW <- (as.data.frame(ts_forcaste2))[1][c(1:2),]
```

```
sum_year_2019 = sum(c(year_2019,year_2019_predict_HW))
year_2020 = (as.data.frame(ts_forcaste2))[1][c(3:14),]
growth_HW <- growth(sum(year_2020),sum_year_2019)
growth_HW
```

```
## [1] 0.05145076
```

We can investigate whether the predictive model can be improved upon by checking whether the in-sample forecast errors show non-zero autocorrelations at lags 1-20, by making a correlogram and carrying out the



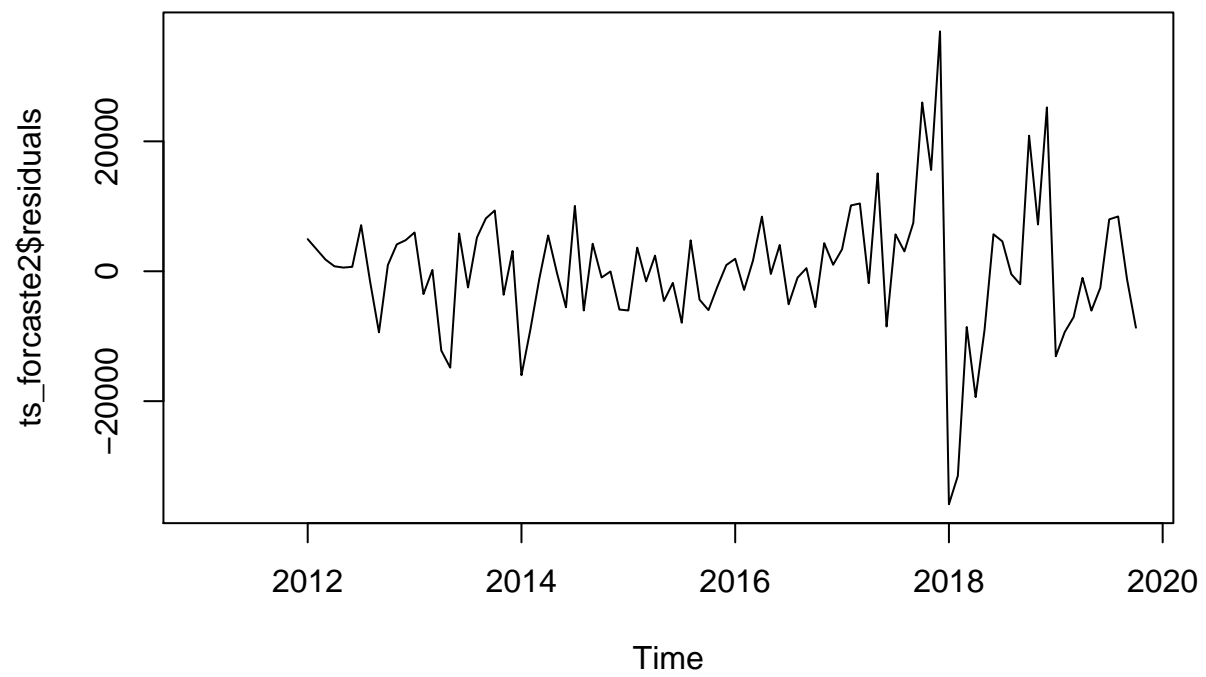
Ljung-Box test:

```
##
## Box-Ljung test
##
## data: ts_forcaste2$residuals
## X-squared = 26.501, df = 20, p-value = 0.1499
```

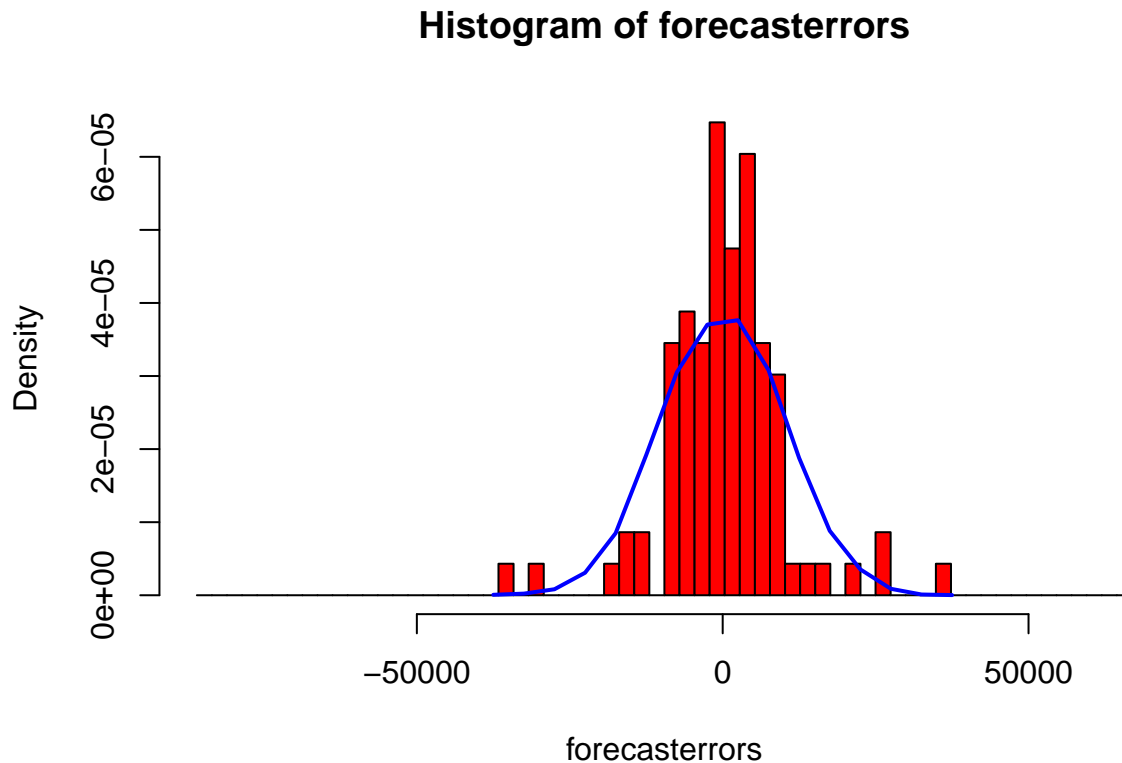
The correlogram shows that the autocorrelations for the in-sample forecast errors do not exceed the significance bounds for lags 1-20. Furthermore, the p-value for Ljung-Box test is 0.2, indicating that there is little evidence of non-zero autocorrelations at lags 1-20.

We can check whether the forecast errors have constant variance over time, and are normally distributed with mean zero, by making a time plot of the forecast errors and a histogram (with overlaid normal curve):

```
plot.ts(ts_forcaste2$residuals)
```

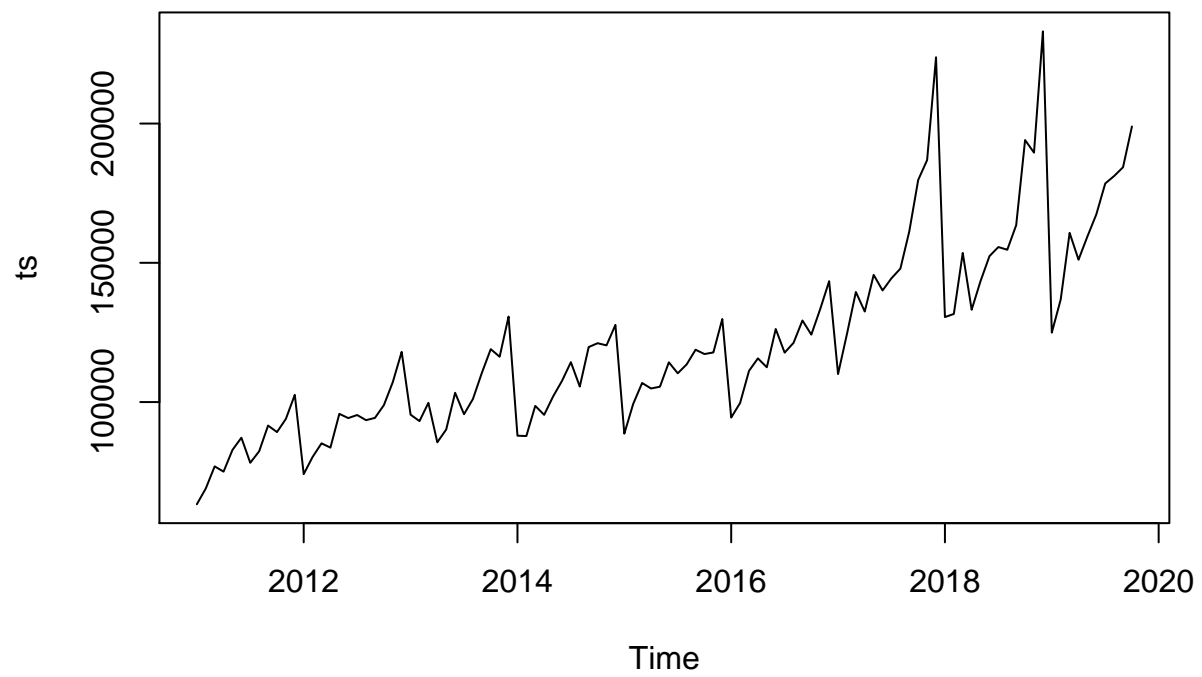
```
plotForecastErrors(ts_forcaste2$residuals)
```



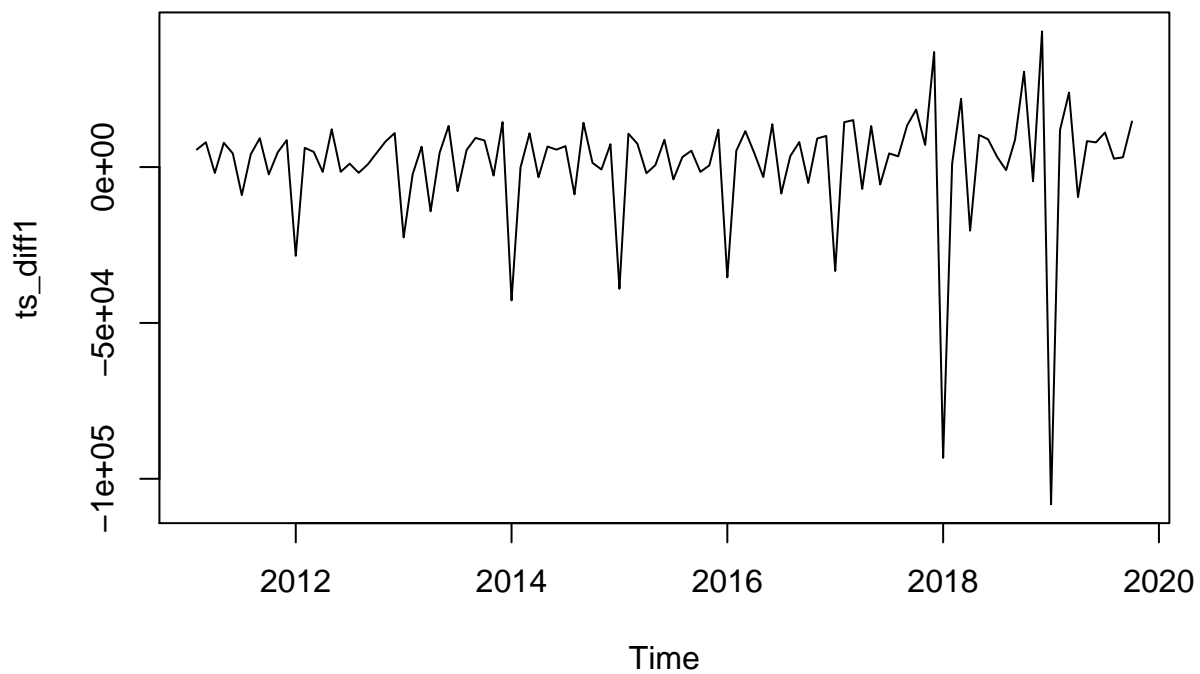
From the time plot, it appears plausible that the forecast errors have constant variance over time. From the histogram of forecast errors, it seems plausible that the forecast errors are normally distributed with mean zero.

Thus, there is little evidence of autocorrelation at lags 1-20 for the forecast errors, and the forecast errors appear to be normally distributed with mean zero and constant variance over time. This suggests that Holt-Winters exponential smoothing provides an adequate predictive model of the log of total productivity, which probably cannot be improved upon. Furthermore, the assumptions upon which the prediction intervals were based are probably valid.

```
plot.ts(ts)
```

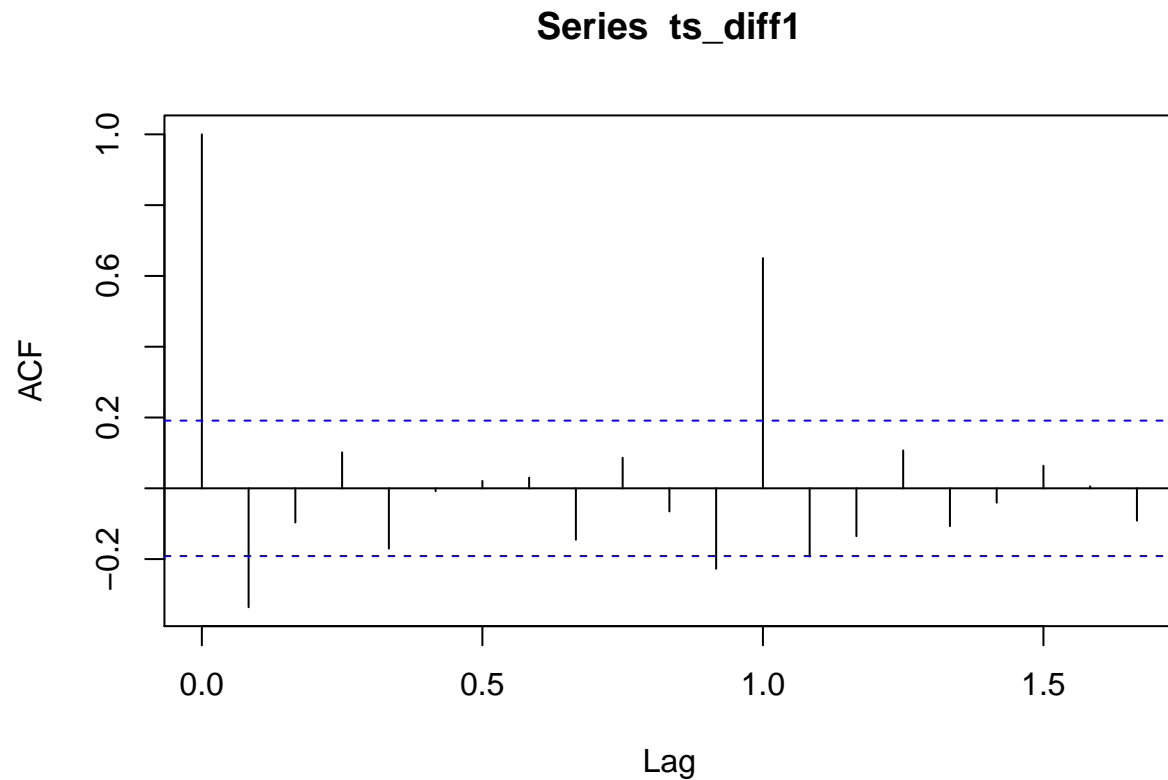


```
ts_diff1 <- diff(ts, differences = 1)
plot.ts(ts_diff1)
```



The time series of differences (above) does appear to be stationary in mean and variance, as the level of the series stays roughly constant over time, and the variance of the series appears roughly constant over time

```
acf(ts_diff1, lag.max=20) # plot a correlogram
```

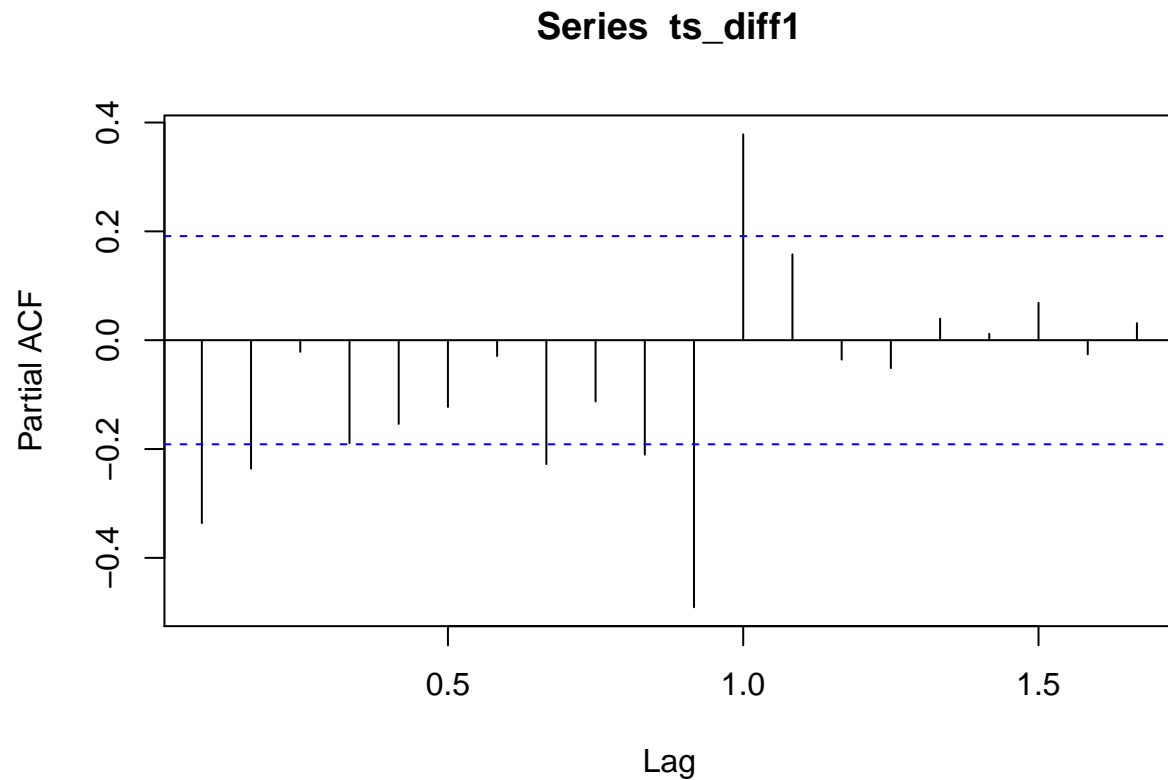


We see from the correlogram that the autocorrelation exceeds the significance bound 3 times but all the others do not exceed

```
acf(ts_diff1, lag.max=20, plot=FALSE) # get the autocorrelation values
```

```
##
## Autocorrelations of series 'ts_diff1', by lag
##
## 0.0000 0.0833 0.1667 0.2500 0.3333 0.4167 0.5000 0.5833 0.6667 0.7500 0.8333
## 1.000 -0.336 -0.097 0.102 -0.170 -0.008 0.021 0.030 -0.145 0.086 -0.065
## 0.9167 1.0000 1.0833 1.1667 1.2500 1.3333 1.4167 1.5000 1.5833 1.6667
## -0.227 0.650 -0.193 -0.135 0.107 -0.107 -0.041 0.064 0.005 -0.091
```

```
pacf(ts_diff1, lag.max=20) # plot a partial correlogram
```



```
pacf(ts_diff1, lag.max=20, plot=FALSE) # get the partial autocorrelation values
```

```
##
## Partial autocorrelations of series 'ts_diff1', by lag
##
## 0.0833 0.1667 0.2500 0.3333 0.4167 0.5000 0.5833 0.6667 0.7500 0.8333 0.9167
## -0.336 -0.236 -0.021 -0.189 -0.154 -0.123 -0.029 -0.228 -0.113 -0.210 -0.491
## 1.0000 1.0833 1.1667 1.2500 1.3333 1.4167 1.5000 1.5833 1.6667
## 0.378 0.158 -0.036 -0.051 0.040 0.012 0.069 -0.026 0.031
```

Arima, 1,1,1

```
ts_arima = Arima(ts, order=c(1,1,1),seasonal = list(order = c(1,1,1)))
ts_arima
```

```
## Series: ts
## ARIMA(1,1,1)(1,1,1)[12]
##
## Coefficients:
```

```
## Warning in sqrt(diag(x$var.coef)): NaNs produced
```

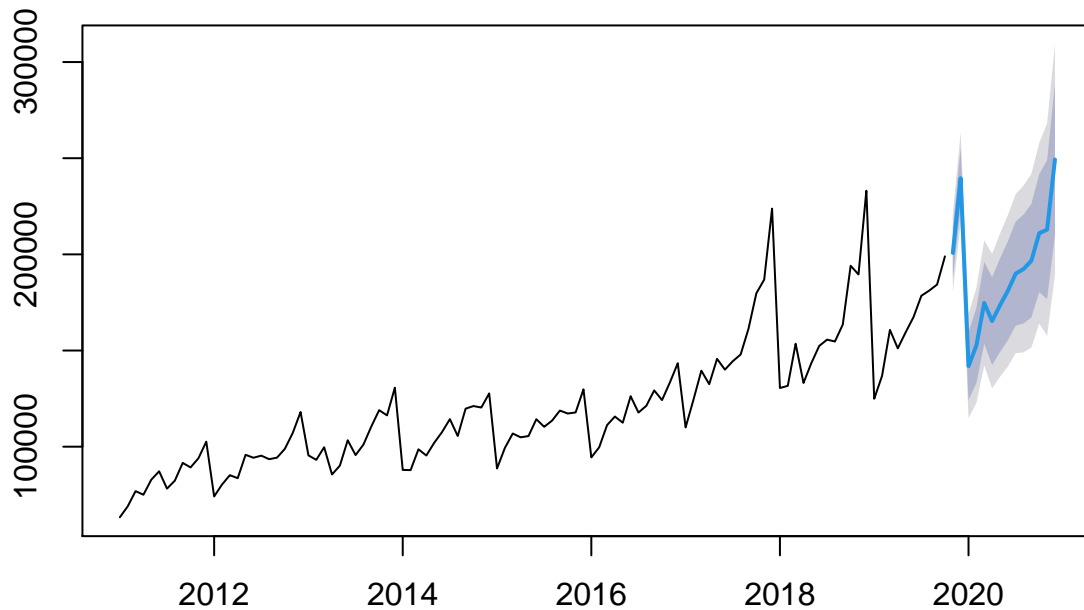
```
##          ar1      ma1      sar1      sma1
##      -0.0038 -0.3631  0.5113 -0.6981
## s.e.      NaN    0.0047  0.0060  0.0059
##
## sigma^2 estimated as 105590908:  log likelihood=-989.51
## AIC=1989.02   AICc=1989.71   BIC=2001.68
```

```
ts_arima_forecast = forecast(ts_arima,h = 14)
ts_arima_forecast
```

```
##          Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## Nov 2019      200747.7 187577.0 213918.5 180604.8 220890.6
## Dec 2019      239599.9 224011.9 255187.9 215760.2 263439.6
## Jan 2020      141878.3 124191.9 159564.6 114829.3 168927.2
## Feb 2020      152638.6 133077.9 172199.4 122723.1 182554.2
## Mar 2020      174734.4 153463.8 196005.0 142203.9 207264.9
## Apr 2020      165337.3 142484.5 188190.1 130386.9 200287.6
## May 2020      173469.8 149137.4 197802.2 136256.6 210683.0
## Jun 2020      181002.0 155274.9 206729.0 141655.8 220348.1
## Jul 2020      189982.2 162932.4 217032.1 148613.0 231351.4
## Aug 2020      192378.5 164067.6 220689.5 149080.6 235676.4
## Sep 2020      196699.0 167180.8 226217.2 151554.8 241843.2
## Oct 2020      211008.4 180330.4 241686.4 164090.5 257926.3
## Nov 2020      212839.1 176717.8 248960.4 157596.4 268081.8
## Dec 2020      249303.2 210138.3 288468.0 189405.7 309200.6
```

```
forecast::plot.forecast(ts_arima_forecast)
```

Forecasts from ARIMA(1,1,1)(1,1,1)[12]



Growth

```
this_year_predict_ARIMA <- (as.data.frame(ts_arma_forecast))[1]

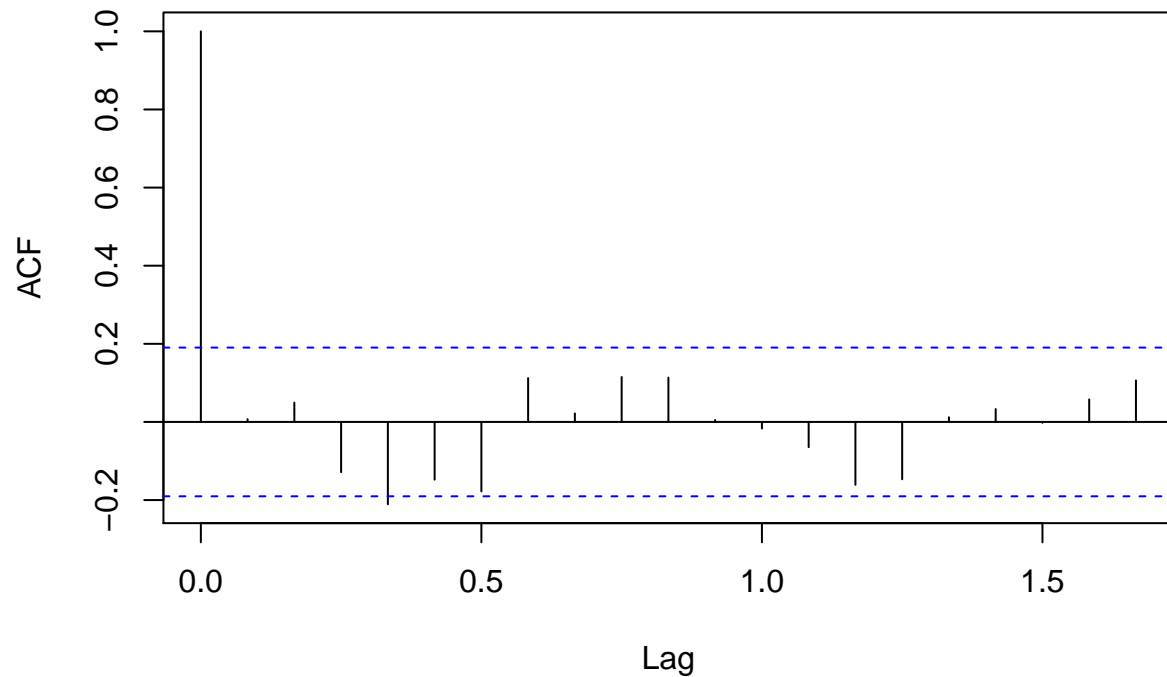
year_2019_predict_ARIMA <- (as.data.frame(ts_arma_forecast))[1][c(1:2),]
sum_year_2019 = sum(c(year_2019,year_2019_predict_ARIMA))
year_2020 = (as.data.frame(ts_arma_forecast))[1][c(3:14),]
growth_ARIMA <- growth(sum_year_2019, sum(year_2020))
growth_ARIMA=-growth_ARIMA
```

As in the case of exponential smoothing models, it is a good idea to investigate whether the forecast errors of an ARIMA model are normally distributed with mean zero and constant variance, and whether there are correlations between successive forecast errors.

For example, we can make a correlogram of the forecast errors for our ARIMA(0,1,1) model, and perform the Ljung-Box test for lags 1-20, by typing:

```
acf(ts_arma_forecast$residuals, lag.max=20)
```


Series ts_arma_forecast\$residuals

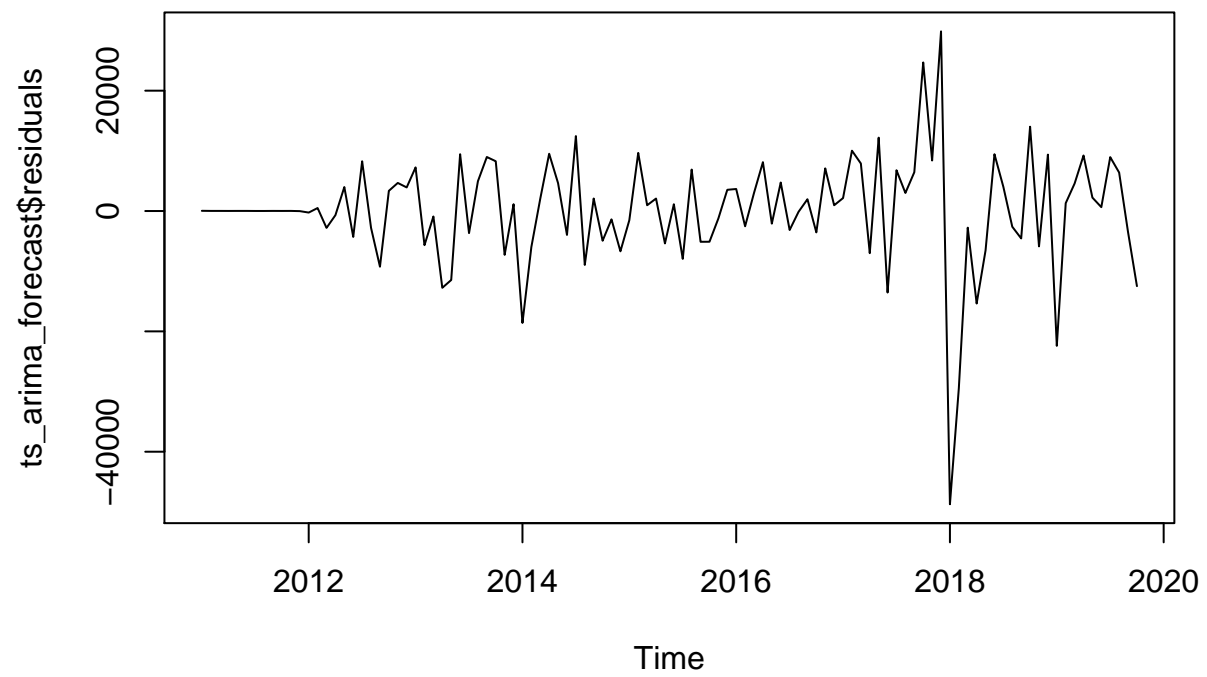


```
Box.test(ts_arma_forecast$residuals, lag=20, type="Ljung-Box")
```

```
##  
## Box-Ljung test  
##  
## data: ts_arma_forecast$residuals  
## X-squared = 26.409, df = 20, p-value = 0.1527
```

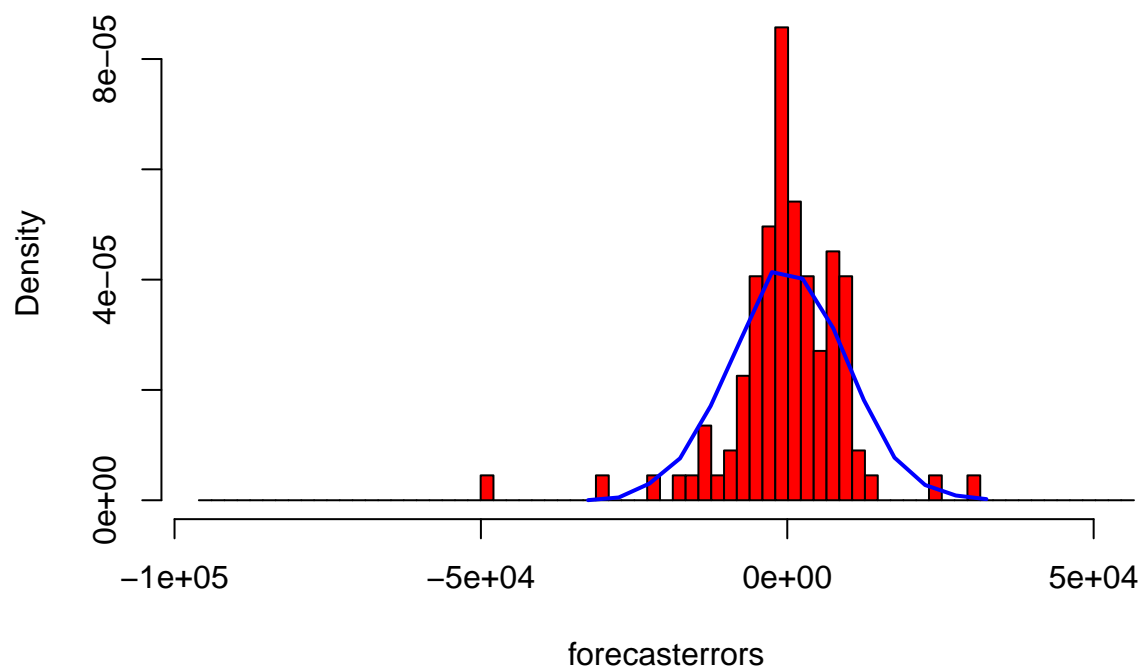
we can reject the null hypothesis, it's rather similar to the HW

```
plot.ts(ts_arma_forecast$residuals)           # make time plot of forecast errors
```



```
plotForecastErrors(ts_arma_forecast$residuals)
```

Histogram of forecasterrors



Arima, 0,1,0 as given from the loop

```
ts_arima = Arima(ts, order=c(2,1,1),seasonal = list(order = c(2,1,0)))
ts_arima
```

```
## Series: ts
## ARIMA(2,1,1)(2,1,0)[12]
##
## Coefficients:
##          ar1      ar2      ma1      sar1      sar2
##          0.5187  0.1770 -0.9699 -0.1603 -0.1667
## s.e.      0.1138  0.1093  0.0578  0.1085  0.1423
##
## sigma^2 estimated as 96893567:  log likelihood=-985.7
## AIC=1983.39  AICc=1984.37  BIC=1998.59
```

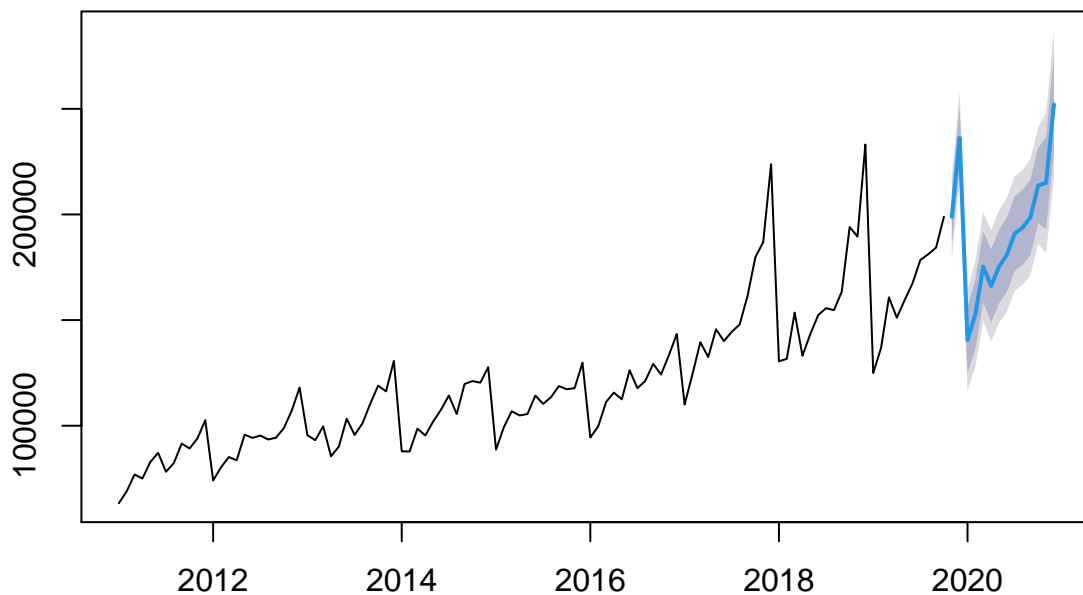
```
ts_arima_forecast = forecast(ts_arima,h = 14)
ts_arima_forecast
```

```
##          Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## Nov 2019      198884.4 186268.0 211500.8 179589.3 218179.5
## Dec 2019      236259.6 221866.9 250652.4 214247.8 258271.4
## Jan 2020      140545.4 124870.8 156220.0 116573.2 164517.7
## Feb 2020      152866.6 136465.3 169267.9 127782.9 177950.3
## Mar 2020      175294.4 158415.9 192172.8 149481.0 201107.7
## Apr 2020      166146.1 148947.6 183344.6 139843.2 192449.0
```

```
## May 2020      175261.5 157836.7 192686.4 148612.5 201910.6
## Jun 2020      180913.6 163321.8 198505.3 154009.3 207817.8
## Jul 2020      190922.9 173202.7 208643.1 163822.2 218023.6
## Aug 2020      193755.1 175932.0 211578.2 166497.0 221013.2
## Sep 2020      198569.6 180660.8 216478.3 171180.5 225958.6
## Oct 2020      213742.6 195760.3 231725.0 186241.0 241244.2
## Nov 2020      214895.4 193202.7 236588.2 181719.2 248071.6
## Dec 2020      252153.2 229270.1 275036.2 217156.5 287149.8
```

```
forecast::plot.forecast(ts_arima_forecast)
```

Forecasts from ARIMA(2,1,1)(2,1,0)[12]



```
## Growth
```

```
this_year_predict_ARIMA <- (as.data.frame(ts_arima_forecast))[1]

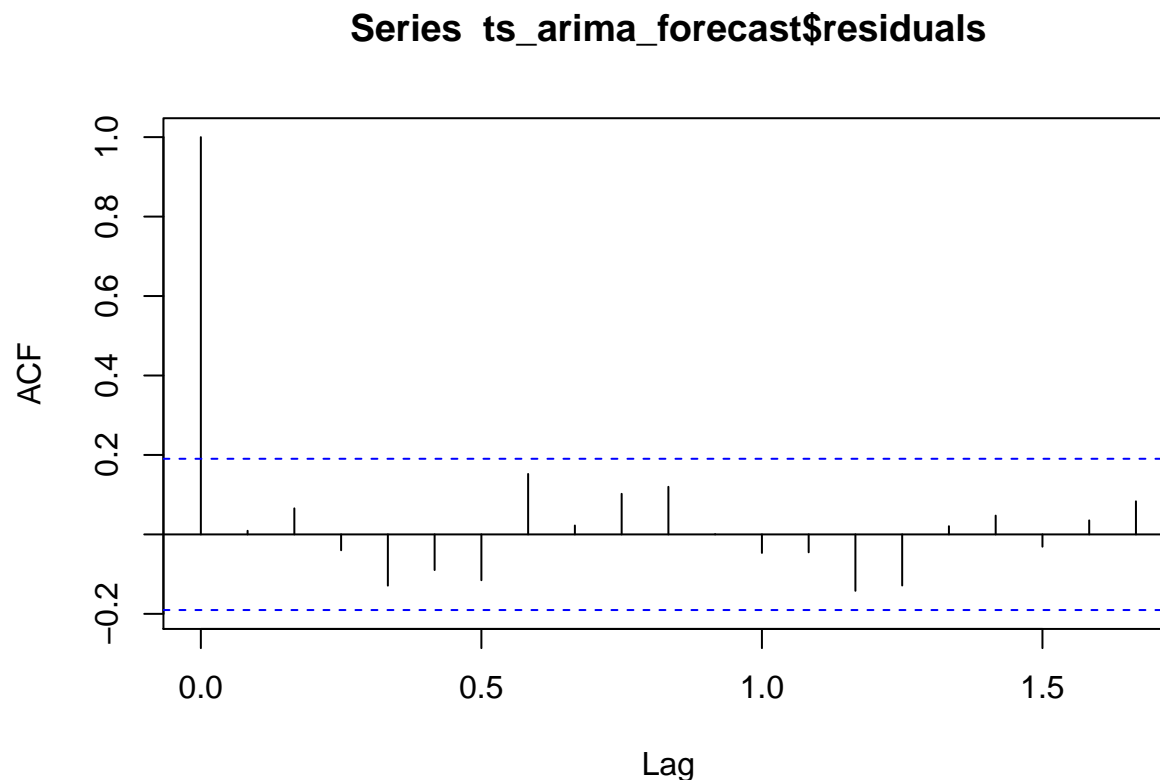
year_2019_predict_ARIMA <- (as.data.frame(ts_arima_forecast))[1][c(1:2),]
sum_year_2019 = sum(c(year_2019,year_2019_predict_ARIMA))
year_2020 = (as.data.frame(ts_arima_forecast))[1][c(3:14),]
growth_ARIMA2 <- growth(sum(year_2020),sum_year_2019)
growth_ARIMA2
```

```
## [1] 0.08492606
```

As in the case of exponential smoothing models, it is a good idea to investigate whether the forecast errors of an ARIMA model are normally distributed with mean zero and constant variance, and whether the are correlations between successive forecast errors.

For example, we can make a correlogram of the forecast errors for our ARIMA(0,1,1) model, and perform the Ljung-Box test for lags 1-20, by typing:

```
acf(ts_arima_forecast$residuals, lag.max=20)
```

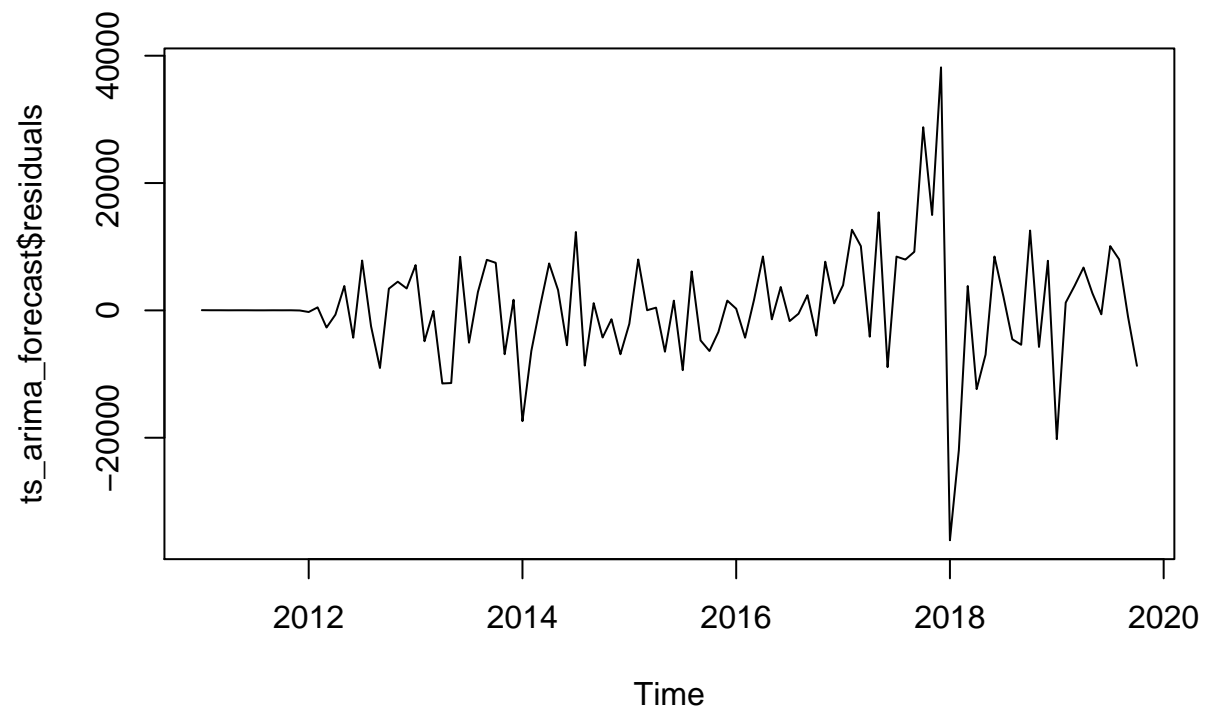


```
Box.test(ts_arima_forecast$residuals, lag=20, type="Ljung-Box")
```

```
##  
## Box-Ljung test  
##  
## data: ts_arima_forecast$residuals  
## X-squared = 17.298, df = 20, p-value = 0.6335
```

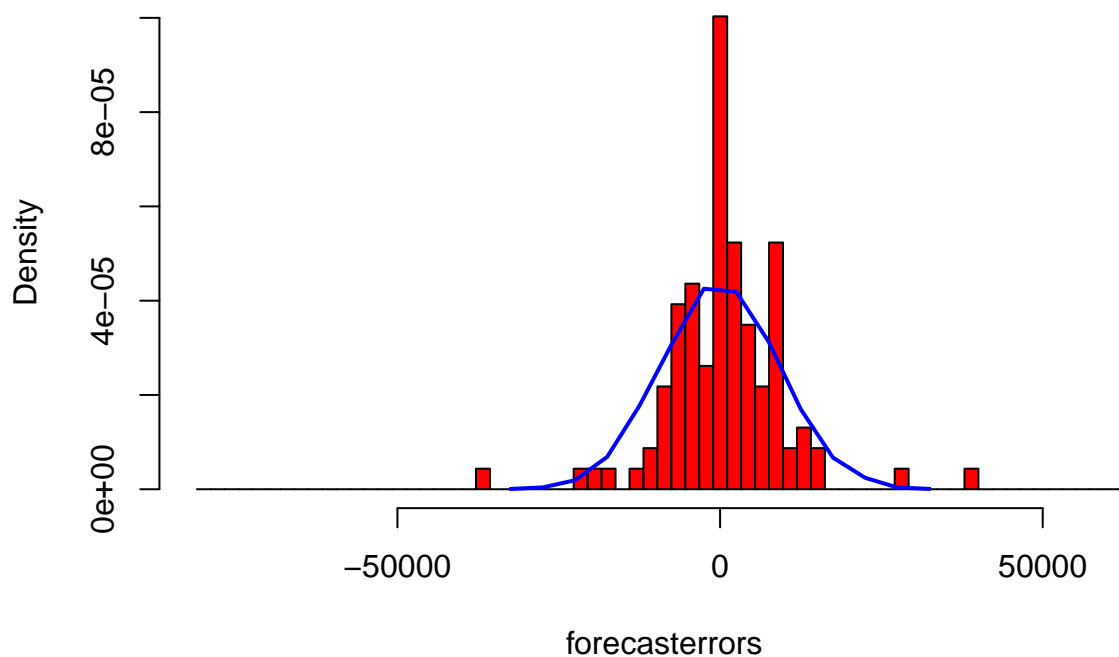
we can reject the null hypothesis, it's rather similar to the HW

```
plot.ts(ts_arima_forecast$residuals)           # make time plot of forecast errors
```



```
plotForecastErrors(ts_arma_forecast$residuals)
```

Histogram of forecasterrors



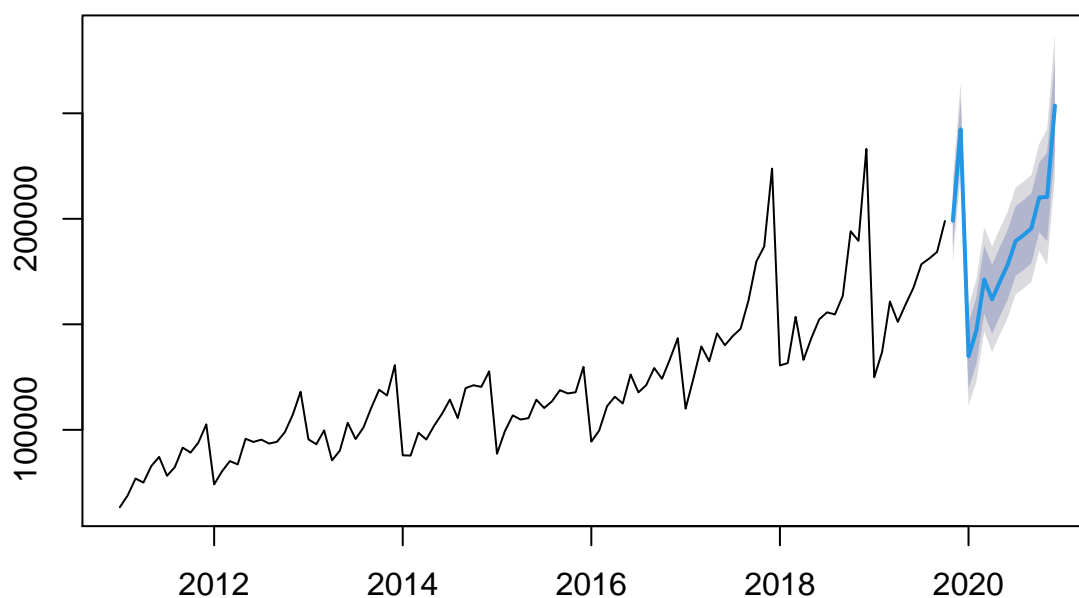
A model chosen automatically

```
fit <- auto.arima(ts,max.p = 5,max.q = 5,max.P = 5,max.Q = 5,max.d = 3,seasonal = TRUE)
fit
```

```
## Series: ts
## ARIMA(2,0,0)(0,1,0)[12] with drift
##
## Coefficients:
##          ar1      ar2      drift
##          0.5279  0.1712  942.9126
## s.e.    0.1016  0.1017  267.7789
##
## sigma^2 estimated as 96306581:  log likelihood=-996.15
## AIC=2000.3   AICc=2000.75   BIC=2010.47
```

```
fit_forecast = forecast(fit,h=14)
plot(fit_forecast)
```

Forecasts from ARIMA(2,0,0)(0,1,0)[12] with drift



```
# str(fit)
```

Growth

```
year_2019_predict_auto.arima <- (as.data.frame(fit_forecast))[1][c(1:2),]
year_2019_predict_auto.arima_95_low <- (as.data.frame(fit_forecast))[4][c(1:2),]
year_2019_predict_auto.arima_95_high <- (as.data.frame(fit_forecast))[5][c(1:2),]

sum_year_2019 = sum(c(year_2019,year_2019_predict_auto.arima))
sum_year_2019_low = sum(c(year_2019,year_2019_predict_auto.arima_95_low))
sum_year_2019_high = sum(c(year_2019,year_2019_predict_auto.arima_95_high))

year_2020_predict_auto.arima <- (as.data.frame(fit_forecast))[1][c(3:14),]
year_2020_predict_auto.arima_95_low <- (as.data.frame(fit_forecast))[4][c(3:14),]
year_2020_predict_auto.arima_95_high <- (as.data.frame(fit_forecast))[5][c(3:14),]

growth_auto.arima <- growth(sum(year_2020_predict_auto.arima),sum_year_2019)
growth_auto.arima_95_low <- growth(sum(year_2020_predict_auto.arima_95_low),sum_year_2019_low)
growth_auto.arima_95_high <- growth(sum(year_2020_predict_auto.arima_95_high),sum_year_2019_high)

growth_auto.arima
```

```
## [1] 0.06246265
```



```
growth_auto.arima_95_low
```

```
## [1] -0.07022863
```

```
growth_auto.arima_95_high
```

```
## [1] 0.1900376
```

all the growths

```
# growth_ARIMA = -growth_ARIMA  
#  
# growth_ARIMA2 = -growth_ARIMA2  
#  
# growth_auto.arima  
#  
# growth_HW
```