Linear regression

Carl Freidrich Gauss

Andrey Markov

All models are wrong
but some are useful

George E.P. Box

- Regression analysis is used to:
  - Predict the value of a dependent variable based on the value of at least one independent variable
  - Explain the impact of changes in an independent variable on the dependent variable

Dependent variable: the variable we wish to predict or explain

Independent variable: the variable used to explain the dependent variable

$$response = f(explanatory) + noise$$

Y (response)- dependent variable

X (explanatory)– independent variable

$$response = intercept + (slope * explanatory) + \ noise$$

$$Y = \beta_0 + \beta_1 * X + \epsilon \qquad\qquad \epsilon \sim N(0, \sigma_\epsilon)$$

Multiple linear regression

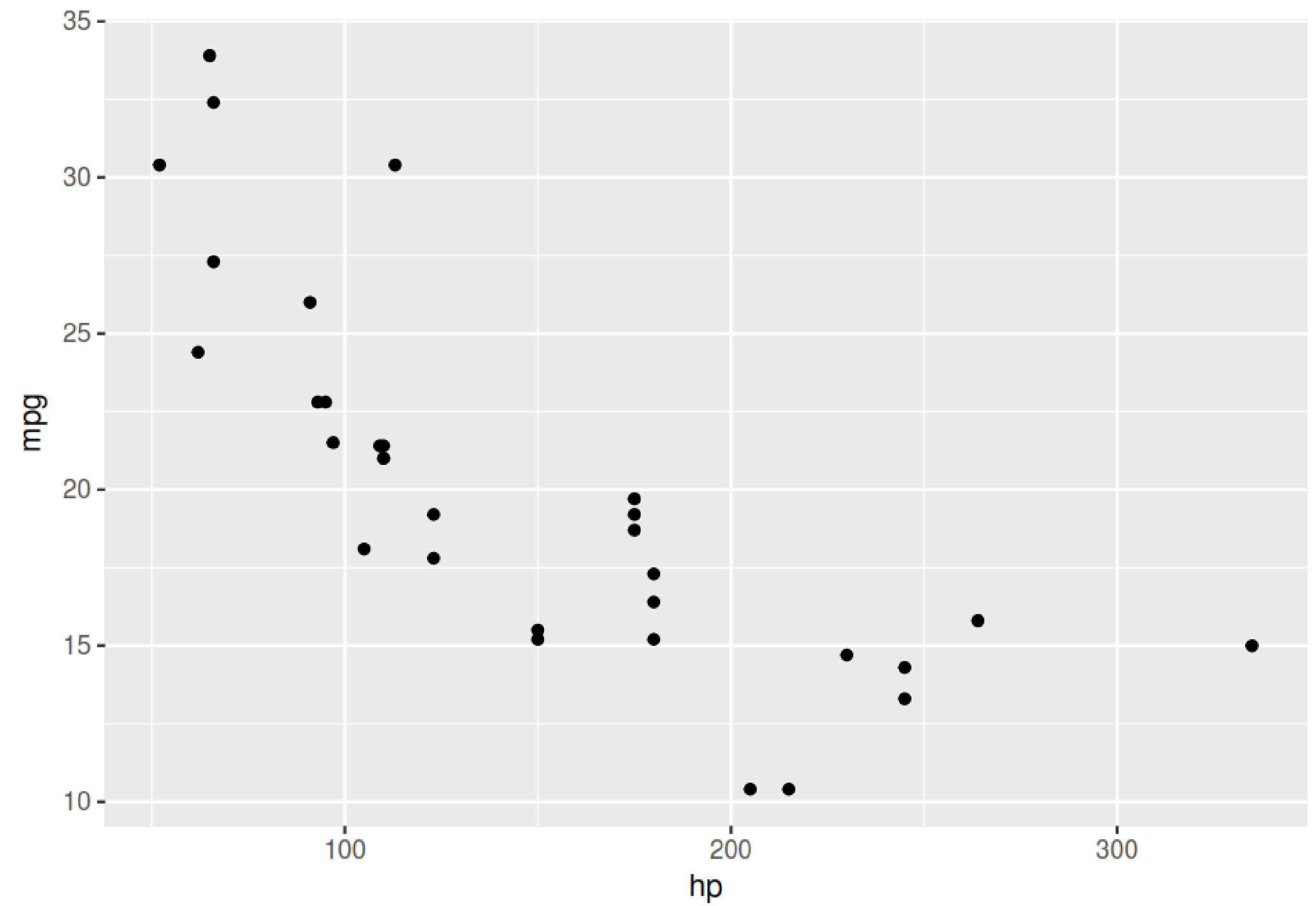
$$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \beta_3 * X_3 + \cdots \epsilon$$

The goal of the modeling is to find optimal estimates for $\beta_0,\ \beta_1 \ldots.., \beta_n$

$$\hat{Y} = \widehat{\beta_0} + \widehat{\beta_1} * X_1 + \widehat{\beta_2} * X_2 + \widehat{\beta_3} * X_3$$

Scatterplot

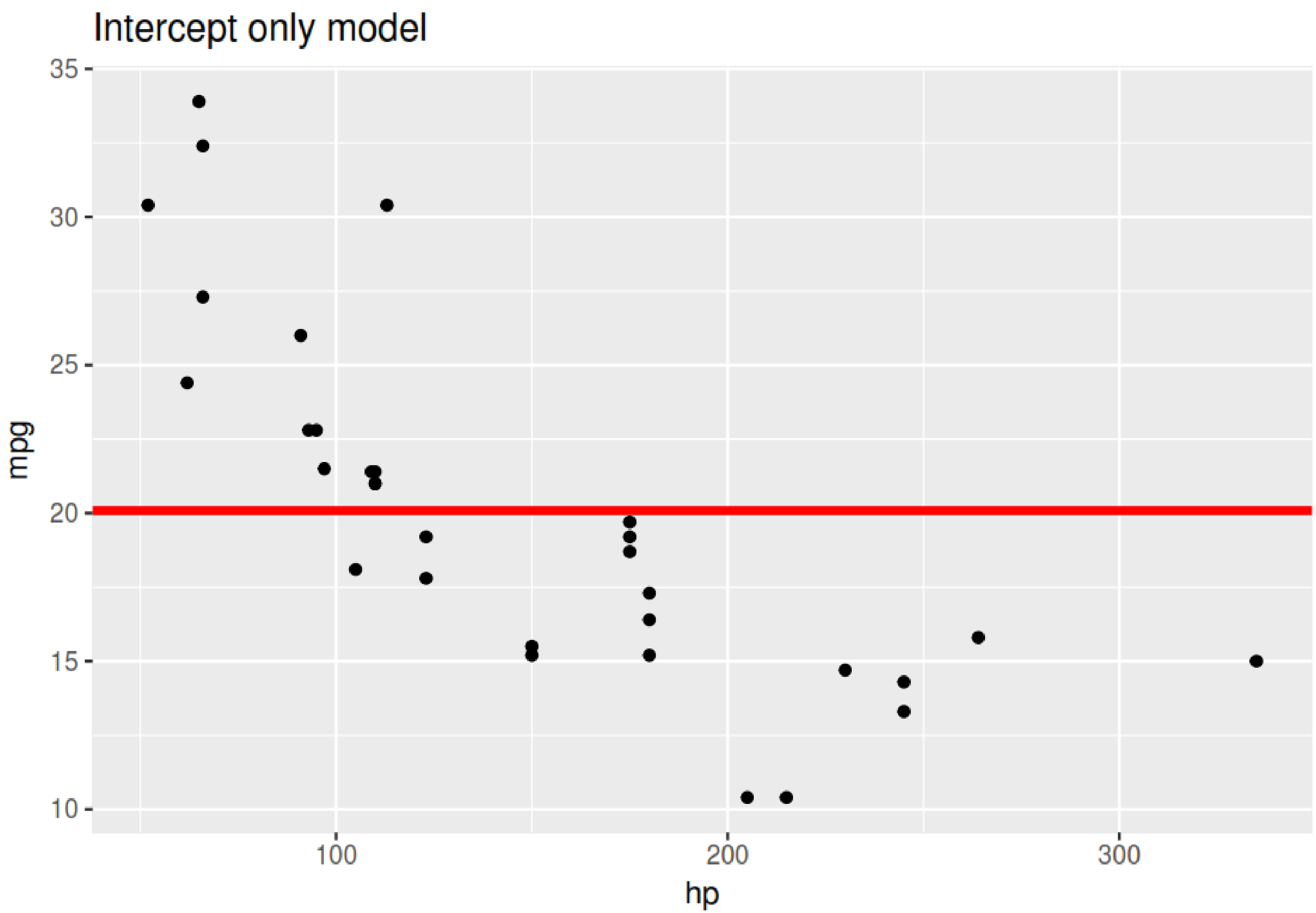
```
library(ggplot2)
data(mtcars)
ggplot(mtcars, aes(x=hp, y=mpg))+geom_point()
```



Intercept only model

Intercept only model

```
ggplot(mtcars, aes(x=hp, y=mpg))+geom_point()+
  geom_hline(yintercept = mean(mtcars$mpg), col="red", size=1.5)+
  ggtitle("Intercept only model")
```



- for linear regression we will use lm() function.
- The formula in R is defined with ~
- For regression
  - on the left side of ~ is your dependent variable,
  - on the right side of ~ are your independent variables

Intercept only model

```
model<-lm(mpg~1, data=mtcars)
summary(model)
```

```
##
## Call:
## lm(formula = mpg ~ 1, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.6906 -4.6656 -0.8906  2.7094 13.8094
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   20.091      1.065   18.86   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.027 on 31 degrees of freedom
```

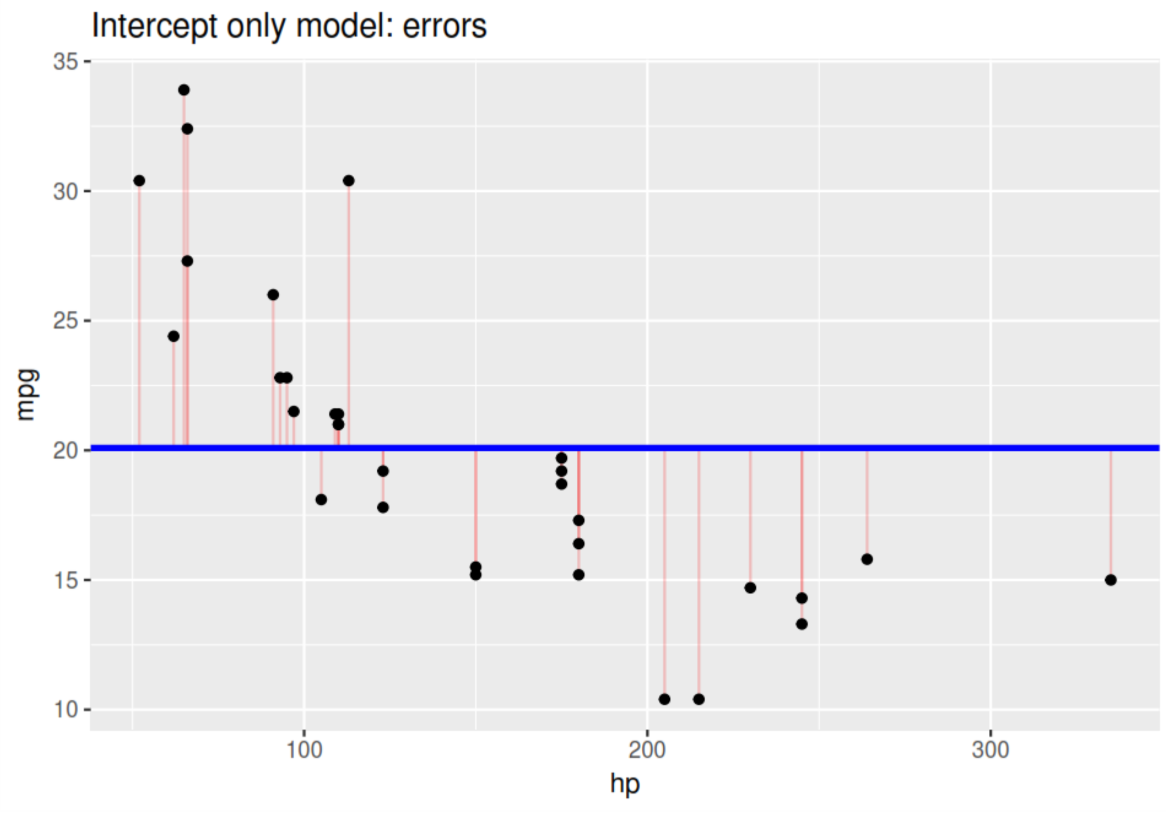
```
mean(mtcars$mpg)
```

```
## [1] 20.09062
```

Intercept only model: errors

Errors with Intercept only model

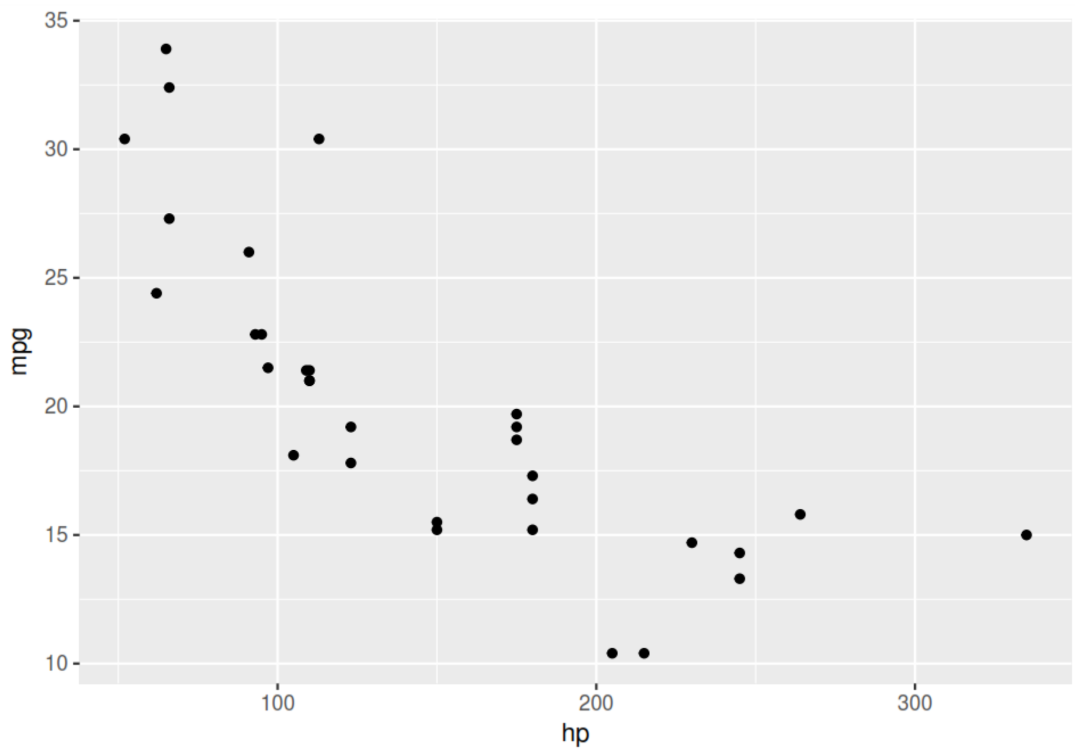
```
ggplot(mtcars, aes(x = hp, y = mpg)) +
geom_segment(aes(xend = hp, yend = mean(mtcars$mpg)), alpha = .2, col="red") +
  geom_point()+
geom_hline(yintercept=mean(mtcars$mpg), col="blue", size=1.2)+
  ggtitle("Intercept only model: errors")
```



Try to find the best line

ggplot(mtcars, aes(x=hp, y=mpg))+geom_point()+
geom_abline(intercept=, slope=)

```
library(ggplot2)
data(mtcars)
ggplot(mtcars, aes(x=hp, y=mpg))+geom_point()
```



Regression

```
mod3<-lm(mpg~hp, data=mtcars)
names(mod3)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

Regression

```
summary(mod3)
```

```
##
## Call:
## lm(formula = mpg ~ hp, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.7121 -2.1122 -0.8854  1.5819  8.2360
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.09886    1.63392  18.421  < 2e-16 ***
## hp          -0.06823    0.01012  -6.742 1.79e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.863 on 30 degrees of freedom
## Multiple R-squared:  0.6024, Adjusted R-squared:  0.5892
## F-statistic: 45.46 on 1 and 30 DF,  p-value: 1.788e-07
```

```
coefficients(mod3)
```

```
## (Intercept)          hp
## 30.09886054 -0.06822828
```

The regression formula

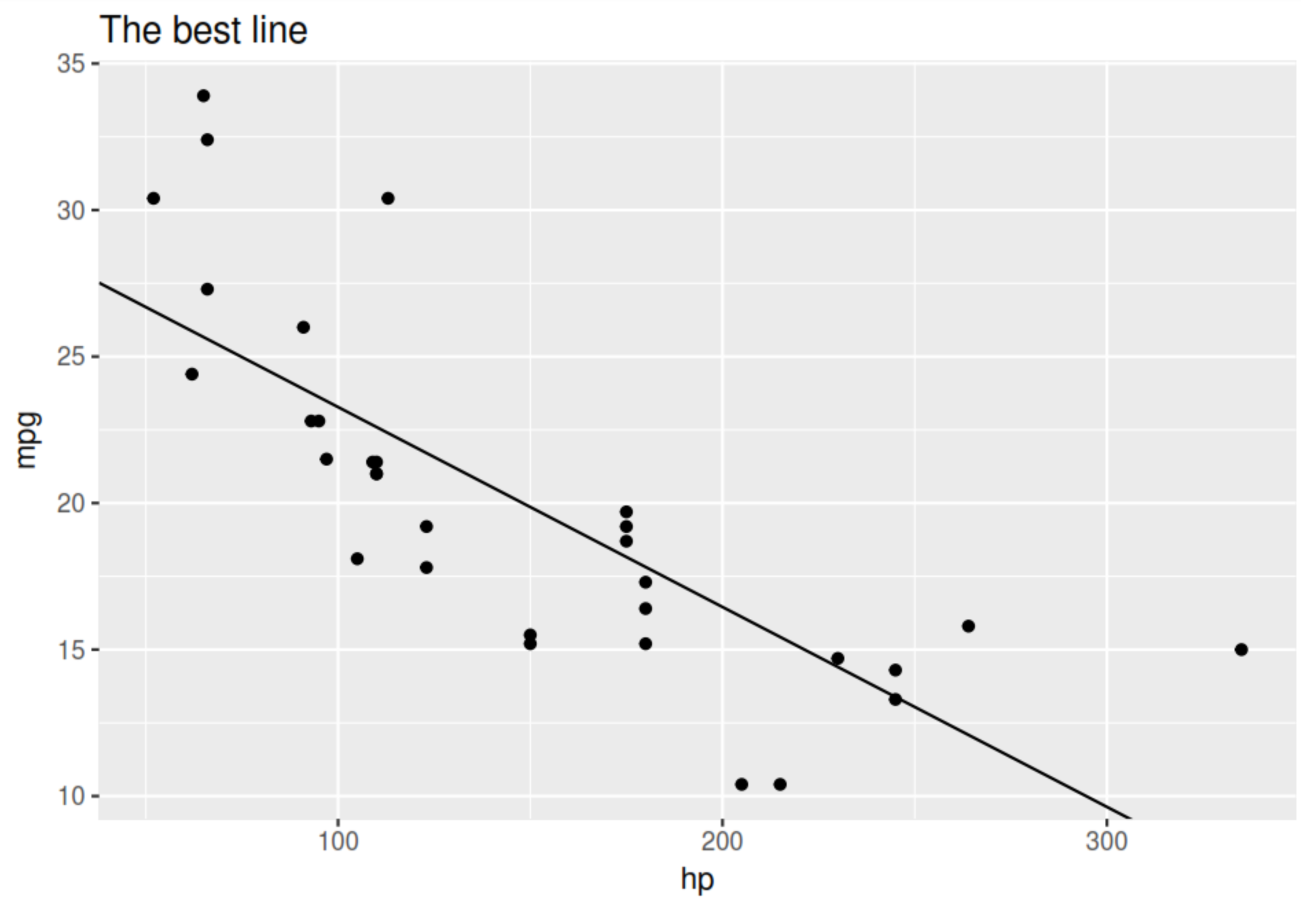
```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.09886    1.63392  18.421  < 2e-16 ***
## hp          -0.06823    0.01012  -6.742 1.79e-07 ***
```

$$mpg = 30.9886 + (-0.06823) * hp$$

The best line

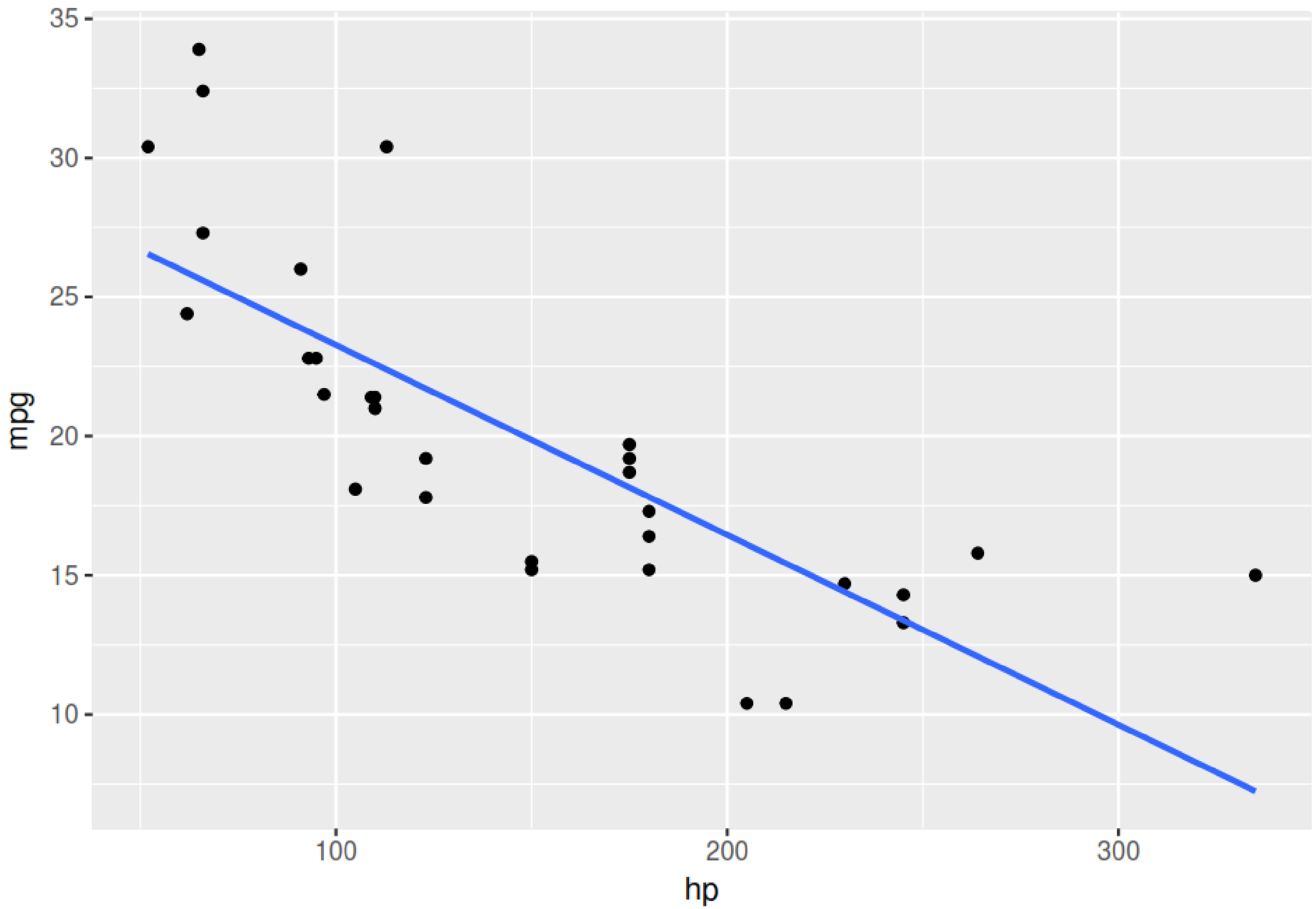
The best line

```
ggplot(mtcars, aes(x=hp, y=mpg))+geom_point()+
geom_abline(intercept=30.09886054, slope=-0.06822828)+
  ggtitle("The best line")
```



The best fit

```
ggplot(mtcars, aes(x=hp, y=mpg))+geom_point()+
  geom_smooth(method="lm", se=F)
```



The fitting process

$$Y = \beta_0 + \beta_1 * X + \epsilon$$

The goal of the modeling is to find optimal estimates for β_0, β_1

$$\hat{Y} = \widehat{\beta_0} + \widehat{\beta_1} * X$$

$\hat{Y}$ is called fitted or predicted value

Residual

$$e = Y - \hat{Y}$$

Fitting procedure

- Given n observations of pairs (x_i, y_i)
- Find $\widehat{\beta_0}$, $\widehat{\beta_1}$ that minimize $\sum_{i=1}^{n} e_i^2$

Least Squares method

- Easy, unique solution
- Residuals sums to zero
- Line must pass through $\left(\bar{X}, \bar{Y}\right)$
- There are several other assumptions as well

- **L**inearity
  - The underlying relationship between X and Y is linear
- **I**ndependence of Errors
  - Error values are statistically independent
- **N**ormality of Error
  - Error values (ε) are normally distributed for any given value of X
- **E**qual Variance (Homoscedasticity)
  - The probability distribution of the errors has constant variance

How good is the model ?

- Inference about the slope: Is there a significant linear relationship between independent and dependent variables?

- How good is the model in explaining the relationship ?

- t test for a population slope
  - Is there a linear relationship between X and Y?
- Null and alternative hypotheses

H_0: $\beta_1 = 0$ (no linear relationship)

H_1: $\beta_1 \neq 0$ (linear relationship does exist)

Testing: look at the p-value. If it is small (less than 0.05) then the relationship is significant

Hypothesis testing for the slope

```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.09886    1.63392  18.421  < 2e-16 ***
## hp          -0.06823    0.01012  -6.742 1.79e-07 ***
## ---
```

2e-16 is a scientific notation

This means $2 * 10^{-16} = \frac{2}{10^{16}}$

Very very small number

How good is the model ?

Total variation is made up of two parts:

$$SST \quad = \quad SSR \quad + \quad SSE$$

Total Sum of Squares	Regression Sum of Squares	Error Sum of Squares
$SST = \sum (Y_i - \bar{Y})^2$	$SSR = \sum (\hat{Y}_i - \bar{Y})^2$	$SSE = \sum (Y_i - \hat{Y}_i)^2$

where:

= Average value of the dependent variable

Y_i = Observed values of the dependent variable

$_i$ = Predicted value of Y for the given X_i value

How good is the model ?

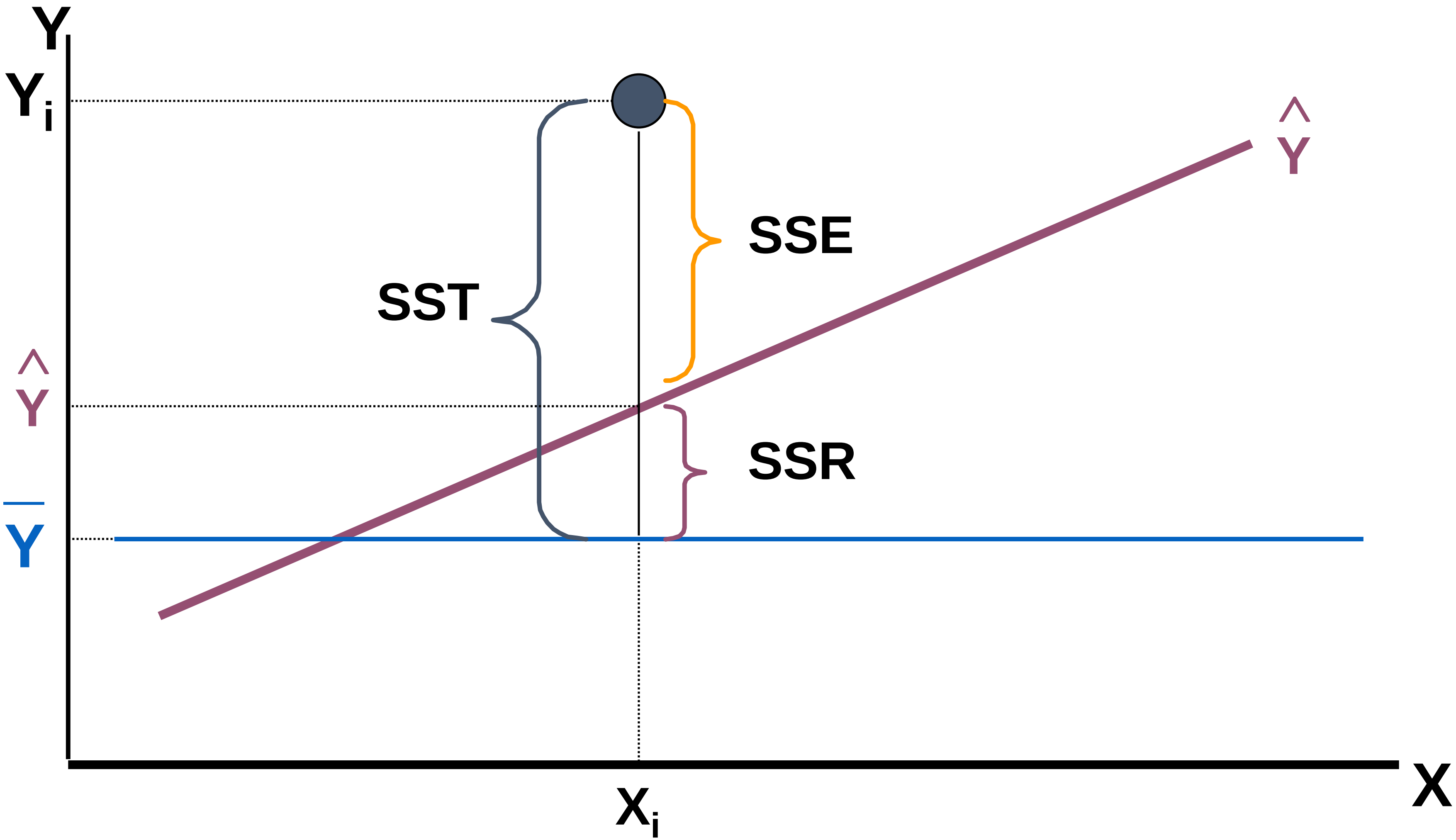
- SST = total sum of squares
  - Measures the variation of the Y_i values around their mean $\bar{Y}$
- SSR = regression sum of squares
  - Explained variation attributable to the relationship between X and Y
- SSE = error sum of squares
  - Variation attributable to factors other than the relationship between X and Y

R square

$$SSR = \sum_{i=1}^{n} (\hat{y} - \bar{y})^2 \qquad SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \qquad SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

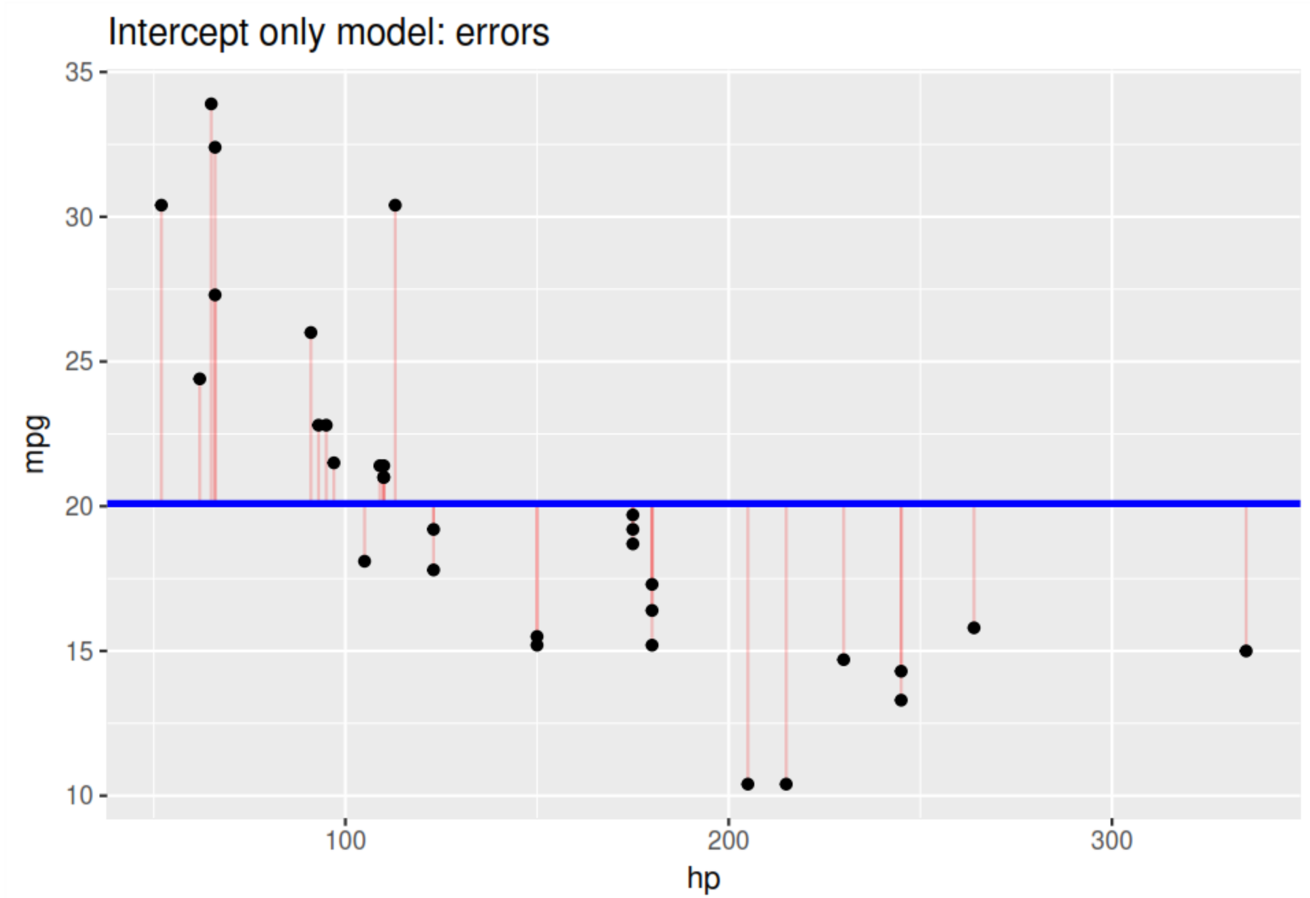
$$R^2 = \frac{SSR}{SST} \qquad R^2 = 1 - \frac{SSE}{SST}$$

Measures of Variation



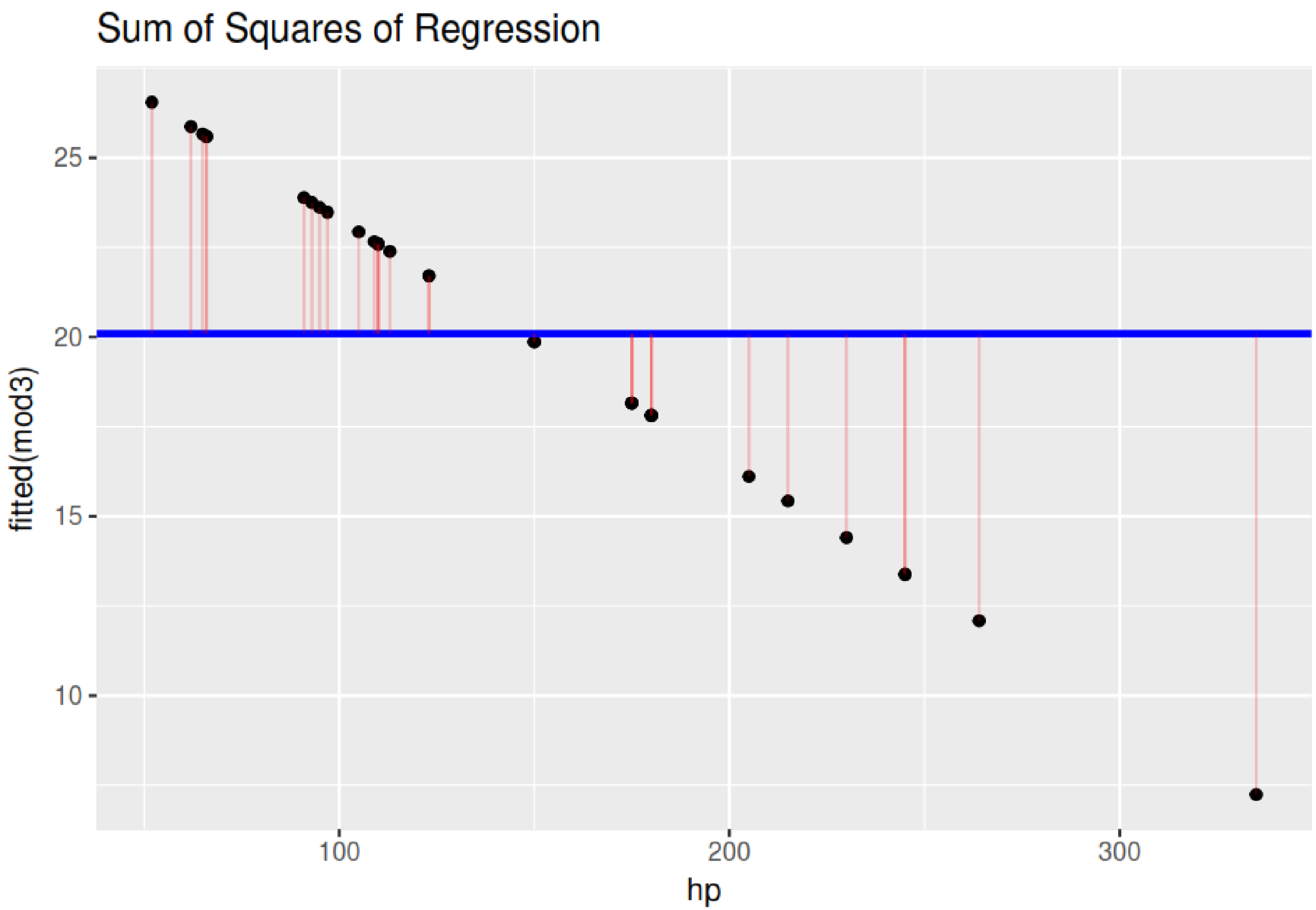
Sum of Squares of Total (intercept only model)

```
ggplot(mtcars, aes(x = hp, y = mpg)) +
geom_segment(aes(xend = hp, yend = mean(mtcars$mpg)), alpha = .2, col="red") +
  geom_point()+
geom_hline(yintercept=mean(mtcars$mpg), col="blue", size=1.2)+
  ggtitle("Intercept only model: errors")
```



Sum of Squares of Regression

```
ggplot(mtcars, aes(x = hp, y = fitted(mod3))) + geom_point()+
geom_hline(yintercept=mean(mtcars$mpg), col="blue", size=1.2)+
geom_segment(aes(xend = hp, yend = mean(mtcars$mpg)), alpha = .2, col="red")+
  ggtitle("Sum of Squares of Regression")
```



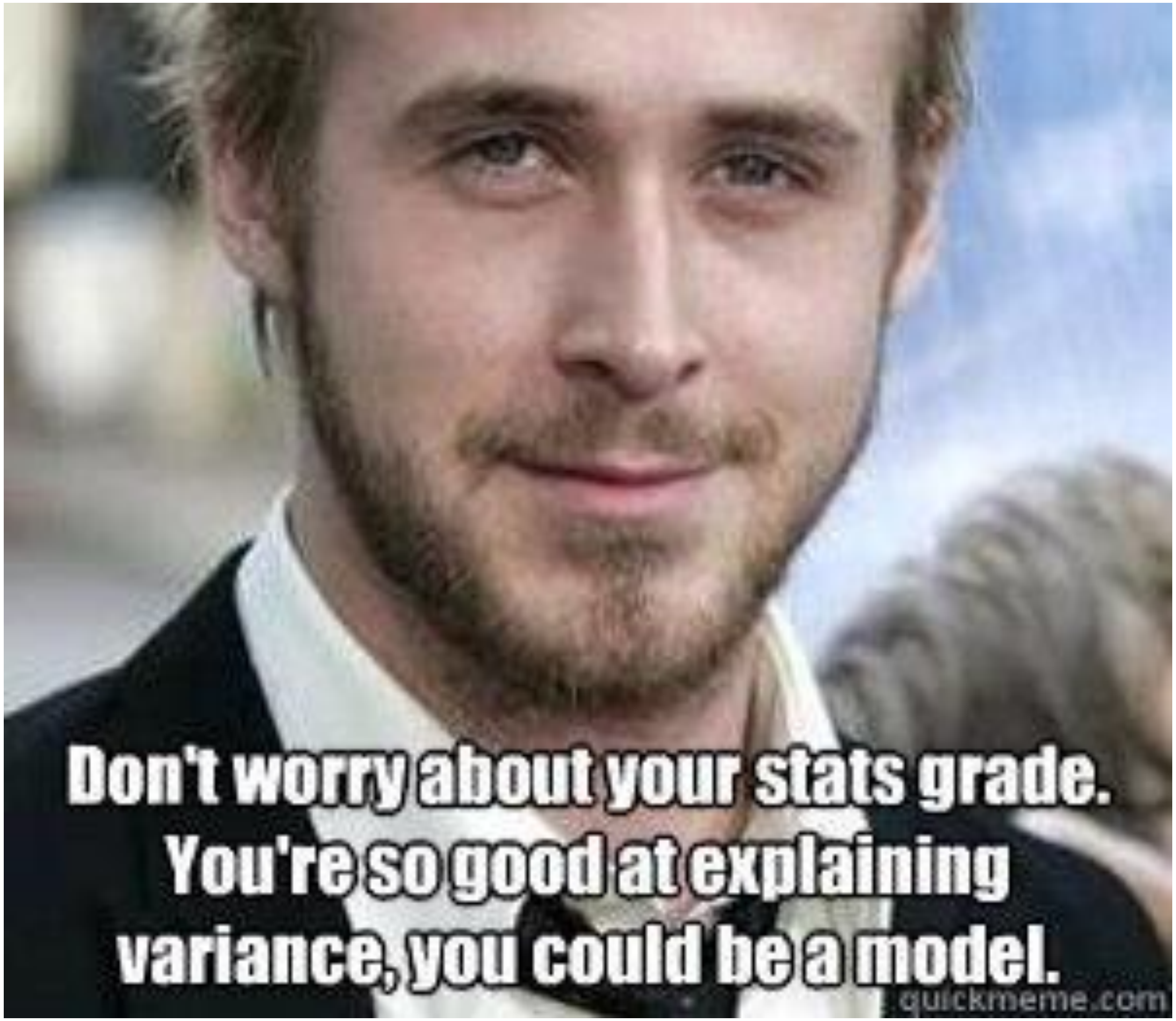
Sum of Squares of Errors

```
ggplot(mtcars, aes(x = hp, y = mpg)) +
geom_smooth(method = "lm", se = FALSE, color = "blue") +
geom_segment(aes(xend = hp, yend = fitted(mod3)), alpha = .2, col="red") +
geom_point()+geom_point(aes(y = fitted(mod3)), shape = 1)+
ggtitle("Sum of Squares of errors")
```

Interpretation

$$0 \leq R^2 \leq 1$$

- R_{square} The percentage of variation in Y (dependent variable) that is explained by the X (independent variable).
- Closer to 1, better is the model
- Is showing how good is the model doing compared to baseline (intercept only model).
- For the simple linear regression model (1 independent variable) is the square of correlation coefficient.


Don't worry about your stats grade.
You're so good at explaining
variance, you could be a model.
quickmeme.com

Multiple regression model

Multiple regression

$$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \beta_3 * X_3 + \cdots \epsilon$$

The goal of the modeling is to find optimal estimates for $\beta_0,\ \beta_1 \ldots ..,\beta_n$

$$\hat{Y} = \widehat{\beta_0} + \widehat{\beta_1} * X_1 + \widehat{\beta_2} * X_2 + \widehat{\beta_3} * X_3$$

```r
seriea <- read.csv("seriea.csv")
str(seriea)
```

```
## 'data.frame':    8775 obs. of  15 variables:
##  $ POS    : Factor w/ 3 levels "D","F","M": 1 1 3 3 3 1 1 1 3 3 ...
##  $ Name   : Factor w/ 2817 levels "Aaron Mattia Tabacchi",..: 1753 946 1859 853 455 1125 26
##  $ Age    : int  27 29 23 20 18 26 31 25 26 23 ...
##  $ APP    : int  32 34 20 1 30 25 8 27 18 27 ...
##  $ SUBIN  : int  0 0 6 1 9 2 1 2 1 6 ...
##  $ G      : int  2 6 3 0 1 0 0 0 0 2 ...
##  $ A      : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ SH     : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ ST     : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ FC     : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ FA     : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ YC     : int  8 1 1 0 7 2 0 8 2 5 ...
##  $ RC     : int  0 0 0 0 1 0 0 0 0 0 ...
##  $ team_id: int  116 116 116 116 116 117 117 117 117 117 ...
##  $ Season : int  2002 2002 2002 2002 2002 2002 2002 2002 2002 2002 ...
```

Variables:

- POS: Players position: D-Defender, F-Forward, M-Midfielder
- Name: Players name
- Age: Player age at that season
- APP: Number of appearances
- SUBIN: Substitute Appearances
- G: Goals
- A: Assists
- SH: Shots
- ST: Shots on target
- FC: Fouls committed
- FA: Fouls Suffered
- YC: Yellow cards
- RC: red cards

- First we run regression model with G – number of goals as dependent variable and POS position of the player as an independent variable

- Note that POS is a categorical variable

```
model1 <- lm(G~POS, data=seriea)
summary(model1)
```

```
##
## Call:
## lm(formula = G ~ POS, data = seriea)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -3.863 -1.324 -0.527  0.473 32.137
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.52731    0.05395   9.775   <2e-16 ***
## POSF         3.33551    0.08654  38.543   <2e-16 ***
## POSM         0.79699    0.07351  10.842   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.019 on 8772 degrees of freedom
## Multiple R-squared:  0.1497, Adjusted R-squared:  0.1495
## F-statistic: 771.9 on 2 and 8772 DF,  p-value: < 2.2e-16
```

- POS is a categorical variable with 3 levels, D,F,M
- D is the reference level, as it comes alphabetically first
- POSF is the coefficient for F, POSM is the coefficient for M

```
coef(model1)
```

```
## (Intercept)        POSF        POSM
##   0.5273076   3.3355065   0.7969946
```

- Interpretations:
  - The average number of goals for Forwards is by 3.3355 more than the average for **Defenders.**
  - The average number of goals for Midfielders is by 0.79699 more than the average for **Defenders.**

- Calculate the average number of the goals for Forwards, Midfielders and Defenders
- You can see that the average for Defenders is the same as our intercept

```r
F <- mean(seriea$G[seriea$POS=='F'])
D <- mean(seriea$G[seriea$POS=='D'])
M <- mean(seriea$G[seriea$POS=='M'])
```

```r
F
```

```
## [1] 3.862814
```

```r
D
```

```
## [1] 0.5273076
```

```r
M
```

```
## [1] 1.324302
```

```r
F
```

```
## [1] 3.862814
```

```r
D
```

```
## [1] 0.5273076
```

```r
M
```

```
## [1] 1.324302
```

```r
coef(model1)
```

```
## (Intercept)        POSF        POSM
##   0.5273076   3.3355065   0.7969946
```

POSF = 3.335

Mean(Forwards) = 0.5273076+3.3355065 = 3.862814

- The resulting object from lm() function is a list
- It contains different information about our model and can be assessed by names as well

```r
typeof(model1)
```

```
## [1] "list"
```

```r
names(model1)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "contrasts"     "xlevels"       "call"          "terms"
## [13] "model"
```

```r
model1$coefficients
```

```
## (Intercept)        POSF        POSM
##   0.5273076   3.3355065   0.7969946
```

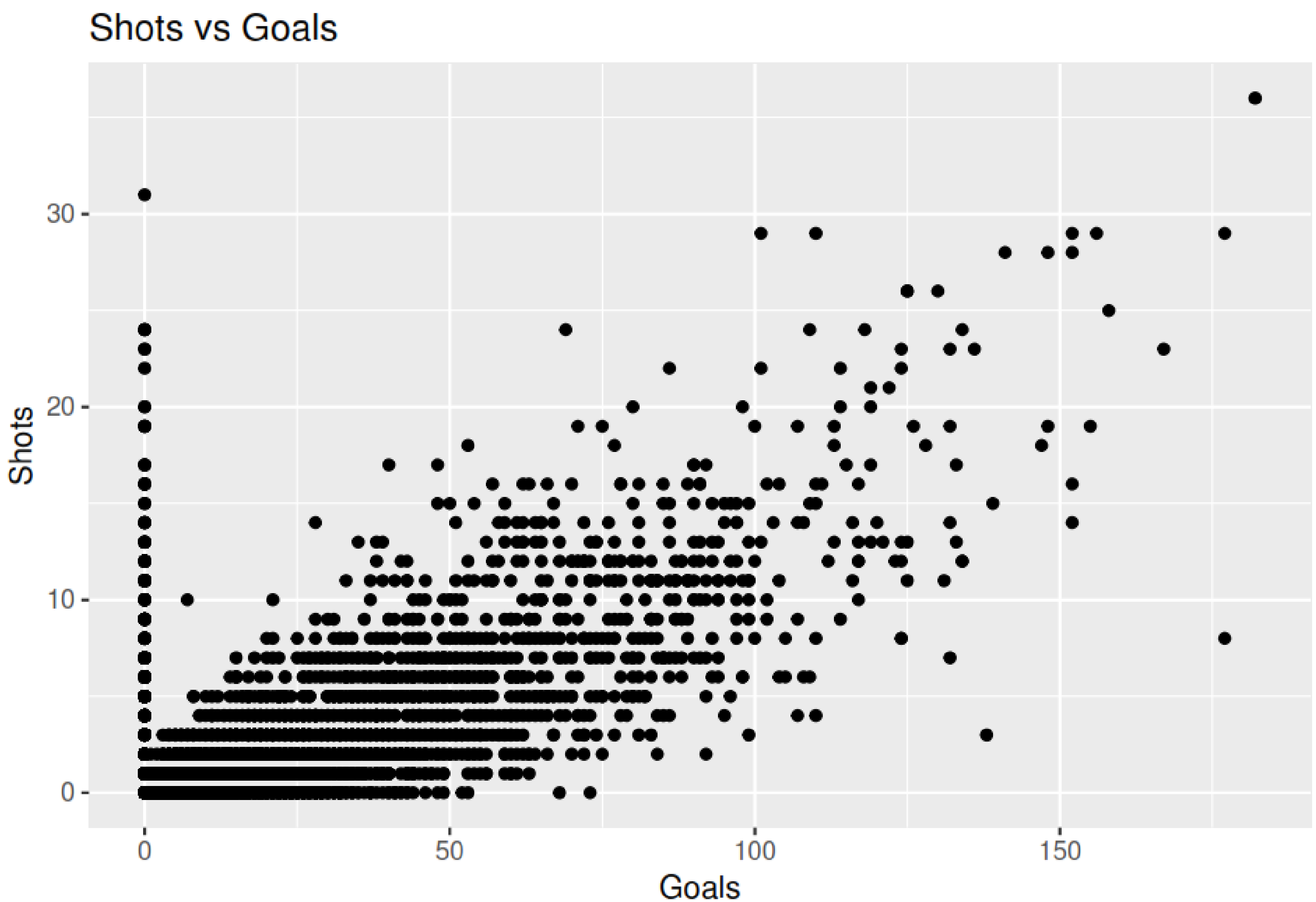
```r
model1$residuals[1:20]
```

```
##          1          2          3          4          5          6
##  1.4726924  5.4726924  1.6756979 -1.3243021 -0.3243021 -0.5273076
##          7          8          9         10         11         12
## -0.5273076 -0.5273076 -1.3243021  0.6756979 -1.3243021  2.6756979
##         13         14         15         16         17         18
##  5.6756979 -1.3243021  3.1371859 -0.5273076 -0.5273076  0.4726924
##         19         20
##  0.4726924  0.4726924
```

Now lets add more variables to our model

Variable SHOTS on goal, fist lets visualize the data, do you thing the relationship is linear? anything strange ?

```
ggplot(seriea, aes(x=SH, y=G))+ geom_point()+
  labs(x="Goals", y="Shots", title="Shots vs Goals")
```



Lets run the model

```
model2 <- lm(G~POS+SH, data=seriea)
summary(model2)
```

```
##
## Call:
## lm(formula = G ~ POS + SH, data = seriea)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.4015  -0.9138  -0.0026   0.0977  29.4301
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.003864   0.039370  -0.098   0.9218
## POSF         1.573751   0.065446  24.047   <2e-16 ***
## POSM        -0.093801   0.053959  -1.738   0.0822 .
## SH           0.100229   0.001115  89.881   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.178 on 8771 degrees of freedom
## Multiple R-squared:  0.5574, Adjusted R-squared:  0.5572
## F-statistic:  3681 on 3 and 8771 DF,  p-value: < 2.2e-16
```

- How good is the model ?

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.178 on 8771 degrees of freedom
## Multiple R-squared:  0.5574, Adjusted R-squared:  0.5572
## F-statistic:  3681 on 3 and 8771 DF,  p-value: < 2.2e-16
```

~56% of the variance explained

- Are the variables significant ?

```
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.003864   0.039370  -0.098   0.9218
## POSF         1.573751   0.065446  24.047   <2e-16 ***
## POSM        -0.093801   0.053959  -1.738   0.0822 .
## SH           0.100229   0.001115  89.881   <2e-16 ***
## ---
```

Look at the p-values

- SH is significant ($\alpha = 0.05$)
- Categorical variable is treated as significant if at least one of its categories is significant

Interpret the coefficient for SH (SHOT)

```
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.003864   0.039370  -0.098   0.9218
## POSF         1.573751   0.065446  24.047   <2e-16 ***
## POSM        -0.093801   0.053959  -1.738   0.0822 .
## SH           0.100229   0.001115  89.881   <2e-16 ***
## ---
```

Lets build another model now with ST (Shots on target).
Are the variables significant?
How good is the model ?

```
model3 <- lm(G~POS+ST, data=seriea)
summary(model3)
```

```
##
## Call:
## lm(formula = G ~ POS + ST, data = seriea)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.3920  -0.6871  -0.1113   0.0361  29.6404
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.111263   0.036359   3.060  0.00222 **
## POSF         1.248364   0.061358  20.346  < 2e-16 ***
## POSM        -0.011135   0.049855  -0.223  0.82328
## ST           0.287939   0.002774 103.815  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.022 on 8771 degrees of freedom
## Multiple R-squared:  0.6185, Adjusted R-squared:  0.6183
## F-statistic:  4739 on 3 and 8771 DF,  p-value: < 2.2e-16
```

Add both variables SH and ST to model.
Anything strange ?

```r
model4 <- lm(G~POS+ST+SH, data=seriea)
summary(model4)
```

```
##
## Call:
## lm(formula = G ~ POS + ST + SH, data = seriea)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.3358  -0.7131  -0.1903   0.1475  29.6211
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.167984   0.036621   4.587 4.56e-06 ***
## POSF         1.210879   0.061146  19.803  < 2e-16 ***
## POSM         0.048768   0.049960   0.976    0.329
## ST           0.379793   0.009752  38.945  < 2e-16 ***
## SH          -0.035746   0.003640  -9.820  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.011 on 8770 degrees of freedom
## Multiple R-squared:  0.6226, Adjusted R-squared:  0.6224
## F-statistic:  3617 on 4 and 8770 DF,  p-value: < 2.2e-16
```

The sign of the variable SH changes from + to – when we add variable ST to the model

```
coef(model2)
```

```
##   (Intercept)          POSF          POSM            SH
## -0.003864495   1.573751402 -0.093801396   0.100228996
```

```
coef(model4)
```

```
## (Intercept)        POSF        POSM          ST          SH
##  0.16798351  1.21087861  0.04876766  0.37979301 -0.03574640
```

This is called a multicollinearity problem

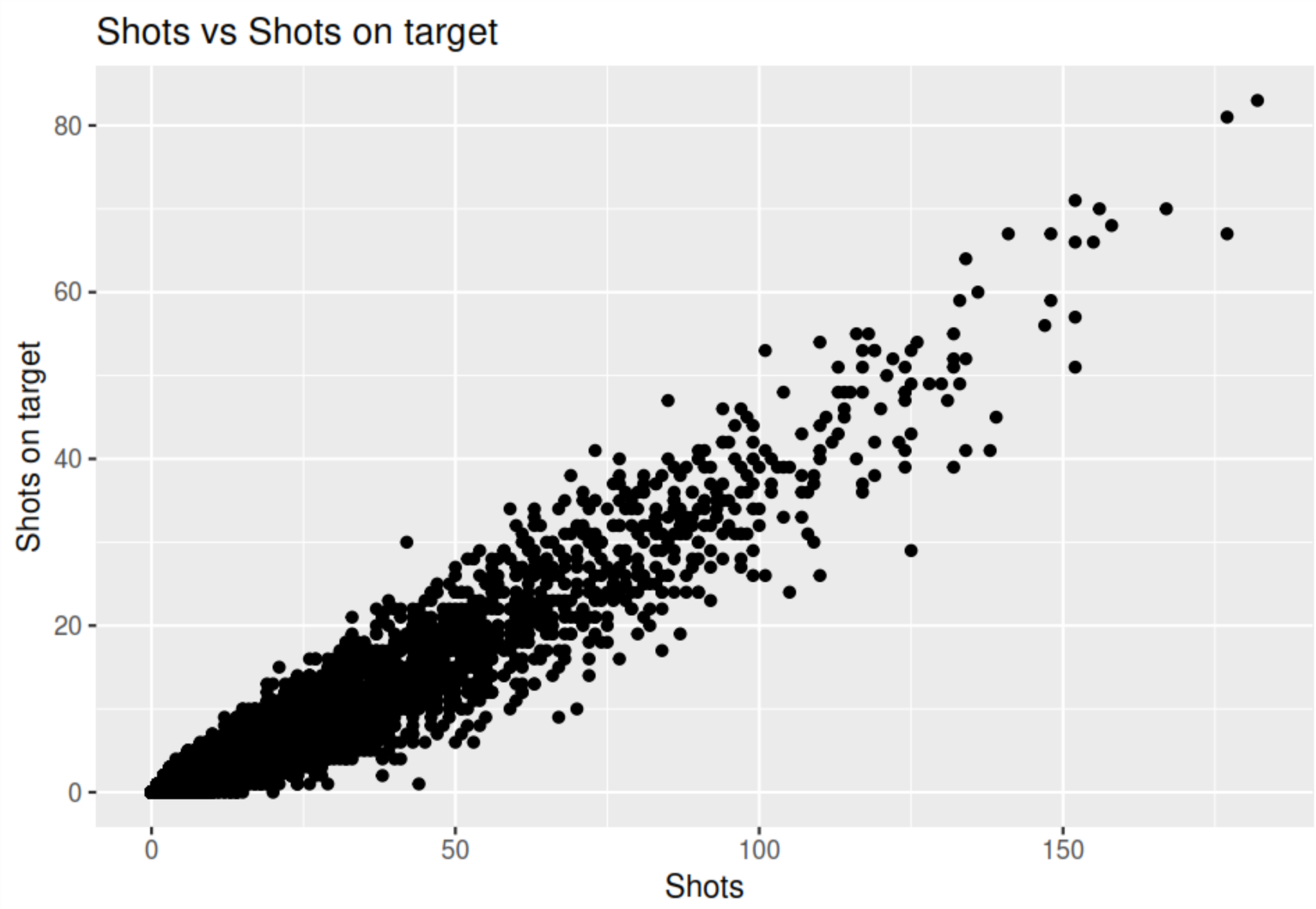
Multicollinearity is present when independent x variables are highly correlated with each other

Look at the correlation coefficient first

```
cor(seriea$SH, seriea$ST)
```

```
## [1] 0.9616804
```

```
ggplot(seriea, aes(x=SH, y=ST))+geom_point()+
  labs(x="Shots", y="Shots on target",
       title="Shots vs Shots on target")
```



Model Selection

Learning process: Supervised Learning

Problem	Solution
$x^2 - 10x + 24 = 0$	(6,4)
$x^2 - 2x - 8 = 0$	(4,-2)
$x^2 - 4x - 21 = 0$	(7,-3)
$x^2 - 6x + 5 = 0$	(5,1)
$x^2 - 5x + 4 = 0$	(4,1)

Problem

$$x^2 - 10x + 24 = 0$$

$$x^2 - 2x - 8 = 0$$

$$x^2 - 4x - 21 = 0$$

$$x^2 - 6x + 5 = 0$$

$$x^2 - 5x + 4 = 0$$

Learning process: How to know if one has really learned how to solve the problems?

Problem	Solution
$x^2 - 10x + 24 = 0$	(6,4)
$x^2 - 2x - 8 = 0$	(4,-2)
$x^2 - 4x - 21 = 0$	(7,-3)
$x^2 - 6x + 5 = 0$	(5,1)
$x^2 - 5x + 4 = 0$	(4,1)

Give another set of problems, with no solutions

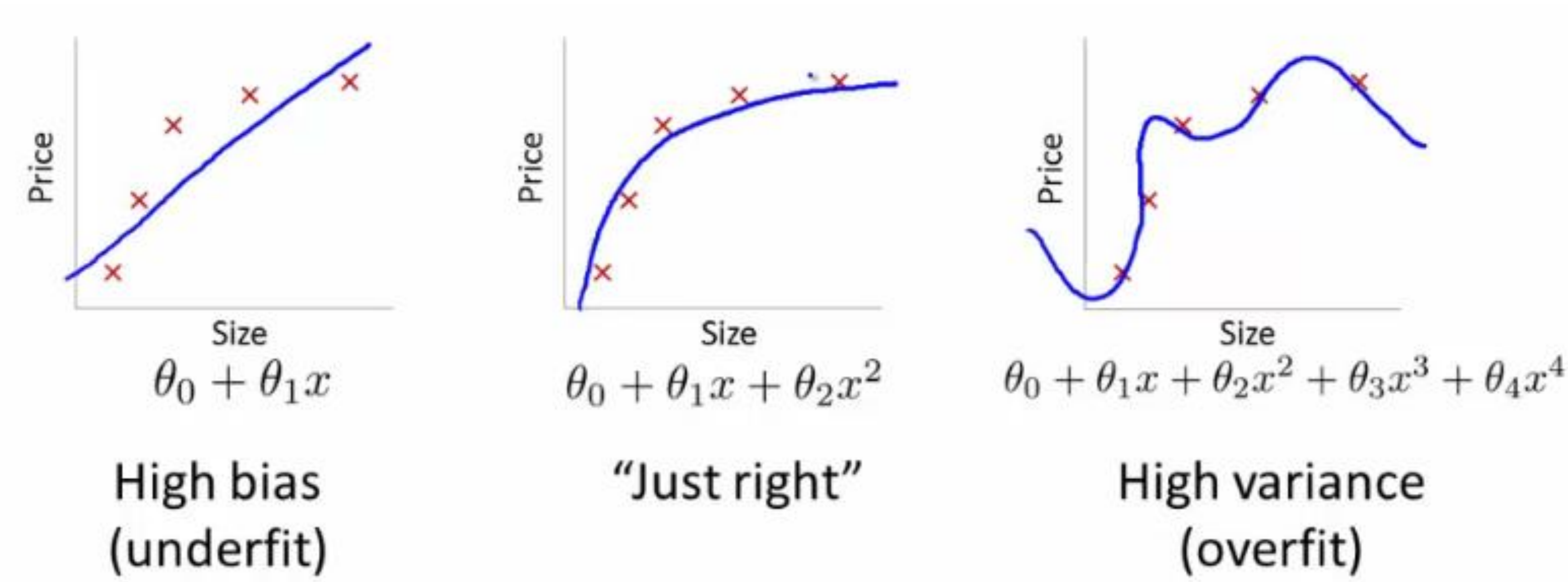
General formula for the error

$$Mean\ Error = Bias^2 + Variance + \sigma^2$$

- The bias is error from erroneous assumptions in the learning algorithm. High bias can cause an algorithm to miss the relevant relations between features and target outputs (underfitting).

- The variance is error from sensitivity to small fluctuations in the training set. High variance can cause overfitting: modeling the random noise in the training data, rather than the intended outputs.

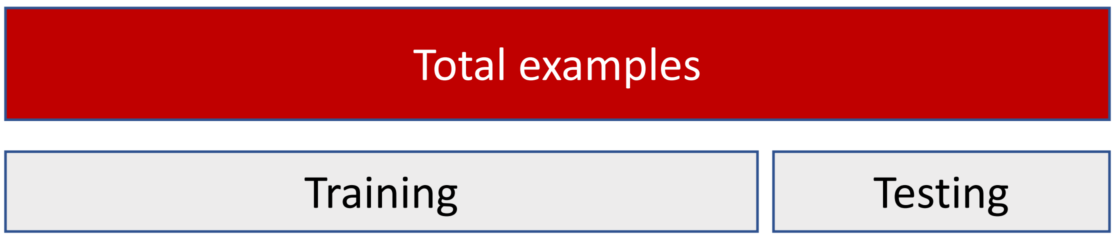
Overfiting and Underfiting



Bias vs Variance trade-off

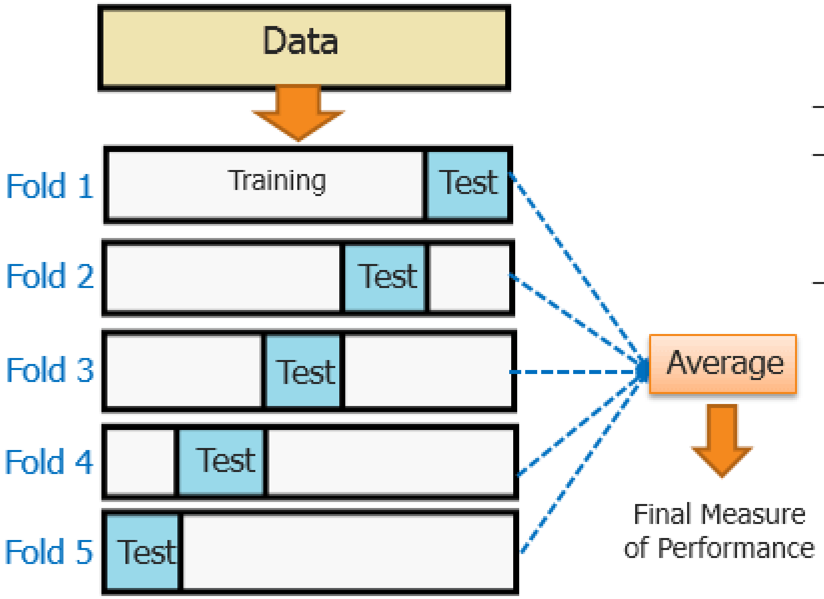
Holdout

- Reserve 2/3 for training and 1/3 for testing



Use model evaluation metric, such as RMSE, on Testing dataset. Choose the model with the optimum evaluation metric.

Cross Validation



- → Technique to validate models/classifiers
- → Method to estimate how accurately the model generalizes to unseen data i.e., how well it performs/predicts
- → K-fold CV
  - » Most popular
  - » k is typically set to 10
  - » Every sample/record is used both in training and test sets

Evaluation metrics

Root Mean Square of Errors

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y} - y)^2}{n}}$$

Mean Absolute Error

$$MAE = \frac{\sum_{i=1}^{n}|\hat{y} - y|}{n}$$

Bias vs Variance tradeoff

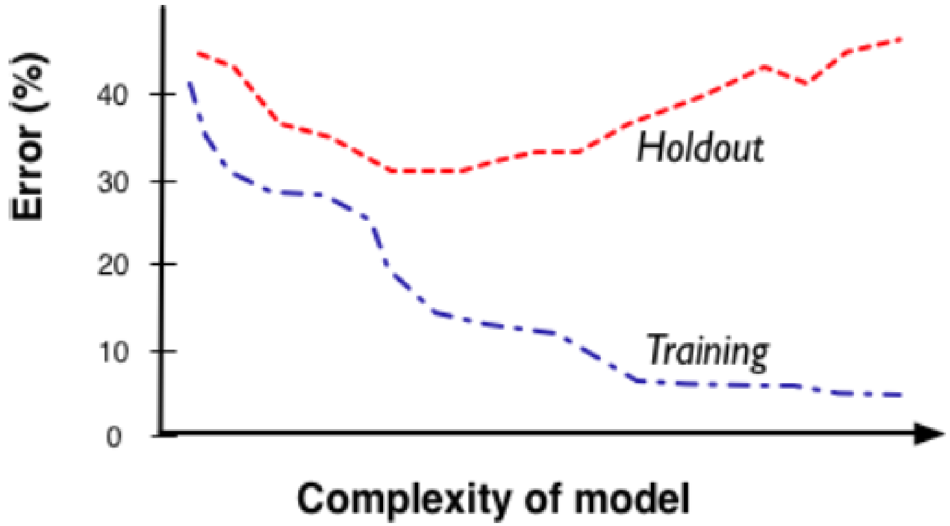
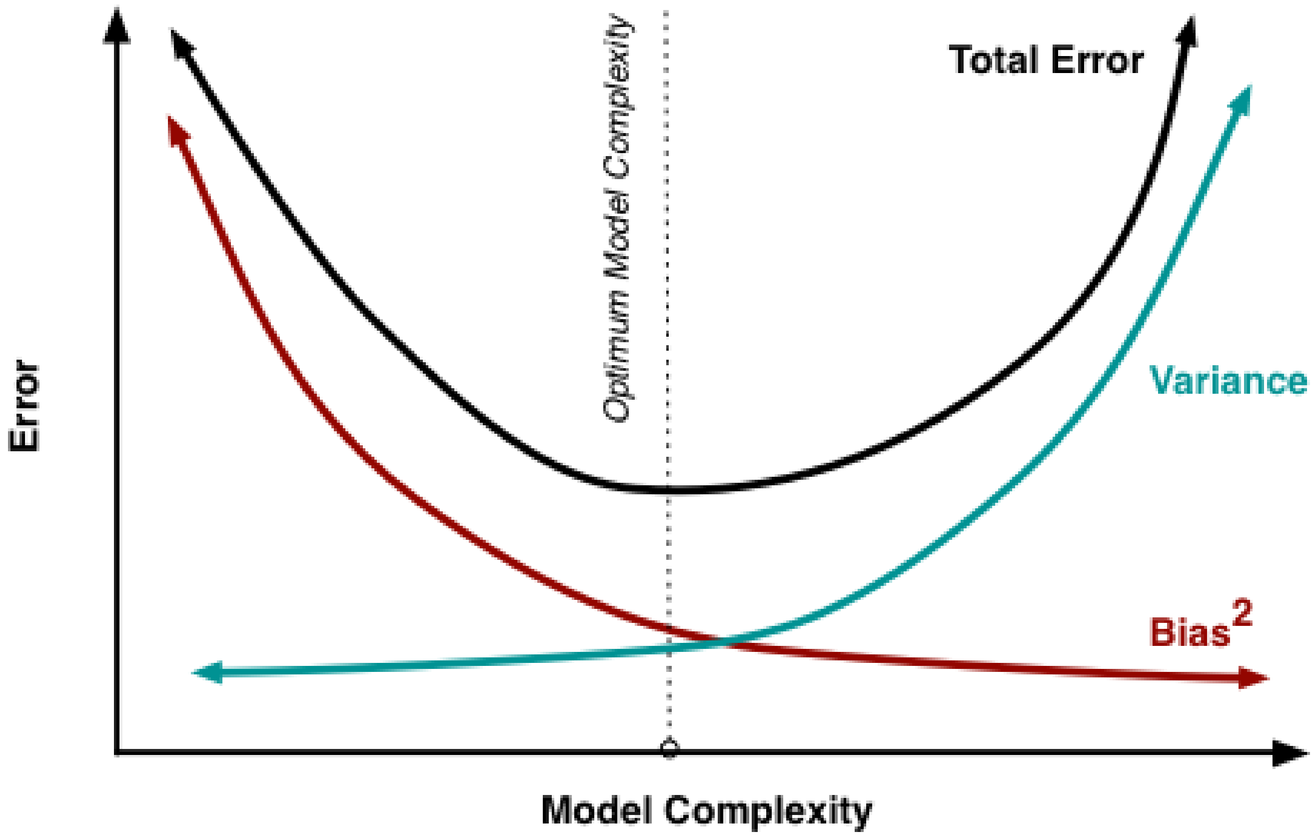
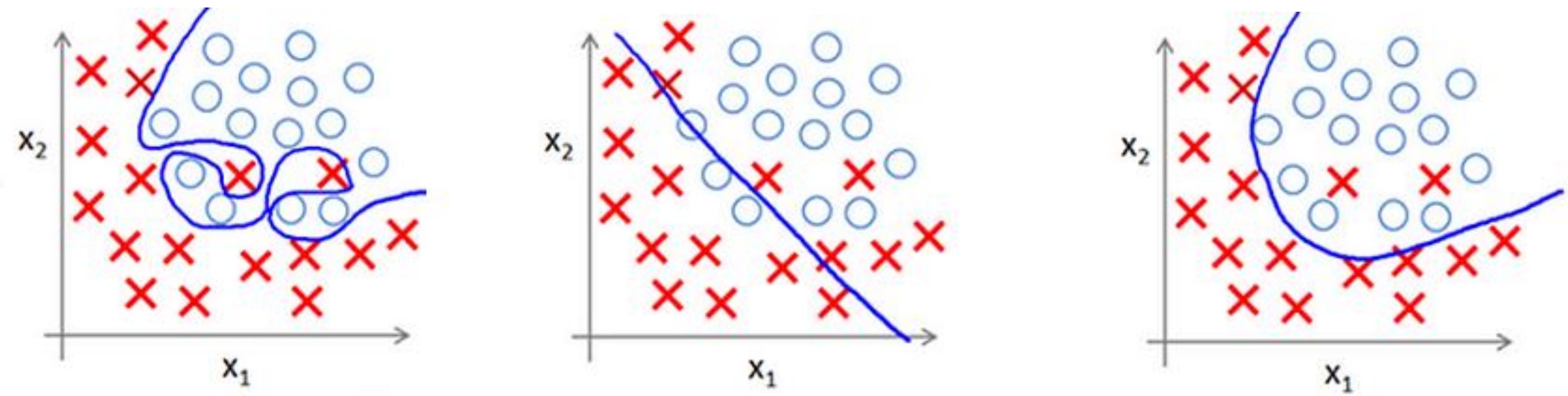


Figure 5-1. A typical fitting graph. Each point on a curve represents an accuracy estimation of a model with a specified complexity (as indicated on the horizontal axis). Accuracy estimates on training data and testing data vary differently based on how complex we allow a model to be. When the model is not allowed to be complex enough, it is not very accurate. As the models get too complex, they look very accurate on the training data, but in fact are overfitting—the training accuracy diverges from the holdout (generalization) accuracy.

Bias-Variance tradeoff



Where is what ?



Baseline estimation, model selection

- Compare your model with the baseline accuracy
    - What will be your prediction and error if you haven't done any modeling?
- Compare the performance of different models on the testing data set. Chose the one with highest performance.

Model selection in practice

General approach

- Create Testing and Training datasets
- Build your models on Training data set
- Evaluate your model performance on Testing set
- Chose the model that does the best on testing set

- Model performance is always evaluated based on testing dataset
- For different types of models (regression, classification), different types of performance measures exists
- For regression analysis, we will use **Root Mean Squares of Errors (RMSE)**

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n}}$$

- The goal is to predict median household value (medv)
- Dataset Boston from library MASS

```
library(MASS)
```

```
## Warning: package 'MASS' was built under R version 3.5.1
```

```
data(Boston)
```

```
## ?MASS::Boston
```

- Training and testing sets need to be generated at random.
- There is no true randomness in computer science, so we use pseudorandom numbers
- When you use set.seed(Number) with the same Number as an argument you are going to get the same “random” output

```
set.seed(1)
sample <- sample(nrow(Boston), floor(nrow(Boston) * 0.7))
sample[1:5]
```

```
## [1] 135 188 289 457 102
```

```
length(sample)
```

```
## [1] 354
```

Create training and testing datasets

- The vector sample contains rownumber that you want to sample
- Boston[sample,] will give a dataframe with given rownumbers
- Boston[-sample,] will give a dataframe with wverything else (that is not sample)

```
Train<-Boston[sample,]
Test<-Boston[-sample,]
```

Here in the formula we use ~.
This means use medv as dependent variable and ALL other variables as independent

```r
model <- lm(medv~., data=Train)
```

- To do predictions on the Test set, we will use generic function (method) predict.
- In case of linear regression model, it takes at least two
    - model
    - newdata

```r
pred1 <- predict(model, newdata = Test)
pred1[1:10]
```

```
##        1        2        3       20       22       23       24       26
## 30.04244 25.08534 30.88295 18.65169 17.73878 15.75622 13.66044 13.48754
##       27       30
## 15.58626 21.03303
```

Calculate Root Mean Square

```r
sqrt(mean((Test$medv-pred1)^2))
```

```
## [1] 4.256622
```

On average our model is wrong by 4.25

```r
summary(model)
```

```
##
## Call:
## lm(formula = medv ~ ., data = Train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.7559  -2.8044  -0.8952   1.6246  26.4958
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  34.693176   6.380806   5.437 1.04e-07 ***
## crim         -0.136079   0.040298  -3.377 0.000818 ***
## zn            0.025453   0.018090   1.407 0.160353
## indus        -0.018390   0.076381  -0.241 0.809876
## chas          2.598633   1.186039   2.191 0.029129 *
## nox         -14.126596   4.771804  -2.960 0.003288 **
## rm            3.820468   0.534460   7.148 5.40e-12 ***
## age          -0.006563   0.016846  -0.390 0.697074
## dis          -1.359358   0.249090  -5.457 9.33e-08 ***
## rad           0.321100   0.088730   3.619 0.000341 ***
## tax          -0.013205   0.004912  -2.688 0.007537 **
## ptratio      -0.907350   0.164474  -5.517 6.86e-08 ***
## black         0.009071   0.003450   2.630 0.008935 **
## lstat        -0.547723   0.063813  -8.583 3.35e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.001 on 340 degrees of freedom
## Multiple R-squared:  0.7154, Adjusted R-squared:  0.7045
## F-statistic: 65.74 on 13 and 340 DF,  p-value: < 2.2e-16
```

Variable indus is not significant, lets remove it from the model

Look at how the formula is specified:
We want all variables as independent variables except indus

```
model2 <- lm(medv~.-indus, data=Train)
pred2 <- predict(model2, newdata = Test)
sqrt(mean((Test$medv-pred2)^2))
```

```
## [1] 4.25144
```

We see slight improvement in RMSE

Remove variable age as well (another non-significant variable)

```r
model3 <- lm(medv~.-indus-age, data=Train)
pred3 <- predict(model3, newdata = Test)
sqrt(mean((Test$medv-pred3)^2))
```

```
## [1] 4.242682
```

What of we take only two variables (2 that have highest correlation with the dependent variable)

```
model4 <- lm(medv~rm+lstat, data=Train)
pred4 <- predict(model4, newdata = Test)
sqrt(mean((Test$medv-pred4)^2))
```

```
## [1] 5.275777
```

What if we have used different Train and Test sets ?

```
sample1 <- sample(nrow(Boston), floor(nrow(Boston) * 0.7))
sample2 <- sample(nrow(Boston), floor(nrow(Boston) * 0.7))
sample3 <- sample(nrow(Boston), floor(nrow(Boston) * 0.7))
```

```
Train1 <- Boston[sample1,]
Test1 <- Boston[-sample1,]

Train2 <- Boston[sample2,]
Test2 <- Boston[-sample2,]

Train3 <- Boston[sample3,]
Test3 <- Boston[-sample3,]
```

```r
pred1 <- predict(model1, newdata = Test1)
pred2 <- predict(model1, newdata = Test2)
pred3 <- predict(model1, newdata = Test3)
```

```r
sqrt(mean((Test1$medv-pred1)^2))
```

```
## [1] 5.372828
```

```r
sqrt(mean((Test2$medv-pred2)^2))
```

```
## [1] 4.76811
```

```r
sqrt(mean((Test3$medv-pred3)^2))
```

```
## [1] 5.317002
```