CSE 4082 / PROJECT 1 --- DIAGONAL 15 PUZZLE

150116048 – Kevser İldeş

150117011 – Buse Batman

In this project, we implemented those search methods (using Python) to solve the Diagonal 15-Puzzle:

a. Uniform Cost Search (UCS)

b. Iterative Lengthening Search (ILS)

c. A* Heuristic Search with the admissible heuristic h1

-> h1(n) = The number of misplaced tiles at node n

d. A* Heuristic Search with the admissible heuristic h2

-> h2(n) = The sum of the city-block distances of each misplaced tile from its current location to its goal location

USED CLASSES / METHODS:

- Node Class: Attributes of this class are:

State: 2 dimensional 4*4 array that contains the order of the tiles

Parent: Points to the parent node of the current node (root node has no parent)

Depth: Number of edge from that node to the root node (root's depth is 0)

Action: Maded action from parent node to reach to that node (up – down – left – right - upper right - down left - upper left – down right) (root's action is none)

Cost: Total cost to reach to that node (Cost of up – down – left – right actions are 1, other diagonal actions are 3)

est_tot_cost: cost + heuristic cost (number of misplaced tiles for h1, manhattan distance for h2)

- display_state (state):

This method, displays the given state of the puzzle in 4 rows - 4 columns.

- swap (state, index_x1, index_y1, index_x2, index_y2)

This method swaps tile-0 with the given tile (x1, y1 -> coordinates of 0 --- x2, y2 -> coordinates of the other tile)

- findIndex (state)

This methods finds the x-y coordinates of the tile-0 of the given state.

- move_up (state)

This method moves tile-0 up (which means -> (y coordinate of 0) - 1), if tile-0 is not in the first row (y coordinate is not 0), by using swap method

- move_down (state)

This method moves tile-0 down (which means -> (y coordinate of 0) + 1), if tile-0 is not in the last row (y coordinate is not 3), by using swap method

- move_left (state)

This method moves tile-0 to left (which means -> (x coordinate of 0) - 1), if tile-0 is not in the first column (x coordinate is not 0), by using swap method

- move_right (state)

This method moves tile-0 to right (which means -> (x coordinate of 0) + 1), if tile-0 is not in the first column (x coordinate is not 3), by using swap method

- move_diag_ur (state)

This method moves tile-0 to upper right (which means -> (x + 1, y - 1), if tile-0 is not in the first row and last column (x is not 3 and y is not 0), by using swap method

- move_diag_ul (state)

This method moves tile-0 to upper left (which means -> (x - 1, y - 1), if tile-0 is not in the first row and first column (x and y is not 0), by using swap method

- move_diag_dr (state)

This method moves tile-0 to down right (which means -> (x + 1, y + 1), if tile-0 is not in the last row and last column (x and y is not 3), by using swap method

- move_diag_dl (state)

This method moves tile-0 to down left (which means -> (x - 1, y + 1), if tile-0 is not in the last row and first column (x is not 0 and y is not 3), by using swap method

- expand_node(node)

This method expands the given node by creating new nodes. So this method calls Node class with state as all of the allowed move operation's return state, parent as given node, depth as given node's depth + 1, action as up-left-right.., cost as given node's cost + 1, est_tot_cost as given node's est_tot_cost + 3 for diagonal moves, + 1 for other moves.

- misplaced (state, goal_state)

This method calculates the number of misplaced tiles by visiting each index of the given state. It controls whether the visited index is in the right place or not. If not, then it increments the est_tot_cost of the node by the number of the misplaced tiles.

- h1 (node_list, goal_state)

This method takes the frontier as node_list and for each node in the frontier, it calculates the est_tot_cost with misplaced() method.

- city_block (state, goal_state)

This method finds the tiles that are not in the right order. And then it increments the est_tot_cost of the node by calculating the city-block distance of these tiles from current index to goal state's index.

City-block distance = dist (a, b) = |x_goal – x_curr| + |y_goal – y_curr|

- h2 (node_list, goal_state)

This method takes the frontier as node_list and for each node in the frontier, it calculates the est_tot_cost with city_block() method.

- General_Graph_Search (init_state, goal_state, strategy)

At first, it creates a root node and adds it to the frontier. And while the current visited node from the frontier is not the goal state, it makes these operations:

It adds the current node to the explored set. And then it expands that node with expand_node(), then adds the expanded nodes to the frontier if it is not in both explored or frontier.

-For UCS: It sorts the frontier by the total cost.

-For A* h1: It calculates the est_tot_cost of the frontier with h1() and sorts those values.

-For A* h2: It calculates the est_tot_cost of the frontier with h2() and sorts those values.

And then it pops the first node from the sorted frontier and at first it checks whether the frontier is empty or not. Empty frontier means, solution could not found for that puzzle.

If it reaches to the goal state, it exists from the while loop and it creates the solution path by going back from current node to the root node. And in each turn, it adds the current state's action to the path. Finally it returns the solution path, total cost, max number of stored nodes and total number of expanded nodes.

- ucs_il (init_state, goal_state, cost_limit)

Like general_graph_search(), this method is also creates a root node at first. And while the current node's state is not the goal state, it makes these operations:

It adds the current node to the explored set. And then it expands that node with expand_node(), then adds the expanded nodes to the frontier if it is not in both explored or frontier and also cost is not greater than the given cost limit.

And again like ucs in general_graph_search, it sorts the frontier by the total cost. Pops the first node from the frontier. Checks whether it's state is goal state or not. If not, makes these operations again. If goal state, then returns the same values like the general_graph_search.

- ils (init_state, goal_state, max_cost)

This method starts cost limit as 0. And while cost_limit is not greater than max_cost, it calls ucs_il() and if the return value of that method is an array, that means it has found the solution and it returns the result to its caller method.

- random_generator (goal_state, depth)

This method is used for creating puzzles that has depth value of given depth. There are 8 possible moves which explained before. At first it creates two random integers between 0 and 7 (both included). The reason of the using two random integers is that preventing returning back to starting state by do not letting up operation after down, left after right, upper left after upper right, down left after down right. And also the vice versa.

- print_information (method_name, puzzle, goal_state, method_num)

This method calls general_graph_search() method with given method number (0-ucs, 2-a*h1, 3-a*h2) (for ils, it calls ils() method) and then prints the total cost, solution path, total number of expanded nodes and maximum number of nodes stored in memory in a nice view to better understand the result.

- Main ()

Main method, creates 10 different puzzles with the given depth (2-4-6...28) with random_generator(). And for each puzzle, it calls print_information() method with UCS, ILS, A*h1 and A*h2 methods.

RESULTS

1- Total number of expanded nodes:

Depth of the solution:	UCS	ILS	A* h1	A* h2
2	95.6	219	4.3	2.5
4	1780	4031.6	16.5	6.4
6	1407.1	2258.1	6.2	6
8	830.8	1277.4	8.4	5.8
10			28.8	10.9
12			133.7	22.4
14			179.2	21.7
16				72.6
18				175
20				115.3
22				416
24				159.7
26				425
28				75

2- Maximum number of nodes stored in memory:

Depth of the solution:	UCS	ILS	A* h1	A* h2
2	464.3	217.5	16.6	12.7
4	8638.8	3930.1	83.8	31.9
6	6808.7	1498	54.5	28.2
8	4035.1	759.7	21.9	30.3
10			154.3	67.2
12			742.2	107.1
14			966.9	89.5
16				324.8
18				821.8
20				540.8
22				1467
24				784.3
26				1909
28				383

RESULTS FOR THOSE 3 PUZZLES:

- Puzzle number 1:

0 1 3 4 / 12 13 2 5 / 11 14 15 6 / 10 9 8 7

---------- UCS ----------

Total cost: 7

Solution path: ['right', 'down right', 'down left']

Total number of expanded nodes: 1121

Maximum number of nodes stored in memory: 5444

---------- ILS ----------

Total cost: 7

Solution path: ['right', 'down right', 'down left']

Total number of expanded nodes: 2690

Maximum number of nodes stored in memory: 2732

---------- A* h1 ----------

Total cost: 7

Solution path: ['right', 'down right', 'down left']

Total number of expanded nodes: 15

Maximum number of nodes stored in memory: 70

---------- A* h2 ----------

Total cost: 7

Solution path: ['right', 'down right', 'down left']

Total number of expanded nodes: 7

Maximum number of nodes stored in memory: 31

- Puzzle number 2:

1 3 5 4 / 2 13 14 1 / 11 12 9 6 / 0 10 8 7

---------- A* h2 ----------

Total cost: 17

Solution path: ['right', 'upper right', 'upper right', 'upper left', 'left', 'down left', 'down right']

Total number of expanded nodes: 277

Maximum number of nodes stored in memory: 1363

- Puzzle number 3:

1 13 3 4 / 12 11 2 5 / 9 8 15 7 / 10 6 14 0

---------- A* h2 ----------

Total cost: 20

Solution path: ['up', 'left', 'down', 'left', 'up', 'up', 'up', 'down right', 'down', 'right', 'down left', 'left', 'upper left', 'right']

Total number of expanded nodes: 1458

Maximum number of nodes stored in memory: 7587

RESULTS IN DETAIL: 1- DEPTH 2

```
moves: upper left -> down
****************** DEPTH 2 / PUZZLE 1 ******************

 1      2       3       4

 11     13      14      5

 0      12      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 4
Solution path: ['up', 'down right']
Total number of expanded nodes: 45
Maximum number of nodes stored in memory: 215

------------ ILS --------------
Total cost: 4
Solution path: ['up', 'down right']
Total number of expanded nodes: 107
Maximum number of nodes stored in memory: 107

------------ A* h1 --------------
Total cost: 4
Solution path: ['up', 'down right']
Total number of expanded nodes: 3
Maximum number of nodes stored in memory: 16

------------ A* h2 --------------
Total cost: 4
Solution path: ['up', 'down right']
Total number of expanded nodes: 3
Maximum number of nodes stored in memory: 16
```

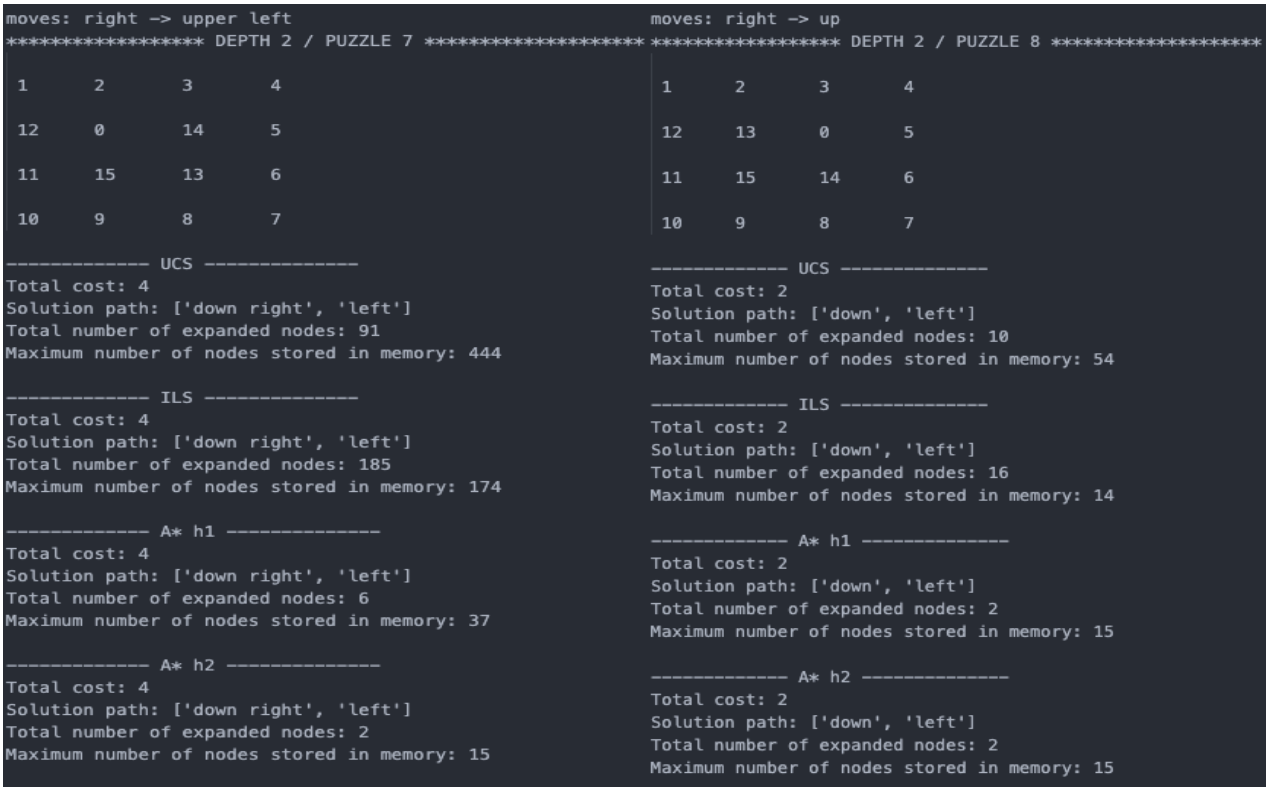
```
moves: right -> up
****************** DEPTH 2 / PUZZLE 2 ******************

 1      2       3       4

 12     13      0       5

 11     15      14      6

 10     9       8       7

------------ UCS --------------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 10
Maximum number of nodes stored in memory: 54

------------ ILS --------------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 16
Maximum number of nodes stored in memory: 14

------------ A* h1 --------------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 15

------------ A* h2 --------------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 15
```

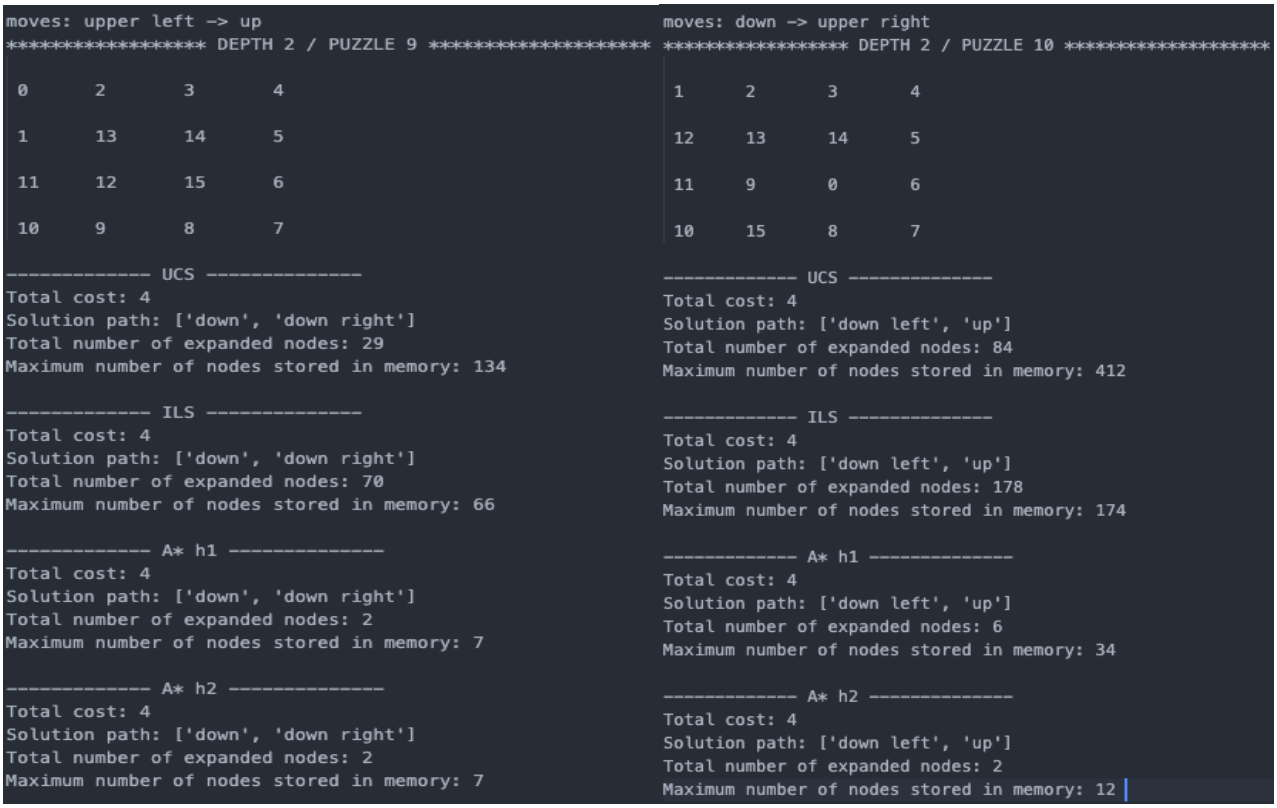
```
moves: down -> upper right
****************** DEPTH 2 / PUZZLE 3 ********************

 1      2       3       4

 12     13      14      5

 11     9       0       6

 10     15      8       7

------------- UCS --------------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 84
Maximum number of nodes stored in memory: 412

------------- ILS --------------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 178
Maximum number of nodes stored in memory: 174

------------- A* h1 --------------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 34

------------- A* h2 --------------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 12
```

```
moves: down right -> left
****************** DEPTH 2 / PUZZLE 4 ********************

 1      2       3       4

 12     13      14      5

 11     8       15      6

 10     0       9       7

------------- UCS --------------
Total cost: 4
Solution path: ['right', 'upper left']
Total number of expanded nodes: 50
Maximum number of nodes stored in memory: 239

------------- ILS --------------
Total cost: 4
Solution path: ['right', 'upper left']
Total number of expanded nodes: 112
Maximum number of nodes stored in memory: 107

------------- A* h1 --------------
Total cost: 4
Solution path: ['right', 'upper left']
Total number of expanded nodes: 3
Maximum number of nodes stored in memory: 16

------------- A* h2 --------------
Total cost: 4
Solution path: ['right', 'upper left']
Total number of expanded nodes: 3
Maximum number of nodes stored in memory: 16
```

```
moves: down right -> upper right
****************** DEPTH 2 / PUZZLE 5 ********************

 1      2       3       4

 12     13      14      5

 11     8       15      0

 10     9       6       7

------------- UCS --------------
Total cost: 6
Solution path: ['down left', 'upper left']
Total number of expanded nodes: 522
Maximum number of nodes stored in memory: 2534

------------- ILS --------------
Total cost: 6
Solution path: ['down left', 'upper left']
Total number of expanded nodes: 1256
Maximum number of nodes stored in memory: 1279

------------- A* h1 --------------
Total cost: 6
Solution path: ['down left', 'upper left']
Total number of expanded nodes: 11
Maximum number of nodes stored in memory: 49

------------- A* h2 --------------
Total cost: 6
Solution path: ['down left', 'upper left']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 22
```

```
moves: down right -> right
****************** DEPTH 2 / PUZZLE 6 ********************

 1      2       3       4

 12     13      14      5

 11     8       15      6

 10     9       7       0

------------- UCS --------------
Total cost: 4
Solution path: ['left', 'upper left']
Total number of expanded nodes: 31
Maximum number of nodes stored in memory: 145

------------- ILS --------------
Total cost: 4
Solution path: ['left', 'upper left']
Total number of expanded nodes: 72
Maximum number of nodes stored in memory: 66

------------- A* h1 --------------
Total cost: 4
Solution path: ['left', 'upper left']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 7

------------- A* h2 --------------
Total cost: 4
Solution path: ['left', 'upper left']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 7
```


moves: right -> upper left
****** DEPTH 2 / PUZZLE 7 ******
1 2 3 4
12 0 14 5
11 15 13 6
10 9 8 7
---------- UCS ----------
Total cost: 4
Solution path: ['down right', 'left']
Total number of expanded nodes: 91
Maximum number of nodes stored in memory: 444
---------- ILS ----------
Total cost: 4
Solution path: ['down right', 'left']
Total number of expanded nodes: 185
Maximum number of nodes stored in memory: 174
---------- A* h1 ----------
Total cost: 4
Solution path: ['down right', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 37
---------- A* h2 ----------
Total cost: 4
Solution path: ['down right', 'left']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 15
moves: right -> up
****** DEPTH 2 / PUZZLE 8 ******
1 2 3 4
12 13 0 5
11 15 14 6
10 9 8 7
---------- UCS ----------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 10
Maximum number of nodes stored in memory: 54
---------- ILS ----------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 16
Maximum number of nodes stored in memory: 14
---------- A* h1 ----------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 15
---------- A* h2 ----------
Total cost: 2
Solution path: ['down', 'left']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 15


moves: upper left -> up
****** DEPTH 2 / PUZZLE 9 ******
0 2 3 4
1 13 14 5
11 12 15 6
10 9 8 7
---------- UCS ----------
Total cost: 4
Solution path: ['down', 'down right']
Total number of expanded nodes: 29
Maximum number of nodes stored in memory: 134
---------- ILS ----------
Total cost: 4
Solution path: ['down', 'down right']
Total number of expanded nodes: 70
Maximum number of nodes stored in memory: 66
---------- A* h1 ----------
Total cost: 4
Solution path: ['down', 'down right']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 7
---------- A* h2 ----------
Total cost: 4
Solution path: ['down', 'down right']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 7
moves: down -> upper right
****** DEPTH 2 / PUZZLE 10 ******
1 2 3 4
12 13 14 5
11 9 0 6
10 15 8 7
---------- UCS ----------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 84
Maximum number of nodes stored in memory: 412
---------- ILS ----------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 178
Maximum number of nodes stored in memory: 174
---------- A* h1 ----------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 34
---------- A* h2 ----------
Total cost: 4
Solution path: ['down left', 'up']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 12

2- DEPTH 4

```
moves: down left -> right
moves: upper left -> right
****************** DEPTH 4 / PUZZLE 1 ******************

 1      2       3       4

 12     13      14      5

 10     0       15      6

 9      11      8       7

------------ UCS --------------
Total cost: 4
Solution path: ['down', 'left', 'up', 'right']
Total number of expanded nodes: 155
Maximum number of nodes stored in memory: 757

------------ ILS --------------
Total cost: 4
Solution path: ['down', 'left', 'up', 'right']
Total number of expanded nodes: 249
Maximum number of nodes stored in memory: 174

------------ A* h1 --------------
Total cost: 4
Solution path: ['down', 'left', 'up', 'right']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 36

------------ A* h2 --------------
Total cost: 4
Solution path: ['down', 'left', 'up', 'right']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 18
```

```
moves: upper right -> right
moves: up -> left
****************** DEPTH 4 / PUZZLE 2 ******************

 1      2       0       3

 12     13      5       4

 11     14      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['right', 'down', 'left', 'down left']
Total number of expanded nodes: 592
Maximum number of nodes stored in memory: 2873

------------ ILS --------------
Total cost: 6
Solution path: ['right', 'down', 'left', 'down left']
Total number of expanded nodes: 1326
Maximum number of nodes stored in memory: 1279

------------ A* h1 --------------
Total cost: 6
Solution path: ['right', 'down', 'left', 'down left']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 25

------------ A* h2 --------------
Total cost: 6
Solution path: ['right', 'down', 'left', 'down left']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 18
```

```
moves: upper left -> up
moves: down right -> left
****************** DEPTH 4 / PUZZLE 3 ******************

 13     2       3       4

 0      1       14      5

 11     12      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 8
Solution path: ['up', 'down right', 'left', 'down right']
Total number of expanded nodes: 6289
Maximum number of nodes stored in memory: 30529

------------ ILS --------------
Total cost: 8
Solution path: ['up', 'down right', 'left', 'down right']
Total number of expanded nodes: 15044
Maximum number of nodes stored in memory: 15277

------------ A* h1 --------------
Total cost: 8
Solution path: ['up', 'down right', 'left', 'down right']
Total number of expanded nodes: 68
Maximum number of nodes stored in memory: 359

------------ A* h2 --------------
Total cost: 8
Solution path: ['up', 'down right', 'left', 'down right']
Total number of expanded nodes: 20
Maximum number of nodes stored in memory: 101
```

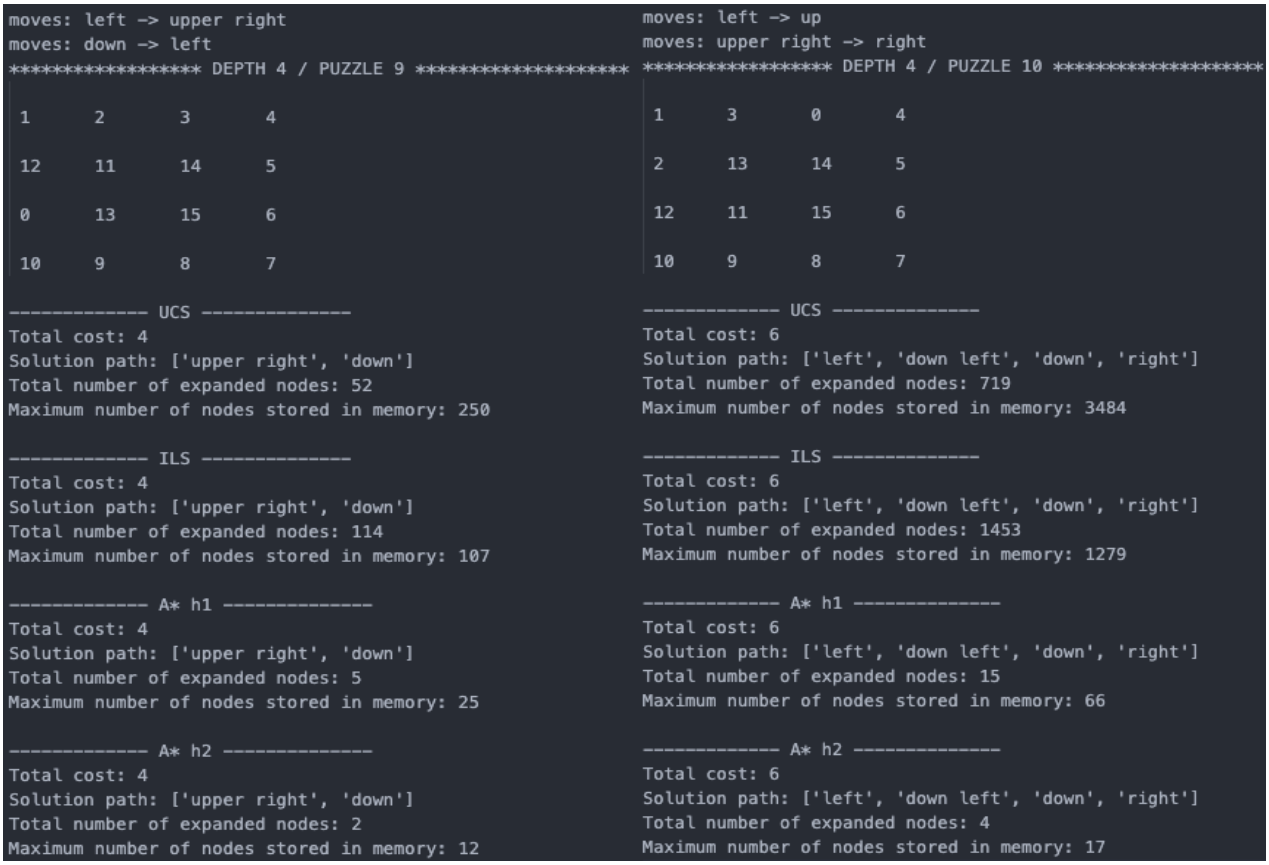
```
moves: upper left -> up
moves: right -> down
****************** DEPTH 4 / PUZZLE 4 ******************

 2      13      3       4

 1      0       14      5

 11     12      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['up', 'left', 'down', 'down right']
Total number of expanded nodes: 865
Maximum number of nodes stored in memory: 4199

------------ ILS --------------
Total cost: 6
Solution path: ['up', 'left', 'down', 'down right']
Total number of expanded nodes: 2045
Maximum number of nodes stored in memory: 2071

------------ A* h1 --------------
Total cost: 6
Solution path: ['up', 'left', 'down', 'down right']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29

------------ A* h2 --------------
Total cost: 6
Solution path: ['up', 'left', 'down', 'down right']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 25
```

```
moves: upper right -> upper left
moves: down -> upper right
****************** DEPTH 4 / PUZZLE 5 ********************

 1      13      0       4

 12     3       2       5

 11     14      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 8
Solution path: ['down', 'left', 'up', 'right', 'down', 'down left']
Total number of expanded nodes: 6618
Maximum number of nodes stored in memory: 32122

------------ ILS --------------
Total cost: 8
Solution path: ['down', 'left', 'up', 'right', 'down', 'down left']
Total number of expanded nodes: 15373
Maximum number of nodes stored in memory: 15277

------------ A* h1 --------------
Total cost: 8
Solution path: ['down', 'left', 'up', 'right', 'down', 'down left']
Total number of expanded nodes: 32
Maximum number of nodes stored in memory: 176

------------ A* h2 --------------
Total cost: 8
Solution path: ['down', 'left', 'up', 'right', 'down', 'down left']
Total number of expanded nodes: 9
Maximum number of nodes stored in memory: 55
```

```
moves: upper left -> up
moves: right -> right
****************** DEPTH 4 / PUZZLE 6 ********************

 2      3       0       4

 1      13      14      5

 11     12      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['left', 'left', 'down', 'down right']
Total number of expanded nodes: 578
Maximum number of nodes stored in memory: 2800

------------ ILS --------------
Total cost: 6
Solution path: ['left', 'left', 'down', 'down right']
Total number of expanded nodes: 1312
Maximum number of nodes stored in memory: 1279

------------ A* h1 --------------
Total cost: 6
Solution path: ['left', 'left', 'down', 'down right']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 15

------------ A* h2 --------------
Total cost: 6
Solution path: ['left', 'left', 'down', 'down right']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 15
```

```
moves: upper left -> down
moves: down -> right
****************** DEPTH 4 / PUZZLE 7 ********************

 1      2       3       4

 11     13      14      5

 10     12      15      6

 9      0       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['left', 'up', 'up', 'down right']
Total number of expanded nodes: 566
Maximum number of nodes stored in memory: 2744

------------ ILS --------------
Total cost: 6
Solution path: ['left', 'up', 'up', 'down right']
Total number of expanded nodes: 1300
Maximum number of nodes stored in memory: 1279

------------ A* h1 --------------
Total cost: 6
Solution path: ['left', 'up', 'up', 'down right']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29

------------ A* h2 --------------
Total cost: 6
Solution path: ['left', 'up', 'up', 'down right']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 22
```

```
moves: up -> upper left
moves: right -> down left
****************** DEPTH 4 / PUZZLE 8 ********************

 2      12      3       4

 0      1       14      5

 11     13      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['right', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 1366
Maximum number of nodes stored in memory: 6630

------------ ILS --------------
Total cost: 6
Solution path: ['right', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 2100
Maximum number of nodes stored in memory: 1279

------------ A* h1 --------------
Total cost: 6
Solution path: ['right', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 17
Maximum number of nodes stored in memory: 78

------------ A* h2 --------------
Total cost: 6
Solution path: ['right', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 36
```


moves: left -> upper right
moves: down -> left
****************** DEPTH 4 / PUZZLE 9 ******************
1 2 3 4
12 11 14 5
0 13 15 6
10 9 8 7
---------- UCS ----------
Total cost: 4
Solution path: ['upper right', 'down']
Total number of expanded nodes: 52
Maximum number of nodes stored in memory: 250
---------- ILS ----------
Total cost: 4
Solution path: ['upper right', 'down']
Total number of expanded nodes: 114
Maximum number of nodes stored in memory: 107
---------- A* h1 ----------
Total cost: 4
Solution path: ['upper right', 'down']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 25
---------- A* h2 ----------
Total cost: 4
Solution path: ['upper right', 'down']
Total number of expanded nodes: 2
Maximum number of nodes stored in memory: 12
moves: left -> up
moves: upper right -> right
****************** DEPTH 4 / PUZZLE 10 ******************
1 3 0 4
2 13 14 5
12 11 15 6
10 9 8 7
---------- UCS ----------
Total cost: 6
Solution path: ['left', 'down left', 'down', 'right']
Total number of expanded nodes: 719
Maximum number of nodes stored in memory: 3484
---------- ILS ----------
Total cost: 6
Solution path: ['left', 'down left', 'down', 'right']
Total number of expanded nodes: 1453
Maximum number of nodes stored in memory: 1279
---------- A* h1 ----------
Total cost: 6
Solution path: ['left', 'down left', 'down', 'right']
Total number of expanded nodes: 15
Maximum number of nodes stored in memory: 66
---------- A* h2 ----------
Total cost: 6
Solution path: ['left', 'down left', 'down', 'right']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 17

3- DEPTH 6

```
moves: right -> right
moves: up -> left
moves: left -> down
****************** DEPTH 6 / PUZZLE 1 ********************

 1      2       3       4

 12     15      13      14

 11     0       6       5

 10     9       8       7

------------- UCS --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 1810
Maximum number of nodes stored in memory: 8780

------------- ILS --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 2990
Maximum number of nodes stored in memory: 2071

------------- A* h1 --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 9
Maximum number of nodes stored in memory: 58

------------- A* h2 --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 37
```

```
moves: right -> down
moves: right -> up
moves: up -> up
****************** DEPTH 6 / PUZZLE 2 ********************

 1      2       3       0

 12     13      14      4

 11     15      8       5

 10     9       7       6

------------- UCS --------------
Total cost: 6
Solution path: ['down', 'down', 'down', 'left', 'up', 'left']
Total number of expanded nodes: 608
Maximum number of nodes stored in memory: 2956

------------- ILS --------------
Total cost: 6
Solution path: ['down', 'down', 'down', 'left', 'up', 'left']
Total number of expanded nodes: 1065
Maximum number of nodes stored in memory: 790

------------- A* h1 --------------
Total cost: 6
Solution path: ['down', 'down', 'down', 'left', 'up', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 24

------------- A* h2 --------------
Total cost: 6
Solution path: ['down', 'down', 'down', 'left', 'up', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 24
```

```
moves: left -> down
moves: right -> right
moves: right -> up
****************** DEPTH 6 / PUZZLE 3 ********************

 1      2       3       4

 12     13      14      5

 10     11      15      0

 9      8       7       6

------------- UCS --------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 1283
Maximum number of nodes stored in memory: 6230

------------- ILS --------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 2017
Maximum number of nodes stored in memory: 1279

------------- A* h1 --------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 28

------------- A* h2 --------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 21
```

```
moves: right -> down
moves: left -> up
moves: right -> up
****************** DEPTH 6 / PUZZLE 4 ********************

 1      2       3       4

 12     13      0       5

 11     8       14      6

 10     15      9       7

------------- UCS --------------
Total cost: 6
Solution path: ['down', 'left', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 1952
Maximum number of nodes stored in memory: 9472

------------- ILS --------------
Total cost: 6
Solution path: ['down', 'left', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 3132
Maximum number of nodes stored in memory: 2071

------------- A* h1 --------------
Total cost: 6
Solution path: ['down', 'left', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 35
Maximum number of nodes stored in memory: 219

------------- A* h2 --------------
Total cost: 6
Solution path: ['down', 'left', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 37
```

```
moves: left -> up
moves: right -> right
moves: right -> up
****************** DEPTH 6 / PUZZLE 5 ********************

 1      2      3      0

 13     14     5      4

 12     11     15     6

 10     9      8      7

------------ UCS -------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'down', 'right']
Total number of expanded nodes: 763
Maximum number of nodes stored in memory: 3706

------------ ILS -------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'down', 'right']
Total number of expanded nodes: 1220
Maximum number of nodes stored in memory: 790

------------ A* h1 -------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'down', 'right']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 36

------------ A* h2 -------------
Total cost: 6
Solution path: ['down', 'left', 'left', 'left', 'down', 'right']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29
```

```
moves: down -> left
moves: up -> right
moves: up -> up
****************** DEPTH 6 / PUZZLE 6 ********************

 1      0      3      4

 12     2      14     5

 9      13     15     6

 11     10     8      7

------------ UCS -------------
Total cost: 6
Solution path: ['down', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 1007
Maximum number of nodes stored in memory: 4890

------------ ILS -------------
Total cost: 6
Solution path: ['down', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 1741
Maximum number of nodes stored in memory: 1279

------------ A* h1 -------------
Total cost: 6
Solution path: ['down', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 9
Maximum number of nodes stored in memory: 47

------------ A* h2 -------------
Total cost: 6
Solution path: ['down', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29
```

```
moves: left -> up
moves: up -> right
moves: down -> right
****************** DEPTH 6 / PUZZLE 7 ********************

 2      13     3      4

 1      14     0      5

 12     11     15     6

 10     9      8      7

------------ UCS -------------
Total cost: 6
Solution path: ['left', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 2177
Maximum number of nodes stored in memory: 10560

------------ ILS -------------
Total cost: 6
Solution path: ['left', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 3357
Maximum number of nodes stored in memory: 2071

------------ A* h1 -------------
Total cost: 6
Solution path: ['left', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 36

------------ A* h2 -------------
Total cost: 6
Solution path: ['left', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29
```

```
moves: down -> left
moves: up -> up
moves: up -> right
****************** DEPTH 6 / PUZZLE 8 ********************

 2      0      3      4

 1      13     14     5

 12     9      15     6

 11     10     8      7

------------ UCS -------------
Total cost: 6
Solution path: ['left', 'down', 'down', 'down', 'right', 'up']
Total number of expanded nodes: 1343
Maximum number of nodes stored in memory: 6522

------------ ILS -------------
Total cost: 6
Solution path: ['left', 'down', 'down', 'down', 'right', 'up']
Total number of expanded nodes: 2077
Maximum number of nodes stored in memory: 1279

------------ A* h1 -------------
Total cost: 6
Solution path: ['left', 'down', 'down', 'down', 'right', 'up']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 28

------------ A* h2 -------------
Total cost: 6
Solution path: ['left', 'down', 'down', 'down', 'right', 'up']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 21
```

```
moves: left -> down
moves: right -> right
moves: up -> right
****************** DEPTH 6 / PUZZLE 9 *********************

 1      2       3       4

 12     13      14      5

 10     11      6       0

 9      8       15      7

------------ UCS --------------
Total cost: 6
Solution path: ['left', 'down', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 1526
Maximum number of nodes stored in memory: 7411

------------ ILS --------------
Total cost: 6
Solution path: ['left', 'down', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 2260
Maximum number of nodes stored in memory: 1279

------------ A* h1 --------------
Total cost: 6
Solution path: ['left', 'down', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 8
Maximum number of nodes stored in memory: 40

------------ A* h2 --------------
Total cost: 6
Solution path: ['left', 'down', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 26
```

```
moves: up -> left
moves: up -> right
moves: right -> down
****************** DEPTH 6 / PUZZLE 10 *********************

 2      3       14      4

 1      12      0       5

 11     13      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['up', 'left', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 1542
Maximum number of nodes stored in memory: 7480

------------ ILS --------------
Total cost: 6
Solution path: ['up', 'left', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 2722
Maximum number of nodes stored in memory: 2071

------------ A* h1 --------------
Total cost: 6
Solution path: ['up', 'left', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29

------------ A* h2 --------------
Total cost: 6
Solution path: ['up', 'left', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29
```

4- DEPTH 8

```
moves: up -> right
moves: down -> left
moves: left -> up
moves: down -> down
****************** DEPTH 8 / PUZZLE 1 ********************

 1      2       3       4

 12     14      15      5

 10     11      13      6

 0      9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'up', 'left', 'down']
Total number of expanded nodes: 773
Maximum number of nodes stored in memory: 3757

------------ ILS --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'up', 'left', 'down']
Total number of expanded nodes: 1230
Maximum number of nodes stored in memory: 790

------------ A* h1 --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'up', 'left', 'down']
Total number of expanded nodes: 9
Maximum number of nodes stored in memory: 56

------------ A* h2 --------------
Total cost: 6
Solution path: ['up', 'right', 'right', 'up', 'left', 'down']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 35
```

```
moves: right -> right
moves: up -> up
moves: down -> left
moves: up -> left
****************** DEPTH 8 / PUZZLE 2 ********************

 1      0       2       4

 12     13      3       14

 11     15      6       5

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['right', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 1633
Maximum number of nodes stored in memory: 7931

------------ ILS --------------
Total cost: 6
Solution path: ['right', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 2367
Maximum number of nodes stored in memory: 1279

------------ A* h1 --------------
Total cost: 6
Solution path: ['right', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 31

------------ A* h2 --------------
Total cost: 6
Solution path: ['right', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 31
```

```
moves: right -> down
moves: right -> up
moves: down -> left
moves: left -> up
****************** DEPTH 8 / PUZZLE 3 ********************

 1      2       3       4

 12     13      14      5

 11     0       8       6

 10     15      9       7

------------ UCS --------------
Total cost: 4
Solution path: ['down', 'right', 'up', 'left']
Total number of expanded nodes: 161
Maximum number of nodes stored in memory: 788

------------ ILS --------------
Total cost: 4
Solution path: ['down', 'right', 'up', 'left']
Total number of expanded nodes: 255
Maximum number of nodes stored in memory: 174

------------ A* h1 --------------
Total cost: 4
Solution path: ['down', 'right', 'up', 'left']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 44

------------ A* h2 --------------
Total cost: 4
Solution path: ['down', 'right', 'up', 'left']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 23
```

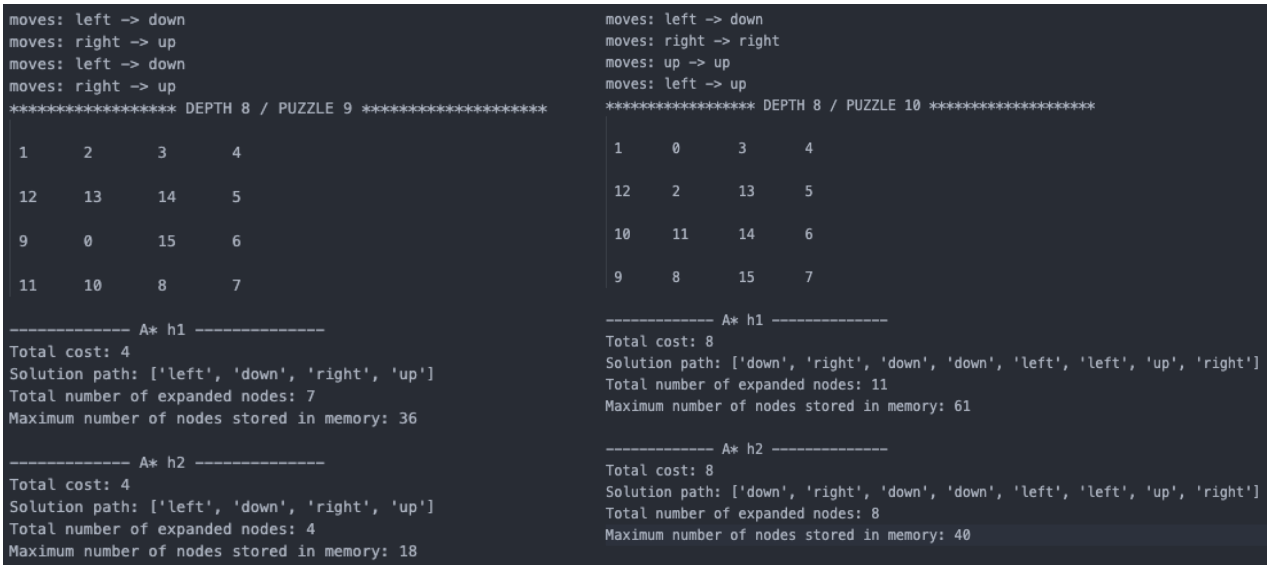
```
moves: left -> up
moves: down -> right
moves: up -> right
moves: up -> left
****************** DEPTH 8 / PUZZLE 4 ********************

 1      0       2       4

 12     14      3       5

 11     13      15      6

 10     9       8       7

------------ UCS --------------
Total cost: 4
Solution path: ['right', 'down', 'left', 'down']
Total number of expanded nodes: 130
Maximum number of nodes stored in memory: 632

------------ ILS --------------
Total cost: 4
Solution path: ['right', 'down', 'left', 'down']
Total number of expanded nodes: 192
Maximum number of nodes stored in memory: 107

------------ A* h1 --------------
Total cost: 4
Solution path: ['right', 'down', 'left', 'down']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 23

------------ A* h2 --------------
Total cost: 4
Solution path: ['right', 'down', 'left', 'down']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 23
```

```
moves: right -> right
moves: up -> left
moves: down -> right
moves: up -> left
****************** DEPTH 8 / PUZZLE 5 ********************

 1      2       3       4

 12     13      0       6

 11     15      5       14

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['down', 'right', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 2054
Maximum number of nodes stored in memory: 9962

------------ ILS --------------
Total cost: 6
Solution path: ['down', 'right', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 3234
Maximum number of nodes stored in memory: 2071

------------ A* h1 --------------
Total cost: 6
Solution path: ['down', 'right', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 19
Maximum number of nodes stored in memory: 113

------------ A* h2 --------------
Total cost: 6
Solution path: ['down', 'right', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 44
```

```
moves: right -> right
moves: up -> up
moves: down -> left
moves: up -> right
****************** DEPTH 8 / PUZZLE 6 ********************

 1      2       4       0

 12     13      3       14

 11     15      6       5

 10     9       8       7

------------ UCS --------------
Total cost: 6
Solution path: ['left', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 945
Maximum number of nodes stored in memory: 4593

------------ ILS --------------
Total cost: 6
Solution path: ['left', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 1402
Maximum number of nodes stored in memory: 790

------------ A* h1 --------------
Total cost: 6
Solution path: ['left', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29

------------ A* h2 --------------
Total cost: 6
Solution path: ['left', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 6
Maximum number of nodes stored in memory: 29
```

```
moves: left -> down
moves: right -> right
moves: right -> up
moves: down -> left
****************** DEPTH 8 / PUZZLE 7 ********************

 1      2       3       4

 12     13      14      5

 10     11      15      6

 9      8       0       7

------------ UCS --------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 120
Maximum number of nodes stored in memory: 583

------------ ILS --------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 182
Maximum number of nodes stored in memory: 107

------------ A* h1 --------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 22

------------ A* h2 --------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 15
```

```
moves: up -> left
moves: up -> right
moves: down -> right
moves: right -> up
****************** DEPTH 8 / PUZZLE 8 ********************

 2      12      3       0

 1      14      5       4

 11     13      15      6

 10     9       8       7

------------ A* h1 --------------
Total cost: 8
Solution path: ['down', 'left', 'left', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 10
Maximum number of nodes stored in memory: 49

------------ A* h2 --------------
Total cost: 8
Solution path: ['down', 'left', 'left', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 9
Maximum number of nodes stored in memory: 45
```



Because the program was solving the puzzle for such a long time (like more than hours) we had to use only up-left-down-right moves. So the solution paths do not contain any diagonal moves. We could not even solve more than 7 seven puzzles. So we left it on there. And solved last 3 puzzles only with A* methods.

This situtaions directly effected the total number of expanded nodes and the maximum number of the stored nodes.

5- DEPTH 10

```
moves: down -> right
moves: right -> up
moves: down -> left
moves: left -> left
moves: up -> right
****************** DEPTH 10 / PUZZLE 1 ********************

 1      2       3       4

 12     13      14      5

 9      0       15      6

 11     10      8       7

-------------- A* h1 ---------------
Total cost: 4
Solution path: ['left', 'down', 'right', 'up']
Total number of expanded nodes: 7
Maximum number of nodes stored in memory: 36

-------------- A* h2 ---------------
Total cost: 4
Solution path: ['left', 'down', 'right', 'up']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 18

moves: down -> right
moves: up -> up
moves: right -> up
moves: left -> left
moves: left -> down
****************** DEPTH 10 / PUZZLE 2 ********************

 12     1       2       3

 0      13      5       4

 11     9       14      6

 10     8       15      7

-------------- A* h1 ---------------
Total cost: 10
Solution path: ['up', 'right', 'right', 'right', 'down', 'left', 'down', 'down', 'left', 'up']
Total number of expanded nodes: 11
Maximum number of nodes stored in memory: 50

-------------- A* h2 ---------------
Total cost: 10
Solution path: ['up', 'right', 'right', 'right', 'down', 'left', 'down', 'down', 'left', 'up']
Total number of expanded nodes: 10
Maximum number of nodes stored in memory: 43
```

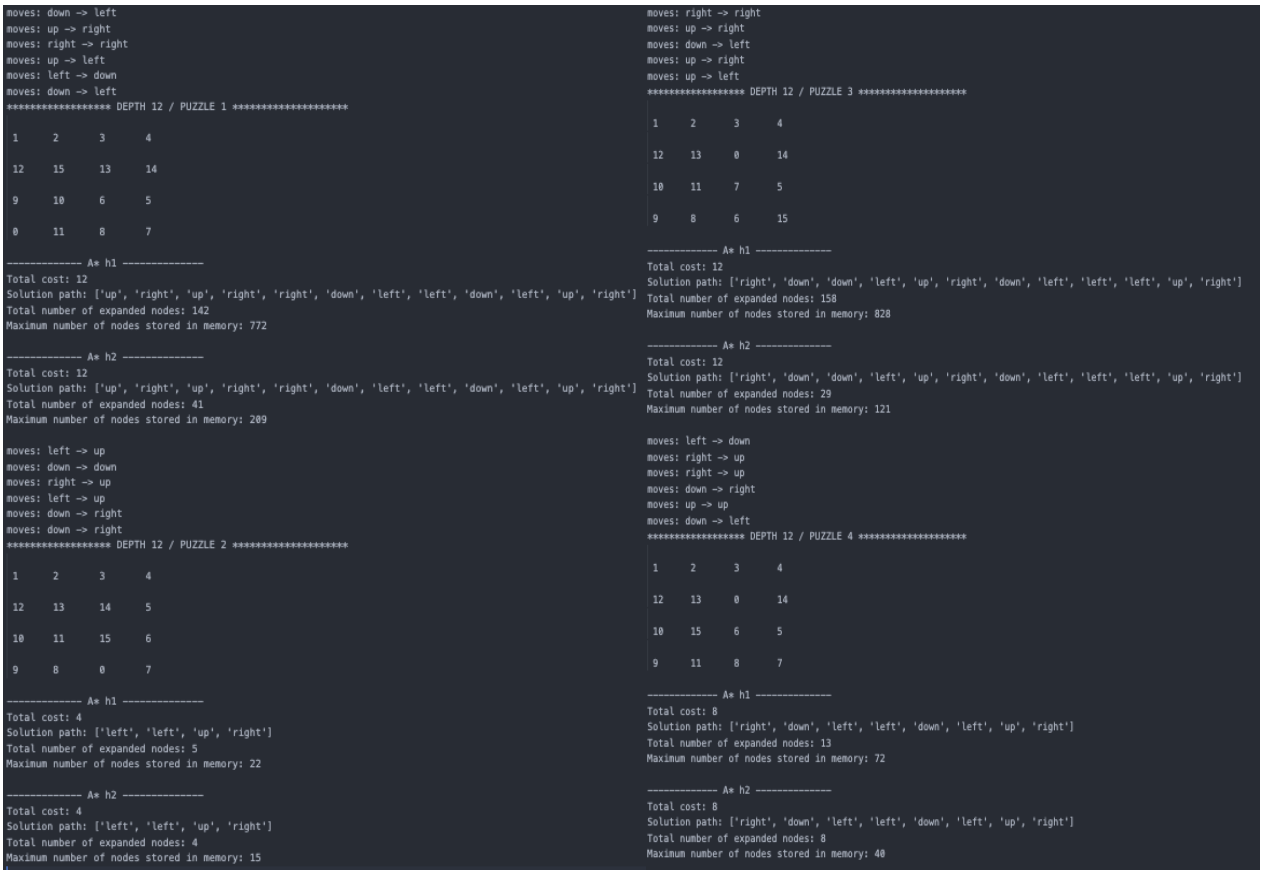
```
moves: right -> right
moves: up -> left
moves: up -> left
moves: down -> down
moves: left -> up
****************** DEPTH 10 / PUZZLE 3 ********************

 1      13      2       4

 0      15      3       14

 12     11      6       5

 10     9       8       7

-------------- A* h1 ---------------
Total cost: 10
Solution path: ['down', 'right', 'up', 'up', 'right', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 21
Maximum number of nodes stored in memory: 118

-------------- A* h2 ---------------
Total cost: 10
Solution path: ['down', 'right', 'up', 'up', 'right', 'down', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 10
Maximum number of nodes stored in memory: 53

moves: down -> right
moves: right -> up
moves: left -> up
moves: left -> left
moves: up -> right
****************** DEPTH 10 / PUZZLE 4 ********************

 2      0       3       4

 1      12      13      5

 11     9       14      15

 10     8       7       6

-------------- A* h1 ---------------
Total cost: 10
Solution path: ['left', 'down', 'right', 'right', 'down', 'right', 'down', 'left', 'left', 'up']
Total number of expanded nodes: 12
Maximum number of nodes stored in memory: 60

-------------- A* h2 ---------------
Total cost: 10
Solution path: ['left', 'down', 'right', 'right', 'down', 'right', 'down', 'left', 'left', 'up']
Total number of expanded nodes: 10
Maximum number of nodes stored in memory: 46
```

```
moves: right -> down
moves: right -> up
moves: up -> left
moves: left -> up
moves: right -> down
****************** DEPTH 10 / PUZZLE 5 ********************

 1      3       13      4

 12     2       0       14

 11     15      8       5

 10     9       7       6

------------ A* h1 -------------
Total cost: 10
Solution path: ['up', 'left', 'down', 'right', 'right', 'down', 'down', 'left', 'up', 'left']
Total number of expanded nodes: 25
Maximum number of nodes stored in memory: 142

------------ A* h2 -------------
Total cost: 10
Solution path: ['up', 'left', 'down', 'right', 'right', 'down', 'down', 'left', 'up', 'left']
Total number of expanded nodes: 16
Maximum number of nodes stored in memory: 79

moves: left -> up
moves: up -> right
moves: right -> down
moves: left -> up
moves: right -> right
****************** DEPTH 10 / PUZZLE 6 ********************

 2      14      4       0

 1      3       13      5

 12     11      15      6

 10     9       8       7

------------ A* h1 -------------
Total cost: 10
Solution path: ['left', 'left', 'down', 'right', 'up', 'left', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 36
Maximum number of nodes stored in memory: 183

------------ A* h2 -------------
Total cost: 10
Solution path: ['left', 'left', 'down', 'right', 'up', 'left', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 14
Maximum number of nodes stored in memory: 60
```

```
moves: up -> right
moves: up -> left
moves: down -> down
moves: left -> up
moves: right -> down
****************** DEPTH 10 / PUZZLE 7 ********************

 1      14      2       4

 13     11      3       5

 12     0       15      6

 10     9       8       7

------------ A* h1 -------------
Total cost: 8
Solution path: ['up', 'up', 'right', 'down', 'left', 'left', 'down', 'right']
Total number of expanded nodes: 21
Maximum number of nodes stored in memory: 121

------------ A* h2 -------------
Total cost: 8
Solution path: ['up', 'up', 'right', 'down', 'left', 'left', 'down', 'right']
Total number of expanded nodes: 11
Maximum number of nodes stored in memory: 60

moves: left -> down
moves: right -> up
moves: left -> up
moves: down -> right
moves: down -> right
****************** DEPTH 10 / PUZZLE 8 ********************

 1      2       3       4

 12     13      14      5

 10     11      15      6

 9      8       0       7

------------ A* h1 -------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 22

------------ A* h2 -------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 15
```

```
moves: up -> left
moves: down -> right
moves: right -> right
moves: down -> left
moves: left -> up
****************** DEPTH 10 / PUZZLE 9 ********************

 1    2     3     4

 11   12    14    5

 13   0     6     7

 10   15    9     8

---------- A* h1 ----------
Total cost: 10
Solution path: ['down', 'right', 'right', 'up', 'left', 'left', 'left', 'up', 'right', 'down']
Total number of expanded nodes: 36
Maximum number of nodes stored in memory: 199

---------- A* h2 ----------
Total cost: 10
Solution path: ['down', 'right', 'right', 'up', 'left', 'left', 'left', 'up', 'right', 'down']
Total number of expanded nodes: 19
Maximum number of nodes stored in memory: 94

moves: up -> left
moves: up -> right
moves: down -> left
moves: down -> right
moves: down -> right
****************** DEPTH 10 / PUZZLE 10 ********************

 2    12    3     4

 11   1     14    5

 13   9     15    6

 10   8     0     7

---------- A* h1 ----------
Total cost: 10
Solution path: ['left', 'up', 'left', 'up', 'right', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 114
Maximum number of nodes stored in memory: 612

---------- A* h2 ----------
Total cost: 10
Solution path: ['left', 'up', 'left', 'up', 'right', 'up', 'left', 'down', 'right', 'down']
Total number of expanded nodes: 11
Maximum number of nodes stored in memory: 55
```

DEPTH 12



```
moves: down -> left
moves: up -> right
moves: right -> right
moves: up -> left
moves: left -> down
moves: down -> left
****************** DEPTH 12 / PUZZLE 1 ********************

 1      2       3       4

 12     15      13      14

 9      10      6       5

 0      11      8       7

------------ A* h1 --------------
Total cost: 12
Solution path: ['up', 'right', 'up', 'right', 'right', 'down', 'left', 'left', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 142
Maximum number of nodes stored in memory: 772

------------ A* h2 --------------
Total cost: 12
Solution path: ['up', 'right', 'up', 'right', 'right', 'down', 'left', 'left', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 41
Maximum number of nodes stored in memory: 209

moves: left -> up
moves: down -> down
moves: right -> up
moves: left -> up
moves: down -> right
moves: down -> right
****************** DEPTH 12 / PUZZLE 2 ********************

 1      2       3       4

 12     13      14      5

 10     11      15      6

 9      8       0       7

------------ A* h1 --------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 5
Maximum number of nodes stored in memory: 22

------------ A* h2 --------------
Total cost: 4
Solution path: ['left', 'left', 'up', 'right']
Total number of expanded nodes: 4
Maximum number of nodes stored in memory: 15
```

```
moves: right -> right
moves: up -> right
moves: down -> left
moves: up -> right
moves: up -> left
****************** DEPTH 12 / PUZZLE 3 ********************

 1      2       3       4

 12     13      0       14

 10     11      7       5

 9      8       6       15

------------ A* h1 --------------
Total cost: 12
Solution path: ['right', 'down', 'down', 'left', 'up', 'right', 'down', 'left', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 158
Maximum number of nodes stored in memory: 828

------------ A* h2 --------------
Total cost: 12
Solution path: ['right', 'down', 'down', 'left', 'up', 'right', 'down', 'left', 'left', 'left', 'up', 'right']
Total number of expanded nodes: 29
Maximum number of nodes stored in memory: 121

moves: left -> down
moves: right -> up
moves: right -> up
moves: down -> right
moves: up -> up
moves: down -> left
****************** DEPTH 12 / PUZZLE 4 ********************

 1      2       3       4

 12     13      0       14

 10     15      6       5

 9      11      8       7

------------ A* h1 --------------
Total cost: 8
Solution path: ['right', 'down', 'left', 'left', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 13
Maximum number of nodes stored in memory: 72

------------ A* h2 --------------
Total cost: 8
Solution path: ['right', 'down', 'left', 'left', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 8
Maximum number of nodes stored in memory: 40
```

```
moves: down -> left
moves: up -> right
moves: up -> up
moves: down -> right
moves: down -> right
moves: up -> left
****************** DEPTH 12 / PUZZLE 5 ********************

 1      2       3       4

 12     14      0       15

 9      13      6       5

 11     10      8       7

------------ A* h1 --------------
Total cost: 10
Solution path: ['right', 'down', 'left', 'up', 'left', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 54
Maximum number of nodes stored in memory: 325

------------ A* h2 --------------
Total cost: 10
Solution path: ['right', 'down', 'left', 'up', 'left', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 16
Maximum number of nodes stored in memory: 85

moves: right -> up
moves: right -> up
moves: down -> down
moves: down -> left
moves: left -> up
moves: right -> down
****************** DEPTH 12 / PUZZLE 6 ********************

 1      2       3       4

 12     13      5       6

 11     14      9       7

 10     15      0       8

------------ A* h1 --------------
Total cost: 10
Solution path: ['up', 'left', 'down', 'right', 'right', 'up', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 196
Maximum number of nodes stored in memory: 1148

------------ A* h2 --------------
Total cost: 10
Solution path: ['up', 'left', 'down', 'right', 'right', 'up', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 15
Maximum number of nodes stored in memory: 72
```

```
moves: down -> left
moves: up -> right
moves: down -> left
moves: up -> right
moves: down -> left
moves: up -> right
****************** DEPTH 12 / PUZZLE 7 ********************

 1    2     3     4

 12   13    14    5

 11   0     15    6

 10   9     8     7

------------ A* h1 --------------
Total cost: 0
Solution path: []
Total number of expanded nodes: 0
Maximum number of nodes stored in memory: 0

------------ A* h2 --------------
Total cost: 0
Solution path: []
Total number of expanded nodes: 0
Maximum number of nodes stored in memory: 0

moves: left -> up
moves: right -> down
moves: right -> right
moves: up -> left
moves: up -> right
moves: down -> left
****************** DEPTH 12 / PUZZLE 8 ********************

 1    2     4     14

 13   11    0     3

 12   15    6     5

 10   9     8     7

------------ A* h1 --------------
Total cost: 12
Solution path: ['right', 'up', 'left', 'down', 'right', 'down', 'left', 'left', 'up', 'left', 'down', 'right']
Total number of expanded nodes: 191
Maximum number of nodes stored in memory: 1068

------------ A* h2 --------------
Total cost: 12
Solution path: ['right', 'up', 'left', 'down', 'right', 'down', 'left', 'left', 'up', 'left', 'down', 'right']
Total number of expanded nodes: 19
Maximum number of nodes stored in memory: 102
```

```
moves: left -> down
moves: right -> up
moves: right -> right
moves: up -> left
moves: down -> right
moves: up -> up
****************** DEPTH 12 / PUZZLE 9 ********************

 1    2     3     0
 12   13    6     4
 10   15    5     14
 9    11    8     7
------------ A* h1 ------------
Total cost: 12
Solution path: ['down', 'down', 'left', 'up', 'right', 'down', 'left', 'left', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 195
Maximum number of nodes stored in memory: 1065

------------ A* h2 ------------
Total cost: 12
Solution path: ['down', 'down', 'left', 'up', 'right', 'down', 'left', 'left', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 32
Maximum number of nodes stored in memory: 158

moves: right -> up
moves: up -> left
moves: down -> right
moves: right -> down
moves: left -> left
moves: down -> right
****************** DEPTH 12 / PUZZLE 10 ********************

 1    13    2     4
 12   3     5     6
 11   9     15    14
 10   8     0     7
------------ A* h1 ------------
Total cost: 12
Solution path: ['left', 'up', 'right', 'right', 'up', 'left', 'left', 'up', 'right', 'down', 'down', 'left']
Total number of expanded nodes: 383
Maximum number of nodes stored in memory: 2122

------------ A* h2 ------------
Total cost: 12
Solution path: ['left', 'up', 'right', 'right', 'up', 'left', 'left', 'up', 'right', 'down', 'down', 'left']
Total number of expanded nodes: 50
Maximum number of nodes stored in memory: 269
```

7- DEPTH 14

```
moves: down -> left
moves: up -> right
moves: right -> up
moves: left -> up
moves: down -> down
moves: right -> right
moves: up -> left
****************** DEPTH 14 / PUZZLE 1 ********************

 1      2       3       4
 12     15      0       13
 9      14      6       5
 11     10      8       7
------------ A* h1 -------------
Total cost: 12
Solution path: ['right', 'down', 'left', 'left', 'up', 'right', 'down', 'left', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 276
Maximum number of nodes stored in memory: 1634

------------ A* h2 -------------
Total cost: 12
Solution path: ['right', 'down', 'left', 'left', 'up', 'right', 'down', 'left', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 25
Maximum number of nodes stored in memory: 137

moves: right -> down
moves: left -> up
moves: right -> up
moves: right -> up
moves: down -> down
moves: down -> left
moves: left -> left
****************** DEPTH 14 / PUZZLE 2 ********************

 1      2       3       4
 12     13      5       6
 11     8       14      7
 0      10      15      9
------------ A* h1 -------------
Total cost: 12
Solution path: ['right', 'right', 'right', 'up', 'up', 'left', 'down', 'left', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 294
Maximum number of nodes stored in memory: 1589

------------ A* h2 -------------
Total cost: 12
Solution path: ['right', 'right', 'right', 'up', 'up', 'left', 'down', 'left', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 36
Maximum number of nodes stored in memory: 180
```

```
moves: down -> right
moves: right -> up
moves: up -> left
moves: up -> left
moves: left -> down
moves: right -> down
moves: down -> left
****************** DEPTH 14 / PUZZLE 3 ********************

 12     1       2       4
 13     9       3       14
 11     8       15      5
 0      10      7       6
------------ A* h1 -------------
Total cost: 14
Solution path: ['right', 'up', 'up', 'left', 'up', 'right', 'right', 'down', 'right', 'down', 'down', 'left', 'left', 'up']
Total number of expanded nodes: 31
Maximum number of nodes stored in memory: 146

------------ A* h2 -------------
Total cost: 14
Solution path: ['right', 'up', 'up', 'left', 'up', 'right', 'right', 'down', 'right', 'down', 'down', 'left', 'left', 'up']
Total number of expanded nodes: 14
Maximum number of nodes stored in memory: 60

moves: right -> right
moves: up -> left
moves: down -> left
moves: down -> right
moves: up -> right
moves: up -> left
moves: up -> right
****************** DEPTH 14 / PUZZLE 4 ********************

 1      2       4       0
 12     13      3       6
 11     9       5       14
 10     8       15      7
------------ A* h1 -------------
Total cost: 10
Solution path: ['left', 'down', 'down', 'right', 'up', 'left', 'down', 'down', 'left', 'up']
Total number of expanded nodes: 39
Maximum number of nodes stored in memory: 205

------------ A* h2 -------------
Total cost: 10
Solution path: ['left', 'down', 'down', 'right', 'up', 'left', 'down', 'down', 'left', 'up']
Total number of expanded nodes: 13
Maximum number of nodes stored in memory: 66
```

```
moves: up -> up
moves: right -> right
moves: down -> down
moves: left -> left
moves: up -> left
moves: up -> right
moves: down -> left
****************** DEPTH 14 / PUZZLE 5 ********************

 3      12      4       5
 8      1       14      6
 11     2       13      15
 10     9       8       7
------------ A* h1 -------------
Total cost: 14
Solution path: ['right', 'up', 'left', 'down', 'right', 'down', 'right', 'right', 'up', 'up', 'left', 'left', 'down', 'down']
Total number of expanded nodes: 695
Maximum number of nodes stored in memory: 3541

------------ A* h2 -------------
Total cost: 14
Solution path: ['right', 'up', 'left', 'down', 'right', 'down', 'right', 'right', 'up', 'up', 'left', 'left', 'down', 'down']
Total number of expanded nodes: 28
Maximum number of nodes stored in memory: 86

moves: left -> up
moves: up -> right
moves: down -> down
moves: right -> right
moves: down -> left
moves: up -> up
moves: down -> down
****************** DEPTH 14 / PUZZLE 6 ********************

 2      13      3       4
 1      11      14      5
 12     15      6       7
 10     9       0       8
------------ A* h1 -------------
Total cost: 10
Solution path: ['right', 'up', 'left', 'left', 'up', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 15
Maximum number of nodes stored in memory: 78

------------ A* h2 -------------
Total cost: 10
Solution path: ['right', 'up', 'left', 'left', 'up', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 10
Maximum number of nodes stored in memory: 46
```

```
moves: right -> up
moves: right -> down
moves: down -> left
moves: left -> up
moves: left -> up
moves: up -> right
moves: right -> right
****************** DEPTH 14 / PUZZLE 7 ********************

 2      3       4       0
 1      13      5       6
 12     11      14      7
 10     15      9       8
------------ A* h1 -------------
Total cost: 14
Solution path: ['left', 'left', 'left', 'down', 'down', 'right', 'down', 'right', 'right', 'up', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 24
Maximum number of nodes stored in memory: 127

------------ A* h2 -------------
Total cost: 14
Solution path: ['left', 'left', 'left', 'down', 'down', 'right', 'down', 'right', 'right', 'up', 'up', 'left', 'down', 'left']
Total number of expanded nodes: 14
Maximum number of nodes stored in memory: 60

 moves: up -> right
moves: down -> right
moves: up -> left
moves: down -> left
moves: down -> left
moves: up -> right
moves: right -> down
****************** DEPTH 14 / PUZZLE 8 ********************

 1      2       3       4
 12     14      6       15
 9      13      8       5
 11     10      0       7
------------ A* h1 -------------
Total cost: 12
Solution path: ['up', 'up', 'right', 'down', 'left', 'up', 'left', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 154
Maximum number of nodes stored in memory: 980

------------ A* h2 -------------
Total cost: 12
Solution path: ['up', 'up', 'right', 'down', 'left', 'up', 'left', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 18
Maximum number of nodes stored in memory: 96
```

```
moves: down -> left
moves: up -> right
moves: up -> right
moves: right -> up
moves: down -> down
moves: down -> left
moves: left -> up
****************** DEPTH 14 / PUZZLE 9 ********************

 1      2       3       4
 12     14      5       6
 9      0       15      7
 11     13      10      8
------------ A* h1 -------------
Total cost: 12
Solution path: ['down', 'right', 'right', 'up', 'up', 'left', 'left', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 184
Maximum number of nodes stored in memory: 544

------------ A* h2 -------------
Total cost: 12
Solution path: ['down', 'right', 'right', 'up', 'up', 'left', 'left', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 14
Maximum number of nodes stored in memory: 63

moves: right -> down
moves: left -> up
moves: up -> left
moves: up -> right
moves: down -> left
moves: up -> right
moves: right -> down
****************** DEPTH 14 / PUZZLE 10 ********************

 12     3       14      4
 2      1       0       5
 11     13      8       6
 10     15      9       7
------------ A* h1 -------------
Total cost: 12
Solution path: ['up', 'left', 'down', 'left', 'up', 'right', 'down', 'down', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 160
Maximum number of nodes stored in memory: 905

------------ A* h2 -------------
Total cost: 12
Solution path: ['up', 'left', 'down', 'left', 'up', 'right', 'down', 'down', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 17
Maximum number of nodes stored in memory: 91
```

8- DEPTH 16

```
moves: up -> right
moves: up -> left
moves: down -> left
moves: down -> down
moves: right -> right
moves: up -> left
moves: left -> down
moves: right -> right
****************** DEPTH 16 / PUZZLE 1 ******************

 1      14      2       4
 11     12      3       5
 9      10      13      6
 8      15      0       7
------------ A* h2 -------------
Total cost: 16
Solution path: ['left', 'left', 'up', 'right', 'right', 'down', 'left', 'left', 'up', 'up', 'right', 'up', 'right', 'down', 'left', 'down']
Total number of expanded nodes: 40
Maximum number of nodes stored in memory: 206

moves: left -> up
moves: up -> right
moves: down -> left
moves: down -> down
moves: right -> right
moves: up -> left
moves: up -> right
moves: up -> left
****************** DEPTH 16 / PUZZLE 2 ******************

 2      0       13      4
 12     14      3       5
 10     1       11      6
 9      8       15      7
------------ A* h2 -------------
Total cost: 16
Solution path: ['right', 'down', 'left', 'down', 'right', 'down', 'left', 'left', 'up', 'up', 'right', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 220
Maximum number of nodes stored in memory: 1032

moves: right -> right
moves: down -> left
moves: left -> up
moves: left -> up
moves: up -> right
moves: down -> left
moves: up -> right
moves: right -> down
****************** DEPTH 16 / PUZZLE 3 ******************

 13     3       14      4
 2      1       0       5
 12     11      6       7
 10     15      9       8
------------ A* h2 -------------
Total cost: 16
Solution path: ['up', 'left', 'down', 'left', 'up', 'right', 'down', 'left', 'down', 'right', 'down', 'right', 'right', 'up', 'left', 'left']
Total number of expanded nodes: 45
Maximum number of nodes stored in memory: 209
```

```
moves: up -> up
moves: down -> left
moves: down -> right
moves: right -> right
moves: up -> up
moves: down -> down
moves: left -> down
moves: left -> left
****************** DEPTH 16 / PUZZLE 4 ******************

 1      2       3       4
 11     12      14      5
 13     15      8       6
 0      10      9       7
------------ A* h2 -------------
Total cost: 8
Solution path: ['right', 'right', 'up', 'left', 'left', 'up', 'right', 'down']
Total number of expanded nodes: 8
Maximum number of nodes stored in memory: 40

moves: left -> up
moves: down -> down
moves: right -> up
moves: up -> up
moves: left -> down
moves: down -> right
moves: down -> right
moves: up -> up
****************** DEPTH 16 / PUZZLE 5 ******************

 12     1       3       4
 10     2       0       5
 13     11      14      6
 9      8       15      7
------------ A* h2 -------------
Total cost: 14
Solution path: ['down', 'down', 'left', 'up', 'left', 'up', 'up', 'right', 'down', 'down', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 80
Maximum number of nodes stored in memory: 352

moves: up -> left
moves: up -> right
moves: down -> down
moves: down -> left
moves: up -> right
moves: up -> right
moves: up -> left
moves: down -> left
****************** DEPTH 16 / PUZZLE 6 ******************

 2      14      12      4
 0      1       3       5
 9      13      15      6
 11     10      8       7
------------ A* h2 -------------
Total cost: 14
Solution path: ['right', 'up', 'right', 'down', 'left', 'up', 'left', 'down', 'right', 'down', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 26
Maximum number of nodes stored in memory: 122
```

```
moves: down -> right
moves: right -> up
moves: left -> up
moves: left -> up
moves: right -> down
moves: left -> left
moves: down -> right
moves: down -> left
****************** DEPTH 16 / PUZZLE 7 ******************

 1      3       13      4
 11     12      2       5
 9      8       14      15
 0      10      7       6
------------ A* h2 -------------
Total cost: 16
Solution path: ['right', 'up', 'left', 'up', 'right', 'right', 'up', 'left', 'down', 'right', 'down', 'right', 'down', 'left', 'left', 'up']
Total number of expanded nodes: 40
Maximum number of nodes stored in memory: 199

moves: left -> up
moves: up -> right
moves: right -> right
moves: down -> down
moves: down -> left
moves: up -> left
moves: left -> up
moves: right -> down
****************** DEPTH 16 / PUZZLE 8 ******************

 2      3       4       5
 13     12      14      6
 1      0       11      7
 10     9       15      8
------------ A* h2 -------------
Total cost: 16
Solution path: ['up', 'left', 'down', 'right', 'right', 'down', 'right', 'up', 'up', 'up', 'left', 'left', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 173
Maximum number of nodes stored in memory: 659

moves: up -> right
moves: right -> down
moves: down -> left
moves: up -> left
moves: down -> left
moves: up -> up
moves: up -> right
moves: down -> down
****************** DEPTH 16 / PUZZLE 9 ******************

 2      14      3       4
 1      9       5       6
 12     0       13      7
 11     10      15      8
------------ A* h2 -------------
Total cost: 16
Solution path: ['up', 'up', 'left', 'down', 'down', 'down', 'right', 'up', 'right', 'down', 'right', 'up', 'up', 'left', 'left', 'down']
Total number of expanded nodes: 22
Maximum number of nodes stored in memory: 105
```

9- DEPTH 18

```
moves: up -> left
moves: down -> right
moves: right -> right
moves: up -> left
moves: left -> left
moves: down -> down
moves: right -> up
moves: up -> up
moves: left -> down
****************** DEPTH 18 / PUZZLE 2 ********************

 13     1       3       4
 0      2       12      14
 10     11      6       5
 9      15      8       7
------------ A* h2 --------------
Total cost: 18
Solution path: ['up', 'right', 'down', 'right', 'right', 'down', 'left', 'down left', 'left', 'up', 'right', 'up', 'left', 'down right']
Total number of expanded nodes: 626
Maximum number of nodes stored in memory: 3000

moves: down -> right
moves: right -> up
moves: left -> left
moves: left -> down
moves: right -> up
moves: up -> right
moves: up -> right
moves: down -> left
moves: down -> down
****************** DEPTH 18 / PUZZLE 3 ********************

 1      2       4       5
 12     14      9       3
 10     13      7       15
 8      11      0       6
------------ A* h2 --------------
Total cost: 18
Solution path: ['up', 'up', 'right', 'up', 'left', 'down', 'left', 'down', 'down', 'left', 'up', 'right', 'right', 'right', 'down', 'left', 'left', 'up']
Total number of expanded nodes: 41
Maximum number of nodes stored in memory: 191

moves: right -> right
moves: up -> left
moves: left -> up
moves: right -> right
moves: down -> down
moves: down -> left
moves: left -> up
moves: down -> left
moves: up -> right
****************** DEPTH 18 / PUZZLE 4 ********************

 1      3       4       14
 12     2       13      5
 15     0       6       7
 11     10      9       8
------------ A* h2 --------------
Total cost: 16
Solution path: ['left', 'down', 'right', 'right', 'right', 'up', 'up', 'up', 'left', 'left', 'down', 'right', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 136
Maximum number of nodes stored in memory: 670
```

```
moves: right -> right
moves: up -> left
moves: left -> down
moves: left -> down
moves: right -> up
moves: right -> down
moves: left -> up
moves: right -> down
moves: right -> up
****************** DEPTH 18 / PUZZLE 5 ******************

 1     2      3      4
 12    15     13     14
 10    8      11     0
 9     6      7      5
------------ A* h2 -------------
Total cost: 16
Solution path: ['down', 'left', 'left', 'up', 'right', 'down', 'left', 'left', 'up', 'right', 'up', 'right', 'right', 'down', 'left', 'left']
Total number of expanded nodes: 22
Maximum number of nodes stored in memory: 107

moves: down -> right
moves: up -> left
moves: down -> right
moves: up -> up
moves: up -> right
moves: down -> left
moves: up -> right
moves: down -> left
moves: left -> left
****************** DEPTH 18 / PUZZLE 6 ******************

 1     2      5      3
 0     12     13     4
 11    8      14     6
 10    15     9      7
------------ A* h2 -------------
Total cost: 12
Solution path: ['right', 'right', 'up', 'right', 'down', 'left', 'down', 'left', 'down', 'right', 'up', 'left']
Total number of expanded nodes: 18
Maximum number of nodes stored in memory: 101

moves: up -> left
moves: down -> down
moves: right -> up
moves: left -> up
moves: right -> down
moves: down -> right
moves: right -> up
moves: left -> up
moves: right -> down
****************** DEPTH 18 / PUZZLE 7 ******************

 1     2      3      4
 12    10     5      15
 11    13     14     0
 9     8      7      6
------------ A* h2 -------------
Total cost: 16
Solution path: ['up', 'left', 'down', 'left', 'up', 'down right', 'right', 'down', 'left', 'left', 'left', 'upper right']
Total number of expanded nodes: 559
Maximum number of nodes stored in memory: 2538
```

```
moves: down -> left
moves: up -> up
moves: down -> right
moves: down -> right
moves: right -> up
moves: left -> up
moves: up -> right
moves: down -> down
moves: left -> down
****************** DEPTH 18 / PUZZLE 8 ******************

 1     2      4      5
 12    13     3      15
 9     10     7      14
 11    8      0      6
------------ A* h2 -------------
Total cost: 16
Solution path: ['up', 'right', 'up', 'up', 'left', 'down', 'down', 'right', 'down', 'left', 'left', 'up', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 30
Maximum number of nodes stored in memory: 133

moves: left -> down
moves: right -> up
moves: up -> right
moves: right -> up
moves: down -> left
moves: up -> right
moves: down -> down
moves: down -> left
moves: up -> up
****************** DEPTH 18 / PUZZLE 9 ******************

 1     2      4      5
 12    14     0      6
 10    13     3      7
 9     11     15     8
------------ A* h2 -------------
Total cost: 14
Solution path: ['down', 'down', 'right', 'up', 'up', 'up', 'left', 'down', 'left', 'down', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 20
Maximum number of nodes stored in memory: 97

moves: down -> left
moves: up -> up
moves: up -> right
moves: down -> left
moves: down -> down
moves: right -> up
moves: left -> down
moves: right -> up
moves: up -> left
****************** DEPTH 18 / PUZZLE 10 ******************

 2     13     3      4
 0     12     14     5
 10    1      15     6
 9     11     8      7
------------ A* h2 -------------
Total cost: 12
Solution path: ['right', 'down', 'down', 'left', 'up', 'up', 'right', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 123
Maximum number of nodes stored in memory: 560
```

10- DEPTH 20

```
moves: right -> up
moves: left -> left
moves: left -> down
moves: down -> right
moves: down -> left
moves: up -> right
moves: down -> right
moves: right -> up
****************** DEPTH 20 / PUZZLE 1 ******************

 12     1       2       3
 11     14      5       4
 9      10      15      0
 13     8       7       6
------------- A* h2 --------------
Total cost: 18
Solution path: ['down', 'left', 'left', 'up', 'left', 'down', 'right', 'up', 'left', 'up', 'up', 'right', 'right', 'right', 'down', 'left', 'left', 'down']
Total number of expanded nodes: 48
Maximum number of nodes stored in memory: 202

moves: down -> left
moves: up -> right
moves: down -> right
moves: right -> up
moves: left -> up
moves: down -> down
moves: right -> up
moves: up -> left
moves: up -> left
moves: left -> down
****************** DEPTH 20 / PUZZLE 2 ******************

 12     1       2       4
 0      13      3       14
 9      10      7       5
 11     8       6       15
------------- A* h2 --------------
Total cost: 18
Solution path: ['up', 'right', 'right', 'down', 'right', 'down', 'down', 'left', 'up', 'right', 'down', 'left', 'left', 'up', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 41
Maximum number of nodes stored in memory: 179

moves: left -> up
moves: right -> up
moves: down -> down
moves: left -> up
moves: up -> right
moves: right -> down
moves: down -> left
moves: left -> down
moves: right -> up
moves: up -> right
****************** DEPTH 20 / PUZZLE 3 ******************

 2      3       14      4
 1      15      0       5
 10     11      12      6
 9      13      8       7
--------------- A* h2 ---------------
Total cost: 16
Solution path: ['left', 'down', 'right', 'up', 'up', 'left', 'left', 'down', 'right', 'down', 'down', 'left', 'up', 'up', 'right', 'down']
Total number of expanded nodes: 117
Maximum number of nodes stored in memory: 555
```

```
moves: up -> up
moves: up -> right
moves: down -> right
moves: down -> right
moves: down -> left
moves: up -> right
moves: up -> left
moves: up -> right
moves: down -> left
****************** DEPTH 20 / PUZZLE 4 ******************

 2      13      4       15
 1      14      0       3
 12     9       7       5
 11     10      6       8
------------ A* h2 ------------
Total cost: 20
Solution path: ['right', 'up', 'left', 'down', 'right', 'down', 'left', 'down', 'right', 'up', 'left', 'up', 'left', 'up', 'left', 'down', 'down', 'down', 'right', 'up']
Total number of expanded nodes: 51
Maximum number of nodes stored in memory: 228

moves: right -> right
moves: down -> left
moves: up -> right
moves: up -> up
moves: down -> left
moves: down -> left
moves: up -> right
moves: down -> down
moves: right -> up
moves: down -> left
****************** DEPTH 20 / PUZZLE 5 ******************

 1      2       3       4
 12     7       15      14
 11     13      6       5
 10     9       0       8
------------ A* h2 ------------
Total cost: 14
Solution path: ['up', 'up', 'left', 'down', 'right', 'up', 'right', 'down', 'left', 'down', 'right', 'up', 'left', 'left']
Total number of expanded nodes: 262
Maximum number of nodes stored in memory: 1361

moves: down -> left
moves: up -> right
moves: right -> down
moves: right -> up
moves: left -> up
moves: left -> left
moves: down -> right
moves: right -> right
moves: down -> left
moves: left -> left
****************** DEPTH 20 / PUZZLE 6 ******************

 1      2       3       4
 9      12      13      5
 15     14      8       6
 0      11      10      7
------------ A* h2 ------------
Total cost: 14
Solution path: ['right', 'right', 'up', 'left', 'left', 'up', 'right', 'right', 'down', 'left', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 16
Maximum number of nodes stored in memory: 77
```

```
moves: up -> right
moves: right -> down
moves: right -> up
moves: left -> left
moves: left -> down
moves: right -> up
moves: up -> up
moves: left -> down
moves: right -> up
****************** DEPTH 20 / PUZZLE 7 ******************

 12     0       3       4
 2      1       14      5
 11     13      15      8
 10     9       7       6
---------- A* h2 ----------
Total cost: 12
Solution path: ['down', 'left', 'up', 'right', 'down', 'down', 'right', 'right', 'down', 'left', 'up', 'left']
Total number of expanded nodes: 113
Maximum number of nodes stored in memory: 594

moves: left -> down
moves: right -> up
moves: up -> up
moves: left -> down
moves: down -> down
moves: right -> up
moves: down -> right
moves: right -> up
moves: left -> left
moves: down -> right
****************** DEPTH 20 / PUZZLE 8 ******************

 12     1       3       4
 10     2       14      5
 9      8       13      15
 11     7       0       6
------------ A* h2 -------------
Total cost: 18
Solution path: ['left', 'up', 'right', 'right', 'down', 'left', 'left', 'left', 'up', 'up', 'up', 'right', 'down', 'down', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 280
Maximum number of nodes stored in memory: 1201

moves: left -> up
moves: right -> up
moves: left -> down
moves: down -> down
moves: right -> right
moves: up -> up
moves: down -> down
moves: right -> up
moves: up -> up
moves: down -> left
****************** DEPTH 20 / PUZZLE 9 ******************

 13     1       3       4
 12     2       0       14
 10     11      15      5
 9      8       7       6
------------ A* h2 -------------
Total cost: 14
Solution path: ['right', 'down', 'down', 'left', 'left', 'left', 'up', 'up', 'up', 'right', 'down', 'left', 'down', 'right']
Total number of expanded nodes: 110
Maximum number of nodes stored in memory: 471
```

11- DEPTH 22

```
moves: up -> right
moves: up -> left
moves: down -> right
moves: down -> left
moves: left -> up
moves: up -> left
moves: down -> right
****************** DEPTH 22 / PUZZLE 1 ******************

 1      2       3       4
 10     12      7       14
 13     0       5       8
 9      15      11      6
------------ A* h2 -------------
Total cost: 22
Solution path: ['left', 'up', 'right', 'down', 'down', 'right', 'upper right', 'left', 'up', 'right', 'down', 'down', 'upper left', 'left', 'down', 'left', 'up', 'right']
Total number of expanded nodes: 322
Maximum number of nodes stored in memory: 1467

moves: right -> right
moves: down -> left
moves: up -> left
moves: down -> left
moves: up -> right
moves: right -> right
moves: up -> left
moves: left -> up
moves: right -> right
moves: down -> down
moves: left -> up
****************** DEPTH 22 / PUZZLE 2 ******************

 1      3       4       14
 12     2       0       5
 9      15      13      7
 11     10      6       8
------------ A* h2 -------------
Total cost: 18
Solution path: ['upper right', 'left', 'left', 'down', 'down right', 'down', 'right', 'up', 'left', 'left', 'left', 'down', 'right', 'up']
Total number of expanded nodes: 308
Maximum number of nodes stored in memory: 1499

moves: right -> up
moves: up -> right
moves: down -> left
moves: left -> left
moves: down -> right
moves: up -> left
moves: down -> down
moves: right -> up
moves: up -> up
moves: right -> down
moves: right -> down
****************** DEPTH 22 / PUZZLE 3 ******************

 1      4       13      5
 15     2       3       6
 10     11      14      0
 9      12      8       7
------------ A* h2 -------------
Total cost: 22
Solution path: ['up', 'left', 'up', 'left', 'down', 'down', 'down', 'left', 'up', 'up', 'right', 'down', 'left', 'up', 'right', 'right', 'right', 'up', 'left', 'down', 'down', 'left']
Total number of expanded nodes: 618
Maximum number of nodes stored in memory: 3061
```

12- DEPTH 24

```
moves: right -> down
moves: left -> left
moves: down -> right
moves: right -> up
moves: down -> left
****************** DEPTH 24 / PUZZLE 1 ******************

 2      4       13      5
 1      15      3       6
 12     9       11      14
 10     8       0       7
------------ A* h2 -------------
Total cost: 20
Solution path: ['left', 'up', 'right', 'right', 'up', 'left', 'up', 'left', 'left', 'down', 'down', 'right', 'up', 'right', 'right', 'up', 'left', 'down', 'down', 'left']
Total number of expanded nodes: 182
Maximum number of nodes stored in memory: 863

oves: left -> down
moves: right -> up
moves: left -> up
moves: up -> right
moves: down -> down
moves: right -> up
moves: down -> down
moves: left -> up
moves: up -> left
moves: down -> right
moves: down -> right
moves: up -> up
****************** DEPTH 24 / PUZZLE 2 ******************

 2      13      3       4
 12     1       0       5
 10     15      14      6
 9      11      8       7
------------ A* h2 -------------
Total cost: 12
Solution path: ['down', 'left', 'down', 'left', 'up', 'up', 'right', 'up', 'left', 'down', 'down', 'right']
Total number of expanded nodes: 106
Maximum number of nodes stored in memory: 524

moves: down -> left
moves: up -> right
moves: right -> down
moves: left -> up
moves: down -> right
moves: up -> left
moves: left -> down
moves: right -> right
moves: right -> up
moves: left -> down
moves: left -> up
moves: left -> down
****************** DEPTH 24 / PUZZLE 3 ******************

 1      2       3       4
 12     13      14      5
 10     11      7       15
 0      9       8       6
------------ A* h2 -------------
Total cost: 10
Solution path: ['up', 'right', 'down', 'right', 'up', 'right', 'down', 'left', 'left', 'up']
Total number of expanded nodes: 191
Maximum number of nodes stored in memory: 966
```

13- DEPTH 26

14- DEPTH 28

For the last depths, program only solved 1 or 2 puzzles. Because it was taking way too much time (again, like more than hours), so this average results shown in table are not so average at all.