

## NUMERICAL AND ALGEBRAIC

Gain in decibels of  $P_2$  relative to  $P_1$

$$G = 10 \log_{10}(P_2/P_1).$$

To within two percent

$$(2\pi)^{1/2} \approx 2.5; \quad \pi^2 \approx 10; \quad e^3 \approx 20; \quad 2^{10} \approx 10^3.$$

Euler-Mascheroni constant<sup>1</sup>  $\gamma = 0.57722$

Gamma Function  $\Gamma(x+1) = x\Gamma(x)$ :

$$\begin{array}{ll} \Gamma(1/6) = 5.5663 & \Gamma(3/5) = 1.4892 \\ \Gamma(1/5) = 4.5908 & \Gamma(2/3) = 1.3541 \\ \Gamma(1/4) = 3.6256 & \Gamma(3/4) = 1.2254 \\ \Gamma(1/3) = 2.6789 & \Gamma(4/5) = 1.1642 \\ \Gamma(2/5) = 2.2182 & \Gamma(5/6) = 1.1288 \\ \Gamma(1/2) = 1.7725 = \sqrt{\pi} & \Gamma(1) = 1.0 \end{array}$$

Binomial Theorem (good for  $|x| < 1$  or  $\alpha =$  positive integer):

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k \equiv 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots$$

Rothe-Hagen identity<sup>2</sup> (good for all complex  $x, y, z$  except when singular):

$$\begin{aligned} \sum_{k=0}^n \frac{x}{x+kz} \binom{x+kz}{k} \frac{y}{y+(n-k)z} \binom{y+(n-k)z}{n-k} \\ = \frac{x+y}{x+y+nz} \binom{x+y+nz}{n}. \end{aligned}$$

Newberger's summation formula<sup>3</sup> [good for  $\mu$  nonintegral,  $\operatorname{Re}(\alpha + \beta) > -1$ ]:

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n J_{\alpha-\gamma n}(z) J_{\beta+\gamma n}(z)}{n+\mu} = \frac{\pi}{\sin \mu\pi} J_{\alpha+\gamma\mu}(z) J_{\beta-\gamma\mu}(z).$$

## VECTOR IDENTITIES<sup>4</sup>

Notation:  $f, g$ , are scalars;  $\mathbf{A}, \mathbf{B}$ , etc., are vectors;  $\mathbf{T}$  is a tensor;  $\mathbf{l}$  is the unit dyad.

- (1)  $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A} = \mathbf{B} \times \mathbf{C} \cdot \mathbf{A} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B} = \mathbf{C} \times \mathbf{A} \cdot \mathbf{B}$
- (2)  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{C} \times \mathbf{B}) \times \mathbf{A} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$
- (3)  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) + \mathbf{B} \times (\mathbf{C} \times \mathbf{A}) + \mathbf{C} \times (\mathbf{A} \times \mathbf{B}) = 0$
- (4)  $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$
- (5)  $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \times \mathbf{B} \cdot \mathbf{D})\mathbf{C} - (\mathbf{A} \times \mathbf{B} \cdot \mathbf{C})\mathbf{D}$
- (6)  $\nabla(fg) = \nabla(gf) = f\nabla g + g\nabla f$
- (7)  $\nabla \cdot (f\mathbf{A}) = f\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla f$
- (8)  $\nabla \times (f\mathbf{A}) = f\nabla \times \mathbf{A} + \nabla f \times \mathbf{A}$
- (9)  $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$
- (10)  $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$
- (11)  $\mathbf{A} \times (\nabla \times \mathbf{B}) = (\nabla \mathbf{B}) \cdot \mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$
- (12)  $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$
- (13)  $\nabla^2 f = \nabla \cdot \nabla f$
- (14)  $\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$
- (15)  $\nabla \times \nabla f = 0$
- (16)  $\nabla \cdot \nabla \times \mathbf{A} = 0$

If  $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$  are orthonormal unit vectors, a second-order tensor  $\mathbf{T}$  can be written in the dyadic form

$$(17) \quad \mathbf{T} = \sum_{i,j} T_{ij} \mathbf{e}_i \mathbf{e}_j$$

In cartesian coordinates the divergence of a tensor is a vector with components

$$(18) \quad (\nabla \cdot \mathbf{T})_i = \sum_j (\partial T_{ji} / \partial x_j)$$

[This definition is required for consistency with Eq. (29)]. In general

$$(19) \quad \nabla \cdot (\mathbf{A}\mathbf{B}) = (\nabla \cdot \mathbf{A})\mathbf{B} + (\mathbf{A} \cdot \nabla)\mathbf{B}$$

$$(20) \quad \nabla \cdot (f\mathbf{T}) = \nabla f \cdot \mathbf{T} + f\nabla \cdot \mathbf{T}$$

Let  $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$  be the radius vector of magnitude  $r$ , from the origin to the point  $x, y, z$ . Then

$$(21) \quad \nabla \cdot \mathbf{r} = 3$$

$$(22) \quad \nabla \times \mathbf{r} = 0$$

$$(23) \quad \nabla r = \mathbf{r}/r$$

$$(24) \quad \nabla(1/r) = -\mathbf{r}/r^3$$

$$(25) \quad \nabla \cdot (\mathbf{r}/r^3) = 4\pi\delta(\mathbf{r})$$

$$(26) \quad \nabla \mathbf{r} = \mathbf{I}$$

If  $V$  is a volume enclosed by a surface  $S$  and  $d\mathbf{S} = \mathbf{n}dS$ , where  $\mathbf{n}$  is the unit normal outward from  $V$ ,

$$(27) \quad \int_V dV \nabla f = \int_S d\mathbf{S} f$$

$$(28) \quad \int_V dV \nabla \cdot \mathbf{A} = \int_S d\mathbf{S} \cdot \mathbf{A}$$

$$(29) \quad \int_V dV \nabla \cdot T = \int_S d\mathbf{S} \cdot T$$

$$(30) \quad \int_V dV \nabla \times \mathbf{A} = \int_S d\mathbf{S} \times \mathbf{A}$$

$$(31) \quad \int_V dV (f \nabla^2 g - g \nabla^2 f) = \int_S d\mathbf{S} \cdot (f \nabla g - g \nabla f)$$

$$(32) \quad \int_V dV (\mathbf{A} \cdot \nabla \times \nabla \times \mathbf{B} - \mathbf{B} \cdot \nabla \times \nabla \times \mathbf{A}) \\ = \int_S d\mathbf{S} \cdot (\mathbf{B} \times \nabla \times \mathbf{A} - \mathbf{A} \times \nabla \times \mathbf{B})$$

If  $S$  is an open surface bounded by the contour  $C$ , of which the line element is  $d\mathbf{l}$ ,

$$(33) \quad \int_S d\mathbf{S} \times \nabla f = \oint_C d\mathbf{l} f$$

$$(34) \quad \int_S d\mathbf{S} \cdot \nabla \times \mathbf{A} = \oint_C d\mathbf{l} \cdot \mathbf{A}$$

$$(35) \quad \int_S (d\mathbf{S} \times \nabla) \times \mathbf{A} = \oint_C d\mathbf{l} \times \mathbf{A}$$

$$(36) \quad \int_S d\mathbf{S} \cdot (\nabla f \times \nabla g) = \oint_C f dg = - \oint_C g df$$

## DIFFERENTIAL OPERATORS IN CURVILINEAR COORDINATES<sup>5</sup>

### Cylindrical Coordinates

Divergence

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Gradient

$$(\nabla f)_r = \frac{\partial f}{\partial r}; \quad (\nabla f)_\phi = \frac{1}{r} \frac{\partial f}{\partial \phi}; \quad (\nabla f)_z = \frac{\partial f}{\partial z}$$

Curl

$$(\nabla \times \mathbf{A})_r = \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}$$

$$(\nabla \times \mathbf{A})_\phi = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}$$

$$(\nabla \times \mathbf{A})_z = \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) - \frac{1}{r} \frac{\partial A_r}{\partial \phi}$$

Laplacian

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Laplacian of a vector

$$(\nabla^2 \mathbf{A})_r = \nabla^2 A_r - \frac{2}{r^2} \frac{\partial A_\phi}{\partial \phi} - \frac{A_r}{r^2}$$

$$(\nabla^2 \mathbf{A})_\phi = \nabla^2 A_\phi + \frac{2}{r^2} \frac{\partial A_r}{\partial \phi} - \frac{A_\phi}{r^2}$$

$$(\nabla^2 \mathbf{A})_z = \nabla^2 A_z$$

Components of  $(\mathbf{A} \cdot \nabla) \mathbf{B}$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_r = A_r \frac{\partial B_r}{\partial r} + \frac{A_\phi}{r} \frac{\partial B_r}{\partial \phi} + A_z \frac{\partial B_r}{\partial z} - \frac{A_\phi B_\phi}{r}$$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_\phi = A_r \frac{\partial B_\phi}{\partial r} + \frac{A_\phi}{r} \frac{\partial B_\phi}{\partial \phi} + A_z \frac{\partial B_\phi}{\partial z} + \frac{A_\phi B_r}{r}$$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_z = A_r \frac{\partial B_z}{\partial r} + \frac{A_\phi}{r} \frac{\partial B_z}{\partial \phi} + A_z \frac{\partial B_z}{\partial z}$$

Divergence of a tensor

$$(\nabla \cdot \mathbf{T})_r = \frac{1}{r} \frac{\partial}{\partial r} (r T_{rr}) + \frac{1}{r} \frac{\partial T_{\phi r}}{\partial \phi} + \frac{\partial T_{zr}}{\partial z} - \frac{T_{\phi\phi}}{r}$$

$$(\nabla \cdot \mathbf{T})_\phi = \frac{1}{r} \frac{\partial}{\partial r} (r T_{r\phi}) + \frac{1}{r} \frac{\partial T_{\phi\phi}}{\partial \phi} + \frac{\partial T_{z\phi}}{\partial z} + \frac{T_{\phi r}}{r}$$

$$(\nabla \cdot \mathbf{T})_z = \frac{1}{r} \frac{\partial}{\partial r} (r T_{rz}) + \frac{1}{r} \frac{\partial T_{\phi z}}{\partial \phi} + \frac{\partial T_{zz}}{\partial z}$$

## Spherical Coordinates

Divergence

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

Gradient

$$(\nabla f)_r = \frac{\partial f}{\partial r}; \quad (\nabla f)_\theta = \frac{1}{r} \frac{\partial f}{\partial \theta}; \quad (\nabla f)_\phi = \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}$$

Curl

$$(\nabla \times \mathbf{A})_r = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{1}{r \sin \theta} \frac{\partial A_\theta}{\partial \phi}$$

$$(\nabla \times \mathbf{A})_\theta = \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi)$$

$$(\nabla \times \mathbf{A})_\phi = \frac{1}{r} \frac{\partial}{\partial r} (r A_\theta) - \frac{1}{r} \frac{\partial A_r}{\partial \theta}$$

Laplacian

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

Laplacian of a vector

$$(\nabla^2 \mathbf{A})_r = \nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r^2} \frac{\partial A_\theta}{\partial \theta} - \frac{2 \cot \theta A_\theta}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$(\nabla^2 \mathbf{A})_\theta = \nabla^2 A_\theta + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} - \frac{A_\theta}{r^2 \sin^2 \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$(\nabla^2 \mathbf{A})_\phi = \nabla^2 A_\phi - \frac{A_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\theta}{\partial \phi}$$

Components of  $(\mathbf{A} \cdot \nabla)\mathbf{B}$

$$(\mathbf{A} \cdot \nabla\mathbf{B})_r = A_r \frac{\partial B_r}{\partial r} + \frac{A_\theta}{r} \frac{\partial B_r}{\partial \theta} + \frac{A_\phi}{r \sin \theta} \frac{\partial B_r}{\partial \phi} - \frac{A_\theta B_\theta + A_\phi B_\phi}{r}$$

$$(\mathbf{A} \cdot \nabla\mathbf{B})_\theta = A_r \frac{\partial B_\theta}{\partial r} + \frac{A_\theta}{r} \frac{\partial B_\theta}{\partial \theta} + \frac{A_\phi}{r \sin \theta} \frac{\partial B_\theta}{\partial \phi} + \frac{A_\theta B_r}{r} - \frac{\cot \theta A_\phi B_\phi}{r}$$

$$(\mathbf{A} \cdot \nabla\mathbf{B})_\phi = A_r \frac{\partial B_\phi}{\partial r} + \frac{A_\theta}{r} \frac{\partial B_\phi}{\partial \theta} + \frac{A_\phi}{r \sin \theta} \frac{\partial B_\phi}{\partial \phi} + \frac{A_\phi B_r}{r} + \frac{\cot \theta A_\phi B_\theta}{r}$$

Divergence of a tensor

$$\begin{aligned} (\nabla \cdot \mathbf{T})_r &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_{rr}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta T_{\theta r}) \\ &\quad + \frac{1}{r \sin \theta} \frac{\partial T_{\phi r}}{\partial \phi} - \frac{T_{\theta\theta} + T_{\phi\phi}}{r} \end{aligned}$$

$$\begin{aligned} (\nabla \cdot \mathbf{T})_\theta &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_{r\theta}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta T_{\theta\theta}) \\ &\quad + \frac{1}{r \sin \theta} \frac{\partial T_{\phi\theta}}{\partial \phi} + \frac{T_{\theta r}}{r} - \frac{\cot \theta T_{\phi\phi}}{r} \end{aligned}$$

$$\begin{aligned} (\nabla \cdot \mathbf{T})_\phi &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_{r\phi}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta T_{\theta\phi}) \\ &\quad + \frac{1}{r \sin \theta} \frac{\partial T_{\phi\phi}}{\partial \phi} + \frac{T_{\phi r}}{r} + \frac{\cot \theta T_{\phi\theta}}{r} \end{aligned}$$

## DIMENSIONS AND UNITS

To get the value of a quantity in Gaussian units, multiply the value expressed in SI units by the conversion factor. Multiples of 3 in the conversion factors result from approximating the speed of light  $c = 2.9979 \times 10^{10}$  cm/sec  $\approx 3 \times 10^{10}$  cm/sec.

Physical Quantity	Sym- bol	Dimensions		SI Units	Conversion Factor	Gaussian Units
		SI	Gaussian			
Capacitance	$C$	$\frac{t^2 q^2}{ml^2}$	$l$	farad	$9 \times 10^{11}$	cm
Charge	$q$	$q$	$\frac{m^{1/2} l^{3/2}}{t}$	coulomb	$3 \times 10^9$	statcoulomb
Charge density	$\rho$	$\frac{q}{l^3}$	$\frac{m^{1/2}}{l^{3/2} t}$	coulomb /m <sup>3</sup>	$3 \times 10^3$	statcoulomb /cm <sup>3</sup>
Conductance		$\frac{t q^2}{ml^2}$	$\frac{l}{t}$	siemens	$9 \times 10^{11}$	cm/sec
Conductivity	$\sigma$	$\frac{t q^2}{ml^3}$	$\frac{1}{t}$	siemens /m	$9 \times 10^9$	sec <sup>-1</sup>
Current	$I, i$	$\frac{q}{t}$	$\frac{m^{1/2} l^{3/2}}{t^2}$	ampere	$3 \times 10^9$	statampere
Current density	$\mathbf{J}, \mathbf{j}$	$\frac{q}{l^2 t}$	$\frac{m^{1/2}}{l^{1/2} t^2}$	ampere /m <sup>2</sup>	$3 \times 10^5$	statampere /cm <sup>2</sup>
Density	$\rho$	$\frac{m}{l^3}$	$\frac{m}{l^3}$	kg/m <sup>3</sup>	$10^{-3}$	g/cm <sup>3</sup>
Displacement	$\mathbf{D}$	$\frac{q}{l^2}$	$\frac{m^{1/2}}{l^{1/2} t}$	coulomb /m <sup>2</sup>	$12\pi \times 10^5$	statcoulomb /cm <sup>2</sup>
Electric field	$\mathbf{E}$	$\frac{ml}{t^2 q}$	$\frac{m^{1/2}}{l^{1/2} t}$	volt/m	$\frac{1}{3} \times 10^{-4}$	statvolt/cm
Electro-motance	$\mathcal{E},$ Emf	$\frac{ml^2}{t^2 q}$	$\frac{m^{1/2} l^{1/2}}{t}$	volt	$\frac{1}{3} \times 10^{-2}$	statvolt
Energy	$U, W$	$\frac{ml^2}{t^2}$	$\frac{ml^2}{t^2}$	joule	$10^7$	erg
Energy density	$w, \epsilon$	$\frac{m}{lt^2}$	$\frac{m}{lt^2}$	joule/m <sup>3</sup>	10	erg/cm <sup>3</sup>



Physical Quantity	Sym- bol	Dimensions		SI Units	Conversion Factor	Gaussian Units
		SI	Gaussian			
Force	<b>F</b>	$\frac{ml}{t^2}$	$\frac{ml}{t^2}$	newton	$10^5$	dyne
Frequency	$f, \nu$	$\frac{1}{t}$	$\frac{1}{t}$	hertz	1	hertz
Impedance	$Z$	$\frac{ml^2}{tq^2}$	$\frac{t}{l}$	ohm	$\frac{1}{9} \times 10^{-11}$	sec/cm
Inductance	$L$	$\frac{ml^2}{q^2}$	$\frac{t^2}{l}$	henry	$\frac{1}{9} \times 10^{-11}$	sec <sup>2</sup> /cm
Length	$l$	$l$	$l$	meter (m)	$10^2$	centimeter (cm)
Magnetic intensity	<b>H</b>	$\frac{q}{lt}$	$\frac{m^{1/2}}{l^{1/2}t}$	ampere–turn/m	$4\pi \times 10^{-3}$	oersted
Magnetic flux	$\Phi$	$\frac{ml^2}{tq}$	$\frac{m^{1/2}l^{3/2}}{t}$	weber	$10^8$	maxwell
Magnetic induction	<b>B</b>	$\frac{m}{tq}$	$\frac{m^{1/2}}{l^{1/2}t}$	tesla	$10^4$	gauss
Magnetic moment	$m, \mu$	$\frac{l^2q}{t}$	$\frac{m^{1/2}l^{5/2}}{t}$	ampere–m <sup>2</sup>	$10^3$	oersted–cm <sup>3</sup>
Magnetization	<b>M</b>	$\frac{q}{lt}$	$\frac{m^{1/2}}{l^{1/2}t}$	ampere–turn/m	$10^{-3}$	oersted
Magneto-motance	$\mathcal{M}, \text{Mmf}$	$\frac{q}{t}$	$\frac{m^{1/2}l^{1/2}}{t^2}$	ampere–turn	$\frac{4\pi}{10}$	gilbert
Mass	$m, M$	$m$	$m$	kilogram (kg)	$10^3$	gram (g)
Momentum	<b>p, P</b>	$\frac{ml}{t}$	$\frac{ml}{t}$	kg–m/s	$10^5$	g–cm/sec
Momentum density		$\frac{m}{l^2t}$	$\frac{m}{l^2t}$	kg/m <sup>2</sup> –s	$10^{-1}$	g/cm <sup>2</sup> –sec
Permeability	$\mu$	$\frac{ml}{q^2}$	1	henry/m	$\frac{1}{4\pi} \times 10^7$	—

Physical Quantity	Sym-bol	Dimensions		SI Units	Conversion Factor	Gaussian Units
		SI	Gaussian			
Permittivity	$\epsilon$	$\frac{t^2 q^2}{ml^3}$	1	farad/m	$36\pi \times 10^9$	—
Polarization	$\mathbf{P}$	$\frac{q}{l^2}$	$\frac{m^{1/2}}{l^{1/2}t}$	coulomb/m <sup>2</sup>	$3 \times 10^5$	statcoulomb/cm <sup>2</sup>
Potential	$V, \phi$	$\frac{ml^2}{t^2 q}$	$\frac{m^{1/2}l^{1/2}}{t}$	volt	$\frac{1}{3} \times 10^{-2}$	statvolt
Power	$P$	$\frac{ml^2}{t^3}$	$\frac{ml^2}{t^3}$	watt	$10^7$	erg/sec
Power density		$\frac{m}{lt^3}$	$\frac{m}{lt^3}$	watt/m <sup>3</sup>	10	erg/cm <sup>3</sup> –sec
Pressure	$p, P$	$\frac{m}{lt^2}$	$\frac{m}{lt^2}$	pascal	10	dyne/cm <sup>2</sup>
Reluctance	$\mathcal{R}$	$\frac{q^2}{ml^2}$	$\frac{1}{l}$	ampere–turn/weber	$4\pi \times 10^{-9}$	cm <sup>-1</sup>
Resistance	$R$	$\frac{ml^2}{tq^2}$	$\frac{t}{l}$	ohm	$\frac{1}{9} \times 10^{-11}$	sec/cm
Resistivity	$\eta, \rho$	$\frac{ml^3}{tq^2}$	$t$	ohm–m	$\frac{1}{9} \times 10^{-9}$	sec
Thermal conductivity	$\kappa, k$	$\frac{ml}{t^3}$	$\frac{ml}{t^3}$	watt/m–deg (K)	$10^5$	erg/cm–sec–deg (K)
Time	$t$	$t$	$t$	second (s)	1	second (sec)
Vector potential	$\mathbf{A}$	$\frac{ml}{tq}$	$\frac{m^{1/2}l^{1/2}}{t}$	weber/m	$10^6$	gauss–cm
Velocity	$\mathbf{v}$	$\frac{l}{t}$	$\frac{l}{t}$	m/s	$10^2$	cm/sec
Viscosity	$\eta, \mu$	$\frac{m}{lt}$	$\frac{m}{lt}$	kg/m–s	10	poise
Vorticity	$\zeta$	$\frac{1}{t}$	$\frac{1}{t}$	s <sup>-1</sup>	1	sec <sup>-1</sup>
Work	$W$	$\frac{ml^2}{t^2}$	$\frac{ml^2}{t^2}$	joule	$10^7$	erg

## INTERNATIONAL SYSTEM (SI) NOMENCLATURE<sup>6</sup>

Physical Quantity	Name of Unit	Symbol for Unit	Physical Quantity	Name of Unit	Symbol for Unit
*length	meter	m	electric potential	volt	V
*mass	kilogram	kg	electric resistance	ohm	$\Omega$
*time	second	s	electric conductance	siemens	S
*current	ampere	A	electric capacitance	farad	F
*temperature	kelvin	K	magnetic flux	weber	Wb
*amount of substance	mole	mol	magnetic inductance	henry	H
*luminous intensity	candela	cd	magnetic intensity	tesla	T
†plane angle	radian	rad	luminous flux	lumen	lm
†solid angle	steradian	sr	illuminance	lux	lx
frequency	hertz	Hz	activity (of a radioactive source)	becquerel	Bq
energy	joule	J	absorbed dose (of ionizing radiation)	gray	Gy
force	newton	N			
pressure	pascal	Pa			
power	watt	W			
electric charge	coulomb	C			

\*SI base unit

†Supplementary unit

## METRIC PREFIXES

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
$10^{-1}$	deci	d	10	deca	da
$10^{-2}$	centi	c	$10^2$	hecto	h
$10^{-3}$	milli	m	$10^3$	kilo	k
$10^{-6}$	micro	$\mu$	$10^6$	mega	M
$10^{-9}$	nano	n	$10^9$	giga	G
$10^{-12}$	pico	p	$10^{12}$	tera	T
$10^{-15}$	femto	f	$10^{15}$	peta	P
$10^{-18}$	atto	a	$10^{18}$	exa	E

# PHYSICAL CONSTANTS (SI)<sup>7</sup>

Physical Quantity	Symbol	Value	Units
Boltzmann constant	$k$	$1.3807 \times 10^{-23}$	$\text{J K}^{-1}$
Elementary charge	$e$	$1.6022 \times 10^{-19}$	C
Electron mass	$m_e$	$9.1094 \times 10^{-31}$	kg
Proton mass	$m_p$	$1.6726 \times 10^{-27}$	kg
Gravitational constant	$G$	$6.6726 \times 10^{-11}$	$\text{m}^3 \text{s}^{-2} \text{kg}^{-1}$
Planck constant	$h$	$6.6261 \times 10^{-34}$	J s
	$\hbar = h/2\pi$	$1.0546 \times 10^{-34}$	J s
Speed of light in vacuum	$c$	$2.9979 \times 10^8$	$\text{m s}^{-1}$
Permittivity of free space	$\epsilon_0$	$8.8542 \times 10^{-12}$	$\text{F m}^{-1}$
Permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	$\text{H m}^{-1}$
Proton/electron mass ratio	$m_p/m_e$	$1.8362 \times 10^3$	
Electron charge/mass ratio	$e/m_e$	$1.7588 \times 10^{11}$	$\text{C kg}^{-1}$
Rydberg constant	$R_\infty = \frac{me^4}{8\epsilon_0^2 ch^3}$	$1.0974 \times 10^7$	$\text{m}^{-1}$
Bohr radius	$a_0 = \epsilon_0 h^2 / \pi m e^2$	$5.2918 \times 10^{-11}$	m
Atomic cross section	$\pi a_0^2$	$8.7974 \times 10^{-21}$	$\text{m}^2$
Classical electron radius	$r_e = e^2 / 4\pi\epsilon_0 m c^2$	$2.8179 \times 10^{-15}$	m
Thomson cross section	$(8\pi/3)r_e^2$	$6.6525 \times 10^{-29}$	$\text{m}^2$
Compton wavelength of electron	$h/m_e c$	$2.4263 \times 10^{-12}$	m
	$\hbar/m_e c$	$3.8616 \times 10^{-13}$	m
Fine-structure constant	$\alpha = e^2 / 2\epsilon_0 h c$	$7.2974 \times 10^{-3}$	
	$\alpha^{-1}$	137.04	
First radiation constant	$c_1 = 2\pi h c^2$	$3.7418 \times 10^{-16}$	$\text{W m}^2$
Second radiation constant	$c_2 = hc/k$	$1.4388 \times 10^{-2}$	$\text{m K}$
Stefan-Boltzmann constant	$\sigma$	$5.6705 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$

Physical Quantity	Symbol	Value	Units
Wavelength associated with 1 eV	$\lambda_0 = hc/e$	$1.2398 \times 10^{-6}$	m
Frequency associated with 1 eV	$\nu_0 = e/h$	$2.4180 \times 10^{14}$	Hz
Wave number associated with 1 eV	$k_0 = e/hc$	$8.0655 \times 10^5$	$\text{m}^{-1}$
Energy associated with 1 eV	$h\nu_0$	$1.6022 \times 10^{-19}$	J
Energy associated with $1 \text{ m}^{-1}$	$hc$	$1.9864 \times 10^{-25}$	J
Energy associated with 1 Rydberg	$me^3/8\epsilon_0^2 h^2$	13.606	eV
Energy associated with 1 Kelvin	$k/e$	$8.6174 \times 10^{-5}$	eV
Temperature associated with 1 eV	$e/k$	$1.1604 \times 10^4$	K
Avogadro number	$N_A$	$6.0221 \times 10^{23}$	$\text{mol}^{-1}$
Faraday constant	$F = N_A e$	$9.6485 \times 10^4$	$\text{C mol}^{-1}$
Gas constant	$R = N_A k$	8.3145	$\text{J K}^{-1} \text{mol}^{-1}$
Loschmidt's number (no. density at STP)	$n_0$	$2.6868 \times 10^{25}$	$\text{m}^{-3}$
Atomic mass unit	$m_u$	$1.6605 \times 10^{-27}$	kg
Standard temperature	$T_0$	273.15	K
Atmospheric pressure	$p_0 = n_0 k T_0$	$1.0133 \times 10^5$	Pa
Pressure of 1 mm Hg (1 torr)		$1.3332 \times 10^2$	Pa
Molar volume at STP	$V_0 = RT_0/p_0$	$2.2414 \times 10^{-2}$	$\text{m}^3$
Molar weight of air	$M_{\text{air}}$	$2.8971 \times 10^{-2}$	kg
calorie (cal)		4.1868	J
Gravitational acceleration	$g$	9.8067	$\text{m s}^{-2}$

# PHYSICAL CONSTANTS (cgs)<sup>7</sup>

Physical Quantity	Symbol	Value	Units
Boltzmann constant	$k$	$1.3807 \times 10^{-16}$	erg/deg (K)
Elementary charge	$e$	$4.8032 \times 10^{-10}$	statcoulomb (statcoul)
Electron mass	$m_e$	$9.1094 \times 10^{-28}$	g
Proton mass	$m_p$	$1.6726 \times 10^{-24}$	g
Gravitational constant	$G$	$6.6726 \times 10^{-8}$	dyne-cm <sup>2</sup> /g <sup>2</sup>
Planck constant	$h$	$6.6261 \times 10^{-27}$	erg-sec
	$\hbar = h/2\pi$	$1.0546 \times 10^{-27}$	erg-sec
Speed of light in vacuum	$c$	$2.9979 \times 10^{10}$	cm/sec
Proton/electron mass ratio	$m_p/m_e$	$1.8362 \times 10^3$	
Electron charge/mass ratio	$e/m_e$	$5.2728 \times 10^{17}$	statcoul/g
Rydberg constant	$R_\infty = \frac{2\pi^2 m e^4}{ch^3}$	$1.0974 \times 10^5$	cm <sup>-1</sup>
Bohr radius	$a_0 = \hbar^2/m_e^2$	$5.2918 \times 10^{-9}$	cm
Atomic cross section	$\pi a_0^2$	$8.7974 \times 10^{-17}$	cm <sup>2</sup>
Classical electron radius	$r_e = e^2/mc^2$	$2.8179 \times 10^{-13}$	cm
Thomson cross section	$(8\pi/3)r_e^2$	$6.6525 \times 10^{-25}$	cm <sup>2</sup>
Compton wavelength of electron	$h/m_e c$	$2.4263 \times 10^{-10}$	cm
	$\hbar/m_e c$	$3.8616 \times 10^{-11}$	cm
Fine-structure constant	$\alpha = e^2/\hbar c$	$7.2974 \times 10^{-3}$	
	$\alpha^{-1}$	137.04	
First radiation constant	$c_1 = 2\pi\hbar c^2$	$3.7418 \times 10^{-5}$	erg-cm <sup>2</sup> /sec
Second radiation constant	$c_2 = \hbar c/k$	1.4388	cm-deg (K)
Stefan-Boltzmann constant	$\sigma$	$5.6705 \times 10^{-5}$	erg/cm <sup>2</sup> - sec-deg <sup>4</sup>
Wavelength associated with 1 eV	$\lambda_0$	$1.2398 \times 10^{-4}$	cm

Physical Quantity	Symbol	Value	Units
Frequency associated with 1 eV	$\nu_0$	$2.4180 \times 10^{14}$	Hz
Wave number associated with 1 eV	$k_0$	$8.0655 \times 10^3$	$\text{cm}^{-1}$
Energy associated with 1 eV		$1.6022 \times 10^{-12}$	erg
Energy associated with $1 \text{ cm}^{-1}$		$1.9864 \times 10^{-16}$	erg
Energy associated with 1 Rydberg		13.606	eV
Energy associated with 1 deg Kelvin		$8.6174 \times 10^{-5}$	eV
Temperature associated with 1 eV		$1.1604 \times 10^4$	deg (K)
Avogadro number	$N_A$	$6.0221 \times 10^{23}$	$\text{mol}^{-1}$
Faraday constant	$F = N_A e$	$2.8925 \times 10^{14}$	statcoul/mol
Gas constant	$R = N_A k$	$8.3145 \times 10^7$	erg/deg-mol
Loschmidt's number (no. density at STP)	$n_0$	$2.6868 \times 10^{19}$	$\text{cm}^{-3}$
Atomic mass unit	$m_u$	$1.6605 \times 10^{-24}$	g
Standard temperature	$T_0$	273.15	deg (K)
Atmospheric pressure	$p_0 = n_0 k T_0$	$1.0133 \times 10^6$	$\text{dyne}/\text{cm}^2$
Pressure of 1 mm Hg (1 torr)		$1.3332 \times 10^3$	$\text{dyne}/\text{cm}^2$
Molar volume at STP	$V_0 = RT_0/p_0$	$2.2414 \times 10^4$	$\text{cm}^3$
Molar weight of air	$M_{\text{air}}$	28.971	g
calorie (cal)		$4.1868 \times 10^7$	erg
Gravitational acceleration	$g$	980.67	$\text{cm}/\text{sec}^2$

## FORMULA CONVERSION<sup>8</sup>

Here  $\alpha = 10^2 \text{ cm m}^{-1}$ ,  $\beta = 10^7 \text{ erg J}^{-1}$ ,  $\epsilon_0 = 8.8542 \times 10^{-12} \text{ F m}^{-1}$ ,  $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$ ,  $c = (\epsilon_0 \mu_0)^{-1/2} = 2.9979 \times 10^8 \text{ m s}^{-1}$ , and  $\hbar = 1.0546 \times 10^{-34} \text{ J s}$ . To derive a dimensionally correct SI formula from one expressed in Gaussian units, substitute for each quantity according to  $\bar{Q} = \bar{k}Q$ , where  $\bar{k}$  is the coefficient in the second column of the table corresponding to  $Q$  (overbars denote variables expressed in Gaussian units). Thus, the formula  $\bar{a}_0 = \bar{\hbar}^2 / \bar{m} \bar{e}^2$  for the Bohr radius becomes  $\alpha a_0 = (\hbar \beta)^2 / [(m \beta / \alpha^2)(e^2 \alpha \beta / 4\pi \epsilon_0)]$ , or  $a_0 = \epsilon_0 \hbar^2 / \pi m e^2$ . To go from SI to natural units in which  $\hbar = c = 1$  (distinguished by a circumflex), use  $Q = \hat{k}^{-1} \hat{Q}$ , where  $\hat{k}$  is the coefficient corresponding to  $Q$  in the third column. Thus  $\hat{a}_0 = 4\pi \epsilon_0 \hbar^2 / [(\hat{m} \hbar / c)(\hat{e}^2 \epsilon_0 \hbar c)] = 4\pi / \hat{m} \hat{e}^2$ . (In transforming *from* SI units, do not substitute for  $\epsilon_0$ ,  $\mu_0$ , or  $c$ .)

Physical Quantity	Gaussian Units to SI	Natural Units to SI
Capacitance	$\alpha / 4\pi \epsilon_0$	$\epsilon_0^{-1}$
Charge	$(\alpha \beta / 4\pi \epsilon_0)^{1/2}$	$(\epsilon_0 \hbar c)^{-1/2}$
Charge density	$(\beta / 4\pi \alpha^5 \epsilon_0)^{1/2}$	$(\epsilon_0 \hbar c)^{-1/2}$
Current	$(\alpha \beta / 4\pi \epsilon_0)^{1/2}$	$(\mu_0 / \hbar c)^{1/2}$
Current density	$(\beta / 4\pi \alpha^3 \epsilon_0)^{1/2}$	$(\mu_0 / \hbar c)^{1/2}$
Electric field	$(4\pi \beta \epsilon_0 / \alpha^3)^{1/2}$	$(\epsilon_0 / \hbar c)^{1/2}$
Electric potential	$(4\pi \beta \epsilon_0 / \alpha)^{1/2}$	$(\epsilon_0 / \hbar c)^{1/2}$
Electric conductivity	$(4\pi \epsilon_0)^{-1}$	$\epsilon_0^{-1}$
Energy	$\beta$	$(\hbar c)^{-1}$
Energy density	$\beta / \alpha^3$	$(\hbar c)^{-1}$
Force	$\beta / \alpha$	$(\hbar c)^{-1}$
Frequency	1	$c^{-1}$
Inductance	$4\pi \epsilon_0 / \alpha$	$\mu_0^{-1}$
Length	$\alpha$	1
Magnetic induction	$(4\pi \beta / \alpha^3 \mu_0)^{1/2}$	$(\mu_0 \hbar c)^{-1/2}$
Magnetic intensity	$(4\pi \mu_0 \beta / \alpha^3)^{1/2}$	$(\mu_0 / \hbar c)^{1/2}$
Mass	$\beta / \alpha^2$	$c / \hbar$
Momentum	$\beta / \alpha$	$\hbar^{-1}$
Power	$\beta$	$(\hbar c^2)^{-1}$
Pressure	$\beta / \alpha^3$	$(\hbar c)^{-1}$
Resistance	$4\pi \epsilon_0 / \alpha$	$(\epsilon_0 / \mu_0)^{1/2}$
Time	1	$c$
Velocity	$\alpha$	$c^{-1}$