Fundamentals of UQ (AM 238)

Homework 1

Consider a real random variable X with PDF

$$p(x) = \begin{cases} \dfrac{2x\cos(x^2)+5}{10+\sin(4)} & x \in [0,2] \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

a) Write a computer code that computes the mean and the standard deviation of X to at least three digits accuracy. Compare your numerical results with analytical results.

b) Write a computer code that computes the cumulative distribution function (CDF) of X and the inverse CDF. Plot the CDF and the inverse CDF. Compare the numerical CDF you obtain from your code with the analytical CDF you obtain by integrating (1).

c) Develop a sampler for the random variable X using the probability mapping method, i.e., the map between a uniform variable in $[0,1]$ and X. Approximate the PDF of X using samples and the method of relative frequencies, with 80 bins over the interval $[0,2]$, and compare it with the given PDF in (1).