Project: Stabilization of a flame

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Outline

- Analysis of the Project
- Method forward euler + 1 st order upwind
- Method forward euler + centered 2nd order
- Method forward euler + 1st downwind
- Method RK2 + 1st order upwind
- Method RK4 + 1st order upwind
- Conclusion

Analyse of the Project

- General: here is a flame, it has an intrinsic propagation speed,the flame propagates against the incoming velocity u, so the project is to analyse the behaviour of flame under different speed u.

- Transport equation: $$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} + Ac^2(1-c),$$

- Boundary condition: $$\frac{\partial c}{\partial x} = 0$$
    - On both boundary, gradient equals to 0

Analyse of the Project

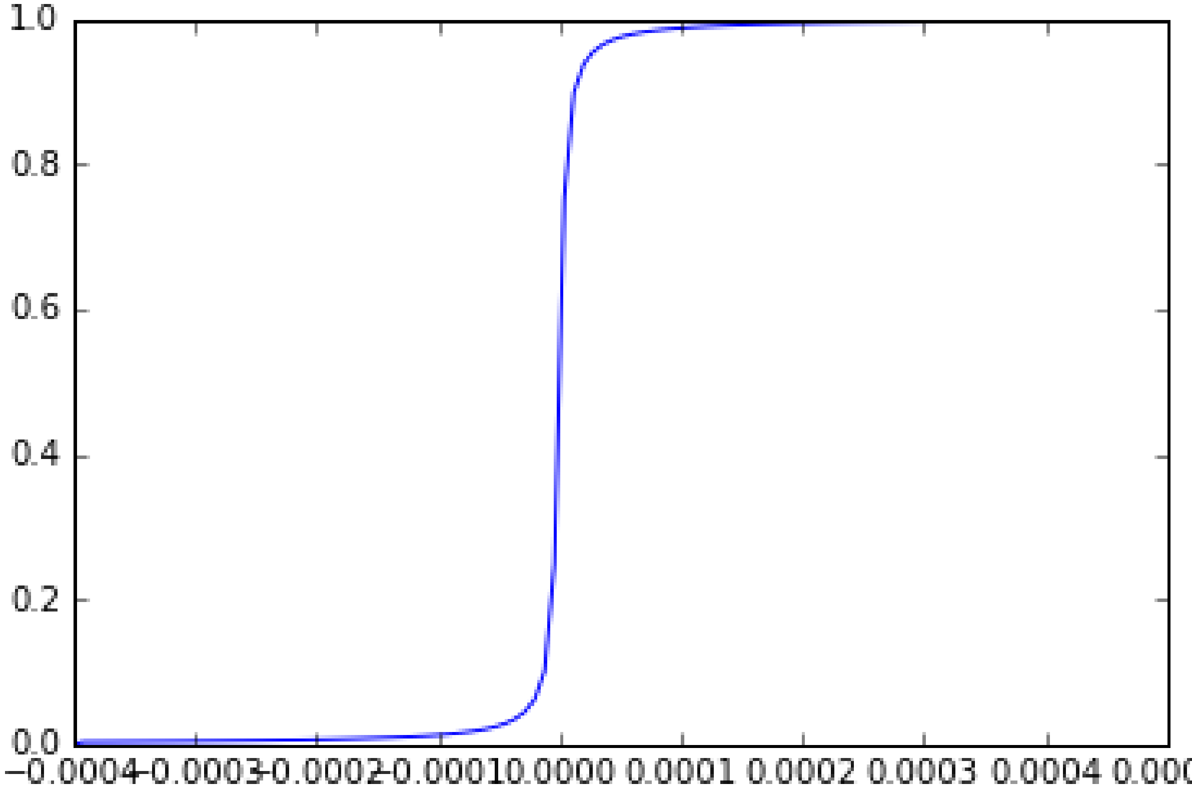
- Initial condition :

Set as arctan(x)/π+0.5

(initial_BC.py -> initialCond(c,u,D))

- C = 0, on the left side
- C = 1, on the right side

- Flame thickness: t_f = D/u



Analyse of the Project

- Critical of project:
  - Optimise the flame
  - Change the incoming velocity u
  - Visualise the output, to see whether the flame flashes back or it is blown out

Analyse of the Project

- Implementation:
- Analyse the flame in range(-10*t_f, 10*t_f), t_f is the thickness of flame.
- Set the dimension of discretization as 100. This allows us to observe the front of the flame.
- Choose CFL number C=u*dt/dx. since the equation is nonlinear, we cannot calculate the exact range of CFL, we try different value to get a stable output.
- Dx depends on the initial condition of flame. Here, dx=1.3e-5
- Set dimTime=100, we can regulate dimTime to observe the flame
- Try different numerical methods

Method forward euler + 1 st order upwind

- Discretization:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} + Ac^2(1-c),$$

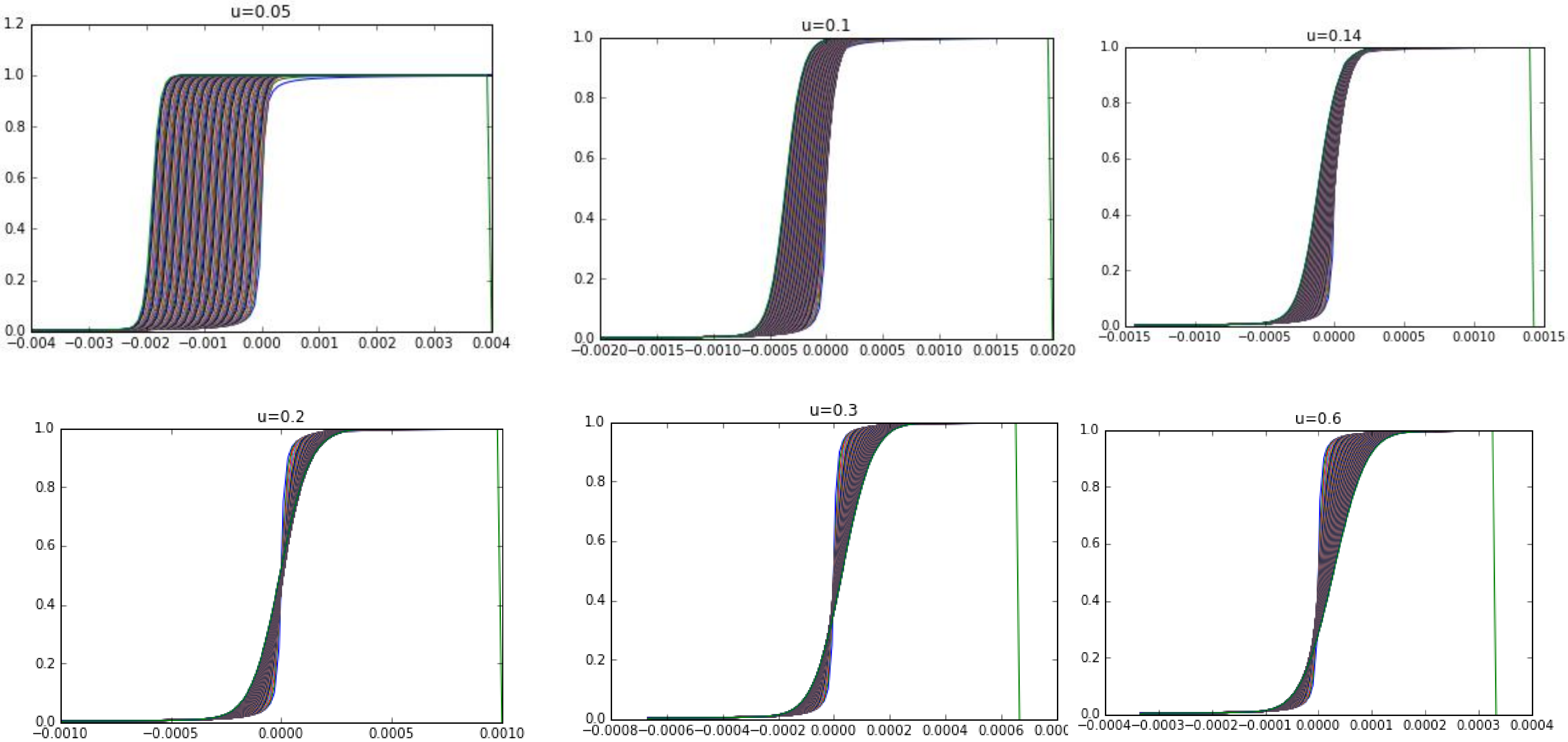
$$\Rightarrow \frac{c_i^{n+1} - c_i^n}{\mathrm{dt}} + u\frac{c_i^n - c_{i-1}^n}{dx} = D\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{dx^2} + A\left(c_i^n\right)^2(1-c_i^n)$$

$$c_i^{n+1} = c_i^n + dt\left[-u\frac{c_i^n - c_{i-1}^n}{dx} + D\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{dx^2} + A\left(c_i^n\right)^2(1-c_i^n)\right]$$

Boundary:

$$\frac{\partial c}{\partial x} = 0 \quad \Rightarrow \quad c_0^n = c_1^n \qquad c_N^n = c_N^n$$

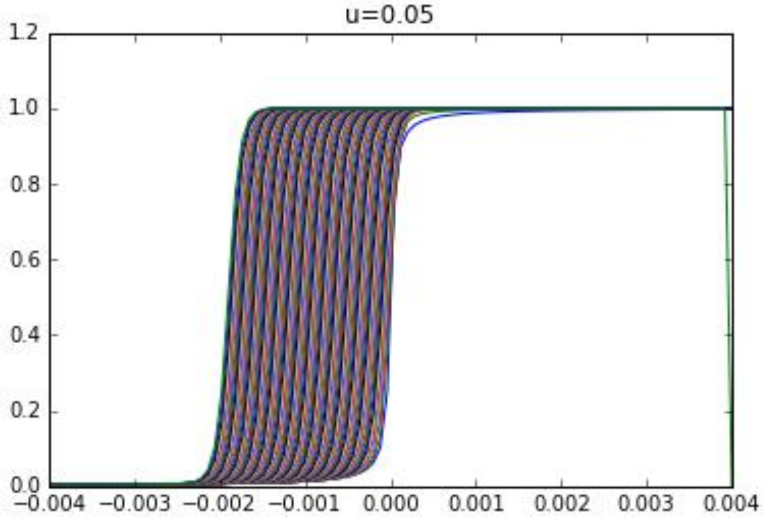
Method forward euler + 1 st order upwind



Method forward euler + 1 st order upwind

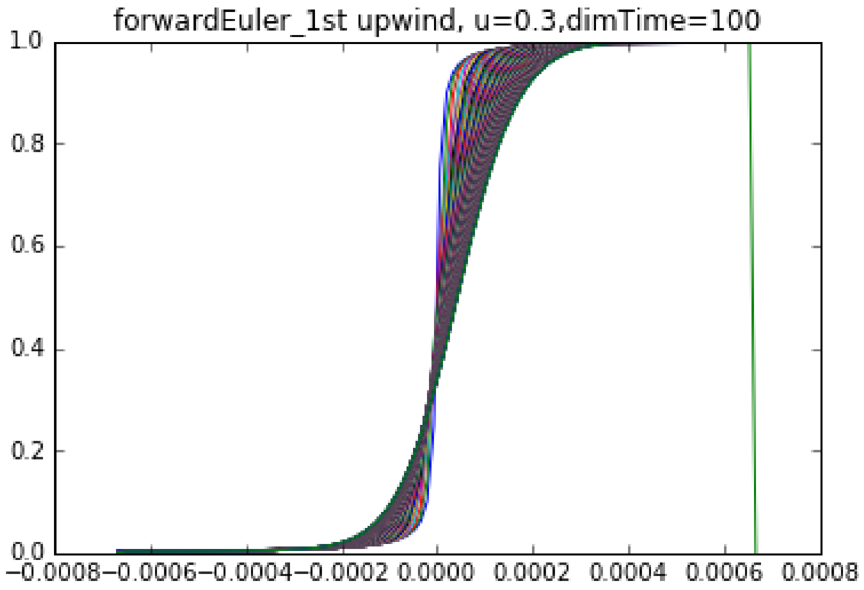
- Result:
- We get the stable output in the condition of CFL<0.1

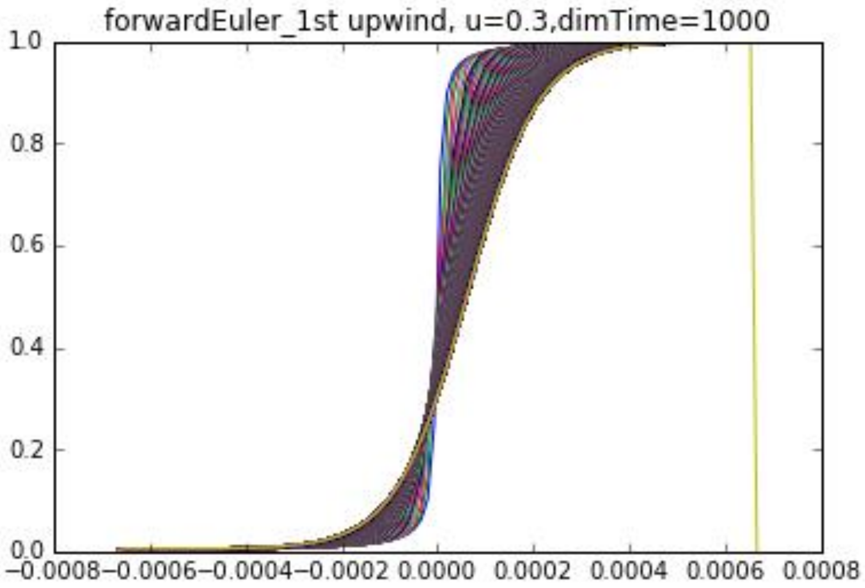
- As we regulate the speed u, we find that:
- When u is too small (u=0.05, u=0.1), the flame propagate to the left, it flashes back



Method forward euler + 1 st order upwind

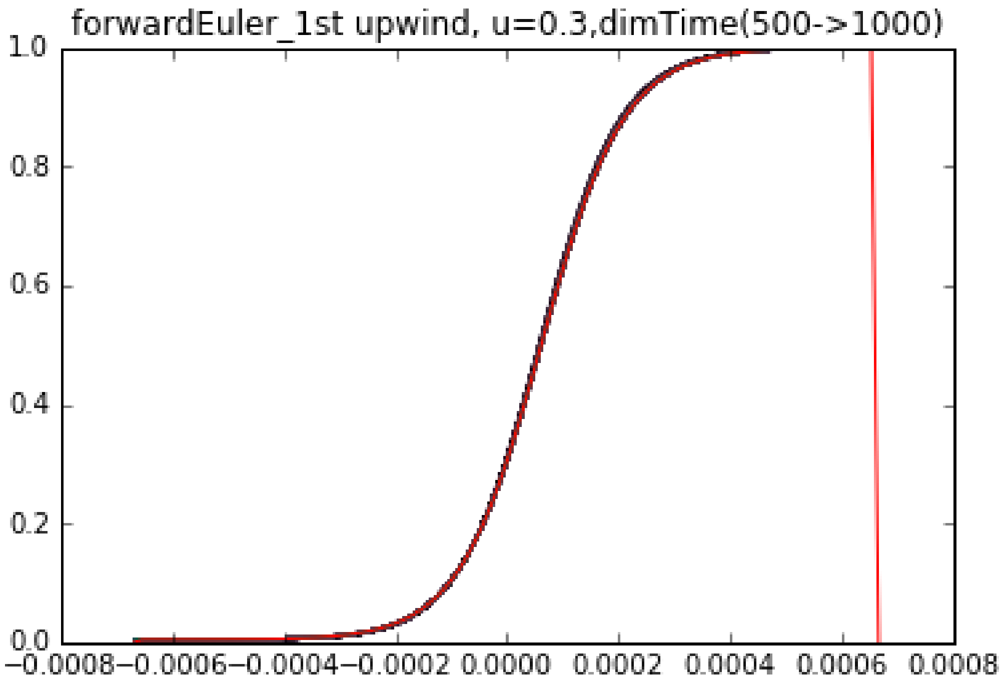
- When u=0.3, we regulate dimTime, T rise, the flame doesnt propagate farther.





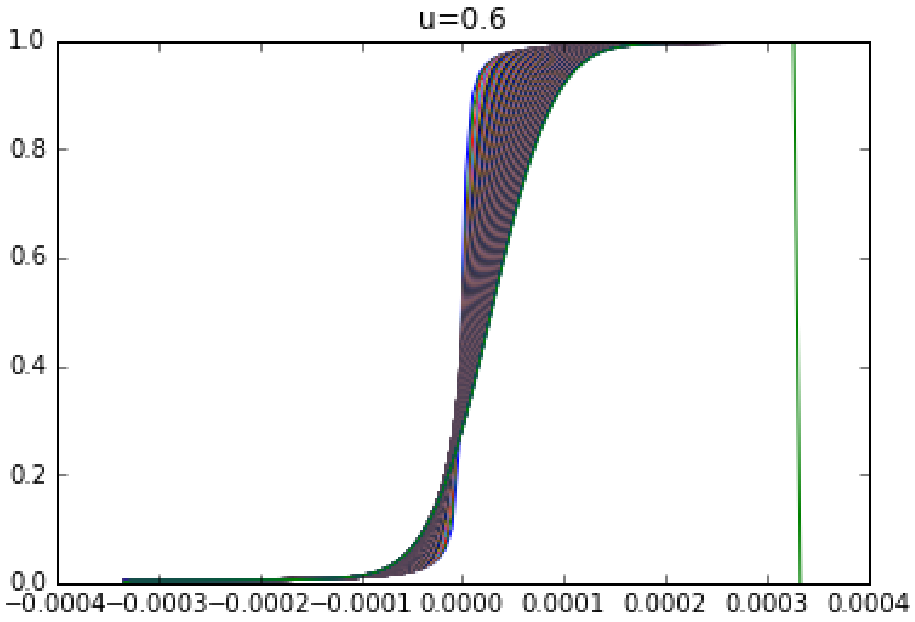
Method forward euler + 1 st order upwind

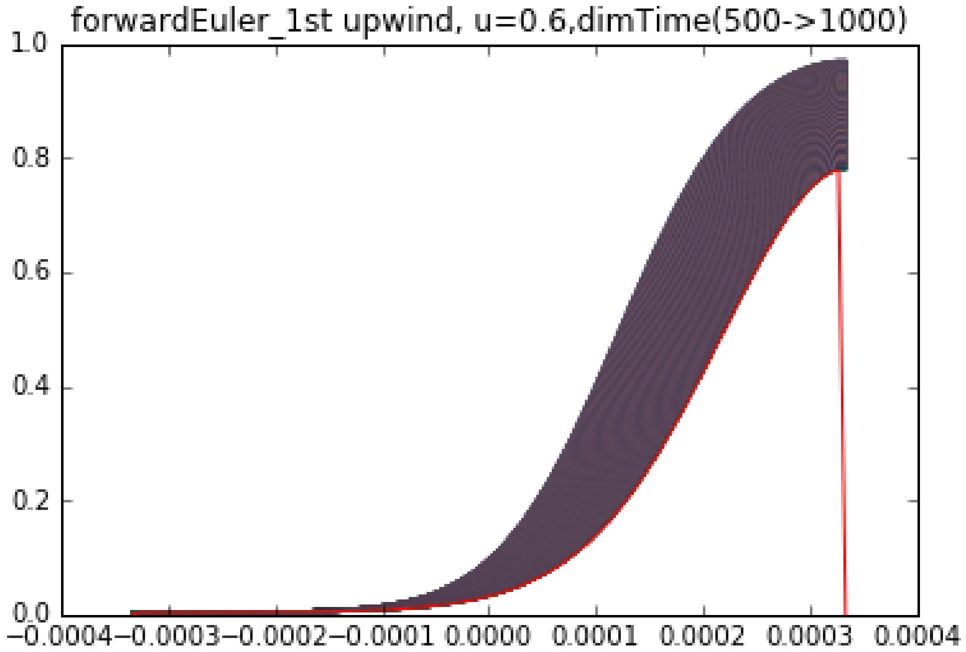
- U=0.3:
- We want to see if the flame get steady, so we change the time range:
- We observe the timestep in range(500,1000), there's almost only one line in the figure.



Method forward euler + 1 st order upwind

- When u is too large, take u=0.6 as example:
- We can see clearly from the fight figure, the flame is blown out to the right side.





- After Method forward euler + 1 st order upwind, we also tried the other methods:
  - Method forward euler + 1 st order upwind
  - Method forward euler + centered 2nd order
  - Method forward euler + 1st downwind
  - Method RK2 + 1st order upwind
  - Method RK4 + 1st order upwind

- and we can the same results, except the different CFL

Method forward euler + centered 2nd order

- Transport equation:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} + Ac^2(1-c),$$

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$$\frac{c_i^{n+1} - c_i^n}{\mathrm{dt}} + u\frac{c_{i+1}^n - c_{i-1}^n}{2dx} = D\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{dx^2} + A\left(c_i^n\right)^2(1 - c_i^n)$$

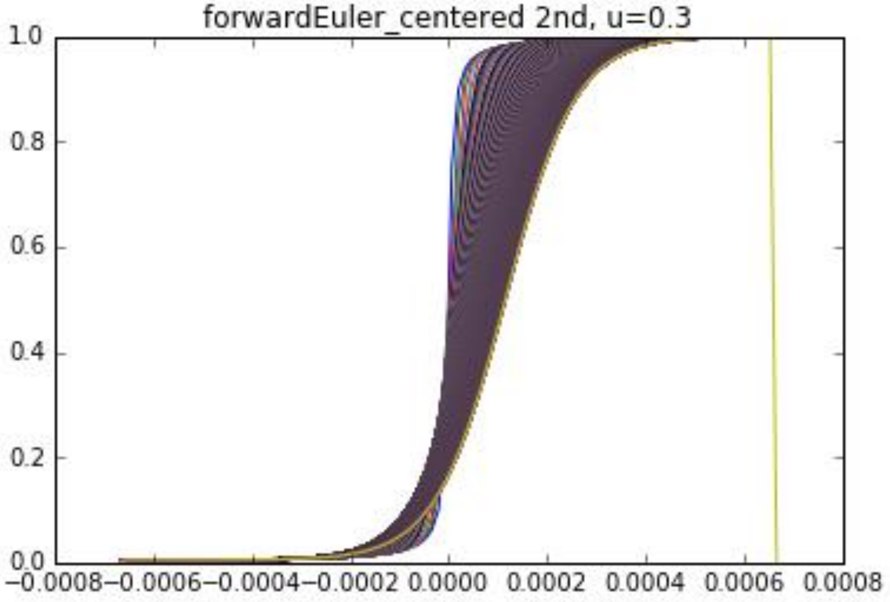
$$c_i^{n+1} = c_i^n + dt\left[-u\frac{c_{i+1}^n - c_{i-1}^n}{2dx} + D\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{dx^2} + A\left(c_i^n\right)^2(1 - c_i^n)\right]$$

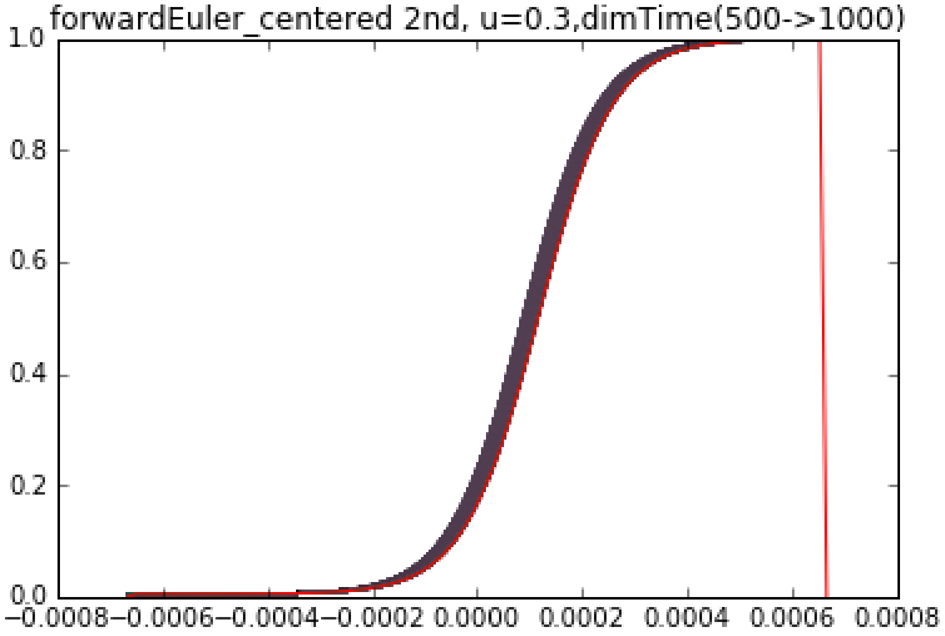
Boundary:

$$c_0^n = c_1^n \qquad c_N^n = c_N^n$$

Method forward euler + centered 2nd order

- Result:
- Stable when CFL<0.1





Method forward euler + 1st downwind

- Transport equation:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} + Ac^2(1-c),$$

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$$\frac{c_i^{n+1} - c_i^n}{\mathrm{dt}} + u\frac{c_{i+1}^n - c_{i-1}^n}{2dx} = D\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{dx^2} + A\left(c_i^n\right)^2(1-c_i^n)$$

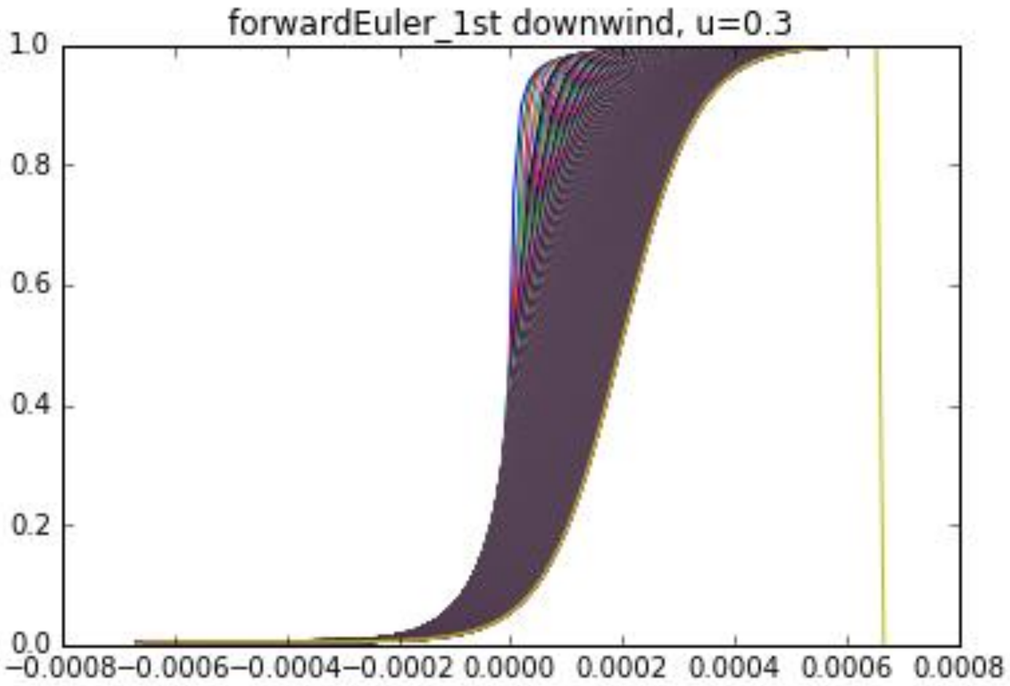
$$c_i^{n+1} = c_i^n + dt\left[-u\frac{c_{i+1}^n - c_{i-1}^n}{2dx} + D\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{dx^2} + A\left(c_i^n\right)^2(1-c_i^n)\right]$$

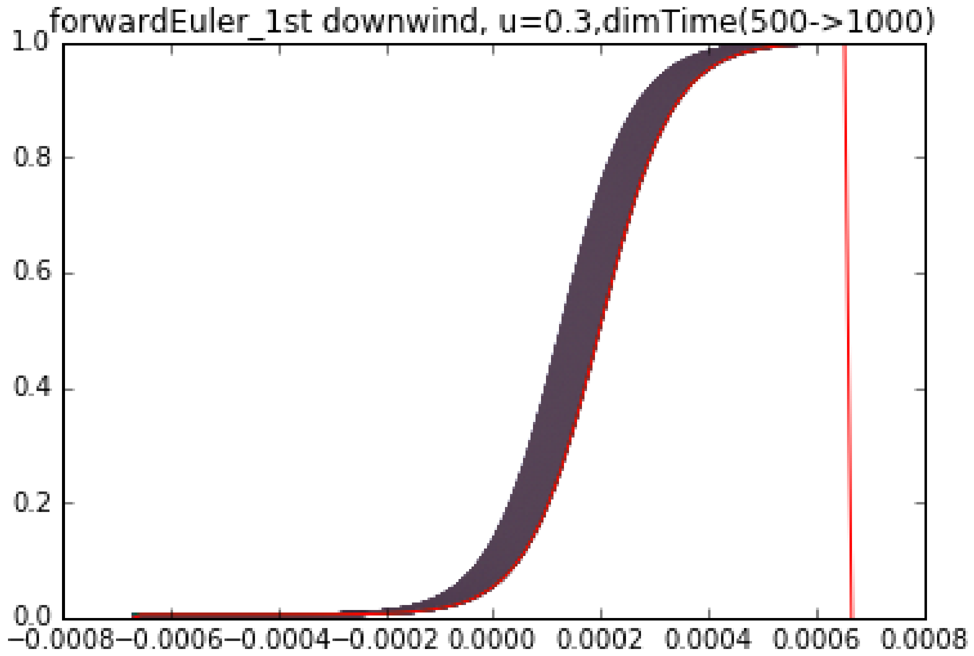
Boundary:

$$c_0^n = c_1^n \qquad c_N^n = c_N^n$$

Method forward euler + 1st downwind

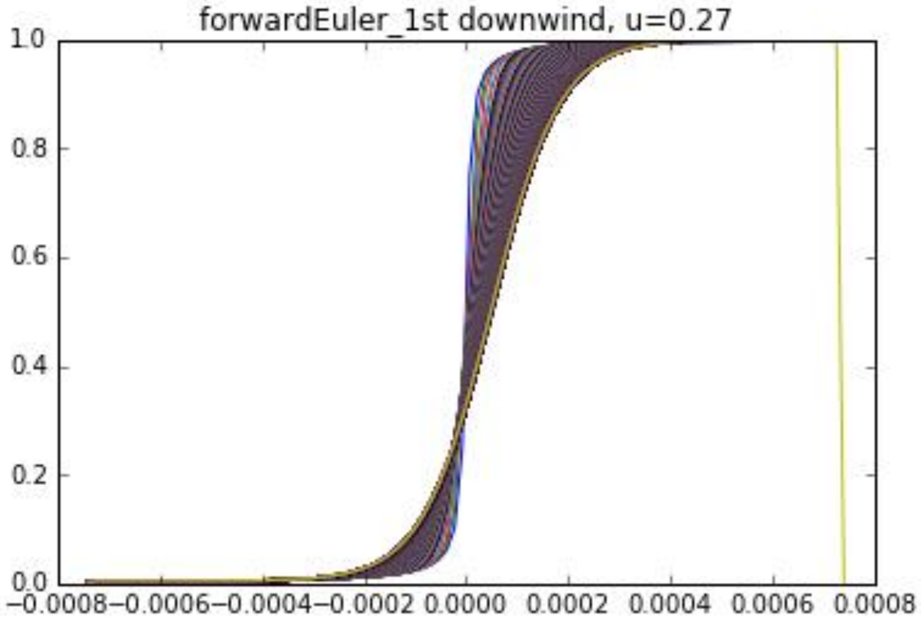
- Stable when CFL<=0.11, this CFL is larger than the former two, we can get a larger dt.
- Observe the right figure, when u=0.3, it still has the tendancy to be blown out.

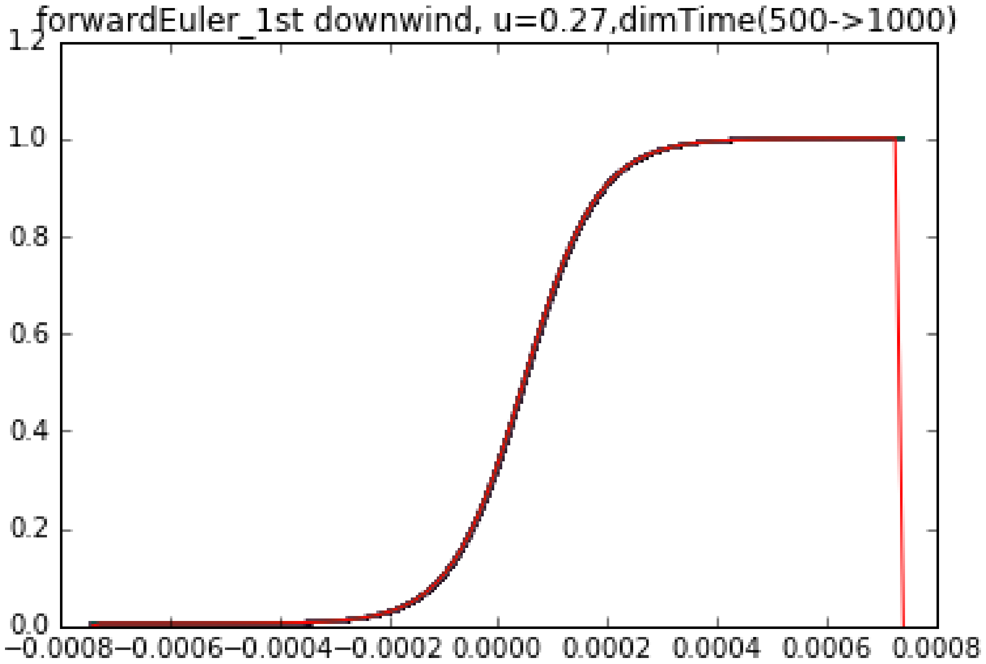




Method forward euler + 1st downwind

- So we regulate the u, trying to find the intrinsic speed Sl
- When u=0.27, the flame is almost steady. (the line is thinest when u=0.27)





Method RK4 + 1st order upwind

- Transport equation: $$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} + Ac^2(1-c),$$

$$f\left(c_i^n\right) = \frac{\partial c_i^n}{\mathrm{dt}} = -u\frac{c_{i+1}^n - c_{i-1}^n}{2dx} + D\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{dx^2} + A\left(c_i^n\right)^2(1-c_i^n)$$

$$\mathrm{k1} = f\left(c^n\right)$$

$$k2 = f\left(c^n + \mathrm{dt}\frac{k1}{2}\right)$$

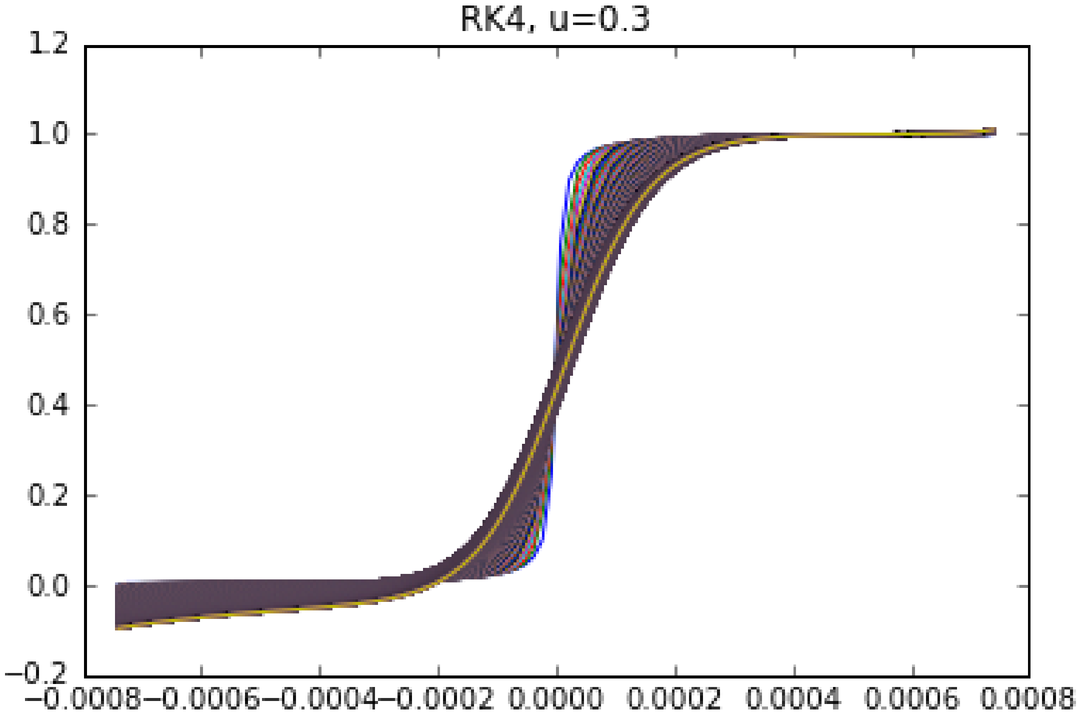
$$k3 = f\left(c^n + \mathrm{dt}\frac{k2}{2}\right)$$

$$k4 = f\left(c^n + \mathrm{dt}\frac{k3}{2}\right)$$

$$c^{n+1} = c^n + \frac{h}{6}(k1 + 2k2 + 2k3 + k4)$$

Method RK4 + 1st order upwind

- Result:
- Stable when CFL <0.1
- Some problem on the boundary



Conclusion

- From the analysis before, we found that:
- The flame get steady when $u=0.27$, so the intrinsic speed Sl=0.27
- When $u<0.27$, the flame flashes back
- When $u>0.27$, the flame is blown away to the right side.

Conclusion

- Question: Is setting u = 0.3 m/s enough to prevent flash-back of the flame?
- Response: Yes, we has observed when u=0.3, it is blown away to the right side.