Numerical Methods in Engineering Applications

Session #05

Parabolic and Hyperbolic PDEs: Explicit Methods

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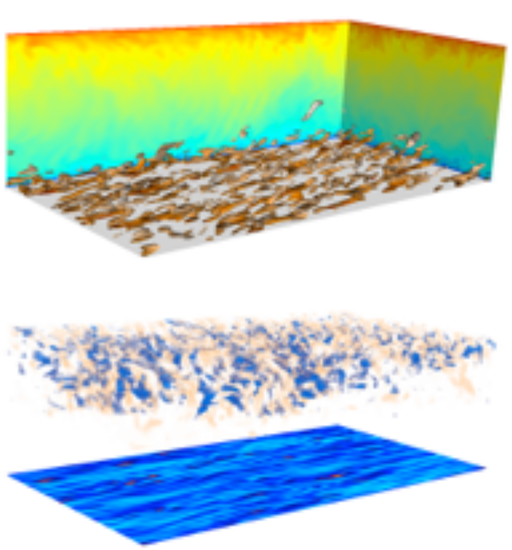
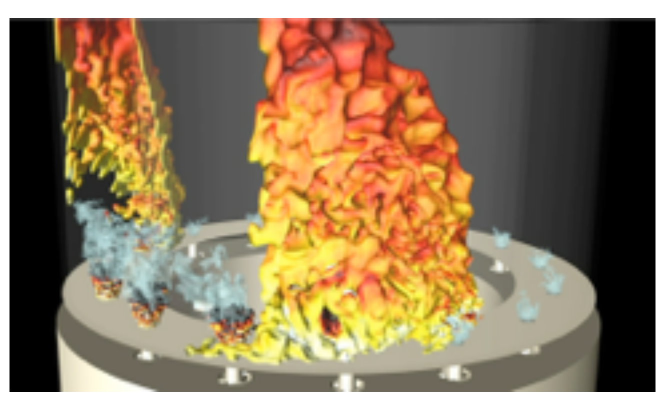
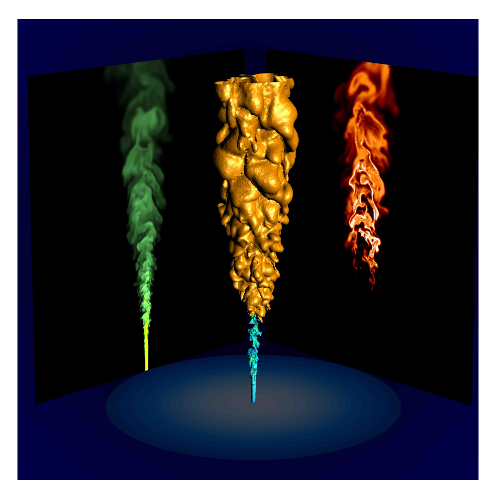
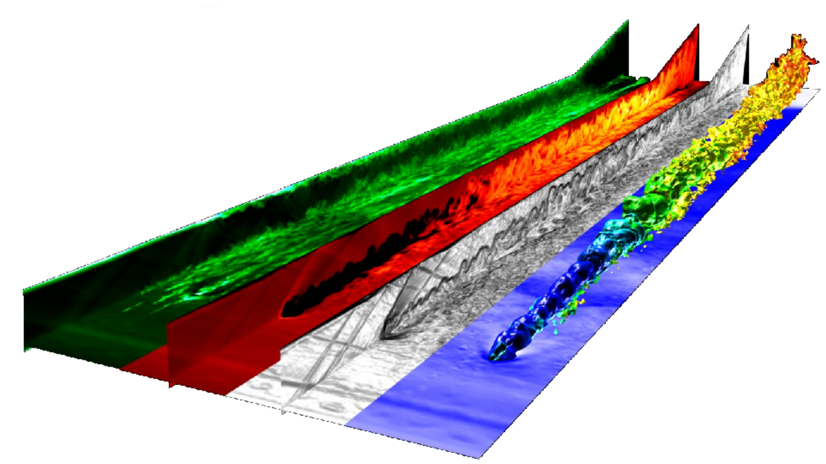

Course contents

● Theoretical lecture
■ Problem-solving workshop

I. Basics on numerical approximations

- Introduction and Finite Differences. ● ■ ■
- Numerical solution of ordinary differential equations. ● ■

II. Solving large linear equations systems: Applications to steady heat equation.

- Elliptic PDE 1. ● ■ ■
- Elliptic PDE 2. ● ■

III. Methods for unsteady advection/diffusion problems

- Hyperbolic and parabolic PDE: Explicit methods. ● ■
- Characterization of numerical errors. ● ■
- Hyperbolic and parabolic PDE: Implicit methods. ● ■

→ Projet #2 deadline: May, 18th
→ Projet #3 deadline: May, 25th

IV. Towards computational fluid dynamics

- Methodology in numerical computations. ■ ■
- Incompressible Flow equations. ● ■
- Semi-Implicit method for incompressible flows. ● ■
- Final project on incompressible flow. ■ ■

Elliptic/Parabolic/Hyperbolic

- **Elliptic**
  - No notion of propagation of physical information
  - Solution at a point depends on the whole field, and reciprocally.
  - Requires BCs on the entire domain border
- **Parabolic**
  - Propagation of information along one preferred direction
  - Solution at a point depends on a half-space domain
  - Requires an “initial condition”
  - Requires BCs for directions different from the preferred one
- **Hyperbolic**
  - Propagation of information at finite speed along one or several directions
  - Solution at a point depends on a region delimited by the PDE characteristics
  - Requires an “initial condition”
  - Requires BCs only where information is incoming

Unsteady Heat Equation (Forward Euler + 2nd order)

- 1D

$$\phi_i^{n+1} = \phi_i^n + \mathrm{Fo}(\phi_i^n - 2\phi_i^n + \phi_{i-1}^n)$$

$$\mathrm{Stability}\ :\ \mathrm{Fo} \leq \frac{1}{2}$$

- 2D

$$\phi_{i,j}^{n+1} = \phi_{i,j}^n + \mathrm{Fo}(\phi_{i,j}^n - 2\phi_{i,j}^n + \phi_{i-1,j}^n) + \mathrm{Fo}(\phi_{i,j+1}^n - 2\phi_{i,j}^n + \phi_{i,j-1}^n)$$

$$\mathrm{Stability}\ :\ \mathrm{Fo} \leq \frac{1}{4}$$

- 3D

$$\phi_{i,j,k}^{n+1} = \ldots$$

$$\mathrm{Stability}\ :\ \mathrm{Fo} \leq \frac{1}{6}$$

Advection (Forward Euler + 1st order Upwind)

- 1D

$$u_i^{n+1} = u_i^n - \mathcal{C}(u_i^n - u_{i-1}^n)$$

$$\text{Stability}: \ \mathcal{C} \leq 1$$

- 2D

$$u_{i,j}^{n+1} = u_{i,j}^n - \mathcal{C}_x(u_{i,j}^n - u_{i-1,j}^n) - \mathcal{C}_y(u_{i,j}^n - u_{i,j-1}^n)$$

$$\text{Stability}: \ \mathcal{C}_x + \mathcal{C}_y \leq 1$$

- 3D

$$u_{i,j,k}^{n+1} = \ldots$$

$$\text{Stability}: \ \mathcal{C}_x + \mathcal{C}_y + \mathcal{C}_z \leq 1$$