**CentraleSupelec** **Numerical Methods, S8**

*Hyperbolic and Parabolic Partial Differential Equations (2)*

***Highlights***:

- Characterization of errors
- Order of convergence
- Dispersion and diffusion

The present workshop focuses on the linear advection equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0 \tag{1}$$

with periodic boundary conditions over the domain $x \in [0, 1]$. The analytical solution of this equation is $u(t, x) = u(t = 0, x - at)$, such that it presents no diffusion (maxima and minima are preserved) nor dispersion (all wave numbers travel at the same speed). It is thus an ideal case for evaluating numerical errors as well as diffusive and dispersive properties of numerical schemes.

# 1 Order of convergence of numerical schemes

In this first problem, the objective is to evaluate the order of accuracy of different schemes by comparing the analytical solution to the numerical one. The initial condition is a Gaussian distribution:

$$u(t = 0, x) = \exp\left(-\frac{(x - 0.5)^2}{0.1^2}\right) \tag{2}$$

- Implement the upwind scheme, Leap-Frog, Lax-Wendroff and Lax-Friedrichs schemes.
- Observe the different behaviors of each scheme, and identify dispersive and diffusive effects.
- Identify the impact of the CFL number on the numerical solution.
- evaluate the error of each scheme as a function of the space discretization for different CFL numbers.
- Quantify the order of accuracy of each scheme.

# 2 Analysis of numerical dispersion and diffusion

In this second problem, the goal is to observe the evolution of a pure sinusoidal perturbation:

$$u(t = 0, x) = \sin(\omega x) \tag{3}$$

- Implement this new initial condition.
- For a given wave number, evaluate the numerical diffusion of all schemes. You have to find an adequate metric.
- Then, evaluate the numerical dispersion of all schemes. You also have to define an adequate metric.
- evaluate theses errors for several wave numbers and several CFL numbers.