# Numerical Methods in Engineering Applications

## Workshop #03

Elliptic equations

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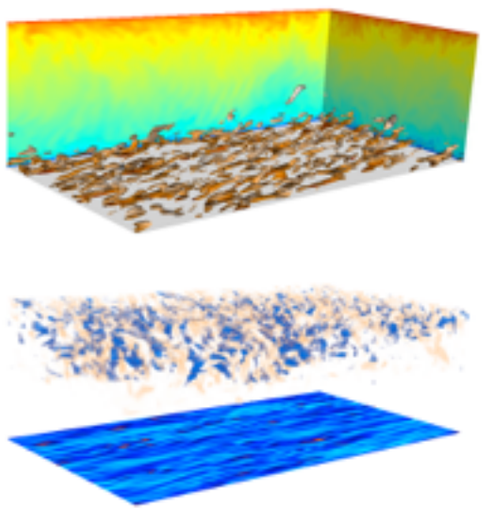
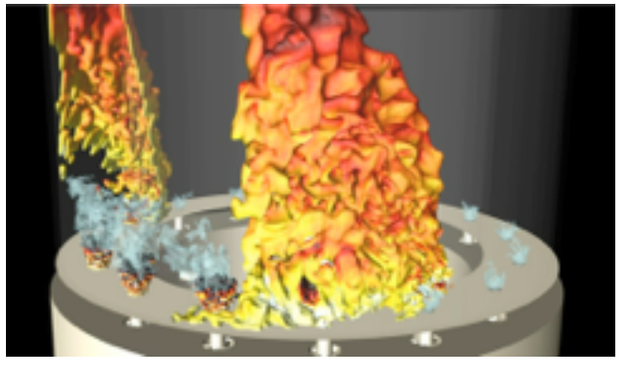
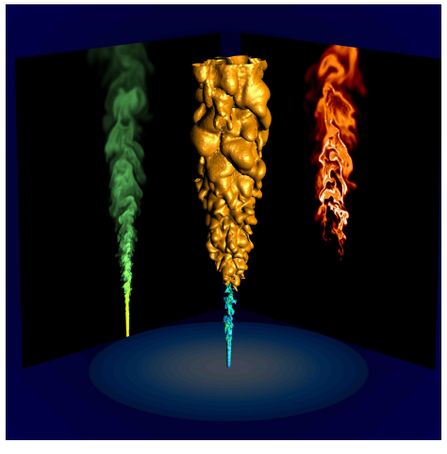
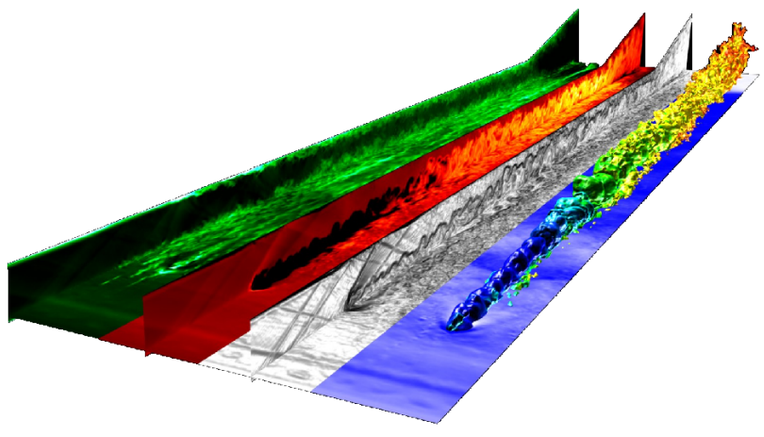




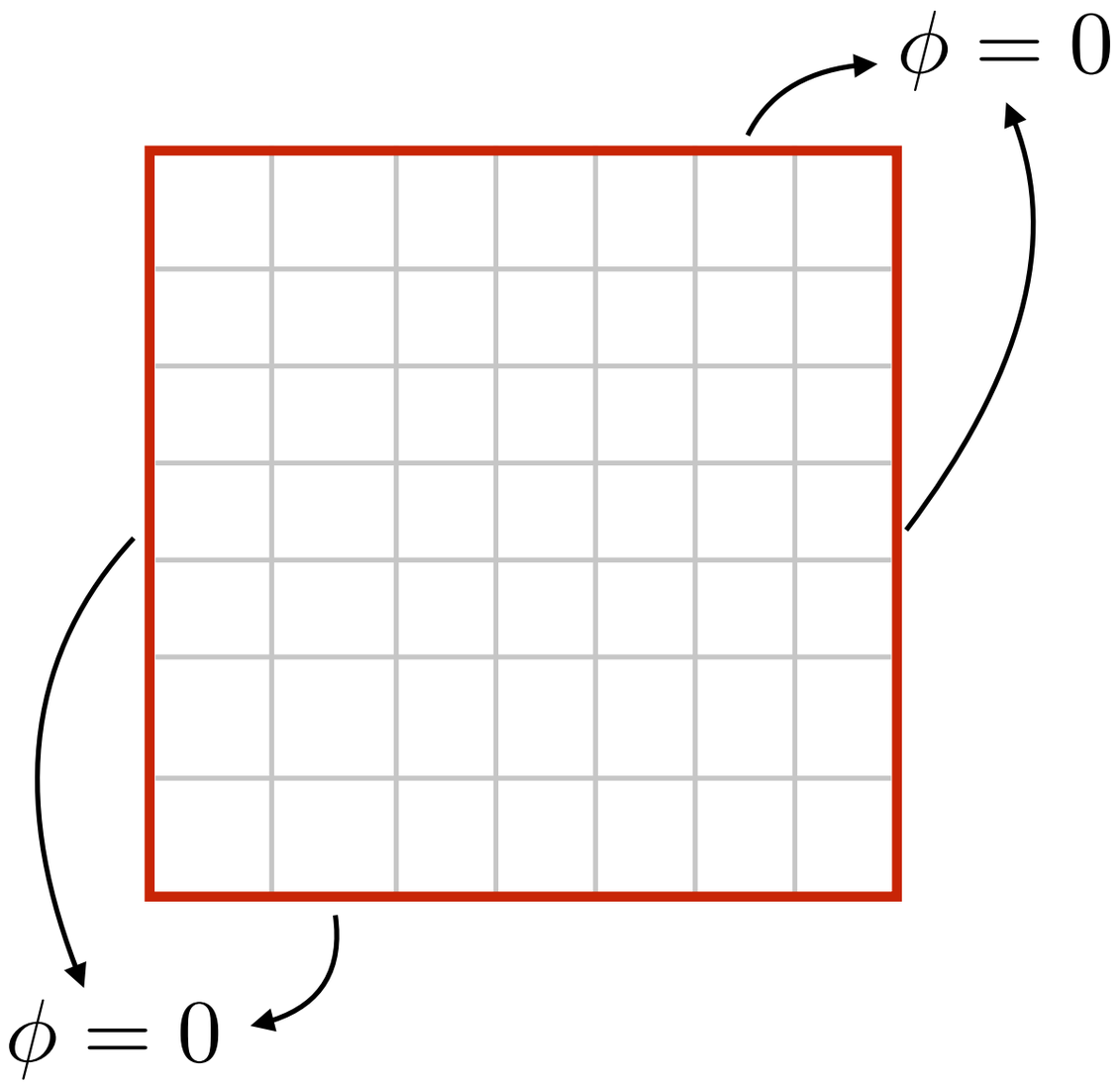




# Objectives of Workshop #3

- **Working with 2D problems**
- **Defining the discrete formulation of an elliptic equation**
- **Solving it using iterative solvers**
- **Treating boundary conditions**

# Poisson equation with Dirichlet boundary conditions



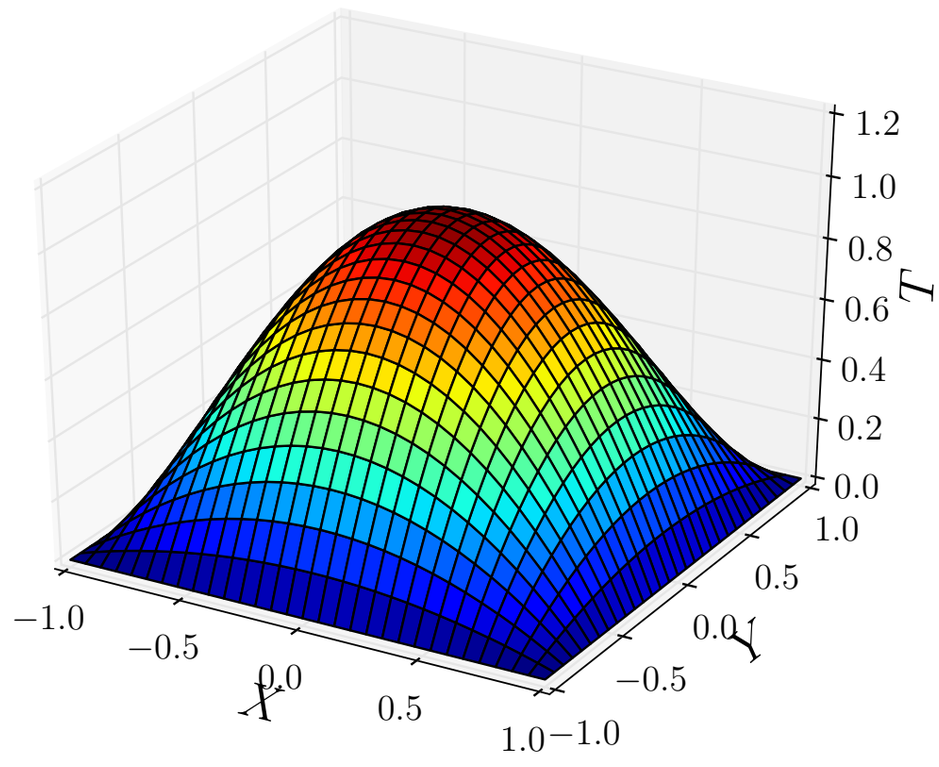

$$\Delta\phi = f(x, y)$$

**with**

$$f(x, y) = 2(x^2 + y^2 - 2)$$

Solution method:

- **Direct** (given)
- **Jacobi**
- **Gauss-Seidel**



# Poisson equation with Dirichlet boundary conditions

$$\eta = 10^{-3}$$

Run time (ms)

| | 10x10 | 20x20 | 40x40 | 80x80 | 160x160 |
|---|---|---|---|---|---|
| **Direct** | 0.7 | 5 | 187 | 13000 | 1060000 |
| **Jacobi** | 25 | 392 | 5510 | 127000 | 1362000 |
| **Gauss-Seidel** | 11 | 200 | 2888 | 78000 | 1034000 |
| **SOR** | 20 | 86 | 392 | 2260 | 30000 |
| **Conjugate Gradient** | 2 | 14 | 103 | 945 | 6950 |

9.9Go

0.263Go

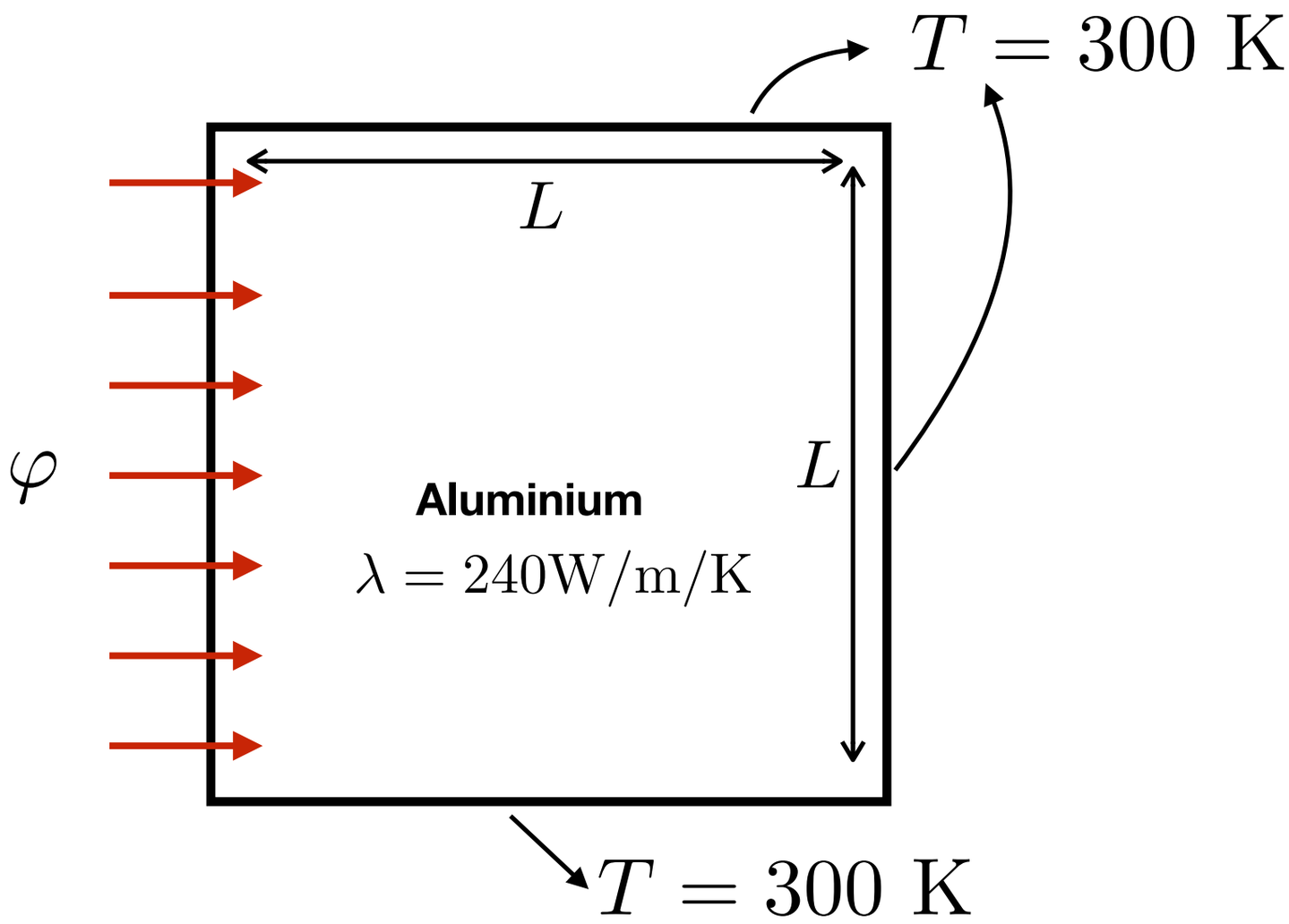
# Poisson equation with Dirichlet boundary conditions

$$\eta = 10^{-3}$$

Iterations

| | 10x10 | 20x20 | 40x40 | 80x80 | 160x160 |
|---|---|---|---|---|---|
| **Jacobi** | 88 | 352 | 1281 | 4400 | 14306 |
| **Gauss-Seidel** | 45 | 177 | 642 | 2200 | 7154 |
| **SOR** | 63 | 65 | 75 | 118 | 382 |
| **Conjugate Gradient** | 6 | 13 | 26 | 53 | 107 |

# Laplace equation with Neumann and Dirichlet B.C.




**Unsteady heat equation**

$$\Delta T = 0$$

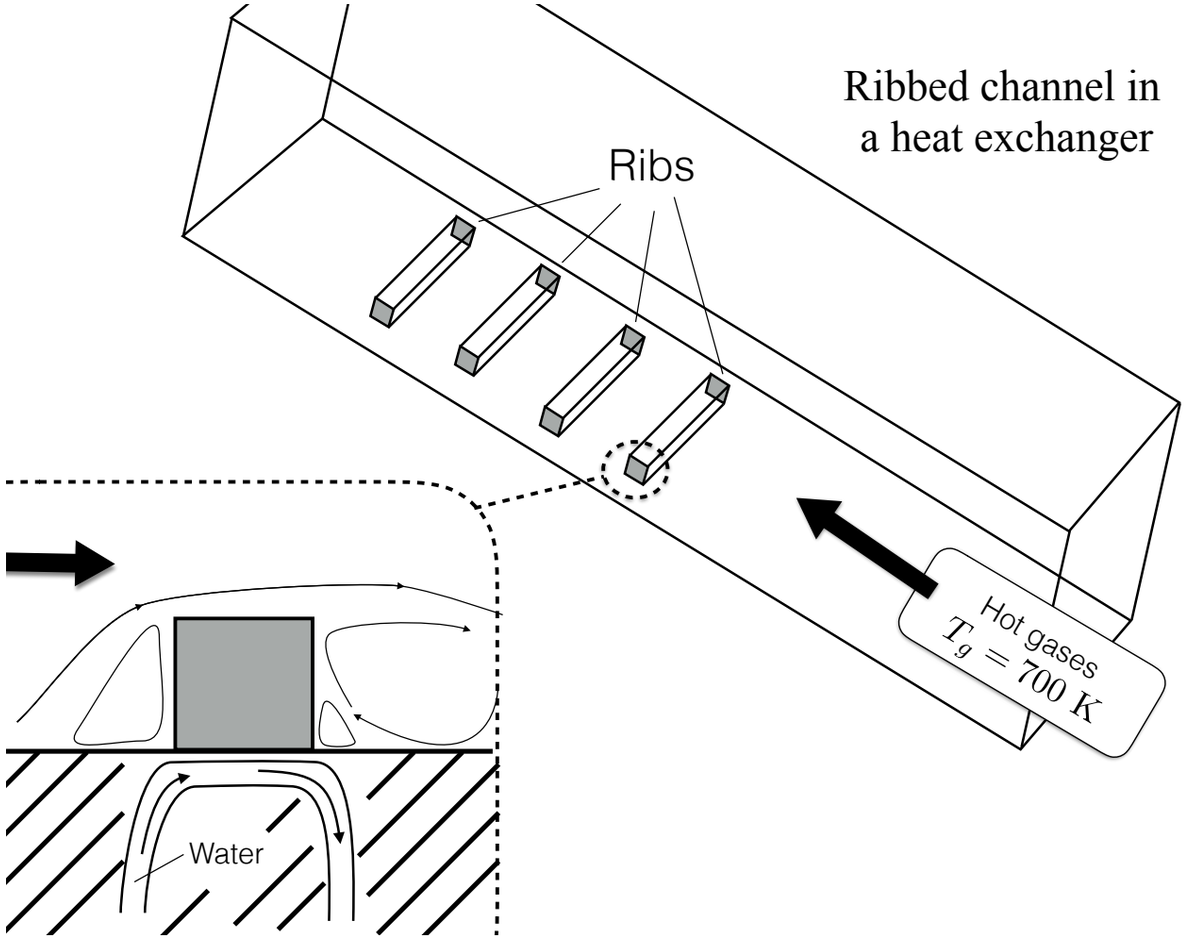
$$\mathrm{L} = 10\ \mathrm{cm}$$

$$\varphi = 2.4\ \mathrm{MW/m^2}$$

**What is the maximum temperature?**

# Project #1: Design of a cooling rib

- **Ribs are often used to maximise the surface in heat exchangers**
- **Water-cooling can be use to cool down the rib**
- **Each individual rib has its own water-cooling channel**



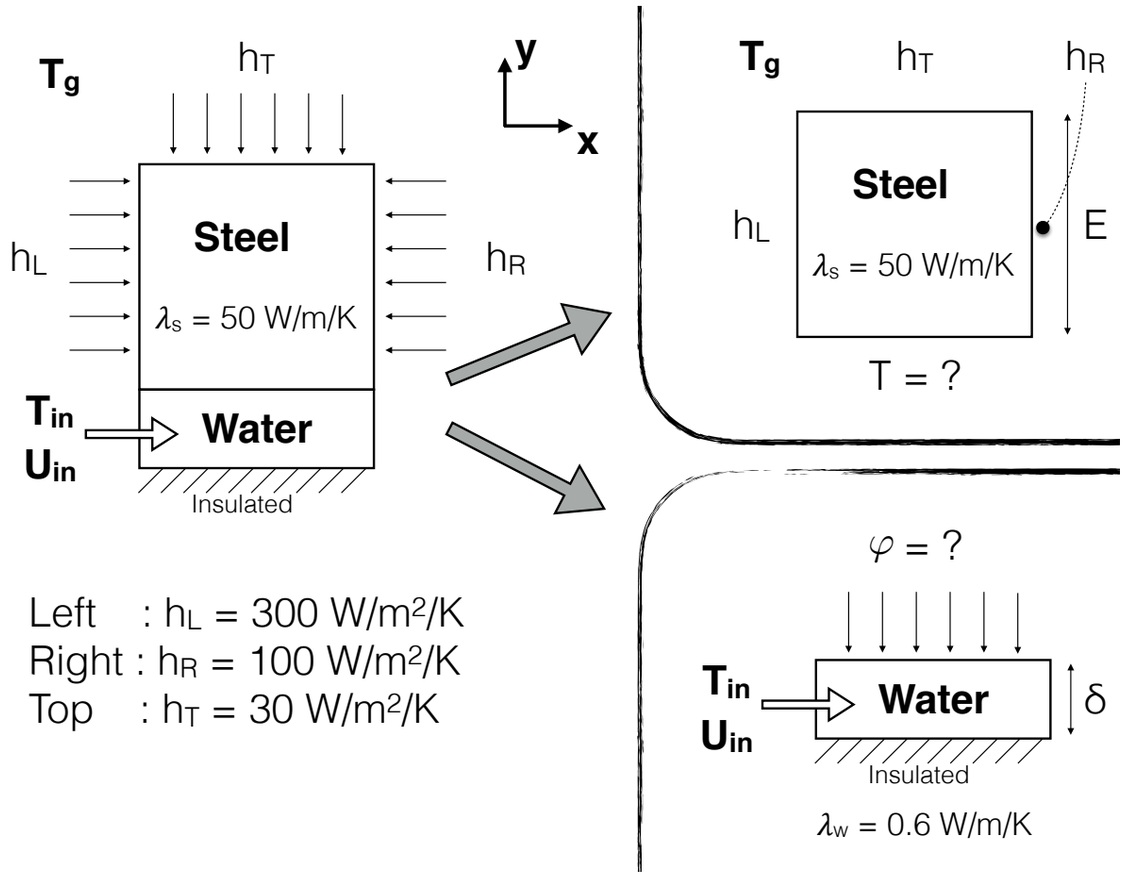
# Project #1: Design of a cooling rib

- **Two domains:**
  - the rib subject to heat exchange

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

  - The channel subject to transport and heat exchange

$$u(x,y)\frac{\partial T}{\partial x} = a_w \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$




Left : $h_L$ = 300 W/m²/K
Right : $h_R$ = 100 W/m²/K
Top : $h_T$ = 30 W/m²/K

- **Project objectives:**
  - Determine the bulk velocity in the channel to cool the rib and prevent water from boiling
  - Estimate the corresponding heat power that can be extracted
  - Determine what happens if the flow is inverted in the channel

# Project #1: Design of a cooling rib

## Project #01 to hand out (slides in PDF) for May, 11th

Send PDF slides to ronan.vicquelin@centralesupelec.fr and aymeric.vie@centralesupelec.fr

- First slide : names (2 people)+ problem title
- Slide #2 : sum up the problem to solve
- Self-sufficient slides => clear, detailed enough, synthetic
- Explain the approach, discuss your choices
- Describe numerical method, very briefly if seen in class, specify details related to the study
- Show and analyse results
- How sure are you that your results are correct ?

- Plots :
  Readable, clear
  axis names
  units
  legend

- Last slide : highlight results and conclusions