Numerical Methods in Engineering Applications

Workshop #05

Hyperbolic and Parabolic PDEs: Explicit methods

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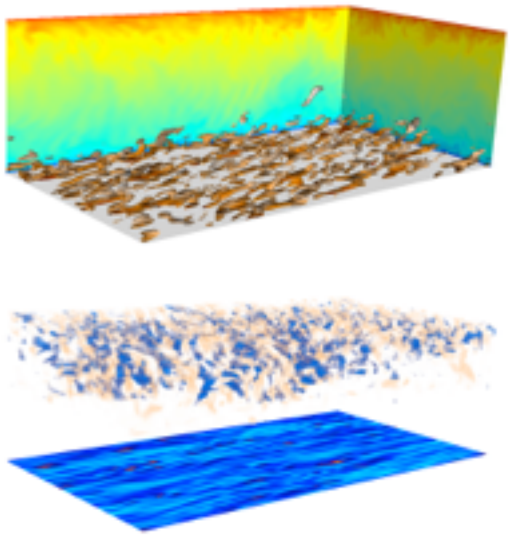
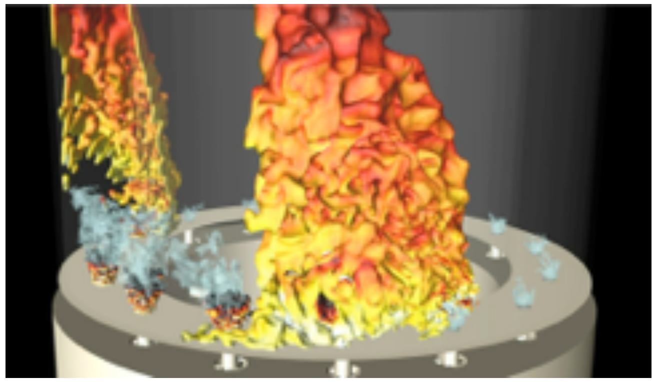
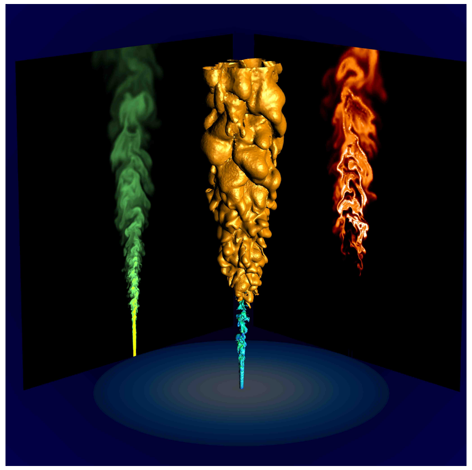
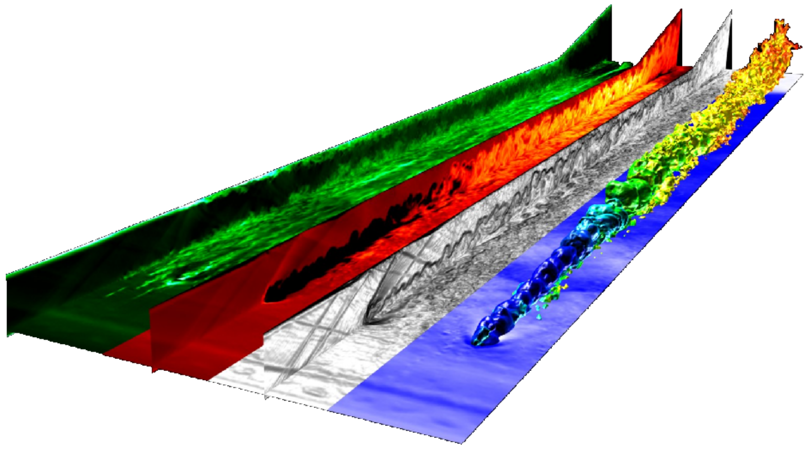




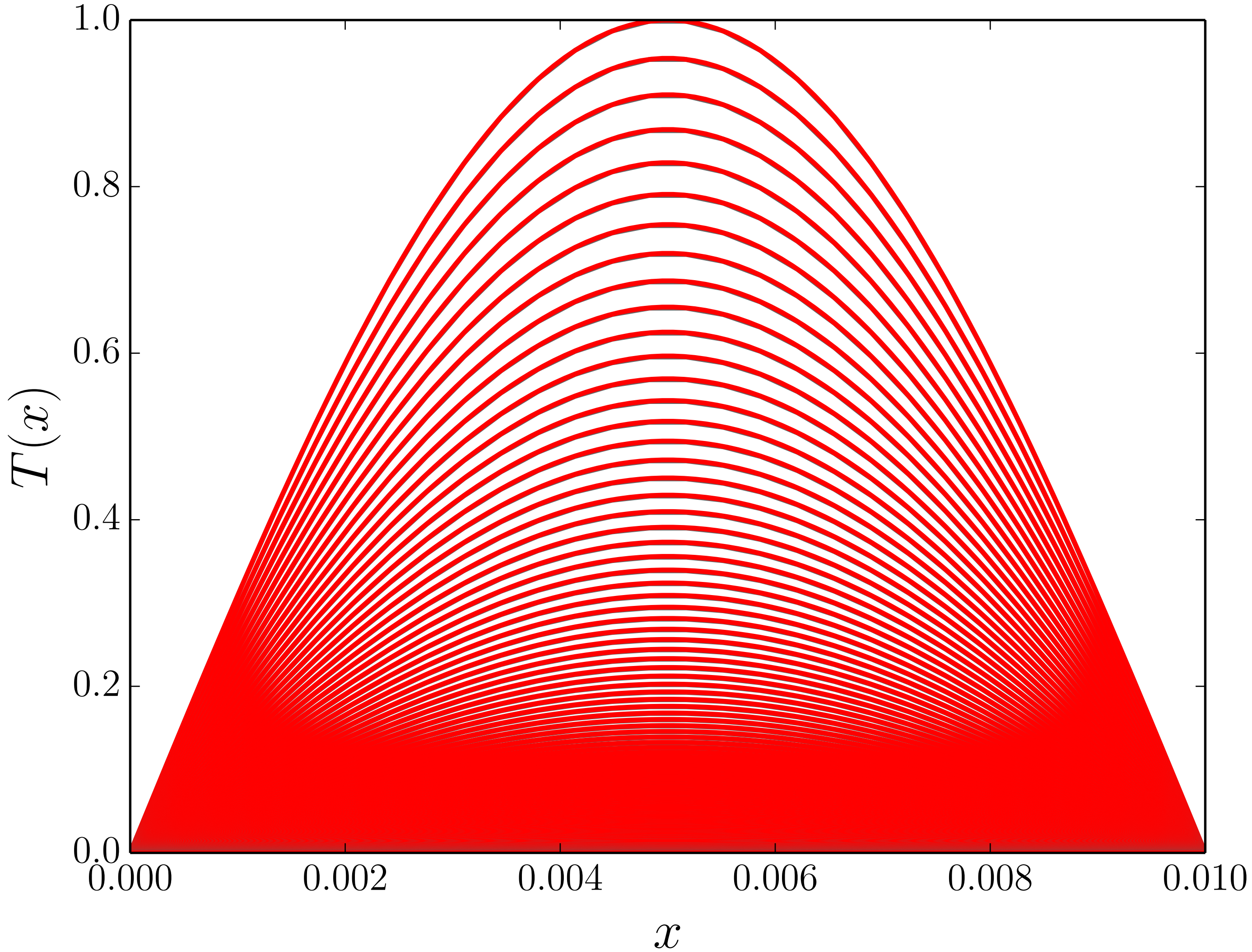




Objectives of Workshop #5

- **Unsteady 1D heat equation** $\longrightarrow$ **Fourier criterion**
- **1D Advection** $\longrightarrow$ **CFL condition**
- **Thin boundary layer**

Unsteady Diffusion: 1D



$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \qquad \text{on } [0, L]$$

with L = 1cm and a = 10^{-4} m^2/s

Initial condition:

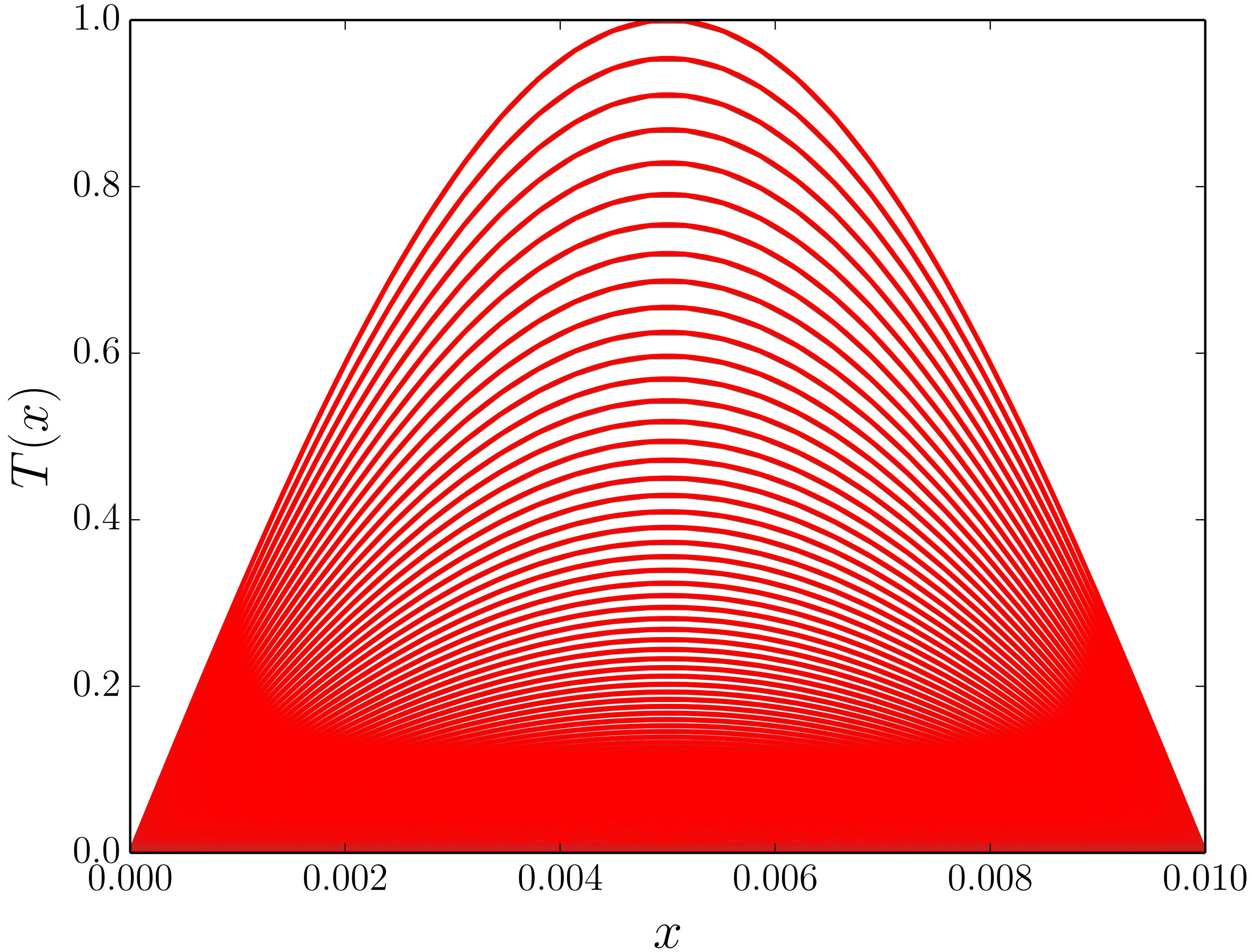
$$T(x,0) = \sin\left(\frac{\pi x}{L}\right)$$

Boundary conditions:

$$\begin{cases} T(x=0,t) & = 0 \\ T(x=L,t) & = 0 \end{cases}$$

Analytical solution: $$T(x,t) = \sin\left(\frac{\pi x}{L}\right) e^{-\frac{\pi^2}{L^2} a t}$$

Unsteady Diffusion: 1D



$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} \quad \text{on } [0, L]$$

with L = 1cm and a = 10^{-4} m^2/s

<u>Initial condition:</u>

$$T(x,0) = \sin\left(\frac{\pi x}{L}\right)$$

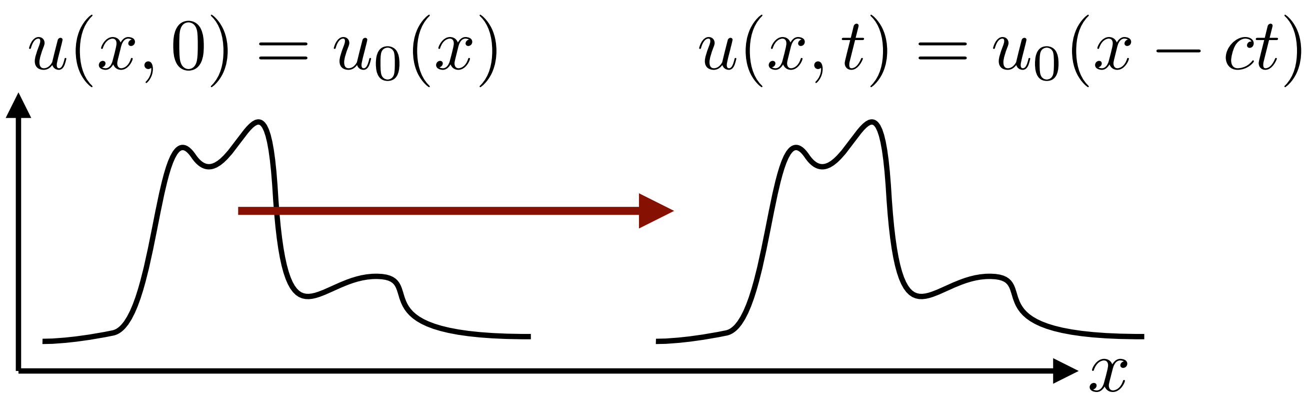
<u>Boundary conditions:</u>

$$\begin{cases} T(x=0,t) & = 0 \\ T(x=L,t) & = 0 \end{cases}$$

Numerical Resolution: **Forward Euler + Centered 2nd-order space discretization**

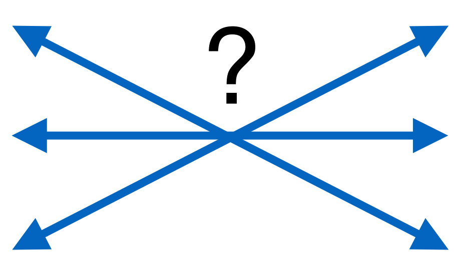
1D Advection

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$



- Solve Forward Euler + 1st order upwind
- Solve Forward Euler + Centered second order
- Then, choose …

Forward Euler
RK2
RK3
RK4
AB2
Leap-Frog



1st order, upwind
1st order, downwind
2nd order, centered
4th order, centered
4th order, centered, Padé scheme

Project #3: Advection/Diffusion/Reaction

Can present intrinsic propagating solutions (ex. : Flames)

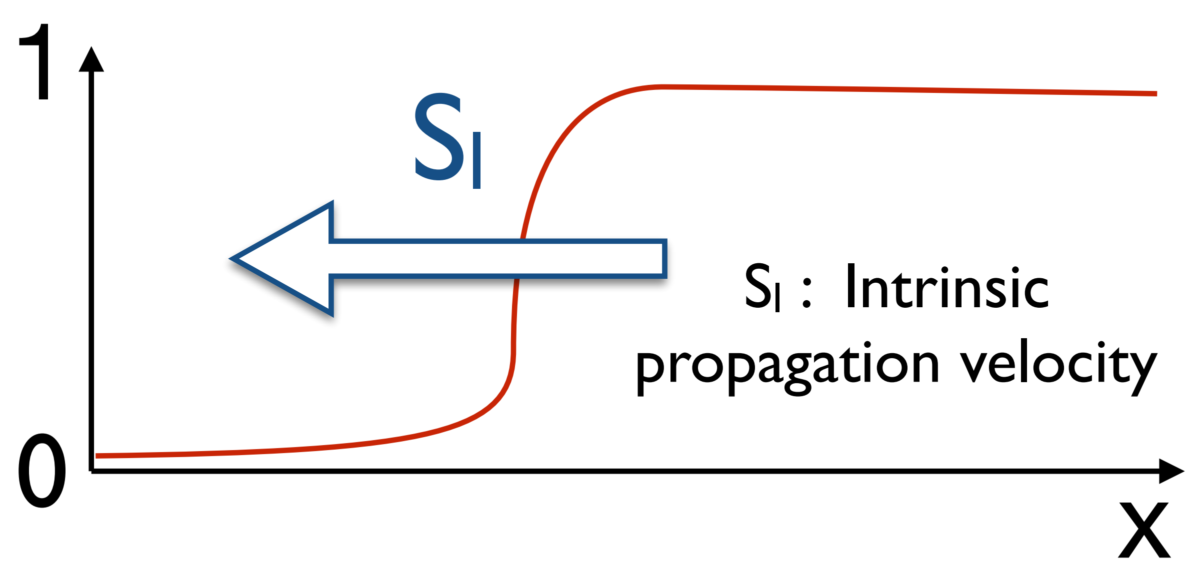
$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2} + A\, c^2 \,(1 - c)$$

Convection
$u = 0.3 \ \mathrm{m/s}$

Diffusion
$D = 2.0 \times 10^{-5} \ \mathrm{m^2/s}$

Reaction
$A = 8\,000.0\,\mathrm{s}^{-1}$

<u>Initial solution</u>



<u>Question</u>

Behavior of the flame:
flash-back or blown-out ?