CentraleSupelec **Numerical Methods, S8**

Project #02: Design of a cooling rib

Ribs, as shown in Fig. 1, are often met in heat exchangers to enhance heat transfer thanks to the increased exchange surface area and the induced flow around the introduced obstacles. The level of extracted heat is sustained by a water-cooling system in the considered system where each rib is associated to one cooling channel. The external temperature encountered in the flow which surrounds the ribs is $T_g = 700$ K. Each rib has a square cross-section with a 2-cm x 2-cm area. $E = 2$ cm denotes the thickness of the ribs.

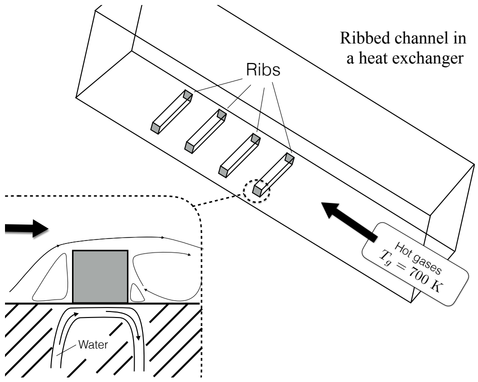

Figure 1: Water-cooled ribs in contact with hot gases

A single rib is studied as depicted in Fig. 2, where the flow applies the conductive heat transfer coefficient h_L, h_R, and h_T on the left, right and top surface of the rib respectively. The water in contact with the steel rib flows from left to write from left to right with the bulk velocity U_{in} and inlet temperature $T_{in} = 300$ K.

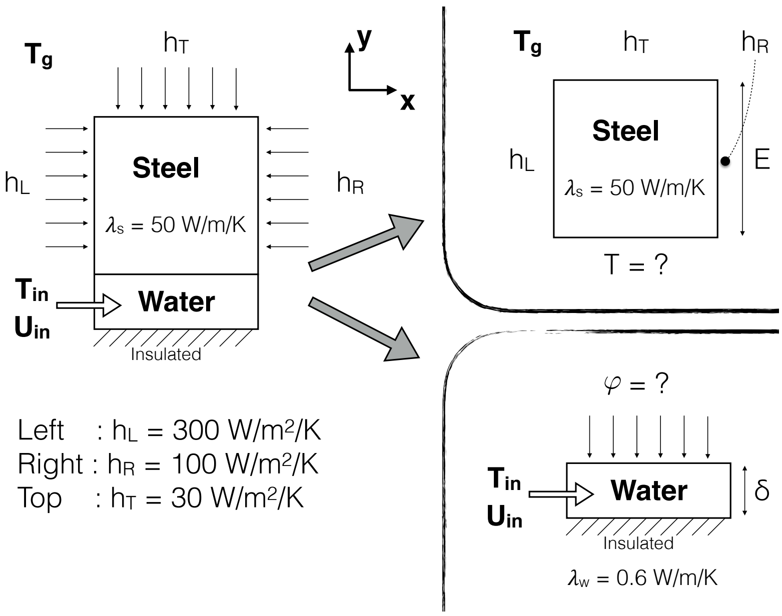

Figure 2: Simplified model of a single water-cooled rib. The dimensions of the rib and the water cooling channel are ExE and Exδ, respectively.

On the onde hand, the temperature in the rib is determined by the steady heat equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,$$

with the given set of boundary conditions. The thermal conductivity of steel is $\lambda_s = 50$ W/m/K. On the other hand, the water temperature is not homogeneous and is determined by the following equation for a parallel flow:

$$u(x, y)\frac{\partial T}{\partial x} = a_w \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right).$$

The water thermal conductivity is $\lambda_w = 0.6$ W/m/K and its thermal diffusivity appearing in the previous equation is $a_w = 1.0\,10^{-6}$ m^2/s. The channel length is identical to the rib's thickness, *i.e.* E, and its width is $\delta = 1$ mm. The flow in the channel is typically laminar and is given by a Poiseuille profile:

$$u(x, y) = 8U_{in}\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right).$$

- **Determine the bulk velocity U_{in} to cool the rib and prevent boiling of the water inside the channel: As a safety measure, water temperature is not to exceed 360 K.**
- **Estimate computationally the corresponding heat power that can be extracted.**
- **What happens in the case of flow reversal (water flowing from right to left with the same bulk velocity U_{in}) due to a wrong setting of the water pumping system?**

For the sake of simplicity, the coupled rib-channel problem should be split into two separate and independent setups as described in the right of Fig. 2 before any attempt to solve the whole problem directly (*not mandatory*).