Numerical Methods in Engineering Applications

<u>Session #4</u>
Elliptic PDEs (2)

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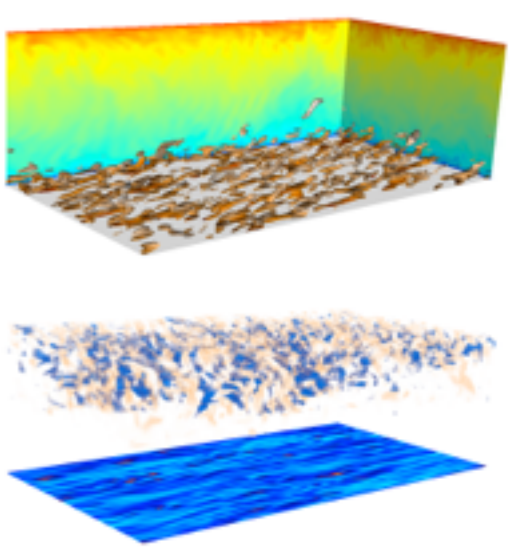
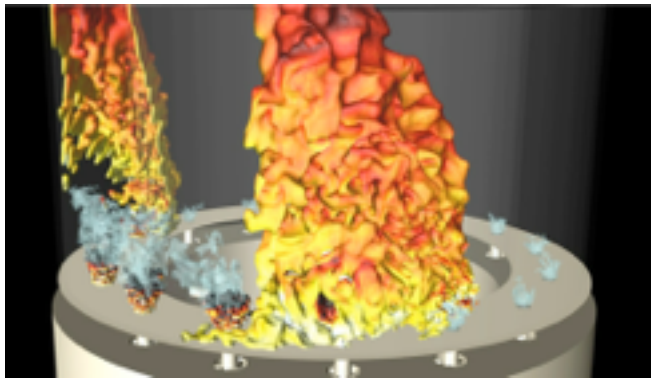
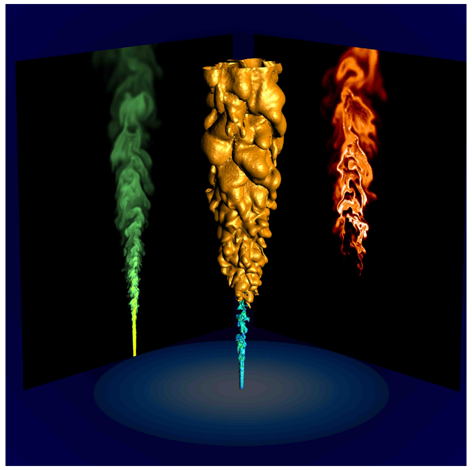
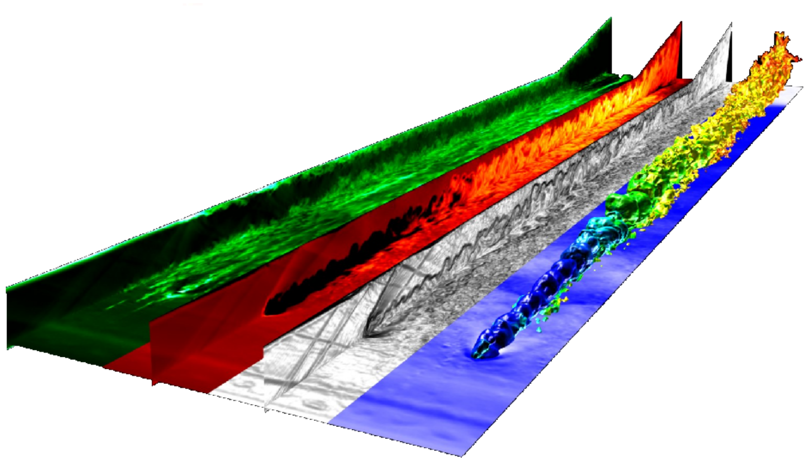

Course contents

● Theoretical lecture
■ Problem-solving workshop

I. Basics on numerical approximations

- Introduction and Finite Differences. ● ■ ■
- Numerical solution of ordinary differential equations. ● ■

II. Solving large linear equations systems: Applications to steady heat equation.

- Elliptic PDE 1. ● ■ ■
- Elliptic PDE 2. ● ■

III. Methods for unsteady advection/diffusion problems

- Hyperbolic and parabolic PDE: Explicit methods. ● ■
- Characterization of numerical errors. ● ■
- Hyperbolic and parabolic PDE: Implicit methods. ● ■

→ Projet #2 deadline: May, 18th

→ Projet #3 deadline: May, 25th

IV. Towards computational fluid dynamics

- Methodology in numerical computations. ■ ■
- Incompressible Flow equations. ● ■
- Semi-Implicit method for incompressible flows. ● ■
- Final project on incompressible flow. ■ ■

Today's contents

- Elliptic PDEs
  - SOR : Succesive OverRelaxation
  - Preconditioning
  - Conjugate Gradient method

- Methods comparison (Hirsch):

Number of iterations to reduce error by k order of magnitude

Jacobi	$\mathcal{O}(n^2)$
Gauss-Seidel	$\mathcal{O}(n^2)$
SOR	$\mathcal{O}(n^{3/2})$
Preconditioned Conjugate Gradient	$\mathcal{O}(n^{5/4})$
MultiGrid	$\mathcal{O}(n)$

n is the number of unknowns

Conjugate Gradient

- Algorithm

Compute $r_0 := b - Ax_0$, $p_0 := r_0$.
For $j = 0, 1, \ldots,$ *until convergence Do:*

$$\alpha_j := (r_j, r_j)/(Ap_j, p_j)$$
$$x_{j+1} := x_j + \alpha_j p_j$$
$$r_{j+1} := r_j - \alpha_j Ap_j$$
$$\beta_j := (r_{j+1}, r_{j+1})/(r_j, r_j)$$
$$p_{j+1} := r_{j+1} + \beta_j p_j$$

EndDo

Preconditioned Conjugate Gradient

- With the preconditioning matrix M

Compute $r_0 := b - Ax_0$, $z_0 = M^{-1}r_0$, *and* $p_0 := z_0$
For $j = 0, 1, \ldots,$ *until convergence Do:*

$$\alpha_j := (r_j, z_j)/(Ap_j, p_j)$$
$$x_{j+1} := x_j + \alpha_j p_j$$
$$r_{j+1} := r_j - \alpha_j Ap_j$$
$$z_{j+1} := M^{-1} r_{j+1}$$
$$\beta_j := (r_{j+1}, z_{j+1})/(r_j, z_j)$$
$$p_{j+1} := z_{j+1} + \beta_j p_j$$

EndDo