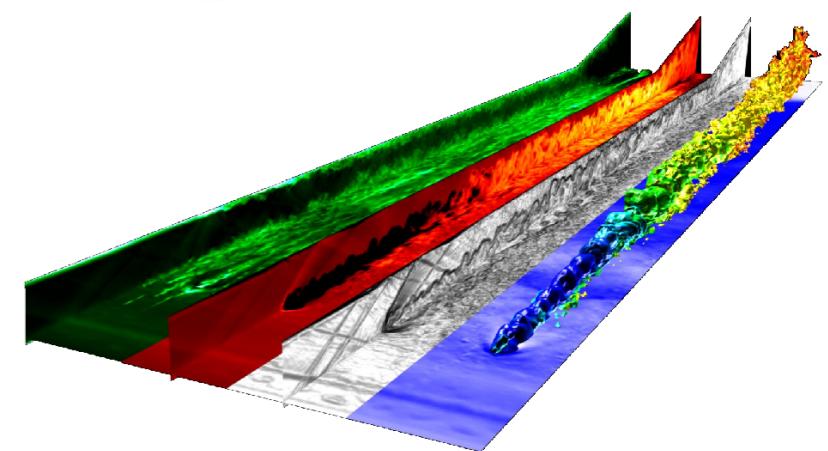
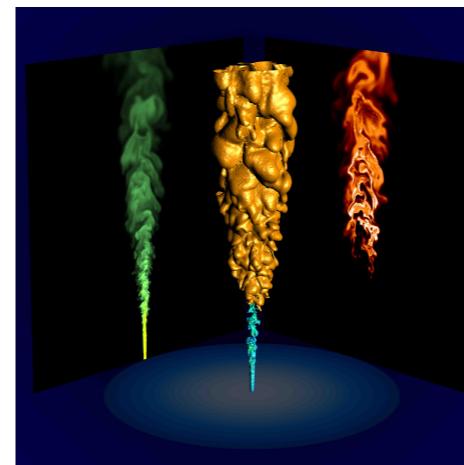
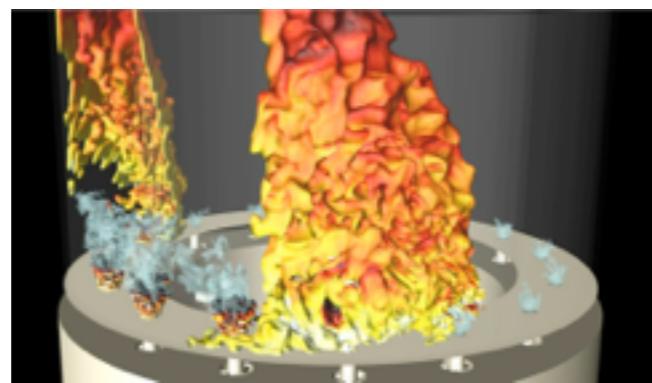
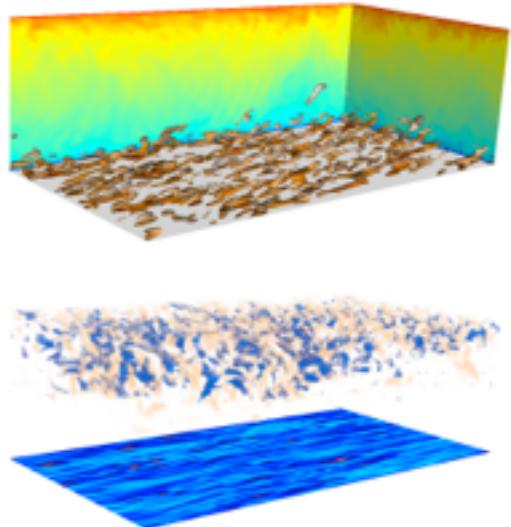


Numerical Methods in Engineering Applications

Session #1 Finite Differences

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Aymeric.Vie@centralesupelec.fr



Course contents

- Theoretical lecture
- Problem-solving workshop

I. Basics on numerical approximations

- Introduction and Finite Differences.
- Numerical solution of ordinary differential equations.



II. Solving large linear equations systems: Applications to steady heat equation.

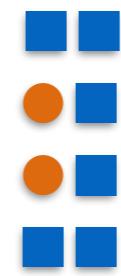
- Elliptic PDE 1.
- Elliptic PDE 2.

III. Methods for unsteady advection/diffusion problems

- Hyperbolic and parabolic PDE: Explicit methods.
- Characterization of numerical errors.
- Hyperbolic and parabolic PDE: Implicit methods.

IV. Towards computational fluid dynamics

- Methodology in numerical computations.
- Incompressible Flow equations.
- Semi-Implicit method for incompressible flows.
- Final project on incompressible flow.

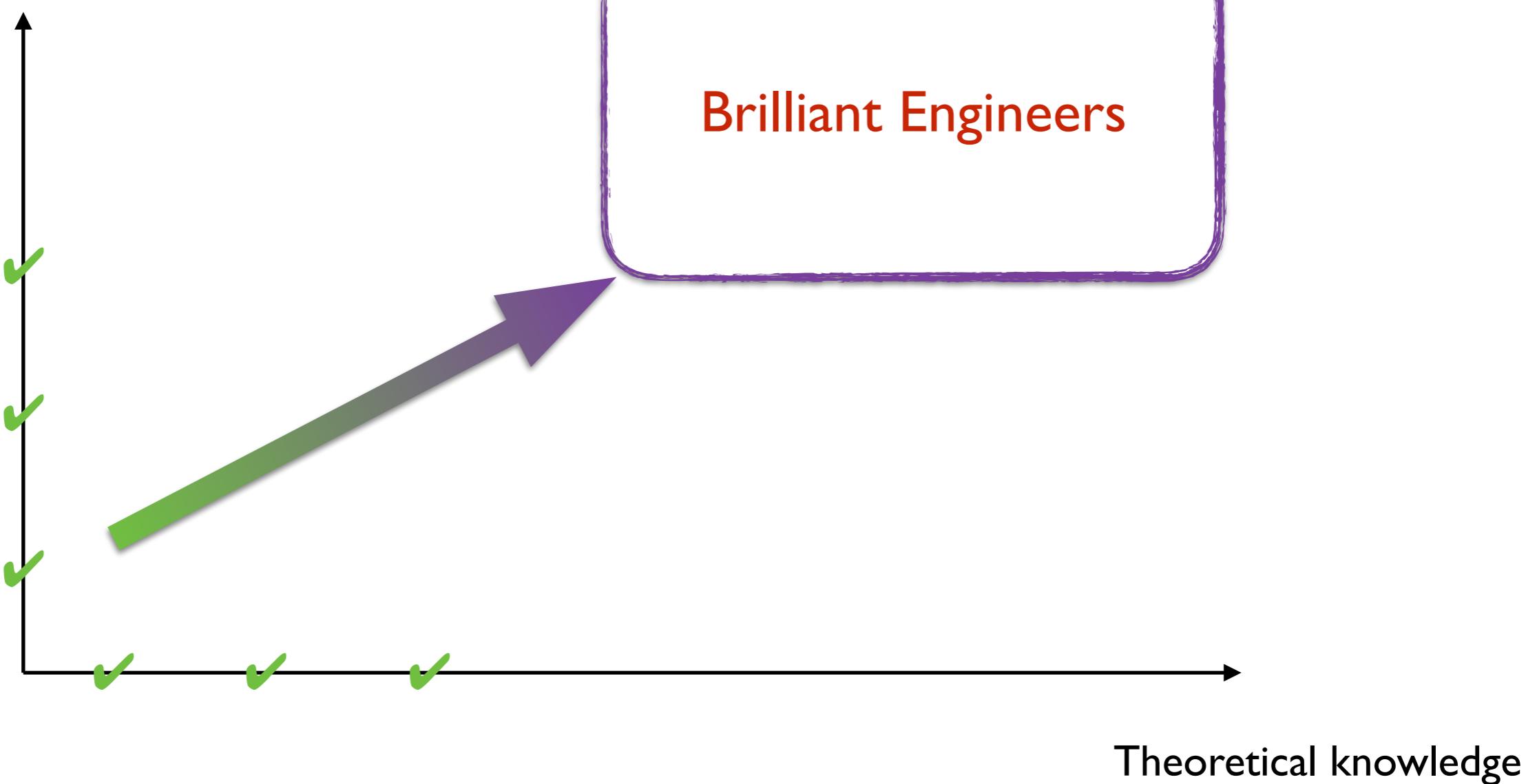


→ Project #4

Pasteur Quadrant

Practical Interest

- Programming
- Numerically sound and accurate
- Physical Analysis



Evaluation

- 1 small project to hand out within a week
- 1 comprehensive mid-term project
- 1 small project to hand out within a week
- 1 comprehensive final project

Send slides and sources at each deadline

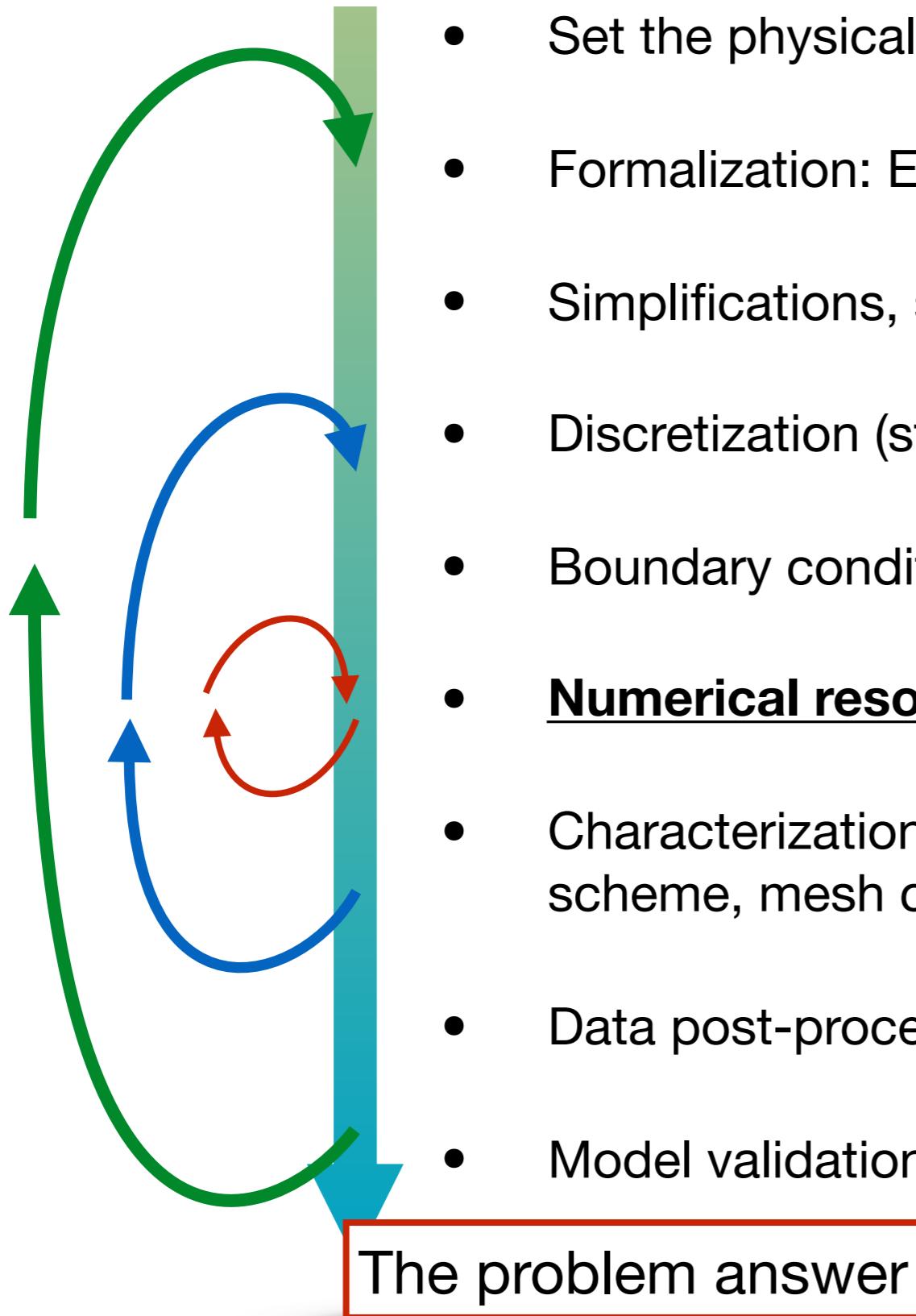


Final deadline: send updated slides + final project

ECP+R

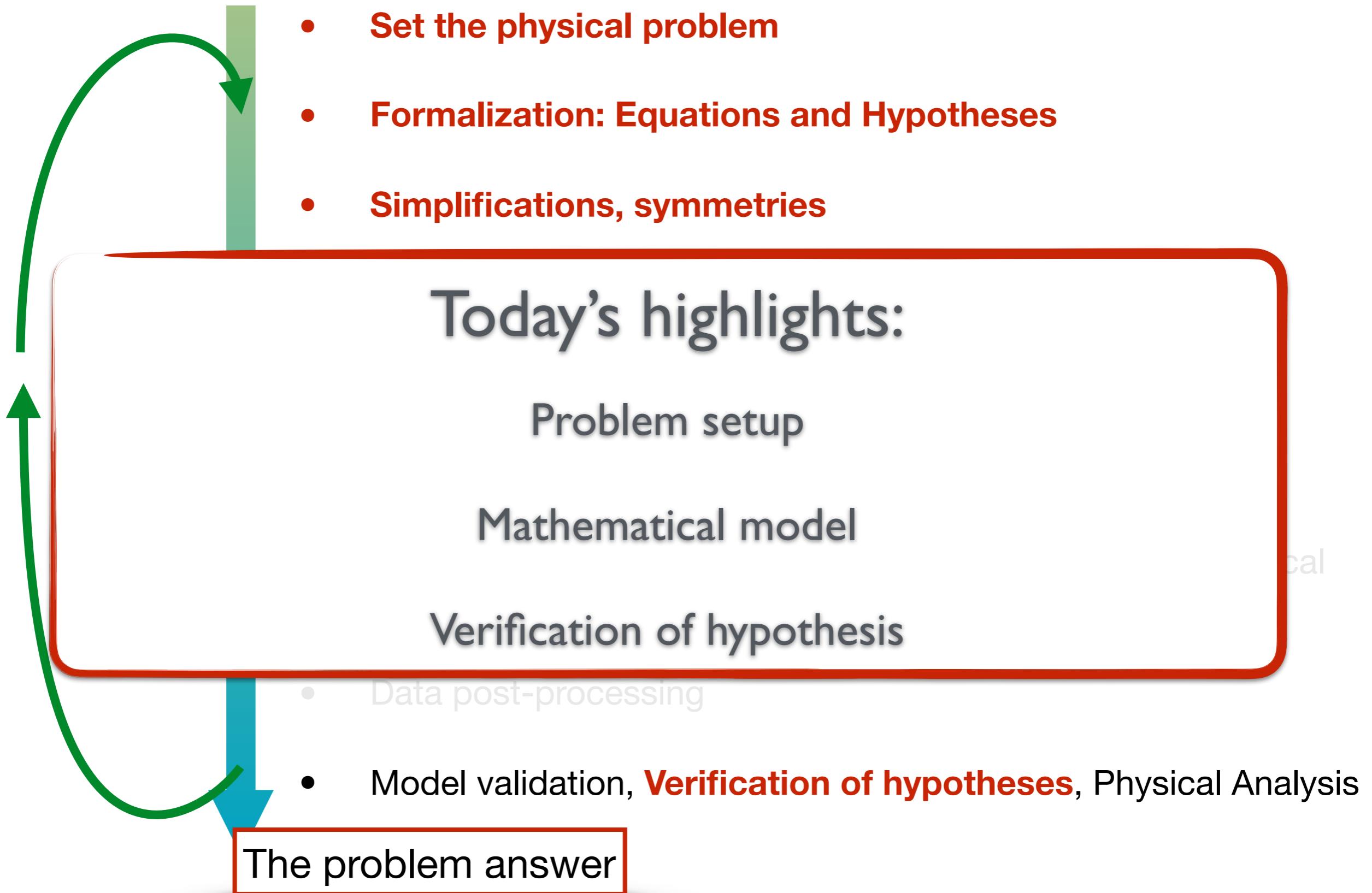
- 5 sessions
- Evaluation on the first 3 projects

Methodology



- Set the physical problem
- Formalization: Equations and Hypotheses
- Simplifications, symmetries
- Discretization (structures, unstructured mesh)
- Boundary conditions. Initial conditions.
- **Numerical resolution**
- Characterization of numerical error (accuracy of the numerical scheme, mesh convergence/adaptation)
- Data post-processing
- Model validation, Verification of hypotheses, Physical Analysis

Methodology



Scalability

Numerics

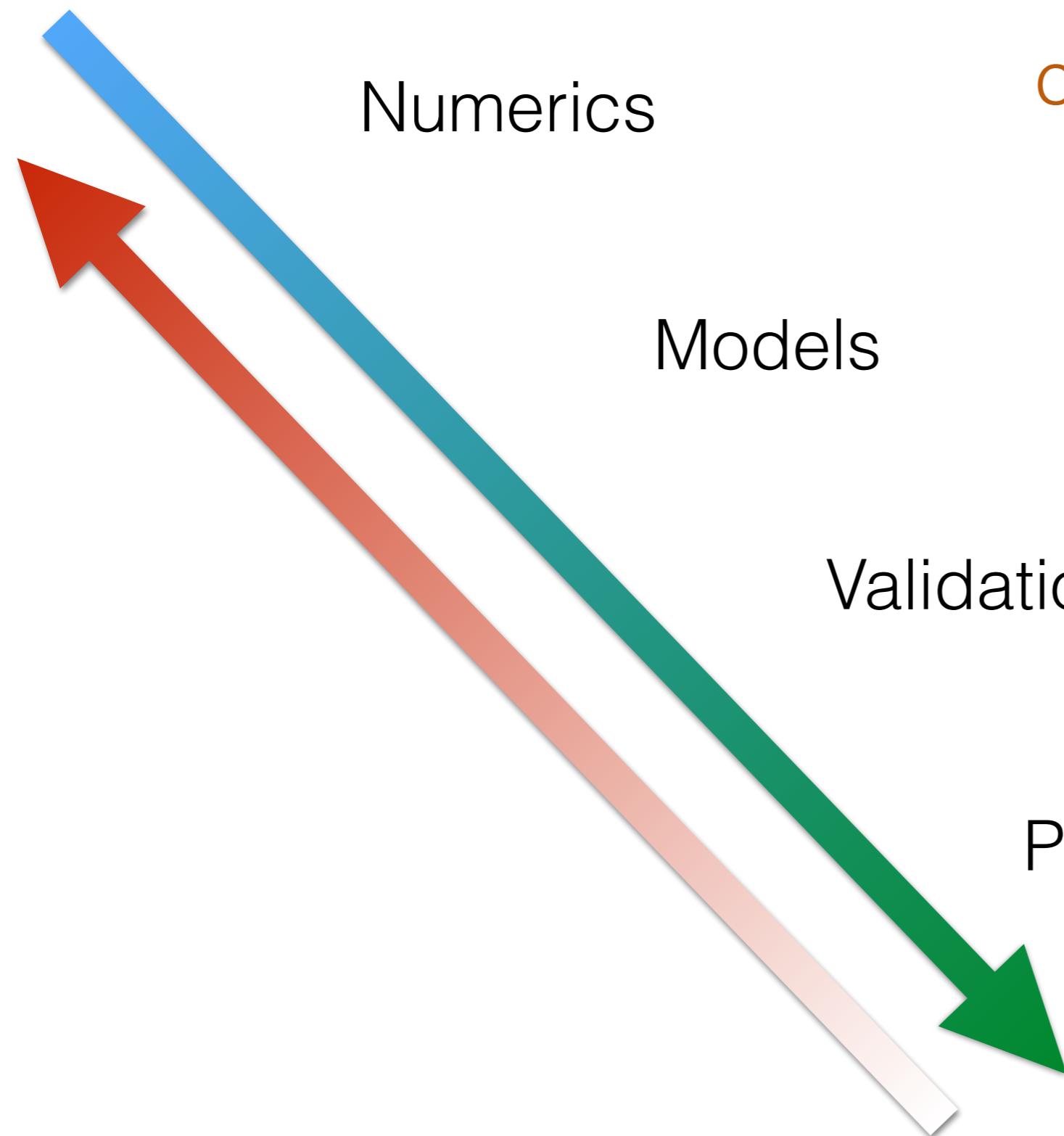
Models

Validation

Physical analysis

Low-order models

A hierarchical view of
developments for
computational physics



Study Case: Light-Round

PhD Thesis: Maxime Philip

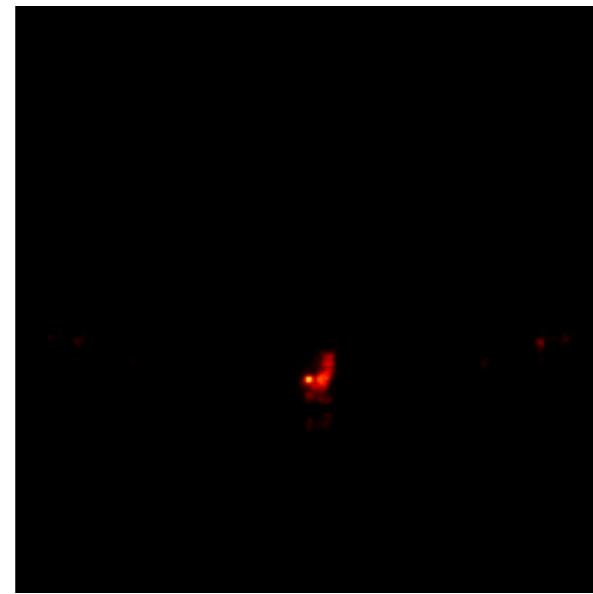
- **L'allumage est une phase critique**
 - Doit être réalisée de façon systématique et sûre
 - Doit conduire à une flamme stabilisée en évitant de générer des oscillations importantes de pression
 - L'extinction partielle doit être évitée
- **Le phénomène peut être décomposé en 4 phases**



Le rallumage en altitude est critique pour les systèmes aéronautiques



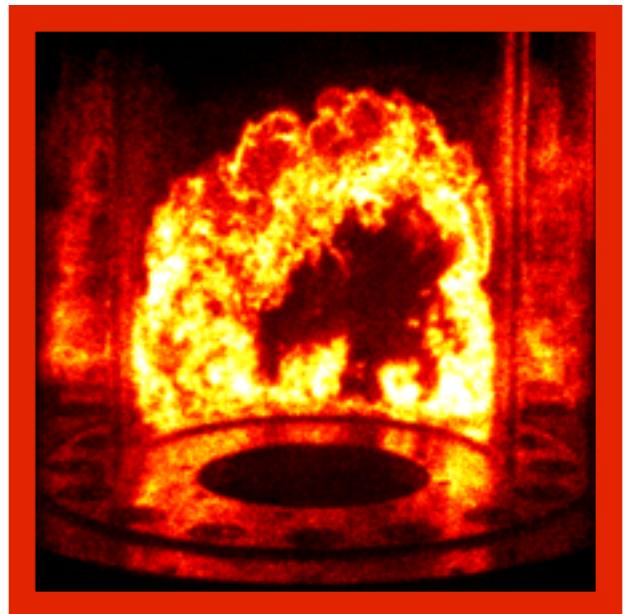
Formation d'un noyau initial (plasma + flamme)



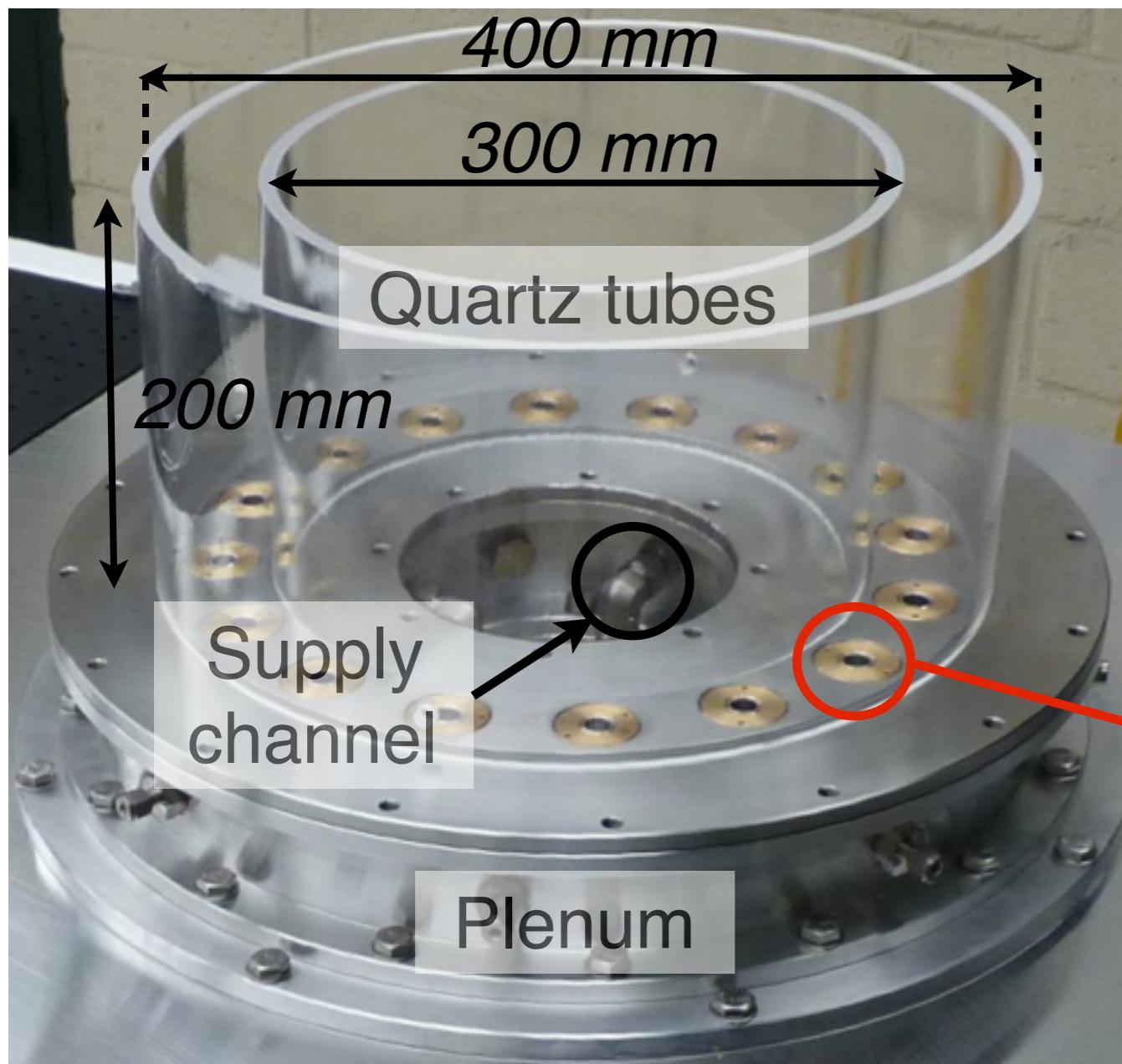
Expansion de la flamme



Allumage du premier brûleur

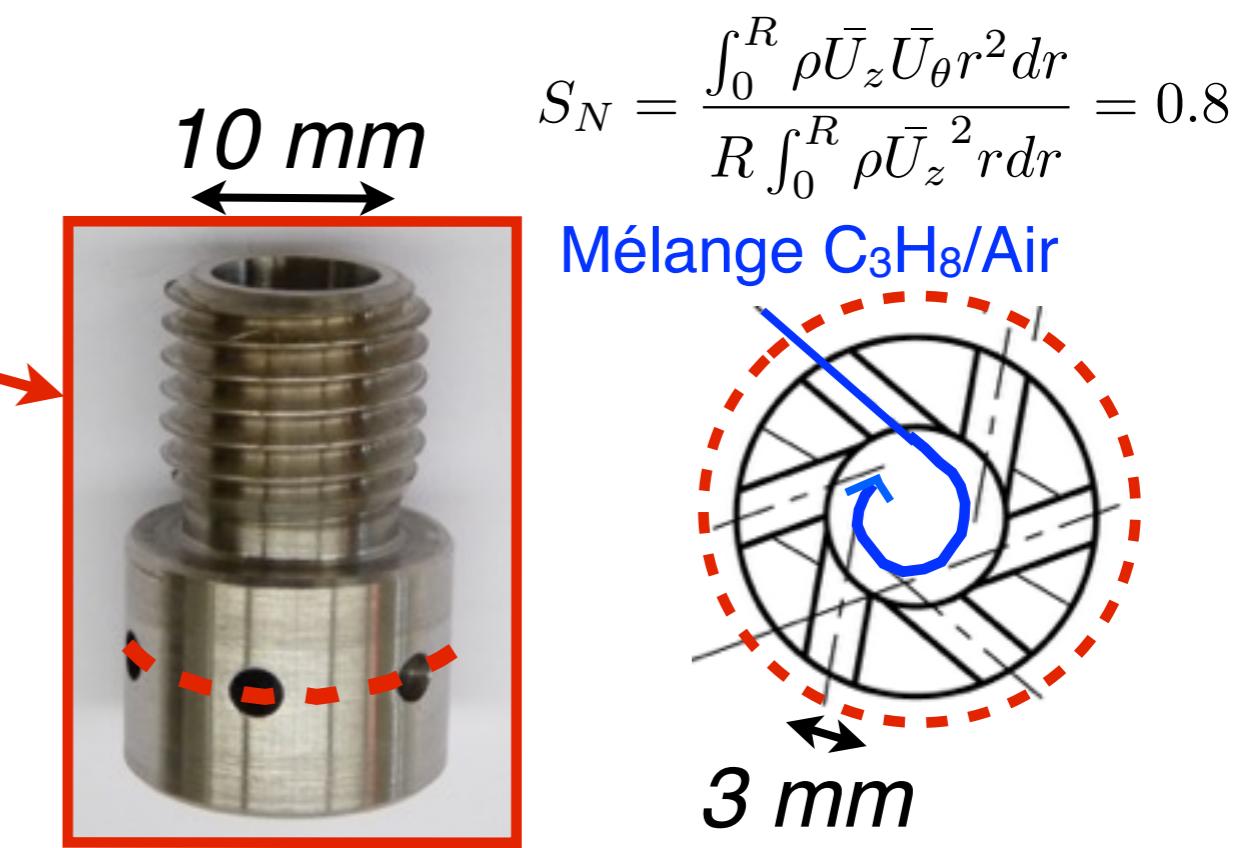


MICCA est une chambre annulaire atmosphérique transparente



La chambre annulaire MICCA

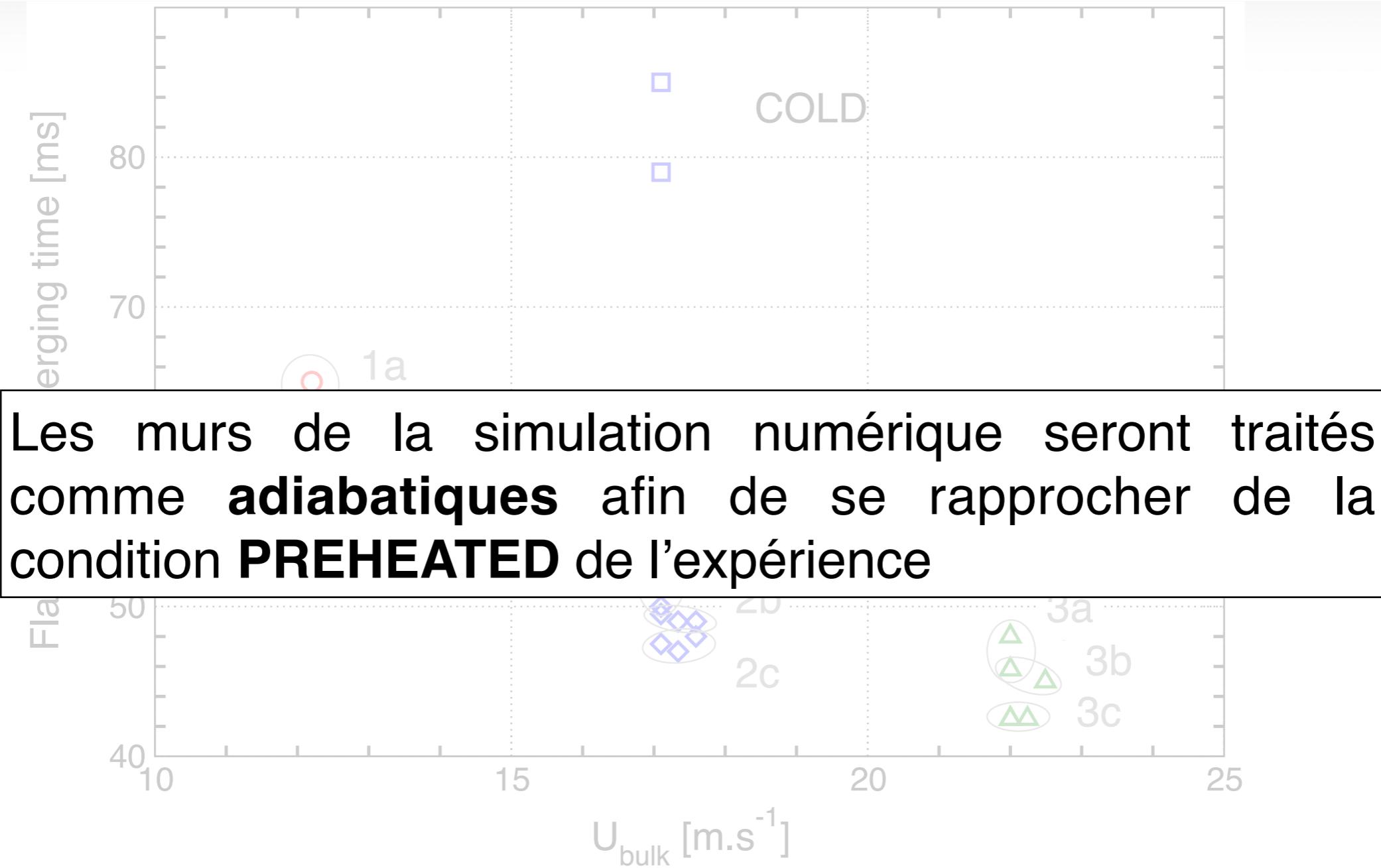
- C₃H₈/Air parfaitement prémélangé
- 16 injecteurs tourbillonneurs
- Dimensionnellement similaire aux chambres de combustion d'hélicoptère
- $P = 100 \text{ kW}$ à pleine puissance



Temps de fusion des fronts de flamme



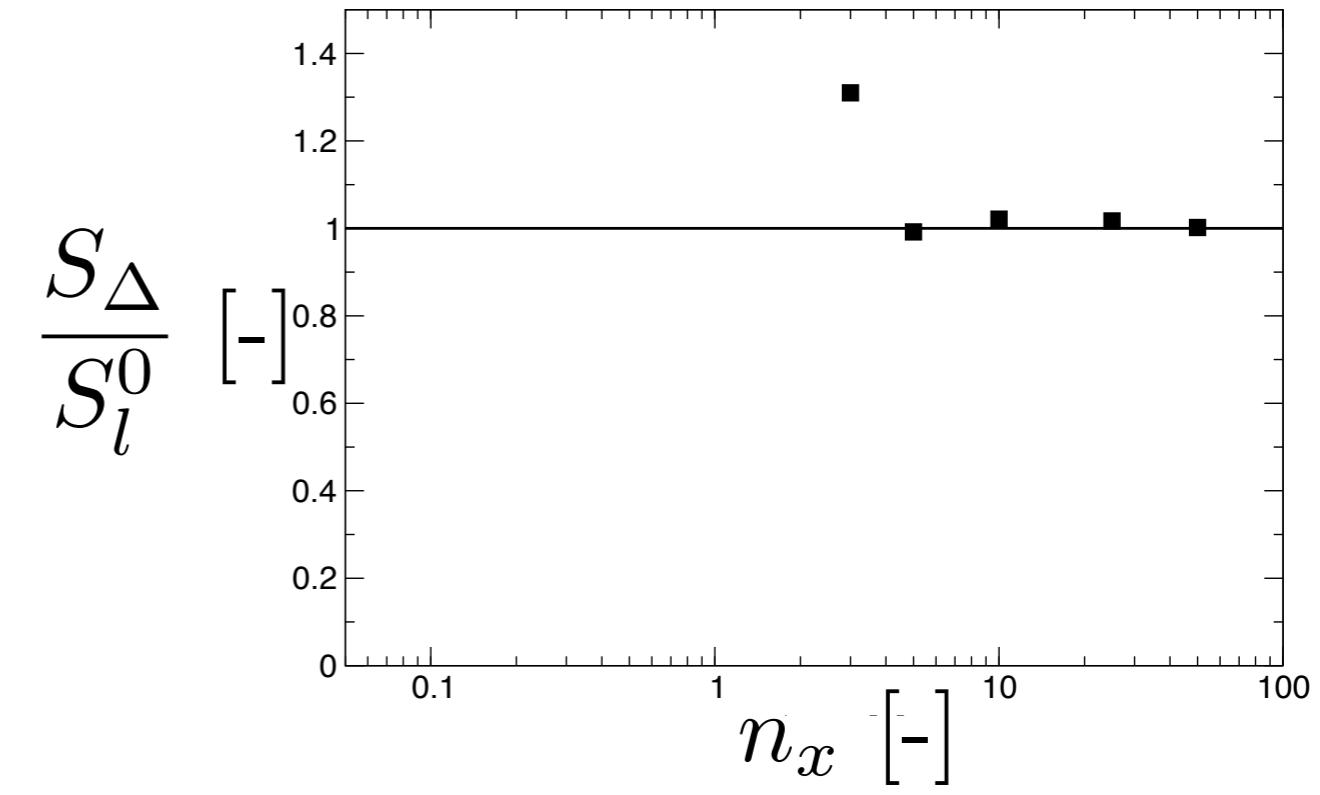
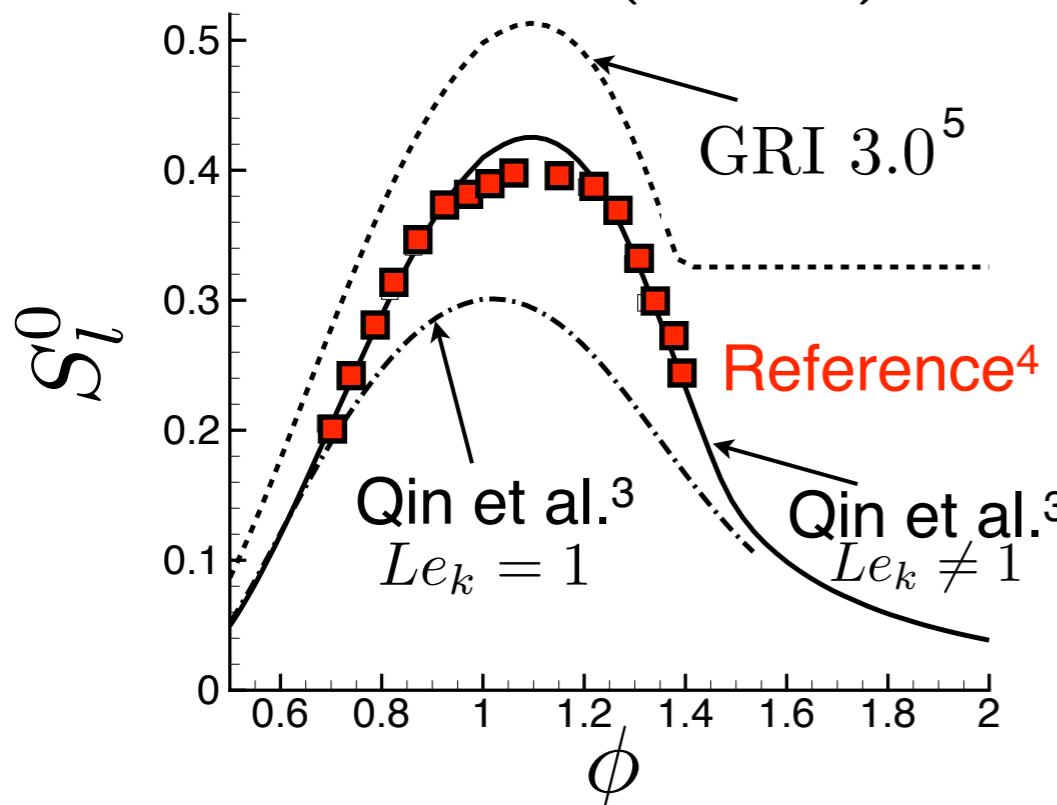
11



- Le temps d'allumage varie inversement avec la vitesse débitante
- Le temps d'allumage est moins rapide avec des parois froides que préchauffées

- Le modèle de combustion est le modèle «**Filtered Tabulated Chemistry for LES**» développé à EM2C (F-TACLES^{1,2})
- Une collection de flammellestes prémélangées est générée en utilisant un solveur 1D
- Les flammettes sont filtrées pour être résolues sur le maillage LES
- La chimie est incluse dans la simulation en utilisant une table chimique

$$\tilde{\psi} = \psi^{tab} \left(\tilde{Y}_c, \Delta \right) \quad \text{avec} \quad Y_c = Y_{CO_2}$$



¹B. Fiorina et al., *Combustion and Flame* (2010)

²R. Vicquelin et al., *Proceedings of the Combustion Institute* (2011)

³Z. Qin et al., *Proceedings of the Combustion Institute* (2000)

⁴C. M. Vagelopoulos et al., *Proceedings of the Combustion Institute* (1998)

⁵G. P. Smith et al., http://www.me.berkeley.edu/gri_mech (1999)

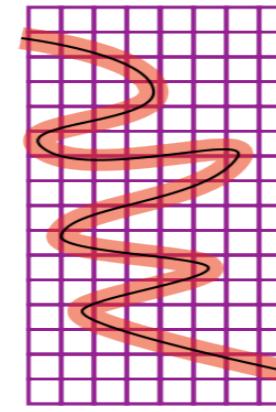
- Le modèle de combustion est le modèle «**Filtered Tabulated Chemistry for LES**» développé à EM2C (F-TACLES^{1,2})

- Une variable de progrès est transportée dans le solveur LES : $c = \frac{Y_{CO_2}}{Y_{CO_2}^{eq}}$

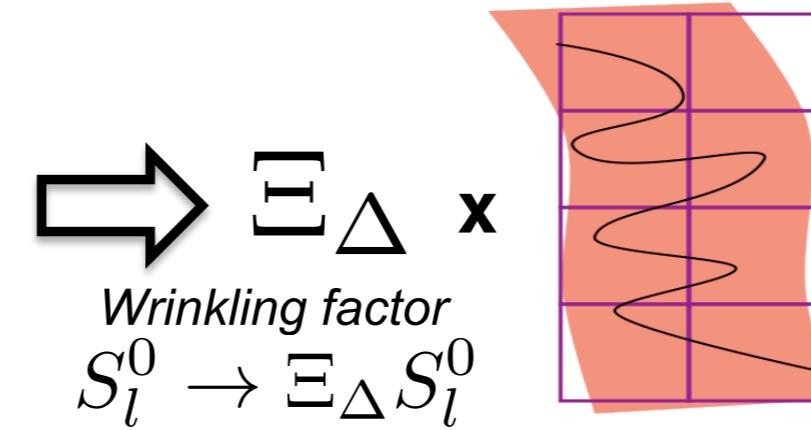
$$\begin{aligned} \frac{\partial \bar{\rho} \tilde{c}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{u}_i \tilde{c}) &= \frac{\partial}{\partial x_i} \left(\Xi_\Delta \alpha_c [\tilde{c}, \Delta] \bar{\rho} D \frac{\partial \tilde{c}}{\partial x_i} \right) \\ &\quad + \Xi_\Delta \bar{\dot{\Omega}}_c [\tilde{c}, \Delta] + \Xi_\Delta \bar{\dot{\omega}}_c [\tilde{c}, \Delta] \end{aligned}$$

- Modèle de sous-maille³

Resolved wrinkling



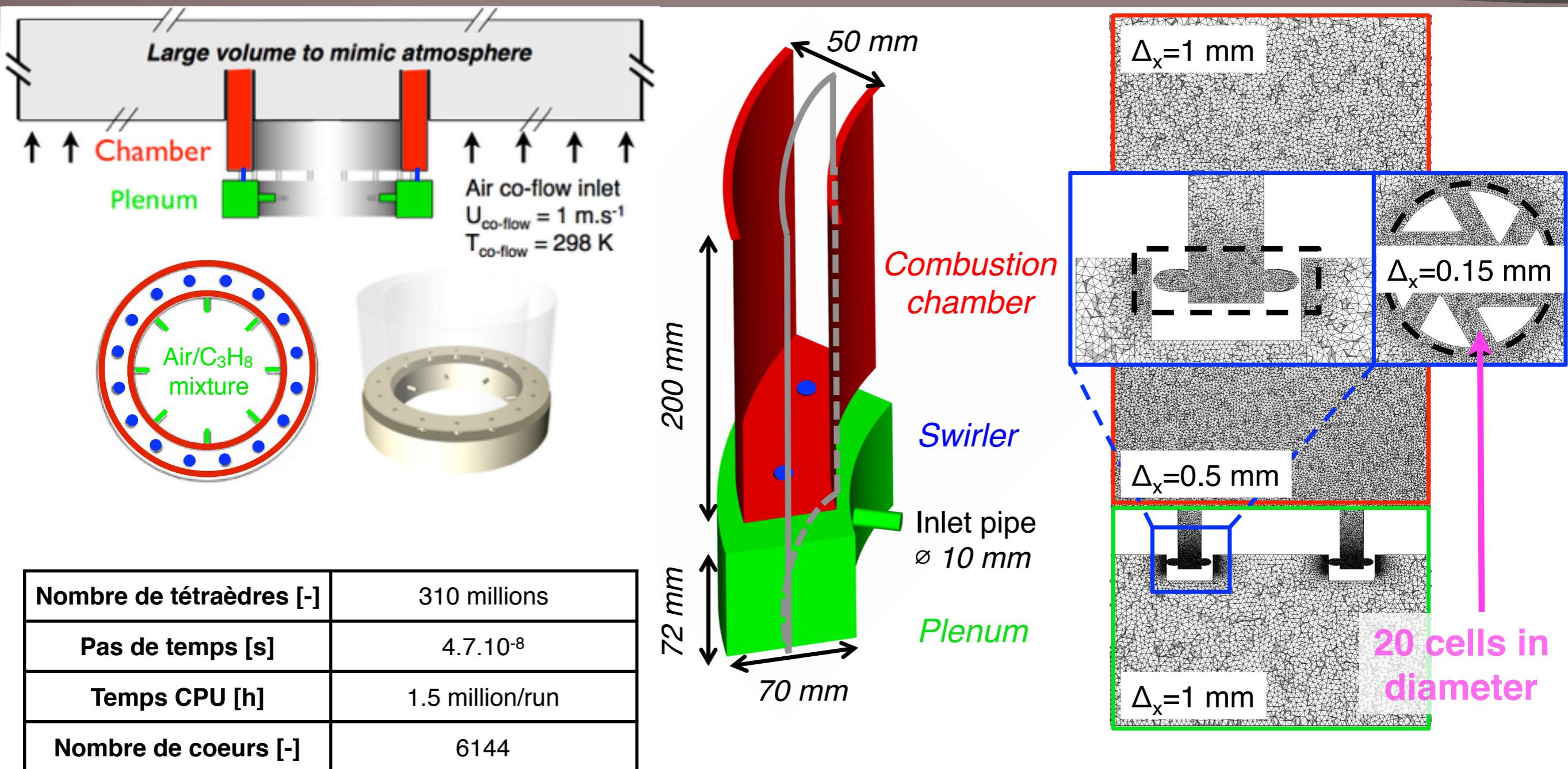
Subgrid wrinkling



¹B. Fiorina et al., A filtered tabulated chemistry model for LES of premixed combustion (2010), *Combustion and Flame*

²R. Vicquelin et al., Coupling tabulated chemistry with compressible CFD solvers (2011), *Proceedings of the Combustion Institute*

³F. Charlette et al., A power-law flame wrinkling model for LES of premixed turbulent combustion (2002), *Combustion and Flame*

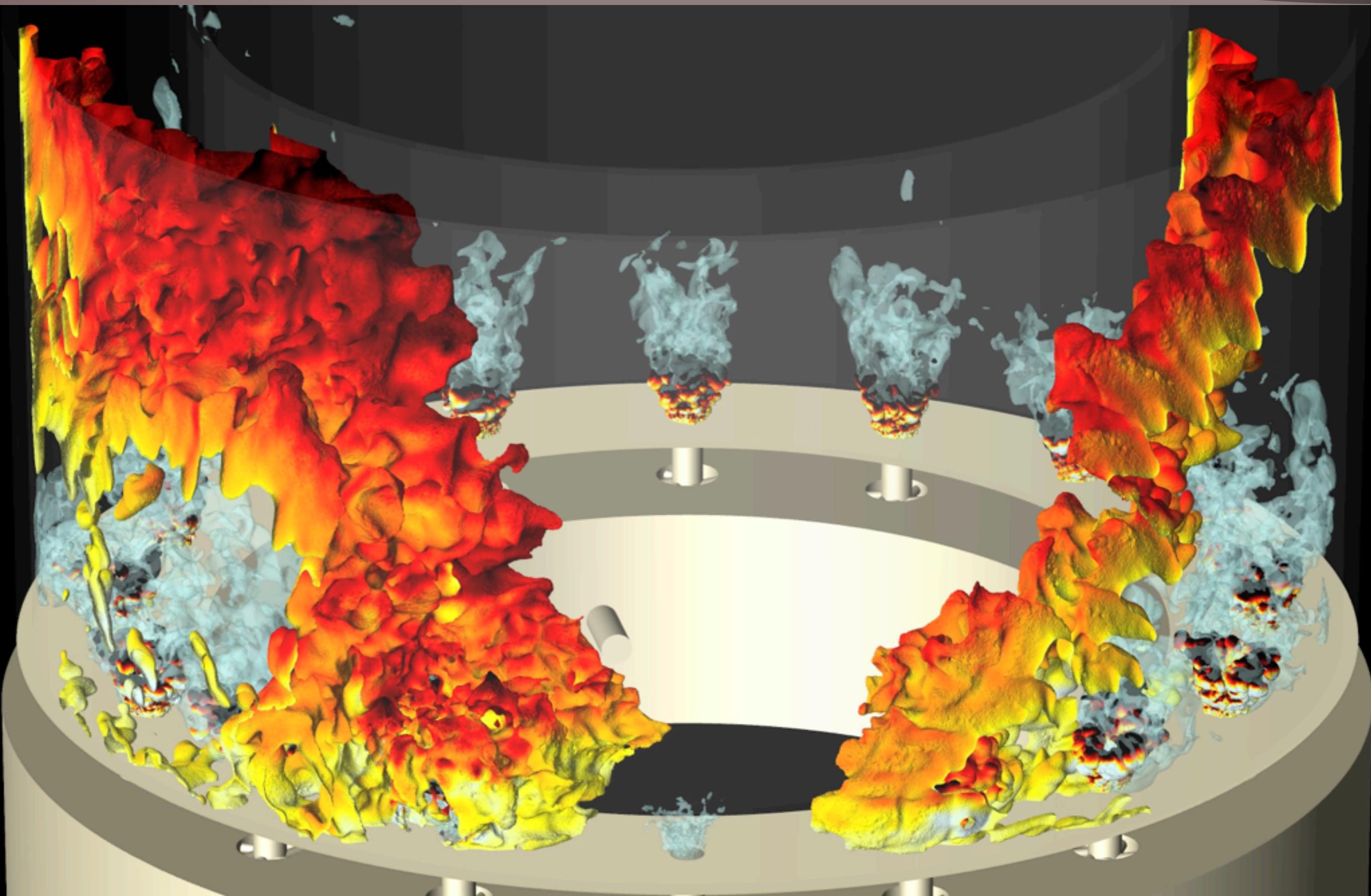


La chambre complète (16 injecteurs) est simulée

Résultats numériques

Point d'opération	#2	#3
U_{bulk} (m.s ⁻¹)	17.1	22.0

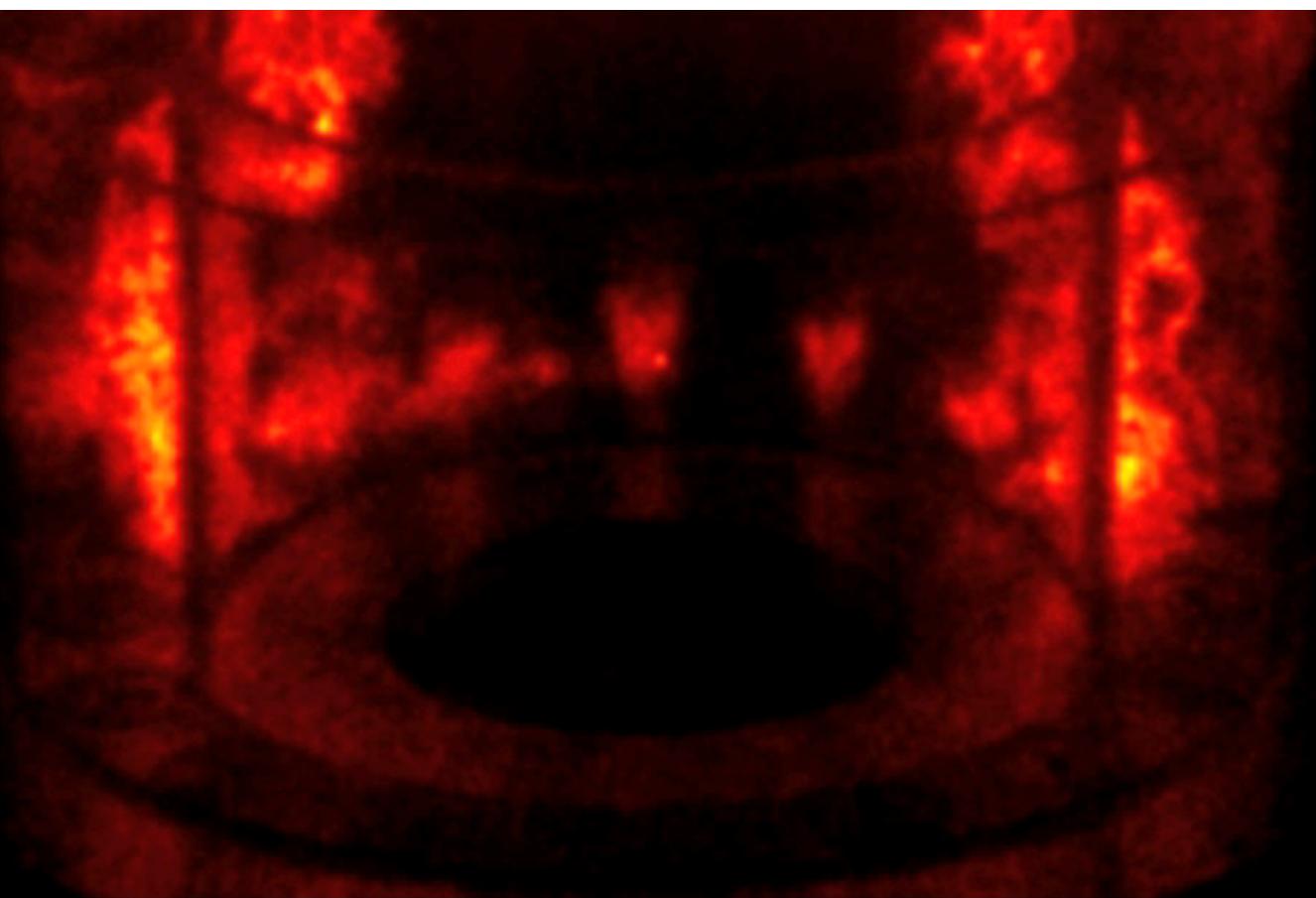
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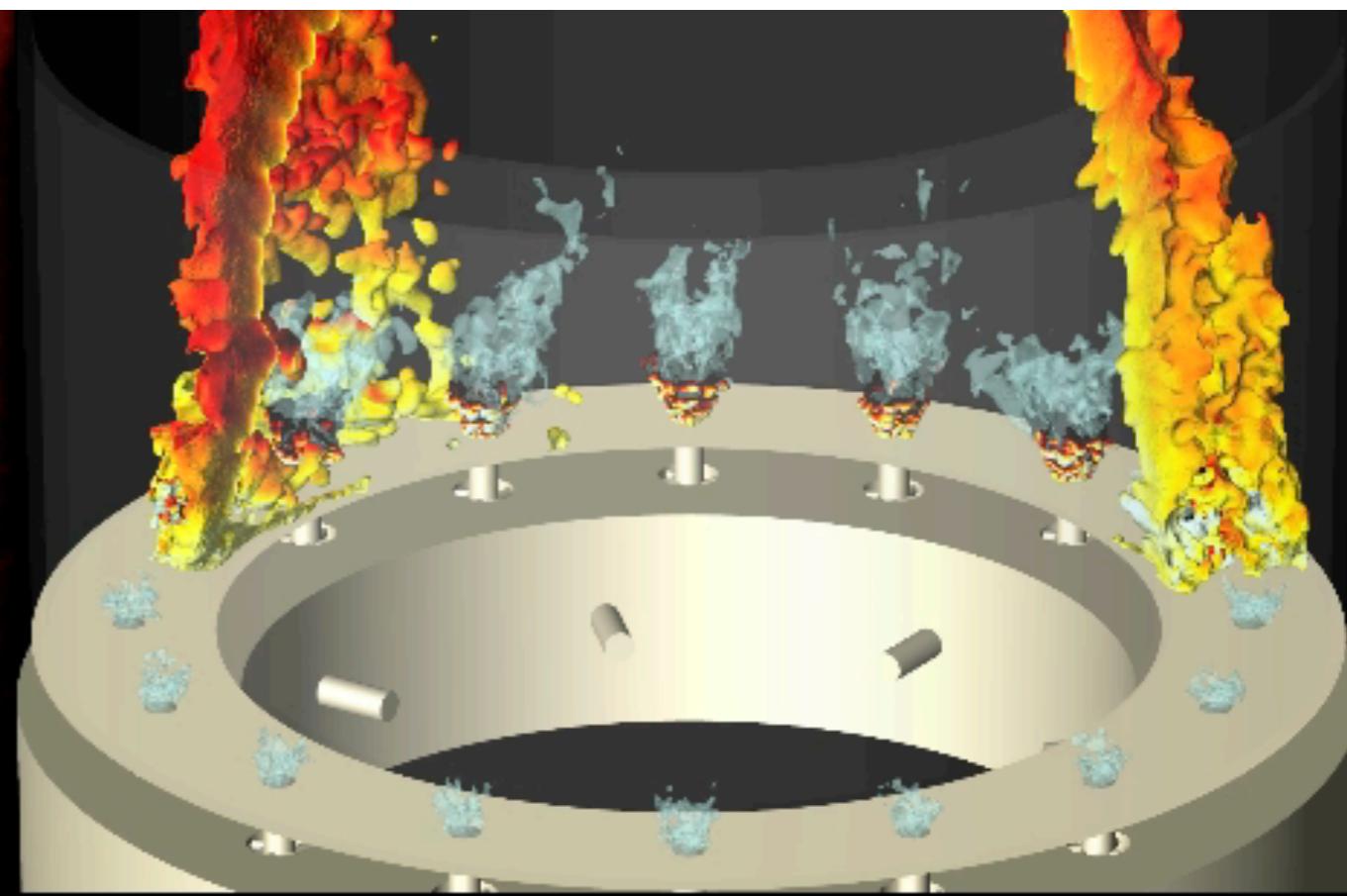
Point d'opération	#2	#3
U _{bulk} (m.s ⁻¹)	17.1	22.0

Comparaison qualitative

Expérience



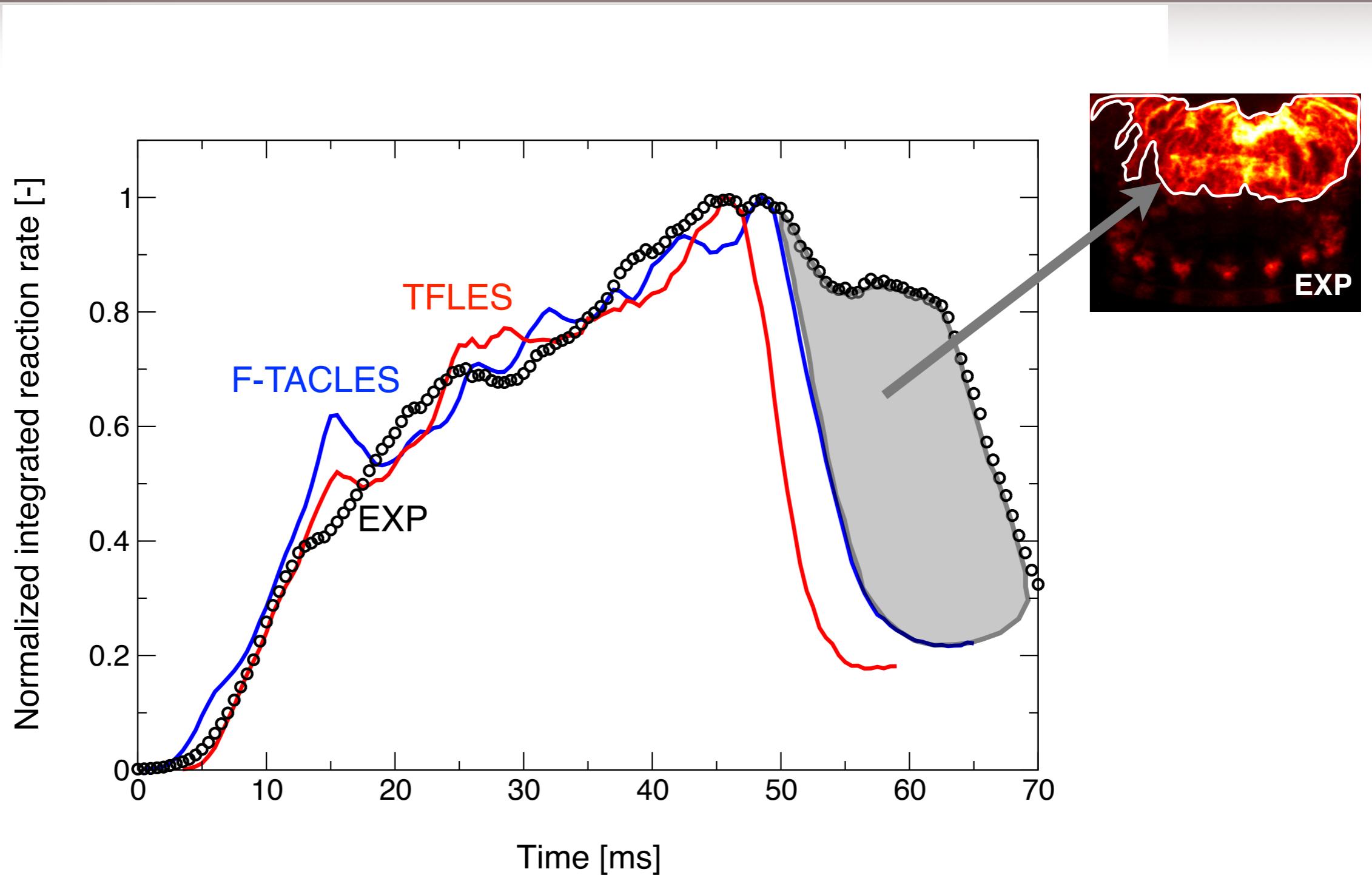
LES



Time = 21.2 ms

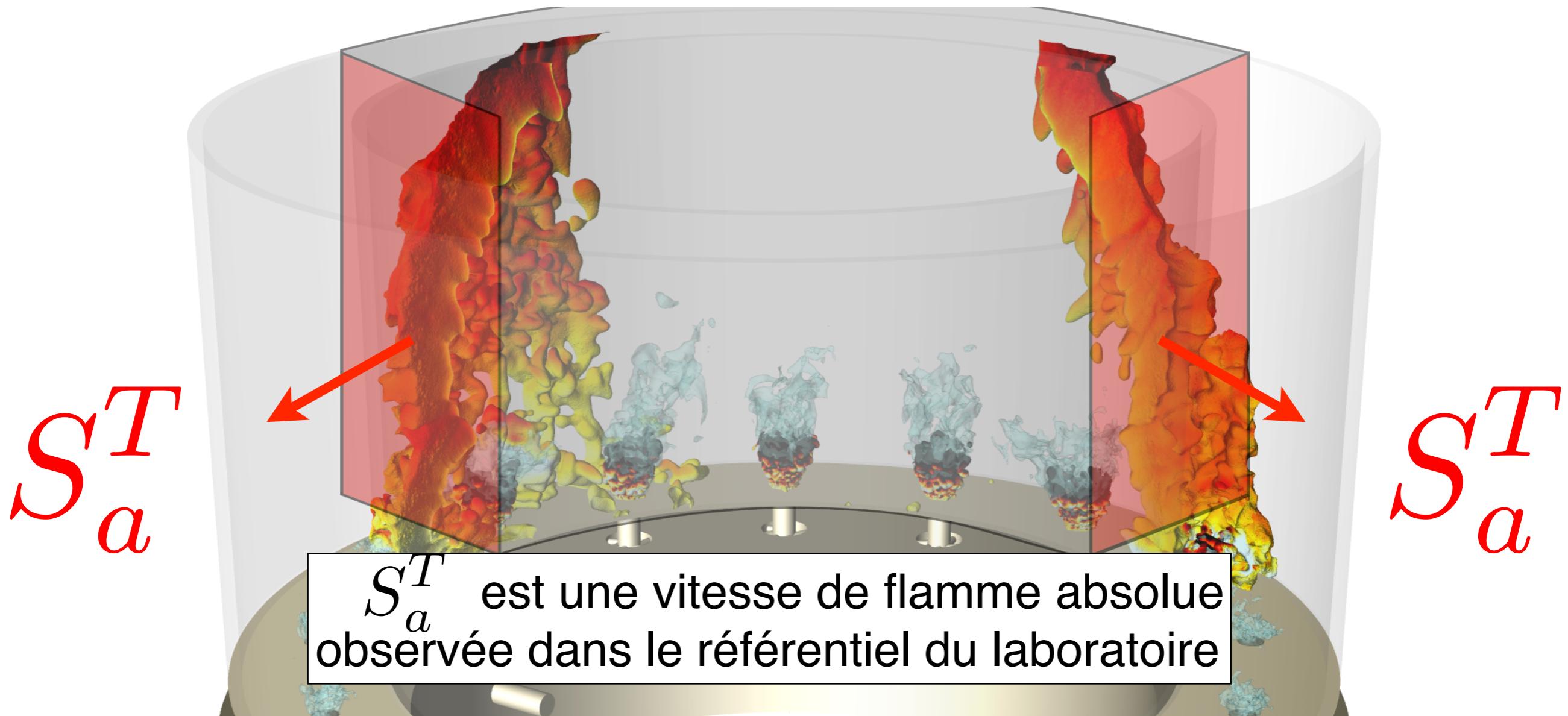
Emission directe de lumière capturée par une caméra intensifiée haute-cadence (en fausses couleurs)

La flamme est mise en évidence par une isosurface de la variable de progrès $c=0.9$ colorée par la vitesse axiale. Les iso-surfaces bleues correspondent au champ de vitesse $U = 25 \text{ m.s}^{-1}$



- Les simulations et l'expérience montrent un comportement similaire sur l'ensemble du processus d'allumage circulaire

- Les fronts sont assimilés à deux fronts verticaux se déplaçant à S_a^T

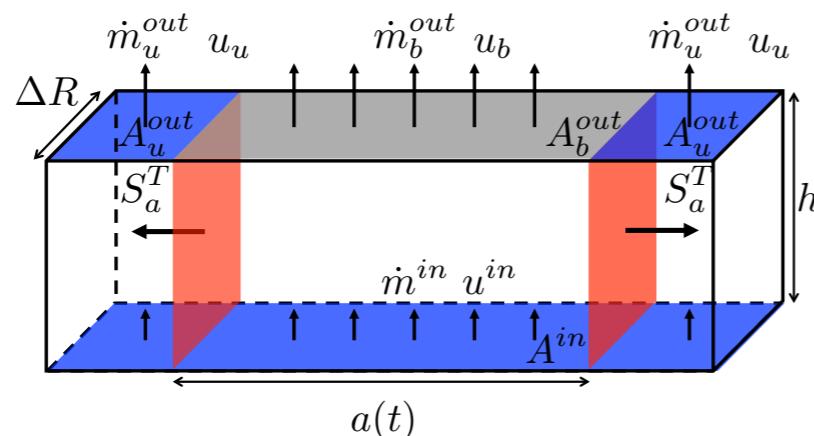


- Quels sont les phénomènes qui contrôlent la vitesse de la flamme sur la phase (III) ?

- Modèle flamme moyenne

	RESOLU (DNS)	FILTRE (LES)	TURBULENT (MOYENNE)
Vitesse consommation S_c [m.s ⁻¹]		$S_c \simeq S_l^0$	$S_c^\Delta = \Xi_\Delta S_l^0$

- Le bilan de masse fait intervenir les débits



Bilan sur la chambre complète :

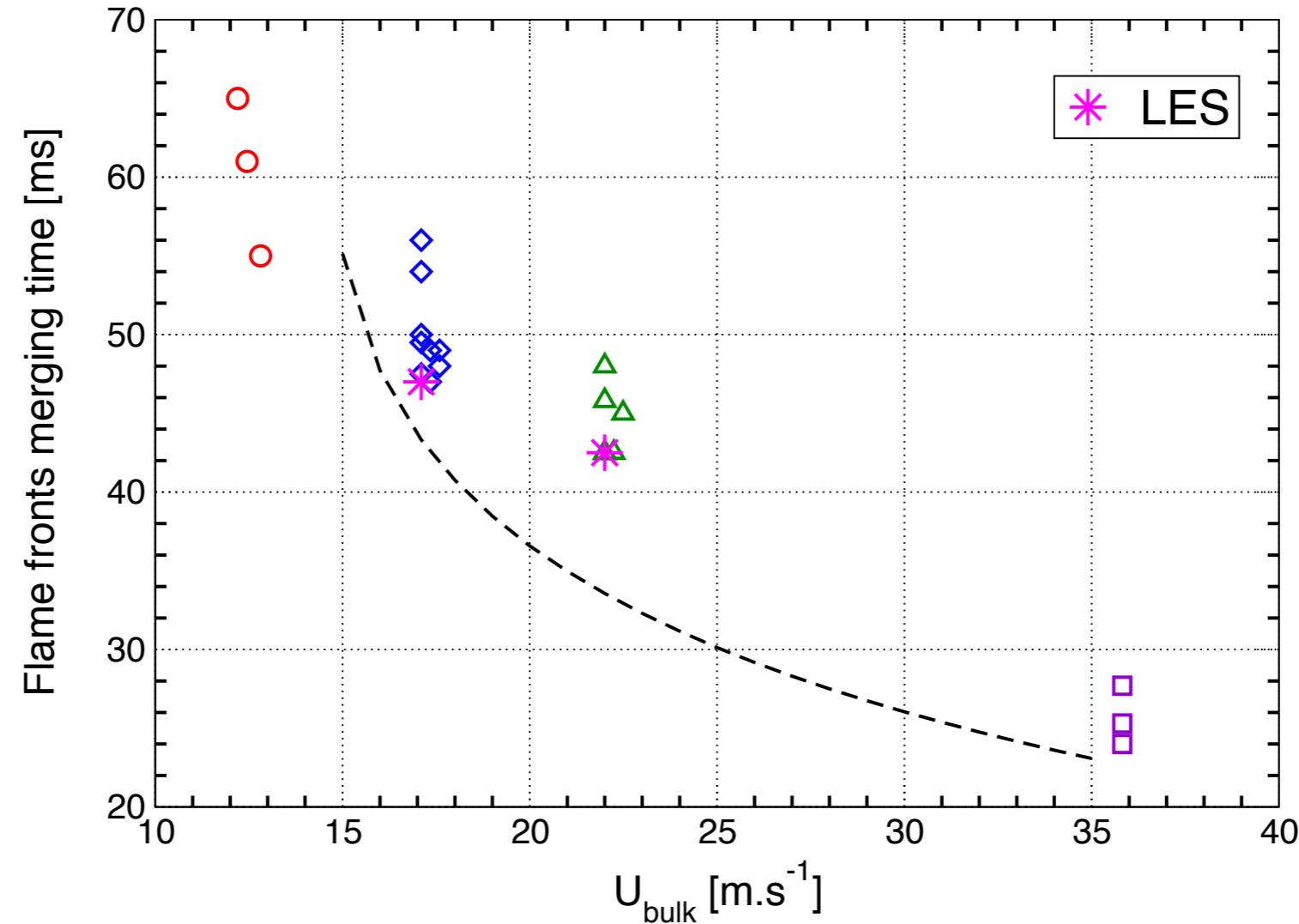
$$\begin{aligned} \dot{m}^{in} + (\rho_u - \rho_b) S_a^T A_0 \\ = \rho_u u_u A_u + \rho_b u_b A_b \end{aligned}$$

- Bilan de quantité de mouvement sur la chambre complète

$$\int_{V_{cc}} \frac{\partial}{\partial t} \rho v dV + \int_{A_{cc}} \rho v (v \cdot n) dA = - \int_{A_{cc}} p n dA + \int_{A_{cc}} \tau \cdot n dA + \int_{V_{cc}} \rho g dV$$

- **Prédiction du temps d'allumage par le modèle**

$$\tau_{merging} \sim \frac{\pi R_m}{S_a^T(t_0)}$$



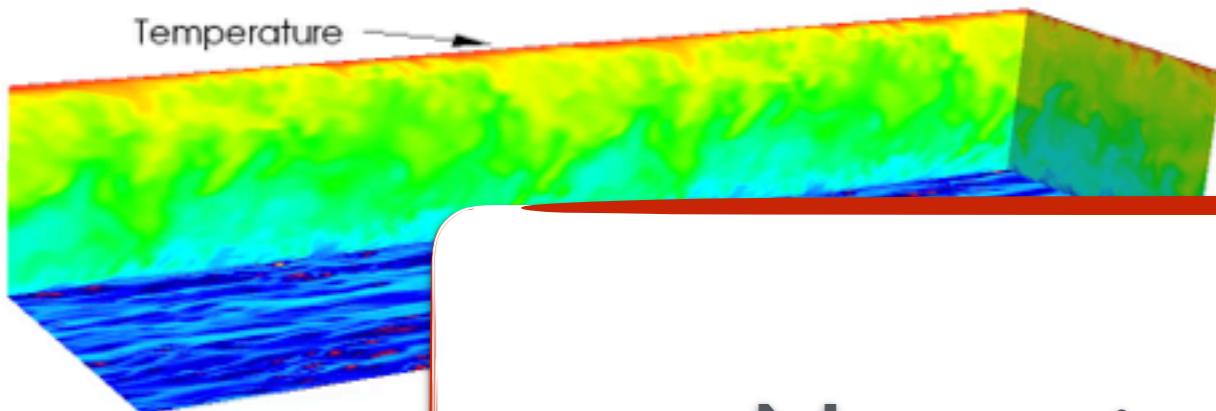
- **Le modèle montre des ordres de grandeur globalement satisfaisants**
- **La tendance expérimentale est retrouvée**

**Study Case:
Radiation Heat Transfer in
Turbulent Boundary Layers**

PhD Thesis: Yufang Zhang

Effects of radiation in Turbulent Boundary layers

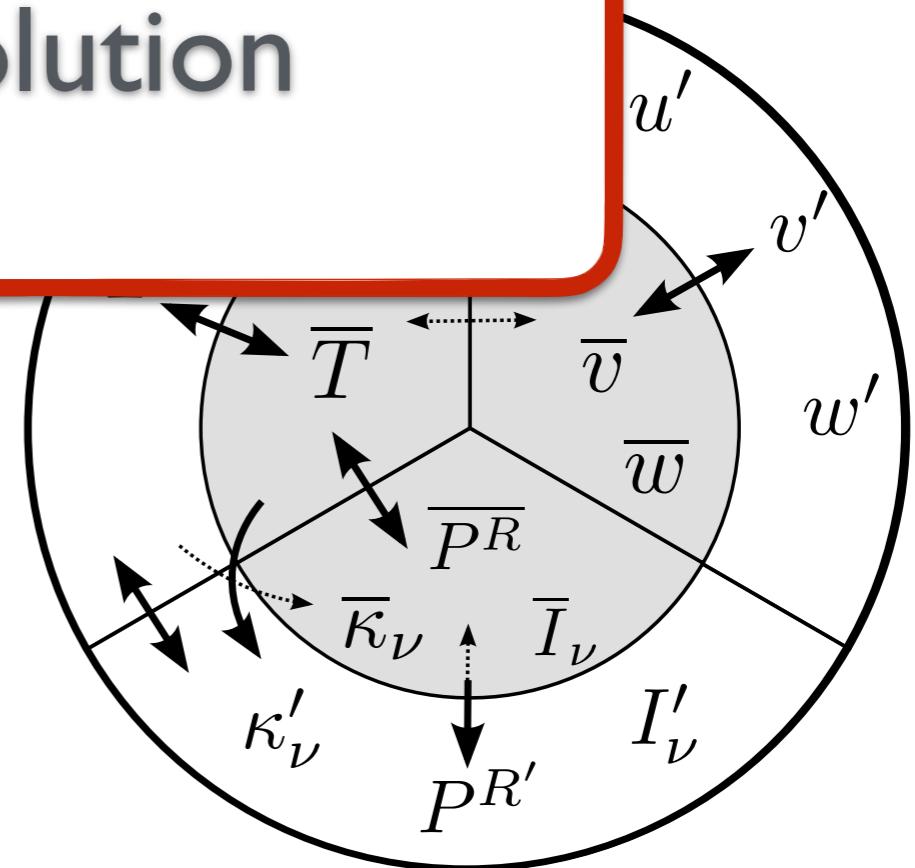
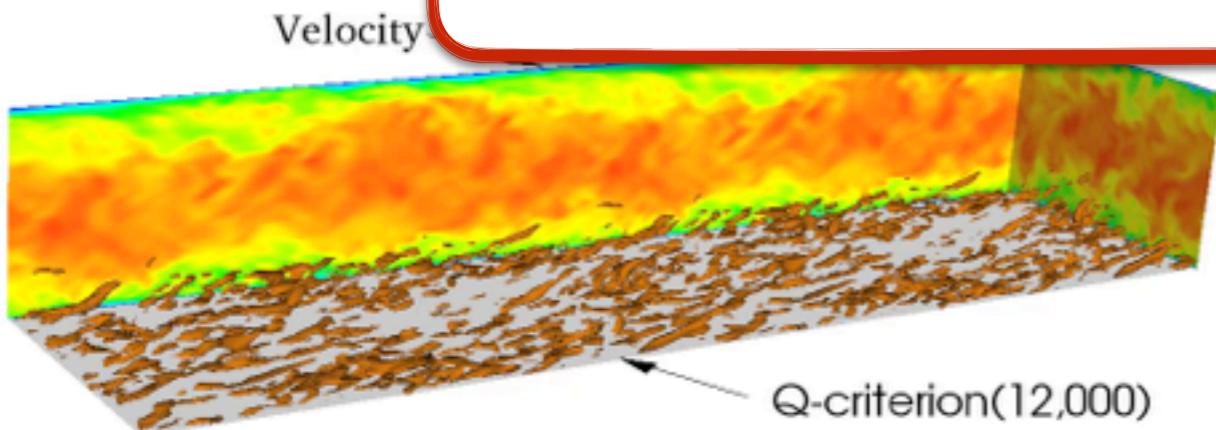
Channel flow with isothermal walls
(streamwise size X2)



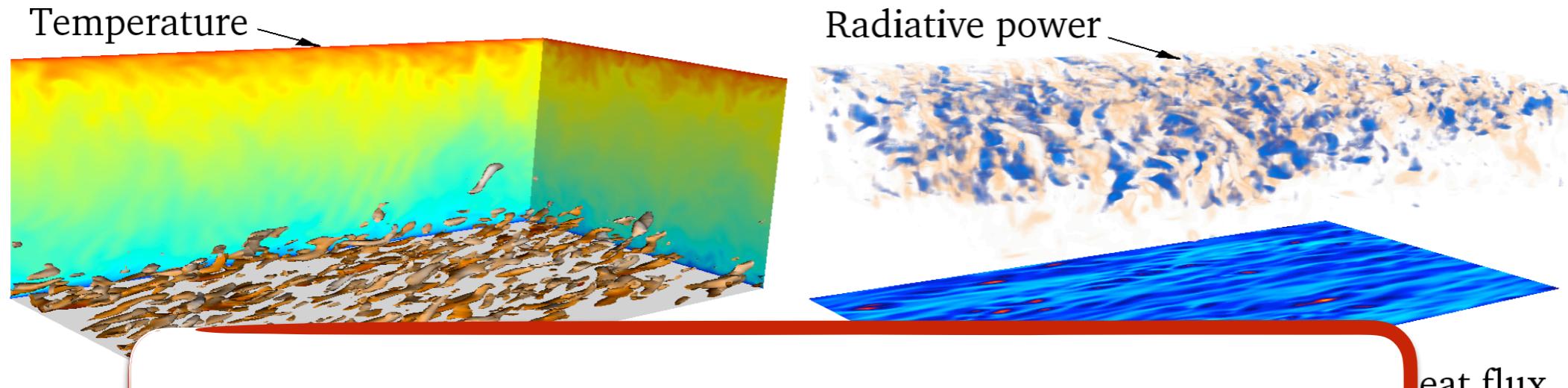
- Direct Numerical Simulations
YALES2 (4th ordre in space and time)
- Reciprocal Monte-Carlo method
Optimized ERM



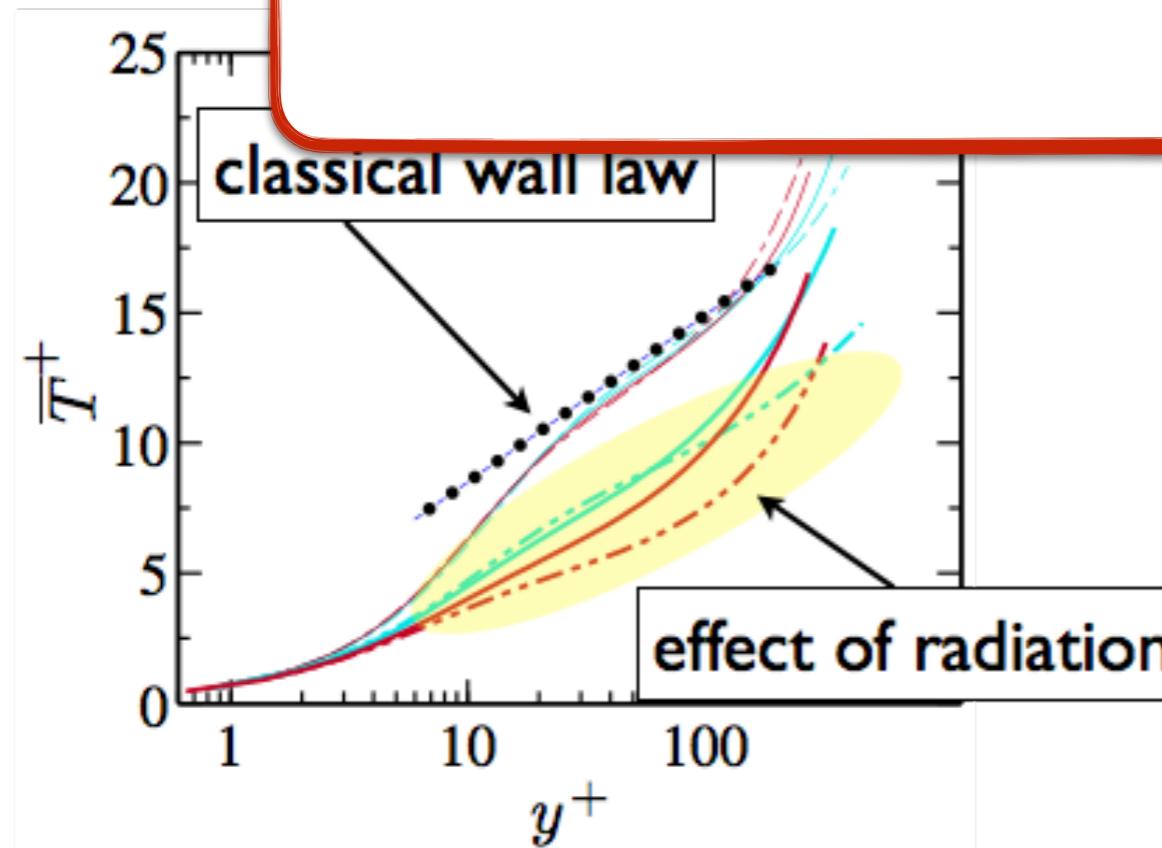
Numerical Resolution



Effects of radiation in Turbulent Boundary layers

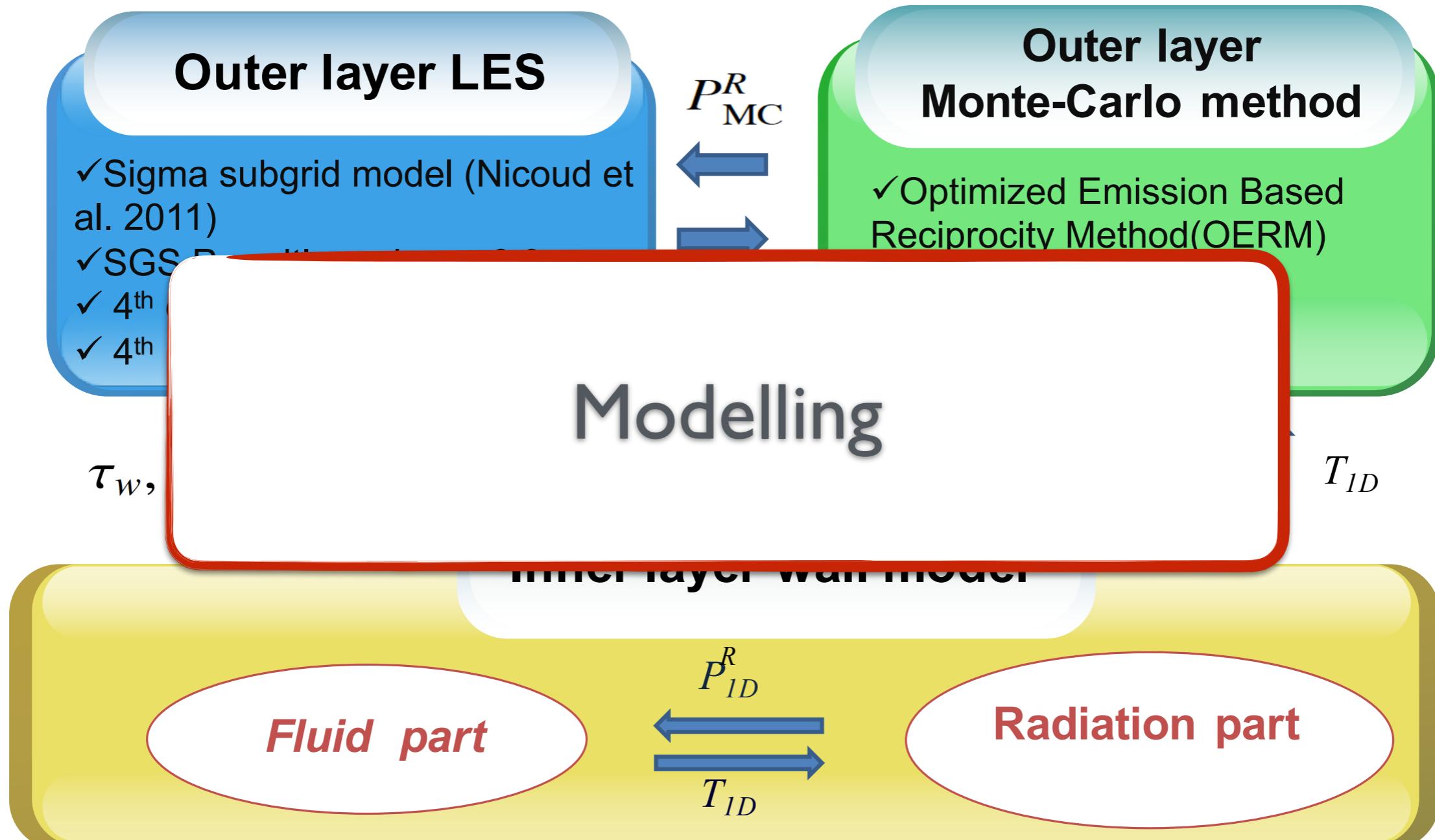


Physical Analysis

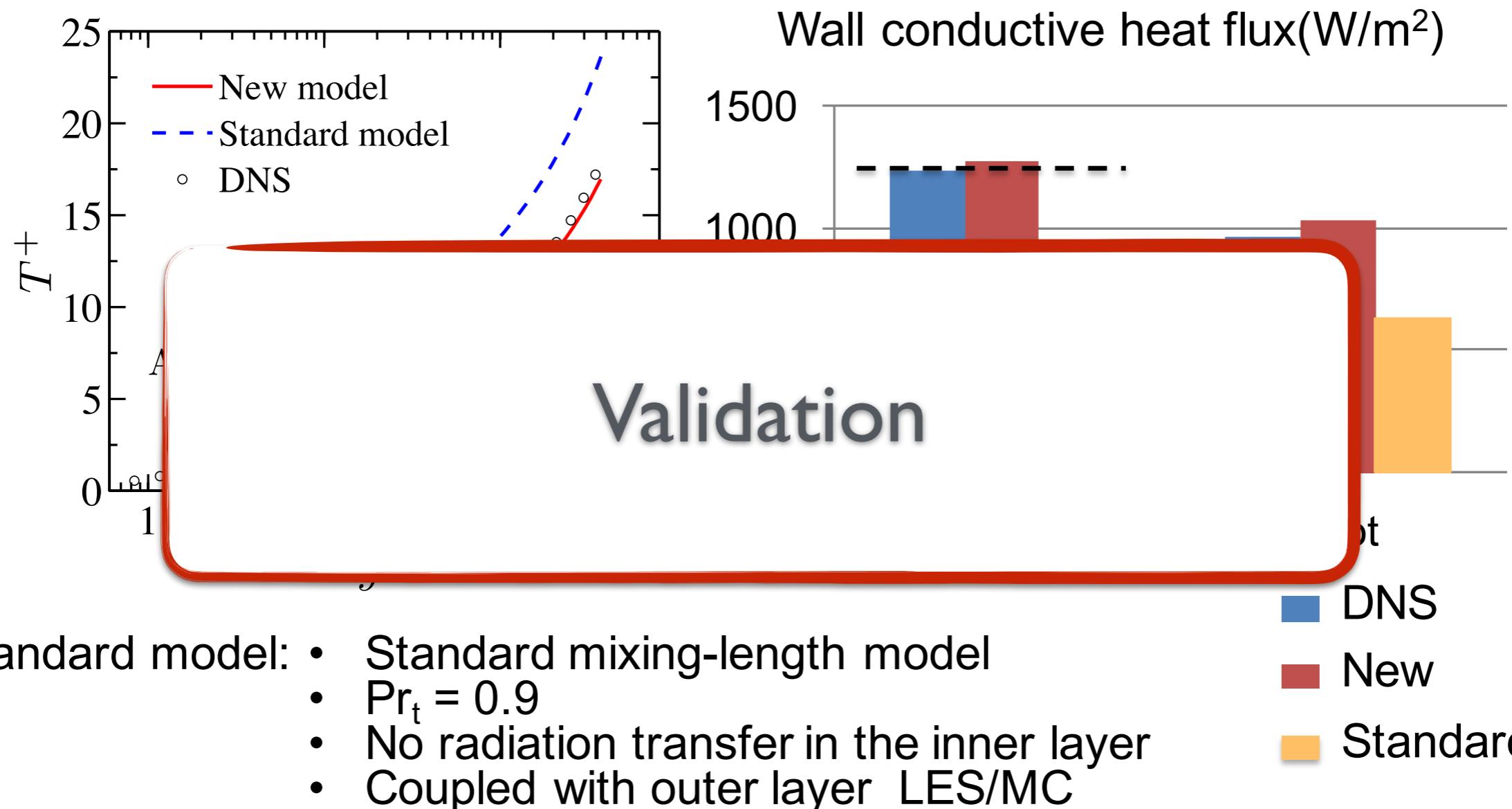


- Opposite effects of gas-gas and gas-wall radiation contributions
- Different effects depending on the conditions => not universal
- A new wall model for large eddy simulation has been developed and validated

Effects of radiation in Turbulent Boundary layers



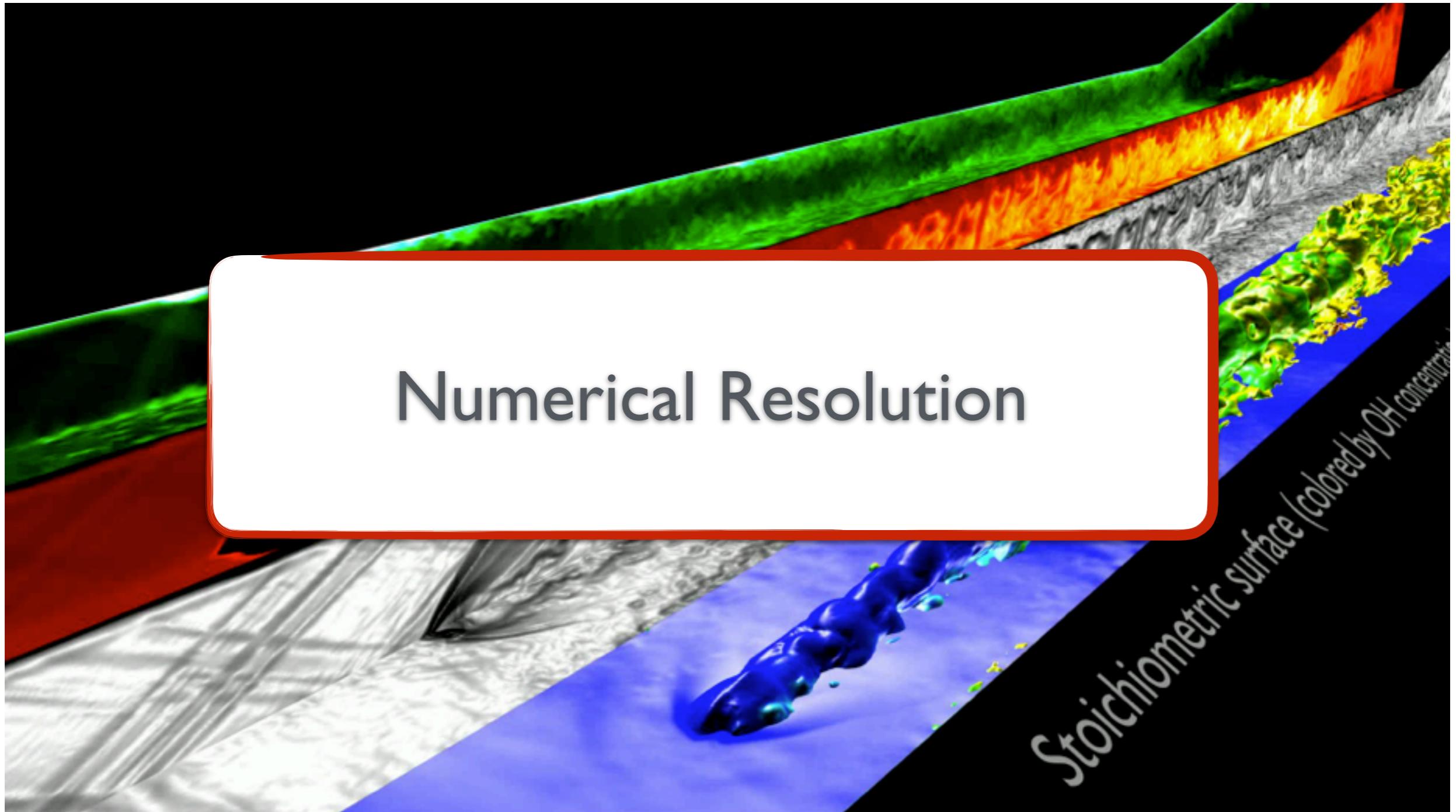
Effects of radiation in Turbulent Boundary layers



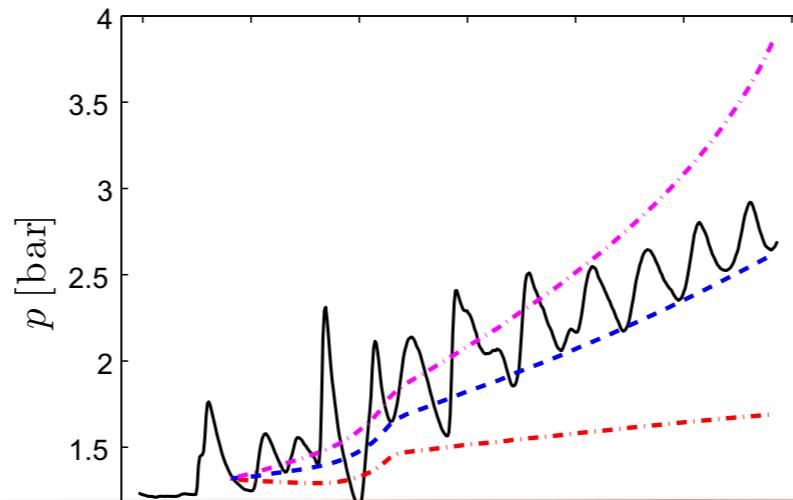
The new model shows a significant improvement in the predicted results.

Study Case: Thermal choking of a Scramjet

Thermal choking of a scramjet



Thermal choking of a scramjet



Physical Analysis

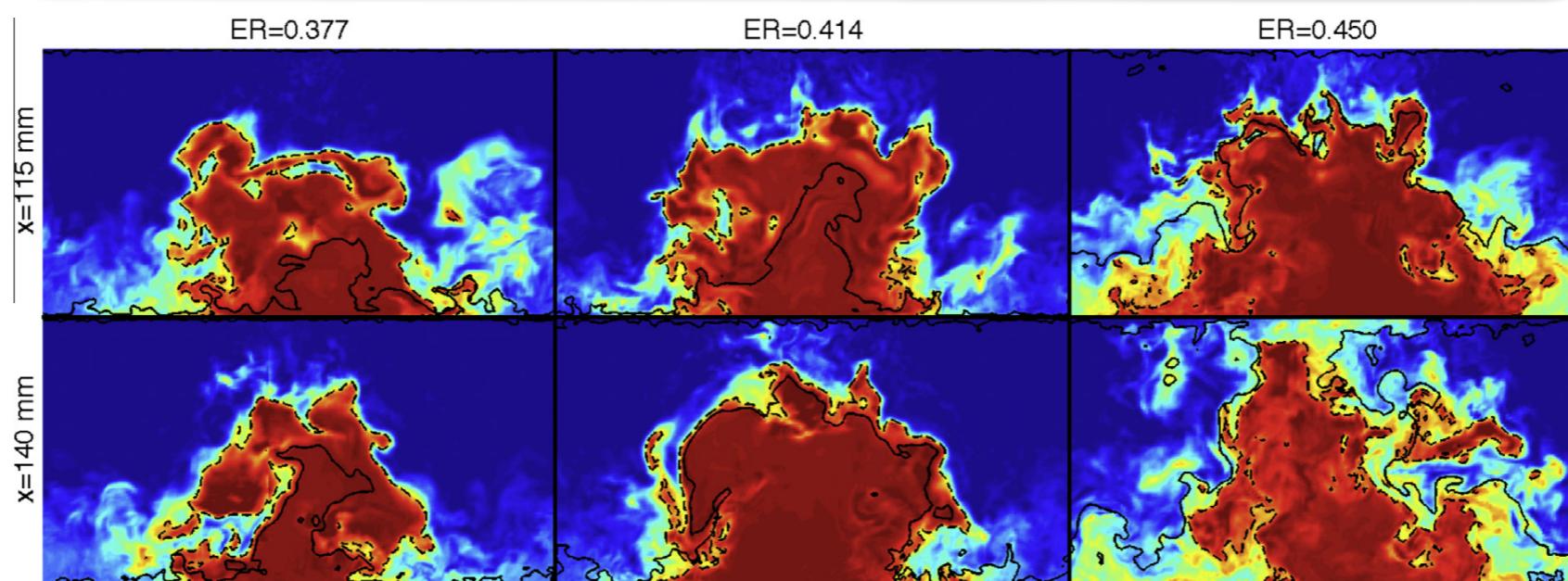
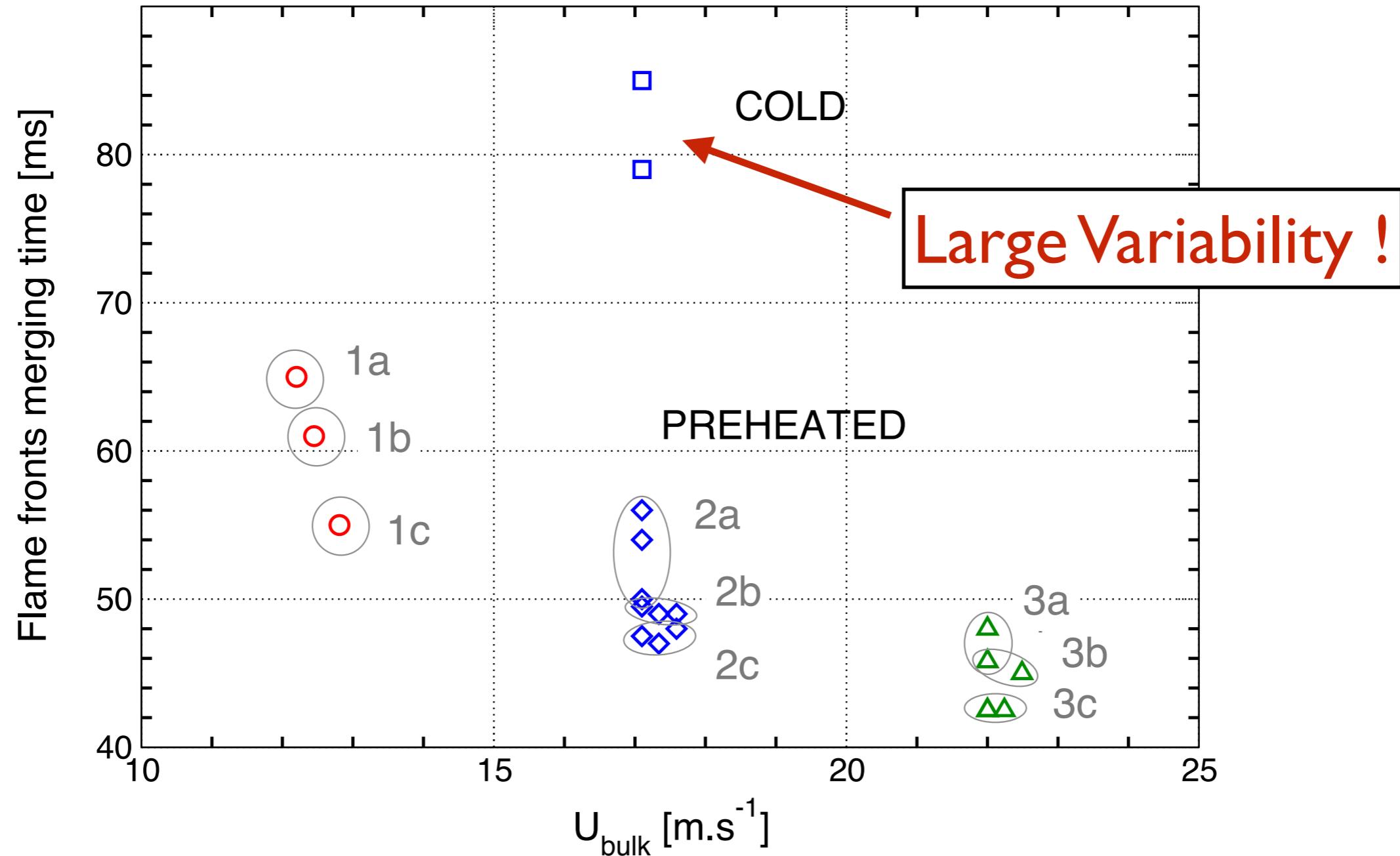


Fig. 16. Contours of instantaneous H_2O mass fraction in cross-sectional yz -slices, colored from 0 to 0.25. ERs of 0.377 (left column), 0.414 (middle column) and 0.450 (right column). Also shown are the stoichiometric line (dashed line) and the sonic line (solid line). Top row: $x = 115$ mm. Bottom row: $x = 140$ mm. Note that the combustor shock-train starts at $x_s \approx 140$ mm (ER = 0.414) and $x_s \approx 75$ mm (ER = 0.450), respectively.

Your Study Case:

**Ignition and Re-ignition Experiments
in the MICCA Burner**

Repeatability of experiments



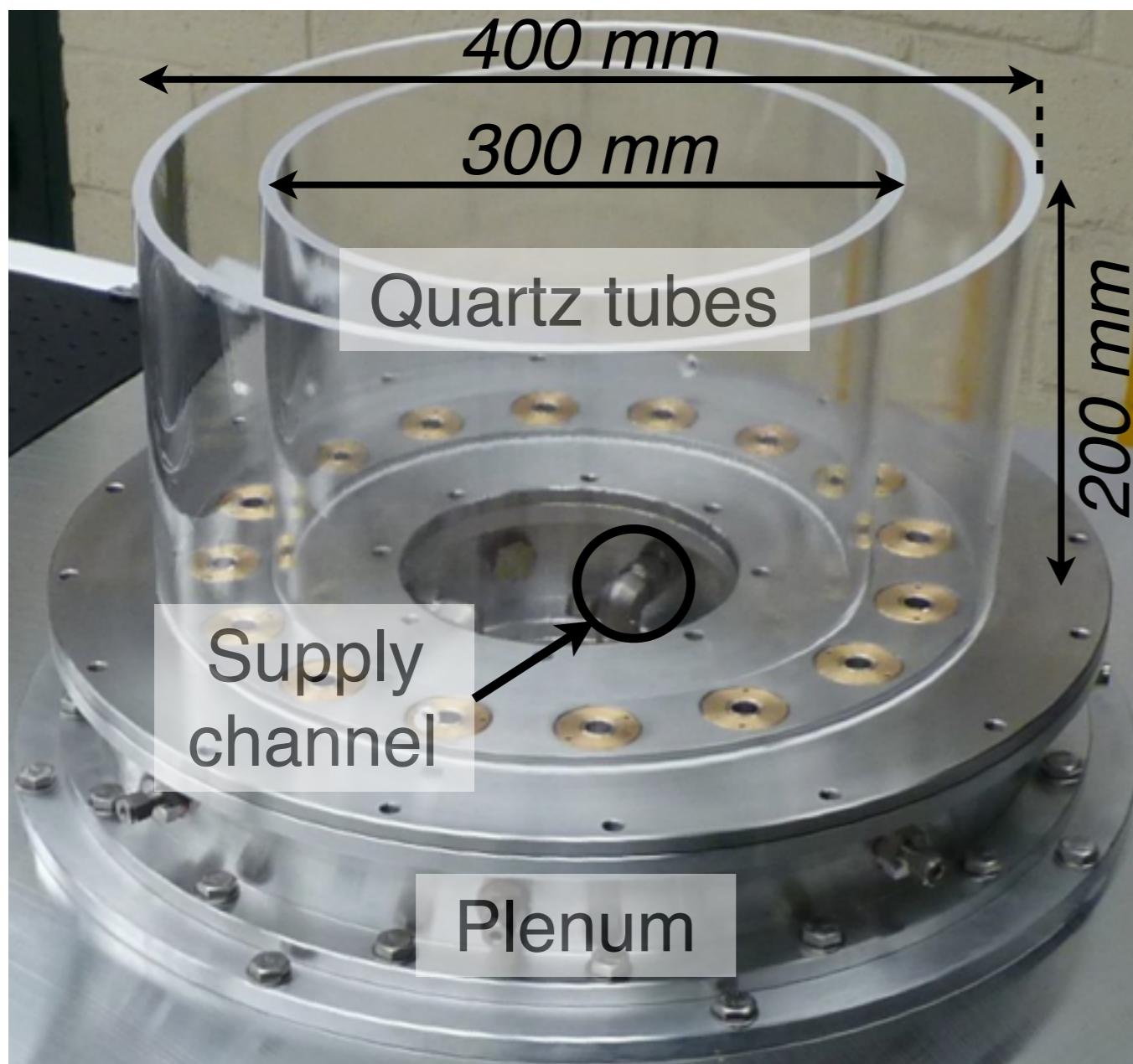
Cooling of the MICCA chamber

- Issues
 - COLD conditions require making sure that the time space between shots is long enough for the walls to be assumed cooled
 - After many shots, the quartz breaks due to repeated thermal loads which generate material fatigue
- Key Quantities
 - The cooling time from steady state heated conditions τ_{cool}
 - The temperature gradient responsible for material stresses

$$|\nabla T_q|_{max}$$

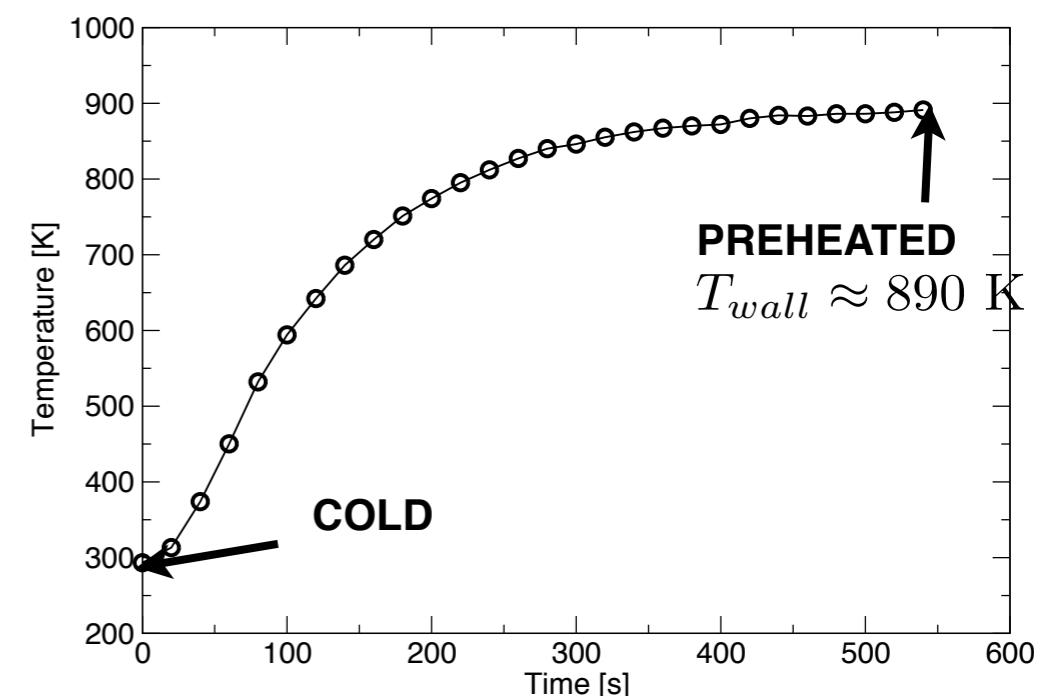
Configuration

The MICCA annular chamber



Quartz thickness: $e = 5 \text{ mm}$

Thermocouple measurement at $x=H/4$ during heating



That's it !!!

Methodology

The full unsteady 3D computation is not affordable to you



Understand the big picture of the physical processes to account for



Build and solve hierarchical models of increasing complexity



List hypotheses and verify them