Study of Collective Behaviors of Atoms

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2020.11.17

Outline

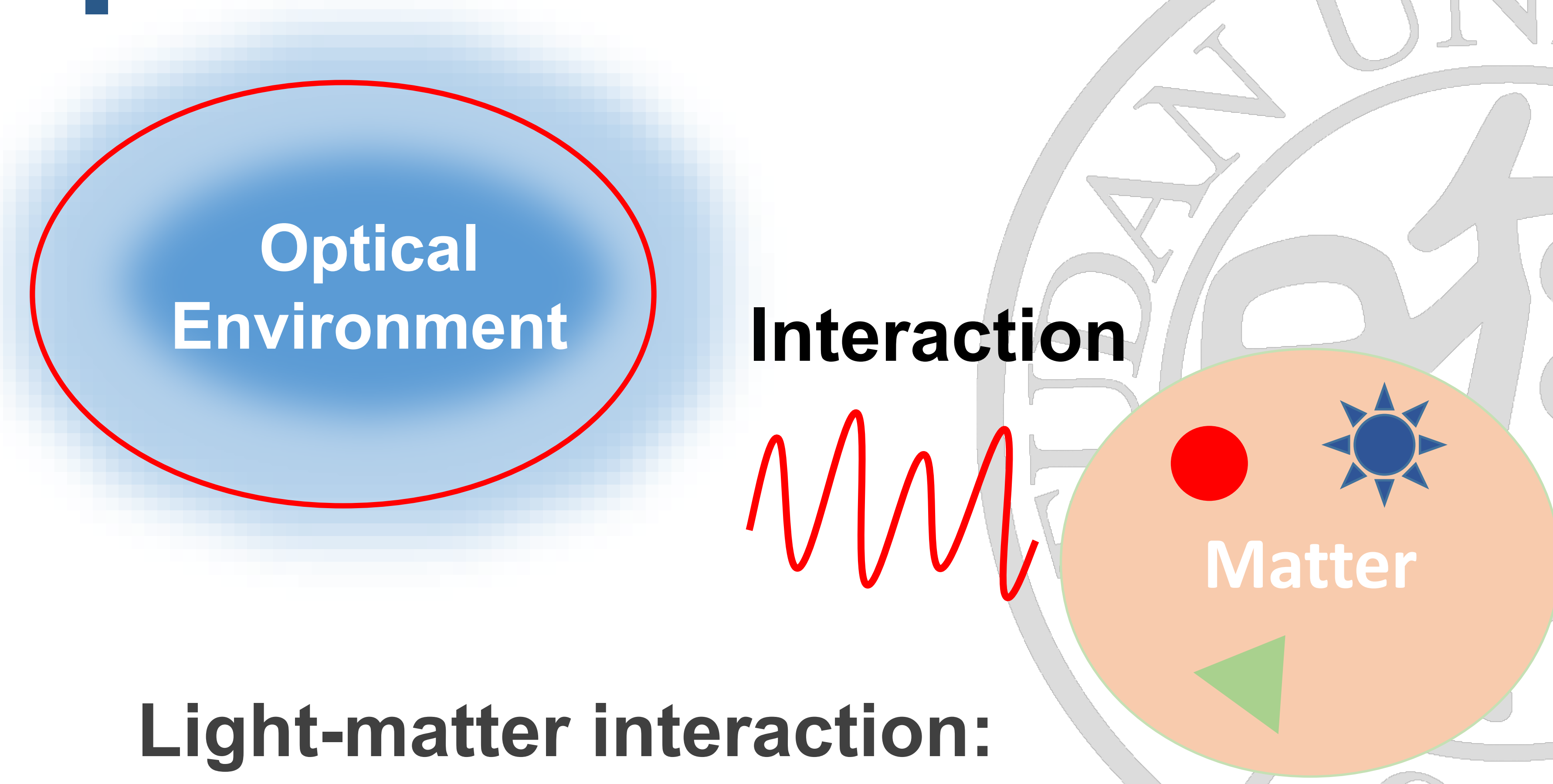
1. Motivation & Background: Why Atoms?
2. Progress: Dipole Model & Array
3. Future Plan: Lattice of Phase-mismatched Atoms
4. Reference & Appendix

Motivation & Background

Why atoms?

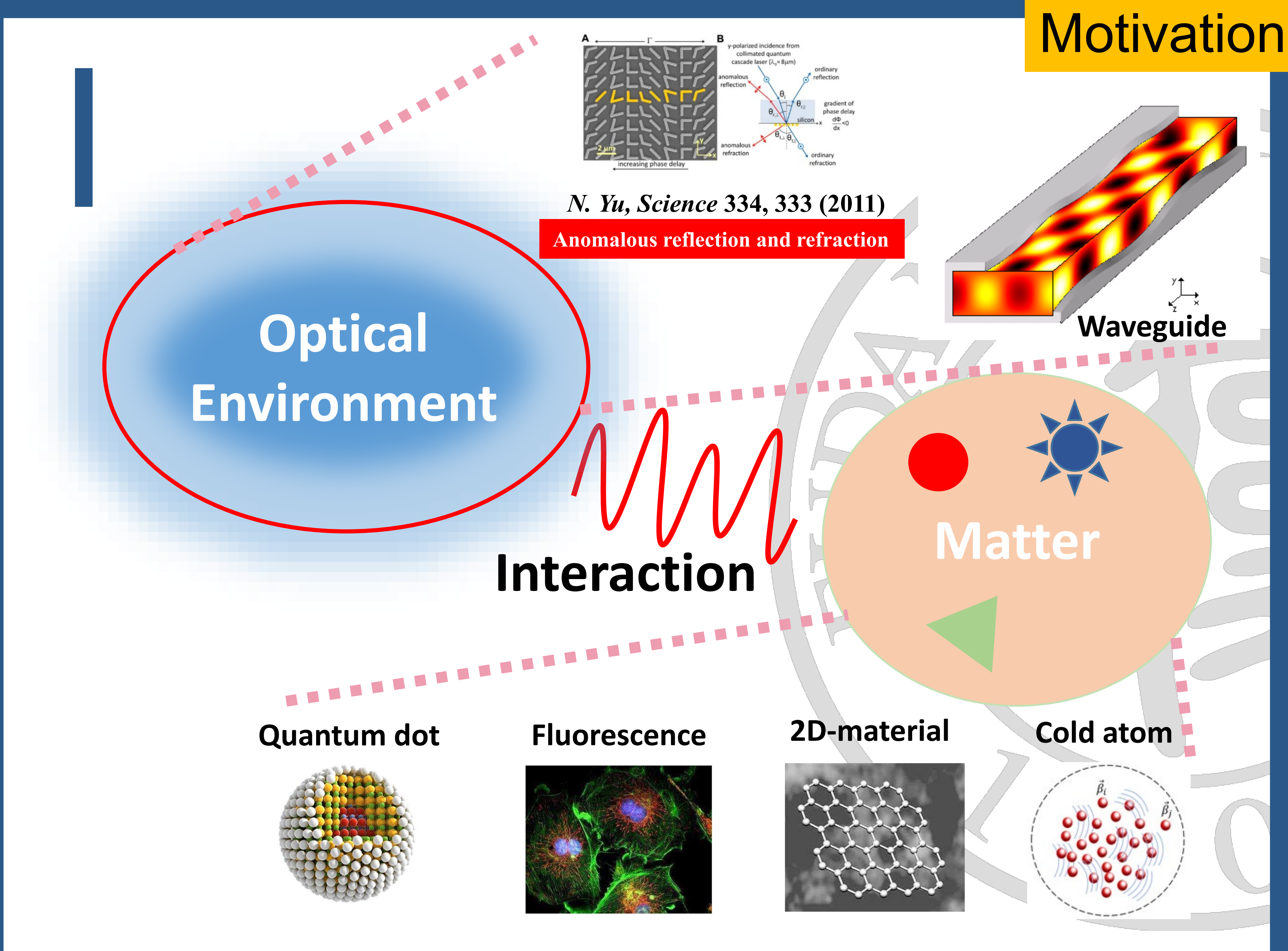
Why lattice?

Light-matter interaction



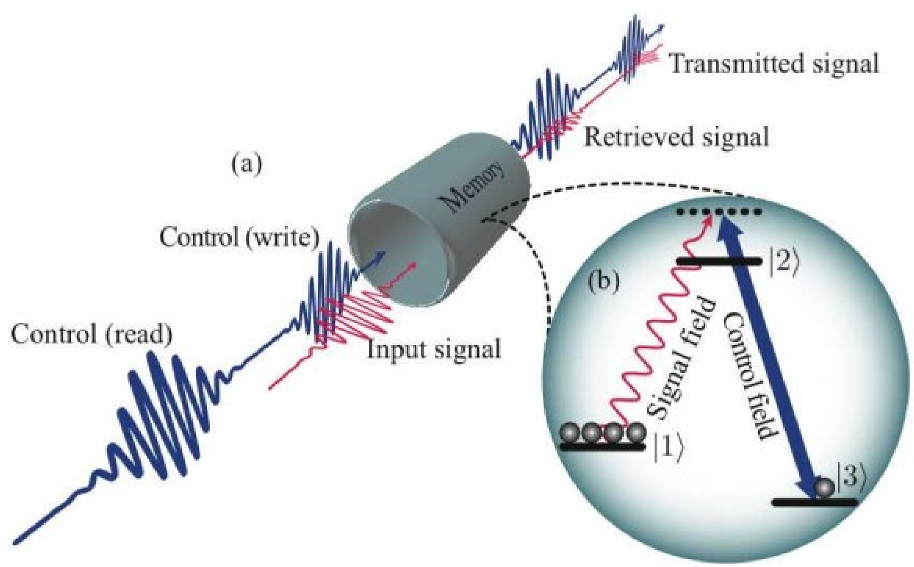
Light-matter interaction:

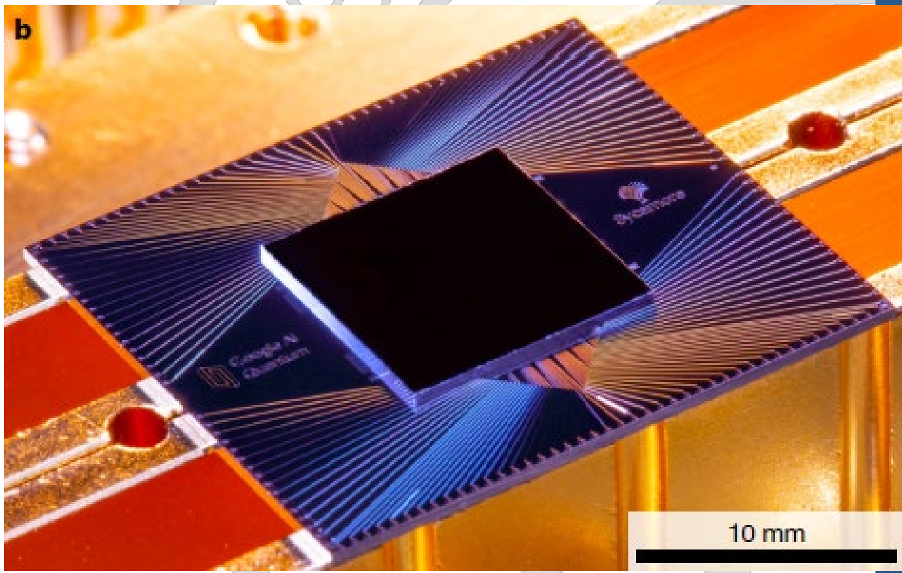
- Scientific issue
- Practical applications


Motivation
A
B
y-polarized incidence from collimated quantum cascade laser
ordinary reflection
anomalous reflection
gradient of phase delay
silicon
anomalous refraction
ordinary refraction
increasing phase delay
N. Yu, Science 334, 333 (2011)
Anomalous reflection and refraction
Waveguide
Optical Environment
Interaction
Matter
Quantum dot
Fluorescence
2D-material
Cold atom

Why atoms?

"The Atom Brick"





Exciting application:
quantum memory

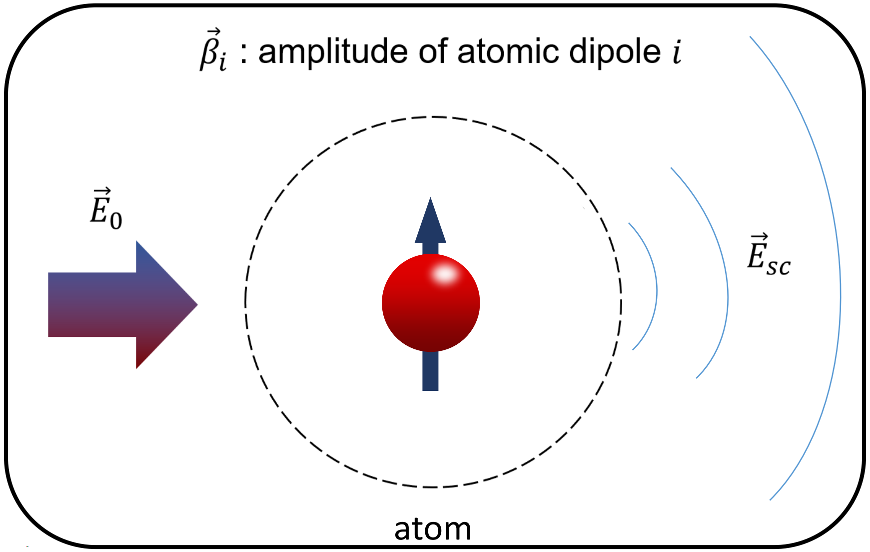
Simon, C. et al. *The European Physical Journal D*, *58*(1), 1-22(2010). .

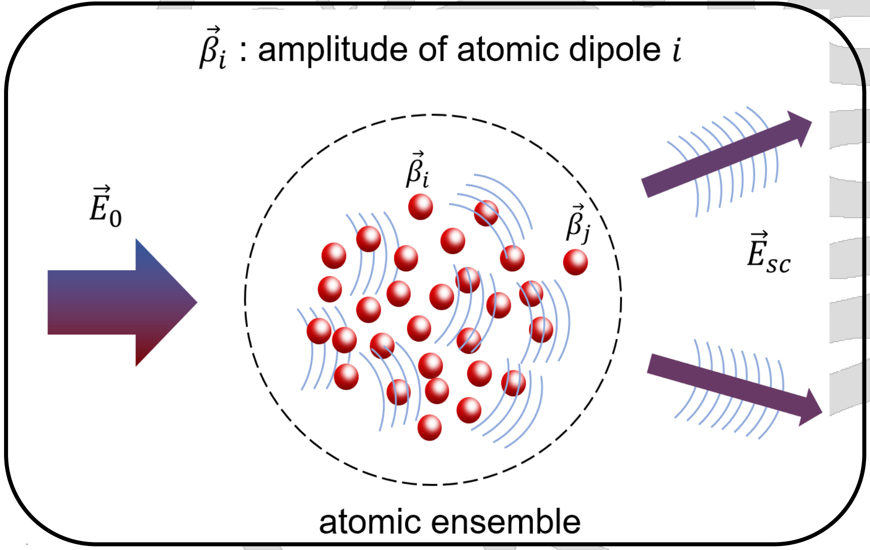
Exciting application:
quantum computation

Arute, F. *et al. Nature* **574,** 505–510 (2019).

Why lattices?

Decay rate: $\gamma \equiv \frac{\Gamma}{2} = \frac{1}{2W}\frac{dW}{dt}$





Single atom:

$$amplitude \propto= \exp(-\gamma_0 t)$$

Atom ensemble:

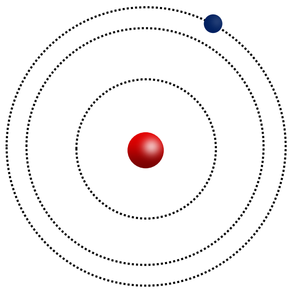
- Superradiance
- Subradiance

Progress

- ✓ Dipole approximation
- ✓ One atom, two dipoles, 1D atom array

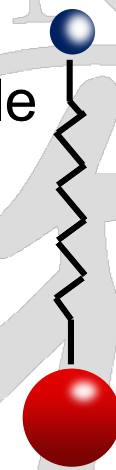
What are atoms? ——dipole approximation

Bohr radius:0.53Å



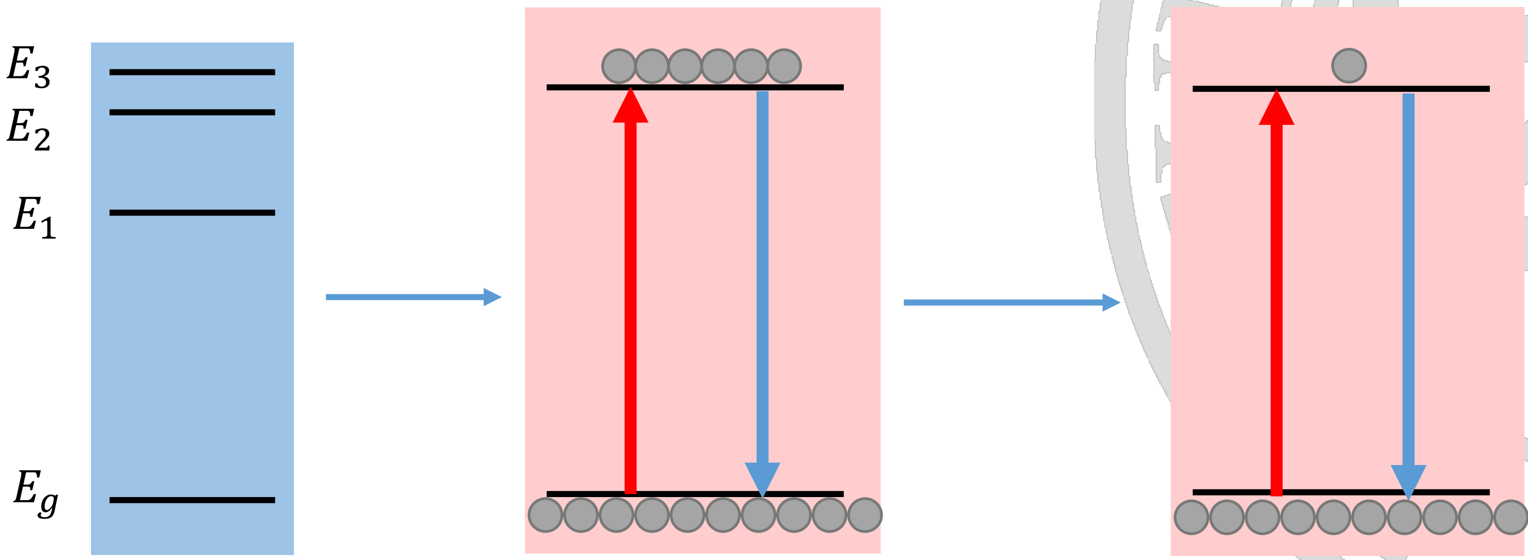
Real atom

Electric dipole



2. Weak excitation approximation

$$\rho_{ee} \ll \rho_{gg}$$



1. Rotating wave approximation

$$|\omega_0 - \omega_k| \ll \omega_0 + \omega_k$$

A Single Atom

- Radiation pattern
  - From Helmholtz's Eq.

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - k^2 \mathbf{E}(\mathbf{r}) = \mathrm{i}\omega\mu_0\mu\, \mathbf{j}(\mathbf{r}).$$

  - Dyadic Green's function

$$\nabla \times \nabla \times \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') - k^2 \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \overleftrightarrow{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}')$$

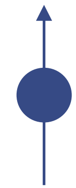
$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_0) = \frac{\exp(\mathrm{i}kR)}{4\pi R}\left[\left(1+\frac{\mathrm{i}kR-1}{k^2R^2}\right)\overleftrightarrow{\mathbf{I}} + \frac{3-3\mathrm{i}kR-k^2R^2}{k^2R^2}\frac{\mathbf{RR}}{R^2}\right]$$

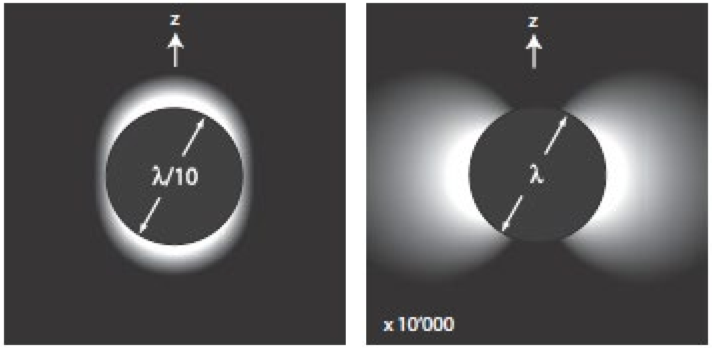
  - The E-field

$$\mathbf{E}(\mathbf{r}) = \omega^2\mu\mu_0\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_0)\mathbf{p}$$

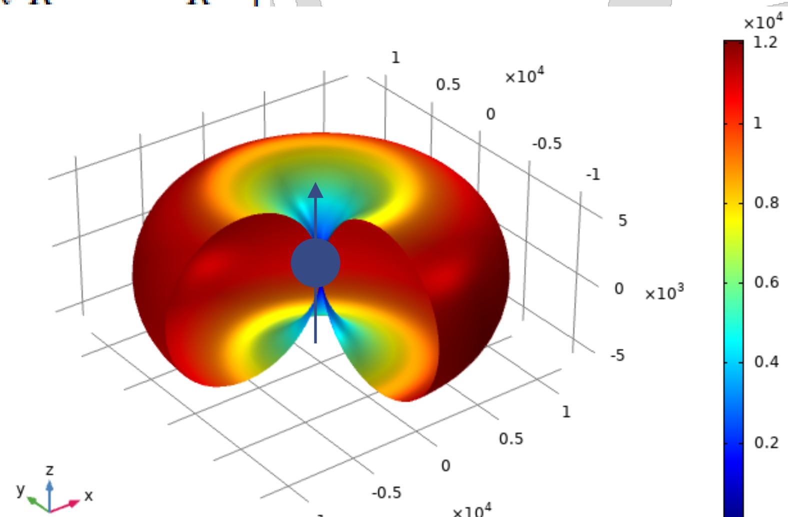
  - Radiation—energy flux

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$





Near field Far field
electric energy density

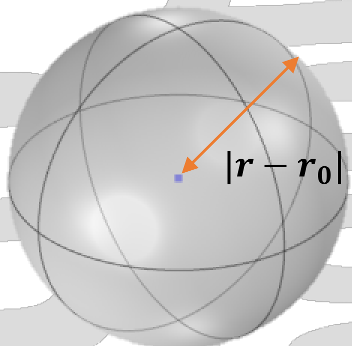


Far field radiation pattern

A Single Atom

- Energy loss
  - By radiation:

$$P(t) = \oiint \boldsymbol{S} \cdot \mathbf{n} da = \frac{|p|^2 \omega}{12\pi\epsilon_0\epsilon} k^3$$



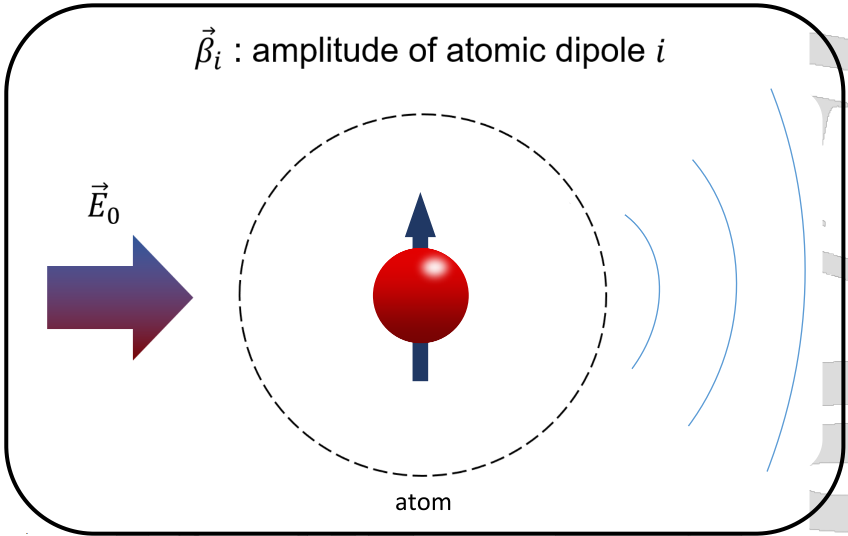
  - By work:

$$\frac{dW}{dt} = -\frac{1}{2} \oiiint Re\{\mathbf{j}^* \cdot \mathbf{E}\} dV \xrightarrow{\text{limiting } |\boldsymbol{r} - \boldsymbol{r}_0| \to 0} \frac{|p|^2 \omega}{12\pi\epsilon_0\epsilon} k^3$$

- Energy radiated by a dipole = Work done by the field on itself

$$P = \frac{dW}{dt}$$

A Single Atom



- Dynamic behaviors :

$$m\frac{d^2}{dt^2}\vec{q}(t) = -m\omega_0^2\vec{q}(t) - e\left(\vec{E}_{incident} + \vec{E}_{reaction}\right)$$

restoring force

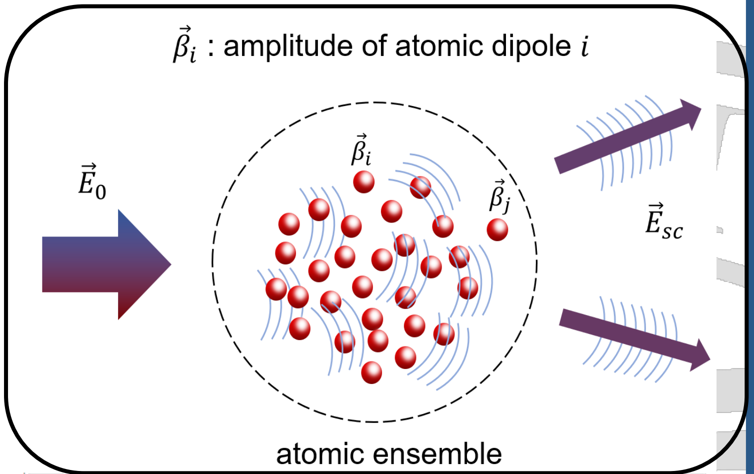
ω_0

incident wave

radiation reaction force

$Im\left[\overleftrightarrow{G}_0\right]$

A Single Atom



■Equation of Motion:

$$m\frac{d^2}{dt^2}\vec{q}(\vec{r}_i,t) = -m\omega_0^2\vec{q}(\vec{r}_i,t) - e\left(\vec{E}_{incident} + \vec{E}_{reaction} + \vec{E}_{inter-atom}\right)$$

restoring force ω_0

incident wave

radiation reaction force $Im[\overleftrightarrow{G}_0(\vec{r}_i,\vec{r}_i)]$

radiation from other dipoles $\overleftrightarrow{G}_0(\vec{r}_i,\vec{r}_j)$

suppose: $\vec{q}(\vec{r}_i,t) = \vec{\beta}(\vec{r}_i,t)e^{-i\omega t}$

■Slow varying approximation : $\omega^2\vec{\beta} \gg \omega\frac{d}{dt}\vec{\beta} \gg \frac{d^2}{dt^2}\vec{\beta}$

Schrödinger-like equation: $\boxed{i\frac{d}{dt}\vec{\beta}(t) = \overleftrightarrow{H}_{eff}\vec{\beta}(t)}$

Progress

Effective Hamiltonian —— of 3 atoms

$$G^{p,q}(u) \equiv \frac{k^3}{4\pi\varepsilon_0} e^{iu}\left[\delta_{p,q}\, g_1(u) + \frac{u_p u_q}{u^2} g_2(u)\right] \times \left(-\frac{3\pi\Gamma_0}{k}\right)$$

$$\left[\begin{array}{ccc|ccc|ccc}
-i\Gamma_0/2 & 0 & 0 & -G_{12}^{xx} & -G_{12}^{xy} & -G_{12}^{xz} & -G_{13}^{xx} & -G_{13}^{xy} & -G_{13}^{xz} \\
0 & -i\Gamma_0/2 & 0 & -G_{12}^{xy} & -G_{12}^{yy} & -G_{12}^{yz} & -G_{13}^{xy} & -G_{13}^{yy} & -G_{13}^{yz} \\
0 & 0 & -i\Gamma_0/2 & -G_{12}^{xz} & -G_{12}^{yz} & -G_{12}^{zz} & -G_{13}^{xz} & -G_{13}^{yz} & -G_{13}^{zz} \\
\hline
-G_{12}^{xx} & -G_{12}^{xy} & -G_{12}^{xz} & -i\Gamma_0/2 & 0 & 0 & -G_{23}^{xx} & -G_{23}^{xy} & -G_{23}^{xz} \\
-G_{12}^{xy} & -G_{12}^{yy} & -G_{12}^{yz} & 0 & -i\Gamma_0/2 & 0 & -G_{23}^{xy} & -G_{23}^{yy} & -G_{23}^{yz} \\
-G_{12}^{xz} & -G_{12}^{yz} & -G_{12}^{zz} & 0 & 0 & -i\Gamma_0/2 & -G_{23}^{xz} & -G_{23}^{yz} & -G_{23}^{zz} \\
\hline
-G_{13}^{xx} & -G_{13}^{xy} & -G_{13}^{xz} & -G_{23}^{xx} & -G_{23}^{xy} & -G_{23}^{xz} & -i\Gamma_0/2 & 0 & 0 \\
-G_{13}^{xy} & -G_{13}^{yy} & -G_{13}^{yz} & -G_{23}^{xy} & -G_{23}^{yy} & -G_{23}^{yz} & 0 & -i\Gamma_0/2 & 0 \\
-G_{13}^{xz} & -G_{13}^{yz} & -G_{13}^{zz} & -G_{23}^{xz} & -G_{23}^{yz} & -G_{23}^{zz} & 0 & 0 & -i\Gamma_0/2
\end{array}\right]$$

$$\boxed{i\frac{d}{dt}\vec{\beta}(t) = \overleftrightarrow{H}_{eff}\vec{\beta}(t)}$$

Stationary solution

$$\overleftrightarrow{H}_{eff}\vec{\beta}(t) = E_0$$

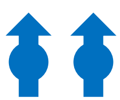
Eigenvalue problem

1D: 2 atoms

- 2 eigenvalues:
  - $\lambda_+ = -\frac{i\Gamma_0}{2} - 3\pi\Gamma_0 G_{12}, \qquad \lambda_- = -\frac{i\Gamma_0}{2} + 3\pi\Gamma_0 G_{12}$
  - decay rate $\gamma_i \propto Im(\lambda_i)$
- Normalized 2 eigenvectors:

In phase mode

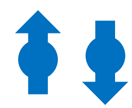
$$\vec{\mathbf{m}}_+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$



Superradiance

Decay rate $\gamma = 2\gamma_0$

Out of phase mode



$$\vec{\mathbf{m}}_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Subradiance

Decay rate $\gamma = 0$

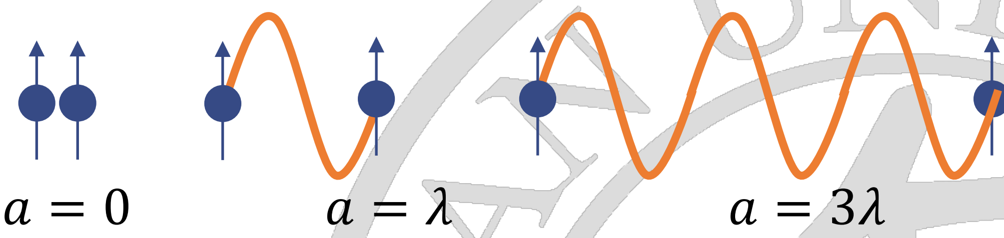
- General solutions:

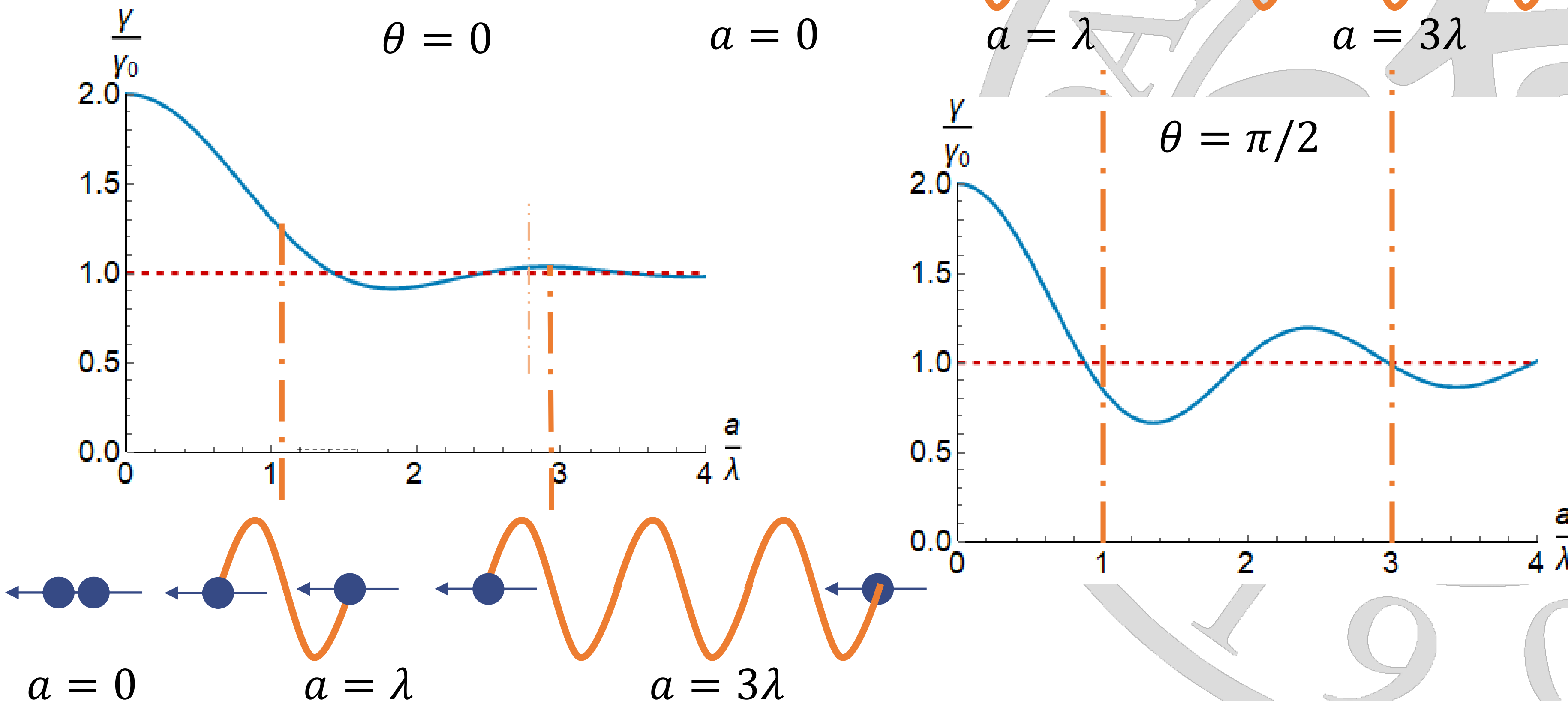
$$\begin{pmatrix}d_1\\d_2\end{pmatrix} = \frac{E_0^{(1)} + E_0^{(2)}}{2}\alpha_+\begin{pmatrix}1\\1\end{pmatrix} + \frac{E_0^{(1)} - E_0^{(2)}}{2}\alpha_-\begin{pmatrix}1\\-1\end{pmatrix}$$

Progress

1D: 2 atoms

■Geometry's effect





■Superradiance in the limit of $r \to 0$ is regardless of geometry

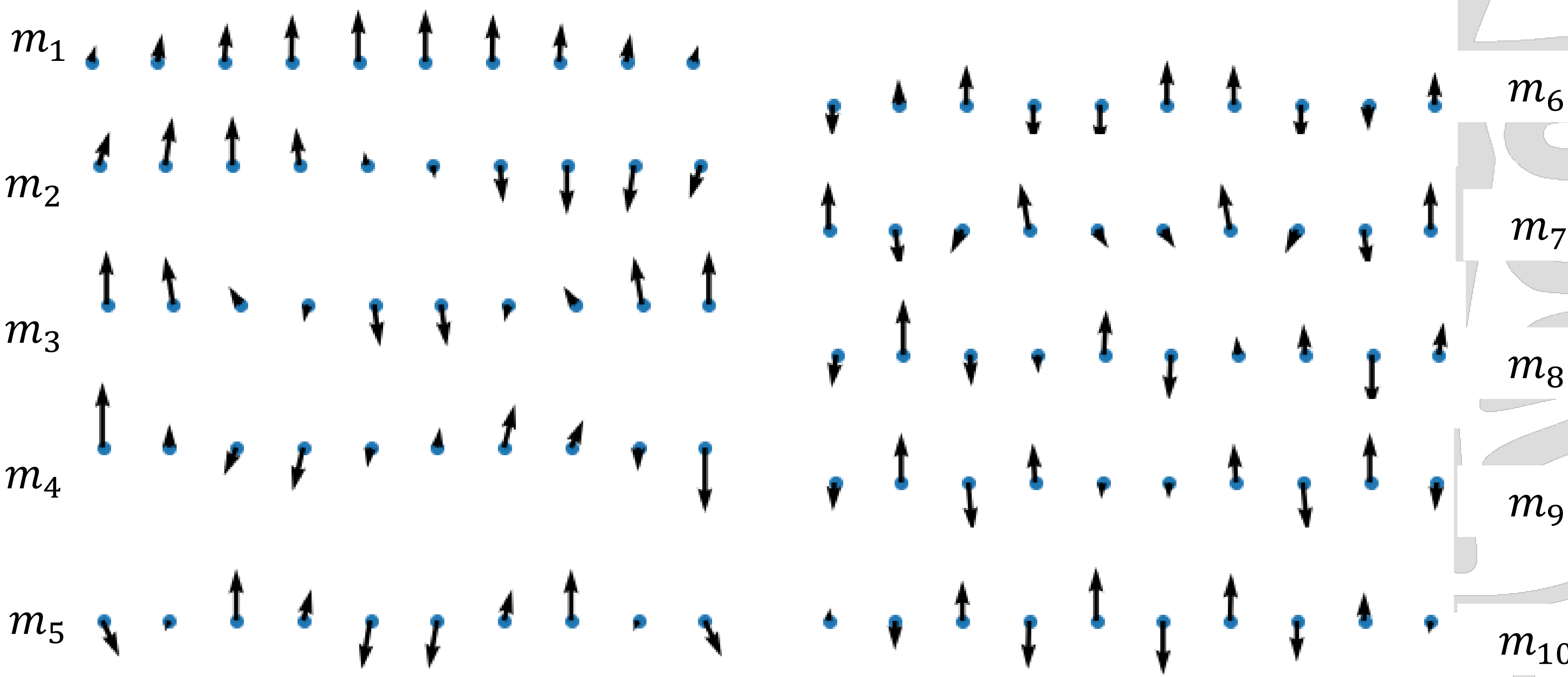
Progress

1D: 10 atoms

Order by correlation function：

$$\langle \cos \varphi_{i,i+1} \rangle = \frac{1}{N-1} \sum_{i=1}^{N-1} \cos(\varphi_i - \varphi_{i+1}).$$

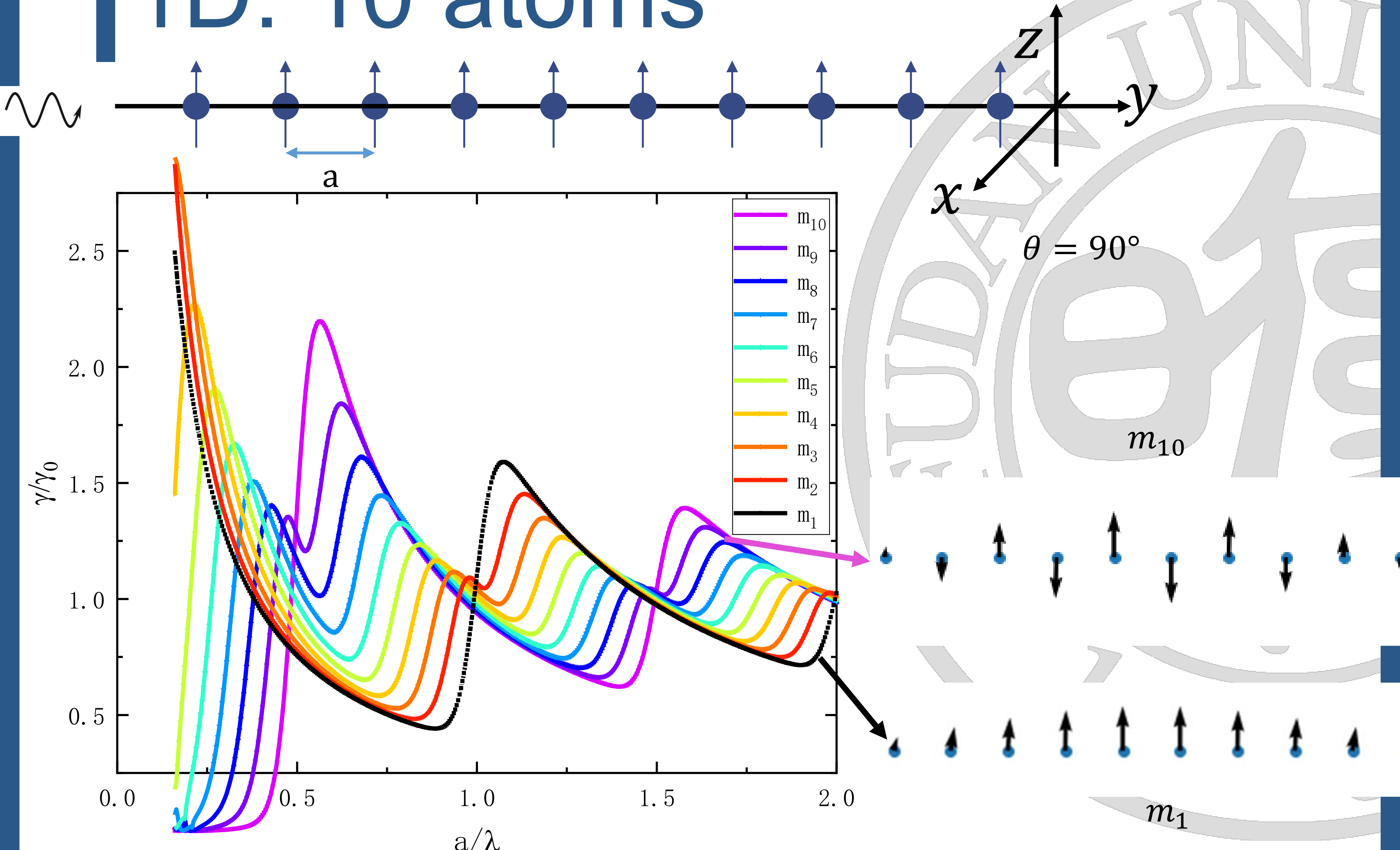
Angular between neighbors: $\varphi_i = \tan^{-1} \frac{Im(g_i)}{Re(g_i)}$



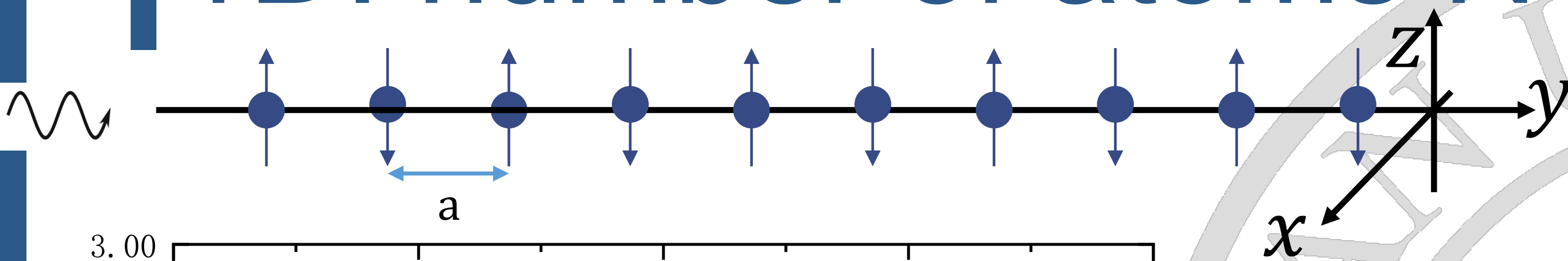
More complex modes →more interesting collective behaviors

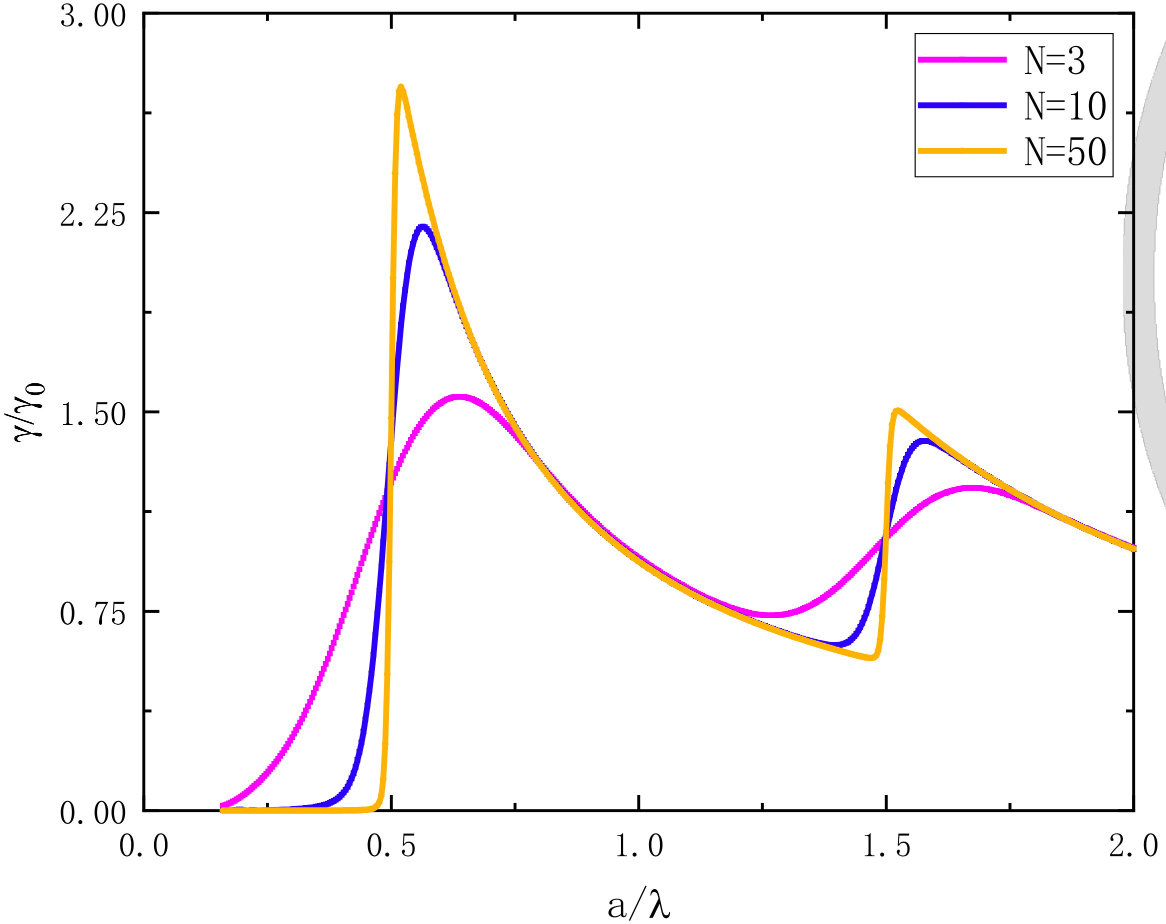
Progress

1D: 10 atoms



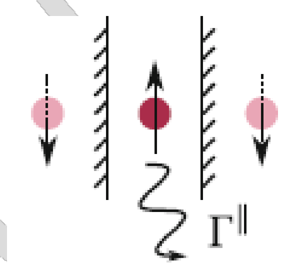
1D: number of atoms N





$\theta = 90°$

Maximally out of phase mode m_N



Conclusion & Future Plan

Atom array

Mismatch

Conclusion

- Schrödinger-like equation

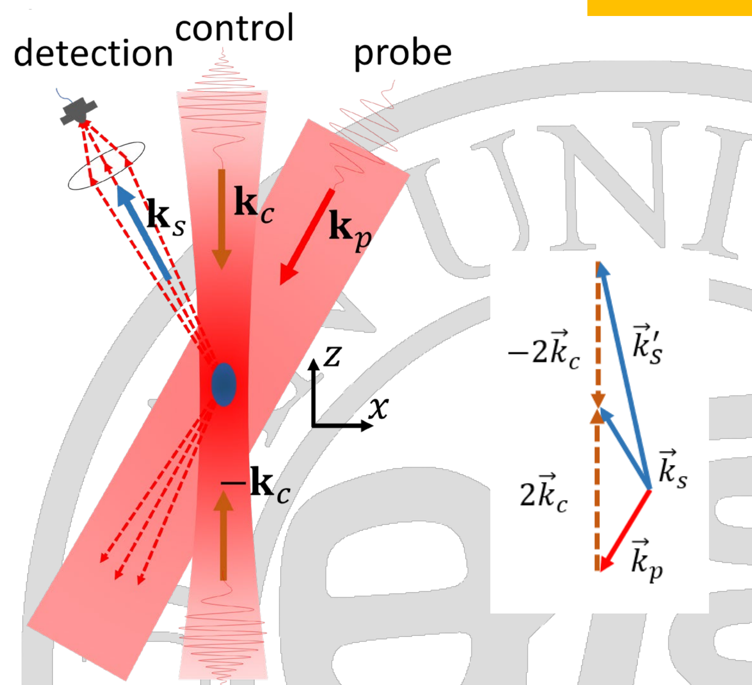
$$\boxed{i\frac{d}{dt}\vec{\beta}(t) = \overleftrightarrow{H}_{eff}\vec{\beta}(t)}$$

- Interaction between 1D array of atoms

✓One atom—dipole apprxoimation
✓Two dipoles—collective behaviors
✓1D atom array—superdadiance & subradiance

Next Step

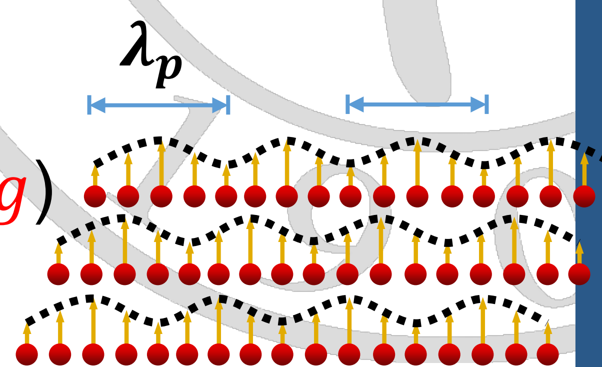
1. Random atom gas(√)
2. Lattice of atoms(√)
3. Mismatched atoms(*ongoing*)



Preparation of phase-mismatched atoms

Yizun He, et al, **Saijun Wu**. *Phys. Rev. Lett.* 125, 213602 – Published **16 November 2020**

4. Lattice of mismatched atoms(*ongoing*)



Thank you for your time!

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Reference

1. He, Y., et al., *Geometric control of collective spontaneous emission.* arXiv: 1910.02289v2, 2019.

2. Dicke, R.H., *Coherence in Spontaneous Radiation Processes.* Physical Review, 1954. **93**(1): p. 99-110.

3. Bettles, R., *Cooperative Interactions in Lattices of Atomic Dipoles*. 2017: Springer International Publishing.

4. Novotny, L., & Hecht, B. (2006). *Principles of Nano-Optics*. Cambridge: Cambridge University Press. doi:10.1017/CBO9780511813535

Appendix A: Didactic Green's function

- Firstly, derive scaler Green’ s function as the solution for vector potential A:

$$\mathbf{E}(\mathbf{r}) = \mathrm{i}\omega\mathbf{A}(\mathbf{r}) - \nabla\phi(\mathbf{r}),$$
$$\mathbf{H}(\mathbf{r}) = \frac{1}{\mu_0\mu}\nabla\times\mathbf{A}(\mathbf{r}).$$

  - Insert with $D = \epsilon E$ into:

$$\nabla\times H = j + \frac{\partial D}{\partial t}$$

  - Simplified with Lorentz gauge:

$$\left[\nabla^2 + k^2\right]\mathbf{A}(\mathbf{r}) = -\mu_0\mu\mathbf{j}(\mathbf{r}).$$

  - And this can be solved by scaler Green’ s function (+ denotes the outflow)

$$\left[\nabla^2 + k^2\right]G_0(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}').\qquad G_0(\mathbf{r},\mathbf{r}') = \frac{\mathrm{e}^{\pm\mathrm{i}k|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

Appendix A: Didactic Green's function

- From the relation of E and A:

$$\mathbf{E}(\mathbf{r}) = \mathrm{i}\omega\left[1 + \frac{1}{k^2}\nabla\nabla\cdot\right]\mathbf{A}(\mathbf{r})$$

- And it can be proved that:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \mathrm{i}\omega\mu_0\mu\int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\,\mathbf{j}(\mathbf{r}')\,\mathrm{d}V' \qquad \mathbf{r} \notin V.$$

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left[\overleftrightarrow{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right]G_0(\mathbf{r},\mathbf{r}').$$

- Thus, the didactic Green' s function writes:

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_0) = \frac{\exp(\mathrm{i}kR)}{4\pi R}\left[\left(1 + \frac{\mathrm{i}kR - 1}{k^2R^2}\right)\overleftrightarrow{\mathbf{I}} + \frac{3 - 3\mathrm{i}kR - k^2R^2}{k^2R^2}\frac{\mathbf{RR}}{R^2}\right]$$

- With the point electric approximation, E can be known:

$$\rho(\mathbf{r}) = \sum_n q_n\delta[\mathbf{r} - \mathbf{r}_n],$$

$$\mathbf{j}(\mathbf{r}) = \sum_n q_n\dot{\mathbf{r}}_n\delta[\mathbf{r} - \mathbf{r}_n].$$

$$\mathbf{E}(\mathbf{r}) = \omega^2\mu\mu_0\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_0)\mathbf{p}$$

Appendix B: Preparation of phase-mismatched atoms

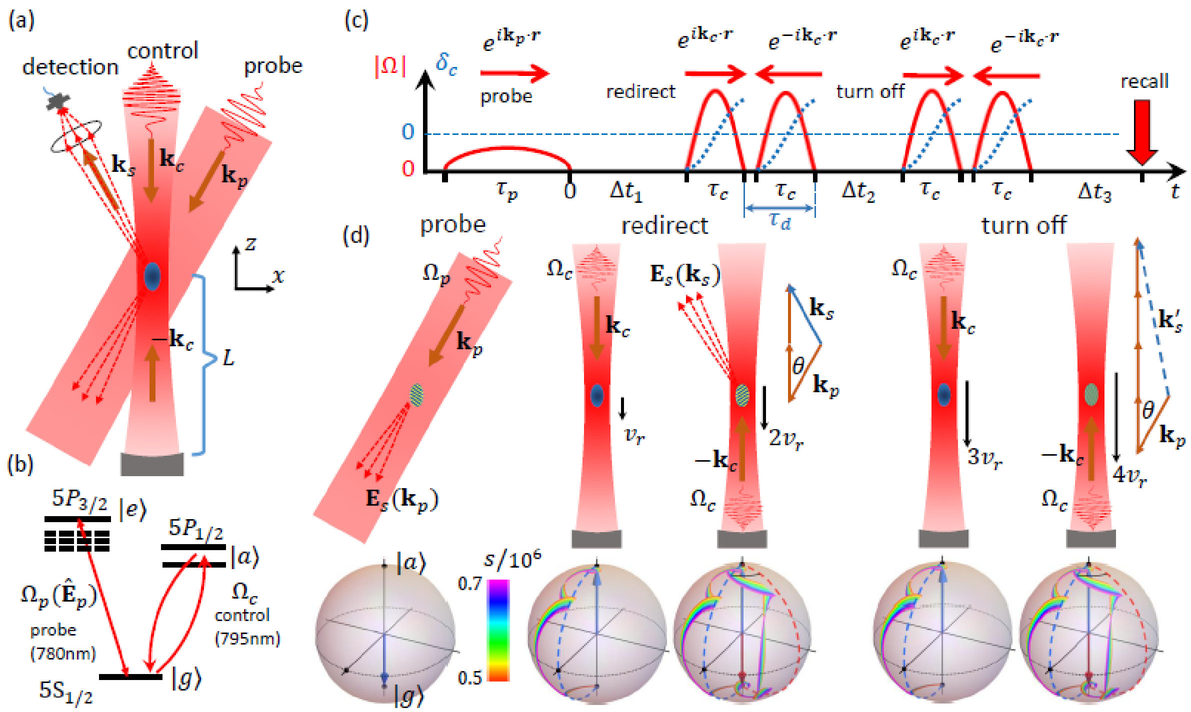


FIG. 1. Schematic of the experiment to demonstrate error-resilient optical spin wave control. (a) The basic setup. (b) The level diagram and the laser coupling scheme. (c) Schematic timing sequence for the amplitudes of the probe and control pulse Rabi frequency $|\Omega|$ (red solid lines), and the instantaneous detuning of the control pulse δ_c (blue dashed lines) from the $|g\rangle - |a\rangle$ transition. (d) Top (from left to right): Generation and control of optical spin wave with probe, and the redirection and turn-off of the collective spontaneous emission. The $|g\rangle - |e\rangle$ electric dipole spin wave is illustrated with fringes in the atomic sample. The optical control is also a spin-dependent kick which leads to momentum transfer with $v_r = \hbar k_c/m \approx 5.8$ mm/sec, m the atomic mass of ^{87}Rb. The drawings are not to actual scales, in particular, the phase-matching angle $\theta = \cos^{-1}\lambda_p/\lambda_c \sim 11.1^\circ$ is exaggerated for clarity. See main text for the "recall" operation. Bottom: Bloch-sphere representation of the projected $|g\rangle - |a\rangle$ state dynamics for atom at position $\mathbf{r}$. Ensemble of trajectories with different control pulse peak intensity parameter s are displayed. The quasi-adiabatic control ensures the geometric phase writing $U_g(\mathbf{r}) = 1 - (e^{2i\mathbf{k}_c\cdot\mathbf{r}} + 1)|g\rangle\langle g|$ insensitive to small deviations of s from $s \sim 0.6 \times 10^6$, for $\tau_c \Gamma_{D1} = 0.03$ in this work.

Appendix C: stationary solution

- From none-source Maxwell' s Eq., the form of electric dipole moment:

$$\mathbf{d}_\ell \equiv \sum_{\mu}^{\{x,y,z\}} \mathbf{d}_\ell^\mu = \alpha\,\mathbf{E}_0(\mathbf{r}_\ell) + \alpha \sum_{i\neq\ell} \mathsf{G}_{i\ell}\,\mathbf{d}_i$$

- Where atomic polarizability α is defined as:

$$\alpha = -\alpha_0 \frac{\gamma_0}{\Delta + \mathrm{i}\gamma_0}, \qquad \alpha_0 \equiv \frac{|\mathbf{d}_{ge}|^2}{\hbar\,\gamma_0}$$

- Thus, it can write:

$$\mathbf{E}(\mathrm{r}) = \left(\frac{1}{\alpha}\overleftrightarrow{\mathrm{I}} - \overleftrightarrow{\mathrm{G}}(\mathrm{r})\right)\mathbf{d}$$

Appendix C: stationary solution

- Element of ma $\mathrm{G}^{p,q}(u) \equiv \frac{k^3}{4\pi\varepsilon_0} e^{iu}\left[\delta_{p,q}\, g_1(u) + \frac{u_p u_q}{u^2} g_2(u)\right]$
- An example of 3 atom system

$$
\begin{bmatrix} E_0^x(\vec{r}_1) \\ E_0^y(\vec{r}_1) \\ E_0^z(\vec{r}_1) \\ E_0^x(\vec{r}_2) \\ E_0^y(\vec{r}_2) \\ E_0^z(\vec{r}_2) \\ E_0^x(\vec{r}_3) \\ E_0^y(\vec{r}_3) \\ E_0^z(\vec{r}_3) \end{bmatrix}
=
\left[\begin{array}{ccc|ccc|ccc}
\alpha^{-1} & 0 & 0 & -G_{12}^{xx} & -G_{12}^{xy} & -G_{12}^{xz} & -G_{13}^{xx} & -G_{13}^{xy} & -G_{13}^{xz} \\
0 & \alpha^{-1} & 0 & -G_{12}^{xy} & -G_{12}^{yy} & -G_{12}^{yz} & -G_{13}^{xy} & -G_{13}^{yy} & -G_{13}^{yz} \\
0 & 0 & \alpha^{-1} & -G_{12}^{xz} & -G_{12}^{yz} & -G_{12}^{zz} & -G_{13}^{xz} & -G_{13}^{yz} & -G_{13}^{zz} \\
\hline
-G_{12}^{xx} & -G_{12}^{xy} & -G_{12}^{xz} & \alpha^{-1} & 0 & 0 & -G_{23}^{xx} & -G_{23}^{xy} & -G_{23}^{xz} \\
-G_{12}^{xy} & -G_{12}^{yy} & -G_{12}^{yz} & 0 & \alpha^{-1} & 0 & -G_{23}^{xy} & -G_{23}^{yy} & -G_{23}^{yz} \\
-G_{12}^{xz} & -G_{12}^{yz} & -G_{12}^{zz} & 0 & 0 & \alpha^{-1} & -G_{23}^{xz} & -G_{23}^{yz} & -G_{23}^{zz} \\
\hline
-G_{13}^{xx} & -G_{13}^{xy} & -G_{13}^{xz} & -G_{23}^{xx} & -G_{23}^{xy} & -G_{23}^{xz} & \alpha^{-1} & 0 & 0 \\
-G_{13}^{xy} & -G_{13}^{yy} & -G_{13}^{yz} & -G_{23}^{xy} & -G_{23}^{yy} & -G_{23}^{yz} & 0 & \alpha^{-1} & 0 \\
-G_{13}^{xz} & -G_{13}^{yz} & -G_{13}^{zz} & -G_{23}^{xz} & -G_{23}^{yz} & -G_{23}^{zz} & 0 & 0 & \alpha^{-1}
\end{array}\right]
\begin{bmatrix} d_1^x \\ d_1^y \\ d_1^z \\ d_2^x \\ d_2^y \\ d_2^z \\ d_3^x \\ d_3^y \\ d_3^z \end{bmatrix}
$$

Appendix C: stationary solution

- Mode expansion of the E-field

$$\vec{\mathbf{E}}_0 = \sum_p b_p \vec{\mathbf{m}}_p \qquad \vec{\mathbf{d}} = \sum_p \frac{b_p}{\mu_p} \vec{\mathbf{m}}_p$$

- The solution:

$$\vec{\mathbf{d}} = \sum_p b_p \vec{\mathbf{m}}_p \frac{-\alpha_0 \gamma_0}{(\Delta - \Delta_p) + \mathrm{i}(\gamma_0 + \gamma_p)} = \sum_p \alpha_p b_p \vec{\mathbf{m}}_p$$

$$\Delta_p \equiv -\alpha_0 \gamma_0 \operatorname{Re}(g_p) \qquad \gamma_p \equiv \alpha_0 \gamma_0 \operatorname{Im}(g_p).$$

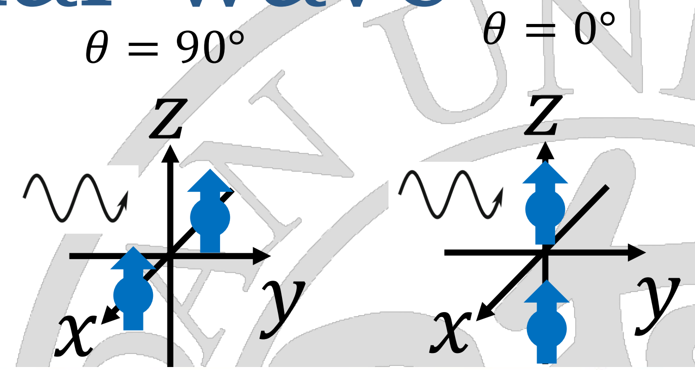
Appendix D: extreme decay rate ~ N

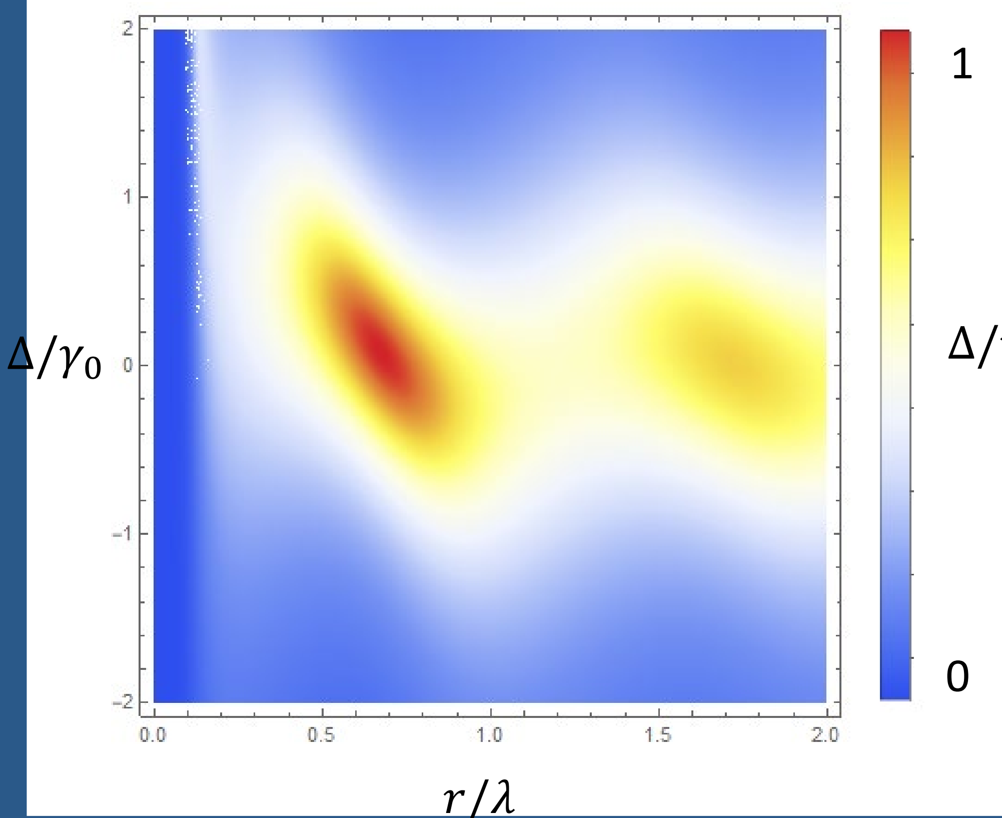
As N grows to infinity, the maximum decay rate of m_N is :

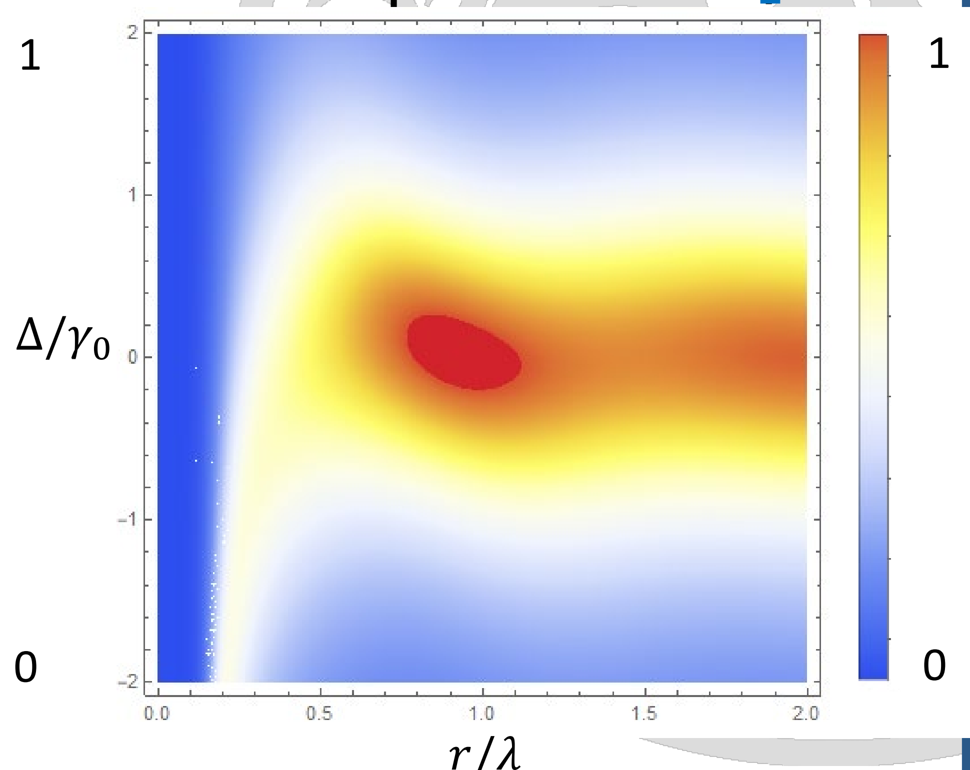
$$\frac{\Gamma^{\parallel}}{\Gamma_0} = \frac{3\pi}{2k_0 L} \sum_{n=1}^{k_0 L/\pi} \left(1 + \frac{n^2\pi^2}{k_0^2 L^2}\right) \sin^2\left(\frac{n\pi}{2}\right)$$

Appendix E: planar wave

- Extinction cross-section







Appendix F: 2D array

- Extinction cross-section