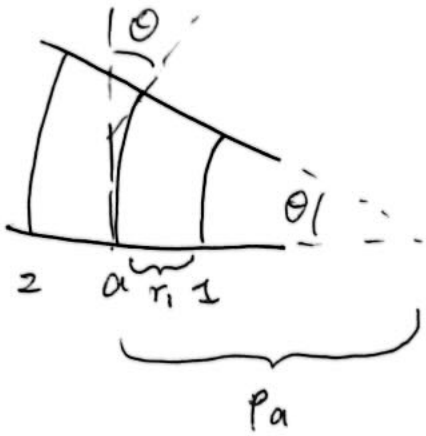

From $\quad$ To

$l_0 \to l_i \quad i=1,2$

$l_0 \to l_a$

$q_a = l_a - l_0$

$q_i = l_i - l_0$

θ

f_i

f_a

1. GeO

$l_a = \rho_a \theta \quad \Rightarrow \quad q_a = l_a - l_0 = \rho_a \theta \quad \Rightarrow \quad q_i = q_a + r_i \theta$

$l_i = \rho_i \theta \qquad q_i = l_i - l_0 = \rho_i \theta$

$\rho_i = \rho_a + r_i$

2. Work Balance.

$$f_a \cdot \Delta q_a = f_1 \cdot \Delta q_1 + f_2 \cdot \Delta q_2 + \Delta \Omega_p$$

(planar)

$$\begin{bmatrix} f_a & 0 \end{bmatrix} \begin{bmatrix} \Delta q_a \\ \Delta \theta \end{bmatrix} = \begin{bmatrix} f_1 & f_2 \end{bmatrix} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \end{bmatrix} + \nabla \Omega_p^T \begin{bmatrix} \Delta q_a \\ \Delta \theta \end{bmatrix}$$

1) $$\begin{bmatrix} \Delta q_1 \\ \Delta q_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & r_1 \\ 1 & r_2 \end{bmatrix}}_{J_{lq}} \begin{bmatrix} \Delta q_a \\ \Delta \theta \end{bmatrix}$$

2) $$\Omega_p = \sum \frac{M^2 \rho}{2EI} \xrightarrow{M = \frac{EI}{\rho}} \sum \frac{EI\theta}{2\rho} \Rightarrow \sum \frac{EI\theta^2}{2l}$$

$$\Omega_p = \frac{E_a I_a \theta^2}{2 l_a} + \sum \frac{E_i I_i \theta^2}{2 l_i}$$

$$= \frac{E_a I_a \theta^2}{2(l_0 + q_a)} + \sum \frac{E_i I_i \theta^2}{2(l_0 + q_a + r_i \theta)}$$

$$\nabla \Omega_{2q} = \begin{bmatrix} \frac{\partial \Omega_p}{\partial q_a} \\ \frac{\partial \Omega_p}{\partial \theta} \end{bmatrix}$$

3) $[f_1 \ f_2] J_{2q} = [f_a \ 0] - \nabla \Omega_{2q}^T$

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = (J_{2q}^T)^{-1} \left(\begin{bmatrix} f_a \\ 0 \end{bmatrix} - \nabla \Omega_{2q} \right)$$

$$= T_{2c} \begin{bmatrix} f_a \\ \nabla \Omega_{2q} \end{bmatrix}$$

with $T_{2c} = (J_{2q}^T)^{-1} \begin{bmatrix} 1 & 1 & \\ & & 1 \end{bmatrix}$

From	To
l_0	l_a, l_i
	θ
	$\delta_i = \delta + \beta_i \rightarrow i \rightarrow 1$
	q_a, q_i

$i = 1, 2, 3$

1. Geo.

$$l_a = \rho_a \theta \qquad \rho_i = \rho_a - r_i \cos\delta_i$$

$$l_i = \rho_i \theta$$

$$q_a = l_a - l_0$$

$$q_i = l_i - l_0 = l_a - r_i \theta \cos\delta_i - l_0$$

$$= q_a - r_i \theta \cos\delta_i$$

2. Work Balance.

$$f_a \cdot \Delta q_a = \Sigma f_i \Delta q_i + \Delta \Omega_s$$

$$\begin{bmatrix} f_a & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta q_a \\ \Delta\theta \\ \Delta\delta \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & f_2 \end{bmatrix} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \end{bmatrix} + \Delta \Omega_s$$

$$\begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -r_1 \cos\delta_i & r_1 \theta \sin\delta_1 \\ 1 & -r_2 \cos\delta_2 & r_2 \theta \sin\delta_2 \\ 1 & -r_3 \cos\delta_3 & r_3 \theta \sin\delta_3 \end{bmatrix}}_{J_{3q}} \begin{bmatrix} \Delta q_a \\ \Delta\theta \\ \Delta\delta \end{bmatrix}$$

$$\Omega_S = \sum \frac{M^2 l \theta}{2EI} \xrightarrow{M = \frac{EI}{\rho}} \sum \frac{EI\theta^2}{2l}$$

$$= \frac{E_a I_a \theta^2}{2 l_a} + \sum \frac{E_i I_i \theta^2}{2 l_i}$$

$$\Delta \Omega_S = \begin{bmatrix} \frac{\partial \Omega_S}{\partial q_a} & \frac{\partial \Omega_S}{\partial \theta} & \frac{\partial \Omega_S}{\partial \delta} \end{bmatrix} \begin{bmatrix} \Delta q_a \\ \Delta \theta \\ \Delta \delta \end{bmatrix}$$

$$= \nabla \Omega_{3q}^T \begin{bmatrix} \Delta q_a \\ \Delta \theta \\ \Delta \delta \end{bmatrix}$$

3. $$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = (J_{3q}^T)^{-1} \left(\begin{bmatrix} f_a \\ 0 \\ 0 \end{bmatrix} - \nabla \Omega_{3q} \right)$$

$$= (J_{3q}^T)^{-1} \begin{bmatrix} 1 & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} f_a \\ \nabla \Omega_{3q} \end{bmatrix}$$

$$= T_{3q} \begin{bmatrix} f_a \\ \nabla \Omega_{3q} \end{bmatrix}$$